USER GUIDE FOR PRACTICAL ACTIVITIES AND LABORATORY EXPERIMENTS

MATHEMATICS

SENIOR FIVE (S5)

Kigali, 2022

FOREWORD

Dear teacher,

Rwanda Basic Education Board (REB) is honoured to present the user guide for Practical Activities and Laboratory Experiments in Mathematics for Senior five(S5). This user guide will supplement competence-based teaching and learning, to ensure consistency and coherence in the learning of Mathematics.

In this user guide, special attention was paid to practical activities that facilitate the learning process in which students can manipulate concrete materials, develop ideas, and make new discoveries during activities carried out individually or in pairs/ small groups.

In a competence-based curriculum, practical activities open students' minds and provide opportunities to interact with the world, use available tools, collect data, and effectively model real-life problems.

For efficient use of this user guide, your role as a teacher is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize groups for students considering the importance of social constructivism.
- Engage students through active learning methods.
- Provide supervised opportunities for students to develop different competences by giving tasks which enhance critical thinking, problem-solving, research, creativity and innovation, communication, and cooperation.
- Support and facilitate the learning process by valuing students' contributions to the practical activities.
- Guide students towards the conclusion of the results of the experiments.
- Encourage individual, peer, and group evaluation of the work done and use appropriate competence-based assessment approaches and methods.

To facilitate your teaching activities, the content of this guide is self-explanatory so that you can easily use it. It is divided into three parts:

- Part I: Structure of this guide and the general introduction on the role of practical activities and laboratory experiments in the implementation of CBC.
- Part II: List of some Mathematics materials distributed to schools.
- Part III: Selected practical activities and laboratory experiments and how you can facilitate them in lessons.

Even though this guide contains practical activities and laboratory experiments, they are not enough; teachers can guide students to carry out more practical activities using improvised teaching resources.

I wish to sincerely extend my appreciation to the people who contributed towards the development of this guide; The African Institute for Mathematical Sciences, Teacher Training Program (AIMS - TTP) in partnership with Mastercard Foundation who provided technical and financial support and REB staff particularly those from the Mathematics and Science Subjects Unit in the Curriculum Teaching and Learning Resources Department who organized the wholeprocess from its inception.

Special appreciation goes also to teachers and independent experts in education who supported the exercise throughout the process. Any comment or contribution would be welcome for the improvement of this booklet for next versions.

Dr. MBARUSHIMANA Nelson

Director General, REB

ACKNOWLEDGEMENT

I wish to express my appreciation to the people who played a major role in the development and editing of the user guide for Practical Activities and Laboratory Experiments in Mathematics for Senior five (S5). It would not have been successful without the active participation of different education stakeholders.

Special thanks are given to AIMS staff, IEE, Independent people, teachers, illustrators, designers and all other individuals whose efforts in one way or the other contributed to the success of the development of this user guide.

I owe gratitude to the Rwanda Basic Education Board staff particularly those from Mathematics and Science subjects Unit in the CTLR Department who were involved in the whole process of the development work of this user guide.

Finally, my word of gratitude goes to AIMS - TTP in partnership with Mastercard Foundation for their support in terms of human and financial resources towards the development of this guide which will strengthen STEM teaching hence improving the quality of education in Rwandan schools.

Joan MURUNGI

Head of CTLR Department

LIST OF ACRONYMS

AIMS : African Institute for Mathematical Sciences

CBC : Competence-Based Curriculum

ICT : Information and Communication Technology

KBC : Knowledge Based Curriculum

Lab : Laboratory

SET : Science and Elementary Technology

STEM : Science Technology Engineering and Mathematics

UR-CE : University of Rwanda- College of Education

TTP : Teacher Training Program

STRUCTURE OF THE USER GUIDE

This user guide for Practical activities and Laboratory experiments in Mathematics for Senior Four is divided into 3 parts:

Part I: General introduction on the role of practical activities and lab experiments in the implementation of CBC.

Part II: List of main Mathematics kit items distributed in schools.

Part III: Practical activities or laboratory experiments and how to facilitate them in lessons.

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PART I: GENERAL INTRODUCTION

1.1. Background

To effectively implement a competence-based curriculum (CBC) Students should apply what they have learnt, in different situations by developing skills, attitudes, and values in addition to knowledge and understanding. This learning process is learner-focused, where a learner is engaged in active and participatory learning activities, and Students finally build new knowledge from prior knowledge. Since 2015, the Rwanda Education system has changed from knowledge-based competence (KBC) to CBC for preparing students that meet the national and international job market requirements and job creation. Therefore, implementing the CBC education system necessitates qualitative laboratory practical works for mathematics and science as more highlighted aspects.

In addressing this necessity, laboratory experiments play a major role. A child is motivated to learn mathematics by getting involved in handling various concrete manipulatives in various activities. In addition to activities, games in mathematics also help the child's involvement in learning by strategizing and reasoning.

For learning mathematical concepts through the above-mentioned approach, child-centred Mathematics kits have been developed for the students of primary and secondary schools. The kits include various kit items along with a manual for performing activities.

The kit broadly covers the activities in the areas of algebra, geometry, trigonometry, and measurement.

The kit has the following advantages:

- Availability of necessary and common materials in one place
- Multipurpose use of items
- The economy of time in doing the activities
- Portability from one place to another
- Provision for teacher's innovation
- Low-cost material and use of indigenous resources.

Apart from the kit, the user guide for laboratory and practical activities to be used by teachers was developed. This laboratory experiment user guide is designed to help mathematics and science teachers to perform high-quality lab experiments for mathematics and science. This user guide structure induces learners' interest, achievement, and motivation through the qualitative mathematics and science lab experiments offered by their teachers and will finally lead to the targeted goals of the CBC education system, particularly in the field of mathematics and science.

In CBC, students hand on the materials and reveal the theory behind the experiment done. Here, experiments are done inductively, where experiments serve as an insight towards revealing the theory. Thus, the experiment starts, and theory is produced from the results of the experiment.

1.2 Why Mathematics Practical Activities and Laboratory Experiments?

Mathematics plays an important role in our daily activities. It provides the vital underpinning of the knowledge of the economy. Mathematics is essential in the physical sciences, technology, business, financial services and many areas of ICT (Roohi, 2015). As a the basis of most scientific and industrial research and development, the teaching and learning of mathematics has to be given much attention by utilizing all possible means to help students to acquire knowledge, skills and understanding of different concepts that are mostly abstract.

The concept of a mathematics laboratory has become very popular in recent years due to its important role in clarifying abstract concepts using real materials. The Mathematics laboratory is a room wherein we find a collection of different kinds of materials and teaching/learning aids, needed to help the students understand the concepts through relevant, meaningful, and concrete activities. These activities may be carried out by the teacher or the students to explore the world of mathematics, to learn, discover and develop an interest in the subject (Maheshwari, 2018).

The majority of students view mathematics as a dull, boring, and stereotyped subject. They think that mathematics is about getting the right or wrong answer. When they get it wrong, they think that they are not good enough for Mathematics and lose interest in learning. Mathematics laboratory helps students understand the principal idea behind Mathematics concepts. Although Mathematics is not an experimental science in the way in which physics, chemistry and biology are, a Mathematics laboratory can contribute greatly to the learning of mathematical concepts and skills. The benefit of using Mathematics laboratories in teaching and learning among others (Adenegan & Balogun, 2010):

- Arousing interest and motivating learning.
- Cultivating favourable attitudes towards mathematics.
- Enriching and varying instructions.
- Encouraging and developing creative problems solving ability.
- Allowing for individual differences in the manner and speed at which students learn.

- Making students see the origin of mathematical ideas and contributing to mathematics innovation.
- Allowing students to engage in the doing rather than being passive observers or recipients of knowledge in the learning process.

In the Mathematics laboratory, the activities help students to visualize, manipulate and reason. They provide an opportunity to make conjectures and test them and generalize observed patterns. Students learn to deal with problems while doing a concrete activity, which lays down a base for more abstract thinking.

1.3 Type of laboratory experiments

The goal of the experiment defines the type of experiment and how it is organized. Therefore, before doing practical work, it is important to have a clear idea of the objective.

The three types of practical works that correspond with its three main goals are:

1. **Equipment-based practical works:** the goal is for students to learn to handle scientific equipment like using a compass, a set square, a thermometer, a protractor, etc.
2. **Concept-based practical works:** the goal is to clarify the new concepts.
3. **Inquiry-based practical works:** the goal is for students to learn process skills. Examples of process skills are the following: defining the problem and good research question(s), installing an experimental setup, observing, measuring, processing data in tables and graphs, identifying conclusions, defining limitations of the experiment, etc.

Note:

- To learn the new concept through practical work, the lesson should start with the practical work, and the theory can be explained afterwards (explore – explain). Starting by teaching the theory and then doing the practical work to prove what they have learned is demotivating and offers little added value for student learning.
- Try to avoid complex arrangements or procedures. Use simple equipment or handling skills to make it not too complicated and keep the focus on learning the new concept.
- The experiments should be useful for all Students and not only for aspiring scientists. Try to link the practical work as much as possible with their daily life and preconceptions.

1.4 Organization, analysis, and interpretation of data

Once data are collected, they must be ordered in a form that can reveal patterns and relationships and allows results to be communicated to others. We list goals for analysing and interpreting data.

By the end of secondary education, students should be able to:

- Analyze data systematically, look for relevant patterns or test whether data are consistent with the initial hypothesis.
- Recognize when data conflict with expectations and consider what revisions in the initial model are needed.
- Use spreadsheets, databases, tables, charts, graphs, statistics, mathematics, and ICT to compare, analyze, summarize, and display data and explore relationships between variables, especially those representing input and output.
- Evaluate the strength of a conclusion that can be inferred from any data set, using appropriate grade-level mathematical and statistical techniques.
- Recognize patterns in data that suggest relationships worth investigating further. Distinguish between causal and correlational relationships.
- Collect data from physical models and analyze the performance of a design under a range of conditions.

1.5 Organising laboratory experiments

i. Methods of organizing a practical work

There are 3 methods of organizing practical work:

- **Each group does the same experiments at the same time**

All Students can follow the logical sequence of the experiments, but this implies that a lot of material is needed. The best group size is 3, as all Students will be involved. With bigger groups, you can ask to experiment twice, where Students change roles.

- **Experiments are divided among groups with group rotation**

Each group does the assigned experiment and moves to the next experiment upon a signal from the teacher. At the end of the lesson, each group has done every experiment. This method saves material but is not perfect when experiments are ordered logically. In some cases, the conclusion of an experiment provides the research question for the next experiment. In that case, this method is not very suitable.

The organization is also more complex. Before starting the lesson, the materials for each experiment should be placed in the different places where the groups will work. Also, the required time for each experiment should be about the same. Use a timer to show Students the time left for each experiment. Provide extra exercise for fast groups.

- **All experiments are divided among groups without group rotation**

Each group does only one or two experiments. The other experiments are done by other groups. Afterwards, the results are brought together and discussed with the whole class. This saves time and materials, but it means that each learner does only one experiment and 'listens' to the other experiments' descriptions. The method is suitable for experiments that are optional or like each other. It is not a good method for experiments that all Students need to master.

ii. Preparation of a practical work

When preparing practical work, do the following:

- Have a look at the available material at school and make a list of what you can use and what you need to improvise.
- Determine the required quantities by determining the method to apply (see above).
- Collect all materials for the experiments in one place. If the Students' group is small, they can come to get the materials on that spot, but it is better for each group to prepare a set of materials and place it on their desk.
- Test all experiments and measure the required time for each experiment.
- Prepare a nice but educational extra task for Students who are ready before the end of the lesson.
- Write on the blackboard how groups of Students are formed.

iii. Preparation of a lesson for practical work

In the lesson plan of a lesson with practical work, there should be the following phases:

1. The introduction of the practical work or the 'excite' phase consists of the formulation of a key question, discrepant event, or a small conversation to motivate Students and make connections with daily life and Students' prior knowledge.
2. The discussion of safety rules for the practical work. For example,
    - Students must work at the assigned place.
    - Long hairs should be tied together, and safety eyeglasses should be worn when dealing with chemical experiments.
    - Only the material needed for the experiment should be on the table.

3. Set the practical work instructions: how groups are formed, where they get the materials, special treatment of materials (if relevant), what they must write down, etc.

4. Set how to conduct practical work:
    - Students do the experiments, while the teacher coaches by asking questions (Explore phase).
    - The practical work should preferably be processed immediately with an explain phase. If not, this should happen in the next lesson.

5. Set how to conclude the lesson of practical work:
    - Students refer to instructions and conduct the experiment,
    - Students record and interpret recorded data,
    - Cleaning the workspace after the practical work (by the Students as much as possible).

1.6 Role and responsibilities of teacher, laboratory technician, and students in the laboratory experiment

1.6.1. The roles and responsibilities of teacher during a laboratory experiment

Before conducting an experiment, the teacher will:

- Decide how to incorporate experiments into class content best,
- Prepare in advance materials needed in the experiment,
- Prepare protocol for the experiment,
- Perform in advance the experiment to ensure that everything works as expected,
- Designate an appropriate amount of time for the experiment - some experiments might be adapted to take more than one class period, while others may be adapted to take only a few minutes.
- Match the experiment to the class level, course atmosphere, and your students' personalities and learning styles.
- Verify the status of lab equipment before lab practices.
- Provide the working sheet and give instructions to Students during lab sessions.

During practical work, the teacher's role is to coach instead of helping with advice or questions. It is better to answer a learner's question with another question than to immediately give the answer or advice. The additional question should help Students to find the answer themselves.

- Prepare some pre-lab questions for each practical work, no matter what the type is.
- Try and start the practical work: start with a discrepant event or questions that help define the problem or questions that link the practical work with students' daily life or their initial conceptions about the topic.
- Use coaching questions during the practical work: 'Why do you do this?', 'What is a control tube?', ''What is the purpose of the experiment?', 'How do you call this product?', 'What are your results?' etc.
- Use some questions to end the practical work: 'What was the meaning of the experiment?', 'What did we learn?', 'What do we know now that we didn't know at the start?', 'What surprised you?' etc.
- Announce the end of the practical work 10 minutes before giving students enough time to finish their work and clean their space.

1.6.2. The Role of a lab technician during a laboratory-based lesson

In schools having laboratory technicians, they assist the science teachers in the following tasks:

- Maintaining, calibrating, cleaning, and testing the sterility of the equipment,
- Collecting, preparing, and/or testing samples,
- Demonstrating procedures.

1.6.3. The students' responsibilities in the laboratory work

During the lab experiment, both students have different activities to do; the table barrow summarizes them. General learner's activities are:

- Experiment and obtain data themselves,
- Record data using the equipment provided by the teacher,
- Analyse the data often this involves graphing it to produce the related graph,
- Interpret the obtained results and deduct the theory behind the concept under the experimentation,
- Discuss the error in the experiment and suggest improvements,
- Cleaning and arranging material after a lab experiment.

1.7 Safety rules, and precautions during lab experiments

Regardless of the type of lab you are in, there are general rules enforced as safety precautions. Each lab member must learn and adhere to the rules and guidelines set, to minimize the risks of harm that may happen to them within the working environment. These encompass dress' code, use of personal protection equipment, and general behaviour in the lab. It is important to know that some laboratories contain certain inherent dangers and hazards. Therefore, when working in a laboratory, you must learn how to work safely with these hazards to prevent injury to yourself and other lab mates around you. You must make a constant effort to think about the potential hazards associated with what you are doing and think about how to work safely to prevent or minimize these hazards as much as possible. Before doing any scientific experiment, you should make sure that you know where the fire extinguishers are in your laboratory, and there should also be a bucket of sand to extinguish fires. You must ensure that you are appropriately dressed whenever you are near chemicals or performing experiments. Please make sure you are familiar with the safety precautions, hazard warnings, and procedures of the experiment you perform on a given day before you start any work. Experiments should not be performed without an instructor in attendance and must not be left unattended while in progress.

1.7.1 Hygiene plan

A laboratory is a shared workspace, and everyone has the responsibility to ensure that it is organized, clean, well-maintained, and free of contamination that might interfere with the lab members' work or safety.

For waste disposal, all chemicals and used materials must be discarded in designated containers. Keep the container closed when not in use. When in doubt, check with your instructor.

1.7.2 Hazard warning symbols

To maintain a safe workplace and avoid accidents, lab safety symbols and signs need to be posted throughout the workplace.

Chemicals pose health and safety hazards to personnel due to innate chemical, physical, and toxicological properties. Chemicals can be grouped into several different hazard classes. The hazard class will determine how similar materials should be stored and handled and what special equipment and procedures are needed to use them safely.

Each of these hazards has a different set of safety precautions associated with them. Annex 1 shows hazard symbols found in laboratories and the corresponding explanations.

1.7.3 Safety rules

Safety is the number one priority in any laboratory. All students are required to know and comply with good laboratory practices and safety norms; otherwise, they will be asked to leave the laboratory. Make sure you understand all the safety precautions before starting your experiments, and you are requested to help your Students to understand too.

The following are some general guidelines that should always be followed:

- **Lab coat**

While working in the lab, everyone must always wear a lab coat (Figure 1) to prevent incidental and unexpected exposures to the skin and clothing. The primary purpose of a lab coat is to protect against splashes and spills.

	The lab coat must be wrist-fitted and must always keep buttoned. A lab coat should be non-flammable and should be easily removed.

- **Safety glasses**

For eyes protection, goggles must always be worn over by all persons in the laboratory while students are working with chemicals. Safety glasses, with or without side-shields, are not acceptable.

The eyes protection safety indicates the possibility of chemical, environmental, radiological, or mechanical irritants and hazards in the laboratory.

- **Breathing Masks**

Respirators are designed to prevent contamination from volatile compounds that may enter in your body through the respiratory system. "Half mask" respirators (Figure3) cover just the nose and mouth; "full face" respirators cover the entire face, and "hood" or "helmet" style respirators cover the entire head.

The breathing mask safety sign lets you know that you are working in an area with potentially contaminated air.

– **Eye Wash Station**

Eyes wash stations consist of a mirror and a set of bottles containing saline solution that can be used to wash the injured eye with water. The eye wash station is intended to flood the eye with a continuous stream of water.

Eyes wash stations provide a continuous, low-pressure stream of aerated water in laboratories where chemical or biological agents are used or stored and in facilities where non-human primates are handled.

The eyewash stations should easily be accessed from any part of the laboratory, and if possible, located near the safety shower so that, if necessary, the eyes can be washed while the body is showered.

– **Footwear**

Shoes that cover entirely the toes, heel, and top of the foot provide the best general protection (Figure 1.5). Closed shoes must always be worn while in the laboratory, regardless of the experiment or curricular activity. Shoes must fully cover your feet up to the ankles, and no skin should be shown.

Socks do not constitute a cover replacement for shoes. Sandals, backless and open shoes are unacceptable.

– **Gloves**

When handling chemical, physical, or biological hazards that can enter the body through the skin, it is important to wear the proper protective gloves.

Butyl, neoprene, and nitrile gloves are resistant to most chemicals, e.g., alcohols, aldehydes, ketones, most inorganic acids, and caustics.

– **Hair dressing**

If hair is long, it must be tied back. It is good to report all accidents including minor incidents to your instructor immediately.

– **Eat and drink**

Never drink, eat, taste, or smell anything in the laboratory unless you are allowed by the lab instructor.

– **Hot objects**

Never hold very hot objects with your bare hands.

Always hold them with a test tube holder, tongs, or a piece of cloth or paper.

1.8. Guidance on the Management of lab materials: Storage Management, repairing and disposal of Lab equipment

Keeping and cleaning up

Working spaces must always be kept neat and cleaned up before leaving. Equipment must be returned to its proper place. Keep backpacks or bags off the floor as they represent a tripping hazard. Open flames of any kind are prohibited in the laboratory unless specific permission is granted to use them during an experiment.

Management of lab materials

A science laboratory is a place where basic experimental skills are learned only by performing a set of prescribed experiments. Safety procedures usually involve chemical hygiene plans and waste disposal procedures. When providing chemicals, you must read the label carefully before starting the experiment. To avoid contamination and possibly violent reaction, do never return unwanted chemicals to their container. In the laboratory, chemicals should be stored in their original containers, and cabinets should be suitably ventilated. It is important to notify students that chemicals cannot be stored in containers on the floor. Sharp and pointed tools should be stored properly. Students should always behave maturely and responsibly in the laboratory or wherever chemicals are stored or handled.

Hot equipment and glassware handling

Hazard symbols should be used as a guide for the handling of chemical reagents. Chemicals should be labelled as explosives, flammable, oxidizers, toxic and infectious substances, radioactive materials, corrosives, etc. All glassware should be inspected before use, and any broken, cracked, or chipped glassware should be disposed of in an appropriate container. All hot equipment should be allowed to cool before storing it.

All glassware must be handled carefully and stored in its appropriate place after use. All chemical glass containers should be transported in rubber or polyethylene bottle carriers when leaving one lab area to enter another. When working in a lab, do never leave a hot plate unattended while it is turned on. It is recommended to handle hot equipment with safety gloves and other appropriate aids but never with bare hands. You must ensure that hands, hair, and clothing are kept away from the flame or heating area and turn heating devices off when they are not in use in the laboratories.

Waste disposal considerations

Waste disposal is a normal part of any science laboratory. As teachers or students perform demonstrations or laboratory experiments, chemical waste is generated.

These wastes should be collected in appropriate containers and disposed of according to local, state, and federal regulations. All schools should have a person with the responsibility of being familiar with this waste disposal. In order to minimize the amount of waste generated and handle it safely, there are several steps to consider.

Sinks with water taps for washing purposes and liquid waste disposal are usually provided on the working table. It is essential to clean the sink regularly. Notice that you should never put broken glass or ceramics in a regular waste container. Use a dustpan, a brush, and heavy gloves to carefully pick-up broken pieces, and dispose of them in a container specifically provided for this purpose. Hazardous chemical waste, including solvents, acids, and reagents, should never be disposed of down sewer drains. All chemical waste must be identified properly before it can be disposed of. Bottles containing chemical waste must be labelled appropriately. Labelling should include the words "hazardous waste." Chemical waste should be disposed of in glass or polyethylene bottles. Plastic-coated glass bottles are best for this purpose. Aluminium cans that are easily corroded should not be used for waste disposal and storage.

Equipment Maintenance

Maintenance consists of preventative care and corrective repair. Both approaches should be used to keep equipment in working order. Records of all maintenance, service, repairs, and histories of any damage, malfunction, or equipment modification must be maintained in the equipment logs. The record must describe hardware and software changes and/or updates and show the dates when these occurred. Each laboratory must maintain a chemical inventory that should be updated at least once a year.

1.9. Student Experiment Work Sheet

There should be a sheet to guide students about how they will conduct the experiment, the materials to be used, the procedures to be followed, and the way of recording data. The following is the structure of the student experiment worksheet. It can be prepared by the teacher or be availed from the other level.

1. Date
2. Name of student/group
3. The title of the experiment
4. Type of experiment (concept, equipment, and inquiry-based)
5. Objective(s) of the experiment
6. Key question(s)
7. Materials (equipment/instrument, resources, etc...)
8. Procedures & Steps of experiment
9. Schematic reference if required.
10. Data recording and presentation

Number of tests	Variables	Results	Comments/Observations
1			
2			
3			
Etc			

11. Reflective questions and answers

Question1

Question 2

Question 3

12. Answer for the key question.

1.10. Report template for students

After conducting a laboratory experiment, students should write a report about their findings and the conclusion they took.

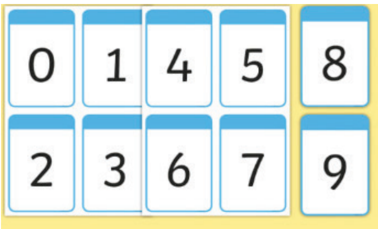
The report to be made depends on the level of students. The report done by primary school Students is not the same us the one to be made by secondary school Students.

The following is a structure of the report to be made by a group of secondary school Students.

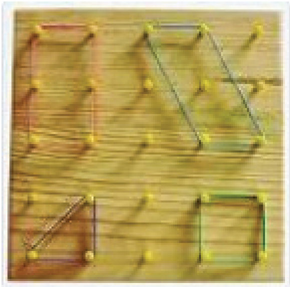
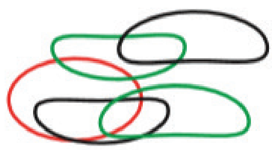
1. Introduction (details related to the experiment: Students identification, date, year, topic area, unit title, and lesson).
2. The title of the experiment.
3. Type of experiment (concept, equipment, and inquiry-based)
4. Objective(s) of the experiment.
5. Key question(s)
6. Materials (equipment/instrument, resources, etc…)
7. Procedures & Steps of experiment
8. Schematic reference if required.
9. Data recording
10. Data analysis and presentation (Plots, tables, pictures, graphs)
11. Interpretation/discussion of the results, student alternative ideas from observation.
12. Theory or main concept, formulas, and application.
13. Conclusion (answer reflective questions and the key question).

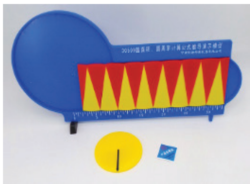
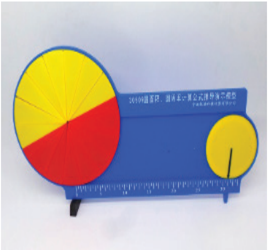
In conclusion, there are safety rules and precautions to consider before, during, and at the end of a practical activity and lab experiment. We hope tutors are inspired to conduct a Mathematics practical activity in a conducive environment as expected by the Competence Based Curriculum.

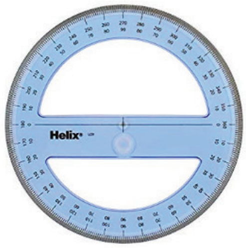
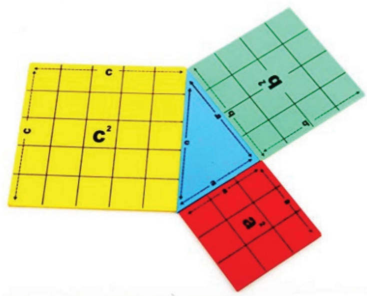
PART II: LIST OF MAIN KIT ITEMS DISTRIBUTED IN SCHOOLS

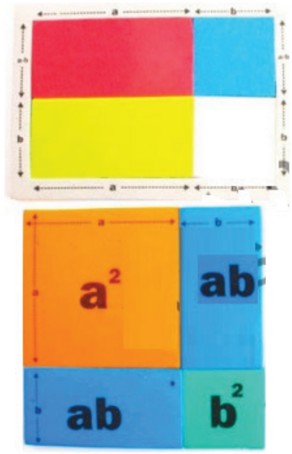
#	Item and description	Picture	Description
1	**Laminated number cards** ***Use***: *Used in game for composition, sorting, factorization of numbers, etc.*	 	A pack of laminated cards numbered from 0 to 9 (9 cards from an A4 paper).
2	**Circle set fraction** ***Use***: *Used for exploring "Area of Circle "and activities related to "Fractions"* and area of a circle		7 Blue (or any other color) colored circular plastic having 3mm thickness and diameter 160 mm. divided into 4, 6, 8, 12, 16 and 32 equal sectors. Each piece is magnetic.
3	**Clock** ***Use***: *To learn to tell the time according to the 24 hours international convention.*	 	1 plastic teaching clock

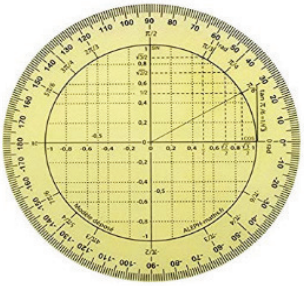
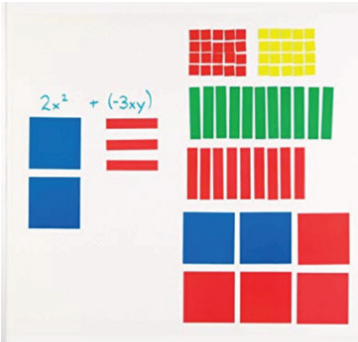
4	**Mathematical set for teachers:** Full circle protractor, meter rule, compass, tape measure, T-square, rope, decameter.		Wooden or plastic
5	**Mathematical set for students**: 2 Metal Study Compasses, 2 T-squares, Ruler, Protractor, Pencil for Compass, Pencil Sharpener, Eraser, Lead Refill.		Geometry 10 Piece Set,
6	**Fest night Stainless Steel 180 Degree Protractor**		Angle Finder Both Arms Stainless Steel Protractor with 0-180 Degrees, Angle10 inch,250mm, 30cm Scale Angle Finder Ruler. Smooth surface, convenient to use, easy to read. 0-180 degree arbitrary rotation. Adjustable screw design, easy operation for fixed reading.

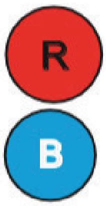
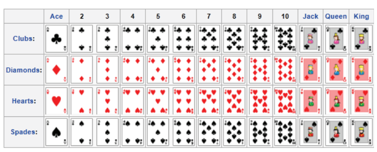
7	**Basic geometric solids**		**6** *pieces* **of** *wooden solids* Includes cube, cylinder, sphere, cone, triangular prism, pyramid . *Use: To demonstrate geometry solids (3D).*
8	**Geoboard** *Geoboard is used to represent planar shapes/ figures and also to find the approximate areas as well as to learn different geometric figures using a rubber band.*		*1 geographic board of 33.5 cm × 53.5 cm. It is printed with 187 grids 3 cm × 3 cm each in alternated colors. Copper pins are nailed on each crossing point of the grids.*
9	**Rubber bands** *for use with geoboard.*		120 rubber bands in 6 colors come with the boards.

10	**Transparent geometric 3D-shapes plus their corresponding fold-up nets:** cylinder, square pyramid, cube, rectangular prism, cone, hexagonal prism, triangular pyramid, and triangular prism. ***Use****: Used to make solid shape.*		Transparent geometric shapes plus their corresponding fold-up net inserts. 16-piece set (8 transparent and 8 folding shapes)
11	**Circle-Area and Diameter Demonstrator**	 	1 plastic demonstrator board of 48 cm × 25 cm. It consists of 17 sectors: 15 sectors that are equal to 1/16 of the cylinder volume; and 2 sectors that are equal to 1/32 of the cylinder volume. Use: To learn how to measure the area and diameter of circles.

12	**Full Protractor**		Helix Professional 360 Degree Protractor 15cm As per sample
13	**Cut Outs for Pythagoras** **Theorem.**		1 plastic right angled triangle. Measure: 3” x 4” x 5” & 3 different size square equal to sides of triangle.

14	**Cut outs for algebraic** **Identities.**		Made up of 3mm thick wooden colored cardboard and includes (1) Square cut-out of side 76 mm (2) 3 cut-outs obtained from another square of side 76mm out of which one is a square of side 38mm and the remaining two are trapezium of dim.38 x76 mm^2 (3) Square cut-out s of side 80 mm and 45mm (4) Rectangular cut-out of dimension 80x45mm^2

15	**Circular Trigonometric Protractor**		CIRCULAR Protractor ROBUST: shatterproof and scratch resistant translucent plastic. Use: direct reading of the angles in degrees and radians and cosine and sine near 0.05.
16	**Algebraic tiles** a) $x^2, x, 1$ b) $-x^2, -x, -1$		Made up of plastic cardboard in different sizes: 40 (20red+20 blue) squares of side 10mm known as unit tiles. 20 (10 red +10 blue) rectangles of 50 x10mm^2 dimension known as x or -x tiles. 10 (5 red + 5 blue) squares of side 50mm known as x^2 or $-x^2$ tiles, etc.

17	**Cubic dice** From 1 sided to 6 sided.	 	6 plastic dices with different edges and different shapes: 8mm, 12mm, 16mm, 19mm and 25mm.
18	**Counters: (20)** ***Use****: Used in activity "Addition and Subtraction of Integers".*		A set of 20 Plastic pieces or lamineted transparent counters whose one side is blue and other side is red.
19	**Scientific Calculator**		Casio Fx-991es Plus Scientific Calculator
20	**Playing cards** to be used in probability		A set of 52 playing cards
21	**The container**: a box in metal to contain all these materials per kit.		A container in metal which can contain or these materials

PART III: PRACTICAL ACTIVITIES AND LABORATORY EXPERIMENTS FOR SENIOR FIVE (S5)

As discussed here above, where possible, every concept developed in Mathematics should start by a practical activity as the concrete stage of learning.

Practical activities given here below were selected as a sample. The teacher will guide students to do more activities depending on the new concept to be developed in a given lesson. The title of the lesson and the unit in which the practical activity takes place are given in the rationale of each practical activity.

Therefore, the teacher and students will verify in this part (rationale) if the given practical activity is selected from the content of their syllabus as expected.

UNIT: 1 SEQUENCES

PRACTICAL ACTIVITY 1: IDENTIFICATION OF ARITHMETIC SEQUENCE AND THE COMMON DIFFERENCE

a) Rationale:

This activity is conducted when teaching the Concept related to arithmetic sequence to be learnt in Unit 2 of S5. In real life, sequences are useful as well as in higher Mathematics. For example, the interest portion of monthly payments made to pay off an automobile or home loan, and the list of maximum daily temperatures in one area for a month are sequences. Sequences are used in calculating interest, population growth, half-life, and decay in radioactivity, construction, and repeated drug dosage.

b) Objective:

Identify the arithmetic sequence and deduce the common difference.

This is a **Concept-based practical activity.**

c) List of required materials:

- Chart paper
- Ruler, Pencil/ Coloured pens, and Eraser
- Strips or post it
- Geometrical Instruments.
- Pair of scissors/cutter

d) Procedures:

Step 1: Take a rectangular manila paper with vertices denoted by ABCD

Step 2: Take strips (better in plastic or use glue) or use post-it of fixed lengths denoted by **a** and **b** respectively.

Step 3: Arrange and paste both types of strips or post them to get terms

$a,$

$a+b,$

$a+2b$

$a+3b$

....

$a+9b$

as shown in the picture below.

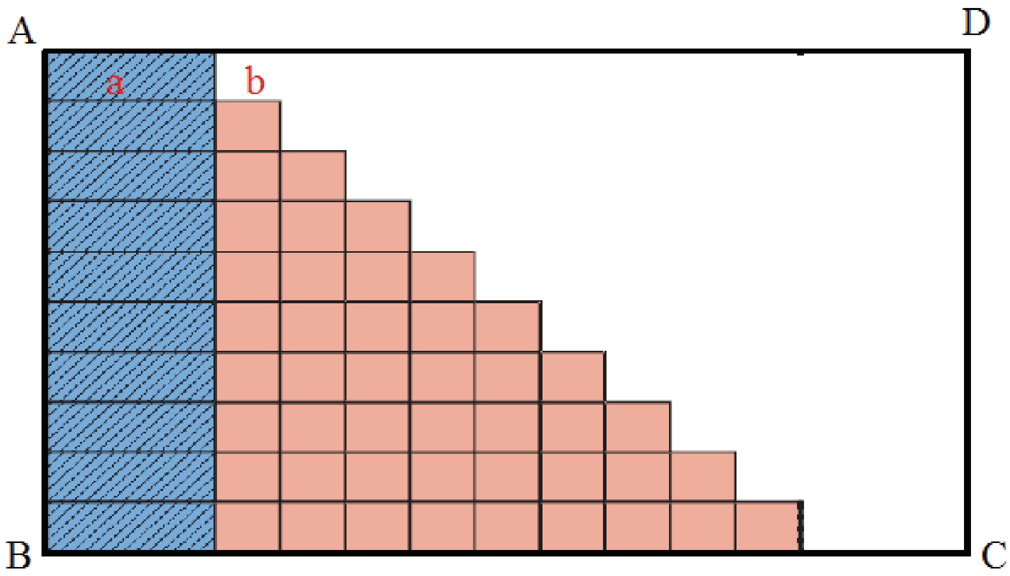


Figure 1: illustration of arithmetic progression and the common difference

Note: With the absence of strips or posts it, draw and combine rectangles having different length with the same width.

Give the length for each strip (row)

How are strips arranged?

How do you get the number of strips (bricks) to be on the next row? Can give a general rule? What is the difference between the 2 consecutive rows?

If the arrangement of strips is a progression, what is the meaning of a progression?

e) Recording of data

Choose a generating rule

i. In the numbers written down, each number could be found by adding a constant number to the previous one. Guess that constant number.

ii. Complete the following table

n^{th} Row	Number of bricks
1	$a+b$
2	$a+2b$
3	?
4	$a+4b$
5	?
6	
7	
8	
9	

iii. From ii) to ii) deduce the formula of finding the term for the n^{th} row.

Expected answer:

i. Each term could be found by adding a constant b to the previous term.

ii.

n^{th} Row	Term
Initial	a
1	$a+b$
2	$a+2b$
3	$a+3b$
4	$a+4b$
5	$a+5b$

6	$a+6b$
7	$a+7b$
8	$a+8b$
9	$a+9b$

iii. At the n^{th} row we have $a+nb$

f) Interpretation of results and Conclusion

- What do you call the list of terms you got in each row?
- In each case, can you find the last requested term?
- What is the main information needed to complete the list of terms as requested?

Expected answers:

- The list of terms we got is called terms of an arithmetic progression. We can find any term of order n of the given arithmetic progression.
- To complete an arithmetic progression, we need a starting term (initial term) and one clear generating rule. If we have two consecutive terms of arithmetic progression, we can also determine a generating rule and then find other terms.
- Every term in an arithmetic progression can be described by the same algebraic term.
- The common term to be added (or subtracted between two consecutive terms) is called the **common difference.**

g) Information for Teachers:

This activity helps students to develop knowledge and skills related to number patterns and arithmetic sequences recommended in the Mathematics Syllabus.

h) Guidance on evaluation

The teacher should provide different tasks related to the determination of the n^{th} term given the initial term and common difference or from the common difference given two terms with their respective order.

Calculate the sum of all strips (or bricks). If we take a and b as the number of pens to be given to Students, for example, let us replace a with 2 pens and b with 5 pens as the common difference. The first learner can have (2+5) pens. How many pens will have the 5^{th} learner?

PRACTICAL ACTIVITY 2: THE SUM OF THE FIRST NTH TERMS OF AN ARITHMETIC PROGRESSION

a) Rationale:

This activity is conducted when teaching the Concept related to the sum of n first terms of the arithmetic sequence. It is learnt in Unit 2 of S5. There is a range of everyday applications involving arithmetic sequences. If you are saving money in equal instalments, for example, the cumulative savings at each savings period form an arithmetic sequence. If you are travelling down a highway at a constant speed, the amount of petrol left in the tank, if measured every minute of the trip, forms another arithmetic progression. Any time you notice a quantity changing in equal amounts at set periods, then you can consider that process as being arithmetic.

b) Objective:

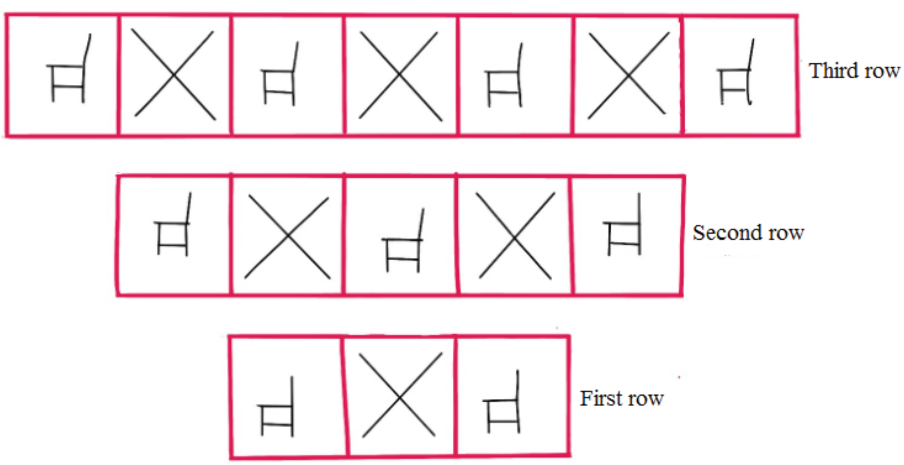
Explore how to find the sum of the first "n" terms of arithmetic sequences.

c) Required materials:

Playground sticks or nails, Pieces of chalk, pieces of paper and pens.

d) Illustration of the activity:

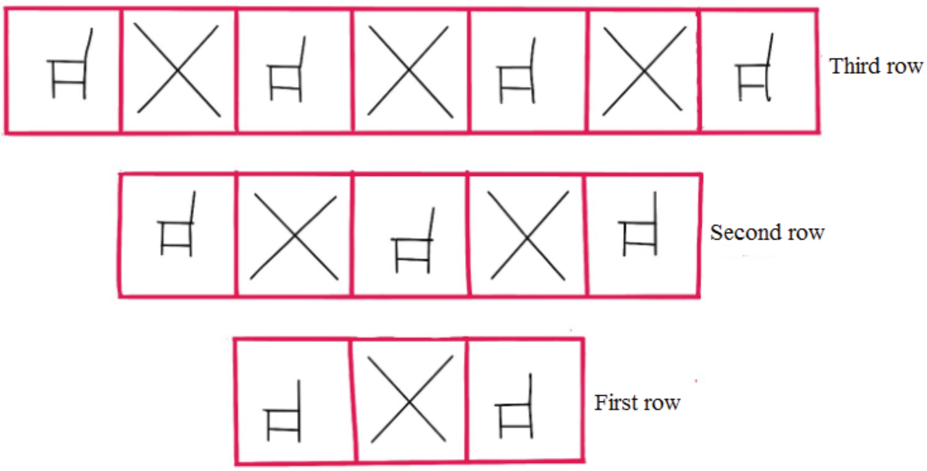
One of the measures for preventing the spread of Covid-19 is physical distancing. Consider a conference hall where people in the meeting are arranged in a way that chairs in each row are increasing by 2 from the previous line and between two people there must be a chair for social distancing as illustrated in the following figure.



There is a need to know the total number of authorized chairs if there are n rows in the room.

e) Procedures:

Step 1: Locate the chairs by small rectangles from 1^{st}.



Step 2: Write down the number of chairs located on each row

Step 3: Observe the pattern and identify the type of progression and related characteristics: first row and the common difference.

Step 4: Count or determine the total number of chairs for the given number of first rows and complete them in the table for data recording.

Step 5: Write down the number of authorized seats on each line

Step 6: Observe the pattern and identify the type of progression and related characteristics: first row and the common difference.

Step 7: Determine the total number of authorized seats (number of inviters) if the room has only 5 rows of chairs.

Can you generalize the total number of authorized chairs in the room if there are n rows? Try to give the rule in terms of the number of chairs in the first row U_1 and the number n of rows.

f) Data recording

Use the table of this form to complete the data

Order of the row	Number of chairs	Total number of chairs from the first row
1		
2		
3		
4		
5		
n		

For more details, complete the following tables:

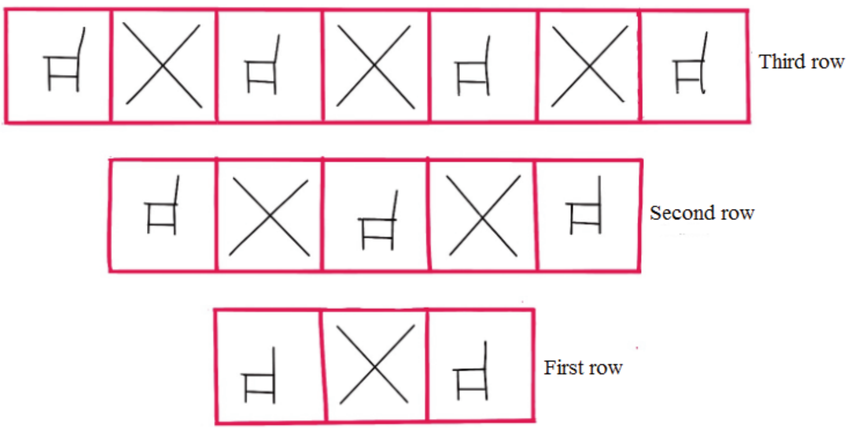
Order of row (n)	Number of chairs (U_n)	Total number of chairs from first row (S_n)	S_n in terms of U_1 and d
1	3	$S_1 = U_1 = 3$	$S_1 = U_1$
2	5	$S_2 = S_1 + U_2 = 8$	$S_2 = S_1 + U_2 = U_1 + U_1 + d = 2U_1 + d$
3	7	$S_3 = S_2 + U_3 = 15$	…………
4	9	$S_4 = S_3 + U_4 = 24$	……….
5	11	$S_5 = S_4 + U_5 = 35$	……………
⋮	⋮	⋮	⋮
n	$3 + 2(n-1)$	………	………

and

Order of row (n)	Number of authorized seats (U_n)	Total number of authorized seats from 1st row (S_n)	S_n in terms of U_1 and d
1	2	$S_1 = U_1 = 2$	$S_1 = U_1$
2	3	$S_2 = S_1 + U_2 = 5$	$S_2 = S_1 + U_2 = U_1 + U_1 + d = 2U_1 + d$
3	4	$S_3 = S_2 + U_3 = 9$	....
4	5	$S_4 = S_3 + U_4 = 14$	....
5	6	$S_5 = S_4 + U_5 = 20$	....
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$2+(n-1)$	....	....

Expected answer:

At step 1:



At step 2-4: Write down the number of chairs located on each row

Order of row (n)	Number of chairs U_n	Total number of chairs from the first row (S_n)
1	3	$S_1 = U_1 = 3$
2	5	$S_2 = S_1 + U_2 = 3 + 5 = 8$
3	7	$S_3 = S_2 + U_3 = 8 + 7 = 15$
4	9	$S_4 = S_3 + U_4 = 15 + 9 = 24$
5	11	$S_5 = S_4 + U_5 = 24 + 11 = 35$
Total	35	

At steps 5-7

Order of row (n)	Number of authorized seats (U_n)	Total number of authorized seats from 1st row
1	2	$S_1 = U_1 = 2$
2	3	$S_2 = S_1 + U_2 = 2 + 3 = 5$
3	4	$S_3 = S_2 + U_3 = 5 + 4 = 9$
4	5	$S_4 = S_3 + U_4 = 9 + 5 = 14$
5	6	$S_5 = S_4 + U_5 = 14 + 6 = 20$
Total	20	

And

Order of row (n)	Number of chairs (U_n)	Total number of chairs from first row (S_n)	S_n in terms of U_1 and d
1	3	$S_1 = U_1 = 3$	$S_1 = U_1$
2	5	$S_2 = S_1 + U_2 = 8$	$S_2 = S_1 + U_2 = U_1 + U_1 + d = 2U_1 + d$
3	7	$S_3 = S_2 + U_3 = 15$	$S_3 = S_2 + U_3 = 2U_1 + d + U_1 + 2d = 3U_1 + 3d$
4	9	$S_4 = S_3 + U_4 = 24$	$S_4 = S_3 + U_4 = 3U_1 + 3d + U_1 + 3d = 4U_1 + 6d$
5	11	$S_5 = S_4 + U_5 = 35$	$S_5 = S_4 + U_5 = 4U_1 + 6d + U_1 + 4d = 5U_1 + 10d$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$U_1 + 2(n-1)$	$S_n = S_{n-1} + U_n$	$s_n = u_1 + u_2 + u_3 + \ldots + u_n$ $= u_1 + (u_1 + d) + (u_1 + 2d) + \ldots + (u_1 + (n-1)d)$ $= (\underbrace{u_1 + u_1 + \ldots u_1}_{n\ terms}) + (d + 2d + \ldots + (n-1)d)$ $= nu_1 + d[1 + 2 + 3 + \ldots + (n-1)] = nu_1 + d[\frac{n(n-1)}{2}]$ $= nu_1 + \frac{n}{2}(n-1)d = \frac{n}{2}[2u_1 + (n-1)d]$ $= \frac{n}{2}[u_1 + u_1 + (n-1)d]$ $= \frac{n}{2}(u_1 + u_n)$

From the sum of chairs of 5 rows (n=5), we generalized the sum of chairs for n rows (terms).

And

Order of line (n)	Number of authorized seats (U_n)	Total number of authorized seats from the first line (S_n)	S_n in terms of U_1 and d
1	2	$S_1 = U_1 = 2$	$S_1 = U_1$
2	3	$S_2 = S_1 + U_2 = 5$	$S_2 = S_1 + U_2 = U_1 + U_1 + d = 2U_1 + d$
3	4	$S_3 = S_2 + U_3 = 9$	$S_3 = S_2 + U_3 = 2U_1 + d + U_1 + 2d = 3U_1 + 3d$
4	5	$S_4 = S_3 + U_4 = 14$	$S_4 = S_3 + U_4 = 3U_1 + 3d + U_1 + 3d = 4U_1 + 6d$
5	6	$S_5 = S_4 + U_5 = 20$	$S_5 = S_4 + U_5 = 4U_1 + 6d + U_1 + 4d = 5U_1 + 10d$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$U_1 + 2(n-1)$	$S_n = S_{n-1} + U_n$	$s_n = u_1 + u_2 + u_3 + \ldots + u_n$ $= u_1 + (u_1 + d) + (u_1 + 2d) + \ldots + (u_1 + (n-1)d)$ $= (\underbrace{u_1 + u_1 + \ldots u_1}_{n\ terms}) + (d + 2d + \ldots + (n-1)d)$ $= nu_1 + d[1 + 2 + 3 + \ldots + (n-1)] = nu_1 + d[\frac{n(n-1)}{2}]$ $= nu_1 + \frac{n}{2}(n-1)d = \frac{n}{2}[2u_1 + (n-1)d]$ $= \frac{n}{2}[u_1 + u_1 + (n-1)d]$ $= \frac{n}{2}(u_1 + u_n)$

From the sum of authorized seats of 5 rows (n=5), we generalized the sum of authorized seats for n rows (terms).

g) Interpretation of results and Conclusion

i. What do you have to know for finding the sum of n first terms of an arithmetic sequence?

ii. How to find the sum of n first term if you know the first term and common difference?

iii. How could you find the sum of n first term of an arithmetic sequence if you know the first and last terms?

Expected answer:

i. To find the sum of n first terms of the arithmetic sequence you need its first term and common difference or its first and last term

ii. $S_n = nU_1 + \frac{n^2-n}{2}d = \frac{2nU_1 + (n^2-n)d}{2} = \frac{n}{2}[2U_1 + (n-1)d]$

iii. $S_n = \frac{n}{2}(2U_1 + (n-1)d) = \frac{n}{2}(U_1 + U_1 + (n-1)d) = \frac{n}{2}(U_1 + U_n)$

The sum of n first terms of the arithmetic sequence is given by

$$S_n = \frac{n}{2}(2U_1 + (n-1)d) \text{ or } S_n = \frac{n}{2}(U_1 + U_n)$$

h) Information for Teachers:

Arithmetic sequences are used in daily life for different purposes, such as determining the number of audience members an auditorium can hold. They help us predict, evaluate and monitor the outcome of a situation or event and are helpful in decision-making.

Consider an arithmetic sequence consisting of "n" terms:

- From the first term and common difference, the sum formula to find the sum of n terms in series is $S_n = \frac{n}{2}(2U_1 + (n-1)d)$
- From the first term and the last one, the sum formula to find the sum of n terms in series is $S_n = \frac{n}{2}(U_1 + U_n)$.

i) Guidance on the evaluation

The teacher facilitates Students while doing exercises and may check the performance of each learner.

The Teacher must figure out several cases where we use arithmetic sequences in our daily lives.

PRACTICAL ACTIVITY 3:	EXPLORING THE SUM OF THE FIRST N ODD NATURAL NUMBERS

a) Rationale:

Odd numbers are numbers that are not multiples of 2. Assuming we take the natural numbers, the odd numbers among them would be 1, 3, 5, 7, etc. This activity will help students understand how to confidently find the sum of n given first odd numbers. This activity is conducted when teaching the sum of n first terms of an arithmetic sequence in unit 2 of S5.

b) Objectives:

Find the sum of n first odd numbers provided.

c) List of materials required:

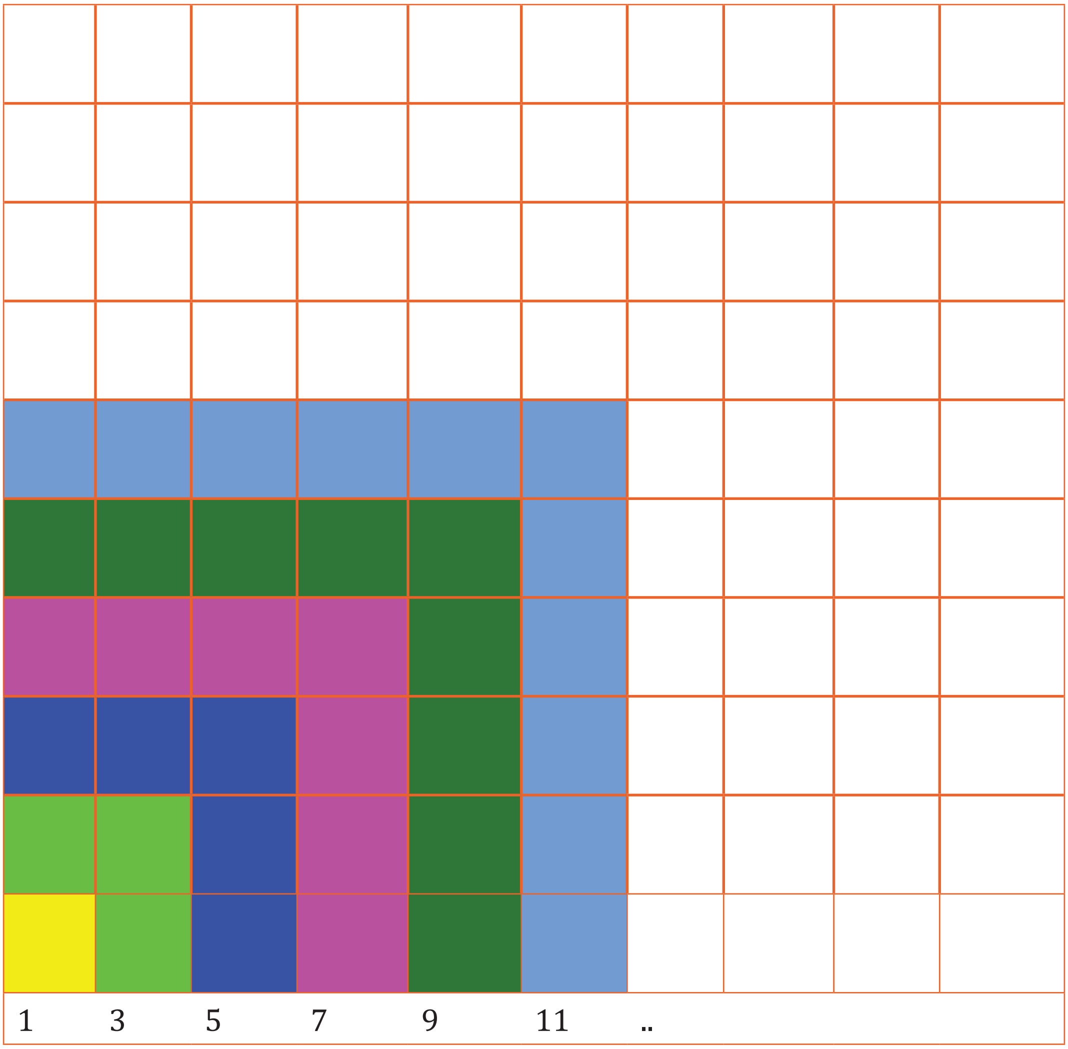
i. Manila paper
ii. Ruler, pencil and eraser
iii. Coloured ballpoint pens
iv. Pair of scissors
v. Geometrical instruments

d) Procedure

Step 1: Take a manila paper and cut out a square of size unit squares from it and mark the boundary of the square as A, B, C and D.

Step2: On the square in, draw horizontal and vertical lines in the square to make unit squares or grids (of size) as shown in the figure given below.

Step3: Colour the small squares with different colours with the help of coloured pens as shown in the figure below. How many squares with the same colour?



Step 4: Determine the area of the squares enclosed by each colour and record the data in the table for data recording.

Step 5: Express the area in (4) in terms of the sum of the square coloured in the same colour enclosed by the given colour. Record the data in the appropriate table. **For example, the a**rea enclosed by Green = Yellow + Green

Step 6: Using the result in the table, relate the item numbers in the table, the sum of the square enclosed and the area to come up with a conclusion.

Step 7: Record the data in (4) and (5) in the table for data recording for item number $n = 1, 2, 3, 4, 5, 6$.

Step 8: Using the same pattern, find the number of squares, the area enclosed and the relationship between them for item number $n = 10, 15, 20$.

Step 9: Use the pattern obtained to get a common formula that helps to find the sum of the first n odd natural number

What is your conclusion?

e) Data recording

Record your data in the table of results below:

Item's number (n)	Colour type	# of squares with the same colour	# of squares counted up to the current colour: for example Yellow + Blue + Green + ...	Area (A) enclosed by the current colour provided that the area of the small square is 1*1 **Hint:** write A=number of the square on Horizontal * Number of the square on vertical.
1	yellow			
2	Bright green			
3	Blue			
4	Purple			
5	Dark green			
6	Sky blue			

f) Result, interpretation, and conclusion

If we consider how the colours are arranged, we can deduce the following:

1. The area enclosed by yellow is 1×1
2. The area enclosed by green is $1+3=2\times 2=2^2$
3. area enclosed by Dark Green =# square in yellow + # square in Green +# square in Purple + #square in Blue + #square in dark Green =1+3+5+7+9=#square in horizontal up to dark green * #square in vertical =5*5=5^2

The left hand of the equation shows the sum of the number of squares with the same colour from yellow and the right hand shows the area calculated in terms of the horizontal and vertical squares. The extension of the pattern shows that the sum of odd numbers can be obtained simply by calculating the square of the corresponding natural number.

Position of odd number (n)	1	2	3	4	5	6	7	8	9
Odd number as generated from the figure	1	3	5	7	9	11	13	15	17
sum of odd number	1^2								
	$1+3=2^2$								
	$1+3+5=3^2$								
	$1+3+5+7=4^2$								
	$1+3+5+7+9=5^2$								
	$1+3+5+7+9+11=6^2$								
	$1+3+5+7+9+11+13=7^2$								
	$1+3+5+7+9+11+13+15=8^2$								
	$1+3+5+7+9+11+13+15+17=9^2$								
the sum of n first odd number	$1+3+5+7+....+(2n-1)=n^2$								

Conclusion:

The observation shows that the sum of n's first odd numbers is equal to the square of n. This means $1+3+5+7+.... + (2n-1) = n^2$

g) More information for the teacher

The sum of n's first odd numbers could be deduced from the sum of the first n terms of an arithmetic sequence. Considering that the first term $U_1 = 1$ and the nth term is $U_n = 2n - 1$, The required sum is given by $\frac{n}{2}\left(U_1 + U_n\right) = \frac{n}{2}(1 + 2n - 1) = n^2$

It can also be calculated using the first term and the common difference d=2.

$$S_n = \frac{n}{2}\left(2U_1 + \left(n-1\right)d\right) = \frac{n}{2}(2 + \left(n-1\right)2) = n^2$$

h) Guidance for evaluation

You can ask students to find the sum of 15 first odd numbers by counting the number of unit squares for a square of sides with 15-unit squares.

PRACTICAL ACTIVITY 4: EXPLORE THE MEANING OF A GEOMETRIC SEQUENCE

a) Rationale:

This activity is conducted when teaching the concept related to Geometric sequence to be learnt in Unit 2 of S5. Geometric sequences are found in many real-life scenarios such as population growth and the growth of an investment. Geometric growth is found in many real-life scenarios such as population growth and the growth of an investment. Geometric growth occurs when the common ratio is greater than 1, that is . The common ratio can be found by adding the percentage increase and 1, that is increase. Say that the percentage increase was 3%. The common ratio can be found by adding the percentage increase (of 3%) and 1. This would be the original amount plus an extra 3%. The reasoning for this is as follows:

Original amount + 3% of the original amount
= original amount (1+3%)
= original amount (1+0.03)
= 1.03 x original amount.

b) Objective:

Demonstrate a geometric sequence through a guided sequence of activity.

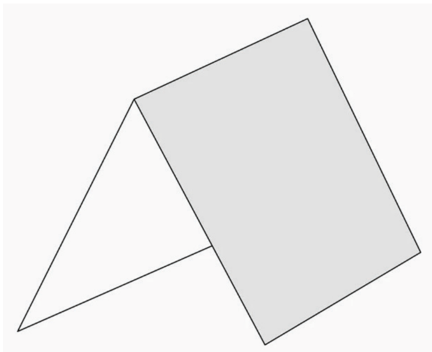
c) List of required materials:

- A4 papers
- pair of scissors
- pens or markers

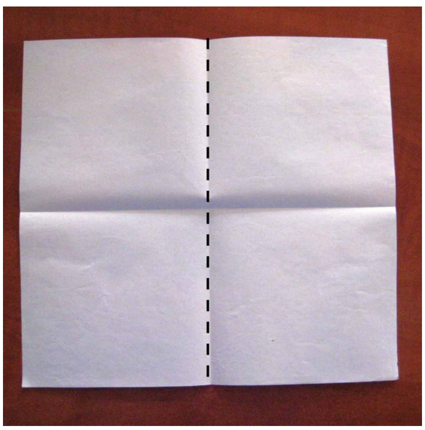
d) Procedures:

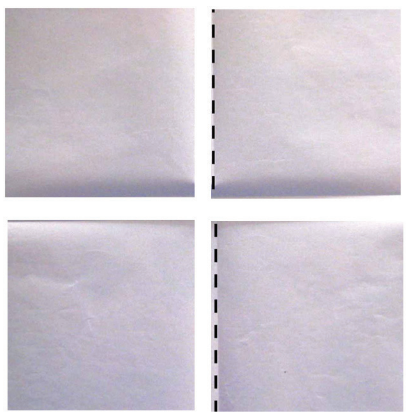
Step 1: Label the provided paper as in the figure below

Step 2: Fold the first A4 paper once in such a way that A lay on B and D on C as in the figure below and cut it down through the line formed with a scissor. Count and fill the number of equal parts obtained in the table provided. What fraction does each part represent?



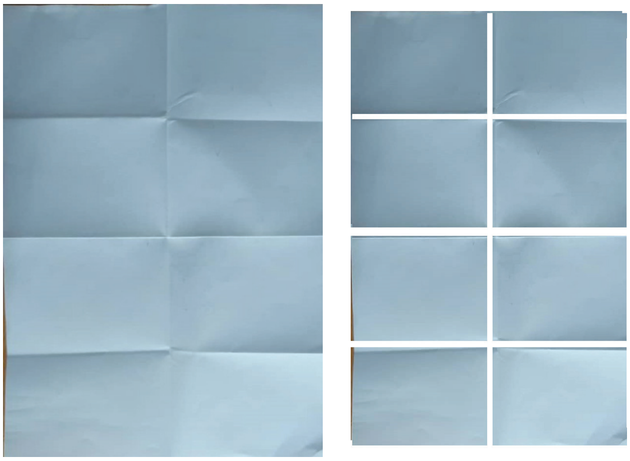
Step 2: Take another A4 paper and fold it twice. Cut it into small pieces through the perpendicular lines formed. Keep the cut small paper aside.





What fraction does each part represent?

Step 3: Take another A4 paper and fold it **three times** as shown in the figure below. Cut the small part through the line formed and keep them aside.



What fraction does each part represent?

Step 4: As done in steps 2 to step 4, fold the paper 4 times, 5 times, and 6 times and cut out the small parts formed in each case.

Step 5: In each case of steps 2 to step 5, count the number of cut out and fill your data in the table of data recording below.

- How many cut out are found in each folding of the paper in steps 2 to step 5?
- How many cut out would be generated if the paper is folded 100 times?
- Referring to the data collected in the table, how can you characterise the number and the size of the small cut-out as the number of folds increases?
- What is the common factor in those series generated?
  - Counting the number of cut out?
  - Looking at the change in the size of the cut-out in terms of folding

e) Recording of data

From your observations, complete the following table:

Number of folding times (n)	number of cut out at each fold	The size of one small cut-out compared to the size of a whole paper $\frac{1}{?}$	Size of one small cut-out compared to the original size written in terms of $\frac{1}{2}$
0	1	1/1	$\left(\frac{1}{2}\right)^0$
1			
2			
3			
4			
5			
6			
...	...	...	...
n			

From the data recorded, relate the number of folding times with the corresponding fraction.

Expected answer:

Number of folding times (*n*)	**Corresponding fraction**
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

3	$\frac{1}{8}$
4	$\frac{1}{16}$
5	$\frac{1}{32}$
6	$\frac{1}{64}$
7	$\frac{1}{128}$
n	

Size of one small cut-out compared to the original size written in terms of $\frac{1}{2}$

Expected answer:

Number of folding times	**Corresponding fraction**	**Relation**
0	1	$\left(\frac{1}{2}\right)^0 = \frac{1}{2^0}$
1	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1 = \frac{1}{2^1}$
2	$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2 = \frac{1}{2^2}$
3	$\frac{1}{8}$	$\left(\frac{1}{2}\right)^3 = \frac{1}{2^3}$

4	$\frac{1}{16}$	$\left(\frac{1}{2}\right)^4 = \frac{1}{2^4}$
5	$\frac{1}{32}$	$\left(\frac{1}{2}\right)^5 = \frac{1}{2^5}$
6	$\frac{1}{64}$	$\left(\frac{1}{2}\right)^6 = \frac{1}{2^6}$
7	$\frac{1}{128}$	$\left(\frac{1}{2}\right)^7 = \frac{1}{2^7}$

Suppose you fold a piece of paper n times, determine the fraction related to the number of folding n times.

Expected answer:

The fraction related to folding n times a piece of paper is $\frac{1}{2}$.

If the pattern continues, determine the value the last term will approach.

If the pattern continues, the last pattern will be closer to zero.

f) Interpretation of results and conclusion

While folding a piece of paper, note that we are multiplying the first term (for our case 1 piece of paper) by $\frac{1}{2}$.

The terms read in the table above show that we have: become:1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \frac{1}{2^n}$

From any two consecutive terms $U_n = \frac{1}{2^n}$ and $U_{n+1} = \frac{1}{2^{n+1}}$, we have a common ratio

$\frac{U_{n+1}}{U_n} = \frac{1}{2}$.

This is called the common ratio of the geometric sequence $U_n = U_{n-1} \times \frac{1}{2}$, we get that the general term of our geometric sequence is $U_n = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$

That is $U_{n+1} = \frac{1}{2}U_n$ or $U_n = 1\times\left(\frac{1}{2}\right)^n$

If we consider r as a common ratio, we get that the general term of a geometric sequence is $U_n = U_0 r^n$.

h) Guidance on the evaluation

While Students present their work, a teacher, moderates them and shows the steps in detail.

You can give for example the following problem-solving, and check how Students are performing it:

A mathematical child negotiates a new pocket money deal with her father which she receives on the first day of the month, on the second day, on the third day, on the fourth day, on the fifth day, ... until the end of the month. How much would the child receive at the end of the first two weeks of the first month?

Expected answer:

n^{th} day	Amount in dollars
1	1
2	2
3	4
4	8
...	...
14	8192

At the end of the 2 first weeks means at the 14^{th} day. Simply $U_{14} = 2^{13}\times 2 = 8192\$$
The child would receive8192$ at the end course for the first two weeks of the first month.

PRACTICAL ACTIVITY 5: THE SUM OF THE FIRST N TERMS OF A GEOMETRIC PROGRESSION

a) Rationale:

This activity is conducted when teaching the Concept related to the sum of n first terms of a geometric sequence. It is taught in Senior 5, unit 2. In real life, sequences are used in calculating interest, population growth, half-life, and decay in radioactivity, construction, and repeated drug dosage.

Conducting this experiment when teaching sequences will facilitate students to link the theoretical concept to real-life situations. The experiment will motivate students to deeply think about other examples related to the sum of n terms of geometric sequence observed in their daily life.

b) Objective:

This experiment aimed at practices of finding the sum of the first “n” terms of geometric sequences. This is a concept-based experiment

c) List of required materials:

Matchsticks boxes, Flipcharts (Manila paper or piece of paper), marker pens, ruler, glue.

d) Illustration of the activity:

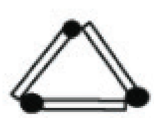
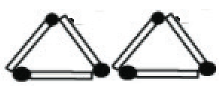
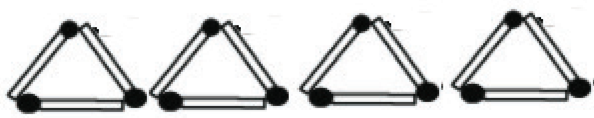
		

Figure1.	Figure 2.	Figure 3.

Arrangement of figures illustrating the three first terms of a sequence whose first term is 3 and the common ratio is 2.

e) Procedures:

Step 1: Use matchsticks and glue to construct figures 1, 2 and 3 as illustrated in figures 1, 2 and 3 above.

Step 2: Count the number of matchsticks used in each figure to generate a pattern and use the information gathered to construct the 4th, 5th and 6th figures.

Step 3: Use the information in steps 1 and step 2 to fill in the total number of matchsticks used in each figure in the table provided for recording data.

f) Recording of data

Could you determine the generating rule for the sum of the first n terms of a geometric sequence and compute the sum of the ten first terms of the given geometric sequence?

The following table will help students to record all data needed to understand the sum of n first terms.

Figure number (n)	Common ratio	Number of matchsticks in the group	Total number of matchsticks for all n first groups	
			The sum of all matchsticks for all n first groups	Alternative method
1				
2				
3				
4				
5				
6				

What name is given to the sequence of numbers for the matchsticks' pattern generated in the table of results?

- Calculate the common ratio in the sequence generated.
- Find the number of all matchsticks needed for the 2 first groups, 3 first groups, 5 first groups, 10 first groups, and generate a formula to calculate the sum of all matchsticks needed to construct any number n of groups.

- Try to generate that formula in the function of the number U_1 of matchsticks for the first group, the number n of groups, and the common ratio r ?

Expected answers:

We will relate the figure number and the number of matchsticks.

It is clear that each time a figure number is added by 1 (i.e. n+1), we get the next term of a geometric sequence whose first term is 3 with common ratio 2.

From formula of nth term of a geometric sequence we have

$U_n = U_1 r^{n-1} \Rightarrow U_5 = 3\times 2^4$ as the number of matchsticks to be used on figure 5.

At Step 2: The number of matchsticks to be used from 1st figure to 2nd figure is 3+6=9

at Step 3: The number of matchsticks to be used from 1st figure to 3rd figure is 3+6+12=9+12=21

At Step 4: The number of matchsticks to be used from 1st figure to 4th figure is 3+6+12+24= 21+24=45

At the 5th figure: The number of matchsticks to be used from 1st figure to 5th figure is 3+6+12+24+48= 45+48=93.

At 6th figure: The number of matchsticks to be used from 1st figure to 10th figure is

3+6+12+24+48+96+192+384+768+1536=3069.

Figure number (n)	**Common ratio**	**Number of matchsticks of the group**	**Total number of matchsticks for all n first groups**	
			sum of all matchsticks for all n first groups	**Alternative method** $S_n = U_1\left(\frac{1-r^n}{1-r}\right)$
1				
2				

3				
4				
5				
6				

g) Interpretation of results and Conclusion

From the table, it is clear that the sum of **n** first terms of a geometric sequence

is calculated from $S_n = U_1\left(\frac{1-r^n}{1-r}\right), r \neq 1$, hence $S_{10} = 3\left(\frac{1-2^2}{1-}\right) = 3069.$

To find the sum of n first terms of a geometric sequence, we identify the following:

- First term
- The common ratio
- The number of terms

And then apply the sum of the Geometric Sequence Formula

$$S_n = U_n\left(\frac{1-r^n}{1-r}\right), r \neq 1$$

h) Information for teacher

To get the general formula for the sum of the first n terms of a geometric sequence (also known as geometric series) you proceed as follows:

Suppose a Geometric Series for n terms:

$$S_n = U_1 + U_1 r + U_1 r^2 + U_1 r^3 + \cdots + U_1 r^{n-1} \qquad (1)$$

Multiplying both sides by the common ratio(r) yields

$$rS_n = U_1 r + U_1 r^2 + U_1 r^3 + U_1 r^4 + \cdots + U_1 r^n \qquad (2)$$

Subtracting equation (2) from equation (1) gives

$$S_n - rS_n = \left(U_1 + U_1r + U_1r^2 + U_1r^3 + \cdots + U_1r^{n-1}\right) - \left(U_1r + U_1r^2 + U_1r^3 + U_1r^4 + \cdots + U_1r^n\right)$$

$$S_n - rS_n = U_1 - U_1r^n \Leftrightarrow S_n(1-r) = U_1\left(1-r^n\right)$$

$$\Leftrightarrow S_n = U_1\left(\frac{1-r^n}{1-r}\right) \qquad (3)$$

i) Guidance on the evaluation

Provide various exercises to facilitate students to find the common ratio; for example, they can be asked to determine the sum of n first terms of a geometric sequence, given the first term and n^{th} term.

UNIT: 3

LOGARITHMIC AND EXPONENTIAL EQUATIONS

PRACTICAL ACTIVITY 6: USING LOGARITHMIC AND EXPONENTIAL FUNCTIONS TO SOLVE PROBLEMS RELATED TO THE COMPOUND INTEREST

a) Rationale:

This activity is conducted when teaching the concept related to applications of logarithmic and exponential functions in real life or other sciences. It is learnt in Unit 3 of S5. Much of the power of logarithms is their usefulness in solving exponential equations. Some examples of this include **sound (decibel measures), earthquakes (Richter scale), the brightness of stars, and chemistry (pH balance, a measure of acidity and alkalinity).**

b) Objective:

Apply logarithmic or exponential functions to solve interest rate problems and population growth problems. Through this practical activity, Students will understand the importance of saving in real life. This is a concept-based practical activity.

c) List of required materials:

Piece of paper, scientific calculator or excel sheet, pen, ruler.

d) Illustration of the activity:

Suppose you deposit 800,000 Frw into your saving account at a compound interest of 15% per annum. After how many years will you be able to buy a car for 6 million frw using only your saving account?

e) Procedures:

Step 1: Calculate the interest at the end of 1st year and the total amount in his/her account at the beginning of 2nd year.

Step 2: Calculate the interest at the end of 2nd year and the amount at the beginning of 3rd year.

Step 3: Calculate the interest at the end of 3rd year and the amount at the beginning of 4th year.

Step 4: Calculate the interest at the end of the 4th year and the amount at the beginning of the 5th year.

Step 5: Calculate the interest at the end of the 5th year and the amount at the beginning of the 6th year.

Step 6: Continue and calculate interest at the end of the year to get the amount at the beginning of the next year.

Step 7: Record all data in the table of data recording. Try to find the formula that can help the client to find the number **n** of years necessary to get the expected amount of money (6000000Frw by using the deposited money P_0 (800,000Frw) and the interest rate r.

f) Recording of data:

Record your data in the table according to your target:

Year t	Principal deposit (FRW) at the beginning P_0	Interest (FRW) $I = P_0 \times r$	Principal deposit with interest (FRW) $P = P_0(1+r)^t$	Determination of the time using the principal P $\frac{\log P}{\log\left(P_0(1+r)\right)}$	is t equal to the nbr of years?
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					

Expected answer:

Year t	Principal deposit (FRW) at the beginning P_0	Interest (FRW) $I = P_0 \times r$	Principal deposit with interest (FRW) $P = P_0(1+r)^t$	Determination of the time using the principal P $\frac{\log\left(\frac{P}{P_0}\right)}{\log(1+r)}$	is t equal to the nbr of years?
1	800,000	120,000	920,000	1	yes
2	920,000	138,000	1,058,000	2	yes
3	1,058,000	159,000	1,217,000	3	yes
4	1,217,000	183,000	1,399,000	4	yes
5	1,399,000	210,000	1,609,000	5	yes
6	1,609,000	241,000	1,850,000	6	yes
7	1,850,000	278,000	2,128,000	7	yes
8	2,128,000	319,000	2,447,000	8	yes
9	2,447,000	367,000	2,814,000	9	yes
10	2,814,000	422,000	3,236,000	10	yes
11	3,236,000	485,000	3,722,000	11	yes
12	3,722,000	558,000	4,280,000	12	yes
13	4,280,000	642,000	4,922,000	13	yes
14	4,922,000	738,000	5,661,000	14	yes
15	5,661,000	849,000	6,510,000	15	yes

g) Interpretation of results and Conclusion

We see that at the end of the 15th year, the accumulated amount will be 6,510,000 Frw as it exceeds 6 million, it is possible to buy a car of 6 million Frw using only your saving account.

To find this number of years we see that we can use $n = \dfrac{\log\left(\dfrac{P}{P_0}\right)}{\log(1+r)}$.

The use of logarithms can help us to get the number of years.

h) Information for Teachers:

$P = P_0 + P_0 r = P_0(1+r)$ The total amount accumulated is found by adding the current principle and current simple interest;

t/n years	Accumulated amount at the end of the year	
1	$P_1 = P_0 + P_0 \text{r} = P_0(1+r)$	$1 = \dfrac{\log\left(\dfrac{P}{P_0}\right)}{\log(1+r)}$
2	$P_2 = P_1 + P_1 r = P_0(1+r) + P_0(1+r)r = P_0(1+r)^2$	$2 = \dfrac{\log\left(\dfrac{P}{P_0}\right)}{\log(1+r)}$
3	$P_3 = P_2 + P_2 r$ $= P_0(1+r)^2 + P_0(1+r)^2 r = P_0(1+r)^3$	$3 = \dfrac{\log\left(\dfrac{P}{P_0}\right)}{\log(1+r)}$
n	$P_n = P_{n-1} + P_{n-1} r$ $= P_0(1+r)^{n-1} + P_0(1+r)^{n-1} r = P_0(1+r)^n$	$n = \dfrac{\log\left(\dfrac{P}{P_0}\right)}{\log(1+r)}$

i) Guidance on the evaluation

The teacher provides different exercises related to interest and helps Students not confuse compound interest with continuous compound interest.

UNIT: 4

SOLVING EQUATIONS BY NUMERICAL METHODS

PRACTICAL ACTIVITY 7: SOLVING EQUATIONS BY BISECTION METHODS.

a) Rationale:

This activity is conducted when teaching the numerical methods to get approximated roots of the given equation. Students determine all intervals where the root is located. This is learnt in Unit 4 of S5. The Bisection method is one of the most straightforward and dependable iterative approaches for solving nonlinear equations. This method, sometimes known as binary chopping or the half-interval method, is based on binary chopping. Since f(x) is in the interval, a x b is real and continuous, and f(a) and f(b) have the same sign. In general, Numerical methods are used in almost all real-life implementations: **The bisection method and Newton-Raphson methods** are used to find the roots and fixed points of equations.

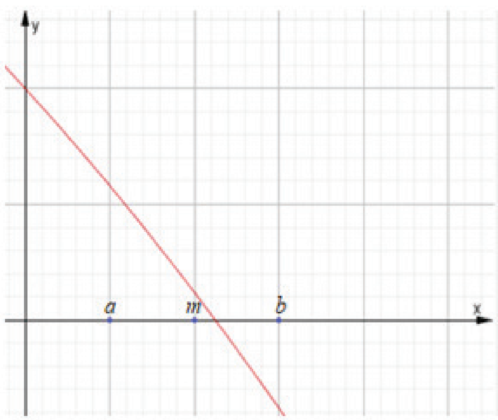
b) Objective:

Use the numerical bisection method to obtain approximated roots for a given equation. This is a conceptual-based practical activity.

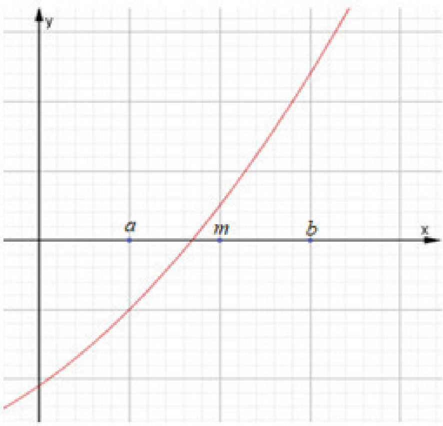
c) List of required materials:

Piece of paper, ruler, calculator and pen.

d) Illustration of the activity:



(a)



(b)

By trial and error, it is known that a single zero of a function f lies between $x = a$ and $x = b$. The root is said to be bracketed by a and b when $f(a) \cdot f(b) < 0$.

Let $m = \frac{a+b}{2}$, the mid-point of the interval $a < x < b$ and evaluating $f(m)$, we can see in which half (the left or the right) of the interval $a < x < b$, the zero is located.

In the previous figure, if $f(m) \cdot f(b) < 0$, then we are in the situation shown in (a) and we replace $a < x < b$ with the smaller bracketing interval $m < x < b$.

If, on the other hand, $f(a) \cdot f(m) < 0$, then we are in the situation shown in (b), and we replace $a < x < b$ with smaller bracketing interval $a < x < m$.

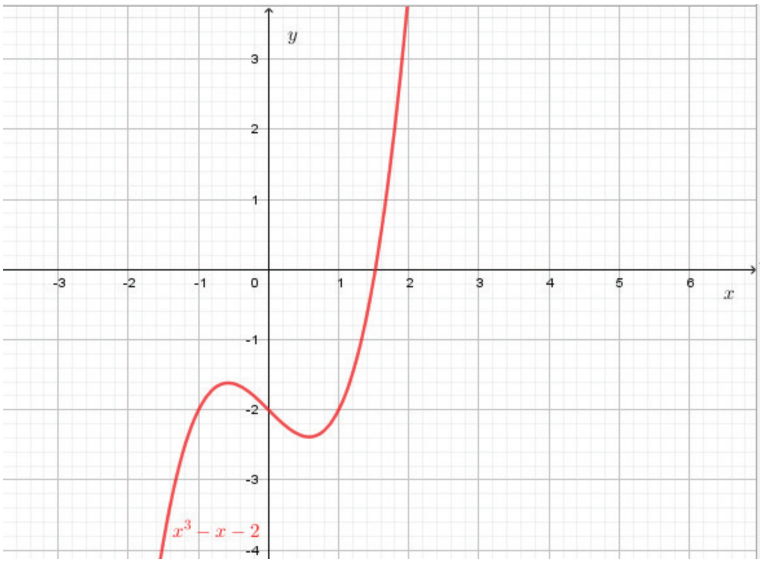
By repeatedly applying the above approach, we are requested to find the root of $f(x) = x^3 - x - 2$ correctly at 2 decimal places.

e) Procedures:

Step 1: Consider $f(x) = x^3 - x - 2$ and find $f(0), f(1), f(2), f(3)$.

Step 2: Compare the result obtained in step 1 to find which one is closer or equal to zero.

Step 3: Identify the interval containing the solution.



The interval should be made by two end points a and b such that $f(a)f(b)<0$.

For example, since $f(0)$ and $f(1)$ are less than 0, while $f(2)$ and $f(4)$ are greater than zero, then you choose the smallest interval fulfilling the condition that $f(a)f(b)<0$.

Step 4: Name the lower end value by a and upper end value by b

Step 5: Find the first approximation of the root $c_1 = \dfrac{a+b}{2}$

Step 6: Find $f(c_1)$ by replacing c_1 in the original function.

Step 7: Compare $f(c_1)$ with zero. Is it closer or equal to zero? If not go, to the next step.

Step 8: Locate the next interval containing the zero by finding $f(a)f(c_1)$ and $f(c_1)f(b)$.

Which one is giving you a negative value? Then, this is your next interval.

Step 9: Find $c_2 = \dfrac{a+c_1}{2}$ or $c_2 = \dfrac{c_1+b}{2}$ depending on what gave you the negative value.

Step 10: Repeat the step 6-9 several times up to 10^{th} iteration and fill your findings in the table of result below as records.

f) Recording of data

From the given function, complete the following table:

Iterations	The interval containing the root: Lower end value a_n	Upper end value b_n	Midpoint $c_n = \frac{a_n + b_n}{2}$	$f(c_n)$	Determination of new interval containing the root: Is $f(a_n)f(c_n) < 0$?	Is $f(c_n)f(b_n) < 0$?
1	1	2	1.5	−0.125	No	Yes
2	1.5	2	1.75	1.609375	Yes	No
3						
4						
5						
6						
7						
8						
9						
10						

Expected answer

Iteration	Interval containing the root: Lower end value a_n	Upper end value b_n	Midpoint $c_n = \frac{a_n + b_n}{2}$	$f(c_n)$	Determination of new interval containing the root: Is $f(a_n)f(c_n) < 0$?	Is $f(c_n)f(b_n) < 0$?
1	1	2	1.5	−0.125	No	Yes
2	1.5	2	1.75	1.609375	Yes	No
3	1.5	1.75	1.625	0.6660156	Yes	No
4	1.5	1.625	1.5625	0.2521973	Yes	No
5	1.5	1.5625	1.53125	0.0591125	Yes	No
6	1.5	1.53125	1.515625	−0.0340538	No	Yes
7	1.515625	1.53125	1.5234375	0.0122504	Yes	No

8	1.515625	1.5234375	1.5195313	−0.0109712	No	Yes
9	1.5195313	1.5234375	1.5214844	0.0006222	Yes	No
10	1.5195313	1.5214844	1.5205078	-0.005179	No	Yes

g) Interpretation of results and conclusion

From 10 iterations we have that the root of $f(x)=x^3-x-2$ with bracketing interval $1<x<2$ correctly to 2 decimal place is 1.52.

The aim with the bisection method is to repeatedly reduce the width of the bracketing interval $a<x<b$ so that it pinches the required zero of f to some desired accuracy.

There are three conditions:

(i) $f(a)=0$, we have a zero at $x=a$.

(ii) If $f(a)\cdot f(m)<0$, the root is located between a and m.

(iii) If $f(m)\cdot f(b)<0$, the root is located between m and b.

h) Guidance on evaluation

The teacher provides various exercises for different types (polynomial, rational, trigonometric...) for which the students get approximate roots and remind them that those solutions are not exact.

UNIT: 5 TRIGONOMETRIC FUNCTION AND THEIR INVERSES

PRACTICAL ACTIVITY 8: GRAPHICAL ILLUSTRATION OF THE TRIGONOMETRIC FUNCTION $y = \arcsin x$ AND ITS BEHAVIOUR RELATIVE TO $y = \sin x$

a) Rationale:

This activity is conducted while teaching the inverse trigonometric functions It is learnt in Unit 5 of S5 Mathematics. It involves the concept of inverse functions behaviour where a function and its inverse are related in such a way that one is the image of another under a mirror reflection line $y = x$.

One example of an inverse trigonometric function is the angle of depression and angle of elevation. The above two fit as a plausible example because they use sine or cosine or tangent to determine the angle of a person's view to the top of a building for example. Or another example would be if a person is standing on a building that is a certain height and looking down at an object a certain distance away, what would be the angle of depression that they're looking down?

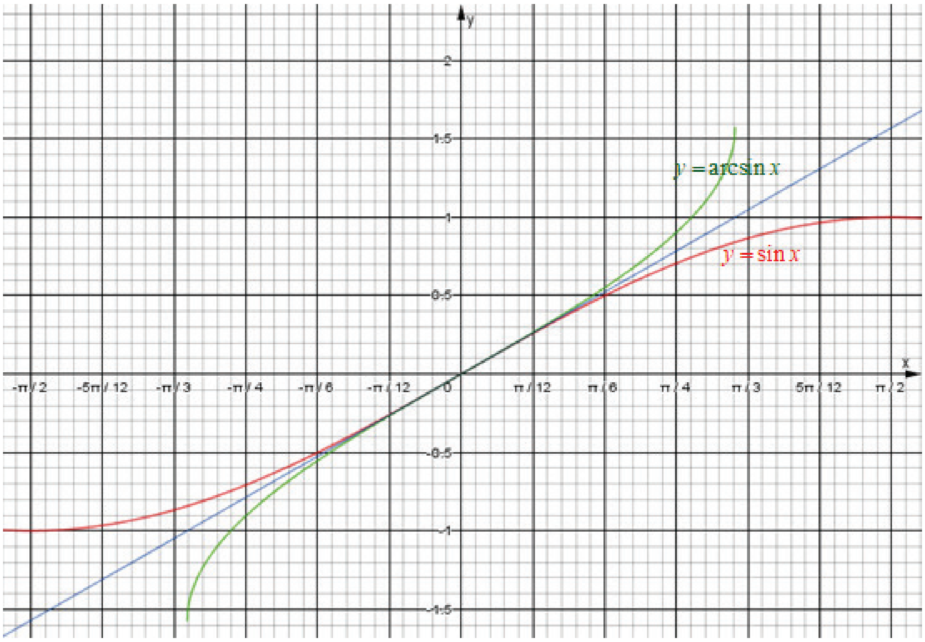
b) Objective:

Describe the curve of $y = \arcsin x$ using the curve of $y = \sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$). This is a combined concept and inquiry-based practical activity.

c) List of required materials:

Cardboard, white chart paper, ruler, coloured pens, scientific calculator, pencil, eraser, marker, reflective plane mirror.

d) Illustration and set up:



e) Procedure:

Step 1: Take a cardboard of suitable dimensions 30cm x 30cm

Step 2: On the cardboard, paste a white graph paper of 25cm x 25cm

Step 3: On the graph paper, draw the Cartesian plan, with centre 0 and orthogonal axes X and Y.

Step 4: Graduate the axes approximately by $\frac{\pi}{12}$ unit scales on the abscissa axis, 0.1 unit scales on the ordinates axis and plot in the same Cartesian plan.

Step 5: In the same Cartesian plan, plot the points in the ranges $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\left[-1,1\right]$ for $y=\sin x$.

Step 6: Join points for the function, draw the graph passing through the appropriate points

Step 7: In the same Cartesian plan, draw a straight line $y=x$.

Step 8: Place the plane mirror on the line $y=x$ so that the graph of $y=\sin x$ is on the reflective Surface.

Step 9: Observe the image of the graph of $y = \sin x$ in the mirror. How is the image? Try to think about how you can draw that image on the same graph paper using the properties of the image obtained under the mirror (Reflection of axis with equation $y = x$)

Step 9: Use your conclusion on step 9 to draw the image on the same Cartesian plane.

Step 10: Use the math software to draw the graph of $y = \arcsin x$ and compare its form with the graph you got on the step 9. How are they? Can you conclude on how you can find the graph of $y = \arcsin x$ when you have the graph of $y = \sin x$? Discuss how you can get the value of $\arcsin x$ basing on the values of $\sin x$ given in the table you found on step 9

f) Recording of data:

For each of the functions complete the following tables:

$y = \sin x$:

x	$-\frac{\pi}{2}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
$y = \sin x$													

Expected results:

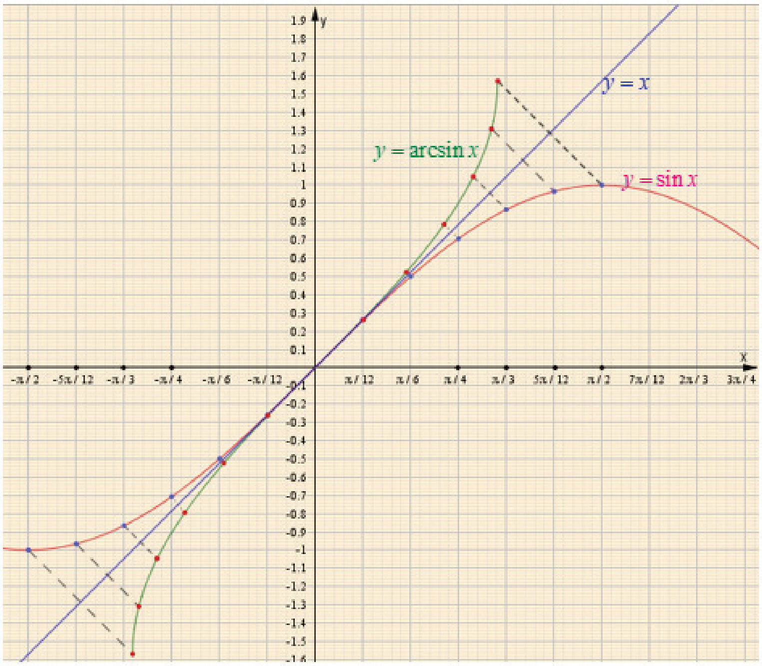
$y = \sin x$

x	$-\frac{\pi}{2}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
$y = \sin x$	-1	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{6}-\sqrt{2}}{4}$	0	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	1

$y = \arcsin x$

x	-1	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{6}-\sqrt{2}}{4}$	0	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	1
$y=\arcsin x$	$\frac{\pi}{2}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$

Each plotted point on $y=\sin x$ has an image on $y=\arcsin x$ under a reflection about $y=x$



- -The two curves are images from one another with respect to reflection under the straight line $y=x$ and vice versa.
- The functions $y=\arcsin x$ and $y=\sin x$ are inverses of each other and one is the image of another under a plane mirror through a straight line $y=x$.

The function $y=f(x)=\arcsin x$:

x	-1	$-\frac{\sqrt{6}+\sqrt{2}}{4}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{6}-\sqrt{2}}{4}$	0	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	1
$y=\arcsin x$													

g) Interpretation of results and Conclusion

1. Placing a plane mirror along the straight line $y = x$:

In the reflecting side of the mirror, the part of the curve of $y = \sin x$ obtained in the mirror has its image on $y = \arcsin x$ in the same direction. Similarly, the part of the curve of $y = \arcsin x$ reflected in the mirror has the image the part of $y = \sin x$ in the same direction. This means that by a plane mirror each one is the image of another.

1. Since, the domain of definition of $y = \sin x$ is the range of $y = \arcsin x$ and vice versa , the two functions are inverses.
2. The function $y = \sin x$ is the inverse function of $y = \arcsin x$, and the function $y = \arcsin x$ is the inverse of the function $y = \sin x$ because one is the image of the other under the reflection line $y = x$.
3. To draw the curve of the function $y = \arcsin x$, Students should first draw the curve of $y = \sin x$ and make a reflection through the line $y = x$. The obtained image is the curve of $y = \arcsin x$.
4. This practical activity should be used for all inverse functions.
5. Inverse functions have an identity that:

$$f \circ f^{-1}(x) = x = f^{-1} \circ f(x)$$

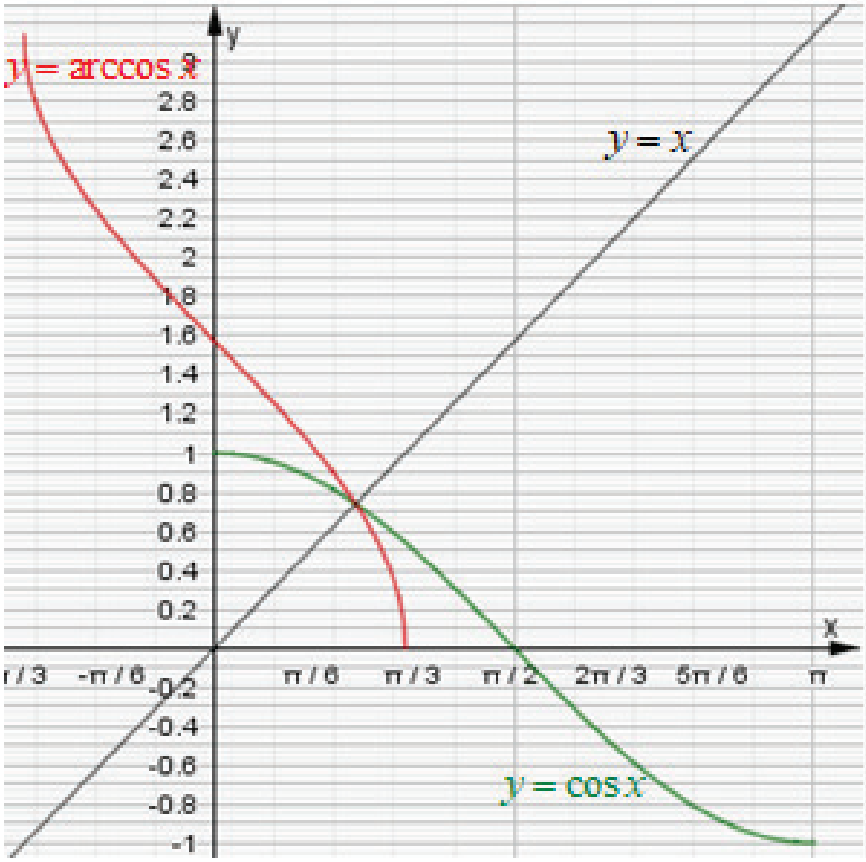
h) More information to the teacher:

- In the tables Students should use approximated decimal values.
- In general the two functions are said to be inverses, iff the domain of definition of one is the range of another, graphically these functions behave like images of one another under the reflection line $y = x$.

i) Guidance on evaluation

Teacher provides various exercises involving the graphical illustration of inverse functions such as: $y = \cos x$ **Vs** $y = \arccos x$

Expected graphs:



Graph of $y = \cos x$ and the graph of $y = \arccos x$

UNIT: 6

VECTOR SPACE OF REAL NUMBERS

ACTIVITY 9: DESCRIBING THE RESULTANT OF FORCES AS THE SUM OF VECTORS

a) Rationale:

This practical activity is done when teaching the concept related to operations of vectors in the vector space $\left(\mathbb{R}^3,+,\bullet\right)$ to be learnt in Unit 6 of S5. Since forces are vectors, they can be added according to the rules of vector addition.

In real life, the addition of vectors is observed in the unification of different forces to get stronger.

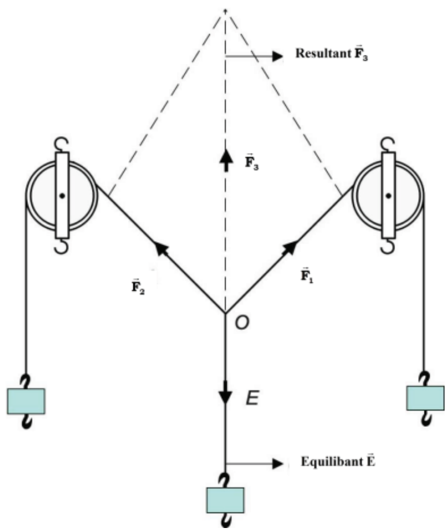
b) Objective:

Determine the resultant force using the sum of two vectors graphically and using the force table to verify the result. This is a concept-based practical activity.

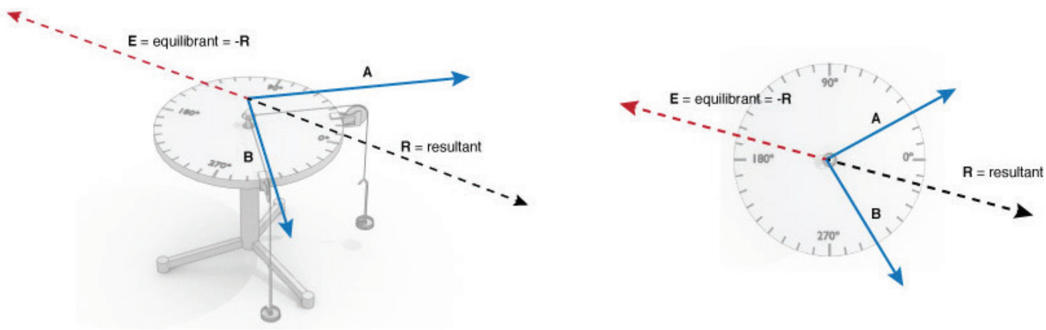
c) List of required materials:

Parallelogram law of forces apparatus (Gravesand's apparatus), plumb line, two hangers with slotted weights, a body (a wooden block) whose weight is to be determined, thin strong thread, white drawing paper sheet, drawing pins, mirror strip, sharp pencil, half meter scale, set squares, protractors, stands.

d) Illustration:



A point to be aware of is that the force needed to balance the system is **not** the resultant of the weights, but the negative of that vector also called the equilibrant.



e) Procedure:

Step 1: Set up the 2 pulleys on 2 fixed positions.

Step 2: Put a thread over the pulleys and attach to each extremity a full set of slotted weights: One extremity has the mass m1, and the other has the mass m2.

Step 3: Calculate the vertical force acting on each mass.

Step 4: Take another mass m3 and put it on the thread between the two pulleys. What happens? Does the mass m1 stay at the initial position? Does it move and becomes stable after a while? What is the angle made by the thread between the 2 pulleys?

Step 5: Consider the F1 as the vertical force acting on the mass m1, F2 as the vertical force acting on the mass m2 and F3 as the vertical force acting on the mass m3 and explain the characteristics of the resultant force when the system becomes stable. Draw this situation.

Step 6: Consider the case in which the mass $m_1 = 2kg$, $m_2 = 5kg$ and $m_3 = 4kg$, calculate the weights F_1, F_2 and F_3, measure the angle θ between the weights F_1 and F_2 when the system is stable.

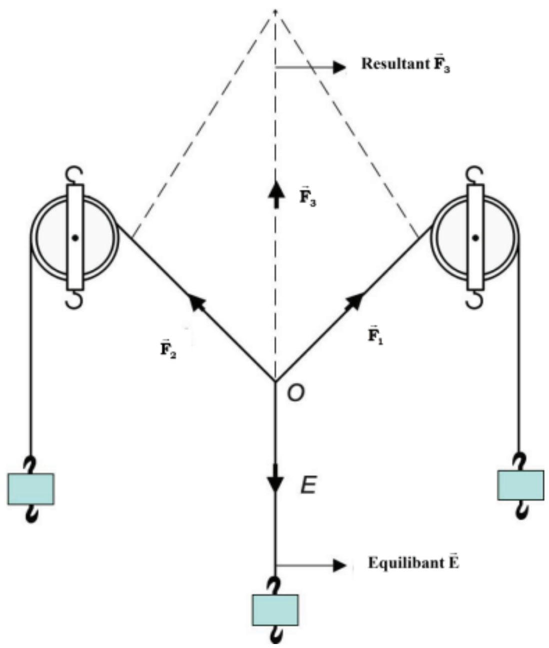
Step 7: Use the formula and calculate the angle that should be between the position of the weights F_1 and F_2 depending on the resultant force

f) Recording of data

Complete the following table:

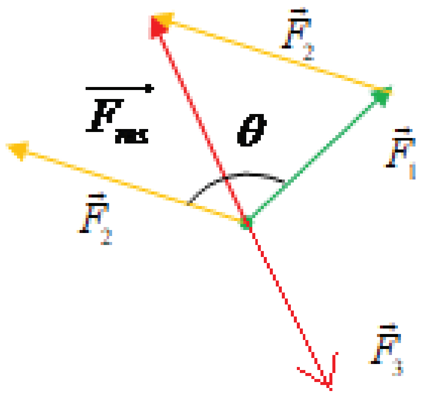
m1	F1	m2	F2	m3	F3	Angle θ

Expected result:



g) Results, interpretation, and conclusion

In this context, we have two concurrent forces $\vec{F}_1$ and $\vec{F}_2$, their resultant force $\vec{F}_3$ is graphically illustrated as follows:



Means that: $\vec{F}_{res} = \vec{F}_1 + \vec{F}_2$

But in this context, the system will be in equilibrium under another force called equilibrant force $\vec{F}_{res}$ that is opposite to the force $\vec{F}_3$ acting on the thread by pulling downwards.

Therefore, we have: $\vec{F}_3 + \overrightarrow{F_{res}} = \vec{0}$

By using the formula,

$\left\|\vec{F}_{res}\right\| = \left\|\vec{F}_1 + \vec{F}_2\right\|^2 = \left\|\vec{F}_1\right\|^2 + \left\|\vec{F}_2\right\|^2 + 2\left\|\vec{F}_1\right\|\left\|\vec{F}_2\right\|\cos\theta$ where θ is the angle between $\vec{F}_1$ and $\vec{F}_2$

Students will calculate the angle θ using this formula and verify if the angle found is the same as the angle measured between $\vec{F}_1$ and $\vec{F}_2$ at the system of the 2 pulleys.

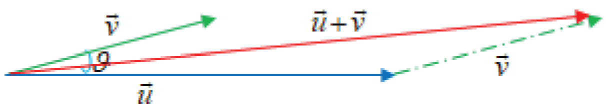
h) Additional information to the teacher

An analysis of the experimental data in the table shows that the three forces are initially represented by their magnitudes and angles as lines (vectors) pointing outward from the central point of action. The lines representing forces F_2

and F_3 are shifted so that the start of each vector is the end of the previous vector while maintaining its direction. The length of the resulting vector force

$\vec{F}_3 = \vec{F}_1 + \vec{F}_2$ opposes the equilibrant vector force $\overrightarrow{F_{res}}$. Therefore the system is in an equilibrium state and the first newton.

Consider two vectors $\vec{u}$ and $\vec{v}$ directed from a common point and the angle θ the angle between the two vectors, the sum $\vec{u}+\vec{v}$ of two vectors is graphically given by:



$\vec{u}+\vec{v}$

The magnitude of the sum (resultant) vector is given by:

$$\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\|\vec{u}\|\|\vec{v}\|\cos\theta$$

In our case of concurrent forces,

$$\|\vec{F}_{res}\| = \|\vec{F}_1 + \vec{F}_2\|^2 = \|\vec{F}_1\|^2 + \|\vec{F}_2\|^2 + 2\|\vec{F}_1\|\|\vec{F}_2\|\cos\theta$$

$$\vec{F}_{res} + \vec{E} = \vec{0}$$

i) Guidance to evaluation

Provide various exercises involving the sum of vector forces and students deduce the required resultant force and equilibrant one.

PRACTICAL ACTIVITY 10: USING THE DOT PRODUCT TO DEDUCE THE ANGLE BETWEEN TWO VECTORS

a) Rationale:

A dot product is a way of multiplying two or more vectors. The result of the dot product of vectors is a scalar quantity. Therefore, it is also called the scalar product. Algebraically, it is the sum of the products of corresponding entries in two sequences. Geometrically, it is the product of the Euclidean magnitudes of two vectors and the cosine of the angle between them. The dot product of vectors has many uses in geometry, mechanics, engineering, and astronomy.

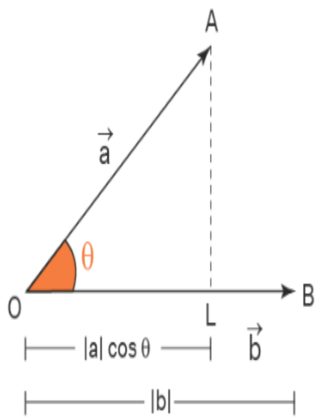
b) Objective:

Find the dot product of two vectors and determine the angle between those two vectors. This is an inquiry-based experiment.

c) List of required materials:

Flip chart (Manila paper or piece of paper), marker, pen, ruler, protractor and scientific calculator.

d) Illustration of the activity:



$$a \cdot b = |a|.|b| \cos\theta$$

e) Procedures:

Step 1: Using the dot product formulae $\vec{u}.\vec{v} = u_1.v_1 + u_2.v_2$, calculate the dot product of the vectors provided below and record the results in the table of result below

i) $\vec{u} = 4\vec{i} + 7j$ and $\vec{v} = 2\vec{i} - 2\vec{k}$

ii) $\vec{u} = \vec{i} + \vec{j}$ and $\vec{v} = -2\vec{i} + 2\vec{j}$

iii) $\vec{u} = 2\vec{i} + 4\vec{j}$ and $\vec{v} = -2\vec{i} + \vec{j}$

	Coordinates of $\vec{u}$	**Coordinates of** $\vec{v}$	$\vec{u}.\vec{v} = u_1.v_1 + u_2.v_2$
$\vec{u} = 4\vec{i} + 7j$ and $\vec{v} = 2\vec{i} - 2\vec{k}$			

Step 2: On a manila paper, use ruler and pen to draw a Cartesian coordinate. Make sure to use a readable and clear scale

Step 3: In the Cartesian plane from step 2, use ruler and pen to draw the vectors $\vec{u} = 4\vec{i} + 7j$ and $\vec{v} = 2\vec{i} - 2\vec{k}$

Step 4: Using ruler, measure the length $\left(\|\vec{u}\|\right)$ of $\vec{u}$ and $\left(\|\vec{v}\|\right)$ of $\vec{v}$

Step 5: Use the formulae $\vartheta = \cos^{-1}\left(\frac{\overrightarrow{u.v}}{\|\vec{u}\|.\|\vec{v}\|}\right)$ to calculate the angle between the two vectors

Step 6: Using a protractor, measure the angle θ between $\vec{u}$ and $\vec{v}$, compare it with ϑ the calculated result in step 5. What is your observation?

Step 7: Repeat step 3 to step 6 for the vector:

i. $\vec{w} = \vec{i} + \vec{j}$ and $\vec{k} = -2\vec{i} + 2\vec{j}$

ii. $\vec{h} = 2\vec{i} + 4\vec{j}$ and $\vec{k} = -2\vec{i} + \overrightarrow{2j}$ tabulate the results in the table of results provided below.

Step 8: What is your interpretation when the dot product is equal to zero?

f) Data recording:

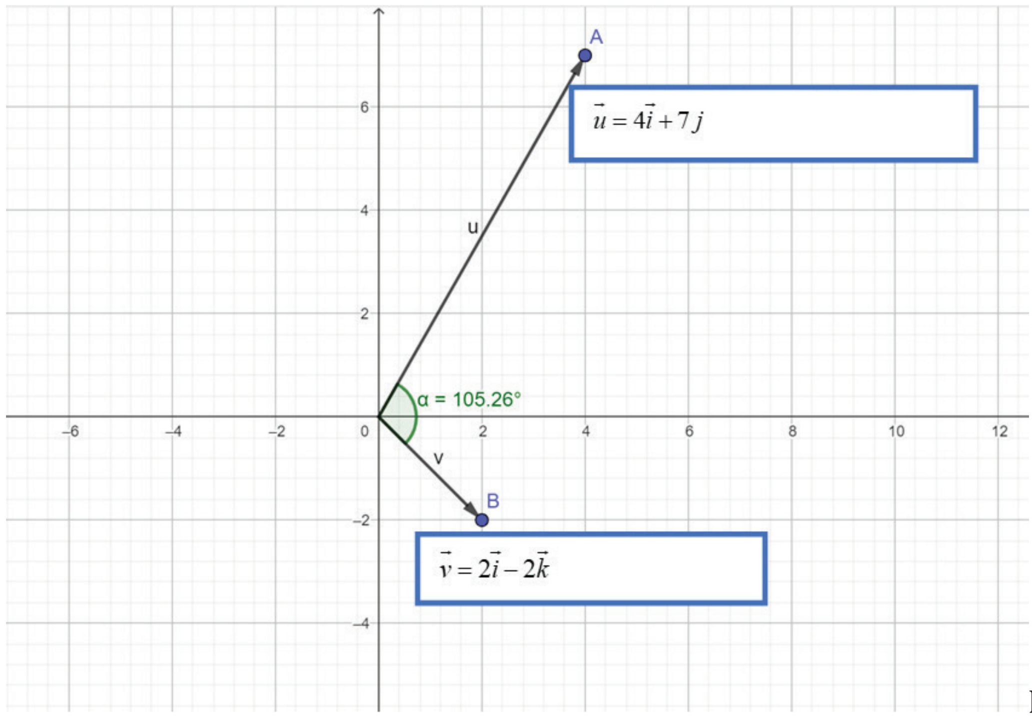
Vectors	Length $(\|\vec{u}\|)$ of $\vec{u}$	Length $(\|\vec{v}\|)$ of $\vec{v}$	Angle θ between $\vec{u}$ and $\vec{v}$	$Q = \|\vec{u}\|.\|\vec{v}\|$	$\vartheta = \cos^{-1}\left(\frac{\overrightarrow{u.v}}{\|\vec{u}\|.\|\vec{v}\|}\right)$
$\vec{u} = 4\vec{i} + 7\vec{j}$ and $\vec{v} = 2\vec{i} - 2\vec{k}$					
$\vec{w} = \vec{i} + \vec{j}$ and $\vec{k} = -2\vec{i} + 2\vec{j}$					
$\vec{h} = 2\vec{i} + 4\vec{j}$ and $\vec{k} = -2\vec{i} + \overrightarrow{2j}$					

Expected answer:

Step 1: Lets calculate the dot product using the formulae $\vec{u}.\vec{v} = u_1.v_1 + u_2.v_2$

	Coordinates of $\vec{u}$	Coordinates of $\vec{v}$	$\vec{u}.\vec{v} = u_1.v_1 + u_2.v_2$
$\vec{u} = 4\vec{i} + 7j$ and $\vec{v} = 2\vec{i} - 2\vec{k}$	$\begin{pmatrix} 4 \\ 7 \end{pmatrix}$	$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$	$\vec{u}.\vec{v} = (4 \times 2) + (7 \times (-2)) = -6$

$\vec{w}=\vec{i}+\vec{j}$ and $\vec{k}=-2\vec{i}+2\vec{j}$	$\begin{pmatrix}1\\1\end{pmatrix}$	$\begin{pmatrix}-2\\2\end{pmatrix}$	$\vec{u}.\vec{v}=(1\times-2)+(1\times2)=0$
$\vec{h}=2\vec{i}+4\vec{j}$ and $\vec{k}=-2\vec{i}+2\overrightarrow{j}$	$\begin{pmatrix}2\\4\end{pmatrix}$	$\begin{pmatrix}-2\\2\end{pmatrix}$	$\vec{u}.\vec{v}=(2\times(-2))+(4\times2)=4$



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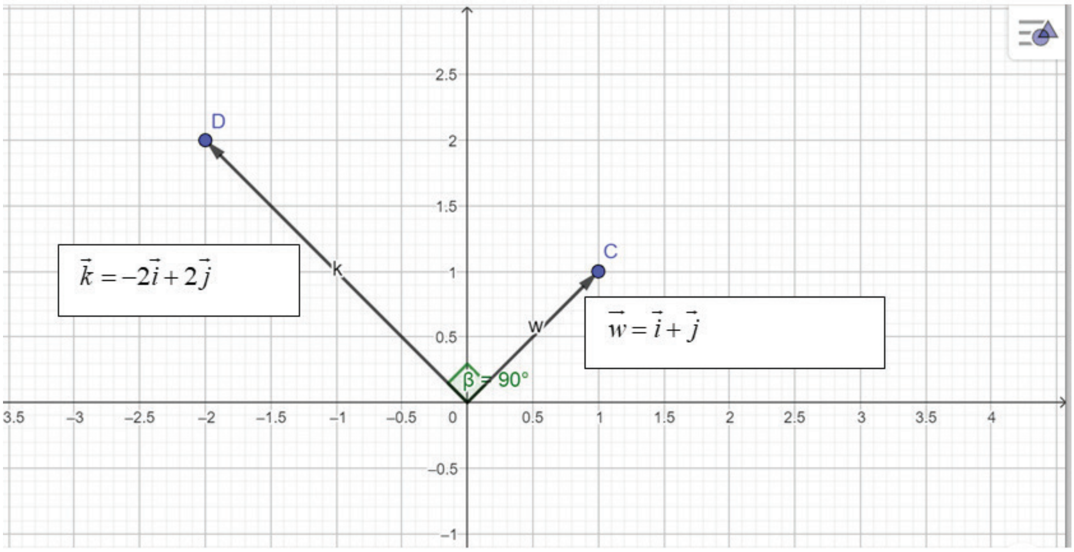
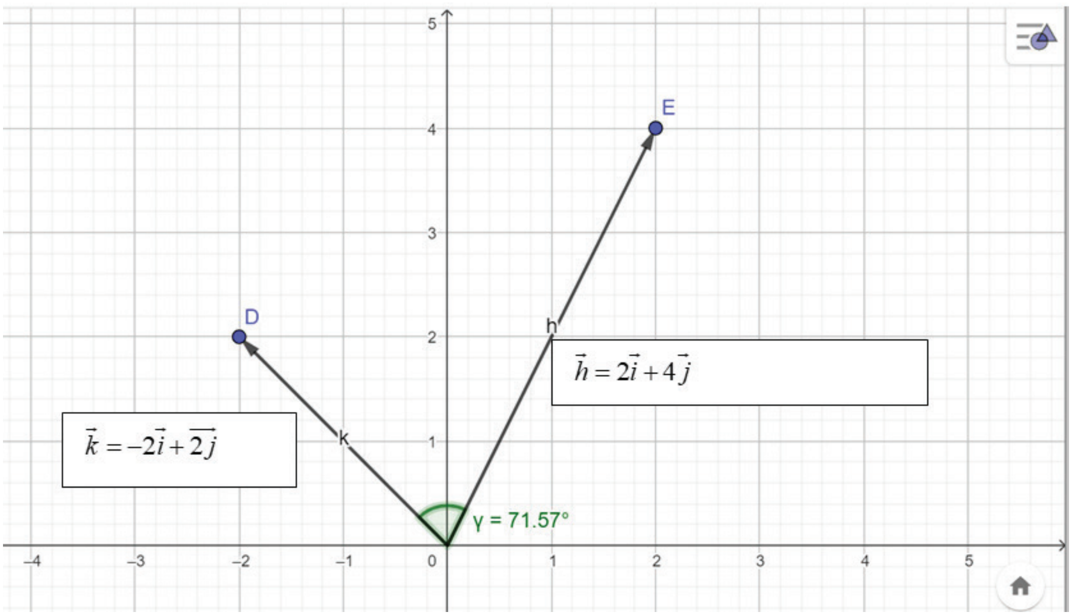




Vectors	**Length** ($\|\vec{u}\|$) **of** $\vec{u}$	**Length** ($\|\vec{v}\|$) **of** $\vec{v}$	**Angle** θ **between** $\vec{u}$ **and** $\vec{v}$	$Q = \|\vec{u}\|.\|\vec{v}\|$	$\vartheta = \cos^{-1}\left(\frac{\overrightarrow{u.v}}{\|\vec{u}\|.\|\vec{v}\|}\right)$
$\vec{u} = 4\vec{i} + 7\vec{j}$ and $\vec{v} = 2\vec{i} - 2\vec{k}$	8.06	2.83	105.26	22.8098	105.26

$\vec{w}=\vec{i}+\vec{j}$ and $\vec{k}=-2\vec{i}+2\vec{j}$	1.41	2.83	90	3.9903	90
$\vec{h}=2\vec{i}+4\vec{j}$ and $\vec{k}=-2\vec{i}+\overline{2\vec{j}}$	4.47	2.83	71.57	12.6501	71.57

Angle between vector $\vec{u}=4\vec{i}+7\vec{j}$ and vector $\vec{v}=2\vec{i}-2\vec{k}$ is 105.26°.

Angle between vector $\vec{u}=\vec{i}+\vec{j}$ and vector $\vec{k}=-2\vec{i}+2\vec{j}$ is 90^{0}.

Angle between vector $\vec{u}=2\vec{i}+4\vec{j}$ and vector $\vec{k}=-2\vec{i}+\overline{2\vec{j}}$ is 71.57^{0}.

f) Interpretation of results and Conclusion

The dot product is calculated using the components of vectors namely, $\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z$ where (a_x,a_y,a_z) and (b_x,b_y,b_z) are the components of vectors $\vec{a}$ and $\vec{b}$ respectively.

-In order to find an angle between two vectors $\vec{a}$ and $\vec{b}$ using dot product you have to remember that $\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos\theta$ which leads to $\theta=\cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}\right)$.

g) Information for Teachers:

Dot product measures how similar two vectors are.

In fact, dot product of two vectors makes sense to multiply their lengths together ***but only when they point in the same direction.*** So, we make one "point in the same direction" as the other by multiplying by $\cos\theta$.

h) Guidance on the evaluation:

Given that $\vec{a}=6\vec{i}+2\vec{j}-\vec{k}$ and $\vec{b}=5\vec{i}-8\vec{j}+2\vec{k}$, determine the dot product $\vec{a}\cdot\vec{b}$

. Using your observations from 4) determine angle θ between vector $\vec{a}$ and vector $\vec{b}$.

Solution:

$$\vec{a}\cdot\vec{b} = 6\times5 + 2\times(-8) + (-1\times2) = 30 - 16 - 2 = 12$$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta \Rightarrow \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

Or $|\vec{a}| = \sqrt{36+4+1} = \sqrt{41}$ and $|\vec{b}| = \sqrt{25+64+4} = \sqrt{93}$.

$$\cos\theta = \frac{12}{\sqrt{93}\sqrt{41}} = 0.19433357648 \quad \theta = 78.8^0.$$

PRACTICAL ACTIVITY 11: DISTRIBUTIVE PROPERTY OF VECTOR PRODUCT OVER ADDITION OF VECTORS

a) Rationale:

This activity will be done when evaluating properties of the vector product especially the distributive property over addition. This topic is learnt in Unit 6 of S5 Mathematics. One of the applications of cross product of vectors is to find the area of a parallelogram. In the other hand, vectors have many real-life applications, including situations involving force or velocity. For example, consider the forces acting on a boat crossing a river. The boat's motor generates a force in one direction, and the current of the river generates a force in another direction. Both forces are vectors. Some other examples include:

- Figuring out the direction of rain and holding your umbrella in that direction.
- To move an object in a particular direction, we will have to apply requisite force in that specific direction.

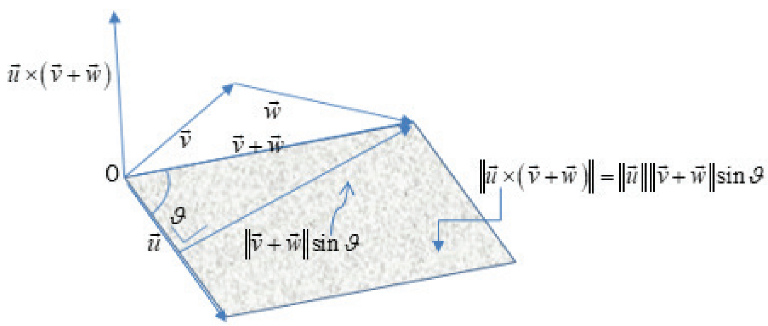
b) Objective:

To verify geometrically that if $\vec{u}$, $\vec{v}$ and $\vec{w}$ are vectors of the vector space $\left(\mathbb{R}^2,+,.\right)$, $\vec{u}\times\left(\vec{v}+\vec{w}\right)=\left(\vec{u}\times\vec{v}\right)+\left(\vec{u}\times\vec{w}\right)$ and apply the cross product of vectors in determining the area of a parallelogram. This is a concept-based experiment.

c) Material required:

Geometry box, cardboard, white paper, cutter, sketch pen, cello tape.

d) Illustration and setup:



e) Procedures:

Step 1: Fix a white paper on the cardboard.

Step 2: Draw a line segment $OA = 5\,cm$ to represent the vector $\vec{u}$.

Step 3: Draw another line segment $OB = 3\,cm$ at an angle 60^0 with OA and let $\overrightarrow{OB} = \vec{v}$.

Step 4: From the vector $\overrightarrow{OB}$ and $\overrightarrow{OA}$ complete the parallelogram $OAQB$ and mark the point Q

Step 5: Starting from B, draw $\text{BC} = 3\text{cm}$ at an angle of 30^0 with $\overrightarrow{BQ}$ and let $\overrightarrow{BC} = \vec{w}$.

Step 6: Draw a line segment BM from B and perpendicular to OA

Step 7: Draw a line segment CL from C perpendicular to OA

Step 8: Make a projection of line segment OB on A and call it AQ

Step 9: Find the sum of vectors $\vec{u}$ and $\vec{v}$ and let the new vector be called $\overrightarrow{OC}$

Step 10: From the vector $\overrightarrow{OC}$ and $\overrightarrow{OA}$ complete parallelogram $OAPC$

Step 11: Join the points P and Q to make a line segment PQ

f) Recording data:

From your work, complete the following gaps

1. $\|\vec{u}\| = 5\,cm = \cdots$
2. $\|\vec{v} + \vec{w}\| = \cdots$
3. $\overrightarrow{\text{OC}} = \overrightarrow{\text{OB}} + \overrightarrow{\text{BC}} = \cdots$ and let $\angle COA = \vartheta$
4. Area of parallelogram $OAPC$: $\|\vec{u} \times (\vec{v} + \vec{w})\| = \cdots$
5. Area of parallelogram $OAQB = \ldots$

6. Area of parallelogram $OAPC = \overline{OA} \cdot \overline{CL}$

$$= \cdots = \cdots$$

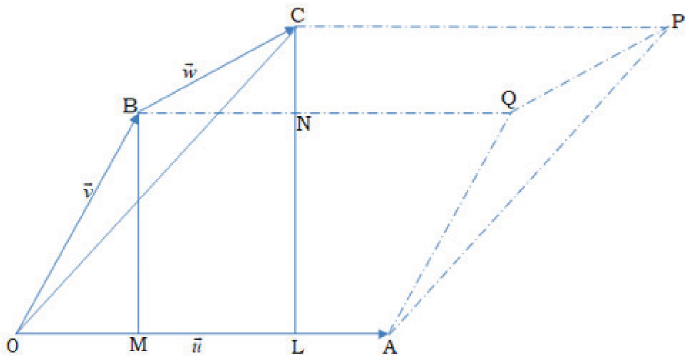
$$= \overline{OA} \cdot \overline{BM} + \overline{OA} \cdot \overline{NC}$$

$$= \text{Area of parallelogram} \cdots + \text{Area of parallelogram} \cdots$$

$$= \cdots + \cdots$$

7. How are vectors $\vec{u} \times (\vec{v} + \vec{w})$, $\vec{u} \times \vec{v}$ and $\vec{u} \times \vec{w}$.

Expected results:



- $\|\vec{u}\| = 5\,cm = \|\overrightarrow{OA}\|$
- $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \vec{v} + \vec{w}$ and let $\angle COA = \theta$
- $\|\overrightarrow{CL}\| = \|\overrightarrow{OC}\| \times \sin\theta$
- $\left\|\vec{u} \times \left(\vec{v} + \vec{w}\right)\right\| = \|\vec{u}\| \|\vec{v} + \vec{w}\| \sin\theta$ = area of parallelogram 0APC
- Area of parallelogram $OAPC = \overline{OA} \cdot \overline{CL} = \overline{OA} \times \left[\overline{LN} + \overline{NC}\right] = \overline{OA} \times \left[\overline{BM} + \overline{NC}\right]$

$$= \overline{OA} \cdot \overline{BM} + \overline{OA} \cdot \overline{NC}$$

$$= \text{Area of parallelogram 0AQB} + \text{Area of parallelogram BQPC}$$

$$= \|\vec{u} \times \vec{v}\| + \|\vec{u} \times \vec{w}\|$$

So, $\left\|\vec{u} \times \left(\vec{v} + \vec{w}\right)\right\| = \|\vec{u} \times \vec{v}\| + \|\vec{u} \times \vec{w}\|$

Direction of each of these vectors $\vec{u}\times(\vec{v}+\vec{w})$, $\vec{u}\times\vec{v}$ and $\vec{u}\times\vec{w}$ is perpendicular to the same plane. Therefore, $\vec{u}\times(\vec{v}+\vec{w})=(\vec{u}\times\vec{v})+(\vec{u}\times\vec{w})$

- Area of parallelogram $OAQB=\|\vec{u}\times\vec{v}\|$

g) Interpretation of results and Conclusion

The vector product of two vectors $\vec{u}$ and $\vec{v}$ is denoted by $\vec{u}\times\vec{v}$ and is orthogonal to two vectors. The same the vector $\vec{u}\times(\vec{v}+\vec{w})$ is orthogonal to both $\vec{u}$ and $\vec{v}+\vec{w}$. The magnitude $\|\vec{u}\times\vec{v}\|$ is the area of parallelogram formed by two vectors $\vec{u}$ and $\vec{v}$, therefore $\|\vec{u}\times(\vec{v}+\vec{w})\|$ is the area of the parallelogram formed by vectors $\vec{u}$ and $\vec{v}+\vec{w}$.

In general, $\vec{u}\times(\vec{v}+\vec{w})=(\vec{u}\times\vec{v})+(\vec{u}\times\vec{w})$

h) Information for teacher:

The vector product of two vectors is a vector whose direction is orthogonal to two vectors. When $\vec{u}=u_1\vec{e}_1+u_2\vec{e}_2+u_3\vec{e}_3$, $\vec{v}=v_1\vec{e}_1+v_2\vec{e}_2+v_3\vec{e}_3$ are vectors in space,

we have: $\vec{u}\times\vec{v}=\begin{vmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix}$, where $\vec{e}_{1,2,3}$ are unit direction vectors of $\mathbb{R}^3$.

Students can analytically show that: $\vec{u}\times(\vec{v}+\vec{w})=(\vec{u}\times\vec{v})+(\vec{u}\times\vec{w})$ (using coordinates)

$$\vec{u}\times(\vec{v}+\vec{w})=\begin{vmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\ u_1 & u_2 & u_3\\ v_1+w_1 & v_2+w_2 & v_3+w_3\end{vmatrix}$$

$$= \begin{vmatrix} \overrightarrow{e_1} & \overrightarrow{e_2} & \overrightarrow{e_3} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} + \begin{vmatrix} \overrightarrow{e_1} & \overrightarrow{e_2} & \overrightarrow{e_3} \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$= \left(\vec{u} \times \vec{v}\right) + \left(\vec{u} \times \vec{w}\right)$$

i) Guidance to evaluation

Provide different exercises related to determination of area of parallelogram in terms of cross product and remind them that from formula $A = B \cdot H$, the height (H) is given by magnitude of vector perpendicular to the base.

UNIT: 7 MATRICES AND DETERMINANT OF ORDER 3

PRACTICAL ACTIVITY 12: MULTIPLYING MATRICES

a) Rationale:

This activity is conducted when teaching topics related to the product of two or more matrices of any order to be learnt in Unit7. Matrices are used to encode data for security reasons; in economics very, large matrices are used for the optimization of problems, for example in making the best use of assets, whether labour or capital, in the manufacturing of a product and managing very large supply chains, etc. In Geology, matrices are used for making seismic surveys. They are used for plotting graphs, and statistics and also to do scientific studies and research in almost different fields. Matrices are also used to change and define the structure of buildings and in the gaming industry to alter the object, in 3D. Matrices have also come to have important applications in computer graphics, where they have been used to represent rotations and other transformations of images.

b) Objective:

Perform the product of two or more matrices and relate them to our daily life situation. This is a Concept based experiment.

c) List of required materials:

Manila paper, marker, scientific calculator.

d) Illustration of the activity

On the 20th day of Rwanda Premier League 2021/2022, the scores of the 3 first football teams among 16 teams, are listed below.

Team	Number of games		
	Won	Drawn	Lost
TEAM 1	13	5	2
Team 2	13	5	2
Team 3	10	6	4

- Considering that 3 points, 1 point and zero point are awarded to a win, a drawn and a lost game respectively, what are the total points of each team?
- Record the given data (score games and number of awarded points) as two matrices and the total points as the third matrix
- Using mathematical operations, establish a relation among those three matrices.

d) Procedures

Step 1: Determine the total points of Team 1

Step 2: Determine the total points of Team 2

Step 3: Determine the total points of Team 3

Step 4: Write the matrix representing the scoring results

Step 5: Write the matrix representing the score awards

Step 6: Write the matrix representing the total results.

Step 7: Find the relationship among the above three matrices. Do you think that matrices are used in real life? Explain your answer.

e) Recording of data

Record your data in the table by indicating names of teams, scoring results (combining number of won, drawn and lost games), score award and total points.

Team	Scoring results			Score award	Total points
	Won	Drawn	Lost		
Team 1					
Team 2					
Team 3					

- Express your data, in matrix form
- Establish a relation among the three matrices.

How many rows (items) for matrix indicating scoring results?

How many columns for the matrix indicating scoring results?

How many rows (items) for matrix indicating points to be awarded according to scoring results?

How many columns for this matrix?

How to proceed when finding the product of two matrices?

Expected answers

Team	Scoring results			Score award	Total points
	Won	Drawn	Lost		
Team 1	13	5	2	3	$13\times3+5\times1+2\times0=44$
Team 2	13	5	2	1	$13\times3+5\times1+2\times0=44$
Team 3	10	6	4	0	$10\times3+6\times1+4\times0=36$

- For the matrix indicating score results, we have 3 rows as there is a selection of first 3 teams from different teams playing Rwanda Premier League 2021/2022 and 3 columns (as possibilities of scoring results).
- For the matrix indicating points to be awarded according to score results, we have 3 rows and one column.
- The matrices found are the following:

Teams	Scoring results	Score award	Total point
Team 1 *Team* 2 *Team* 3	$\begin{pmatrix}13 & 5 & 2\\13 & 5 & 2\\10 & 6 & 4\end{pmatrix}$	$\times\begin{pmatrix}3\\1\\0\end{pmatrix}$	$=\begin{pmatrix}44\\44\\36\end{pmatrix}$

f) Interpretation of results and Conclusion

Matrices are used in real life situation. They can be added and multiplied depending on the values of a phenomenon that depends on different variables.

In a product matrix $A_{m\times n}\times B_{n\times p}=C_{m\times p}$, the element c_{ij} (ith row and jth column) of

the product matrix is obtained by multiplying every element in row **i** of matrix A by each element of column **j** of matrix B and then adding them together.

From recorded data, we deduce the following matrices.

Teams	Scoring results	Score award	Total point
Team 1 *Team* 2 *Team* 3	$\begin{pmatrix} 13 & 5 & 2 \\ 13 & 5 & 2 \\ 10 & 6 & 4 \end{pmatrix}$	$\times \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$	$= \begin{pmatrix} 44 \\ 44 \\ 36 \end{pmatrix}$

A matrix is a rectangular arrangement of numbers, expressions, symbols which are arranged in rows and columns. Two matrices A and B can be multiplied together if and only if the number of columns of A is equal to the number of rows of B : $A_{m\times n} \times B_{n\times p} = C_{m\times p}$

g) Guide on evaluation

Prepare the various exercises on multiplication of matrices of order 3 and check whether students are able to figure out the existence of product of matrices before calculations.

UNIT: 8

POINTS, STRAIGHT LINES, PLANES, AND SPHERE IN 3D

PRACTICAL ACTIVITY 13: REPRESENTING TWO POINTS IN 3D AND DETERMINING THE DISTANCE BETWEEN THEM

a) Rationale:

This activity is conducted when teaching topics related to the location of points and lines in 3D to be learnt in Unit8. The geometry of space is about how everything fits together. For example, if you have a packing box, it is the geometry of space that determines just how many items can fit inside the box. It is also the geometry of space that lets you fit more items in a box if they are placed in a certain way. Architects use geometry to aid them in designing buildings and cities that work; the construction of various buildings or monuments has a close relationship with geometry. Space geometry is employed in the field of astronomy to map the distances between stars & planets and between different planets. It also aids in the determination of a relationship between the movements of different bodies in the celestial environment. Apart from mapping distances between celestial bodies, space geometry also plays a vital role in surveying and navigation. In the case of surveying, measurement of the area of land is a result of the accurate determination of the shape of the land. Moreover, the use of coordinate geometry in the Global Positioning System (GPS) provides precise information about the location and time. GPS uses coordinates to calculate the distance between any two places. The coordinate geometry helps GPS to track transportation accidents and carry out rescue operations. The coordinate geometry also aids in enhancing flight security weather forecasting, earthquake monitoring, and environmental protection. Moreover, various facets of military operations are equipped with GPS.

b) Objective:

To locate the points or objects in 3D and determine the distance between two points/objects by measuring or using the distance formula. This is a Concept based experiment.

c) Required materials

Geometry set, squares paper, wire, ropes of 3 different colours, scotch, a pair of scissors, paper arrows, meter ruler, sticks of different known length and nails.

d) Procedures

Step1: Fix two perpendicular ropes using nail through O (the origin) to represent $x-axis$ and $y-axis$ respectively and make sure to label them using paper arrow and clearly show the positive and negative direction.

Step2: Using meter rule, graduate your axis and mark key points using a scotch or more nails.

Step4: Fix a graduated meter ruler (here some other materials like graduated stick or wire etc can be used) through O, in the vertical direction to represent the $z-axis$. Consider the figure below for reference.

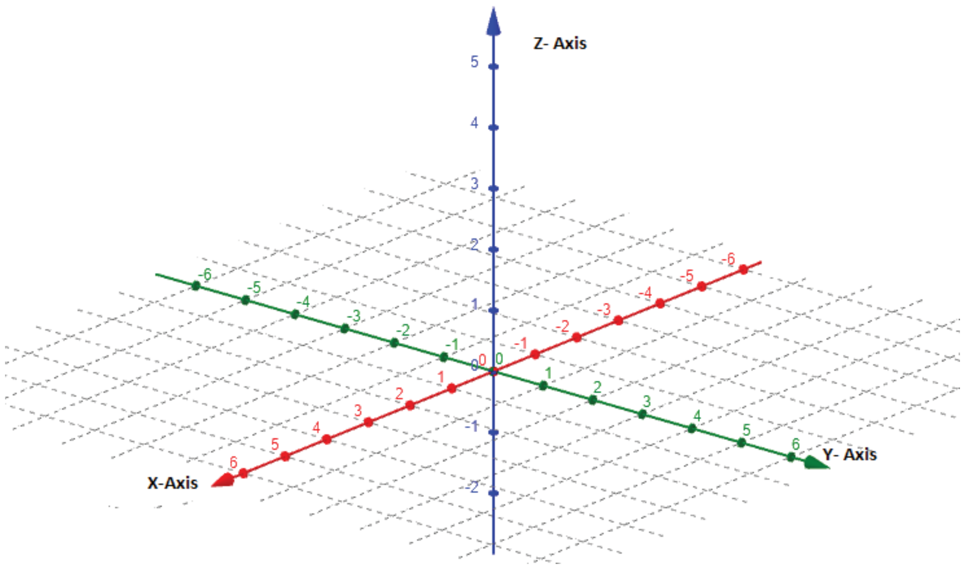


Figure: Representation of X, Y and Z axes in space

Step4: Starting from the origin O measure in negative direction 1 unit on x-axis and measure 2 units in positive direction on y-axis respectively. Mark the two new points with a_1 and b_1 respectively.

Step 5: Using a scotch, complete the rectangle from points O, a_1 and b_1 through point $A(\mathrm{a}_1, \mathrm{b}_1)$ in XY plane. What is the coordinate of the A?

Step6: Fix vertically a stick (wire/rope) of 2 units long at point $A(a_1, b_1)$

Step7: What is the coordinate of the point $P(a_1, \mathrm{b}_1, \mathrm{z}_1)$ represented by the top of the stick positioned at $A(a_1, b_1)$?

Step8: Repeat the process (iv) to (vii) for the measurements -2,2,3 and -3 units on x-axis, 1, 3, -3 and 4 units on y-axis and 1,2,3 and 4 units' on z-axis respectively to form points Q, R, S, T and fill the table below

Step9: Using a rope join the top end of the points whose distance need to be found and ruler measure its length to find the distance $d(P,Q)$, $d(Q,S)$, $d(P,T)$, $d(Q,T)$

How do you compare the result in (viii) and in (ix)? If any difference is found, to what do you attribute it?

Point name Q_i	Measure on x-axis (unit of measure) a_i	Measure on y-axis (unit of measure) b_i	Measure on z-axis (unit of measure) z_i	Coordinate of $A_i(a_i, b_i)$	Coordinate of $Q_i(a_i, b_i, z_i)$
P	-1	2	2		$(-1,2,2)$
Q					
R					
S					
T					

Sn	**Measured distance**	**Calculated distance** $d(X,Y)=\sqrt{(a_2-a_1)^2+(b_2-b_1)^2+(z_2-z_1)^2}$	**Difference**
$d(P,Q)$			
$d(Q,S)$			
$d(P,T)$			
$d(Q,T)$			

Step 6: Finding out the distance between points P and Q

- With a rope, join the top end stick representing the points P and Q. Make sure that the two stick are held vertical and the rope is straight.
- Use a meter rule to measure the length of the rope between P and Q to find the distance PQ. Do the same for other points as requested.

- Calculate the same distance using the formula $d(X,Y)=\sqrt{(a_2-a_1)^2+(b_2-b_1)^2+(z_2-z_1)^2}$ to Record the obtained result as the distance from point P to point Q denoted by $P(-1,2,2)$ and $Q(-2,1,1)$ respectively.

e) Results, their interpretation and Conclusion

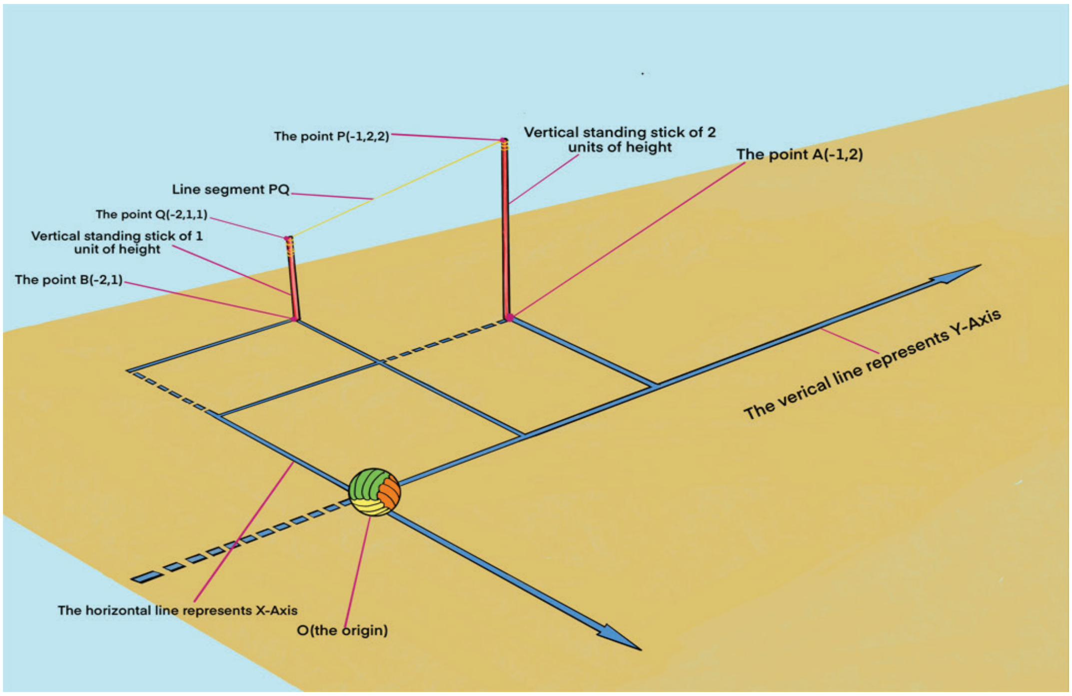


Figure: Location of two points in space and its distance.

The points to be presented in space are P and Q with their respective coordinates $P(-1,2,2)$ and $Q(-2,1,1)$. To present them is 3D, we first present their corresponding points in $xy-plane$ by omitting $z-component$; for our case the corresponding points are $(-1,2)$ and $(-2,1)$ respectively for the first two point.

From their corresponding points in 2D, we establish the height as the $z-component$. Finally, the top of the height indicates the location/coordinates of a given point. By actual measurements (after measuring the wire/ rope connecting P and Q using a scale) the distance $PQ = 1.3 unit (length)$.

By using distance formula,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(-2+1)^2 + (1-2)^2 + (1-2)^2} = \sqrt{3} = 1.333 \text{ unit of length}$$

Thus, the distance PQ, obtained by actual measurement is approximately the same as the distance obtained by using the distance formula. However, a slight difference may be recorded due error in measurement or wrong vertical position.

Thus, the distance between two points in space obtained on actual measurement and by using distance formula is approximately the same.

f) Information for teacher

To locate a point in a plane, two numbers are necessary. We know that any point in a plane can be represented as an ordered pair (a,b) of real numbers, where a is the $x-coordinate$ and b is the $y-coordinate$. For this reason, a plane is called two-dimensional. To locate a point in space, three numbers are required. We represent any point in space by an ordered triple (a,b,c) of real numbers where a is the $x-coordinate$, b is the $y-coordinate$ and c is the $z-coordinate$

g) Guidance on the evaluation

Provide different exercises related to representation of points in 3D such as the coordinates of the midpoint of the line segment. Let students do more practices related to this lesson.

PRACTICAL ACTIVITY 14: FINDING THE MID-POINT OF A LINE SEGMENT

a) Rationale:

In this experiment we are going to prove the midpoint of a line segment using local available materials. The midpoint will be proved using a line segment. This lesson is taught in unit 8 of Senior 5 Mathematics. There are actually quite a few real-life examples of a midpoint. When two end points of a distance are known, it is possible to be in a need of locating the object that is equidistant from both ends.

b) Objective of the experiment

Correctly locate the midpoint and apply it in solving a problem from real life situation. This is a concept-based experiment.

c) Materials

Meter ruler, sticks (a stick of $50cm$ of length at least and $1.5m$ at most), 10 ropes with length greater than $50cm$ (around $1.5m$), paper, manila paper, marker, a pair of scissors, etc.

d) Procedures & Steps of experiment

Step 1: Given two points $A(2,4,0)$ and $B(4,4,0)$ locate them in a Cartesian plane.

Step 2: Measure take a rope and put use it to measure the length of the line segment AB.

Step 3: Take the rope with length equal to distance AB and fold it in such a way that A falls on B, halving the length of AB.

Step 4: Mark the point of intersection of the line segment and the crease formed by folding the rope. This gives the mid-point E of the line segment AB.

What is a line segment?

What is a midpoint of a line segment?

Step 5: Measure the distance AE and put it from A and identify the mid-point $M(x', y', z')$.

Step 6: Find the coordinate of the point M and deduce the related formula.

Step 7: Do the same for 2 more pairs of points and compare the calculated coordinates and observed coordinates of the midpoints for each pair of points.

Step 8: Consider the general case of two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ and explain how to locate the coordinates of the midpoint of the line segment AB.

e) Data recording

	Coordinates of A	**Coordinates of B**	**Observed Coordinate of M**	**Calculated coordinates of M** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
First Line segment				
Second Line segment				
Third Line segment				

f) Interpretation of the results and conclusion

The Midpoint of a line segment AB where $A(x_1, y_1, z_1)$and $B(x_2, y_2, z_2)$ is $\frac{1}{2}(x_1 + x_2, y_1 + y_2, z_1 + z_2)$. By folding a paper, students will be able to find themselves the midpoint of a line segment and will be able to link it to real-life situations.

PRACTICAL ACTIVITY 15: POSITION AND RELATIONSHIP OF TWO LINES IN 3D

a) Rationale:

This experiment is done when teaching space geometry, the Unit 8 in Senior 5 mathematics. We live in a three-dimensional space. Numerous things in the real world are comparable to lines in geometry. The relative position of two straight lines can also be determined from a more geometric and not such an algebraic point of view from its respective director vectors. In actuality, geometry-based figures are used to construct many things around us. Students' awareness of the geometry that is all around you every day can be greatly increased by knowing what real-world things represent. In relation to the position of lines, this experiment will facilitate students to find themselves the linkage between what have been taught using formulae and daily life situations.

b) Objective:

To construct the lines passing through located points in space and then indicate their relative positions. This is an inquiry-based experiment

c) Materials

Drawing board, geometry box, squares paper, wire/ tree/ropes of 3 different colours, scotches, scissors, paper arrows, ruler meter, T-square.

d) Procedures

Step 1: Plot the following points in three dimensions Follow the procedures used in previous experiments and locate the following points in space using wires ropes:

$A(1,2,2), \mathrm{B}(0,3,7), \mathrm{C}(6,-3,1), D(8,-3,2), \mathrm{E}(2,1,3), \mathrm{F}(3,3,6), G(-1,2,1)$ and $H(0,4,4)$

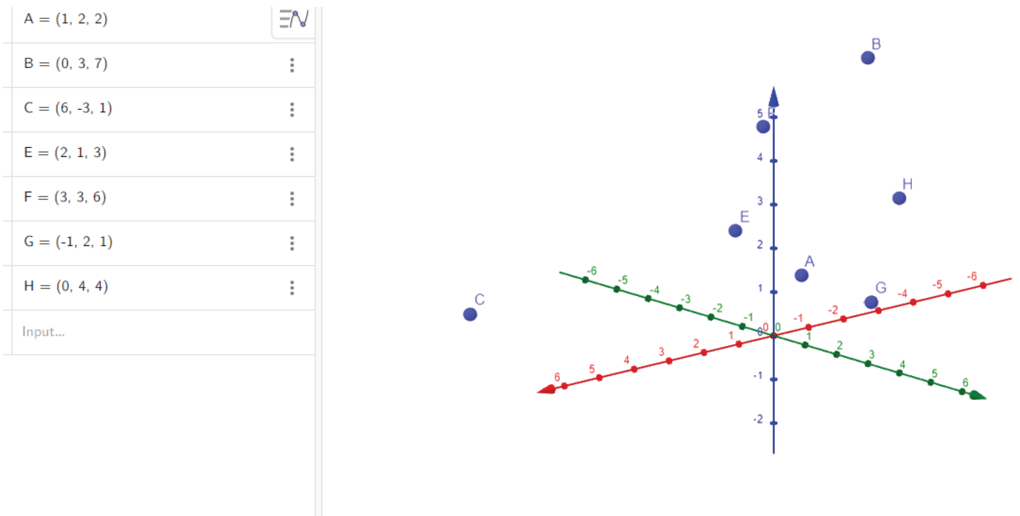


Figure 15.a: Representation of points in space

Now, the top of different trees/wires/ropes represent the points A, B, C, D, E, F, G and H.

Step 2: Join the top end of wires/trees/ropes representing the points E and F to make a line passing through E and F.

Step 3: Join the top end of wires/trees/ropes representing the points G and H to make a line passing through G and H.

- Using a flat object (paper or a book) check whether the lines EF and GH lie on the same plane or do not.
- Check whether these lines do intersect or do not.

Step 4: Join the top end of wires/trees/ropes representing the points C and D to make a line passing through C and D.

Step 5: Join the top end of wires/trees/ropes representing the points E and D to make a line passing through E and D.

- Using a flat object (paper or a book) check whether the lines CD and ED lie on the same plane or do not.
- Check whether these lines do intersect or do not.

Step 6: Join the top end of wires/trees/ropes representing the points A and B to make a line passing through A and B.

- Using a flat object (paper or a book) check whether the lines AB and CD lie on the same plane or do not.
- Check whether these lines do intersect or do not.

e) Recording of data

From your observations, record your results in the following table.

Item No	Pair of lines	Lie on the same plane	Parallel	Intersect	Neither parallel, nor intersect
1	EF, GH				
2	ED, CD				
3	AB, CD				
4					
5					

Expected answers

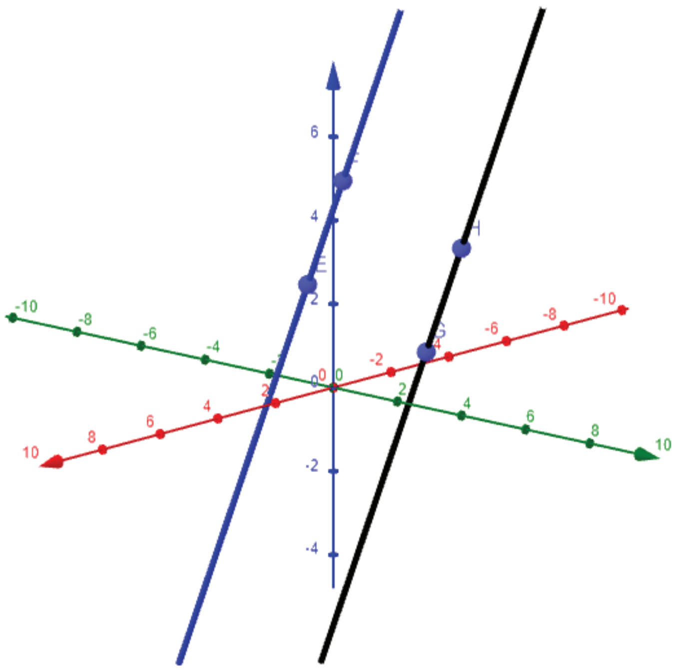


Figure 15.a. Two parallel lines (EF and GH)

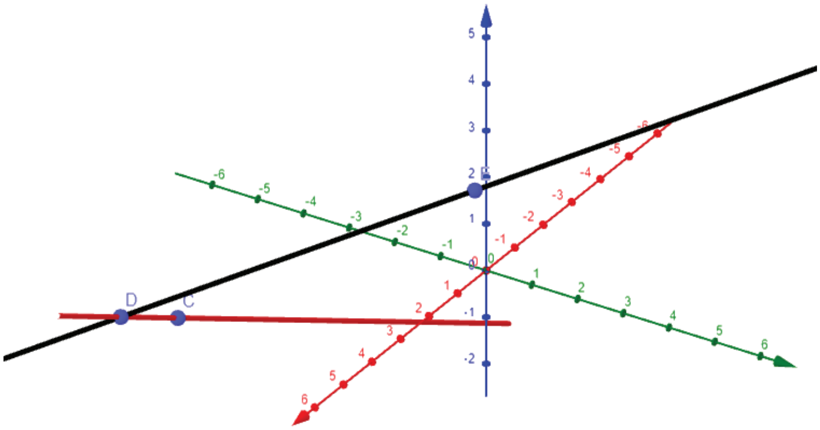


Figure 15.b. Intersecting lines (CD and ED)

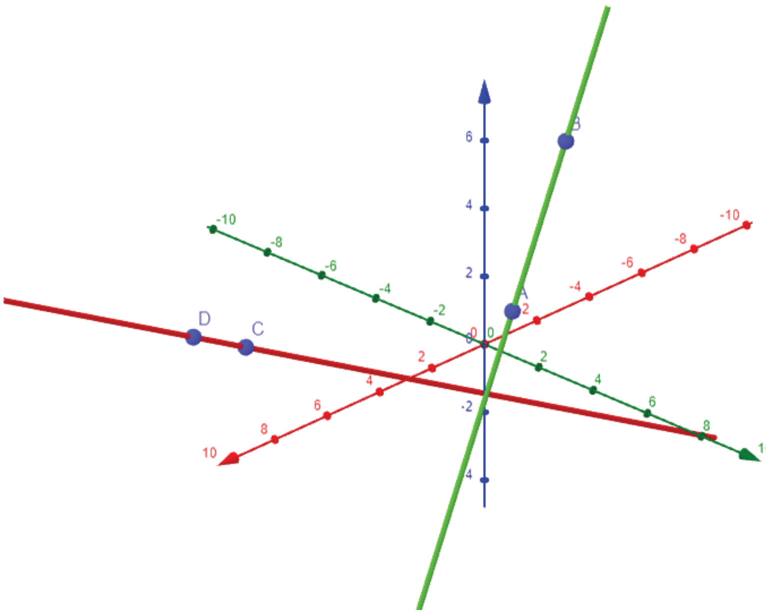


Figure 15.c. Skew lines (AB and CD)

The table below shows different positions of lines.

Item No	Pair of lines	Lie on the same plane	Parallel	Intersect	Neither parallel, nor intersect
1	EF, GH	Yes	Yes	No	No
2	ED, CD	Yes	No	Yes	No
3	AB, CD	No	No	No	Yes
4					
5					

f) Results, interpretation and Conclusion

- Generally, two or more lines that lie in the same plane and never intersect. Parallel lines will always have the same slope. Therefore, lines EF and GH are parallel.
- **Skew** lines are lines that are in different planes and never intersect. They are different from **parallel lines** because parallel lines lie in the same plane. Skew lines are a pair of lines that are non-intersecting, non-parallel, and non-coplanar. This implies that skew lines can never intersect and are not parallel to each other.

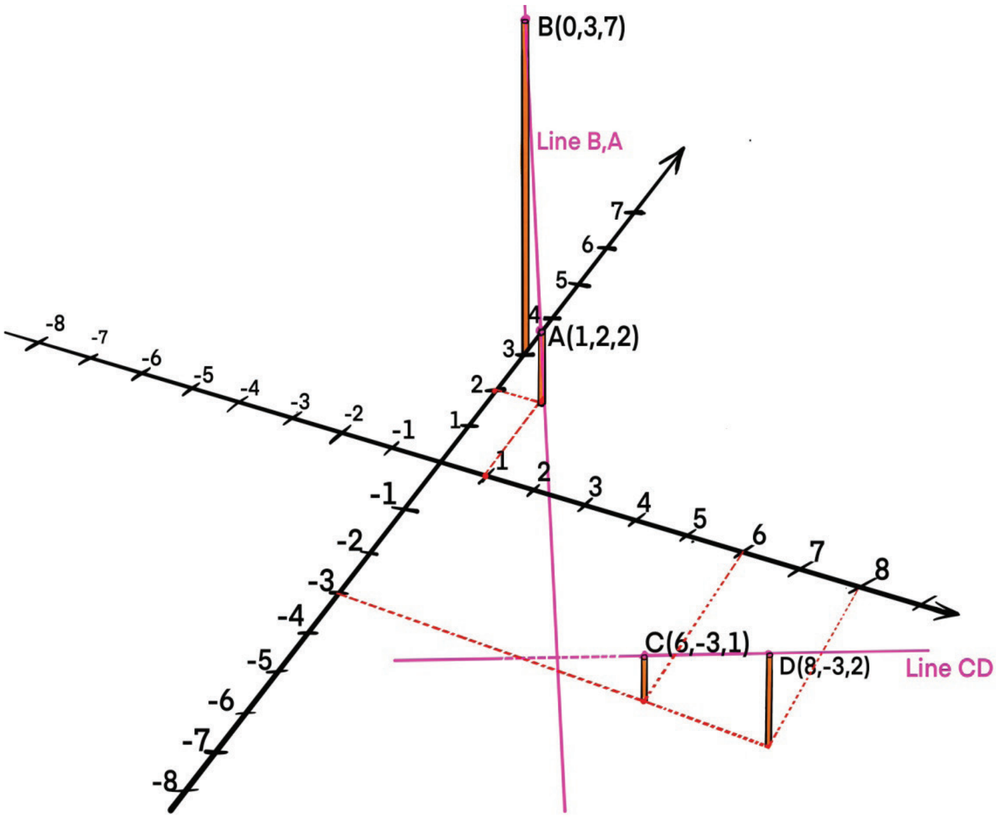


Figure 15.c'. Skew lines (AB and CD)

- Lines are said to be intersecting when they meet at any point. They are not parallel nor skew

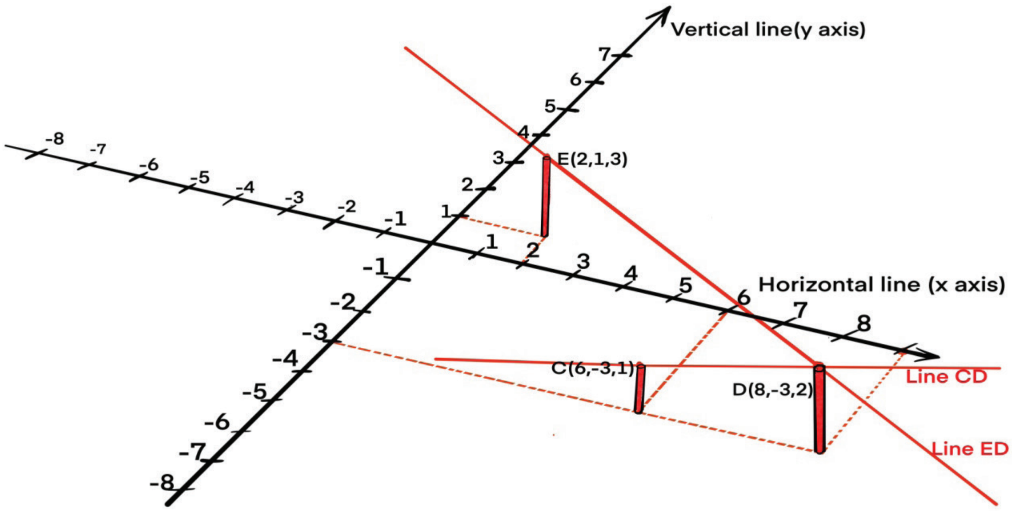


Figure 15.b'. Intersecting lines (CD and ED)

g) Information for teacher

To locate a point in space, two numbers are necessary. We know that any point in a plane can be represented as an ordered pair (a, b) of real numbers, where a is the x-coordinate and b is the y-coordinate. For this reason, a plane is called two-dimensional. To locate a point in space, three numbers are required. We represent any point in space by an ordered triple (a, b, c) of real numbers where a is the x-coordinate, b is the y-coordinate and c is the z- coordinate. In two dimensions, the two lines are either parallel or intersecting. In 3-dimensional space, there is one more possibility: two lines may be skew, which means that they do not intersect, but are not parallel.

h) Guidance on the evaluation

Provide different exercises related to representation of points in 3D such as the coordinates of the midpoint of the line segment. Let students do more practices related to this lesson.

PRACTICAL ACTIVITY 16: USE REAL OBJECTS TO PREDICT THE POSITION OF A POINT TO THE SPHERE (POINT-SPHERE)

a) Rationale:

This activity is done when teaching in Senior 5 Unit 8. This experiment comes to facilitate both teachers and students to understand the concept of the position of a point and a sphere in space. Normally, students failed to understand and perform this concept and used to develop negative attitude towards space geometry and this experiment was developed to help them to do hands on activities which will show the logic behind formulae used when learning this lesson. As the experiment will be done using local available materials, students may be motivated and this can also enhance their innovation, creativity and teamwork spirit. Students will be able to find the linkage between the theory taught in classroom and the real-life situations. From this experiment, teachers will be able to assess students' generic competences like critical thinking, communication, research and innovation and creativity.

b) Objective:

Use local available materials and find the position of a point and a sphere in space.

This is a Concept based experiment

c) Materials

- To conduct this experiment, the following materials will be used:
- Full orange to represent a sphere, Balloons / Tennis balls/ spheres (these spheres also may be printed out using 3D printers).
- A stick
- 2 Sheets of paper
- Wire/ rope of 2 different colours
- Vernier calliper
- Scotch
- Pens
- Cutter

d) Illustration

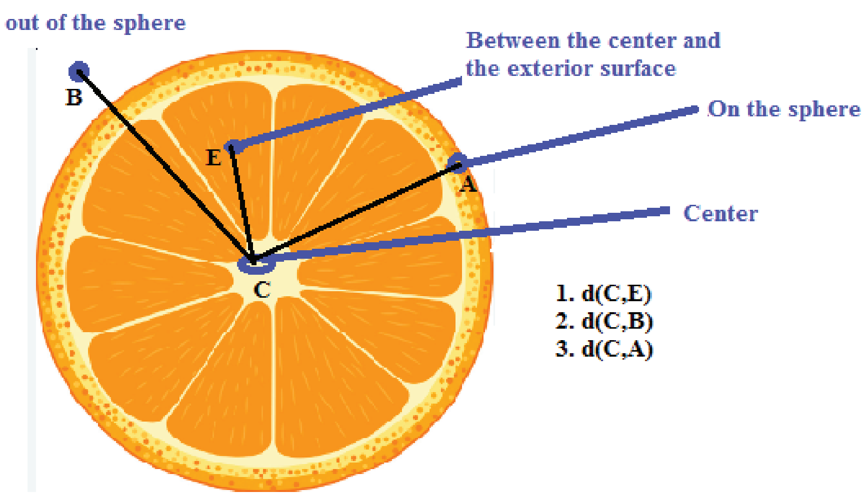
Given co-ordinates of the centre of a sphere (x_0, y_0, z_0) and its radius r our task is to check whether a point (x, y, z) lies inside, outside or on this sphere.

Figure 16.a. A Sphere

e) Procedures

Step 1: Take a full orange, observe it and tell the solid similar to it.

Step 2: Cut in halves, observe one half and try to think about its centre C, a point A of the sphere, A point E located in the sphere and a point B located out of the sphere.



Step 3: Observe the distance from each point and compare them with the radius r of the orange.

Is CA greater than the radius? What about CB? What about CE?

Step 4: Let us consider a sphere in the space; first choose. On the ground or a tiled floor, a fixed-point O (the origin).

Step 5: Draw two lines X'OX and Y'OY that represent X-axis and Y-axis respectively and choose unit scale.

Step 6: Fix a graded wire (here some other materials like graded tree, rope, etc can be used) through O, in the vertical direction to represent the z-axis.

Step 7: Plot these different points named A (1,3,2), B (-2,1,4), D (3,2,3), E (1,1,1) and C (2,0,1) in the space.

Step 8: Consider a sphere S of the centre C and the radius $r=3$ units of length.

It is compared as the orange that was observed above.

Step 9: By using a tape measure, find the distance between a given point and the centre of the sphere or join the top end for each point A, B, E, D and the top end of the centre C of that Sphere by using a wire/rope to make the corresponding line segment CA, CB, CD and CE.

Step 9: Measure those line segments and record their lengths using a meter ruler. The obtained results are distances between the given points and the centre of the sphere.

What are possible positions of a point and a sphere?

Step 10: Calculate the distance CA, CB, CD, and CE and compare them with the radius of the sphere. Try to discuss if each point A, B, D and E is inside, outside or at the surface of the given sphere of centre C and $r=3$.

f) Results, their interpretation and Conclusion

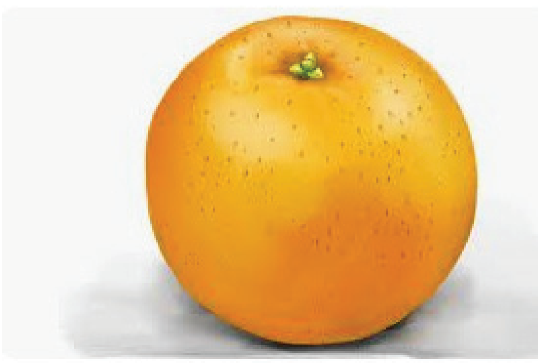


Figure 16.b. A full orange and half orange representing a full & half sphere.

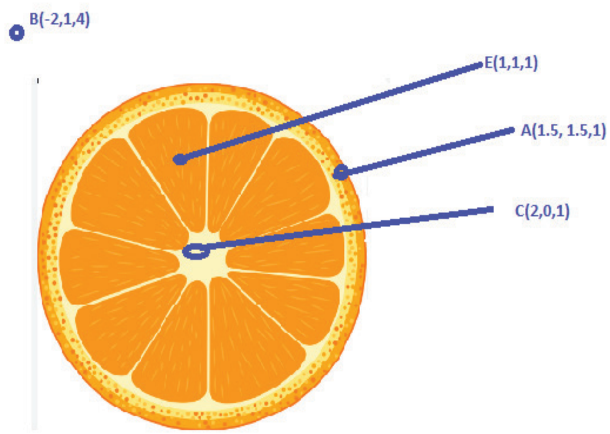


Figure 16.c. location of points in a sphere

- The half sphere above shows possible position of points and sphere
- A point may be on the sphere, a point may be inside the sphere or outside of a sphere.

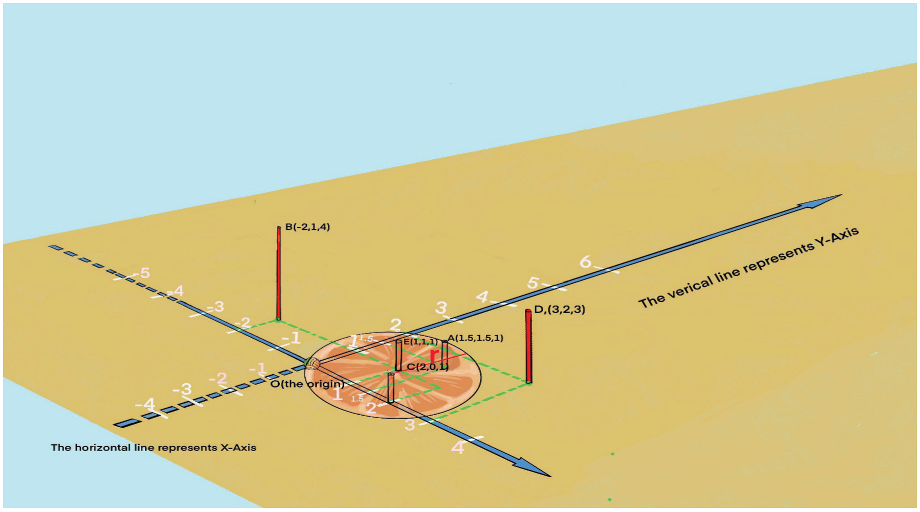


Figure 16.d presentation of points and sphere in space

In the space we can plot the given points and place the half orange on the point C by considering it as the centre of sphere and also consider the radius r = 3.

Considering a sphere with radius r = 3and centre $\boldsymbol{C}(2,0,1)$ and any points

$B(-2,1,4), D(3,2,3), E(1,1,1)$

The distance from the centre of the sphere with the centre $\boldsymbol{C}(\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{z}_0)$ and the given point $P(a,b,c)$ is $d = \sqrt{(a-x_0)^2 + (b-y_0)^2 + (c-z_0)^2}$.

- If $d(C,P) < r$ where P is a point and C is the centre of a sphere, the point lies inside the sphere
- If $d(C,P) = r$, the point lies on the sphere
- If $d(C,P) > r$, the point lies outside of the sphere.

In all cases, $d(C,P)$ is the distance between point P and the centre C of the sphere S.

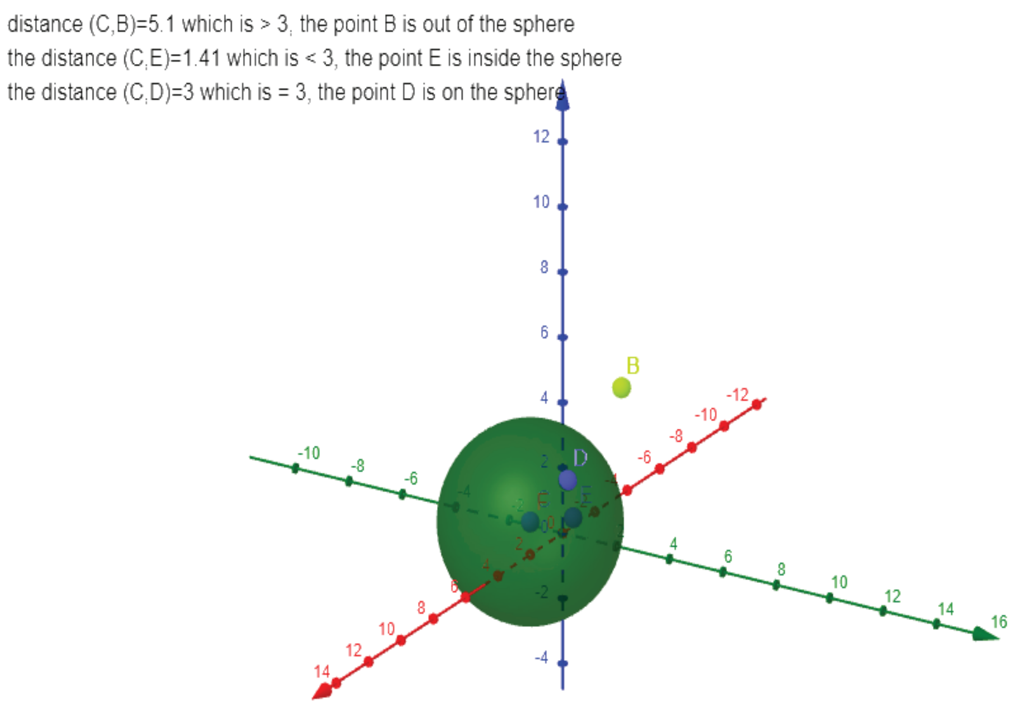


Figure 16.b: Graphical representation of sphere and points in space

g) Information for Teachers

When performing this experiment, let students cooperate and raise questions as it will help them to develop their critical thinking. The experiment itself can help students to master the content related to previous concept of space geometry. It is advised to make different groups and provide different exercises.

h) Guidance on the evaluation

The teacher will propose to the students' similar exercises where they will determine the location of a point relating to the given sphere.

UNIT: 9 BIVARIATE STATISTICS

PRACTICAL ACTIVITY 17: PLOTTING A SCATTER DIAGRAM, REPRESENTING GIVEN DATA AND THE REGRESSION LINE

a) Rationale:

This activity will be done when teaching the concept or topic related to bivariate data to be learnt in Unit 9 of senior 5 Mathematics. The Scatter diagrams (also called scatter plots) are used to show the possible relationship between two variables that both relate to the same "event". Scatter diagrams have a specific purpose to show how much one variable affects another. Therefore, this experiment will help students to link the content to the real-life situation. Doing more practices related to this content will help Students to develop some competences such as critical thinking, research, communication skills and this will help students to know the reason behind the theory taught in classroom.

b) Objective:

Present a regression line and use it to predict about value of a variable.

This is a concept-based experiment.

c) Materials:

Graph paper, ruler, scientific calculator.

d) Illustration of the activity:

Let us consider the following bivariate data for this situation. A farmer sold hens in different periods and received interest in thousands of Rwandan francs. The following table shows the number x of hens sold and the corresponding interest y in thousands of Rwandan francs earned in different periods:

x	3	5	6	8	9	11
y	2	3	4	6	5	8

e) Procedures

Step 1: On a paper choose a fixed-point O (the origin)

Step 2: Draw two lines $X'OX$ and $Y'OY$ representing $x-axis$ and $y-axis$ respectively and choose unit scale.

Step 3: Observe the given data and present them in a Cartesian plan.

Step 4: Calculate the mean for the two variables ($\bar{x}$ and $\bar{y}$).

Step 5: Complete the table of data recording given below.

Step 6: Write down the equation of the regression line

Step 7: Draw the regression line $y(x)$

f) Data recording

Complete this table

							Some
x	3	5	6	8	9	11	$\sum_{i=1}^{6} x_i =$
y	2	3	4	6	5	8	$\sum_{i=1}^{6} y_i =$
$x-\bar{x}$							
$y-\bar{y}$							
$(x-\bar{x})^2$							$\sum_{i=1}^{6} (x_i-\bar{x})^2 =$
$(y-\bar{y})^2$							$\sum_{i=1}^{6} (y_i-\bar{y})^2 =$

$\left(x-\bar{x}\right)\left(y-\bar{y}\right)$							$\sum_{i=1}^{6}\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)=$

Expected answer

The data recorded

							Some
x	3	5	6	8	9	11	$\sum_{i=1}^{6} x_i = 42$
y	2	3	4	6	5	8	$\sum_{i=1}^{6} y_i = 28$
$x-\bar{x}$	-4	-2	-1	1	2	4	
$y-\bar{y}$	-2.6	-1.6	-0.6	1.4	0.4	3.4	
$\left(x-\bar{x}\right)^2$	16	4	1	1	4	16	$\sum_{i=1}^{6}\left(x_i-\bar{x}\right)^2 = 42$
$\left(y-\bar{y}\right)^2$	6.76	2.56	0.36	1.96	0.16	11.56	$\sum_{i=1}^{6}\left(y_i-\bar{y}\right)^2 = 23.36$
$\left(x-\bar{x}\right)\left(y-\bar{y}\right)$	10.4	3.2	0.6	1.4	0.8	13.6	$\sum_{i=1}^{6}\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)=30$

The equation of the regression line of y on x

$$L_{y/x} \equiv y = \frac{5}{7}x - 0.3$$
.

x	4	7	16
y	2.5	4.7	11.1

By joining the obtained points (number of hens, interest), we can get the following figure: Since the data appears to be linearly related, we can find a straight-line model that fits the data better than all other possible straight-line models.

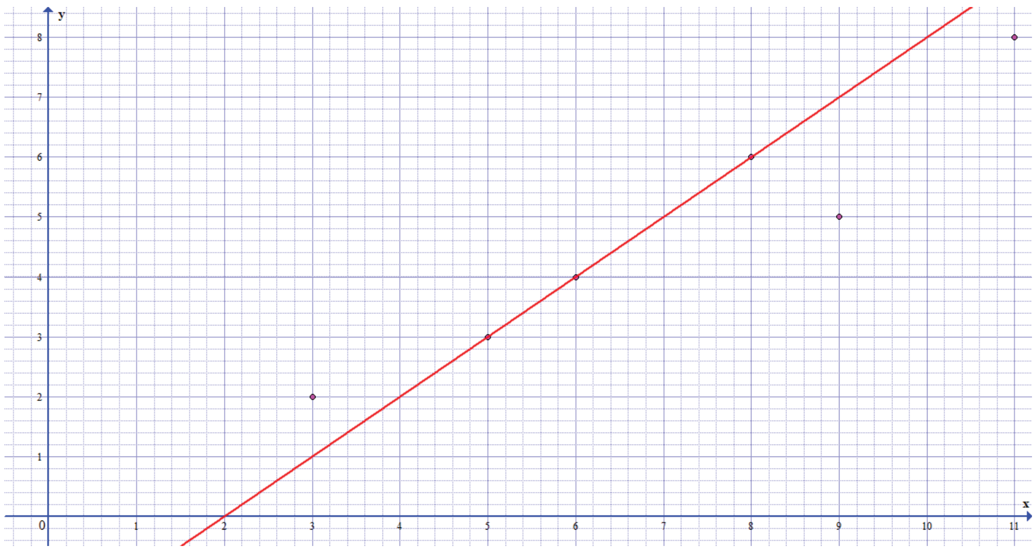


Figure 16: *Scatter diagram*

On the other hand, when we have the interest y, we can predict the number of hens sold using the equation of the regression line of x on y. This is $L_{x/y} \equiv x-\bar{x}=\frac{Cov(x,y)}{\sigma_y^2}(y-\bar{y})$.

Hence, $L_{x/y} \equiv x = 1.3y+1$.

Using this equation, we can predict the number of hens to be sold if the farmer received the interest of 7000Frw, 9000Frw or 16000Frw

y	7	9	16
x	10.1	12.7	21.8

Note: As x represents the number of hens, it should be an integer.

Therefore, the interest of 7000Frw, 9000Frw and 16000Frw correspond to 10 hens, 13 hens and 22 hens respectively.

g) Interpretation of results and conclusion

The equation of the regression line of y on x:

$$L_{y/x} \equiv y-\bar{y}=\frac{Cov(x,y)}{\sigma_x^2}(x-\bar{x})$$

$$\bar{x}=\frac{42}{6}=7,\ \bar{y}=\frac{28}{6}=4.7$$

$$Cov(x,y)=\frac{1}{n}\sum_{i=1}^{6}(x_i-\bar{x})(y_i-\bar{y})=\frac{30}{6}=5$$

$$\sigma_x^2 = \frac{1}{n}\sum_{i=1}^{6}\left(x_i - \bar{x}\right) = \frac{42}{6} = 7 \text{ and } \sigma_y^2 = \frac{1}{n}\sum_{i=1}^{6}\left(y_i - \bar{y}\right) = \frac{23.36}{6} = 3.89$$

The equation of the regression line of x on y:

$$L_{y/x} \equiv y - 4.7 = \frac{5}{7}(x - 7)$$

Finally, the line of y on x is $L_{y/x} \equiv y = \frac{5}{7}x - 0.3$

$$L_{x/y} \equiv x - \bar{x} = \frac{Cov(x,y)}{\sigma_y^2}\left(y - \bar{y}\right)$$

$$L_{x/y} \equiv x - 7 = \frac{5}{3.89}(y - 4.7)$$

Finally, the line of x on y is $L_{x/y} \equiv x = 1.3y + 1$

Two variables have a positive association when above-average values of one tend to accompany above-average values of the other, and when below-average values also tend to occur together. Two variables have a negative association when above-average values of one tend to accompany below-average values of the other.

f) Information for teacher

The regression lines help to make prediction of y for any given x and vice versa.

The regression line y on x is given by: $L_{y/x} \equiv y - \bar{y} = \dfrac{Cov(x,y)}{\sigma_x^2}\left(x - \bar{x}\right)$

The regression line x on y is given by: $L_{x/y} \equiv x - \bar{x} = \dfrac{Cov(x,y)}{\sigma_y^2}\left(y - \bar{y}\right)$

g) Guidance on evaluation

Provide to students more related exercises related to real life situations. Let students perform in groups.

UNIT: 10

CONDITIONAL PROBABILITY AND BAYES THEOREM

PRACTICAL ACTIVITY 18: EXPLAINING AND CALCULATING THE CONDITIONAL PROBABILITY

a) Rationale:

This activity will be conducted when teaching the conditional probability which is taught in Unit 10 of Senior 5 Mathematics. Generally probability is a branch of mathematics that deals with the occurrence of a random event. Conditional probability is defined as the likelihood of an event or outcome occurring, based on the occurrence of a previous event or outcome. It is the probability that requires additional conditions for being evaluated , this involves independent events and the Bayes theorem. Conditional probability is well connected to the real life situations. Its applications are like in sport related games of chance, card games and other games of chance. Note that insurance, traffic signals, medical diagnosis, election results, use also probability rules. The better performance of this experiment will enhance students' probability conceptual understanding, and this will help them to know the reasons behind formulae used in conditional probability. This is a concept based experiment.

b) Objective:

To explain the conditional probability of the event A , when event B has already occurred, through an example of throwing a pair of dice.

c) Material:

Two dice of different colours (orange and blue), piece of plywood , white paper, flipchart or manilla paper ,pen/pencil .

d) Illustration and setup:

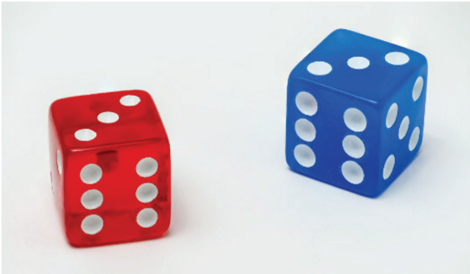


Figure 18 a: 2 dice

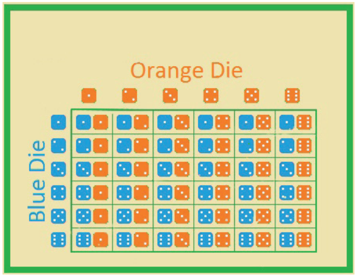


Figure 18.b: Possible outcomes

e) Procedures:

Step 1: Take a plywood of a convenient size

Step 2: On a plywood paste a white paper/ flipchart/ manilla paper

Step 4: Throw two dice simultaneously and record results in the table of data recording below.

Step 5: On a white paper/ flipchart/ manilla paper, determine all possible outcomes obtained by throwing two dice , each has faces labeled 1, 2, 3, 4, 5, 6.

Step 6: Complete the table by **(** Blue die result, Orange die result **)** with all possible obtained results for two dice thrown.

a) Write down the sample space of the experiment.

b) Write down outcomes for the number 3 appears on at least one of dice.

c) Write down the outcomes for the number 3 appears on both of dices

d) Given he events A and B such that A is " a number 3 appears on both the dice" and B is "the event "3 has appeared on at least one of the dice"; Find the probability of an event A if an event B has already occurred. Does the event B depend on the occurrence of the event A? If yes, note it $\Pr(A\setminus B)$ and discuss how you can call such types of probability?

The following sentences will help you to find answers:

Total number of outcomes:

- Outcomes with 3 appears on one of dice : $(3,\ldots),\ldots(\cdots,3)$**,...**

Number of outcomes with 3 appears on one of dices :...

- **Outcomes with 3 appears on both of dice :** $(3,3)$
- Number of outcomes with 3 on both of dices :...
- **Number of outcomes with 3 appears on atleast one of dice:...**

$\Pr(A) = \cdots$

$\Pr(B) = \cdots$

$\Pr(A \cap B) = \cdots$

f) Recording of data:

Orange Die		1	2	3	4	5	6
Blue Die	1						
	2						
	3						
	4						
	5						
	6						

Expected answers"

Total outcomes:

Table 18. a. Possible outcomes

Orange Die		1	2	3	4	5	6
Blue Die	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Total outcomes:

Here the totol number of outcomes means the sum of all possible outcomes

S ={(1,1),(2,2),....,....,(6,6)}= $6^2 = 6\times 6 = 36$

If two different dice are thrown simultaneously (being number 1, 2, 3, 4, 5 and 6 on their faces). We know that in a single thrown of two different dice, the number of total possible outcomes is equal to the total number of the first die (6) multiplied by the total numbers of the second die (6), which is 36. So, the total of possible outcomes when two dice are thrown together is 36.

Outcomes with 3 on one of dices :

$(3,1)_,(3,2)_,(3,4)_,(3,5)_,(3,6)$

$(1,3)_,(2,3)_,(4,3)_,(5,3)_,(6,3)$

Number of outcomes with 3 appears on one of dices : **10**. Here we only consider pair with 3 and then count the total number of those pairs.

Outcomes with 3 appears on both of dice: (3,3)

Number of outcomes with 3 appears on both of dice : 1. Here we consider a pair that contains 3 on the two sides.

Number of outcomes with 3 appears on atleast one of dice

This will Outcomes with 3 on one of dices and Outcomes with 3 appears on both of dice. It means that number of outcomes with 3 appears on atleast one of dice:11.

Note that:

$$\Pr(A) = \frac{1}{36}$$

$$\Pr(B) = \frac{11}{36}$$

$$\Pr(A \cap B) = \frac{1}{36}$$

$$\Pr(A \setminus B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{1}{11}$$

h) Interpretation of results and Conclusion

The existence of the event A is based on the existence of B that already occurred. The the conditional probability of an event A if an event B has already occurred , is given by:

$$\mathbf{Pr}(\mathbf{A}\setminus\mathbf{B})=\frac{\mathbf{Pr}(\mathbf{A}\cap\mathbf{B})}{\mathbf{Pr}(\mathbf{B})}$$

The the conditional probability of an event B if an event A has already occurred , is given by:

$$\mathbf{Pr}(\mathbf{B}\setminus\mathbf{A})=\frac{\mathbf{Pr}(\mathbf{A}\cap\mathbf{B})}{\mathbf{Pr}(\mathbf{A})}$$

Information for teacher:

The conditional probability is the basic of the Bayes theorem such that:

$$\mathbf{Pr}(\mathbf{A}\setminus\mathbf{B})=\frac{\mathbf{Pr}(\mathbf{A}\cap\mathbf{B})}{\mathbf{Pr}(\mathbf{B})} \quad \text{and} \quad \mathbf{Pr}(\mathbf{B}\setminus\mathbf{A})=\frac{\mathbf{Pr}(\mathbf{A}\cap\mathbf{B})}{\mathbf{Pr}(\mathbf{A})}$$

It implies that we have:

$$\Pr(A\setminus B)\Pr(B)=\Pr(B\setminus A)\Pr(A)$$

Bayes formula:

$$\Pr(B\setminus A)=\frac{\Pr(A\setminus B)\Pr(B)}{\Pr(A)}\xrightarrow{\textit{Extended to}}\Pr(B_r\setminus A)=\frac{\Pr(A\setminus B_r)\Pr(B_r)}{\sum_{i=1}^{n}\Pr(A\setminus B_i)\Pr(B_i)}$$

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Annex 1: Name of commonly hazard symbols useful in the laboratory

S/N	Name	Hazard Symbol	Explanation
1	Flammable and combustible		The flammable and combustible symbol signifies substances that will ignite and continue to burn in air. Substances in this category may be gases, aerosols, liquids, or solids, and include many solvents and cleaning materials that are commonly used in the laboratory.
2	Oxidizing agents		The symbol for oxidizing materials indicates the presence of chemicals that readily give off oxygen or other oxidizing substances. Oxidizing materials may intensify fires and cause explosions, and also may be toxic or corrosive. Some common oxidizing liquids and solids found in laboratories are bromine, chlorates, nitrates, perchloric acid, and peroxides.

3	Toxic		A substance known to pose that is classified as posing skin corrosion or irritation; serious eye damage or eye irritation; respiratory or skin sensitization; germ cell mutagenicity; carcinogenicity; reproductive toxicity, and other toxicity is classified as hazardous or toxic substance. These substances can cause death or damage to health by inhalation, ingestion or skin absorption. Example: acid
4	Irritants		Irritants are substances that cause reversible inflammatory effects on living tissue at the site of contact.

5	Magnetic Field		Certain pieces of laboratory equipment generate strong magnetic fields. The strong magnetic field sign alerts lab members to the dangers that this type of equipment can pose. The risks are especially imminent for people wearing pacemakers and implants, which will tend to align themselves with the magnetic field lines, as will watches, clipboards, and certain tools. Magnetic fields result from the flow of current through wires or electrical devices. Examples of sources: machines, electrical wiring (such as power lines)

6	Exit		It is good to know where all of the exits are located, especially when working in a laboratory environment where you may need to get out quickly. Labs are required to mark exits routes from the area with clearly identifiable signs.
7	Fire extinguisher		Fires can happen anywhere, but lab fires can be even more dangerous due to Bunsen burners, flammable liquids, research documents, laptops, and lab equipment that might be present at any given time. It is essential that the occupants of a laboratory are fully aware of the risks and the appropriate extinguishing media. A fire extinguisher safety sign indicates the exact location of a lab's fire extinguisher.

8	Electrical hazard		The electrical hazard safety symbol, which typically includes a frayed wire and a hand with a lightning bolt across it, indicates any electrical hazards in the lab. If an electrical hazard is suspected, the device in question should be disconnected immediately and the cause determined by a qualified technician. Equipment should always be turned off and unplugged when any work is being done on it.