MATHEMATICS FOR TTCs

TUTOR'S GUIDE

YEAR

2

OPTION:

LANGUAGE EDUCATION(LE)

FOREWORD

Dear Tutor,

Rwanda Education Board is honoured to present the tutor's guide for Year two Mathematics in the option of Language Education (LE). This book serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of the mathematics content. The Rwandan educational philosophy is to ensure that learners achieve full potential at every level of education which will prepare them to be well integrated in society and exploit employment opportunities.

Specifically, TTC curriculum was reviewed to train quality teachers who will confidently and efficiently implement the Competence Based Curriculum in pre-primary and primary education. The rationale of the changes is to ensure that TTC leavers are qualified for job opportunities and further studies in Higher Education in different programs under education career advancement.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate their learning process. Many factors influence what they learn, how well they learn and the competences they acquire. Those factors include the relevance of the specific content, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials.

The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market has necessitated the shift to a competence-based curriculum. After a successful shift from knowledge to a competence-based curriculum in general education, TTC textbooks also were elaborated to align them to the new curriculum.

The book provides active teaching and learning techniques that engage student teachers to develop competences. In view of this, your role is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize group discussions for students considering the importance of social constructivism suggesting that learning occurs more effectively when the students works collaboratively with more knowledgeable and experienced people.
- Engage students through active learning methods such as inquiry methods, group discussions, research, investigative activities and

group and individual work activities.

- Provide supervised opportunities for students to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation.
- Support and facilitate the learning process by valuing students' contributions in the class activities.
- Guide students towards the harmonization of their findings.
- Encourage individual, peer and group evaluation of the work done in the classroom and use appropriate competence-based assessment approaches and methods.

To facilitate you in your teaching activities, the content of this book is self explanatory so that you can easily use it. It is divided in 3 parts:

The part I explain the structure of this book and give you the methodological guidance;

The part II gives a sample lesson plan;

The part III details the teaching guidance for each concept given in the student book.

Even though this Tutor's guide contains the guidance on solutions for all activities given in the learner's book, you are requested to work through each question before judging student's findings.

I wish to sincerely express my appreciation to the people who contributed towards the development of this book, particularly, REB staff, UR-CE Lecturers, TTC Tutors, Teachers from general education and experts from Local and international Organizations for their technical support. A word of gratitude goes also to the Head Teachers and TTCs principals who availed their staff for various activities.

Dr. NDAYAMBAJE Irénée

Director General, REB

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I wish to express my appreciation to the people who played a major role in the development of this tutor`s guide for Year two Mathematics in the option of Language Education (LE). It would not have been successful without active participation of different education stakeholders.

I owe gratitude to different universities and schools in Rwanda that allowed their staff to work with REB in the in-house textbooks production initiative.

I wish to extend my sincere gratitude to lecturers and teachers whose efforts during writing exercise of this tutor`s guide was very much valuable.

Finally, my word of gratitude goes to the Rwanda Education Board staffs who were involved in the whole process of in-house textbook writing.

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PART I. GENERAL INTRODUCTION

1.1 The structure of the guide

The tutor's guide of Mathematics is composed of three parts:

The Part I concerns general introduction that discusses methodological guidance on how best to teach and learn Mathematics, developing competences in teaching and learning, addressing cross-cutting issues in teaching and learning and Guidance on assessment.

Part II presents a sample lesson plan. This lesson plan serves to guide the tutor on how to prepare a lesson in Mathematics.

The Part III is about the structure of a unit and the structure of a lesson. This includes information related to the different components of the unit and these components are the same for all units. This part provides information and guidelines on how to facilitate student teachers while working on learning activities. More other, all application activities from the textbook have answers in this part.

1.2 Methodological guidance

1.2.1 Developing competences

Since 2015 Rwanda shifted from a knowledge based to a competency-based curriculum for pre-primary, primary, secondary education and recently the TTC curriculum. This called for changing the way of learning by shifting from teacher centred to a learner centred approach. Teachers are not only responsible for knowledge transfer but also for fostering learners' learning achievement and creating safe and supportive learning environment. It implies also that learners have to demonstrate what they are able to transfer the acquired knowledge, skills, values and attitude to new situations.

The competence-based curriculum employs an approach of teaching and learning based on discrete skills rather than dwelling on only knowledge or the cognitive domain of learning. It focuses on what learner can do rather than what learner knows. Learners develop competences through subject unit with specific learning objectives broken down into knowledge, skills and attitudes through learning activities.

In addition to the competences related to Mathematics, student teachers also develop generic competences which should promote the development of the higher order thinking skills and professional skills in Mathematics teaching. Generic competences are developed throughout all units of Mathematics as follows:

Generic competences	**Ways of developing generic competences**
Critical thinking	All activities that require learners to calculate, convert, interpret, analyse, compare and contrast, etc have a common factor of developing critical thinking into learners
Creativity and innovation	All activities that require learners to plot a graph of a given algebraic data, to organize and interpret statistical data collected and to apply skills in solving problems of economics have a common character of developing creativity into learners
Research and problem solving	All activities that require learners to make a research and apply their knowledge to solve problems from the real-life situation have a character of developing research and problem solving into learners.
Communication	During Mathematics class, all activities that require learners to discuss either in groups or in the whole class, present findings, debate …have a common character of developing communication skills into learners.
Co-operation, interpersonal relations and life skills	All activities that require learners to work in pairs or in groups have character of developing cooperation and life skills among learners.

Lifelong learning	All activities that are connected with research have a common character of developing into learners a curiosity of applying the knowledge learnt in a range of situations. The purpose of such kind of activities is for enabling learners to become life-long learners who can adapt to the fast-changing world and the uncertain future by taking initiative to update knowledge and skills with minimum external support.
Professional skills	Specific instructional activities and procedures that a teacher may use in the class room to facilitate, directly or indirectly, students to be engaged in learning activities. These include a range of teaching skills: the skill of questioning, reinforcement, probing, explaining, stimulus variation, introducing a lesson; illustrating with examples, using blackboard, silence and nonverbal cues, using audio – visual aids, recognizing attending behaviour and the skill of achieving closure.

The generic competences help learners deepen their understanding of Mathematics and apply their knowledge in a range of situations. As students develop generic competences, they also acquire the set of skills that employers look for in their employees, and so the generic competences prepare students for the world of work.

1.2.2 Addressing cross cutting issues

Among the changes brought by the competence-based curriculum is the integration of cross cutting issues as an integral part of the teaching learning process-as they relate to and must be considered within all subjects to be appropriately addressed. The eight cross cutting issues identified in the national curriculum framework are: *Comprehensive Sexuality Education, Environment and Sustainability, Financial Education, Genocide studies, Gender, Inclusive Education, Peace and Values Education, and Standardization Culture*.

Some cross-cutting issues may seem specific to particular learning areas/ subjects but the teacher need to address all of them whenever an opportunity arises. In addition, learners should always be given an opportunity during the learning process to address these cross-cutting issues both within and out of the classroom.

Below are examples of how crosscutting issues can be addressed:

Cross-Cutting Issue	Ways of addressing cross-cutting issues
Comprehensive Sexuality Education: The primary goal of introducing Comprehensive Sexuality Education program in schools is to equip children, adolescents, and young people with knowledge, skills and values in an age appropriate and culturally gender sensitive manner so as to enable them to make responsible choices about their sexual and social relationships, explain and clarify feelings, values and attitudes, and promote and sustain risk reducing behaviour.	Using different charts and their interpretation, Mathematics tutor should lead students to discuss the following situations: “Alcohol abuse and unwanted pregnancies” and advise student teachers on how they can instil learners to fight those abuses. Some examples can be given when learning statistics, powers, logarithms and their properties.
Environment and Sustainability: Integration of Environment, Climate Change and Sustainability in the curriculum focuses on and advocates for the need to balance economic growth, society well-being and ecological systems. Learners need basic knowledge from the natural sciences, social sciences, and humanities to understand to interpret principles of sustainability.	Using Real life models or students’ experience, Mathematics Tutor should lead student teachers to illustrate the situation of “population growth” and discuss its effects on the environment and sustainability.
Financial Education: The integration of Financial Education into the curriculum is aimed at a comprehensive Financial Education program as a precondition for achieving financial inclusion targets and improving the financial capability of Rwandans so that they can make appropriate financial decisions that best fit the circumstances of one’s life.	Through different examples and calculations on interest rate problems, total revenue and total cost, Mathematics Tutor can lead student teachers to discuss how to make appropriate financial decisions.

Gender: At school, gender will be understood as family complementarities, gender roles and responsibilities, the need for gender equality and equity, gender stereotypes, gender sensitivity, etc.	Mathematics Tutor should address gender as cross-cutting issue through assigning leading roles in the management of groups to both girls and boys and providing equal opportunity in the lesson participation and avoid any gender stereotype in the whole teaching and learning process.
Inclusive Education: Inclusion is based on the right of all learners to a quality and equitable education that meets their basic learning needs and understands the diversity of backgrounds and abilities as a learning opportunity.	Firstly, Mathematics Tutors need to identify/recognize students with special needs. Then by using adapted teaching and learning resources while conducting a lesson and setting appropriate tasks to the level of students, they can cater for students with special education needs. They must create opportunity where student teachers can discuss how to cater for learners with special educational needs.
Peace and Values Education: Peace and Values Education (PVE) is defined as education that promotes social cohesion, positive values, including pluralism and personal responsibility, empathy, critical thinking and action in order to build a more peaceful society.	Through a given lesson, a tutor should: • Set a learning objective which is addressing positive attitudes and values, • Encourage students to develop the culture of tolerance during discussion and to be able to instil it in colleagues and cohabitants; • Encourage students to respect ideas for others.
Standardization Culture: Standardization Culture in Rwanda will be promoted through formal education and plays a vital role in terms of health improvement, economic growth, industrialization, trade and general welfare of the people through the effective implementation of Standardization, Quality Assurance, Metrology and Testing.	With different word problems related to the effective implementation of Standardization, Quality Assurance, Metrology and Testing, students can be motivated to be aware of health improvement, economic growth, industrialization, trade and general welfare of the people.

1.2.3 Guidance on how to help students with special education needs in classroom

In the classroom, students learn in different way depending to their learning pace, needs or any other special problem they might have. However, the teacher has the responsibility to know how to adopt his/her methodologies and approaches in order to meet the learning need of each student in the classroom. Also teachers need to understand that student with special needs, need to be taught differently or need some accommodations to enhance the learning environment. This will be done depending to the subject and the nature of the lesson.

In order to create a well-rounded learning atmosphere, teachers need to:

- Remember that learners learn in different ways so they have to offer a variety of activities (e.g. role-play, music and singing, word games and quizzes, and outdoor activities);
- Maintain an organized classroom and limits distraction. This will help learners with special needs to stay on track during lesson and follow instruction easily;
- Vary the pace of teaching to meet the needs of each child. Some learners process information and learn more slowly than others;
- Break down instructions into smaller, manageable tasks. Learners with special needs often have difficulty understanding long-winded or several instructions at once. It is better to use simple, concrete sentences in order to facilitate them understand what you are asking.
- Use clear consistent language to explain the meaning (and demonstrate or show pictures) if you introduce new words or concepts;
- Make full use of facial expressions, gestures and body language;
- Pair a learner who has a disability with a friend. Let them do things together and learn from each other. Make sure the friend is not over protective and does not do everything for the one with disability. Both learners will benefit from this strategy;
- Use multi-sensory strategies. As all learners learn in different ways, it is important to make every lesson as multi-sensory as possible. Learners with learning disabilities might have difficulty in one area, while they might excel in another. For example, use both visual and auditory cues.

- Below are general strategies related to each main category of disabilities and how to deal with every situation that may arise in the classroom. However, the list is not exhaustive because each child is unique with different needs and that should be handled differently.

Strategy to help learners with developmental impairment:

- Use simple words and sentences when giving instructions;
- Use real objects that learners can feel and handle. Rather than just working abstractly with pen and paper;
- Break a task down into small steps or learning objectives. The learner should start with an activity that she/he can do already before moving on to something that is more difficult;
- Gradually give the learner less help;
- Let the learner with disability work in the same group with those without disability.

Strategy to help learners with visual impairment:

- Help learners to use their other senses (hearing, touch, smell and taste) and carry out activities that will promote their learning and development;
- Use simple, clear and consistent language;
- Use tactile objects to help explain a concept;
- If the learner has some sight, ask him/her what he/she can see;
- Make sure the learner has a group of friends who are helpful and who allow him/her to be as independent as possible;
- Plan activities so that learners work in pairs or groups whenever possible;

Strategy to help learners with hearing disabilities or communication difficulties

- Always get the learner 's attention before you begin to speak;
- Encourage the learner to look at your face;
- Use gestures, body language and facial expressions;
- Use pictures and objects as much as possible.
- Keep background noise to a minimum.

Strategies to help learners with physical disabilities or mobility

difficulties:

- Adapt activities so that learners, who use wheelchairs or other mobility aids, can participate.
- Ask parents/caregivers to assist with adapting furniture e.g. the height of a table may need to be changed to make it easier for a learner to reach it or fit their legs or wheelchair under;
- Encourage peer support when needed;
- Get advice from parents or a health professional about assistive devices if the learner has one.

Adaptation of assessment strategies:

At the end of each unit, the tutor is advised to provide additional activities to help students achieve the key unit competence. These assessment activities are for remedial, consolidation and extension designed to cater for the needs of all categories of students; slow, average and gifted students respectively. Therefore, the tutor is expected to do assessment that fits individual student.

Remedial activities	After evaluation, slow students are provided with lower order thinking activities related to the concepts learnt to facilitate them in their learning. These activities can also be given to assist deepening knowledge acquired through the learning activities for slow students.
Consolidation activities	After introduction of any concept, a range number of activities can be provided to all students to enhance/ reinforce learning.
Extended activities	After evaluation, gifted and talented students can be provided with high order thinking activities related to the concepts learnt to make them think deeply and critically. These activities can be assigned to gifted and talented students to keep them working while other students are getting up to required level of knowledge through the learning activity.

1.2.4. Guidance on assessment

Assessment is an integral part of teaching and learning process. The main purpose of assessment is for improvement of learning outcomes. Assessment for learning/ Continuous/ formative assessment intends to improve students' learning and tutor's teaching whereas assessment of learning/summative assessment intends to improve the entire school's performance and education system in general.

Continuous/ formative assessment

It is an on-going process that arises during the teaching and learning process. It includes lesson evaluation and end of sub unit assessment. This formative assessment should play a big role in teaching and learning process. The teacher should encourage individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods.

Formative assessment is used to:

- Determine the extent to which learning objectives are being achieved and competences are being acquired and to identify which students need remedial interventions, reinforcement as well as extended activities. The application activities are developed in the learner book and they are designed to be given as remedial, reinforcement, end lesson assessment, homework or assignment
- Motivate students to learn and succeed by encouraging students to read, or learn more, revise, etc.
- Check effectiveness of teaching methods in terms of variety, appropriateness, relevance, or need for new approaches and strategies. Mathematics tutors need to consider various aspects of the instructional process including appropriate language levels, meaningful examples, suitable methods and teaching aids/ materials, etc.
- Help students to take control of their own learning.

In teaching Mathematics, formative or continuous assessment should compare performance against instructional objectives. Formative assessment should measure the student's ability with respect to a criterion or standard. For this reason, it is used to determine what students can do, rather than how much they know.

Summative assessment

The assessment can serve as summative and informative depending to its purpose. The end unit assessment will be considered summative when it is done at end of unit and want to start a new one.

It will be formative assessment, when it is done in order to give information on the progress of learners and from there decide what adjustments need to be done.

The assessment done at the end of the term, end of year, is considered as summative assessment so that the teacher, school and parents are informed of the achievement of educational objective and think of improvement strategies. There is also end of level/ cycle assessment in form of national examinations.

When carrying out assessment?

Assessment should be clearly visible in lesson, unit, term and yearly plans.

- Before learning (diagnostic): At the beginning of a new unit or a section of work; assessment can be organized to find out what students already know / can do, and to check whether the students are at the same level.
- During learning (formative/continuous): When students appear to be having difficulty with some of the work, by using on-going assessment (continuous). The assessment aims at giving students support and feedback.
- After learning (summative): At the end of a section of work or a learning unit, the Mathematics Tutor has to assess after the learning. This is also known as Assessment of Learning to establish and record overall progress of students towards full achievement. Summative assessment in Rwandan schools mainly takes the form of written tests at the end of a learning unit or end of the month, and examinations at the end of a term, school year or cycle.

Instruments used in assessment.

- **Observation:** This is where the Mathematics tutor gathers information by watching students interacting, conversing, working, playing, etc. A tutor can use observations to collect data on behaviours that are difficult to assess by other methods such as attitudes, values, and generic competences and intellectual skills. It is very important because it is used before the lesson begins and throughout the lesson since the tutor has to continue observing each and every activity.

- **Questioning**

(a) Oral questioning: a process which requires a student to respond verbally to questions

(b) Class activities/ exercises: tasks that are given during the learning/ teaching process

(c) Short and informal questions usually asked during a lesson

(d) Homework and assignments: tasks assigned to students by their tutors to be completed outside of class.

Homework assignments, portfolio, project work, interview, debate, science fair, Mathematics projects and Mathematics competitions are also the different forms/instruments of assessment.

1.2.5. Teaching methods and techniques that promote active learning

The different learning styles for students can be catered for, if the teacher uses active learning whereby learners are really engaged in the learning process.

The main teaching methods used in mathematics are the following:

- **Dogmatic method** (the teacher tells the students what to do, What to observe, How to attempt, How to conclude)
- **Inductive-deductive method:** Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples.
- **Analytic-synthetic method:** Analytic method proceeds from unknown to known, 'Analysis' means 'breaking up' of the problem in hand so that it ultimately gets connected with something obvious or already known. Synthetic method is the opposite of the analytic method. Here one proceeds from known to unknown.
- **Skills Laboratory method:** Laboratory method is based on the maxim "learning by doing." It is a procedure for stimulating the activities of the students and to encourage them to make discoveries through practical activities.
- **Problem solving method, Project method and Seminar Method.**

The following are some active techniques to be used in Mathematics:

- Group work
- Research
- Probing questions
- Practical activities (drawing, plotting, interpreting graphs)
- Modelling
- Brainstorming
- Quiz Technique
- Discussion Technique
- Scenario building Technique

What is Active learning?

Active learning is a pedagogical approach that engages learners in doing things and thinking about the things they are doing. Learners play the key role in the active learning process. They are not empty vessels to fill but people with ideas, capacity and skills to build on for effective learning. Thus, in active learning, learners are encouraged to bring their own experience and knowledge into the learning process.

The role of the teacher in active learning	The role of learners in active learning
• The teacher engages learners through active learning methods such as inquiry methods, group discussions, research, investigative activities, group and individual work activities. • He/she encourages individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods.	A learner engaged in active learning: • Communicates and shares relevant information with peers through presentations, discussions, group work and other learner-centred activities (role play, case studies, project work, research and investigation); • Actively participates and takes responsibility for his/her own learning; • Develops knowledge and skills in active ways;

• He provides supervised opportunities for learners to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation. • Teacher supports and facilitates the learning process by valuing learners' contributions in the class activities.	• Carries out research/investigation by consulting print/online documents and resourceful people, and presents their findings; • Ensures the effective contribution of each group member in assigned tasks through clear explanation and arguments, critical thinking, responsibility and confidence in public speaking • Draws conclusions based on the findings from the learning activities.

Main steps for a lesson in active learning approach

All the principles and characteristics of the active learning process highlighted above are reflected in steps of a lesson as displayed below. Generally, the lesson is divided into three main parts whereby each one is divided into smaller steps to make sure that learners are involved in the learning process. Below are those main part and their small steps:

1) Introduction

Introduction is a part where the teacher makes connection between the current and previous lesson through appropriate technique. The teacher opens short discussions to encourage learners to think about the previous learning experience and connect it with the current instructional objective. The teacher reviews the prior knowledge, skills and attitudes which have a link with the new concepts to create good foundation and logical sequencings.

2) Development of the new lesson

The development of a lesson that introduces a new concept will go through the following small steps: discovery activities, presentation of learners' findings, exploitation, synthesis/summary and exercises/application activities.

- **Discovery activity**

Step 1

- The teacher discusses convincingly with learners to take responsibility of their learning
- He/she distributes the task/activity and gives instructions related to the tasks (working in groups, pairs, or individual to instigate collaborative learning, to discover knowledge to be learned)

Step 2

- The teacher let learners work collaboratively on the task;
- During this period the teacher refrains to intervene directly on the knowledge;
- He/she then monitors how the learners are progressing towards the knowledge to be learned and boosts those who are still behind (but without communicating to them the knowledge).

- **Presentation of learners' findings/productions**
  - In this episode, the teacher invites representatives of groups to present their productions/findings.
  - After three/four or an acceptable number of presentations, the teacher decides to engage the class into exploitation of learners' productions.
- **Exploitation of learner's findings/ productions**
  - The teacher asks learners to evaluate the productions: which ones are correct, incomplete or false
  - Then the teacher judges the logic of the learners' products, corrects those which are false, completes those which are incomplete, and confirms those which are correct.
- **Institutionalization or harmonization (summary/conclusion/ and examples)**
  - The teacher summarizes the learned knowledge and gives examples which illustrate the learned content.
- **Application activities**
  - Exercises of applying processes and products/objects related to learned unit/sub-unit
  - Exercises in real life contexts

- Teacher guides learners to make the connection of what they learnt to real life situations. At this level, the role of teacher is to monitor the fixation of process and product/object being learned.

3) Assessment

In this step the teacher asks some questions to assess achievement of instructional objective. During assessment activity, learners work individually on the task/activity. The teacher avoids intervening directly. In fact, results from this assessment inform the teacher on next steps for the whole class and individuals. In some cases, the teacher can end with a homework/ assignment. Doing this will allow learners to relay their understanding on the concepts covered that day. Teacher leads them not to wait until the last minute for doing the homework as this often results in an incomplete homework set and/or an incomplete understanding of the concept.

PART II: SAMPLE LESSON

School Name: ………   **Teacher's name**: ……………………

Term	Date	Subject	Class	Unit Nº	Lesson Nº	Duration	Class size
III	……. /……./2020	Mathematics	Year2 LE	3	11 of 12	40 min	…

Type of Special Educational Needs to be catered for in this lesson and number of learners in each category	1slow learner and 1 low vision learners:
Unit title	Points, lines and geometric shapes in 2D
Key unit competency:	To be able to determine algebraic representations of lines and calculate the area of geometric shapes in 2D.
Title of the lesson	Perimeter and Area of Geometric shapes
Instructional Objective	Using a meter ruler, learners will be able to determine the perimeter and area of any geometric shapes in 2D accurately.
Plan for this Class (location: in / outside)	The lesson is held indoor and outdoor, the class is organized into groups ,1 slow learner is working with others in group, and 1 low vision learner seats on the front desk near the blackboard in order to see and participate fully in all activities. Outdoor, student-teachers will measure the perimeter and calculate the area of different playgrounds in their groups
Learning Materials (for ALL learners)	Textbooks, rulers, decameters, working sheets and questionnaires, mathematical sets.
References	• TTC syllabus, • Year2 Mathematics textbook and Teacher's guide • P4 Subsidiary Mathematics textbook and teacher's guide

<table>
<tr><th>Timing for each step</th><th colspan="2">Description of teaching and learning activity</th><th>Generic competences and cross cutting issues to be addressed + a short explanation</th></tr>
<tr><td></td><td colspan="2">• Students work individually the questions in the introduction, and the correction is done on the chalk board by two students, under the guidance of the tutor.
• Then outside in different groups, tutor will distribute meter rulers in their groups student teacher will measure the perimeter of near the class playground of basketball , volley ball and then they will discuss in groups the area, followed by the presentation by a sample group, interaction of students and harmonization of the results under the facilitation of the tutor
• Next, they discuss in pairs the solved examples and compare their results with the answer proposed in the book.
• -Finally, the students are assigned individual tasks, and the correction is done on the chalk board, and the tutor winds up the lesson.</td><td></td></tr>
<tr><td></td><td>Teacher activities</td><td>Learner activities</td><td></td></tr>
<tr><td>Introduction:

5 minutes</td><td>The teacher asks students to work individually:

Find the length of one wire that can be used on one row of the fence in order to protect the plants:
• two points (see 3.7.2)
• The teacher links the introduction to the lesson of the day</td><td>• Students work individually.
• Two students, one after another, write the answers on the chalkboard:</td><td>Communication skills developed through the presentation and sharing of ideas.</td></tr>
</table>

Development of the lesson			
2.1 Discovery activity: **10 minutes**	• The teacher organizes the students into groups • Teacher distribute meter rules in groups • Teacher gives students activity 3.7.2 to measure perimeter and calculate area in groups and gives instructions related to the task • Teacher goes around to monitor the work of each group and provide assistance where needed.	• Learners form groups • Each group analyzes and discuss the activity 3.7.2 under the direction of the task manager of the group. • Students present to the teacher their eventual problems	• **Cooperation** and **communication** skills through discussions. • **Peace and values education**; Cooperation, mutual respect, tolerance through discussions with people with different views and respect one's views
2.2 Presentation of student's findings and exploitation: **15 minutes.**	• Teacher invites the one member of a sample group to present the findings of his/her group • The teacher encourages students to follow attentively • Teacher takes notes on key points from learners' presentation. • The teacher asks students to amend the presentation and to evaluate their work.	• The reporter presents the work on the behalf of the group. **Expected answers** **(Refer to solution of activity 3.7.2, in TG)** • students follow the presentation • students evaluate the findings of other learners • students evaluate their own findings	• **Cooperation** and **communication**/ attentive listening during presentations and group discussions. • **Critical thinking** through evaluating other's findings.

2.4.Conclusion/ **Summary:** **5 minutes** **Assessment** **5 minutes**	• Teacher facilitates the students to elaborate the summary of the presentation • Teacher requests students to write down the main points in their books • Teacher asks students to work out individually the application activity 3.7.2	• The students come to the main point: The perimeter and the area of different two-dimensional shapes appear in student-teacher book. • Students take notes in their books. • Individually students work out the application activity 3.7.2 and finally they make a correction on the chalk board. **Expected answers** **(Refer** to solution of application activity 3.7.2, in TG)	• **Critical thinking** and **problem-solving** skills are developed through analyzing and solving real life Mathematical problem: e.g. finding the area and the perimeter of the playground; • **Financial education** is addressed through good management of the money found when you sell a field given the price of one square meter; • **Standardization culture**: Use correctly the unit of length measurement.
Observation on lesson delivery	To be completed after receiving the feed-back from the students (what did the students like, what challenged them...).		

PART III: UNIT DEVELOPMENT

UNIT 1

GRAPHS AND FUNCTIONS

1.1 Key unit competence:

Apply graphical representation of function in economics models.

1.2 Prerequisite

Student-teachers will perform better in this unit if they refer to linear functions learnt in Unit 6 of S3.

1.3 Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching);
- Peace and value Education (respect others' view and thoughts during class discussions);
- Gender (equal opportunity of boys and girls in the lesson participation);

1.4 Guidance on introductory activity

- In groups, facilitate student-teachers to read and do the introductory activity 1 from Student -teacher's book;
- Facilitate discussions to avoid noise or other unnecessary conversation,
- Move around in the classroom to get aware of straggling groups and provide assistance where necessary;
- Invite group representatives to present their findings and promote gender into presentation;
- Through class discussions, let student-teachers think on different ways of getting solutions.
- Through question-answer, arouse the curiosity of student teacher on the content of unit 1.

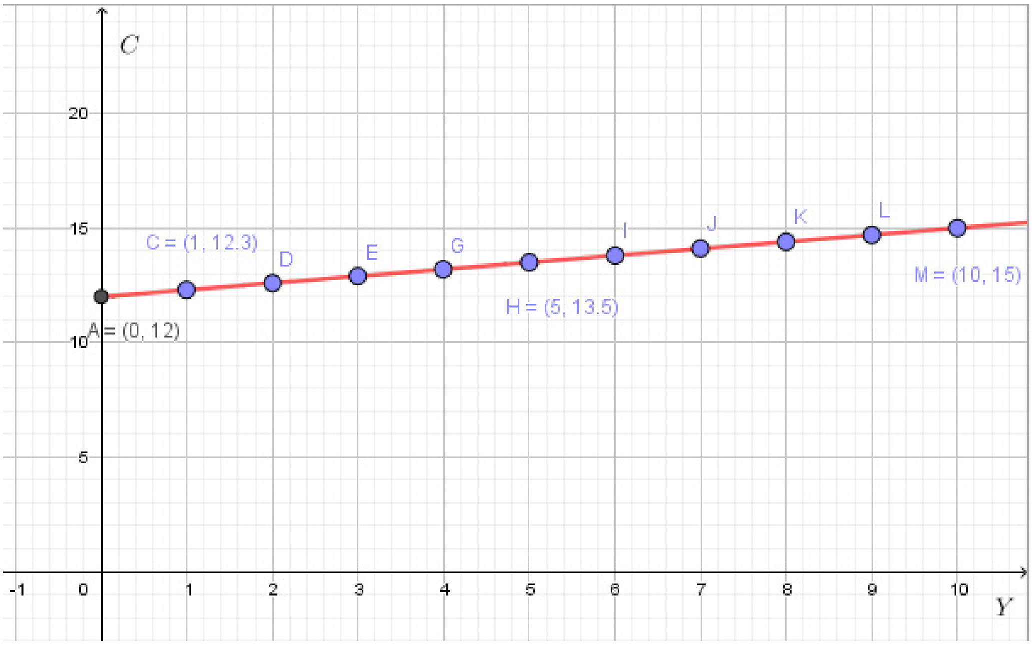
Expected answer for introductory activity 1

Suppose that average weekly household expenditure on food C depends on average net household weekly income Y according to the relationship $C = 12 + 0.3Y$.

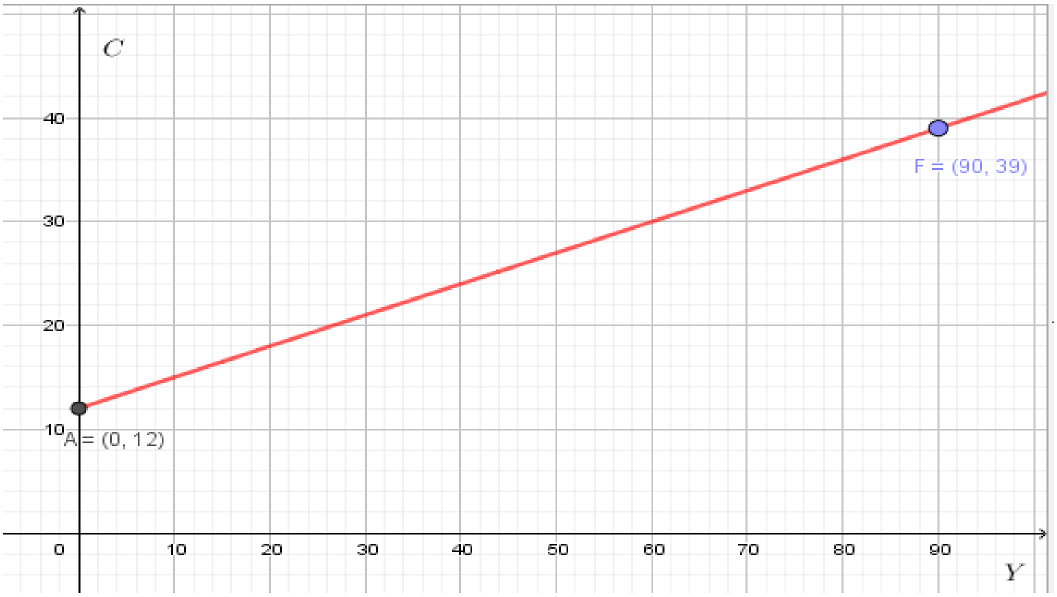
a) For every value of Y, C is a real number. This means that $\forall y \in \mathbb{R}, C \in \mathbb{R}$. The domain of C is $\mathbb{R}$.

b) Table of value from Y= 0 to Y= 10 and use it to draw the graph of $C = 12 + 0.3Y$

Y	0	1	2	3	4	5	6	7	8	9	10
$C(Y)$	12	12.3	12.6	12.9	13.2	13.5	13.8	14.1	14.4	14.7	15



c) If Y=90, the value of C is $C(90) = 12 + 0.3(90) = 39$



1.5. List of lessons/sub-headings

#	Lesson title	Learning objectives	Number of periods
0	Introductory activity	To arouse the curiosity of student teachers on the content of unit 1	1
1	Generalities on numerical functions	Identify a function and recognize relations (rules) that are not functions.	2
2	Types of numerical functions	Differentiate types of numerical functions.	2
3	Domains of definition of numerical functions	Determine the domains of definition of different numerical functions.	4
4	Parity of a function (odd or even).	Differentiate even functions from odd functions.	2

5	Operations on functions	Perform operations on functions: addition, subtraction , multiplication and Division	2
6	Composite function	Use operations of function to determine the composite functions	2
7	Inverse of a function	Use operations of function to determine inverse of a function	2
8	Graphical representation and interpretation of linear and quadratic functions.	Plot the graph of linear quadratic functions and using the table of values	2
9	Graphical representation and interpretation of economics functions.	Use the properties of functions to explain different concepts of Economics and finance.	4
10	End unit Assessment		1
	Total number of periods		24

Lesson 1: Generalities on numerical functions

a) Learning objective

Identify a function and recognize rules that are not functions

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, Manila paper, markers, pens, pencils, etc.

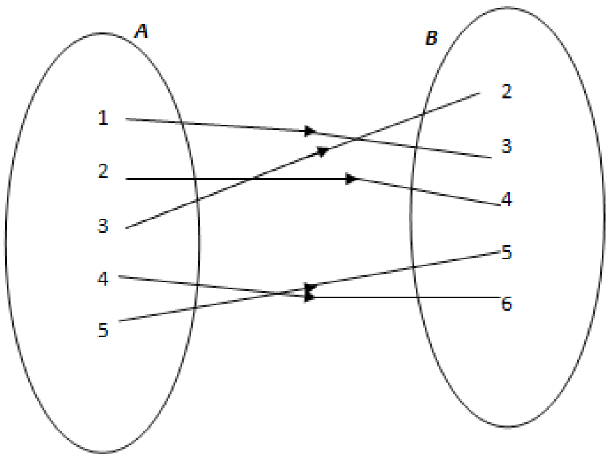
c) Prerequisites/Revision/Introduction

In this lesson, Student-teachers must be skilled in Unit 1 &3 of S1, Unit 2 of S2 and Unit 6 of S3.

d) Learning activities

- Invite student-teachers to work in group and do the activity 1.1 found in their Mathematics books;
- Move around in the class for facilitating where necessary and give more clarification on eventual challenges they may face during their work;
- Invite one member from each group with different working steps to present their work;
- As a tutor, harmonize the findings from presentation of student-teachers and guide them to specify the set of elements which have images and the set of all elements which have antecedents.
- Help them to check whether there exists an element which has more than one image.
- Guide them to explore the content and examples given in the student's book where they will be able to differentiate a function from correspondences (relations) and determine image for a point per a given function.
- After the lesson, guide students to do the application activity 1.1 and evaluate whether lesson objectives were achieved.

Answer for activity 1.1



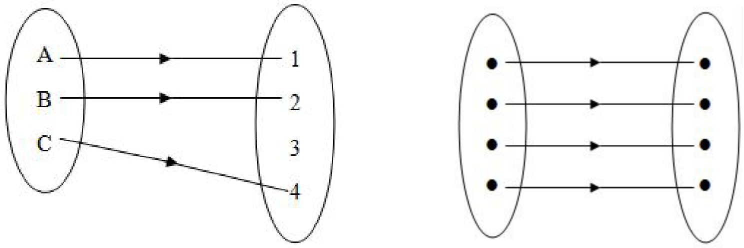
a) The set of elements of A which have images in B is $\{1;2;3;4;5\}$

b) The set of elements in B which have antecedent in A is $\{2;3;4;5;6\}$

c) No, because each element of the set A has one image in the set B.

Answer of application activity 1.1

1) The following arrow diagrams are functions.



2) $Dom=\{a,b,c,d,e\}$ $co-domain=\{1,2,3,4,5,6,7\}$ $Range=\{1,2,3,4\}$

3) $a)f(2)=8$

$b)f(-2)=0$

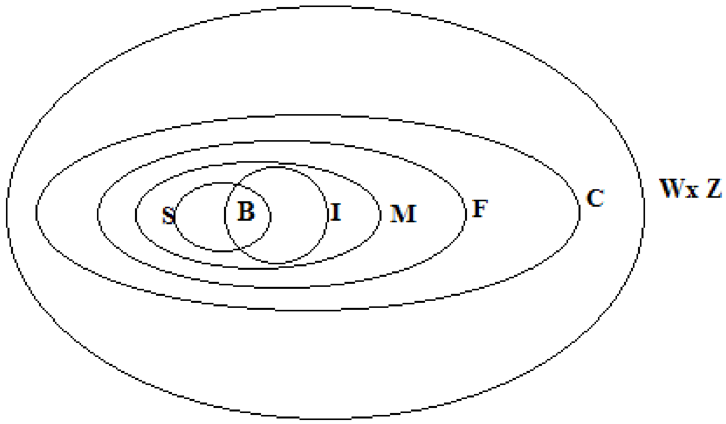
$c)f(d)=2d+4$

$d)f(a)=a,$

$2a+4=a$

$a=-4$

4) Relationship between mappings, functions and correspondences



The following inclusion is true: $S \subset M \subset F \subset C \subset (W \times Z)$.

A bijection is a type of mapping which is at the same time surjective (S) and injective (I) mapping.

All mappings are functions f(x) for which every point x has an image.

All functions are types of correspondences where you cannot observe a point which has more than one image.

Note: For each case, students can give examples.

Lesson 2: Types of numerical functions

a) Learning objective

Differentiate the types of functions.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, Manila paper, markers, pens, pencils, etc.

c) Prerequisites/Revision/Introduction

In this lesson, Student-teachers will perform better if they make a short revision on the content on functions learnt in S2 and S3.

d) Learning activities

- Invite student-teachers to work in groups and do the activity 1.2 found in their Mathematics books;
- Move around in the class for facilitating where necessary and give more clarification on eventual challenges they may face during their work;
- Invite a student from each group to present their findings;
- As a tutor, harmonize the findings from presentation and guide them to explain why they take such type of function.
- Use different probing questions and guide them to explore the content and examples given in the student's book lead them to be able to differentiate different types of functions: Constant function, Identity, Monomial, Polynomial, Rational and Irrational functions.

- Guide them to classify polynomial functions either by number of terms or by degrees and guide them to establish the general form of a polynomial function as $f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+...+a_2x^2+a_1x^1+a_0x^0$.

Degree 0	Degree 1	Degree2	Degree3	Degree4	etc.
Constant polynomial function.	Linear polynomial function/1st degree polynomial function.	quadratic polynomial function/ 2nd degree polynomial function.	Cubic polynomial function/3rd degree polynomial function.	4th degree polynomials (bi-quadratic polynomial functions).	

One term	Two terms	Three terms	Four terms	Etc
Monomial function	Binomial function	Trinomial function	Qua-trinomial Function	

- After this step, guide students to do the application activity 3.2 and evaluate whether lesson objectives were achieved.

Answer for activity 1.2

Polynomial	Rational	Irrational
$f(x)=(x+1)^2$	$h(x)=\dfrac{x^3+2x+1}{x-4}$	$f(x)=\sqrt{x^2+x-2}$

Answers of application activity 1.2

1. $f(x)=x^3+2x^2-2$ is a polynomial function.

2. $g(x)=-2$ is a constant function.

3. $h(x)=\dfrac{x^3+2x^2-2}{x-5}$ is a rational function.

4. $f(x)=\dfrac{x^2-2}{x^2-8x+15}$ is a rational function.

5. Here students can give various functions for example irrational functions, logarithmic functions, trigonometric functions, linear functions, refer to the student's book and see if their examples are correct.

Lesson 3: Domain of definition of numerical functions

a) Learning objective

Determine the domains of definition of different numerical functions.

b) Teaching resources

Student -teacher's book and other reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, graphing software such as Geogebra, etc.

c) Prerequisites/Revision/Introduction

Student-teachers should be able to explain the difference between types of functions learnt in previous lessons of year 1.

Note: Even though the student teachers need to be aware of the meaning and application of the range of a function, insist only on the domain of a function.

d) Learning activities

- Invite student-teachers to work in pairs and do the activity 3.3 found in their Mathematics books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work;
- Let the 2 neighbouring pairs work together and share their works so that they may improve them by highlighting the set for the values obtained;
- Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work.
- As a tutor, harmonize the findings from presentation and guide them to explain why they took such values.
- Use different probing questions and guide them to explore the content and examples given in the student's book and lead them to determine the domain and range for specified functions: Constant, linear, quadratic, Polynomial, Rational and Irrational functions. Note that the range will be determined only for elementary functions.

- After this step, guide students to do the application activity 3.3 and evaluate whether lesson objectives were achieved.

Answer for activity 1.3.1

1) None. It means that for all real numbers $f(x)=x^3+2x+1$ is defined.

2) $f(x)=\frac{1}{x}$ is not defined for $x=0$. It means that for $x=0 \Rightarrow f(0)=\frac{1}{0}\notin\mathbb{R}$ (it is impossible to divide by zero in the set of real numbers).

3) $g(x)=\frac{x+2}{x-1}$ is not defined for $x=1$. It means that for $x=1\Rightarrow f(1)=\frac{3}{0}\notin\mathbb{R}$.

Answer for activity 1.3.2

1) $domf=\mathbb{R}=]-\infty,+\infty[$ in this case f(x) is defined for all real numbers.

2) $domg=\mathbb{R}=]-\infty,+\infty[$ in this case f(x) is defined for all real numbers.

3) $domh=\mathbb{R}-\{5\}\ or\mathbb{R}\setminus\{5\}$ or $]-\infty,5[\cup]5,+\infty[$ in this case f(x) is defined for all real numbers except 5.

4) $domf=\mathbb{R}\setminus\{3,5\}$ in this case f(x) is defined for all real numbers except 3 and 5. Or in interval form: $]-\infty,3[\cup]3,5[\cup]5,+\infty[$

Answer of activity 1.3.3

1) $\forall x\in\left]-\infty,-\frac{1}{2}\right[$, the given function is not defined.

2) The range is $\left[\sqrt[3]{\frac{7}{4}},+\infty\right[$, as $Min(f)\left[-\frac{1}{2},f\left(-\frac{1}{2}\right)\right]$

3) $\forall x\in[-1,2[$, the given function is not defined.

Answers for application activity 1.3

1. $domf=[2,+\infty[$

2. $domg=]-\infty,-6]\cup[1,+\infty[$

3. $domh = \mathbb{R} \setminus \{-4\}$ or $]-\infty, -4[\cup]-4, +\infty[$

4. $domf =]-\infty, -5[\cup]5, +\infty[$

5. $domf =]-4, +\infty[$

Lesson 4: Parity of a function

a) Learning objective

Differentiate even functions from odd functions.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, gridded paper, etc.

c) Prerequisites/Revision/Introduction

Student-teachers must be well skilled in the content of Unit 2 of S2.

d) Learning activities

- Invite student-teachers to work in groups and do the activity 3.4 found in their Mathematics books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work;
- Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work.
- As a tutor, harmonize the findings from presentation and guide them to enhance the characteristics of even functions and odd functions.
- Use graphs for simple functions to illustrate the characteristics of even and odd functions: The graph of even function is symmetric about the vertical axis (the line $x = 0$ is the axis of symmetry) while the graph of odd function looks the same when rotated through half a revolution about 0 (the point $(0,0)$ is the centre of symmetry for its parts).

- Use different probing questions and guide them to explore the content and examples given in the student's book and lead them to verify the parity of different functions.
- After this step, guide students to do the application activity 3.4 and evaluate whether lesson objectives were achieved.

Answer for activity 1.4

1. $f(x) = x^2 + 3$

   - $f(-x) = (-x)^2 + 3 = x^2 + 3$
   - $-f(x) = -(x^2 + 3) = -x^2 - 3$

   $\therefore f(-x) = f(x) \quad and \quad -f(x) \neq f(-x)$

2. $f(x) = \sqrt[3]{x^2 + x}$

   $f(-x) = \sqrt[3]{(-x)^2 + (-x)} = \sqrt[3]{x^2 - x}$

   $-f(x) = -\sqrt[3]{x^2 + x} = \sqrt[3]{-x^2 - x}$,(for odd index it is possible to inter *negative* sign under radical)

   $\therefore f(-x) \neq -f(x)$

3. $f(x) = \dfrac{x^2 - 3}{x^2 + 1}$

   $f(-x) = \dfrac{(-x)^2 - 3}{(-x)^2 + 1} = \dfrac{x^2 - 3}{x^2 + 1}$

   $-f(x) = -\dfrac{x^2 - 3}{x^2 + 1} = \dfrac{-x^2 + 3}{x^2 + 1}$

   $\therefore -f(x) \neq f(-x)$

Answers for application activity 1.4

1) $f(x) = 2x^2 + 2x - 3$

$\circ f(-x) = 2(-x)^2 + 2(-x) - 3 = 2x^2 - 2x - 3$

$\circ - f(x) = -(2x^2 + 2x - 3) = -2x^2 - 2x + 3$

$f(-x) \neq -f(x) and$

$f(-x) \neq -f(x)$

$\therefore f(x) = 2x^2 + 2x - 3$ is neither odd nor even function.

2) $f(x) = \dfrac{3x^3 + 2x^2 + 8}{x - 5}$ is neither odd nor even.

3) $g(x) = x^3 - x$

$\circ g(-x) = (-x)^3 - (-x) = -x^3 + x$

$\circ -g(x) = -(x^3 - x) = -x^3 + x$

$g(-x) = -g(x)$

$\therefore g(x) = x^3 - x$ is odd.

4) $h(x) = \dfrac{x^2 + 4}{x^2 - 4}$ is even function.

5) $g(x) = x(x^2 + x) = x^3 + x^2$

$\circ g(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$

$\circ -g(x) = -(x^3 + x^2) = -x^3 - x^2$

$g(-x) \neq -g(x) \, and$

$g(-x) \neq -g(x)$

$\therefore g(x) = x(x^2 + x)$ is neither odd nor even.

Lesson 5: Addition, subtraction, multiplication and division

a) Learning objective

Perform operations on functions: addition, subtraction, multiplication and Division.

b) Teaching resources

Student-teacher's book and other reference textbooks to facilitate research, Mathematical set, calculator, Manila paper, markers, pens, pencils, etc.

c) Prerequisites/Revision/Introduction

In this lesson, student-teachers must be skilled in unit 6 of S3.

c) Learning activities

- Invite student-teachers to work in groups and do the activity 1.5.1 found in their Mathematics books;
- Move around in the class for facilitating students where necessary and

ask some challenging questions to lead them to work correctly;

- Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work.
- As a tutor, harmonize the findings from presentation and guide them to enhance the operation of functions: addition, subtraction, multiplication and division.
- Use different probing questions and guide them to explore the content and examples given in the student's book and lead them to explore the addition, multiplication and division of functions.
- After this step, guide students to do the application activity 1.5.1 and evaluate whether lesson objectives were achieved.

Answer for activity 1.5.1

1) $f(x)=\frac{x+1}{2x-3}; g(x)=x+1$

$$f(x)+g(x)=\left(\frac{x+1}{2x-3}\right)+\left(\frac{x+1}{1}\right)=\frac{(x+1)+(x+1)\cdot(2x-3)}{2x-3}$$

$$=\frac{x+1+2x^2-3x+2x-3}{2x-3}=\frac{2x^2-3}{2x-3}; x\neq\frac{3}{2}$$

2) $f(x)=\frac{x+1}{2x-3}; g(x)=x+1$

$$f(x)-g(x)=\left(\frac{x+1}{2x-3}\right)-\left(\frac{x+1}{1}\right)=\frac{(x+1)-(x+1)\cdot(2x-3)}{2x-3}$$

$$=\frac{x+1-(2x^2-x-3)}{2x-3}=\frac{-2x^2+2x+4}{2x-3}; x\neq\frac{3}{2}$$

3) $f(x)=\frac{x+1}{2x-3}; g(x)=x+1$

$$f(x)\cdot g(x)=\left(\frac{x+1}{2x-3}\right)\left(\frac{x+1}{1}\right)=\frac{(x+1)\cdot(x+1)}{2x-3}=\frac{x^2+2x+1}{2x-3}; x\neq\frac{3}{2}$$

4) $f(x)=\frac{x+1}{2x-3}; g(x)=x+1$

$$\frac{f(x)}{g(x)}=\left(\frac{\frac{x+1}{2x-3}}{\frac{x+1}{1}}\right)=2x-3$$

Answer for application activity 1.5.1

1) $(2x^3+5x-1)+(3x-4)=2x^3+8x-5$

2) $(3x^3-5x^2+7x-4)(2x^2-x+3)=6x^5-13x^4+23x^3-30x^2+30x-12$

Lesson 6: Composite function

a) Learning objective:

Use operations of function to determine the composite functions.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research Mathematical set, calculator, Manila paper, markers, pens, pencils, sibling relationship, etc.

c) Prerequisites/Revision/Introduction

In this lesson, student-teachers can refer to the content of unit 6 of S3.

c) Learning activities

- Invite student-teachers to work in groups and do the activity 1.5.2 found in their Mathematics books;
- Move around in the class for facilitating students where necessary and ask some challenging questions to lead them to work correctly;
- Verify and identify groups with different working steps;

- Invite one member from each group with different working steps to present their work.
- As a tutor, harmonize the findings from presentation and guide them to enhance how to find image by the composite function.
- Use different probing questions and guide them to explore the content and examples on the composite of functions as it is given in the student's book;
- After this step, guide students to do the application activity 1.5.2 and evaluate whether lesson objectives were achieved.

Answer for activity 1.5.2

1) $f(\mathrm{y}) = 3\,\mathrm{y} + 2$ *and*

$f(x) = 3x + 2; g(x) = x^2 - 1,$ *we have*

$f(g(x)) = 3(x^2 - 1) + 2 = 3x^2 - 3 + 2 = 3x^2 - 1$

2) $f(x) = 3x + 2; g(x) = x^2 - 1$

$g(f(x)) = (3x+2)^2 - 1 = 9x^2 + 12x + 3$

3) $f(g(x)) \neq g(f(x))$

Answer for application activity 1.5.2

1) By the definition of composition, we have

$(fg)(x) = f(g(x)) = f(2x+1)$

$= (2x+1)^2 = 4x^2 + 4x + 1$

$(gof) = g(f(x)) = g(x^2)$

$= 2x^2 + 1$

2) $a) fg(x) = f[g(x)] = f(2x)$

replace x by $2x$ to get $f(2x) = 2x + 3$

hence $fg(x) = 2x + 3$

$b) gf(x) = g[f(x)] = g(x+3)$

replace x by $x+3$ to get

$g(x+3) = 2(x+2)$

hence $gf(x) = 2x + 6.$

3) $a) fg(x) = f[g(x)] = f(x^2+2)$

Replace x by x^2+2 to get

$f(x^2+2)=2(x^2+2)-1$

this gives $fg(x) = 2x^2+4-1$

$= 2x^2+3$

hence $f(g) = 2x^2+3$

$b) gf(x) = g[f(x)] = g(2x-1)$

replace x in x^2+2 by $(2x-1)$ to get

$g(2x-1) = (2x-1)^2+2$

$= (4x^2-4x+1)+2$

$= 4x^2-4x+3$

$c) gf(3) = 4(3)^2-4(3)+3$

$= 4(9)-12+3$

$= 36-12+3$

$= 27$

hence $gf(3) = 27$

4) $a) f.g(x) = -3; \quad g.f(x) = 2$

$b) f.g(x) = 72x^2+6x-3; \quad g.f(x) = 12x^2+6x-18$

Lesson 7: Inverse of an invertible function

a) Learning objective: Use operations of function to determine the inverse of a function

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research Mathematical set, calculator, Manila paper, markers, pens, pencils, sibling relationship, etc.

c) Prerequisites/Revision/Introduction

In this lesson, student-teachers must be skilled in the content of the unit 6 of S3.

c) Learning activities

- Invite student-teachers to work in groups and do the activity 1.5.3 found in their Mathematics books;
- Move around in the class for facilitating students where necessary and ask some challenging questions to lead them to work correctly;
- Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work.
- As a tutor, harmonize the findings from presentation and guide them to enhance the steps of finding the inverse of a function: they have to relate the solved equation in y variable with the usual equations in x variable.
- Use different probing questions and guide them to explore the content and examples on the inverse of a function as it is given in the student's book;
- After this step, guide students to do the application activity 1.5.3 and evaluate whether lesson objectives were achieved.

Answer for activity 1.5.3

1) $x = y - 1$ 2) $x = \frac{y+2}{3}$ 3) $x = \frac{y+3}{2y+1}$

Answers of application activity 1.5.3

1. a) $f^{-1}(x) = \frac{x-2}{5}$ b) $f^{-1}(x) = \frac{-x-2}{7}$ c) $f^{-1}(x) = \frac{2x+1}{x+2}$

Lesson 8: Graphical representation and interpretation of linear and quadratic functions

a) Learning objective

Use a table of values to plot graph of linear or quadratic function

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, Manila paper, markers, pens, pencils, etc.

c) Prerequisites/Revision/Introduction

In this lesson, student-teachers must be skilled in Unit 6 of S3 and all lessons above in this unit.

d) Learning activities

- Invite student-teachers to work in group and do the activity 1.6 found in their Mathematics books;
- Move around in the class for facilitating groups where necessary and ask some guiding questions on eventual challenges they may face during their work;
- Invite groups with different working steps to present their work;
- As a tutor, harmonize the findings from presentation of student-teachers and guide them to use a table of values, ordered pairs (points), and plot a graph of a given function.
- Guide students to explore the content and examples of functions given in the student's book.
- After the lesson, guide students to do the application activity 1.6 and evaluate whether lesson objectives were achieved.

Answer for activity 1.6

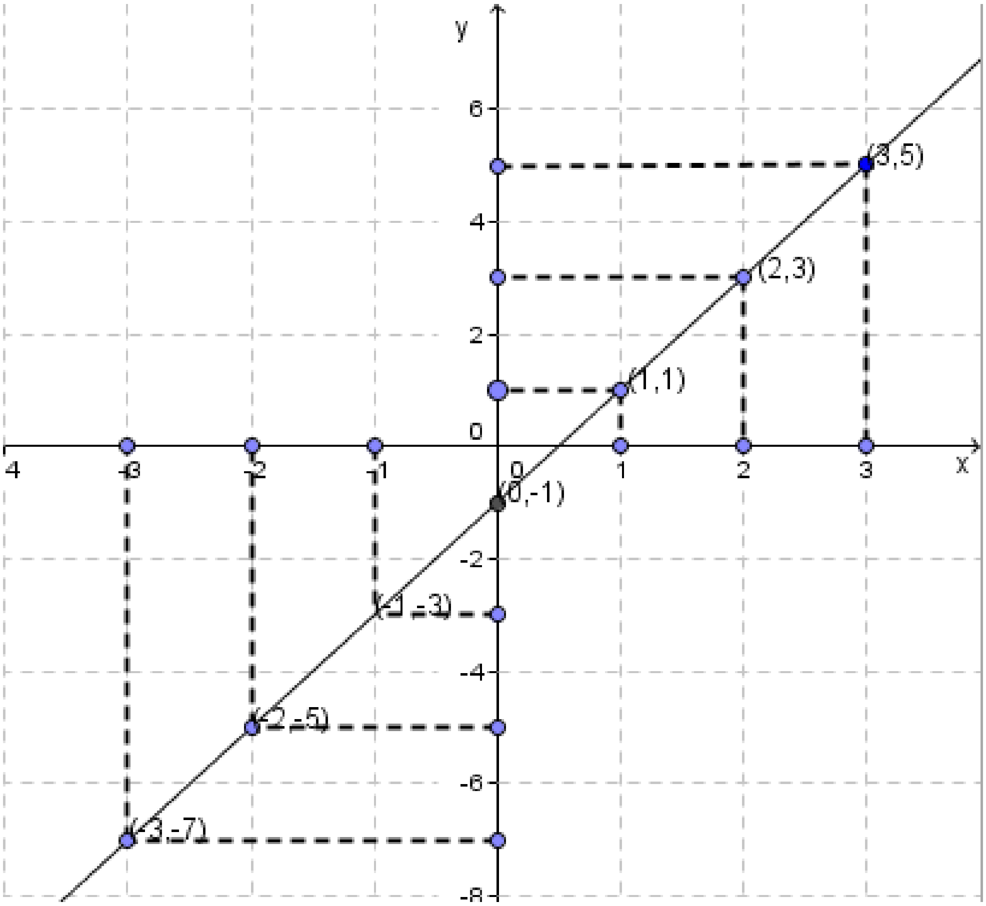
1. a)

x	-3	-2	-1	0	1	2	3
$y=2x-1$	-7	-5	-3	-1	1	3	5

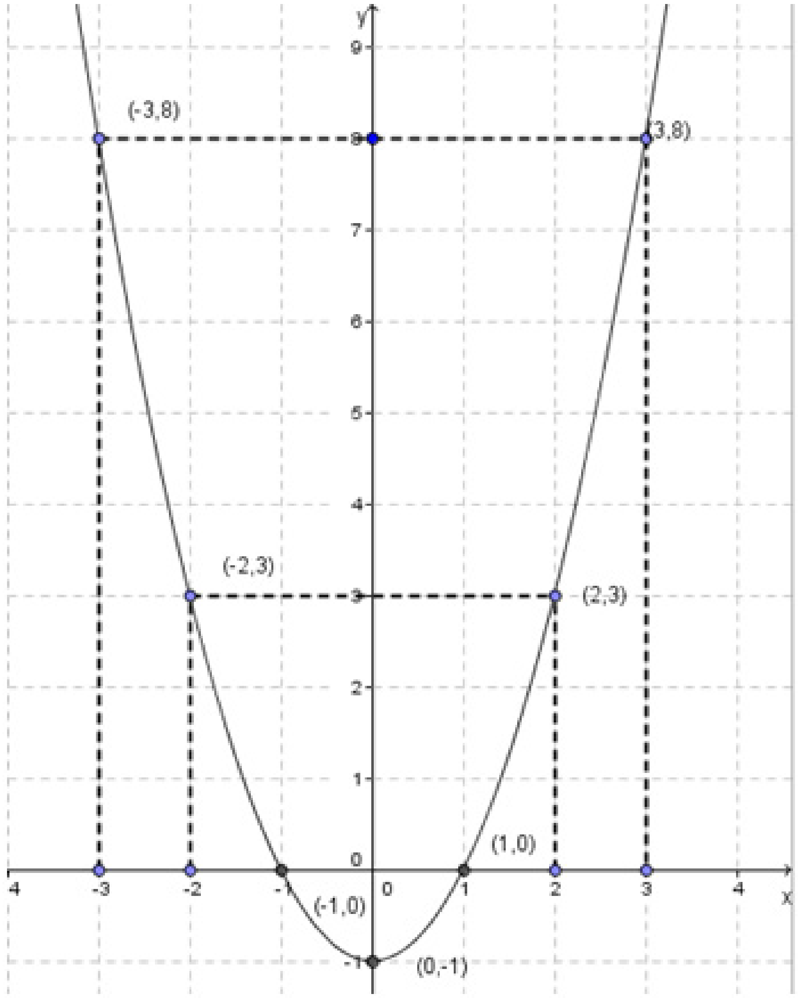
b)

x	-3	-2	-1	0	1	2	3
$y = x^2 - 1$	8	3	0	-1	0	3	8

2. a)



b)

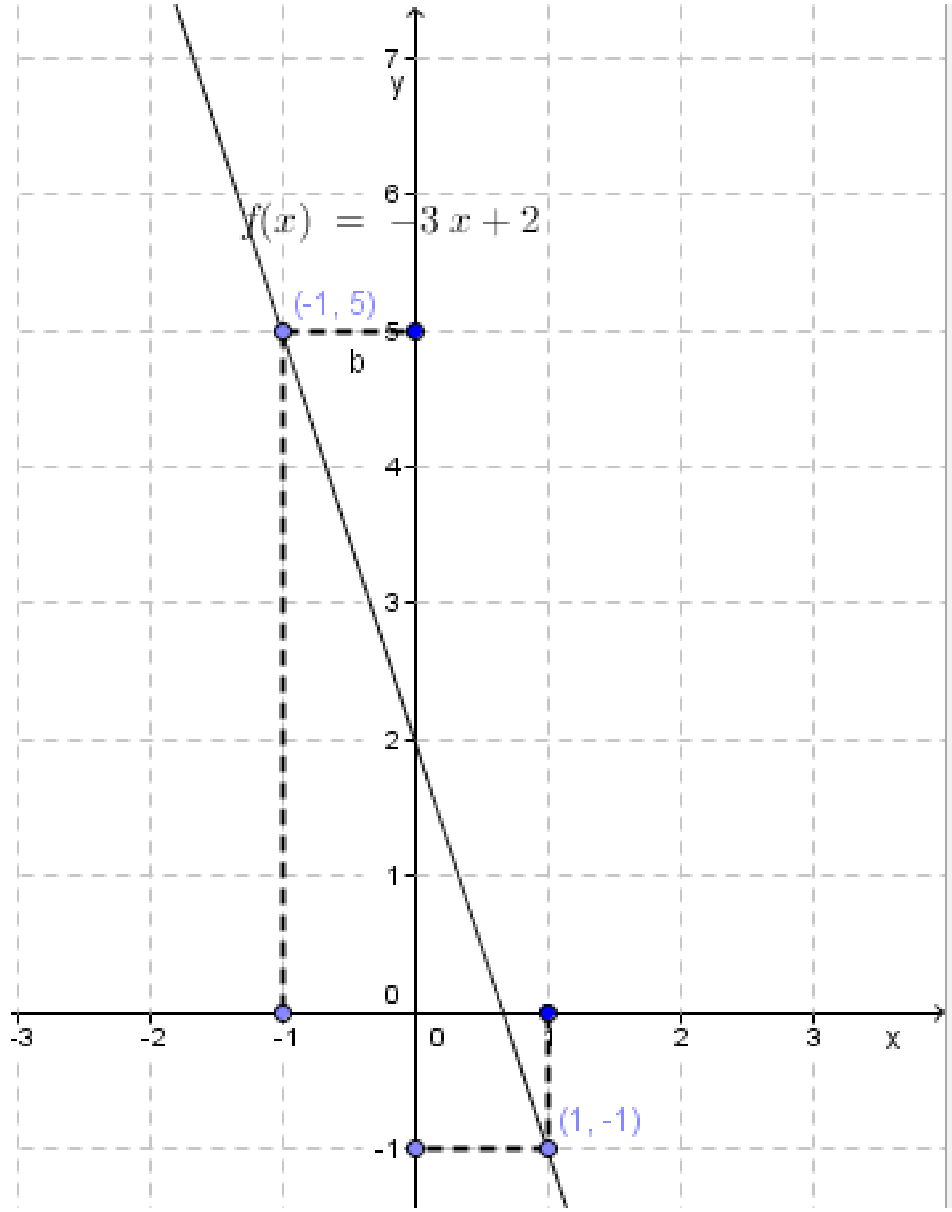


3)Shape of $y=2x-1$ is straight line while shape of $y=x^2-1$ is upward curve.

Answers for Application activity 1.6

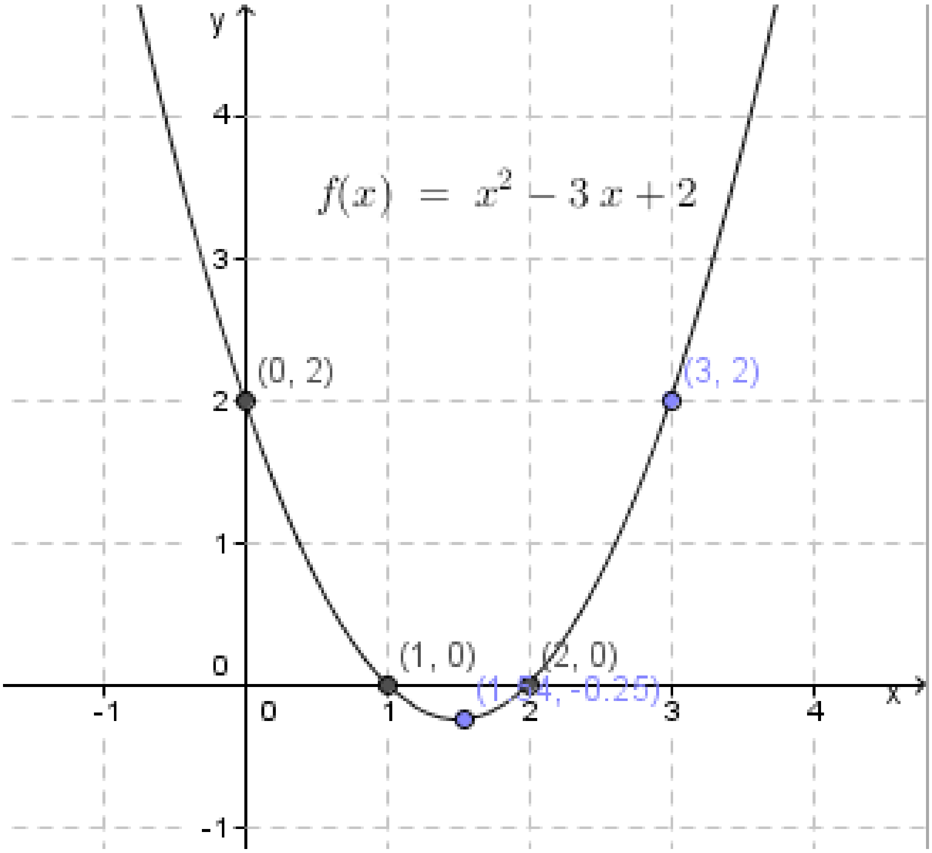
1. **a)**

x	-3	-2	-1	0	1	2	3
$y=-3x+2$	11	8	5	2	-1	-4	-7



b)

x	-3	-2	-1	0	1	2	3
$y = x^2 - 3x + 2$	20	12	6	2	0	0	2



2. $-3x^2 + 6x + 1 = 0$ $a = -3 \quad b = 6 \quad c = 1$

Vertex formula $= \left(-\frac{b}{2a}, f(-\frac{b}{2a})\right)$

The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{-6}{-6} = 1$ which also symmetric axis is

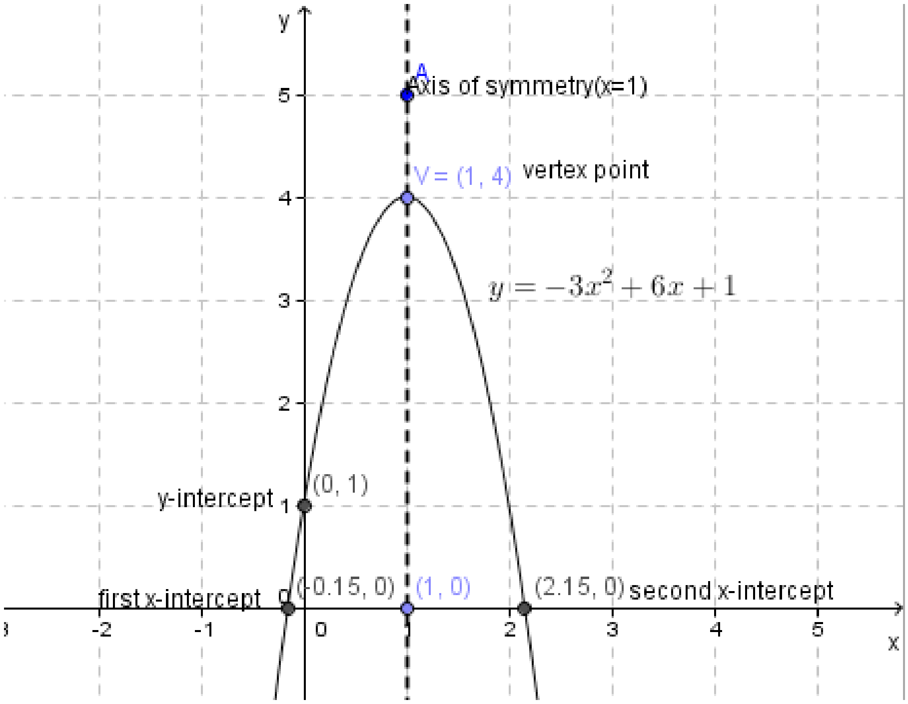
The y-coordinate of the vertex is $y = f\left(\frac{-b}{2a}\right) = f(1) == 3 + 6 + 1 = 4$

Now the vertex is located at the point $(1,4)$. The axis of symmetry is the line $x = 1$

x-intercepts are $(-0.15, 0)$ and $(2.15, 0)$

y-intercept is $(0,1)$

The graph is



Lesson 9: Graphical representation and interpretation of economics functions

a) Learning objective

Use the properties of functions to explain different concepts of Economics or finance.

b) Teaching resources

Student-teacher's book and other reference textbooks to facilitate research, calculator, Manila paper, markers, pens, pencils, etc.

c) Prerequisites/Revision/Introduction

In this lesson, student-teachers should refer to all previous lessons of this unit.

d) Learning activities

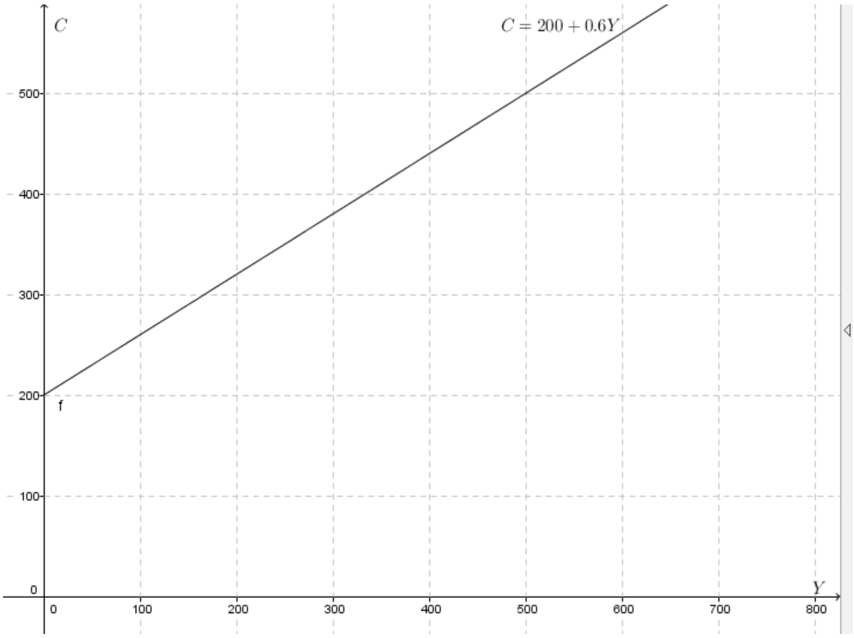
- Invite student-teachers to work in group and do the activity 1.7 found in their Mathematics books;
- Move around in the class for facilitating groups where necessary and

ask some guiding questions on eventual challenges they may face during their work;

- Invite groups with different working steps to present their work;
- As a tutor, harmonize the findings from presentation of student-teachers and guide students to illustrate any simple economic model that uses linear or quadratic function;
- Invite them to brainstorm on other application of functions they have observed in economics or finance;
- Guide them to explore the content and examples given in the student's book where they will be able to explore the following: Price as function of quantity supplied, Consumption as function of income, Price as function of quantity demanded, Point elasticity of demand, The Cost Function, The Profit Function, The Marginal Cost, Marginal Revenue, and Marginal Profit and Equilibrium Price and Quantity.
- After the lesson, guide students to do the application activity 1.7 and evaluate whether lesson objectives were achieved.

Answer for activity 1.7

The function is $C = 200 + 0.6Y$



The point at which the line cuts the vertical axis $(0, 200)$.

Answers for Application activity 1.7

1. $C(Y) = a + bY$

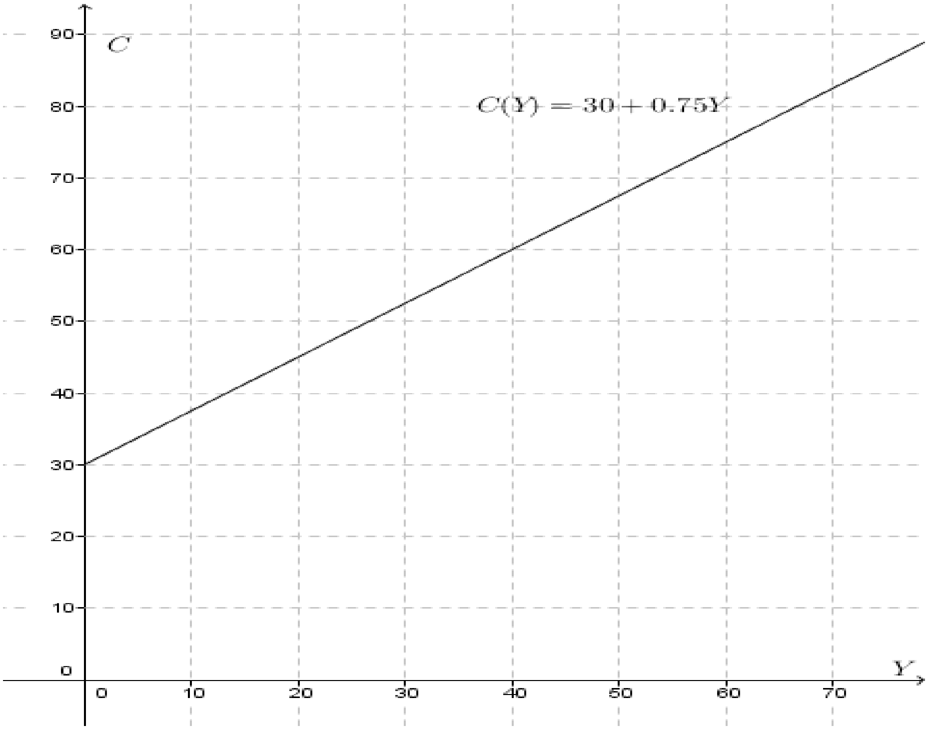
$$\begin{cases} 60 = a + 40b(\times -1) \\ 90 = a + 80b(\times 1) \end{cases}$$

$$\begin{cases} -60 = -a - 40b \\ 90 = a + 80b \end{cases}$$

$$30 = 40b \Rightarrow b = \frac{3}{4} = 0.75$$

$$a = 30$$

The graph of C(Y)



2) a) $f(12) = 60$ means that when the product is 12 units, the price is 60 units of money.

b) Normally the price increases when the quantity is reducing.

1.6 Summary of the unit

Function

A function is a rule that assigns to each element in a set *A* one and only one element in set *B*. We can even define a function as any relationship which takes one element of one set and assigns to it one and only one element of second set

Types of functions:

There are: constant function, Identity, Monomial, Polynomial function, Rational function, Irrational function, etc.

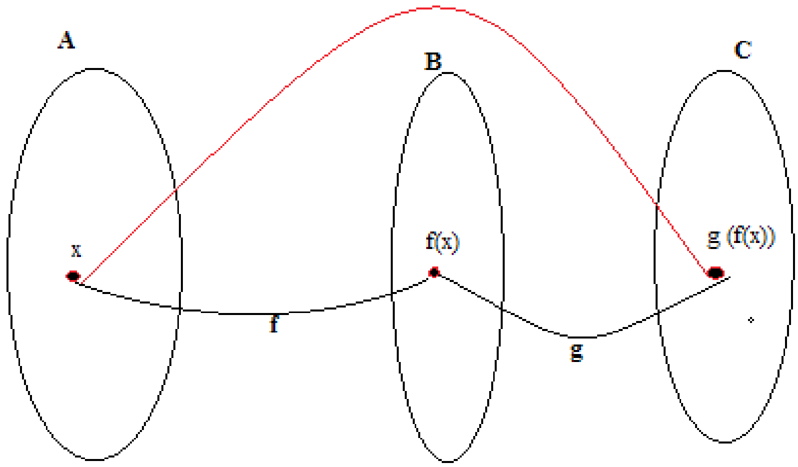
Parity of a function (odd or even

Even function: $f(-x) = f(x)$

Odd function: $f(-x) = -f(x)$

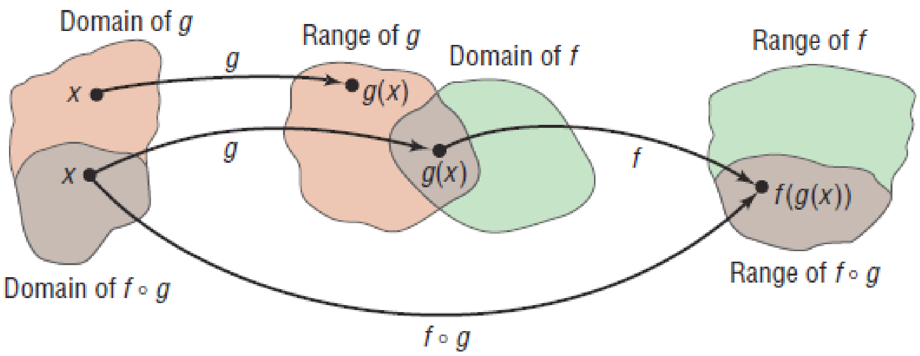
Composite function

This composite function of f(x) and g(x) is written $(g \circ f)(x)$ or $g[f(x)]$ and can be illustrated as follows:



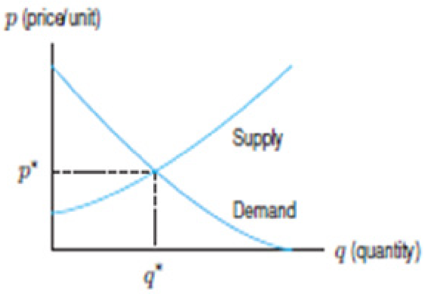
$g_{\circ}f(x) = g(f(x))$

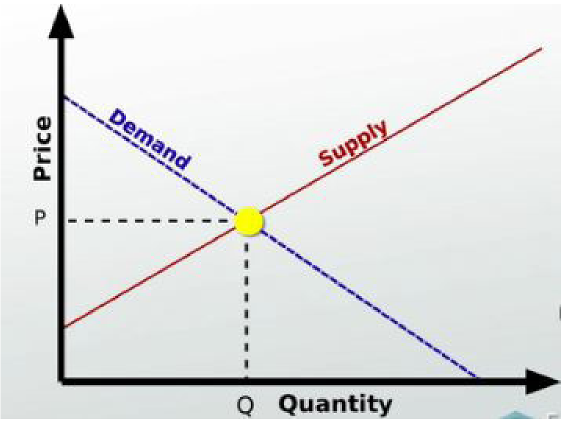
The composite function of f(x) and g(x) is written $(fog)(x) = f(g(x))$ and illustrated as follows:



Equilibrium Price and Quantity

If we plot the supply and demand curves on the same axes, the graphs cross at the *equilibrium point*. The values $p*$ and q* at this point are called the *equilibrium price* and *equilibrium quantity*, respectively. It is assumed that the market naturally settles to this equilibrium point.





1.7. Additional Information for Teachers

Be careful!

- Emphasize the use of graph paper/gridded paper while student-teachers draw the graphs.
- Emphasize and facilitate students to use of geometric materials to ameliorate the quality of graphs.
- Remind students to graduate correctly and name axes (x-axis and y-axis).
- Recall them to mention/highlight the origin/intersection point of axes by 0.

1.8 Answer for end unit assessment 1

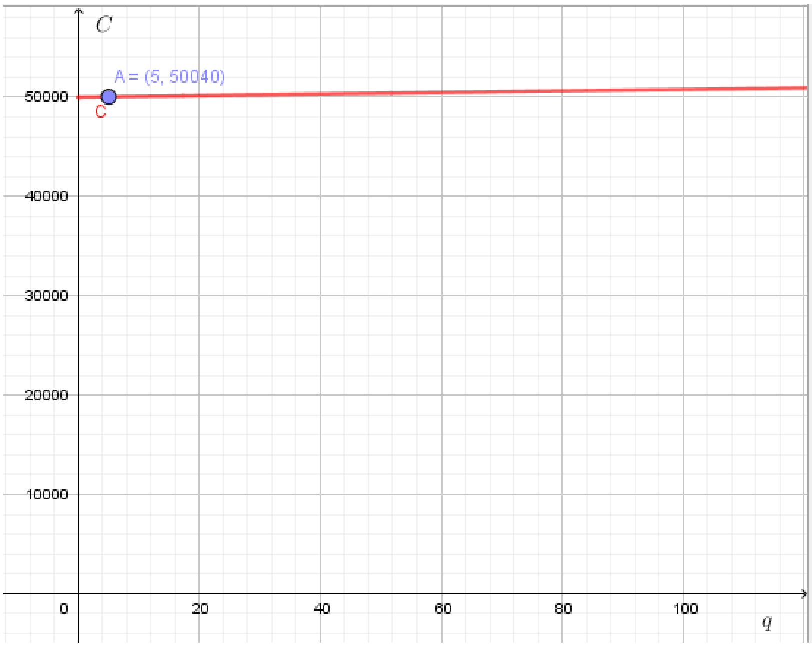
1) The total cost C for units produced by a company is given by $C(q) = 50000 + 7q$ where q is the number of units produced.

Solution:

a) The amount 50000 represents the fixed cost;

b) the number 8 represents the marginal cost (cost of a unit of product);

c) The graph for $C(q) = 50000 + 8q$ is the following:



d) The real domain of C that corresponds to q which is positive is $[0;\infty[$. The range is $[50000;\infty[$

e) C(q) is not an odd function because $C(-q) \neq -C(q)$.

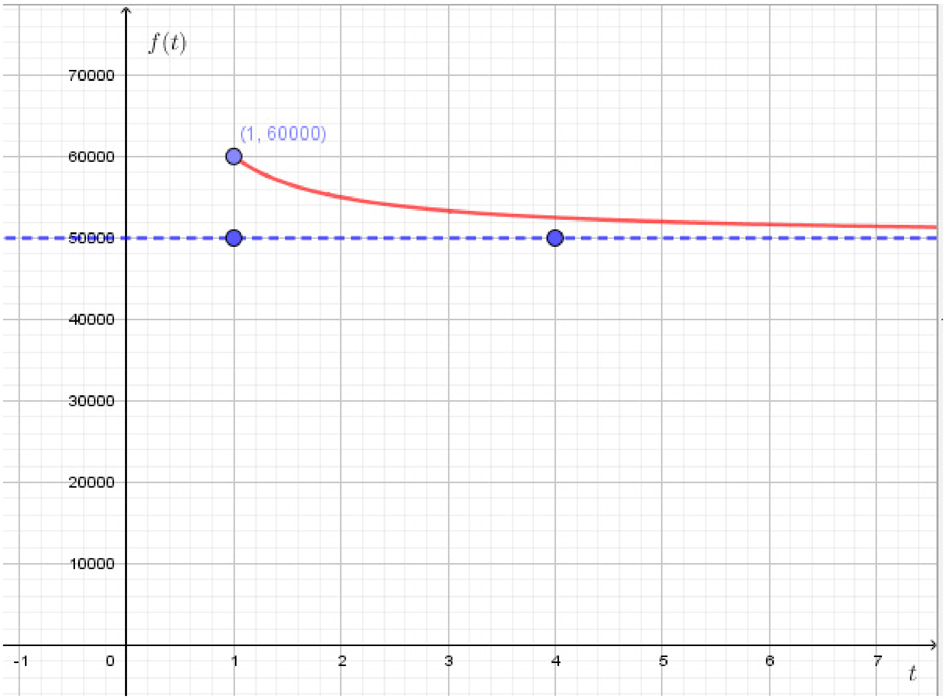
2) a) The function $f(t)$ which models the monthly salary of Bosco is

$$f(t) = 50000 + \frac{10000}{t}$$

b) The value of $f(t)$ will depend on the positive time t and $f(t)$ is possible when $t \neq 0$. $Domf = \mathbb{N} - \{0\}$. This domain is the set of the time t (number of moths) that Bosco can work for Kamana .

c) Supposing that Bosco can continue to work indefinitely,

$$f(t) = 50000 + \frac{10000}{t}$$



The maximum salary of Bosco is the salary gotten after the first month (t=1) this is

50000Frw +10,000Frw=60,000Frw

The salary is decreasing. The minimum salary is 50,000Frw.

d) Even though Bosco has a monthly bonus of 50,000Frw, the salary is reducing after every month. The bonus is good but not motivating as the salary decreases.

e) As the salary is reducing from 60,000Frw to 50,000Frw, the answer will depend on the decision for every student teacher who can decide whether or not he/she can accept a salary which is less than 60,000Frw per month.

1.9 Additional activities

A small ball is projected vertically upward from the top of a building with the initial velocity $v_0 = 144\ m/\sec$. Its distance $s(t)$ in meter above the ground after t seconds is given by the equation $s(t) = -16t^2 + 144t + 100$.

a) What is the distance $s(t)$ at the initial time when $t = 0$?

b) Make a table of variation for $s(t)$ to show the distance from the initial time t=0 to t=10 seconds.

c) Use the table to draw the graph of $s(t)$ and show the position of the ball at t = 5 seconds

d) Discuss the parity of the function $s(t)$.

Solution:

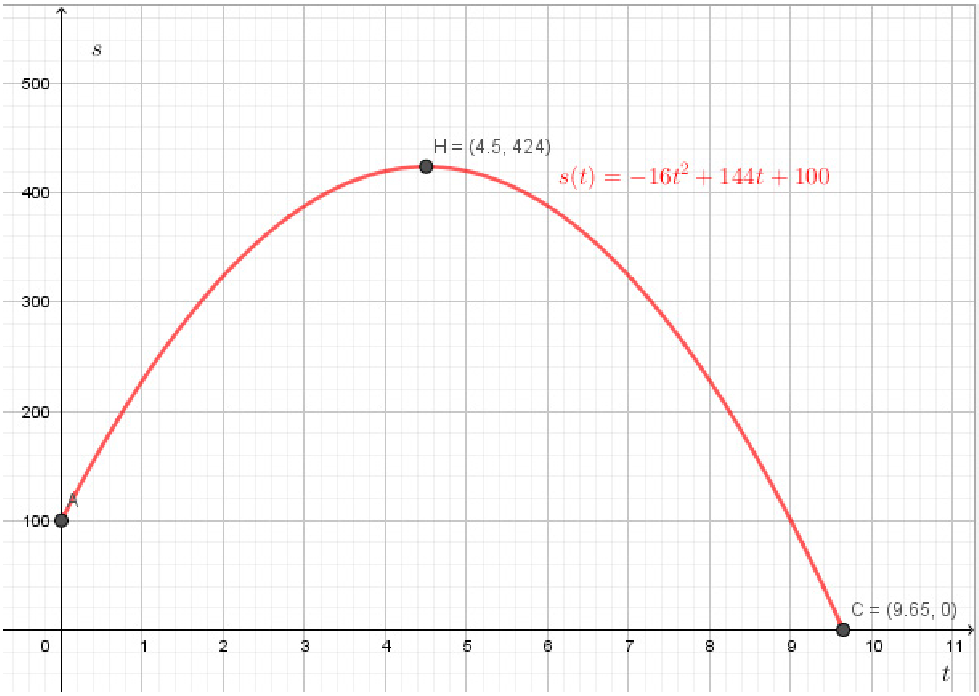
$s(t) = -16t^2 + 144t + 100$ represents the height or the vertical distance above the ground.

a) At the initial time $t = 0$, then the distance is $s(0) = -16(0) + 144(0) + 100 = 100$

This distance is 100m.

b)

t	0	1	2	3	4	5	6	7	8	9	$\frac{483}{50}$
$s(t)$	100	228	324	388	420	420	388	324	328	100	0



The graph shows that the ball falls on the ground between $t=9$ and $t=10$.

To find this time, you must solve the equation $s(t)=-16t^2+144t+100=0$.

$t=\frac{483}{50}$ unit of time.

UNIT 2

INTRODUCTION TO LOGIC

2.1 Key unit competence

Use Mathematical logic as a tool of reasoning and decision making in daily life.

2.2 Prerequisite

Student-teachers will perform well in this unit if they refer to the types of sentences as learnt it in English grammar.

2.3 Cross-cutting issues to be addressed:

- **Inclusive education**: promote the participation of all student-teachers while teaching)
- **Peace and value Education**: During group activities, the teacher will encourage student-teachers to help each other and to respect opinions of colleagues.
- **Gender:** During group activities try to form heterogeneous groups (with boys and girls) or when student-teachers start to present their findings encourage both (boys and girls) to present.

2.4 Guidance on introductory activity

- Form groups of student-teachers that and guide them to work on the introductory activity.
- Walk around all groups to provide pieces of advice where necessary.
- After a given time invite student-teachers to present their findings and harmonize them.
- During the presentation, let student-teachers discover the concept of logic.
- Through class discussions, let student-teachers think of different possible solutions and justify their validity.

Expected answers for introductory activity 2

A) The meaning of propositional statement: In logic, a propositional statement is a sentence or expression that is either true or false. Generally speaking, a statement is proposition because it makes a proposition about the world that it asserts a truth.

B) 1. Don't eat the daisies! NOT PROPOSITION, it is an exclamation.

2. My dog is called Didi. PROPOSITION

3. Do you enjoy reading novels? NOT PROPOSITION, it is a question.

4. The jokes are great. PROPOSITION

5. Mozart composed classical music. PROPOSITION

6. The camera is not a Kodak. PROPOSITION

7. This statement is false. PROPOSITION

8. Use the quadratic formula on that one. NOT PROPOSITION, it is a command.

2.5. List of lessons/sub-heading

#	Lesson title	Learning objectives	Number of periods
0	Introduction activity	To arouse the curiosity of student teachers on the content of unit 2.	1
1	Simple statement and compound statements	Give example of a logical statement Formulate compound statements using simple statements and vice versa.	2
2	Truth values and Truth tables	Deduce the truth values of a given proposition; Draw the truth table of a composite proposition.	1
3	The negation **"Not"**	Use correctly negation of logical statements in daily life	1

4	The conjunction **"and"**	Use correctly conjunction statements in daily life	1
5	The disjunction **"or"**	Use correctly disjunction statements in daily life	2
6	The conditional statement **"if ,..., then"**	Use correctly conditional statements in daily life and Determine the converse, inverse and contrapositive of a conditional statement	2
7	The bi-conditional statement **"if and only if"**	Use correctly bi-conditional statements in daily life statements.	2
8	Tautology and contradiction	Evaluate that a given logic statement is tautology or contradiction.	2
10	Quantifiers: Universal quantifier **"for all"** and **Existence** quantifier **"there exists"**	Use correctly universal quantifiers in logical statements of daily life	2
11	Quantifiers: Existence quantifier **"there exists"**	Use correctly existence quantifiers in logical statements of daily life	2
12	Negation of quantifiers	Use correctly the negation of quantifiers in logical statements from ordinal and official dialogue.	2
13	Applications	Apply conditional statements to make a true conclusion using syllogism.	2
14	End unit Assessment		2
Total periods			24

Lesson 1: Simple statement and compound statements

a) Learning objective:

Give example of a logical statement, and formulate compound statement using simple statements and vice versa.

b) Teaching resources:

Student-teacher's book and other reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they refer to the correct forms of sentences with clear examples as it was learnt in English.

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask students to use their books to discuss the activity 2.1 and motivate them to differentiate simple statement from compound statement. Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize how to form a compound statement;
- Use different probing questions to guide students to explore examples and the content related to **simple statement** and compound statements given in the student's book and help them to define the **truth-value of a statement.**
- Guide student-teachers to perform individually **application activity 2.1** to assess their competences.

Answers for Activity 2.1

1. T
2. F
3. Neither true nor false

4. F
5. Neither true nor false
6. Neither true nor false
7. T

Answers for application activity 2.1

1. Answers of question 1:
   a) A statement, whose truth value is true.
   b) Not a statement. It is an imperative sentence.
   c) A statement, whose truth value is false
   d) A statement, whose truth value is true.
   e) A statement, whose truth value is true.
   f) Not a statement. It is an exclamative sentence.

2. a) F
   b) T
   c) T
   d) F
   e) T

Lesson 2: Truth values and truth tables

a) Learning objective:

- Draw the truth table of a proposition,
- Draw the truth table of a composite proposition.

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of logical statement as learnt in 1st lesson of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 2.2** from student-teacher's book and introduce the concept logical statement.
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings,
- Use different probing questions to guide students to explore examples and the content related to values of a compound statement and then to complete the truth table**.** Students can also give their own examples of statements.
- Guide student-teachers to perform individually **application activity 2.2** to assess their competences.

Answer to application activity 4.5

Are these sentences proposition? If yes, give their truth values

a) Uganda is a member of East African Community. Yes; the truth value is T

b) The sun shines. Yes; the truth value is T

c) Paris is in England. Yes; the truth value is F

d) Come to class! No; no truth value

e) The sum of two prime numbers is even. Yes; the truth value is F

Answers for application activity 2.2

1)

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

2)

p	q	r	s
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

Lesson 3: "Negation" as a logical connective

a) Learning objective:

Use correctly negation of logical statements in daily life

b) Teaching resources:

T-square, ruler, Student-teacher's book and other reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they refer to negative form of a sentence, the concept of logical statement and truth table as it was learnt in the previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and guide them to attempt the activity 2.3.1 from student-teacher's book and to introduce the concept of negation for a statement;
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and guide students to formulate the negation of a given statement;
- Use different probing questions to guide students to explore examples and the content related to the negation of a simple and compound statement and related truth table, students can also give their own examples.
- Guide student-teachers to perform individually application activity 2.3.1 to assess their competences.

Answers of Activity 2.3.1

1) Given statements P: I am strong and Q: I can jump, try to make a compound statement formed by P and Q in different ways. What are the different connecting words that can be used?

Answer:

Various answers can be suggested by students-teacher and the correct are the following

a) I am strong and I can jump.

b) I am strong or I can jump

c) If I am strong then I can jump

d) I am strong if and only if I can jump

2) 1. Jack is not running.

2. Ronald smiles.

3. She is a football player.

4. -3 is not a natural number.

5. Mathematic is not needed in languages Education option.

Answers of Application Activity 2.3.1

1.

a) Today is not raining.

b) Sky is not blue

c) My native country is not Rwanda.

d) Bony is not smart nor healthy.

2.

p	q	r	$\neg p$	$\neg q$	$\neg r$
T	T	T	F	F	F
T	T	F	F	F	T
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	T
F	f	F	T	T	F
F	F	F	T	T	T

Lesson 4: "Conjunction" as a logical connective

a) Learning objective

Use correctly conjunction of logical statements in daily life.

b) Teaching resources:

T-square, ruler, student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding in the concept of compound statement, truth values and truth table as it is learnt in the 1st and 2nd lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the activity 2.3.2 from student-teacher's book and introduce the concept of conjunction of statements;
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and guide students to use correctly the conjunction in a compound statement;
- Use different probing questions to guide students to explore examples and the content related to the conjunction of two statements, its negation and related truth table**.** Students can also give their own examples.
- Guide student-teachers to perform individually application activity 2.3.2 to assess their competences.

Answer for activity 2.3.2

Given two propositions: p: I am at school, q: it is raining; Discuss the truth value of the compound propositions:

a) "I am at school and it is raining" T

b) “I am not at school and it is raining”, F

c)“I am not at school and it is not raining”. F

Answer for application activities 2.3.2

1. If p stands for the statement “ *It is cold*” and q stands for the statement “ *It is raining*”, then what does $\neg q \wedge \neg q$ stands for “ *It is not cold and it is not raining”*

Truth table of $\neg q \wedge \neg q$

p	q	$\neg p$	$\neg q$	$\neg q \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2. .

				a)	b)	c)	d)
p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge q$	$p \wedge \neg q$	$\neg(p \wedge q)$
T	T	F	F	T	F	F	F
T	F	F	T	F	F	T	T
F	T	T	F	F	T	F	T
F	F	T	T	F	F	F	T

3. i) True

ii) False

iii) False

iv) False.

vi) False

vii) False (weight is different from mass).

Lesson 5: "Disjunction" as a logical connective

a) Learning objective:

Use correctly disjunction in logical statements of daily life.

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they refer to previous lessons on: compound statement, truth values and truth table as they were learnt in this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the activity 2.3.3 from student-teacher's book where they are going to work on disjunction of statements;
- Move to every group to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and guide students to make a compound statement using the connective "or";
- Use different probing questions to guide students to explore examples and the content related to the disjunction of two statements, its negation and related truth table. Students can also give their own examples.
- Guide student-teachers to perform individually application activity 2.3.3 to assess their competences.

Answer for activity 2.3.3

1) i) if they ask to choose 2 boys and 2 girls, you will choose 4students.

ii) if they ask to choose two girls or two boys, you will choose 2students of the same sex.

2. a) T

b) F

c) F

Answer for application activity 2.3.3

1. Translation in symbolic form

a) Let p: Bwenge reads News Paper; q: Bwenge reads Mathematics book, Bwenge reads News Paper or Mathematics book is translated symbolically as $p \vee q$

b) Let p: Rwema is a student-teacher, q: Rwema is a book seller, thus Rwema is a student-teacher or not a book seller is translated symbolically as $p \vee \neg q$

2. If p is a false statement, and q is a true statement.

a) The truth-value of the compound statement $\neg p \vee q$ is **true**

b) The truth-value of the compound statement $p \vee \neg q$ is **false**

c) The truth-value of the compound statement $p \vee q$ is **true**; the truth-value of the compound statement $\neg p \vee \neg q$ is **true**.

				1	2	3	4	5
p	q	$\neg p$	$\neg q$	$p \vee q$	$p \vee \neg q$	$p \wedge (p \vee \neg q)$	$\neg(p \vee q) \wedge (\neg p \vee \neg q)$	$(\neg q \vee p) \wedge (\neg p \vee q)$
T	T	F	F	T	T	T	F	T
T	F	F	T	T	T	T	F	F
F	T	T	F	T	F	F	F	F
F	F	T	T	F	T	F	T	T

Lesson 6: Conditional statement

a) Learning objective: Use correctly conditional statements in daily life and determine the converse, inverse and contrapositive of a conditional statement.

b) Teaching resources:

T-square, ruler, Student-teacher’s book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they refer to concepts of truth values, truth table, negation, conjunction and disjunction connectives as they were learnt in the previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 2.3.4** from student-teacher's book and introduce the concept of conditional statements;
- Move around to ensure all student-teachers in groups participate actively;
- Call upon groups to present their findings.
- Harmonize their findings and guide students to use the conditional "if" in a compound statement;
- Use different probing questions to guide students to explore examples and the content related to conditional statements and related truth table**.** Students can also give their own examples.
- Guide student-teachers to perform individually **application activity 2.3.4** to assess their competences.

Answer for activity 2.3.4

1. i) The hypothesis (premise) is "If Samantha's health is good" and the conclusion (consequent) is "she will go to the party".

2) If Samantha's health is not good, then she will not go to the party.

Samantha will go to the party if her health is good.

Samantha will not go to the party if her health is good.

Answer for application activity 2.3.4

1. Let p : Mico is fat

q : Mico is happy

a) If Mico is fat then she is happy. in symbolic form $p \Rightarrow q$

b) Mico is unhappy implies that Mico is thin. In symbolic form as $\neg q \Rightarrow \neg p$.

2. Statements in symbolic form and their truth values

a) Let p be "n is prime", q be "n is odd" and r be "n is 2". We have $p \rightarrow (q \vee r)$

b) Let p be "x is negative", q be "x is positive" and r be "x is 0". We have $\neg p \rightarrow (q \vee r)$

c) Let p be "Tom is Ann's father", q be "Jim is her uncle" and r be "Sue is her aunt". We have $p \rightarrow (q \wedge r)$.

Lesson 7: Bi-conditional statement

a) Learning objective:

Use correctly bi-conditional statements in daily life statements.

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they refer to the concepts of conditional statement as it was learnt in the previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 2.3.5** from student-teacher's book and introduce the concept of bi-conditional statements,
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, insisting on the use of "if close", and "if …. and only if …." expressions;
- Use different probing questions to guide students to explore examples and the content related to bi-conditional statements and related truth table**.** Students can also give their own examples.
- Guide students to explore the equivalent propositions and the related examples;
- Guide student-teachers to perform individually **application activity 2.3.5** to assess their competences.

Answers for activity 2.3.5

1. a) $r \Rightarrow s$ is True b) $s \Rightarrow r$ is True c) $(r \Rightarrow s) \wedge (s \Rightarrow r)$ is True

2. a) $r \Rightarrow s$ is True b) $s \Rightarrow r$ is True c) $(r \Rightarrow s) \wedge (s \Rightarrow r)$ is True

Answer for application activity

1. If r is a false statement, s a true statement, then

   a) the truth-value of the compound statement $(\neg r) \Leftrightarrow s$ is True

   b) the truth-value of the compound statement $r \Leftrightarrow (\neg s)$ is True

   c) the truth-value of the compound statement $r \Leftrightarrow s$ is False

   d) the truth-value of the compound statement $\neg(r \Leftrightarrow (\neg s))$ is **False**

2. Construct the truth table for

a) $p \leftrightarrow q$ And $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$p \leftrightarrow q$ And $(\neg p \vee q) \wedge (\neg q \vee p)$

p	q	$p \Leftrightarrow q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \wedge (\neg q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$\neg(p \leftrightarrow q)$ and $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$\neg(p \Leftrightarrow q)$	$p \vee q$	$\neg(q \wedge p)$	$(p \vee q) \wedge \neg(q \wedge p)$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	T	T	T	T
F	F	F	F	T	F

d) $\neg(p \leftrightarrow q)$ and $(p \wedge \neg q) \vee (\neg p \wedge q)$

p	q	$\neg(p \Leftrightarrow q)$	$p \wedge \neg q$	$(\neg p \wedge q)$	$(p \wedge \neg q) \vee (\neg p \wedge q)$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F

From these results, we note that

a) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are equivalent

b) $p \leftrightarrow q$ and $(\neg p \vee q) \wedge (\neg q \vee p)$ are equivalent

c) $\neg(p \leftrightarrow q)$ and $(p \vee q) \wedge \neg(p \wedge q)$ are equivalent

d) $\neg(p \leftrightarrow q)$ and $(p \wedge \neg q) \vee (\neg p \wedge q)$ Are equivalent

3.

p	q	$\neg p$	$\neg q$	$(\neg p \Rightarrow \neg q)$	$q \Rightarrow p$	$p \Rightarrow q$
T	T	F	F	T	T	T
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	F	T	T	T	T	T
				Inverse	converse	Implication

From the truth table the inverse is not equivalent to implication and the converse is not equivalent to the implication , thus neither the converse nor the inverse of an implication are equivalent to the implication.

4. Practical activity: Students will discuss in small groups guided by the tutor and they will have various conditional statements.

Lesson 8: Tautology and Contradiction

a) Learning objective:

Show that a given logical statement is a tautology or a contradiction

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on all logical connectives learnt in the previous lessons of this unit two.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 2.4** from student-teacher's book and introduce the concepts "tautology" and "contradiction",
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, insisting on the compound statement that is always true and the compound statement that is always **false** regardless of the truth values of the individual statements substituted for statement variables.
- Use different probing questions to guide students to explore examples and the content related to tautology and contradiction and related truth tabl**e.** Students can also give their own examples.
- Guide student-teachers to perform individually **application activity 2.4** to assess their competences.

Answers for Activity 2.4

1.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

2.

p	$\neg p$	$p \wedge \neg p$
T	F	T
F	T	T

3.

p	q	$\neg p$	$p \wedge q$	$\neg p \wedge (p \wedge q)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

4.

p	q	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \wedge q) \vee (p \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	T

Yes; 1, 2 and 4 are showing T in the last column while 3 is showing F

Answers of Application Activity 2.4

A.

1.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \wedge \neg(p \wedge q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	F
F	F	F	T	F

$p \wedge \neg(p \wedge q)$ is neither tautology nor contradiction

2.

q	r	$\neg q$	$q \wedge r$	$\neg q \wedge (q \wedge r)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

$\neg q \wedge (q \wedge r)$ is a contradiction

3.

p	q	$p \wedge q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	F
F	F	F	T	F

$(p \wedge q) \wedge \neg(p \vee q)$ is neither tautology nor contradiction

4.

p	r	$p \vee r$	$\neg r$	$(p \vee r) \vee \neg r$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

$(p \vee r) \vee \neg r$ is a tautology

B. Practical activity; the answers will vary (in small group learners will discuss the question; and the tutor will guide them to get conclusion).

Lesson 9: The universal quantifier "for all"

a) Learning objective

Use correctly universal quantifiers in logical statements of daily life

b) Teaching resources:

Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they refer to previous lessons of this unit 2.

d) Learning activities

- Invite students to work out the activity 2.5.1 in groups;
- Check if everybody is engaged in the given activity;
- Choose groups with different working steps to present their findings to the whole class and then harmonize their works to provide the lesson summary;
- Guide students to explore the examples and motivate them to work out individually the application activity 2.5.1 to check the skills they have acquired.

Answers of Activity 2.5.1

1.a) T

b) F

2. T

3. P "All men eat banana" F

Q "All men are mortal" T

Answers of Application Activity 2.5.1

1. The values of y which make p true are every real number y: $y > 10 - x$

2. The truth value for the following statements:

a) Some rectangles are squares. T

b) All squares are rectangles. T

c) Every language student-teacher must take a logic mathematics course. F

d) $\forall x \in \mathbb{N}, x + 4 > 4.$ T

Lesson 10: The Existence quantifier "there exists"

a) Learning objective:

Use correctly existence quantifiers in logical statements of daily life.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they have a good concept of previous lessons in this unit 2.

d) Learning activities

- Invite students to work out the activity 2.5.2 in groups;
- Check if everybody is engaged in the given activity;

- Choose randomly one group to present their findings to the whole class and then harmonize their works insisting on the truth set of a statement with a quantifier;
- Guide students to work through the examples and motivate them to work out individually the application activity 2.5.2 to check the skills they have acquired.

Answers of Activity 2.5.2

1. a) F

   b) T

2. a) The statement is true for $y \in \{1,2,3\}$ the truth set is $\{1,2,3\}$

b) It is open sentence in x. the truth set is A itself.

Answers for application activity 2.5.2

1. a) $(\exists x)[p(x) \wedge q(x)]$, where $p(x)$ - x cried out for help and $q(x)$ -x called the police

   b) $(\forall x)[\neg p(x)]$, where $p(x)$ - x can ignore her.

2. The symbolic form means the following: There exist real numbers that are greater than 9, and not all real numbers are equal to 9. True.

3. The answers will vary (in small groups student –teachers guided by tutor will discuss the different argumentations).

Example: Some people are very intelligent (True),

There exist women who are priests in Rwanda (False).

Lesson 11: Negation of quantifiers

a) Learning objective

Develop and show mutual respect.

b) Teaching resources

- T-square, ruler.
- Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites / Revision / Introduction

Student-teachers will perform well in this lesson if they refer to the concept of quantifiers learnt in the previous lesson of this unit.

d) Learning activities

- Organize the student-teachers into groups;
- Introduce the topic by making review on negation of compound statement with connectives;
- Let students attempt the activity 2.5.3 from student-teacher's book;
- Check if everybody is engaged;
- Invite group representatives to present their findings to the whole class, and then help students to correct answers which are false and harmonize them.
- Lead student-teachers to work through examples and the content and then work individually the application activities 2.5.3 for assessing their competences.

Answers of Activity 2.3.5

1. Some grapefruits have not red color;
2. No celebrities are ugly (not beautiful).
3. Some people weigh more than five hundred kg.
4. No one is more than ten meters tall.
5. Some snakes are not poisonous.
6. No mammals can stay under water for two days without surfacing for air.

7. Some birds cannot fly;

Note: There are a great variety of different ways for writing the negation.

Answers of Application Activity 2.5.3

1. No student is mathematics major (or All students are not mathematics majors)

   $\forall x\, p(x)$, where $p(x) \sim x$ is not mathematics major

2. Some real numbers are not positive, negative or zero

   $\exists x\, p(x)$, where $p(x) \sim x$ is not positive, negative or zero

3. Some good boys do not do fine

   $\exists x\, p(x)$, where $p(x) \sim x$ is does not do fine

4. All desk in our classroom are not broken (or No desk in our classroom is broken)

   $\forall x\, p(x)$, where $p(x) \sim x$ is not broken

5. Some lockers must not be turned in by the last day of class

   $\exists x\, p(x)$, where $p(x) \sim x$ is not be turned in by the last day of class

6. Some haste does not make waste

   $\exists x\, p(x)$, where $p(x) \sim x$ does not make waste.

Lesson 12: Applications of logic in real life

a) Learning objective

Apply conditional statements to make a true conclusion using syllogism.

b) Teaching resources

T-square, ruler, Student-teacher's book and other reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction

Student-teachers will learn better this lesson if they refer to all previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **activity 2.6** from student-teacher's book and ask them to make the conclusion of premises in given statements;
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings,
- Use different probing questions to guide students to explore examples and the content related to the conclusion of premises through different types of syllogism**.** Students can also give their own examples.
- Guide student-teachers to perform individually **application activity 2.6** to assess their competences.

Answer for activities

It is known that you must have the correct form and true premises to reason deductively toward a true conclusion.

The following are examples of arguments that need true conclusions. Try to conclude:

a) If you live in Nyarugenge, then you live in Kigali.

If you live in Kigali, then you live in Rwanda.

Therefore, ***if you live in Nyarugenge you live in Rwanda.***

b) If a triangle is isosceles, then it has two equal sides.

If a triangle has two equal sides, then it has two equal angles.

Therefore, ***if a triangle is isosceles then it has two equal angles***.

c) If you study the whole student book, then you will pass the exam.

You study the whole student book;

Therefore, ***you will pass the exam***.

Answer for application activity 2.6

1) Rewriting the argument into conditional statements, we get the following:

If one is a good chess player, then one wears glasses.

Sylvia is a good chess player.

Therefore, Sylvia wears glasses.

The conclusion respects the form of premises but this conclusion is not true because the major premise (If one is a good chess player, then one wears glasses) is not always true.

Note: *We need both the correct form true premises to ensure true conclusion.*

2) i) When it is midnight, I am asleep.

I was asleep.

Therefore, it was midnight. ***Converse error because when $A \Rightarrow B$, it does not necessary imply $B \Rightarrow A$.***

In addition, $A \Rightarrow B$ does not imply $\neg A \Rightarrow \neg B$.

ii) All Rhode Island Red hens lay brown eggs.

My hen, Marguerite, is a Rhode Island Red.

Therefore, Marguerite lays brown eggs.

This is related to *affirming the antecedent, a correct argument*

iii) If *ABCD* is a square, it has four sides.

If it has four sides, then it is a quadrilateral.

Therefore, if *ABCD* is a square, it is a quadrilateral.

This is the *Correct hypothetical syllogism*

iv) If a triangle is equilateral, then it has three equal sides.

ABC does not have three equal sides.

Therefore, ABC is not equilateral.

This is the *Denying of the consequent, it is a correct argument*

2.6 Summary of the unit

A proposition (statement or verbal assertion) is a sentence which is either true or false but not both.

Truth table:

We can always summarize the truth values of compound statement in a table called truth table. If the compound statement contains n distinct components, we need to consider 2^n possible combinations of truth values in order to obtain the truth table.

Logical connectives

Given statements p and q, we can combine them with various connectives. The most five useful logical connectives are negation, conjunction, disjunction, conditional and biconditional.

The negation of a statement by introducing the word "not" denoted by prefixing the statement has opposite truth value from the statement. It is denoted by $\neg p$ or $\bar{p}$ or $\sim p$.

The conjunction (and) of two statements p and q is denoted $p \wedge q$. It has the truth value true whenever both p and q have the truth value true; otherwise it has the truth value false.

The disjunction (or) of two statements p and q is denoted $p \vee q$. It has the truth value false only when p and q have truth value false, otherwise it has the truth value true.

The conditional statement $p \Rightarrow q$ (read $"p \, implies \, q"$) has the truth value false when q has the truth value false while p has truth value true, otherwise it has the truth value true.

The biconditional statement $p \Leftrightarrow q$, which we read "p if and only if q" or "p is equivalent to q" is true if both p and q have the same truth values and false if p and q have opposite truth values.

Tautology and contradiction

A tautology is a statement formula that is always true regardless of the truth values of the individual statements substituted for its statement variables.

A contradiction is a statement formula that is always false regardless of the truth values of the individual statements substituted for its statement variables.

Quantifiers

The quantifiers help decide the frequencies with which a predicate becomes true, whether it is satisfied by no element of a domain, or one element, or some elements, or all elements.

There are two basic quantifiers:

The existential quantifiers $\exists$ ("there exist"), and

The universal quantifier $\forall$ **("**for all")

These quantifiers are negated as follows

$\neg\left[\forall xp\left(x\right)\right]\equiv\exists x\left[\neg p\left(x\right)\right]$

$\neg\left[\exists xp\left(x\right)\right]\equiv\forall x\left[\neg p\left(x\right)\right]$

2.7 Additional Information for the tutor

Tutor may facilitate student-teacher by providing **examples related to real life**;

For example, on the conjunction connective, they can use the following statement and find out the rule of conjunction "and";

"Abijuru and Bwenge are going to the market" $\left(p\wedge q\right)$.

They find that it is true that Abijuru is going and true that Bwenge is going, then they would know this statement to be true. But if Abijuru is going, Bwenge is not, then the above statement will be false, because the statement claims both are going, and in this case only Abijuru is going. They deduce also that the above statement will be false in the case Abijuru is not going, even though Bwenge is, and in the case in which neither Abijuru nor Bwenge are going.

This, then, will be the rule for conjunction connective $\left(p\wedge q\right)$that it is true when all the components are true, and false otherwise.

On the disjunction connective, consider the following example when someone says "Abijuru or Bwenge is going to the market". If only one of them is going, the statement would still be true. The only time the disjunction would be false, would be when neither are going. This, then, will be our rule for disjunction. A disjunction statement $\left(p\vee q\right)$is true when either one or both of the simple statement parts are true, and false only when both parts are false.

We defined $p \Rightarrow q$ and $q \Rightarrow p$ according to the $(\Rightarrow)$ rule and then put them together by the $(\wedge)$ rule. This then will be our rule for $(\equiv)$: A biconditional statement $(p \equiv q)$ is true only when both components have the same truth value and false otherwise.

At this point, a little common-sense reflection shows why this is so. When a mother tells her son that he will go out if and only if he cleans his room, the mother's statement would be true in only two cases:

When the son cleans his room and the mother fulfills her promise by letting the son go out, and when the son does not clean his room and suffers the consequence of not going out.

In the other two cases when the son does not clean his room and the mother lets him go out anyway, and when the mother does not let her son go out even after he has cleaned his room the mother's statement would be false.

One can show that two logical expressions are equivalent by showing they have the same truth values in all circumstances. For example, it is not necessary to have the connective ⇒ because $p \Rightarrow q$ is equivalent to $\neg p \vee q$, as can be seen in the following truth table

p	q	$\neg p$	$p \Rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

An equivalent way of writing p implies q is "If p, then q." the form usually used in English writing, and is called an implication. There are three statements that are gotten from the implication:

- Converse: If q, then p.
- Inverse: If $\neg p$, then $\neg q$.
- Contrapositive: If $\neg q$, then $\neg p$.

We note that the implication and the contrapositive are equivalent because both are equivalent to $\neg p \vee q$. Similarly, the inverse and the converse are equivalent. But the converse and inverse are not equivalent to the implication and the contrapositive. A simple example is the implication "If there is light coming through the window, then there is light in the room." The converse is "If there is light in the room, then there is light coming through the window." a false statement at night or in a room without windows. So be careful with your implications but remember that you can prove the contrapositive and still have proven the implication.

The existential quantifier (there exists) has many forms in English: "some or for some or there is $a(n)$".

The universal quantifier (for all) may be expressed as – for each or for any or any or every or for every. These are used in sentences with variables in them.

2.8. Answers for the end unit assessment

1. a) Proposition;

   b) Not a proposition;

   c) Not a proposition;

2. The negation is "Today is not Monday".

3. The conjunction of the given propositions is the proposition "Today is Sunday and the moon are made of cheese".

4. The disjunction of the given propositions is the proposition "Today is Sunday or the moon is made of cheese".

5. For a compound statement made from n statements, 2^n rows, not counting the top one, are needed to construct the truth table

6. Make a truth table for this proposition:

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

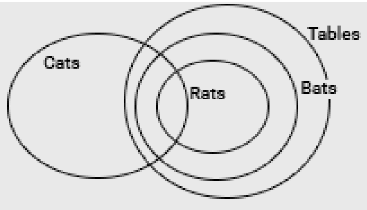
We see that $[p \wedge (p \to q)] \to q$ is a tautology.

7. The moon is made of cheese if and only if 1=2". It is False.

8. The truth table of $(\neg p \to q) \wedge r$ is

p	q	r	$\neg p \to q$	$(\neg p \to q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

9. Answer Key: 4

Explanation: 

Clearly, from the diagram Conclusion I is true.

2.9 Additional activities

2.9.1 Remedial activities

1. Find out which of the following sentences are statements and which are not.

    i. The moon revolves around the sun.

    ii. A triangle has four sides.

    iii. $\sqrt{3}$ is an irrational number.

    iv. What is your age?

    v. Africa is a continent.

    vi. $x+7=10$.

    vii. The earth is a planet.

    viii. $7+9>12$.

2. Write down the truth value of the following statements.

i. 1 is a prime number.

ii. Every square is a rectangle.

iii. All real numbers are rational.

iv. Karongi is in western province.

v. Uganda is north-west of Rwanda.

Solution

1. To find out which of the following sentences are statements and which are not.

   i. "The moon revolves around the sun". This is false and so it is a statement.

   ii. A triangle has four sides. This is false and so it is a statement.

   iii. $\sqrt{3}$ is an irrational number. This is true and so it is a statement.

   iv. What is your age? This is not a statement as it is an interrogative sentence.

   v. Africa is a continent. This is true and so it is a statement

   vi. The earth is a planet. This is true and so it is a statement

   vii. $7+9>12$. This is true and so it is a statement.

2. The truth values

   I. The statement has truth value False

   II. True

   III.False.

   IV. True.

   V. False.

2.9.3 Consolidation activities

1) If the statements p, q, r all have truth value "True" and u, v, w are false statements, which of the following are true and which are false?

1	$(r \vee w) \wedge (v \vee q)$	13	$u \Rightarrow (v \Rightarrow r)$
2	$(p \wedge q) \vee (u \wedge v)$	14	$(p \Rightarrow q) \Rightarrow w$
3	$\neg(q \vee u) \wedge \neg(v \vee w)$	15	$[(u \Rightarrow v) \Rightarrow q] \Rightarrow w$
4	$\neg(r \vee q) \vee \neg(\neg u \wedge v)$	16	$[(q \Rightarrow w) \Rightarrow q] \Rightarrow w$
5	$\neg q \vee r$	17	$u \Rightarrow (q \Rightarrow w)$
6	$\neg(q \vee u)$	18	$[(u \Rightarrow p) \Rightarrow u] \Rightarrow u$
7	$\neg u \vee p$	19	$[u \Rightarrow (v \Rightarrow w)] \Rightarrow [(u \Rightarrow v) \Rightarrow w]$
8	$\neg(u \vee v)$	20	$\{ p \Rightarrow (q \Rightarrow r) \equiv \neg u\} \Rightarrow \{x \Rightarrow [(p \wedge q) \Rightarrow r]\}$
9	$\neg[(q \vee p) \vee (\neg p \vee q)]$	21	$\{[u \Rightarrow (v \Rightarrow w)] \Rightarrow [(u \wedge v) \Rightarrow w]\}$ $\equiv [(u \Rightarrow p) \Rightarrow (q \Rightarrow v)]$
10	$\neg[(\neg v \vee w) \vee (\neg w \vee v)]$	22	$(q \Rightarrow p) \equiv (\neg p \Rightarrow \neg q)$
11	$[p \wedge (q \vee r)] \wedge [(p \wedge q) \vee (p \wedge r)]$	23	$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$
12	$\neg[u \wedge (\neg p \vee w)] \vee [(u \wedge \neg p) \vee (u \wedge w)]$	24	$[p \Rightarrow (q \Rightarrow r)] \equiv [(p \wedge q) \Rightarrow r]$

2) In the question below are given three statements, followed by conclusions: I, II, III, IV. You have to take the given statements to be true even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusions logically follows from the given statements disregarding commonly known facts.

Statements: Some ships are boats. All boats are submarines. Some submarines are yatches.

Conclusion:

I. Some yatches are boats.

II. Some submarines are boats.

III. Some submarines are ships.

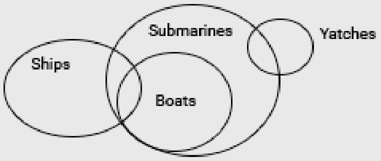
IV. Some yatches are ships

1. All follow
2. Only II and III follows
3. Only III follows
4. Only IV follows

Solution

Answer Key: 2

Explanation:



From the diagram we can infer that some submarines are boats and some submarines are ships. So 2^{nd} option.

3) Show, by constructing its truth table, that $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology.

Solution

To show that $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

Therefore, $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology.

4) Show, by constructing its truth table, that $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology.

Solution

To show that $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

Therefore, $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology

5) Consider the predicate $p(x, y)$: "$y = x + 3$". What are the truth values of the propositions $p(1,2)$ and $p(0,3)$?

Solution

$p(1,2)$ is the proposition "2 = 1 + 3" which is false. The statement $p(0,3)$ is the proposition "3 = 0 + 3" which is true.

6) Express the statement "Every student in this class has seen a computer" as a universal quantification.

Solution

Let $q(x)$ be the predicate "x has seen a computer". Then the statement "Every student in this class has seen a computer" can be written as $\forall x\, q(x)$, where the universe of discourse consists of all students in this class. Also, this statement can be expressed as $\forall x\,(p(x) \to q(x))$, where $p(x)$ is the predicate " x is in this class" and the universe of discourse consists of all students.

7) Express the negations of the following propositions using quantifiers. Also, express these negations in English.

a) "Every student in this class likes mathematics".

b) "There is a student in this class who has been in at least one room of every building on campus".

Solution

a) Let $l(x)$ be the predicate "x likes mathematics", where the universe of discourse is the set of students in this class. The original statement is $\forall x\, l(x)$ and its negation is $\exists x \neg l(x)$. In English, it reads "Some student in this class does not like mathematics".

b) Consider the predicates $p(z, y)$: "room z is in building y" and $q(x, z)$: "student x has been in room z ". Then the original statement is $\exists x \forall y \exists z \left(p(z, y) \wedge q(x, z)\right)$. To form the negation, we change all the quantifiers and put the negation on the inside, then apply De Morgan's law. The negation is therefore $\forall x \exists y \forall z \left(\neg p(z, y) \vee \neg q(x, z)\right)$, which is also equivalent to $\forall x \exists y \forall z \left(p(z, y) \rightarrow \neg q(x, z)\right)$. In English, this could be read "For every student there is a building on the campus such that for every room in that building, the student has not been in that room".

2.9.3 Extended activities

Determine which of the pairs of statements in the following are logically equivalent

a) $\neg\left[(p \wedge q) \wedge r\right]$ and $\neg\left[p \wedge (q \wedge r)\right]$

b) $(p \vee q) \vee r$ and $p \vee (q \wedge r)$

c) $\left[(\neg q \wedge p)\right] \wedge \left[p \wedge (\neg p)\right]$ and $(p \vee q) \wedge c$, where c is a contradictory

d) $\left[(\neg p \vee q)\right] \wedge \left[p \wedge (\neg q)\right]$ and $(p \wedge q) \vee t$, where t is a tautology

e) $(p \wedge q) \vee \left\{(\neg p) \vee \left[p \wedge (\neg q)\right]\right\}$ and $\left[(\neg q) \wedge q\right] \vee t$, where t is a tautology

Solution

a) Truth table of $\neg[(p \wedge q) \wedge r]$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$\neg[(p \wedge q) \wedge r]$
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

Truth table of $\neg[p \wedge (q \wedge r)]$

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$\neg[p \wedge (q \wedge r)]$
T	T	T	T	T	F
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	F	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

As the truth values in the last column of both tables are the same, the statement $\neg[(p \wedge q) \wedge r]$ is logically equivalent to $\neg[p \wedge (q \wedge r)]$

b) Truth table of $(p \vee q) \vee r$

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T

F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Truth table of $p \vee (q \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Since the truth values in the last column of both tables are not the same, the statement $(p \vee q) \vee r$ is not logically equivalent to $p \vee (q \wedge r)$.

c) Truth table of $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$

p	q	$\neg p$	$\neg q$	$\neg q \wedge p$	$p \wedge (\neg p)$	$[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

Truth table of $(p \vee q) \wedge c$, where c is a contradictory

p	q	c	$p \vee q$	$(p \vee q) \wedge c$
T	T	F	T	F
T	F	F	T	F
F	T	F	T	F
F	F	F	F	F

Getting that the truth values in the last column of both tables are the same, then statement $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$ is logically equivalent to $(p \vee q) \wedge c$, where c is a contradictory.

d) Truth table of $[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \wedge (\neg q)$	$[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

Truth table of $(p \wedge q) \vee t$, where t is a tautology

p	q	t	$p \wedge q$	$(p \vee q) \wedge c$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

As the truth values in the last column of both tables are not the same, the statement $[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$ is not logically equivalent to $(p \wedge q) \vee t$, where t is a tautology.

e) Truth table of $(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \wedge (\neg q)$	$(\neg p) \vee [p \wedge (\neg q)]$	$(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T

Truth table of $\left[(\neg p)\wedge q\right]\vee t$, where t is a tautology

p	q	t	$\neg p$	$(\neg p)\wedge q$	$\left[(\neg p)\wedge q\right]\vee t$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	T	T	F	T

As the truth values in the last column of both tables are the same, the statement $(p\wedge q)\vee\{(\neg p)\vee\left[p\wedge(\neg q)\right]\}$ is logically equivalent to $\left[(\neg p)\wedge q\right]\vee t$, where t is a tautology.

UNIT 3

POINTS, LINES AND GEOMETRIC SHAPES IN 2D

3.1 Key Unit competence

To be able to determine algebraic representations of lines and calculate the area of geometric shapes in 2D

3.2. Prerequisite

Student - teachers will perform well in this unit if they have a good background on:

- Linear equations learnt in S1;
- Vectors and their properties learnt in S2;
- Translations learnt in S3;
- Solving algebraically and graphically simultaneous linear equations in two unknowns learnt in year one.

3.3. Cross-cutting issues to be addressed

- **Inclusive education**: promote the participation of all student-teachers while teaching;
- **Peace and value Education**: During group activities, the teacher will encourage student teachers to help each other and to respect opinions of colleagues;
- **Financial education**: Guide students to discuss how to establish an equation illustrating the variation of money;
- **Gender:** Give equal opportunities to all students (girls and boys) to participate actively in all learning activities from the beginning to the end of the lesson.

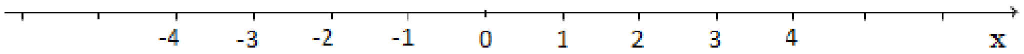
3.4. Guidance on introductory activity 3

- Invite student teachers to form groups and guide them to work on the introductory activity.
- Give time to students to analyse the activity;
- Invite group representatives to present findings in a whole class discussion;
- Harmonize students' answers and guide them to enhance how to make effective graduation of lines and axes for a Cartesian plane;
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, use different questions to arouse their curiosity on what is going to be leant in this unit.

Answer for introductory activity 2

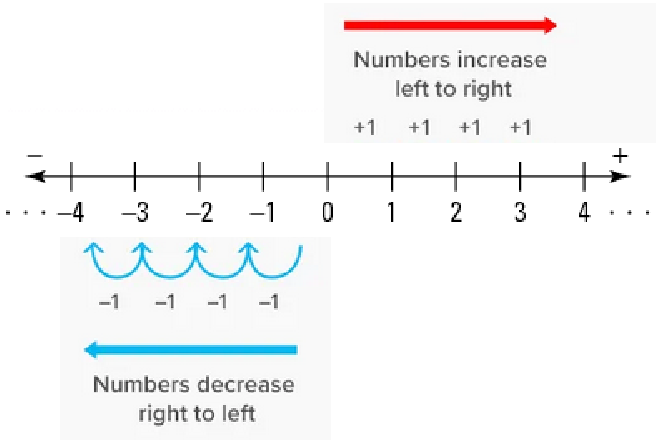
a) We all know that, numbers on the number line increase as one moves from left to right and decrease on moving from right to left.

Basing on this, Rukundo made a mistake. The correct number line he should have done is the following:



b) A number line is a straight with numbers placed at equal intervals or segments along its length. A number line can be extended infinitely in any direction.

c) The numbers on the number line increase as one moves from left to right and decrease on moving from right to left.



d) The Cartesian plane is a plane made by two fixed and perpendicular number lines (i.e. two directions) in which every point P is characterized by P (x, y) where x is measured as horizontal displacement on the axis OX and y is measured as vertical displacement on the y-axis.

3.5 List of lessons and sub-heading

#	Lesson title	Learning objectives	Number of periods
0	Introductory activity	To arouse the curiosity of student teachers on the content of unit 3.	1
1	Cartesian coordinate of a point in 2D.	-Define and plot the coordinates of a point in 2D.	2
2	Distance between two points in 2D	Calculate the distance between two points	2
3	Mid-point and Distance between two points in 2D	Determine the coordinate for the mid-Point of a segment in 2D.	1
4	Vector in 2D and Dot product	Describe a vector in 2 D and determine the dot product of 2 vectors.	2

5	Vector equation, parametric equation and Cartesian equation of a straight line	Determination of equation of a straight line given 2 points and Direction vector	2
6		Determination of equation of a straight line passing through a point $P(x_0, y_0)$ and parallel to a direction vector $\vec{v} = (a,b)$.	2
7		Equation of a straight line given the gradient and a point	2
8	Problems on points and straight lines in 2D: Positions, angles and Distance	Perform operations to determine the intersection, perpendicularity or parallelism of lines	2
9		Determine the distance between two lines	2
10	Geometric shapes in 2D	Identify the Geometric shapes in 2D	2
11	Area of Geometric shapes	Determine the perimeter and area of geometric shapes in 2D.	3
12	End unit Assessment		1
Total number of periods			24

Lesson 1: Cartesian coordinates of a point

a) Learning objectives:

Define and plot the coordinates of a point in 2D

b) Teaching resources

Learner's book and other reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, Math Type and GeoGebra software, smart class, …

c) Prerequisites / Revision / Introduction

Student - teacher will easily learn this unit, if they refer to:

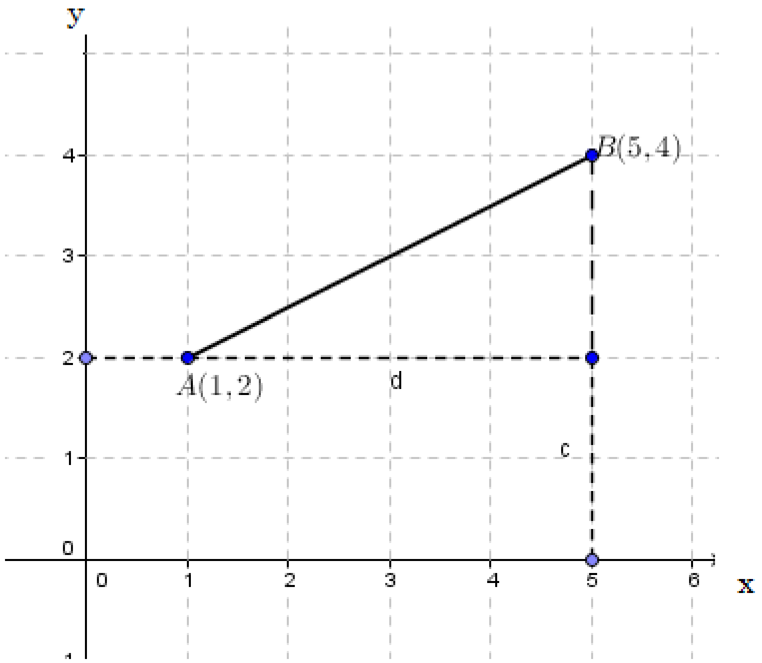
- Define the coordinate of a point in 2D as it is taught in S1 (UNIT3),
- Calculate the distance between two points in 2D S2 (UNIT7)

- Determine equations of a straight line S3 (UNIT6),
- Appreciate the importance of a point and a line in a plane (S2 and S3).

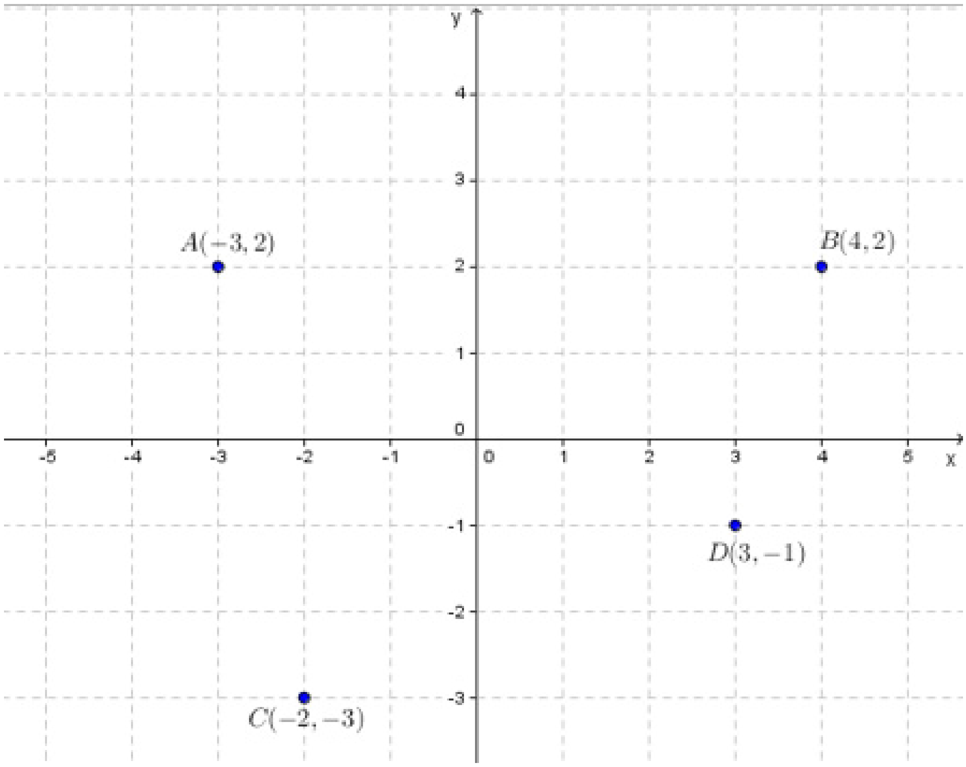
d) Learning activities

- Organize the student-teachers into small groups;
- Ask them to do the activity 3.1 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, help student-teachers plot a point and draw a line joining two points in the Cartesian plan;
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance the coordinate of a point and identify the point when coordinates were given;
- After this step, guide students to do the application activity 3.1, assess their competences and evaluate whether lesson objectives were achieved.

Answers for Activity 3.1



Answers for application activity 3.1



Lesson 2: Distance between two points in 2D

a) Learning objectives:

Calculate the distance between two points.

b) Teaching resources

Learner's book and other reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils.

c) Prerequisites / Revision / Introduction:

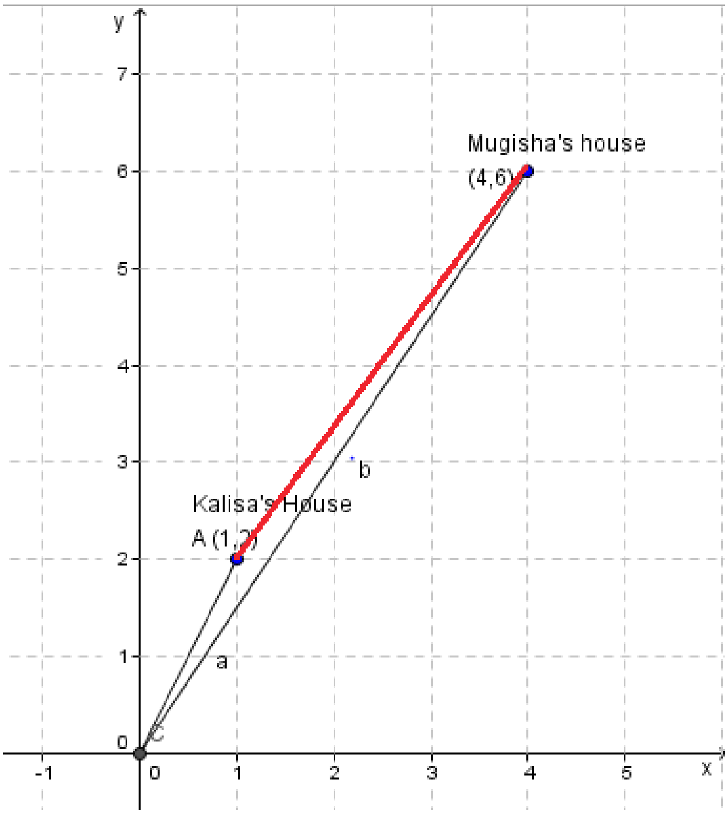
Student - teacher will easily learn this unit, if they are able to:

- Define the coordinate of a point in 2D as it is taught in S1 (UNIT3),
- Calculate the distance between two points in 2D learnt in S2 (UNIT7)
- Determine equations of a straight line. S3 (UNIT6),
- Appreciate the importance of a point and a line in a plane.

d) Learning activities

- Organize the student-teachers into small groups;
- Ask them to do the activity 3.2 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, help student-teachers to measure and to calculate the distance between two points in the Cartesian plane;
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance how to find the distance "*d*" between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the Cartesian plane;
- After this step, guide students to do the application activity 3.2, assess their competences and evaluate whether lesson objectives were achieved.

Answers for Activity 3.1



a) Here the distance is

$$d_{AB} = \sqrt{(4-1)^2 + (6-2)^2} \qquad d_{AB} = \sqrt{25} = |5| = 5$$

Answers for Application activity 3.2

1. a). $d_{SQ} = \sqrt{(-2-7)^2 + (-5-(-2))^2} = \sqrt{90} = 9.5$

   b). $d_{SQ} = \sqrt{(-2-7)^2 + (-5-(-2))^2} = \sqrt{90} = 9.5$

   c). $d_{AB} = \sqrt{(2-(-3))^2 + (7-5)^2} = \sqrt{29}$

   d). $d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{x-(x+4)^2 + (y-(y-1))^2} = \sqrt{17}$

2. The missing coordinates given CD, are the following

a). $d_{CD} = \sqrt{(6-x)^2 + (-2-2)^2}$

$$5^2 = 36 - 12x + x^2 + 16, \quad 0 = (x-3)\ (x-9) \quad \text{therefore} \quad \begin{vmatrix} x_1 = 3 \\ x_2 = 9 \end{vmatrix}$$

b). $5 = \sqrt{(4-1)^2 + (y+1)^2}$

$$5^2 = 9 + y^2 + 2y + 1, \quad 0 = (y-3)\ (y+5) \quad \text{therefore} \quad \begin{vmatrix} y_1 = 3 \\ y_2 = -5 \end{vmatrix}$$

Lesson 3: Mid-point and Distance between two points in 2D

a) Learning objectives:

Determine the coordinate for the mid-Point of a segment in 2D.

b) Teaching resources

Student's book and other reference textbooks to facilitate research, Mathematical set, calculator, Manila paper, markers, pens, pencils.

c) Prerequisites / Revision / Introduction:

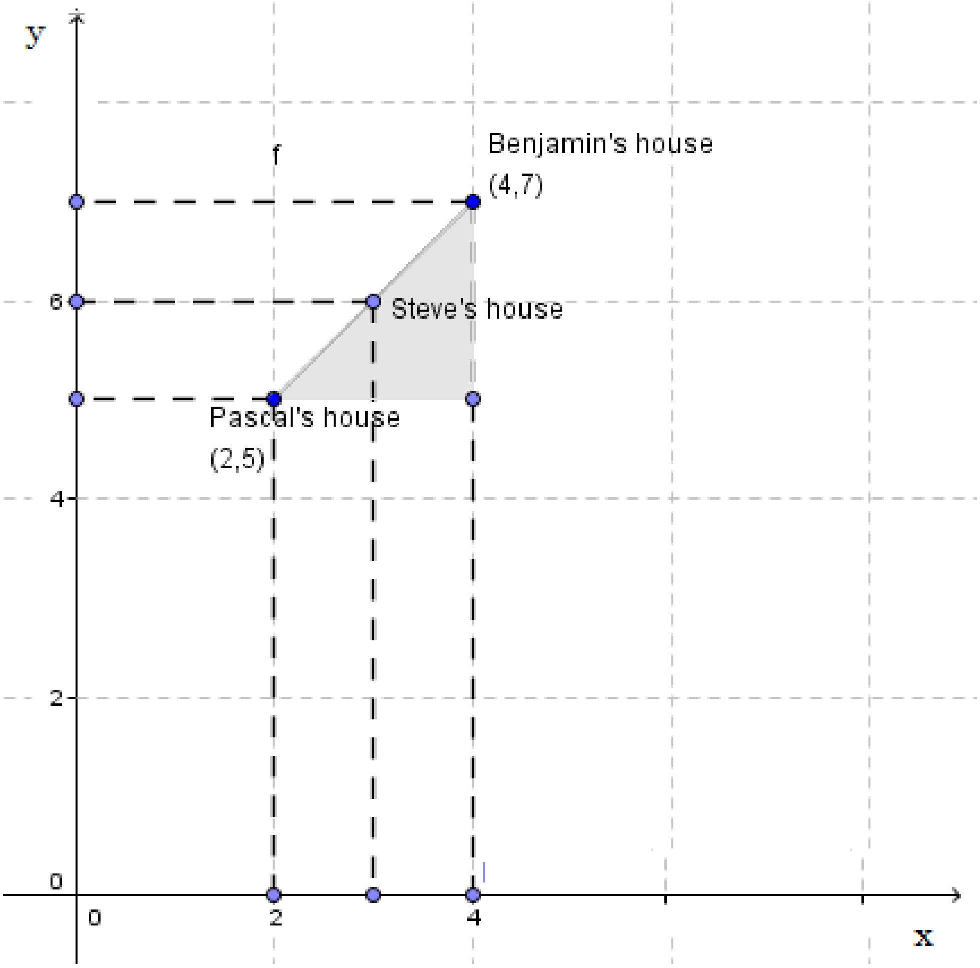
Student-teacher will easily learn this unit, if they are able to:

- Define the coordinate of a point in 2D as it is taught in S1 (UNIT3),
- Calculate the distance between two points in 2D S2 (UNIT7)
- Appreciate the importance of a point and a line in a plane.

d) Learning activities

- Organize the student-teachers into groups;
- Ask them to do the activity 3.3 from student-teacher's book;
- After a given time, ask some groups to present their findings to the whole class;
- During the harmonization, help student-teachers identify the middle point of a line segment and its coordinates;
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance how to determine the coordinate of midpoint of a line segment;
- After this step, guide students to do the application activity 3.3, assess their competences and evaluate whether lesson objectives were achieved.

Answers for Activity 3.3



Steve's location is given by: $\left(\frac{2+4}{2}, \frac{5+7}{2}\right) \Rightarrow (3, 6)$

Answers for Application activity 3.3

Michael and Sarah live in different cities and one day they decided to meet up for lunch. Because they both wanted to travel as little as possible, they will have to meet at:

$$\left(\frac{3100+5120}{2},\frac{500+125}{2}\right) \Rightarrow (4110\,,312.5)$$

a) $(4110,312.5)$

b) $(4110,375)$

c) $(2020,375)$

d) $(8220,625)$

The right answer is sub question $a)$ $(4110\,,312.5)$, the rest answers are false.

Lesson 4: Vector in 2D and dot product

a) Learning objectives:

Describe a vector in 2 D and determine the dot product of 2 vectors.

b) Teaching resources

Student's book and other reference textbooks to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- Define different concepts of a vector and their properties, as it is taught in S2 (UNIT 7),
- Plot vectors in a Cartesian plane
- Effectively do operations on vectors
- Calculate the magnitude of vectors as its length.
- Calculate the distance between two points in 2D S2 (UNIT7)

d) Learning activities

- Organize the student-teachers into groups;
- Ask them to do the activity 3.4.1and activity 3.4.2 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, help student-teachers to have the ability of plotting and describing a vector in a Cartesian plane;
- Guide students to do the activity 2.4.2 related to dot product;
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance the concepts: vector position, components of a vector, characteristics of a vector in 2D, operation of vectors, dot product of two vectors related properties and the magnitude of a vector;
- After this step, guide students to do the application activity 3.4.1 and 3.4.2, assess their competences and evaluate whether lesson objectives were achieved.

Answers for Activity 3.4.1

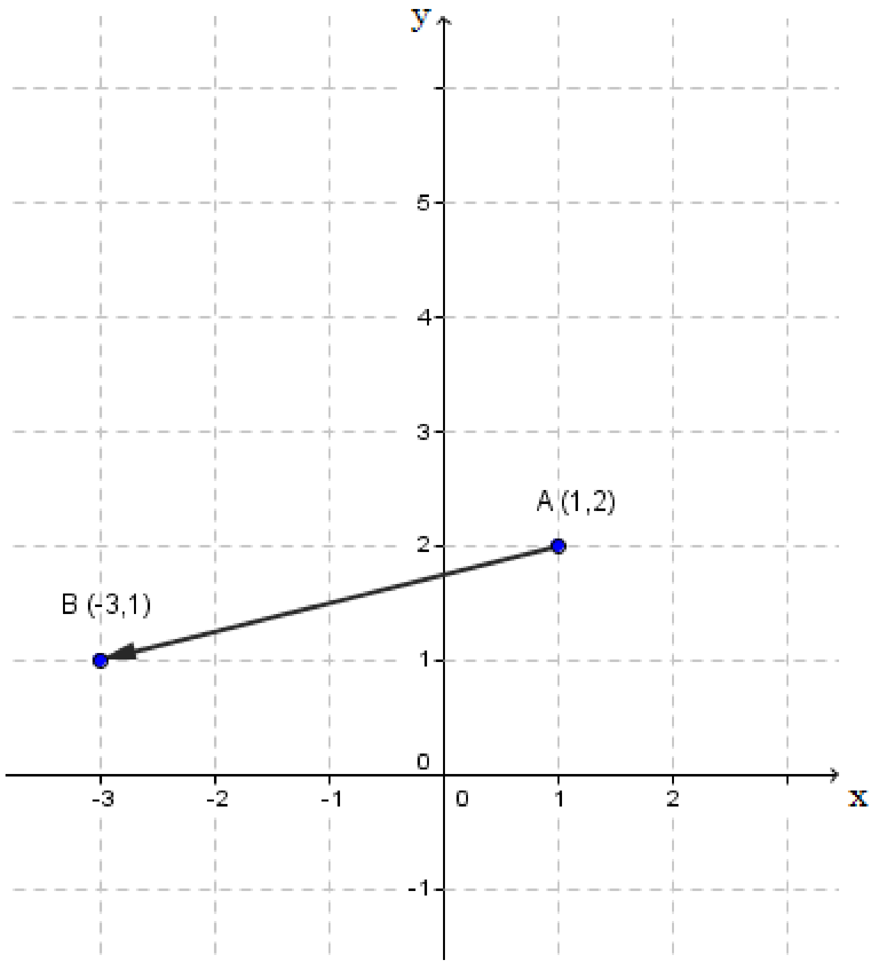
A vector is any quantity that has both magnitude and direction. Two examples of vectors are force and velocity. Both force and velocity are in a particular direction. The magnitude of the force indicates the strength of the force. For velocity, the speed is the magnitude. Other examples include displacement, acceleration.

Note that magnitude and direction are the two properties of a vector.

Quantities with magnitude only are only called scalars.

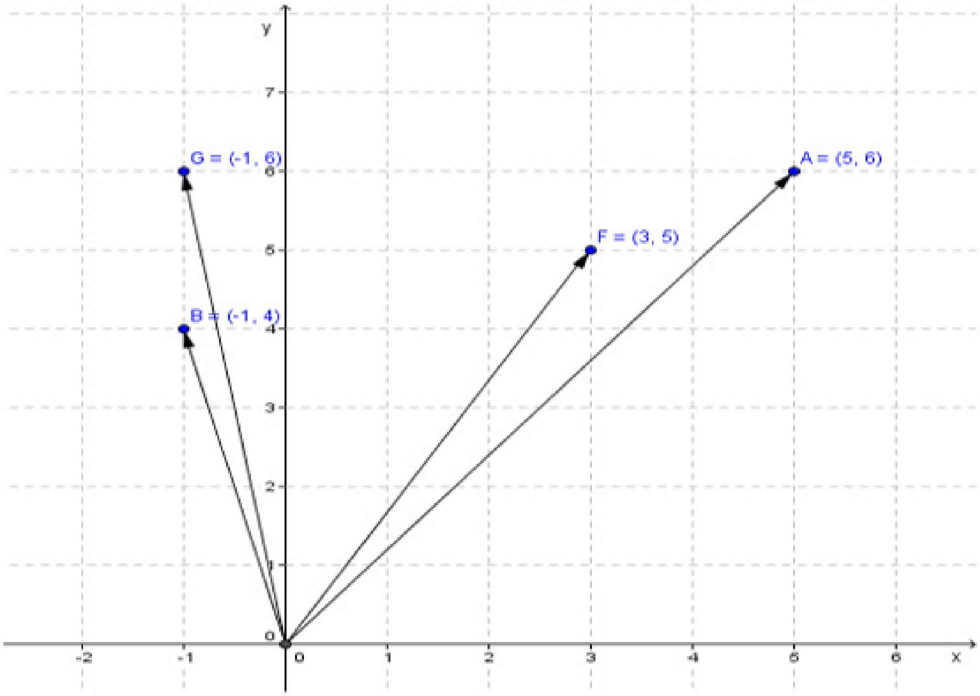
Examples of scalar quantities are: distance, mass, time.

Geometrically, we represent a vector as a directed line segment, whose length is proportional to the magnitude of the vector and with an arrow indicating the direction.

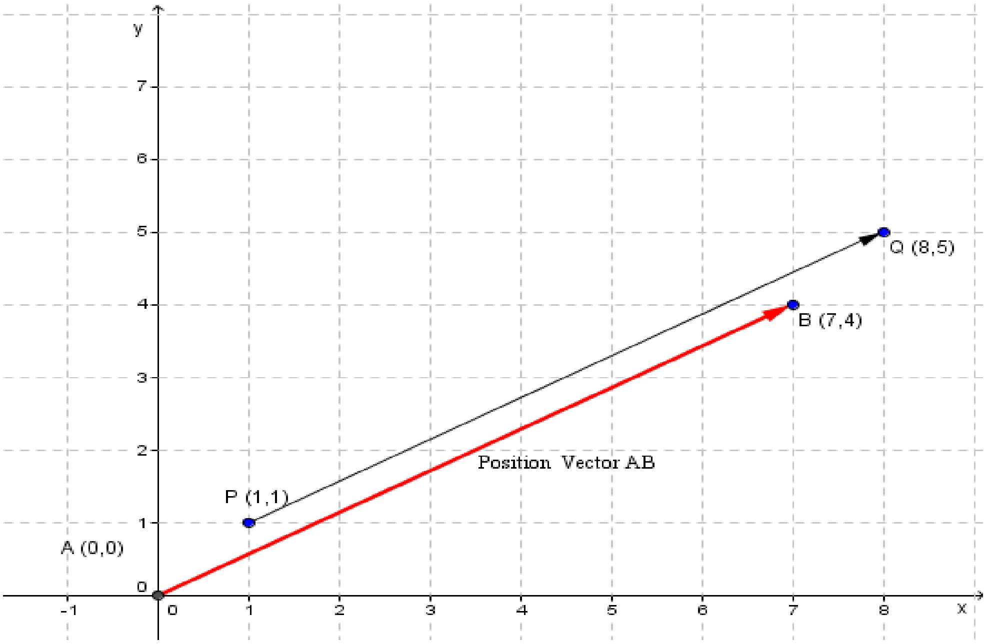

y
x
A (1,2)
B (-3,1)
5
4
3
2
1
0
-1
-3
-2
-1
0
1
2

Answers for application activity 3.4.1

a)

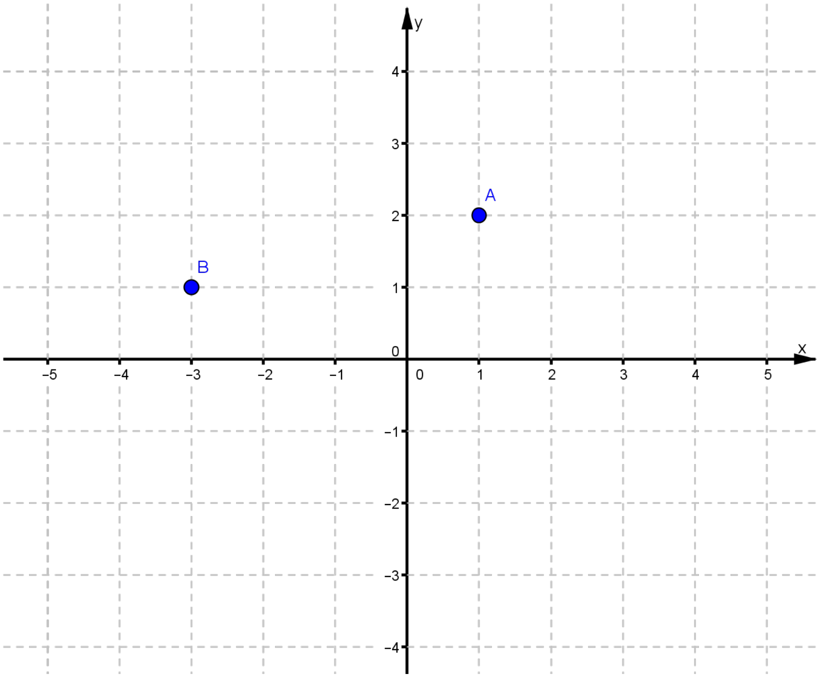


b)



Answers for activity 3.4.2

1. a).



b). $\sqrt{17}$

2. a) $a)\ -1$ $b)\ 0$ $c)\ \overline{u}.\vec{v}=\left(a_1b_1+a_2b_2\right)$

since $\vec{u}(a_1,a_2)$ and $\vec{v}(b_1,b_2)$

Answers for application activity 2.4.2

1. a) Given the vector $\vec{v}=(3,4)$.

   The norm is $\left\|\vec{v}\right\|=\sqrt{9+16}=5$

2. b) Given the vector $\vec{v}=(3,4)$.

   The norm is $\left\|\vec{v}\right\|=\sqrt{9+1}=\sqrt{10}$

3. $\overrightarrow{AB}=(-5,-1)$ b) $(-25,-16)$ c) $\left\|\overrightarrow{AB}\right\|=\sqrt{26}$, $\left\|\vec{w}\right\|=\sqrt{881}$ d) 7 e) 59.

Lesson 5: Straight line passing through a point and parallel to a direction vector

a) Learning objectives

Determination of equation of a straight line given 2 points and a direction vector.

b) Teaching resources:

Student's book and other Reference Textbooks to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- Define different concepts of vectors and its properties seen in unit 7 and parallel projection of a point to a line in unit 8 leant in S2
- Effectively Plot lines and vectors in a Cartesian plane
- Appreciate the importance of a point and a line in a plane.

d) Learning activities

- Organize the student-teachers into small groups;
- Ask them to do the activity 3.5.1 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, help student-teachers establish a vector parallel to a given vector and translated to a given point;
- Use different probing questions and guide students to explore examples and the content given in the student's book to establish equations of a line given a direction vector and a through it passes that line;
- After this step, guide students to do the application activity 3.5.1, assess their competences and evaluate whether lesson objectives were achieved.

Answers for activity 3.5.1

a) Given director vector $\vec{u}(2.-3)$ and $\vec{w} \parallel \vec{v}$, it means there exist a parameter r such that $\vec{w} = r\vec{v}$.

As $\vec{w}$ is translated (passes) to the point $(1,6)$, we have $\vec{w} = r\vec{v} + (1,6)$.

Thus, the form of a vector $\vec{w} = \begin{pmatrix} x \\ y \end{pmatrix}$ is $(x, y) = r(2,-3) + (1,6)$

b) Examples of such a vector $\vec{w}$

If $r = 1, \vec{w} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

If $r = 2, \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

Here the answer may vary depending on the choice of the value of the parameter r.

c) If D is a line with the direction vector $\vec{w}$, the equation D should be given by

$\begin{cases} x = 2r + 1 \\ y = -3r + 6 \end{cases}$ or $\frac{x-1}{2} = r$ and $\frac{y-6}{-3} = r$

Thus as $r = r$, The Cartesian equation of D becomes

$\frac{x-1}{2} = \frac{y-6}{-3}$. This equation can be written in the form of $y = ax + b$ as it was learnt in S1.

$-3(x-1) = 2(y-6)$

$\Rightarrow -3x + 3 = 2y - 12$

$\Rightarrow 2y + 3x - 15 = 0$

$2y + 3x = 15$

$y = \frac{-3}{2}x + \frac{15}{2}$

Answers for application activity 3.5.1

1) Vector equation of a given line is $(x,y)=(3,5)+r(1,6)$

A line L parallel to this line has the same direction vector $\vec{v}=\begin{pmatrix}1\\6\end{pmatrix}$.

As L passes to the point (2,3), every point P (x, y) of L is such that $(x,y)=(2,-3)+r(1,6)$

Therefore, the vector equation of L is $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}2\\-3\end{pmatrix}+r\begin{pmatrix}1\\6\end{pmatrix}$ or

$x\vec{i}+y\vec{j}=(2+r)\vec{i}+(6r-3)\vec{j}$

Parametric equations:

$$\begin{cases}x=2+r\\y=-3+6r\end{cases}$$

Cartesian equation:

$x-2=\frac{y+3}{6}$ or $y=6x-15$

2) a. $(x,y)=(-1,2)+r(-2,3)$

$\begin{cases}x=-1+3r\\y=2+3r\end{cases}$ $\qquad r=\frac{x+1}{3}$ and $r=\frac{y-2}{3}$

Thus, $\frac{x+1}{3}=\frac{y-2}{3}$

$\Rightarrow y=x+3$

b) $\begin{cases}x=3+3r\\y=2-r\end{cases}$ $\qquad r=\frac{x-3}{3}$ and $r=2-y$

Thus, $\frac{x-3}{3}=2-y$ $\qquad \Rightarrow y=2-\frac{x-3}{3}\Rightarrow y=\frac{9-x}{3}\Rightarrow y=-\frac{1}{3}x+3$

Lesson 6: Equation of a straight line given 2 points and a direction vector

a) Learning objectives:

Determination of equation of a straight line given 2 points and a direction vector.

b) Teaching resources:

Student's book and other Reference Textbooks to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

Note: The Tutor can help student-teachers to use GeoGebra software to draw lines.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- Define different concepts of vectors and its properties seen in unit 7 and parallel projection of a point to a line in unit 8 leant in S2
- Effectively plot lines and vectors in a Cartesian plane.

d) Learning activities

- Organize the student-teachers into small groups;
- Ask them to do the activity 3.5.2 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization guide students to determine the vector AB passing joining two given points (A and B) and another vector V parallel to the vector AB;
- Use different probing questions and guide students to explore examples and the content given in the student's book to determine the equation of a straight line given 2 points and a direction vector;
- After this step, guide students to do the application activity 3.5.2, assess their competences and evaluate whether lesson objectives were achieved.

Note: Where it is possible, the tutor can help student-teachers to use GeoGebra software to plot lines.

Answers for activity 3.5.2

1\. a) The vector $\overrightarrow{AB}=\begin{pmatrix}2\\-6\end{pmatrix}$.

b). The vector $\vec{w}=\begin{pmatrix}x\\y\end{pmatrix}=r\begin{pmatrix}2\\-6\end{pmatrix}+\begin{pmatrix}3\\-2\end{pmatrix}$

c) Here the answer may vary depending on the choice of each and every one

For example, one may choose $r=1$ or another value $r=2$ and the answer differ.

d) $\begin{cases}x=2r+3\\y=-6r-2\end{cases}$

$r=\frac{x-3}{2}$ *and* $r=\frac{y+2}{-6}$,

therefore $\frac{x-3}{2}=\frac{y+2}{-6}$

$\Rightarrow -6(x-3)=2(y+2)\Rightarrow -6x+18=2y+4$

$\Rightarrow 2y+6x=14$

Answers for application activity 3.5.2

1\) The direction vector is $\overrightarrow{PB}=(1,-6)$,

If r is a parameter, the vector equation is $(x,y)=\overrightarrow{OP}+r\overrightarrow{PB}=(2,4)+r(1,-6)$

Parametric equations: $\begin{cases}x=2+r\\y=4-6r\end{cases}$

The Cartesian equation:

$\frac{x-2}{1}=\frac{y-4}{-6}$

or $-6(x-2)=(y-4)\Rightarrow y+6x=16$

2) We need the equation whose vector equation is $a\vec{i}+b\vec{j}$ such that $\left(a\vec{i}+b\vec{j}\right)\cdot\left(\vec{i}-2\vec{j}\right)=0$

$$\left(a\vec{i}+b\vec{j}\right)\cdot\left(\vec{i}-2\vec{j}\right)=0$$

$$a-2b=0 \Rightarrow a=2b$$

The direction vector of the needed line is $2\vec{i}+\vec{j}$. The required equation is

$$x\vec{i}+y\vec{j}=2\vec{i}+3\vec{j}+r\left(2\vec{i}+\vec{j}\right).$$

Lesson 7: Equation of a straight line given its gradient

a) Learning objectives:

Determination of equation of a straight line given its gradient.

b) Teaching resources:

Student's book and other Reference Textbooks to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

Note: Where it is possible, the tutor can help student-teachers to use GeoGebra software to plot lines.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- Previous lessons of this unit;
- Effectively Plot lines and vectors in a Cartesian plane;
- Appreciate the importance of a point and a line in a plane.

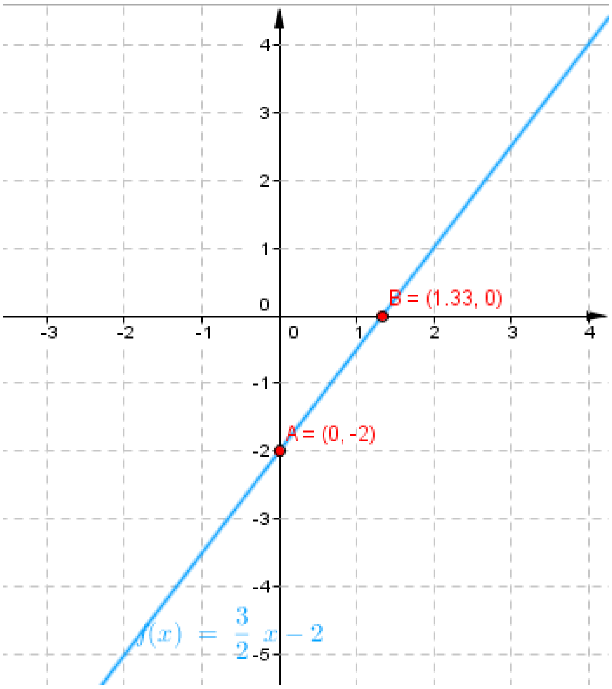
d) Learning activities

- Organize the student-teachers into groups;
- Ask them to do the activity 3.5.3 from student-teacher's book;
- After a given time, ask groups with different working steps to present their findings to the whole class;
- During the harmonization, help student-teachers plot graph of a line passing to a point given its gradient;

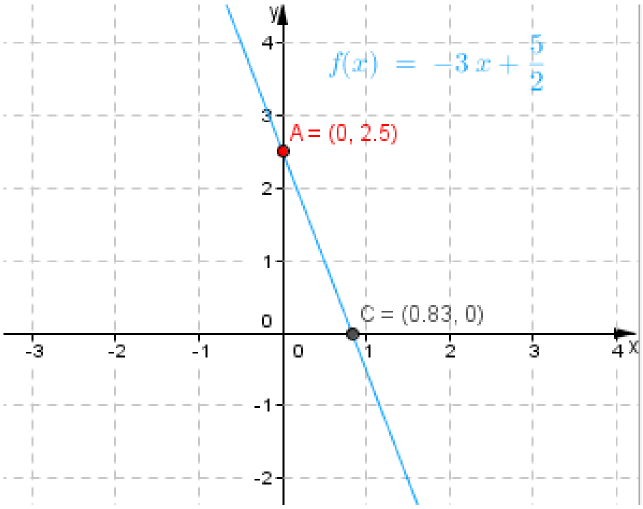
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance how to plot a line parallel to a given line (as its gradient is known);
- After this step, guide students to do the application activity 3.5.3, assess their competences and evaluate whether lesson objectives were achieved.

Answers for activities 3.5.3

a) The slope is $m=\frac{3}{2}$ for $y=\frac{3}{2}x-2$ and y-intercept is $y=-2$ and $x=0$



b)The slope $m=-3$ for $y=-3x+\frac{5}{2}$ and y-intercept is $y=\frac{5}{2}$ and $x=0$



Answer for application activities 3.5.3

1. a) $y+4=6(x-3)$ $y=6x-22$

b) $y+7=\frac{-3}{2}(x+2)$

$\Rightarrow y=\frac{-3}{2}x-\frac{6}{2}-7 \Rightarrow y=\frac{-3}{2}x-10$

c) $y-2=0$

$\Rightarrow y=2$

d) $y-2=\frac{-5}{3}(x-4)$

$y=-\frac{5}{3}x-\frac{20}{3}+2$

$y=-\frac{5}{3}x-\frac{14}{3}$

e) $y=4x$

f) $y+8=-\frac{1}{5}(x-1)$

$$y=-\frac{1}{5}x+\frac{1}{5}-8$$

$$y=-\frac{1}{5}x-\frac{39}{5}$$

Lesson 8: intersection, perpendicularity or parallelism of lines in 2D

a) Learning objectives:

Perform operations to determine the intersection, perpendicularity or parallelism of lines

b) Teaching resources

Student's book and other Reference Textbooks to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

Note: Where it is possible, the tutor can help student-teachers to draw graphs using GeoGebra software.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- Define different concepts of a parallel and / or perpendicular projection in 2D as it is taught in S2 (UNIT 8).
- Effectively Plot lines and vectors in a Cartesian plane
- Appreciate the importance of a point and a line in a plane.

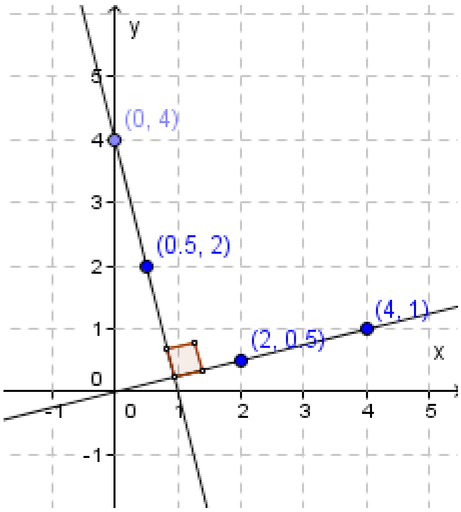
d) Learning activities

- Organize the student-teachers into groups;
- Ask them to do the activity 3.6.1 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, guide student-teachers to determine the equation of a line parallel or perpendicular to a given line and to determine the intersection of lines where it is possible;
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance the development of their competences.

- After this step, guide students to do the application activity 3.6.1, assess their competences and evaluate whether lesson objectives were achieved.

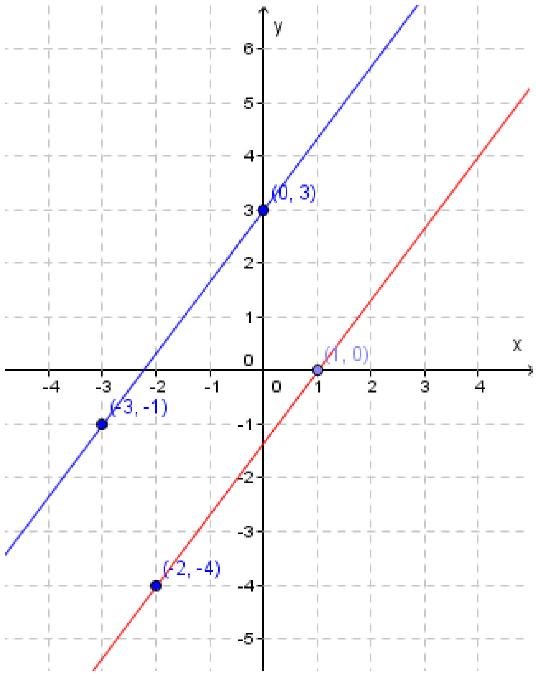
Answer for activity 3.6.1

1)



The two lines are perpendicular

2)



The two lines are parallel

Answers for application activity 3.6.1

1.The equation of a line perpendicular to $2x+4y+7=0$ is $4x-2y+k=0$

(i) Where k is an arbitrary constant.

According to the problem equation of the perpendicular line 4x - 2y + k = 0 passes through the point (-2, 3)

Then,

$(4)(-2) - 2 \cdot (3) + k = 0$

$\Rightarrow -8 - 6 + k = 0$

$\Rightarrow -14 + k = 0 \quad \Rightarrow \quad k = 14$

Now putting the value of k = 14in (i) we get, 4x - 2y + 14 = 0

Therefore, the required equation is 4x - 2y + 14 = 0.

2) The given two equations are x + y + 9 = 0 (i) and

3x - 2y + 2 = 0 (ii)

Multiplying equation (i) by 2 and equation (ii) by 1 we get

$2x + 2y + 18 = 0$

$3x - 2y + 2 = 0$

Adding the above two equations we get, $5x = -20 \quad \Rightarrow \quad x = -4$

Putting $x = -4$ in (i) we get, $y = -5$

Therefore, the co-ordinates of the point of intersection of the lines (i) and (ii) are (- 4, - 5).

Since the required straight line is perpendicular to the line $4x + 5y + 1 = 0$, hence we assume the equation of the required line as

$5x - 4y + \lambda = 0$ (iii)

Where λ is an arbitrary constant.

By problem, the line (iii) passes through the point (- 4, - 5); hence we must have,

$\Rightarrow (5)(-4) - 4 \cdot (-5) + \lambda = 0$

$\Rightarrow -20 + 20 + \lambda = 0$

$\Rightarrow \lambda = 0.$

Therefore, the equation of the required straight line is $5x - 4y = 0$.

3) The equation of any straight line parallel to the line

$3x - 2y + 10 = 0$ is $3x - 2y + k = 0$ (i)

(k is an arbitrary constant).

According to the problem, the line (i) passes through the point (5, - 6) then we shall have,

$(3)\,5 - (2)(-6) + k = 0$

$\Rightarrow \quad 15 + 12 + k = 0$

$\Rightarrow \quad 27 + k = 0$

$\Rightarrow \quad k = -27$

Therefore, the equation of the required straight line is $3x - 2y - 36 = 0$.

4) $\vec{u}.\vec{v}$ =(3)(-8)+4.6=0

$$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{0}{\sqrt{3^2+4^2}\sqrt{(-8)^2+6^2}}\right) = \cos^{-1}(0)$$

$$\Rightarrow \quad \alpha = 90^\circ$$

Lesson 9: Distance between point and lines, lines and lines in 2D

a) Learning objectives: Determine the distance between point and lines, lines and lines in 2D

b) Teaching resources

Student's book and other Reference Textbooks to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- Apply previous lessons in this unit.
- Define different concepts of a parallel and / or perpendicular projection in 2D as it is taught in S2 (UNIT 8),
- Effectively Plot lines and vectors in a Cartesian plane
- Appreciate the importance of a point and a line in a plane.

d) Learning activities

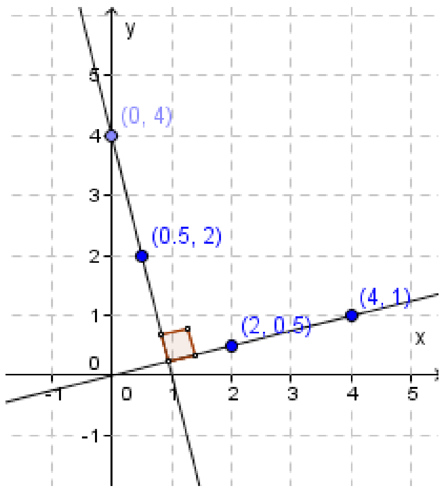
- Organize the student-teachers into groups;
- Ask them to do the activity 3.6.2 from student-teacher's book;
- After a given time, ask groups with different working steps to present their findings to the whole class;
- During the harmonization, help student-teachers establish how to

determine the distance between two lines;

- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance the development of their competences.
- After this step, guide students to do the application activity 3.6.2, assess their competences and evaluate whether lesson objectives were achieved.

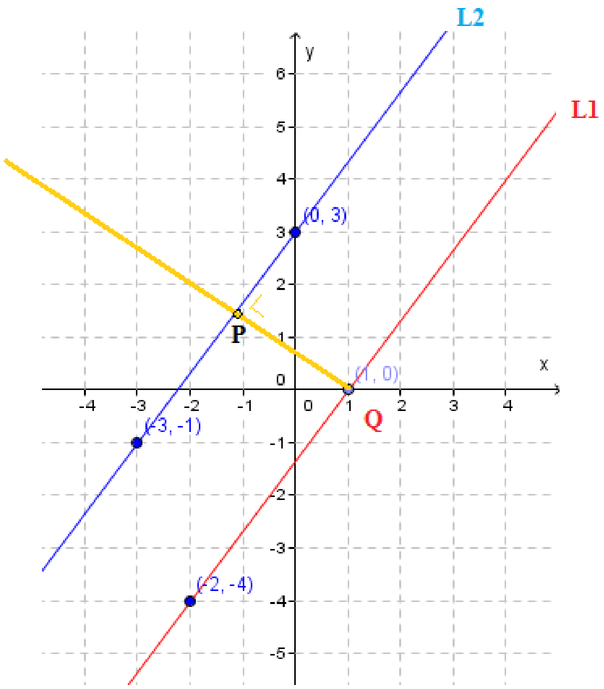
Answers activity 3.6.2

1)



the distance between the two lines is zero because they intersect.

2) When the two lines are parallel: For example, the line L1 and L2, the distance between them is the length of a segment PQ where the line PQ is perpendicular to L1 or L2.



2) Considering one line L1 from the two we can find the normal line QP between the two parallel lines for which the gradient m is $-\frac{1}{m}$ given by $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0+4}{1+2} = \frac{4}{3}$ (red line) is the gradient of L1. Thus, the equation of PQ can be found as it passes through the point Q (1,0): This is $PQ \equiv y = -\frac{1}{m}x + b$

As it passes for example at a point (1, 0), we can find the value of b

$$0 = -\frac{3}{4} + b$$

$$b = \frac{3}{4}$$

Thus, the equation of the Normal line PQ is $y = -\frac{3}{4}x + \frac{3}{4}$.

The equation of line L2 passing through the points (0,3) and (-3,-1) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$y+1=\frac{-1-3}{-3}(x-0)$$

$$y=\frac{4}{3}x-1$$

Now we can determine the point P intersection of L2 and PQ.

$$\begin{cases} y=\frac{4}{3}x-1 \\ y=-\frac{3}{4}x+\frac{3}{4} \end{cases}$$

$$-\frac{3}{4}x+\frac{3}{4}=\frac{4}{3}x-1,$$

$$x=\frac{21}{25},$$

$$y=\frac{4}{3}(\frac{21}{25})-1, \quad y=\frac{3}{25}$$

Now the intersection point is $P\left(\frac{21}{25},\frac{3}{25}\right)$

The distance between $P\left(\frac{21}{25},\frac{3}{25}\right)$ and $Q(1,0)$ is $\sqrt{(1-\frac{21}{25})^2+(0-\frac{3}{25})^2}=\frac{1}{5}$ unit length.

Note: This distance can be shown as the distance between Q(1,0) and the line L1with equation $y=\frac{4}{3}x-1$ that can be written as $-4x+3y+3=0$

And $PQ=\frac{|-4(1)+3(0)+3|}{\sqrt{(-4)^2+(3)^2}}=\frac{1}{5}$.

This result can be generalized as the distance between a line L of equation $L\equiv ax+by+c=0$ and the point $Q(x_0,y_0)$.

This distance $d(Q,L)=\dfrac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$.

3) Applying the same process as the previous question, one can show that the distance between a point and line is given by $d=\dfrac{|ax+by+c|}{\sqrt{a^2+b^2}}$; hence, the distance between the point $(4,2)$ and the line $-2x+3y=7$ is

$$d=\frac{|-2(4)+3(2)-7|}{\sqrt{(-2)^2+(3)^2}}=\frac{|-8+6-7|}{\sqrt{4+9}}=\frac{9\sqrt{13}}{13}\text{ unit of length}$$

The shortest distance is $\dfrac{9\sqrt{13}}{13}\approx 2.5$ unit of length

Answers for applications 3.6.2

1) First of all, we find the slopes of the two lines. We convert the equations into slope intercept form

$$3x+4y=9$$
$$\Rightarrow 4y=-3x+9$$
$$\Rightarrow y=\frac{-3}{4}x+\frac{9}{4}$$

and

$$6x+8y=15$$
$$\Rightarrow 8y=-6x+15$$
$$\Rightarrow y=\frac{-6}{8}x+\frac{15}{8}$$
$$\Rightarrow y=\frac{-3}{4}x+\frac{15}{8}$$

Slope m of the lines $=\dfrac{-3}{4}$

Hence both lines are parallel

Y intercept of the first line $=\dfrac{9}{4}$

Y intercept of the second line $=\dfrac{15}{4}$

Difference between the y- intercepts

$$=\left|\frac{9}{4}-\frac{15}{8}\right|=\left|\frac{18}{8}-\frac{15}{8}\right|=\frac{3}{8}$$

$$\sqrt{1+m^2}=\sqrt{1+\frac{9}{16}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$$

Distance between the lines $= \frac{3}{8} \div \frac{5}{4} = \frac{3}{8} \times \frac{4}{5} = \frac{3}{10}$

2) The slope of the lines $3x + 4y = 5$ and $6x - 8y = 45$ is $\frac{-3}{4}$ and $\frac{3}{4}$ respectively.
So, these lines are not parallel but are perpendicular therefore the distance between the two lines is 0.

3) Look at closely, the second equation is actually first multiplied by 2 on both LHS and RHS.

Then the lines represented by these two equations are also same so the distance between them is 0

4) If $P(x_p, y_p)$, its perpendicular distance from $ax + by + c = 0$ is

$$d = \left| \frac{ax_p + by_p + c}{\sqrt{a^2 + b^2}} \right|$$

So, if 0 $\Leftrightarrow$ $(0,0)$, its perpendicular distance from $3x - 4y + 15 = 0$ is,

$$d = \left| \frac{3 \times 0 - 4 \times 0 + 15}{\sqrt{(3)^2 + (-4)^2}} \right| = \frac{15}{5} = 3$$

5) The distance between the point (-2,3) and the line $x - y = 5$?

line $x - y - 5 = 0$, point (-2,3)

distance $d = \left| \frac{ax_p + by_p + c}{\sqrt{a^2 + b^2}} \right|$

$$= \left| \frac{(1 \times -2) + (-1 \times 3) + (-5)}{\sqrt{(1)^2 + (-1)^2}} \right|$$

$(\because \ a = 1, b = -1, c = -5, x = -2, y = 3)$

$$d = \left| \frac{-2 - 3 - 5}{\sqrt{2}} \right| = \left| \frac{-10}{\sqrt{2}} \right| = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ unit lenght}$$

6) As mentioned, points (-3, -4) to the line $3x - 4y - 1 = 0$

Let $x = -3$ and $y = -4$, so comparing the line with equation

$$Ax + By + C = 0 \qquad A = 3, \ \ B = -4 \text{ and } C = -1$$

Equation for distance between line and coordinate axes points. Is

$$d = \left| \frac{Ax + By + C}{\sqrt{A^2 + B^2}} \right|$$

$$d = \left| \frac{3(-3) + 4(-4) + (-1)}{\sqrt{(-3)^2 + (-4)^2}} \right| \quad d = \frac{6}{5} \text{ unit lenght}$$

7. Distance between the line $3x + 4y - 6 = 0$ and point $(2, -1)$ is given by

$$d = \left| \frac{3(2) + 4(-1) - 6}{3^2 + 4^2} \right| = \left| \frac{6 - 4 - 6}{\sqrt{25}} \right| = \frac{4}{5}$$

Lesson 10: Geometric shapes in 2D

a) Learning objectives:

To identify the Geometric shapes in 2D

b) Teaching resources:

Student's book and other Reference Textbooks to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

Note: Tutor can help student-teachers to identify the geometric shapes using real life materials from different corners.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- Identify 2D shapes and write their names as they have seen them in P4 unit 15 and in S1;
- Identify different objects from their surrounding environment;
- Plot a point in a Cartesian plane as it was seen in lesson one of this unit.

d) Learning activities

- Organize the student-teachers into small groups;
- Ask them to do the activity 3.7.1 from student-teacher's book;

- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, help student-teachers to discover the shapes with two dimensions using Cartesian coordinates: Quadrilaterals (square, rectangle, parallelogram, trapezium etc.), Triangles, circle, pentagon, hexagon, etc.
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance the development of their competences.
- After this step, guide students to do the application activity 3.7.1, assess their competences and evaluate whether lesson objectives were achieved.

Answers for activity 3.7.1

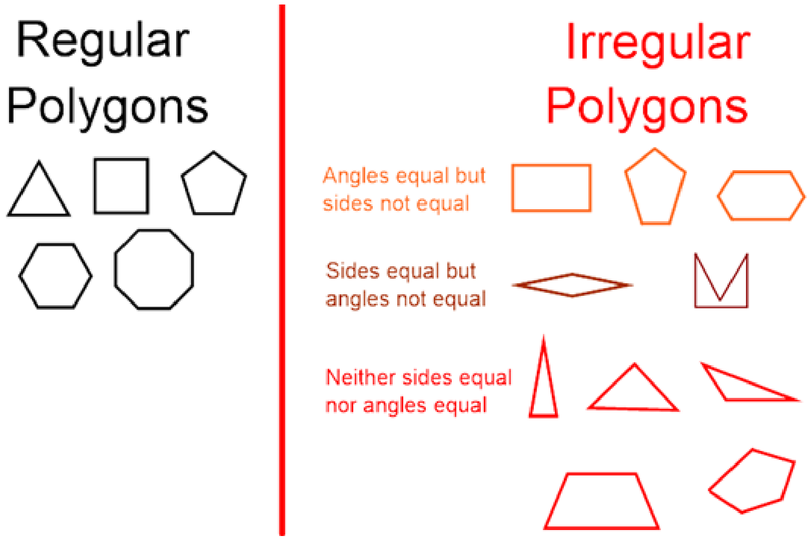
1. Regular and irregular polygons are mutually exclusive. Every polygon is either regular or irregular.

There are 2 conditions, and a ***regular*** polygon must meet both:

All sides must be congruent, the polygon is equilateral

All angles must be congruent; the polygon is equiangular for example square tables' faces, some box faces, sheet of papers etc.

i. If the polygon fails either or both conditions then it is ***irregular.*** For example, some fields are in the form of irregular polygons.



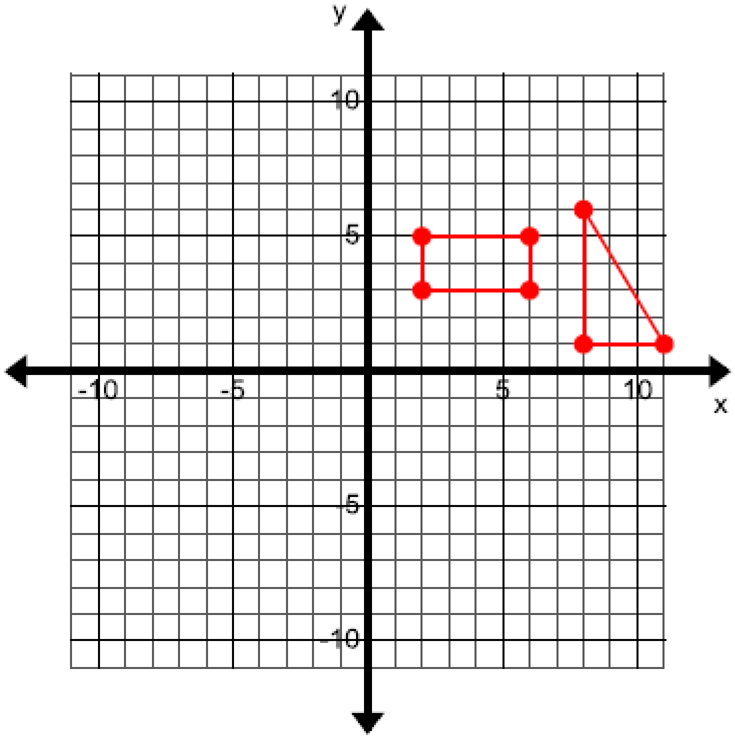
ii. Three-sided polygon is called triangle.

(The student teachers can provide different examples of triangle in real life)

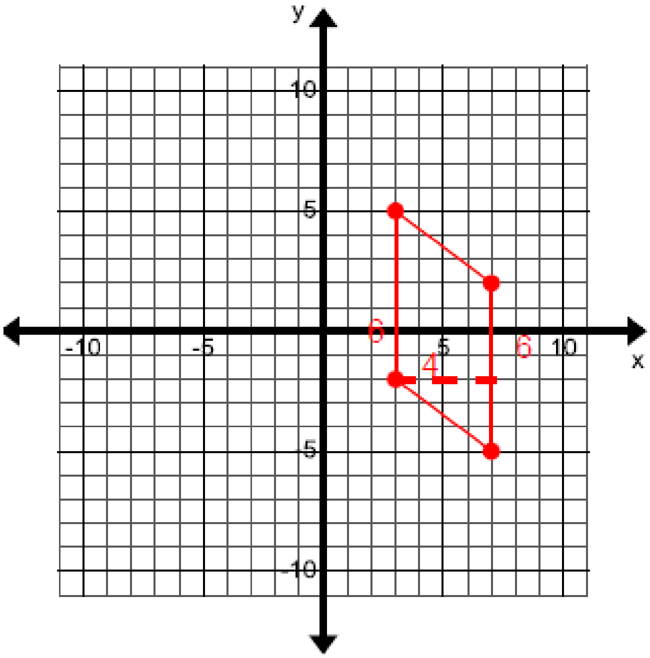
iii. Four-sided polygon is called rectangle.

(The student teachers can provide different examples of triangle in real life)

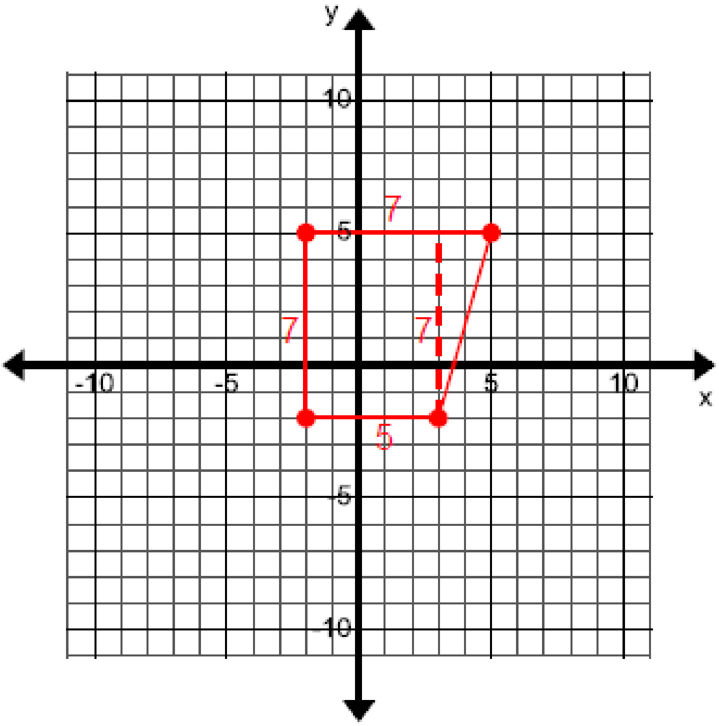
2. **i. Polygon one and two (1)**



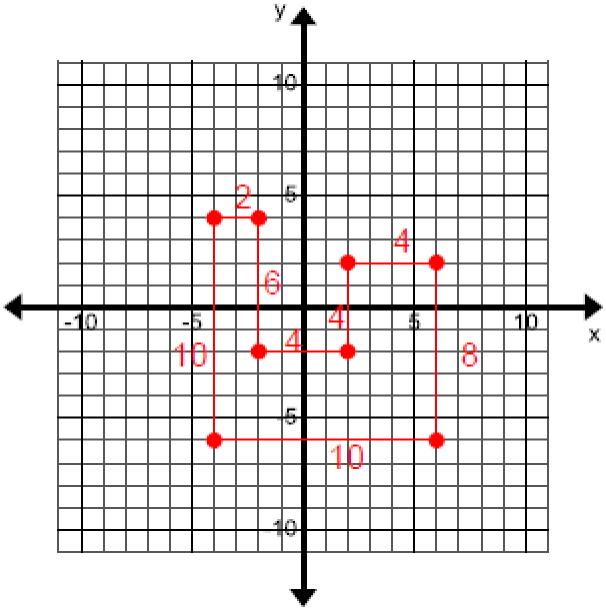
Polygon three(2)



Poygon four (3)



Polygon five (4)



ii. Names of geometric fidgures 1, 2, 3 and 4 are respectively

1. Three-sided polygon is called triangle and Four-sided polygon is called rectangle

2. Four sided polygon is called parallelogram

3. Four sided polygon called trapezium

4. Eight-sided irregular polygon

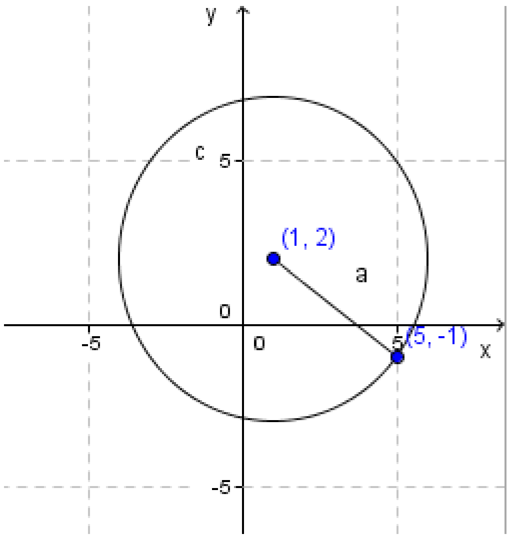
iii. Similarities and differences of these figure(to be provided by student-teachers after their observation)

Answers for application activity 3.7.1

1. We have the point $C(1,2)$ and $P(5,-1)$

a) $r = \overline{CP} = \sqrt{(5-1)^2 + (-1-2)^2} = 5$

b)



c) From any point of the circle to its center is equal to 5 units of length, thus here many solutions are possible because we have many solutions on the circle for example let (x_1, y_1) be the first point where the value of x is between -5 and 5 if the center is in origin then you can find the y by letting x in that interval

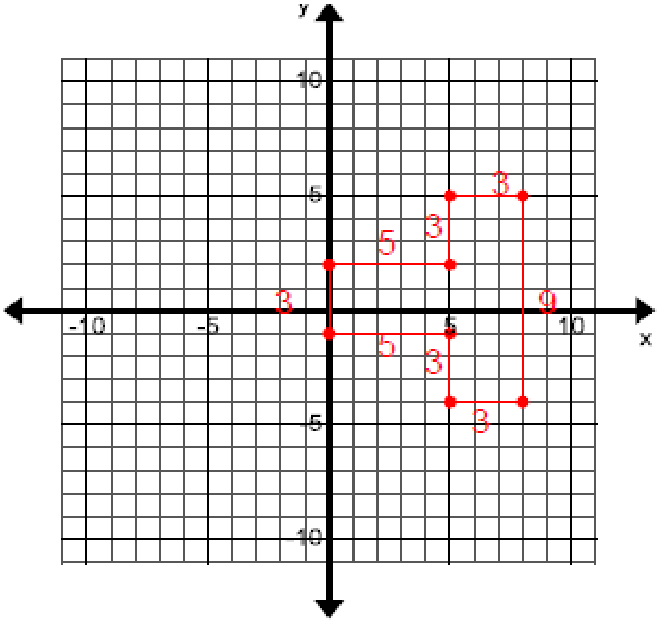
$$25 = (x_1 - 1)^2 + (y_1 - 2)^2$$

If $x_1 = 1$,

$$y_1^2 - 4y + 4 = 25$$
$$y_1^2 - 4y - 21 = 0$$

$y_1 = -3$ or $y_1 = 7$, Thus the point $(1, -3)$ and $(1, 7)$ are others on the circle

2.a)

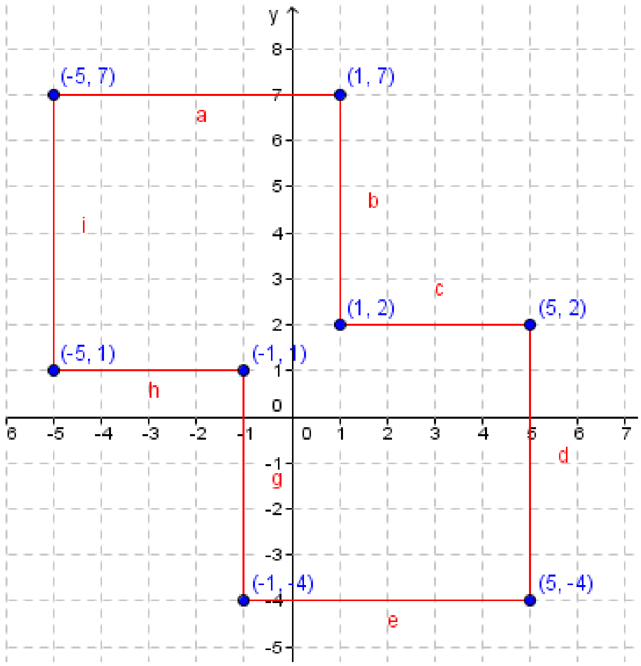


= 33 square units

It is irregular polygon

If one square stand for 1unit square the area of the polygon formed is 33 square units.

b)



It is irregular polygon of 8 sides.

Its area can be found by adding the sum of squares it contains. If each is equivalent to 1 square unit, thus the sum of square is 34+36 =70-unit squares.

Lesson 11: Perimeter and Area of Geometric shapes

a) Learning objectives:

Determine the perimeter and area of geometric shapes in 2D.

b) Teaching resources:

Student's book and other Reference books to facilitate research, Mathematical set, Calculator, Manila paper, Markers, Pens and Pencils.

Note: Tutor helps student-teachers to estimate the geometric shapes' areas and perimeter.

c) Prerequisites / Revision / Introduction:

Student-teacher will easily learn this unit, if they are able to:

- To determine area and perimeter of 2D shapes as they have seen in P4 unit 16;
- Estimate the area and the perimeter of different objects from their surrounding environment.
- Refer to the lesson 1 of this unit: Cartesian coordinates of points.

d) Learning activities

- Organize the student-teachers into groups;
- Ask them to do the activity 3.7.2 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, help student-teachers to discover the shapes with two dimensions, determine their perimeter and area;
- Use different probing questions and guide students to explore examples and the content given in the student's book to enhance the development of their competences.

- After this step, guide students to do the application activity 3.7.2, assess their competences and evaluate whether lesson objectives were achieved.

Answer for activity 3.7.2

1. A) the first figure is clear that two parallel sides are equal therefore the perimeter can be found by the sum of sides.

   $$2\times(30m+40m)=60m+80m=140m$$

   The length of one wire that can be used on one row of the fence in order to protect the plants is **140m.**

   b) The second figure is irregular for having its perimeter we add sides thus the perimeter is

   $$4\times 3dam+2\times 3dam=12dam+6dam=18dam$$

   The length of one wire that can be used on one row of the fence in order to protect the plants is **18dam.**

2. As it is given that one-unit square is used for one square by approximation and counting how many squares are there in polygons you will find that these polygons have the same area which is 24 square units.

   A is a rectangle with an area of 24 square units.

   B is a parallelogram with an area of 24 square units.

   C is a triangle with an area of 24 square units.

   D is a trapezium with an area of 24 square units.

   E is a 5-sided polygon with an area of 24 square units.

   F is a polygon with more than 5 sides with an area of 24 square units.

Answer for application activities 3.7.2

Area = 87 square units

Two different methods are shown:

The first one includes finding the area of three triangles and a rectangle.

The second method circumscribes a rectangle around the pentagon. The area of the pentagon is equal to the difference of the area of the rectangle and the 4 triangles formed.

3.6 Summary of the unit

1. The **Cartesian coordinates** of a point in the plane are written as (x, y).

2. If $A(a_1, b_1)$ and $B(a_2, b_2)$ are two points the **distance between these two points** denoted is

$$d(A,B)=\sqrt{(a_2-a_1)^2+(b_2-b_1)^2}$$

3. The **midpoint *M* of the line segment** from point *A* to point *B* given by

$$M=\frac{1}{2}(A+B).$$

4. **Equation of the line** passing through point $P(x_0, y_0)$ and parallel to the direction vector, $\vec{v}=(a,b)$:

Vector equation

$$(x,y)=(x_0,y_0)+r(a,b)$$

Parametric equation

$$\begin{cases} x=x_0+ra \\ y=y_0+rb \end{cases}$$

Cartesian equations

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}$$

5. The **distance from a point** $D(x_1, y_1)$ **to the line** $ax+by+c=0$ is given by

$$\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

6. **Distance between two parallel lines** $y_1=mx_1+c_1$ **and** $y_2=mx_2+c_2$ is given by $d=\frac{|c_1-c_2|}{\sqrt{1+m^2}}$.

3.7 Additional information for tutors

- The unit vector in two dimensions are:

  $\{\vec{i}=(1,0),\vec{j}=(0,1)\}$

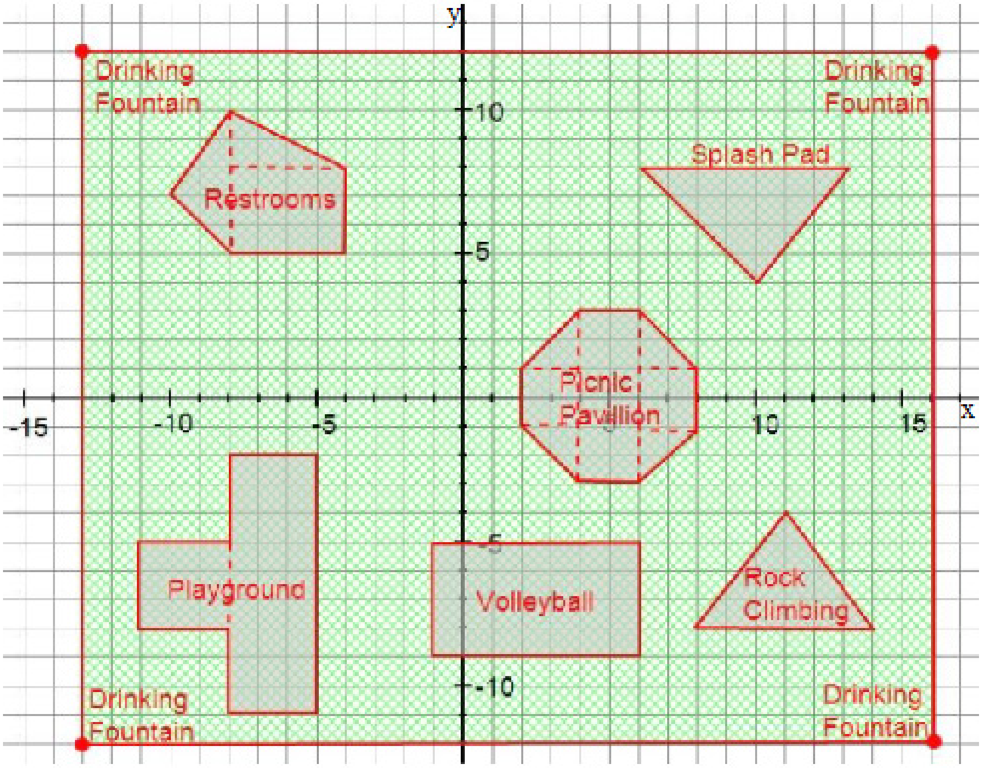
  $\vec{i}$ is on $x-axis$ while $\vec{j}$ is on $y-axis$.

- Emphasizing the unit vectors, the equation of the line can be rewritten

  as $x\vec{i}+y\vec{j}=x_0\vec{i}+y_0\vec{j}+r\left(a\vec{i}+b\vec{j}\right)$

3.8 End unit assessment

1) (a)



b) Graph above

a. The city will need 14 m^2 for the splash pad and 26 m^2 for the restrooms. This makes 40 m^2 of total cement.

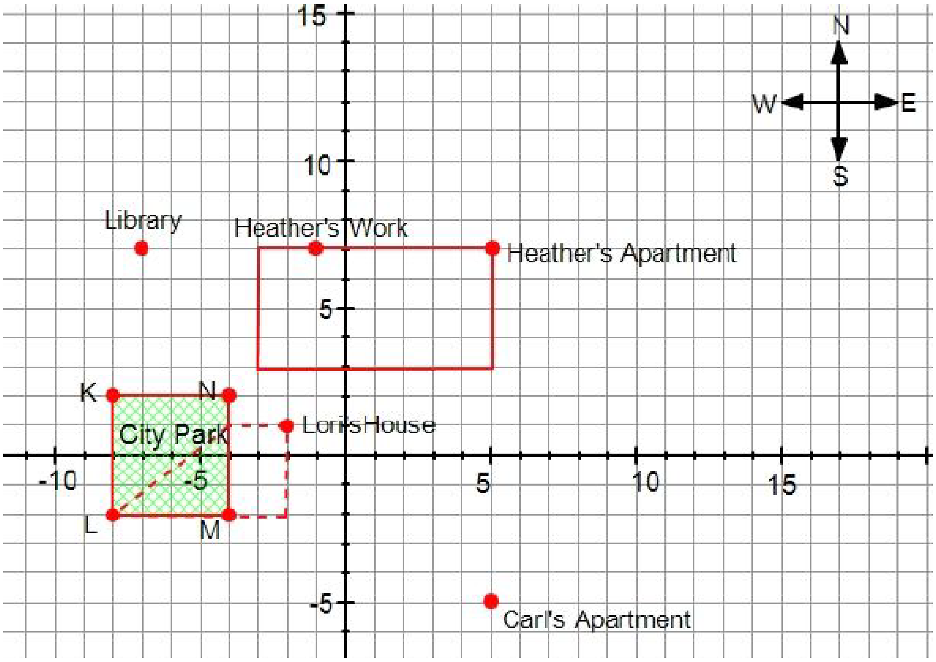
b. The city will need 36 m^2 of bark to cover the playground area.

c. The city will need 30 m of fencing for the playground.

d. The city will need $28\,m^2$ of pavers for the pavilion.

e. The area occupied by volleyball and rock climbing is $28m^2+5m^2=33m^2$

2 .



a) Heather walked 24 blocks. Her path made a rectangle.

b) Any polygon with a perimeter of 20.

c) The distance between Carl and Heather's house is 12 blocks.

d) If the distance between Carl and Heather's house is 12 blocks, then one block has 10m

of length's side the distance between them became $12\times 10m = 120m$

The area of a circle centered at C is given by the formula πr^2

$= \left[(120)^2 \pi \right] m^2 = \left(14400\pi \right) m^2$

3.9 Additional activities

3.9.1 Remedial activities

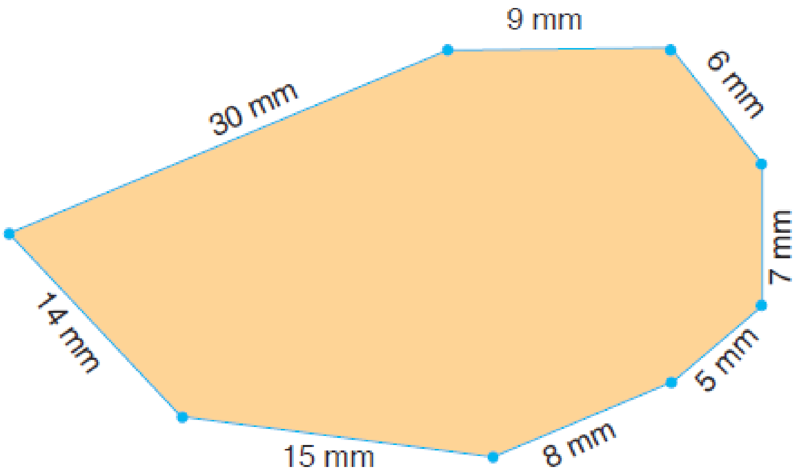
1. If $\vec{v} = 2i + 3j = \langle 2,3 \rangle$ and $\vec{w} = 3i - 4j = \langle 3,-4 \rangle$ find;

$a) 3\vec{v}$

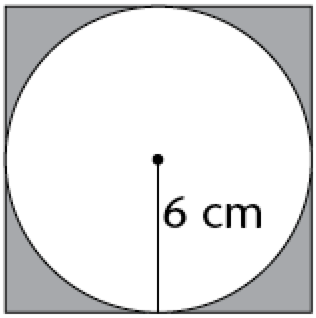
$b) 2\vec{v} - 3\vec{w}$

$c) \left\| \vec{v} \right\|$

2. Calculate the perimeter of the shape given below.



3. Calculate the perimeter of the square and the area of the shaded parts of the square.



Solution:

1)

$a) 3\vec{v} = 3(2i + 3j) = 6i + 9j$

or

$3\vec{v} = 3\langle 2,3 \rangle = \langle 6,9 \rangle$

$b) 2\vec{v} - 3\vec{w} = 2(2i + 3j) - 3(3i - 4j)$

$(-5i + 18j) = \langle -5,18 \rangle$

$c) \left\| \vec{v} \right\| = \left\| 2i + 3j \right\| = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$

2) Perimeter = 30 mm + 9 mm + 6 mm + 7 mm + 5 mm + 8 mm + 15 mm

+ 14 mm = 94 mm

3) The perimeter of the square is $(6 \times 2cm)4 = 48\,\text{cm}$

The area of the square is $s^2 = (12cm)^2 = 144cm^2$

The area of circle is $\pi r^2 = (6)^2 \pi cm^2 = 36\pi cm^2 = 113.04cm^2$

The shaded area

$$square\ area - circle\ area = 144cm^2 - 113.03cm^2$$
$$= 30,97cm^2$$

3.9.2 Consolidation activities

1. Write down the formulae for the following

Perimeter of the square	
Perimeter of the rectangle	
Area of square	
Area of rectangle	
Area of rectangle	
Area of rhombus	
Area of kite	

Area of parallelogram	
Area of trapezium	
Diameter of circle	
Perimeter of circle	
Area of circle	

2) Find the angle θ between

$u = 4i - 3j$ and $v = 2i + 5j$

find $u.v, \|u\|$ and $\|v\|$.

3) a) Show that The vectors $v = 3i - j$ and $w = 6i - 2j$ **are parallel.**

b) **Show that the vectors** $v = 2i - j$ and $w = 3i + 6j$ are orthogonal.

Solutions

1)

Perimeter of the square	$4 \times side$
Perimeter of the rectangle	$2(l + w)$
Area of square	s^2
Area of rectangle	$l \times w$
Area of triangle	$\frac{b \times h}{2}$
Area of rhombus	$\frac{D \times d}{2}$
Area of kite	$\frac{D \times d}{2}$
Area of parallelogram	$b \times h$
Area of trapezium	$\frac{(B + b)\text{h}}{2}$

Diameter of circle	$2r$
Perimeter of circle	D π
Area of circle	πr^2

2) $u = 4i - 3j$ and $v = 2i + 5j$

find $u.v, \|u\|$ and $\|v\|$.

$u.v = 4(2) + (-3)(5) = -7$

$\|u\| = \sqrt{(4)^2 + (-3)^2} = 5$

$\|v\| = \sqrt{(2)^2 + (5)^2} = \sqrt{29}$

By the formula, if θ is the angle between u and v. then

$$\cos\theta = \frac{u.v}{\|u\|\|v\|} = \frac{-7}{5\sqrt{29}} \approx -0.29$$

We find that $\theta \approx 105^0$

3. **a) since** $v = \frac{1}{2}w$. Furthermore, since $\cos\theta = \frac{u.v}{\|u\|\|v\|} = \frac{18+2}{\sqrt{10}\sqrt{40}} = \frac{20}{\sqrt{400}} = 1$

the angle θ between v and w is 0.

b) Since, $v.w = 6 - 6 = 0$

3.9.3 Extended exercises

1) Calculate the angles of the triangle with vertices:

$A = (6,0),\ B = (3,5)$ *and* $C = (-1,-1)$.

2) Find the value of k if the angle between $\vec{u} = (k,3)$ *and* $\vec{v} = (4,0)$ *is* 45°

3) Given the vectors $\vec{u} = (2,k)$ and $\vec{v} = (3,-2)$, calculate the value of k so that the vectors $\vec{u}$ and $\vec{v}$ are:

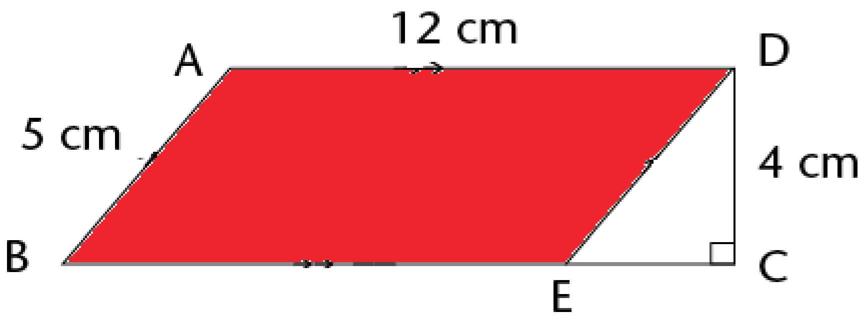
a) Perpendicular.

b) Parallel.

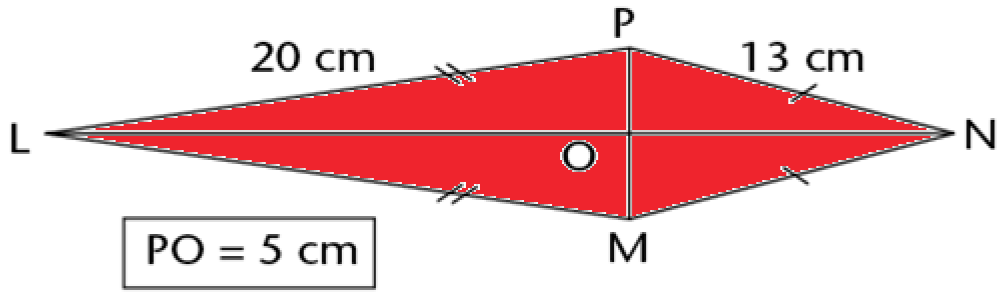
c) Make an angle of 60°.

4. Calculate the areas of the following 2D shapes. Round off your answers to two decimal places where necessary

a)

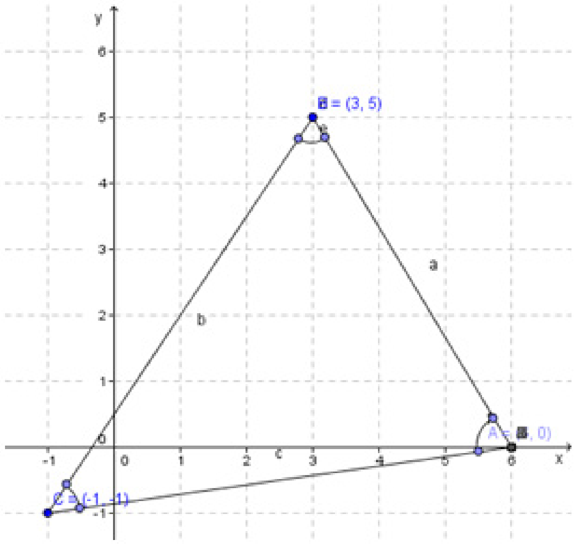


b)



Solution

1. **Theanglesofthetrianglehavingthepoints** $A=(6,0), B=(3,5) and C=(-1,-1)$



The angle at A is given by the vector s $\overrightarrow{AB}$ and $\overrightarrow{AC}$ which are

$\overrightarrow{AB}=\begin{pmatrix}-7\\-1\end{pmatrix}$ and $\overrightarrow{AC}=\begin{pmatrix}-3\\5\end{pmatrix}$

Then the angle A is $\cos^{-1}\left(\dfrac{(-7)\times(-3)+(-1)\times 5}{\sqrt{(-7)^2+(-1)^2}\times\sqrt{(-3)^2+(5)^2}}\right)$

$$=\cos^{-1}\left(\frac{16}{\sqrt{50}\times\sqrt{34}}\right)=67.16^0$$

The angle at B is given by the vector $\overrightarrow{BA}$ and $\overrightarrow{BC}$ which are

$\overrightarrow{BA}=\begin{pmatrix}3\\-5\end{pmatrix}$ and $\overrightarrow{BC}=\begin{pmatrix}-4\\-6\end{pmatrix}$

The angle B is $\cos^{-1}\left(\dfrac{-12+30}{\sqrt{(3)^2+(-5)^2}\times\sqrt{(-4)^2+(-6)^2}}\right)$

$$=\cos^{-1}\left(\frac{18}{\sqrt{34}\times\sqrt{52}}\right)$$

$$=64.65^0$$

The angle at C is given by the vector $\overrightarrow{CB}$ and $\overrightarrow{CA}$ which are

$\overrightarrow{CB}=\begin{pmatrix}-4\\-6\end{pmatrix}$ and $\overrightarrow{CA}=\begin{pmatrix}-7\\-1\end{pmatrix}$

The angle at C is $\cos^{-1}\left(\dfrac{28+6}{\sqrt{(-4)^2+(-6)^2}\times\sqrt{(-7)^2+(-1)^2}}\right)$

$$=\cos^{-1}\left(\frac{34}{\sqrt{52}\times\sqrt{50}}\right)$$

$$=48.17^0$$

2. $\vec{v}=\begin{pmatrix}4\\0\end{pmatrix}$ and $\vec{u}=\begin{pmatrix}k\\3\end{pmatrix}$ the angle between $\overrightarrow{u}\ and \vec{v}$ is 45^0

$\cos(\vec{u},\vec{v})=\left(\dfrac{a.c+b.d}{\sqrt{a^2+b^2}\times\sqrt{c^2+d^2}}\right)$ $\quad\cos 45^0=\left(\dfrac{4k}{\sqrt{(4)^2+(0)^2}\times\sqrt{k^2+3^2}}\right)$

$$\frac{\sqrt{2}}{2}=\frac{4k}{4\sqrt{k^2+9}}$$

$$4\sqrt{2(k^2+9)}=8k$$

$$\sqrt{2(k^2+9)}=2k$$

$$2(k^2+9)=4k^2\Rightarrow 2k^2+18=4k^2$$

$$2k^2-18=0\Rightarrow 2(k^2-9)=0\Rightarrow k=\pm3$$

3. $\vec{v}=\begin{pmatrix}2\\k\end{pmatrix}$ and $\vec{u}=\begin{pmatrix}3\\-2\end{pmatrix}$

a) the angle between $\overrightarrow{u}\ and \vec{v}$ is $90^0, thus$

$$0=\left(\frac{a.c+b.d}{\sqrt{a^2+b^2}\times\sqrt{c^2+d^2}}\right)=\left(\frac{6-2k}{\sqrt{4+k^2}\times\sqrt{9+4}}\right)$$

$$\Rightarrow 0=\left(\frac{6-2k}{\sqrt{4+k^2}\times\sqrt{9+4}}\right)\Rightarrow k=3$$

c) If the two vectors are parallel then $\cos 0 = 1$

$$\Rightarrow 1 = \left(\frac{6-2k}{\sqrt{4+k^2} \times \sqrt{9+4}}\right) \Rightarrow \sqrt{4+k^2} \times \sqrt{9+4} = 6-2k$$

$$\Rightarrow \sqrt{13\left(4+k^2\right)} = 6-2k$$

$$\Rightarrow 13\left(4+k^2\right) = \left(6-2k\right)^2$$

$$\Rightarrow 52+13k^2 = 36-24k+4k^2$$

$$\Rightarrow 9k^2 + 24k + 16 = 0$$

$$\Rightarrow (3k+4)^2 = 0$$

$$\Rightarrow 3k+4 = 0$$

$$\Rightarrow k = \frac{-4}{3}$$

d) $\cos\left(\vec{u}, \vec{v}\right) = \left(\frac{a.\text{c}+\text{b.d}}{\sqrt{a^2+b^2} \times \sqrt{c^2+d^2}}\right)$

$$\cos 60^0 = \left(\frac{6-2k}{\sqrt{4+k^2} \times \sqrt{9+4}}\right)$$

$$\frac{1}{2} = \left(\frac{6-2k}{\sqrt{4+k^2} \times \sqrt{13}}\right) \Rightarrow \sqrt{13(4+k^2)} = 2\left(6-2k\right)$$

$$\Rightarrow \sqrt{13(4+k^2)} = 12-4k$$

$$\Rightarrow 13\left(4+k^2\right) = \left(12-4k\right)^2$$

$$\Rightarrow 52+13k^2 = 144-96k+16k^2$$

$$\Rightarrow 3k^2 - 96k + 92 = 0$$

$$\Rightarrow k = \frac{48-26\sqrt{3}}{3} \text{ or } k = \frac{48+26\sqrt{3}}{3}$$

$$\Rightarrow k \approx 0.9888 \text{ or } k \approx 31.0111$$

4. a) parallelogram area $B \times H = 12cm \times 4cm = 48cm^2$

Triangle area $= \frac{b \times h}{2} = \frac{3cm \times 4cm}{2} = 6cm^2$ where the distance EC $= \sqrt{(5)^2 - (4)^2} = 3cm$ by Pythagoras theorem

Therefore, the area of the given figure $= 48cm^2 + 6cm^2 = 54cm^2$

a) the kite area is $\frac{D \times d}{2}$

The distance LP=20cm, PN=13cm, PO=5cm where PM=2OP=10cm=d

We can find ON by applying Pythagoras theorem;

$\sqrt{(13)^2 - (5)^2} = \sqrt{139 - 25} = \sqrt{114}$

$= 10.67cm$

Also we can find LO; $\sqrt{20^2 - 5^2} = \sqrt{375} = 19.36cm$ now

D=10.67cm+19.36cm=30.03cm

So the area is $\frac{D \times d}{2} = \frac{30.03cm \times 10cm}{2} = 150.15cm^2$.

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