TEACHING METHODS AND PRACTICE FOR MATHEMATICS IN PRIMARY SCHOOLS

TTC Option: SME and ECLPE

Student's book

FOREWORD

Dear Student-teachers,

Rwanda Education Board is honoured to present the Mathematics teaching methods and practice book which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of mathematics. The Rwandan educational philosophy is to ensure that learners achieve full potential at every level of education which will prepare them to be well integrated in society and exploit employment opportunities.

Specifically, TTCs curriculum was reviewed to train quality teachers who will confidently and efficiently implement the Competence Based Curriculum in pre-primary and primary education.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of Mathematics teaching methods and practice aligned with the TMP syllabus to facilitate their learning process.

The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market has necessitated the elaboration of a book which will facilitate and guide student-teachers to understand different methods, techniques, strategies and adequate teaching and learning materials for the effective implementation of a competence-based curriculum.

The book of Mathematics teaching methods and practices provides active teaching and learning strategies that engage student-teachers in doing things and thinking about the things they are doing. It also provides a range of activities which will facilitate student-teachers to:

- Critically analyse and be familiar with all teaching and learning materials used in primary mathematics lessons.
- Carefully explore all teaching and learning methods, techniques and strategies to be used by primary schools teachers in order to effectively deliver an engaging, active and enjoyable Mathematics lessons.
- Critically investigate the importance of teaching and learning materials to effectively deliver an engaging, active and enjoyable Mathematics lessons.
- Set relevant and age appropriate tasks which intend to develop generic competences and address cross-cutting issues.
- Prepare a competence based lesson plan.

To facilitate student-teachers from Science-Mathematics Education (SME) and Early Childhood and Lower Primary Education (ECLPE), the content of this book is self explanatory so that they can easily use it.

I wish to sincerely express my appreciation to the people who contributed towards the development of this book, particularly, REB staff, UR-CE Lecturers and TTC Tutors for their technical support. A word of gratitude goes to the Head Teachers and TTCs principals who availed their staff for various activities.

Dr. NDAYAMBAJE Irénée

Director General, REB

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I wish to express my appreciation to the people who played a major role in development of this student-teacher's book of Mathematics teaching methods and practices. It would not have been successful without active participation of different education stakeholders.

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Joan MURUNGI

Head of CTLR Department

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Unit 1 INTRODUCTION TO TEACHING MATHEMATICS IN PRIMARY SCHOOL

Key unit competence: **Explain the essence of teaching Mathematics in primary and how Mathematical skills are applied in everyday life**

Introductory activity 1

Brainstorm and provide answers on the following:

1. How did you feel about Mathematics when you were studying in primary school?
2. Why do you think Mathematics is important in primary schools?
3. What do you think could be the main topics to be taught in Mathematics in primary school?
4. What topic(s) was too difficult to you? Which one was easiest and interesting? Explain why

1.1. Rationale/importance of teaching and learning Mathematics in basic education.

Activity 1.1

Using lower or upper primary Mathematics syllabus, internet or any other document carry out a research and discuss the rationale of learning and teaching Mathematics in primary education

Mathematics is around us and is present in different forms; Right from getting up in early hours of the day to the ringing of an alarm, reading time on a watch, rounding a date on a calendar, picking up the phone, preparing a recipe in the kitchen, to wait for the counts of whistles of the cooker, manage the money, travel to some place, to exchange currency at a ticket outlet while availing a public conveyance or checking up the mileage of a car, halting at the filling station, attending to a roll call at school, getting scores in the class exams, even meet new friends the list is just endless if one goes on to note down the situations when our computational skills, or more specifically, simple Mathematics comes to play a role, almost every next moment we do the simple calculations at the back of our mind.

Mathematics is also used in natural and applied sciences; the applied statistics and probability play an important role in different games of chance, in the national census process, in scientific research, etc.

In the teaching and learning of Mathematics, some cross-cutting issues can be addressed to improve social and economic welfare of Rwandan society. Addressing these crosscutting issues, one can show the following:

- Business and industry depends upon the knowledge of Mathematics; the change in the social structure with regards to the modern facilities like mode of transport, means of communication and progress in the field of science and technology is due to Mathematics as well;
- Mathematics teaching is very important for intellectual developments. Mathematics is one of the subjects which makes students' brain active;
- Mathematics helps to prepare students for technical and other vocations such as engineering, architecture, accountancy, banking, business, agriculture, tailoring, carpentry, surveying, office management related courses…
- Mathematics develops the skills of reflection.

This leads to the conclusion that life without any knowledge of calculation, computation or in other words Mathematics can fail. Therefore, through CBC, Mathematics is being taught in all levels of Basic Education, from Pre-primary to Secondary education, for the ambition of developing a knowledge-based and technology-led economy in Rwanda, since it provides to learners all required knowledge, skills and attitudes to be used in different learning areas and in real life situations.

Application Activity 1.1

- Give five examples where Mathematics is useful for student teachers in everyday life
- Using examples, explain why Mathematics is important to primary school learners.
- In your understanding what Mathematics topics do you think are more important for primary school learners

1.2. Description and role of primary mathematics syllabus components

Activity 1.2

Read the lower or upper primary Mathematics syllabus and do the following:

1. Identify the structure and main components or elements of Maths syllabus.
2. Identify and discuss Mathematics teaching strategies outlined in the pedagogical approach
3. Discuss the role of the following components or elements:

a. Introduction b. Pedagogical approach c. References	d. Assessment approach e. Resources f. Syllabus units

7. Find the main topic areas of Mathematics taught in primary school

Content summary

The lower or upper primary Mathematics syllabus has the following main components:

1. Preliminary pages

- Foreword
- Acknowledgement
- Table of content
- Introduction
- Pedagogical Approach
- Assessment Approach
- Resources

Each component of the preliminarily pages plays an important role at every step of the teaching and learning process.

- Introduction of the syllabus provides all information related to: Background to curriculum review, Rationale of teaching and learning mathematics (Mathematics and society, Mathematics and learners), Competences (generic and broad subject competences).

A primary mathematics teacher gains the rationale/ importance/ role of mathematics through this part of the syllabus.

- Pedagogical approach part provides all information related to: the role of the learner, the role of the teacher, the special needs education and inclusive approach in the teaching and learning of mathematics. In this part, various teaching strategies and approaches such as direct instruction, discovery learning, investigation, guided discovery or other methods are incorporated. Also, in this pedagogical part, a list of generic and broad subject competences are presented so that a mathematics teacher can consider them while preparing and delivering mathematics lessons.
- Assessment approach part provides information needed to formative assessment and summative assessments, record keeping, Item writing in summative assessment and reporting to parents.
- Resources are the part of the syllabus which provides a non-exhaustive list of materials needed for implementation of Mathematics syllabus.

2. Mathematics syllabus units for lower or upper primary

Syllabus units is composed by the following: Presentation of the structure of the syllabus units, Mathematics units for each grade in lower primary (primary one , two, three) ; Mathematics units for each grade of upper primary (Primary four , five , six), Key competences at the end of primary one, two, three, four, five and six.

At every grade, the syllabus is structured in topic areas, sub-topic Areas where applicable and then further broken down into units and unit content. This breakdown promotes the uniformity, effectiveness and efficiency of teaching and learning mathematics. The Units have the following elements:

1. Each Unit is aligned with the number of periods
2. Each Unit has a key unit competence whose achievement is pursued by all teaching and learning activities undertaken by both the teacher and the learners.
3. Each Key Unit Competency is broken into three types of Learning Objectives as follows:

a. Learning Objectives related to Knowledge and Understanding (also known as Lower Order Thinking Skills).

b. Learning Objectives related to acquisition of skills (also known as Higher Order Thinking skills) .

c. Learning Objectives related to acquisition of Attitudes and Values (also known as Higher Order Thinking Skills)

4. Each Unit has content that indicates the scope of coverage of what is to be taught and learnt in line with the stated Learning Objectives.
5. Each Unit suggests a non-exhaustive list of learning activities that are expected to engage learners in an interactive learning process as much as possible (learner-centered and participatory approach).

- Main topic areas developed in the part of syllabus units are the following:
- Number and operations, Fractions, decimals, and proportional reasoning, Metric measurements, Algebra, Geometry, Statistics and elementary probability (upper primary only)
- Application activity 1.2

Application activity 1.2

Using lower or upper primary Mathematics syllabus, explain the importance of teaching and learning the following topics to primary school learners:

a. Numbers and operations

b. Fractions, decimals and proportional reasoning

c. Metric measurement

d. Algebra

e. Geometry

f. Statistics and Elementary Probability

End unit assesment 1

1. All components of the lower or upper primary Mathematics syllabus are important for teachers while planning his/her lessons. In your understanding, order from the most important to the least important the components of the Mathematics syllabus and justify your order.

Main components of lower or upper primary mathematics syllabus	
• Foreword • Acknowledgement • Introduction • Pedagogical Approach	• Assessment Approach • Resources • Syllabus Units with content • References

Unit 2 TEACHING AND LEARNING RESOURCES FOR MATHEMATICS LESSONS

Key unit competence: **Differentiate available teaching and learning resources and produce more others required in the learning of Mathematics for primary school.**

Introductory activity 2

As we all know, today's age is the age of science and technology and the process of teaching and learning depend upon the different types of equipment available in or outside the classroom. Identify and categorize all Mathematics teaching and learning resources you can use in Mathematics lesson. Are those resources available in our environment? Who can produce them? Who is responsible to use and keep them? When are those materials used?

2.1. Exploration of mathematics primary textbooks and teacher's guides in the context of CBC

Activity 2.1

Take lower primary or upper primary Mathematics textbooks and teacher's guide and explore them to find out their structure (arrangement of units, topics/lessons and activities). How these books can be used in the context of Competence based curriculum (CBC)?

Content summary

The process of teaching and learning depends upon the different type of equipment available. A textbook is an example of printable teaching resource.

In the context of CBC, Mathematics textbook is written with sequencing of units where each unit has a key unit competence, introductory activity, different learning activities, examples and application activities. The unit ends with an end unit assessment to help learners to make auto-evaluation for assessing whether the key unit competence has been achieved.

In Mathematics book, the introductory activity intends to arouse learners' curiosity on what is going to be taught in the unit basing on the prior knowledge of the learners. This can be the reflective activities in problem solving form: objects to count, question to be answered, word problem to be solved, a graph to be interpreted, a picture to be labelled, a table to be completed, a topic to be discussed on etc.

In Mathematics, a learning activity introduces every lesson in order to help learners develop new competences basing on what they acquired before. After the learning activities, the Mathematics book contains the summary of the content and the application activity which helps learners to be engaged in acquiring different skills. Such activities are sometimes exercises, remedial or consolidation activities. Every activity is set depending on the lessons within the unit.

The context of CBC provides also the teacher's guide. Primary Mathematics teacher's guide has three main parts:

The Part 1 concerns general introduction that discusses methodological guidance on how best to teach and learn Mathematics, developing competences in teaching and learning Mathematics, addressing cross-cutting issues in teaching and learning Mathematics and the Guidance on assessment.

Part 2 consists in sample lesson plan per unit developed to guide the teacher on how to prepare a lesson in Mathematics.

Part 3 is about the structure of a unit and the structure of each lesson. This part provides information and guidelines on how to facilitate learners while working during learning activities. All application activities from the textbook have answers in this part. A well elaborated teacher guide (TG) provides also additional information for the teacher on the unit basis and a variety of activities classified in 3 categories (remediation, consolidation and extended activities) to help learners enrich their concepts and content development.

Use of the Mathematics textbooks

The structure of Mathematics textbooks is likely to have an impact on actual classroom instruction. The form and structure of textbooks advance a distinct pedagogical model and thus embody a plan for the particular succession of educational opportunities.

The pedagogical model only becomes effective when the textbook is actually used.

Therefore, Mathematics textbooks should not be a subject to analysis detached from its use. It is an interactive part within the activities of Mathematics teaching and learning.

In order to develop a better understanding of the role of the Mathematics textbooks within the activities of teaching and learning Mathematics a model was developed (Rezat, 2006) as illustrated on the following tetrahedron model.

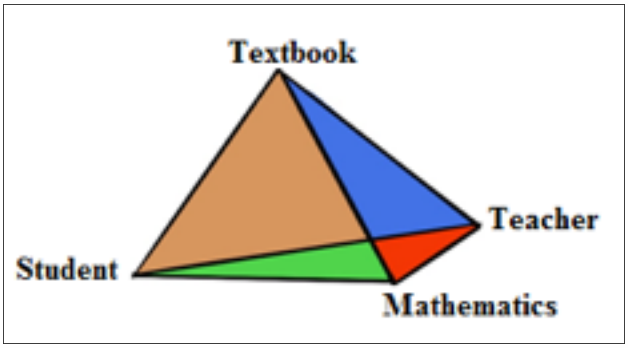


Fig. 1: Tetrahedron model of the use of textbook

This model is based on the fundamental model of didactical system: the ternary relationship between learner, teacher, and Mathematics. The Mathematics textbook is implemented as an instrument at all three sides of the triangle: teachers use textbooks in the lesson and to prepare their lessons; by using the textbook in the lesson teachers also mediate textbook use to students, and finally learners learn from textbooks.

The Mathematics textbook was and still is considered to be one of the most important tools. The activities found in the Mathematics books do not only give an insight into learner's utilization of Mathematics textbooks, but they also give an idea of what learning Mathematics is about for learners. Learning Mathematics with the Mathematics textbook comprises activities as solving tasks and problems, consolidating mathematical knowledge and skills and acquiring new contents and competences.

Advantages of Mathematics textbooks

The following list contains the most frequently stated advantages of using textbooks (Graves 2000):

- It provides the course of the syllabus because it is written basing on Mathematics syllabus provided by the curriculum framework.
- It provides security for the learners because they have a kind of a road map of the course: they know what to expect and they know what is expected from them.
- It provides a set of visuals, activities, readings, etc., and so saves the teacher time in finding or developing such materials.

- It provides teachers with a basis for assessing learners' learning. Some textbooks include different application activities.
- It may include supporting materials (teacher's guide, worksheets, worked examples, solution of activities, videos for simulation, etc)
- It provides consistency within a program across a given level, if all teachers use the same textbook. If textbooks follow a sequence, as within a series, it provides consistency between levels.

Textbooks also have limitations

Some textbooks bought from the publisher who wrote them without the involvement of the future user (teachers) can lead to teachers' and learners' dissatisfaction with the course. The following list contains the most frequently stated disadvantages of using only ready-made textbooks (Graves, 2000):

- The content or examples may not be relevant or appropriate to the group and they may not reflect the learners' needs since textbooks are often written for global markets and often do not reflect the interests and needs of learners. They may contain inauthentic language, since texts, methods of teaching, dialogs and other aspects of content tend to be specially written to incorporate teaching points and are often not developing all Mathematics competences as suggested in the syllabus.
- The content may not be at the right level of learners. There may not be the right mix of activities (too much of it or too little of it), there may be too much focus on one topic and not enough focus on others, or it may not include everything teachers want to include.
- The sequence of units may not be in accordance with the real work-related needs of the learners' experiences.
- The textbook may not take the learners' background knowledge into account.

To overcome such challenges or limitations, Graves (2000) suggests that, in order to minimize difficulties when selecting textbooks, teachers should: use the textbook as a resource for learners, but not the only resource; use a textbook as a guide, be free to modify, evaluate, develop, change, eliminate, or add to the material in the textbook, supplement the textbook with lots of outside readings.

By adding, the teacher may not rely on solved exercises in the teachers' guide, she/he should solve those exercises him/her self and compare his findings with book's answers before marking the learners' works.

In addition the teacher can use and adapt Mathematics activities for the textbooks by choosing some that will be:

- **Warm-up activities** – usually based on previous topics. It can be considered a review activity and it is usually given at the beginning of a class as a creative way to start a class or break the routine of a class.
- **Presentation activities** – introduce new topics.
- **Practice activities** – it is a meaningful opportunity for the learners to practice the taught material and develop more competences in mathematics.
- **Consolidation activity** – it is developed after the practice and these activities reinforce the competence that had already been developed.

Application activity 2.1

1. Using Mathematics books for lower or upper primary, identify the number of units, and highlight learning activities and application activities in each unit.
2. Compare all primary Mathematics books and conclude whether they all have the same structure or not.
3. Discuss the structure of available Mathematics teacher's guides for lower or upper primary and suggest how you can use them and adapt them when preparing your lessons.
4. Explain the roles of Mathematics books (textbooks and teacher's guides); when and how to use them?

2.2. Mathematics teaching and learning materials in primary

Activity 2.2

Using Mathematics book for lower or upper primary, select one lesson on addition of fractions with the same denominator and do the following:

a. Select teaching and learning materials for facilitating that lesson;

b. Differentiate locally made materials, printed and non-printed among them;

c. Is it possible to teach this lesson by the use of ICT resources? If yes, give examples of those resources and explain.

d. Can learners with special educational needs use the materials you selected? Explain.

Content summary

Nowadays teaching and learning requires teaching materials in order to put the methods of teaching into practice and eventually achieve the learning objectives.

Some mathematics teaching materials:

Depending on the learning styles for every child, one can show that every learner needs learning materials:

Visual-spatial: A visual learner learns best Mathematics by using their eyes to see information. They learn by seeing words in printed form or by using graphics and pictures, observing real objects, and other visual aids.

Auditory learners: An auditory learner is someone who learns best Mathematics by listening and talking. They learn by listening to someone who presents information orally (audio) and by being allowed to discuss and ask questions.

Tactile learners: Tactile learners learn best Mathematics through their sense of touch, such as using their hands and fingers. They learn best by writing, drawing, and using hands-on manipulative.

Kinesthetic learners: Kinesthetic learners learn best Mathematics through movement of their large or gross motor muscles. They take in information best while moving and doing, being involved in projects, role playing, learning while standing up and engaging in real life activities.

Therefore, different types of teaching aids are needed to satisfy the need for every learner in Mathematics lesson.

1. Types of teaching aids in Mathematics lesson

2.1. Printed materials

- Textbooks
- Supplementary materials: workbook, teacher prepared study guide, reference book, magazines articles, charts on manila papers, flash cards, number cards, etc.

2. 2. Non- printed materials

Teaching multimedia

- **Audio aids:** The aids that involve the sense of hearing as radio, telephones and memory cards.
- **Visual aids:** The aids which use sense of vision for example: actual objects, models, pictures, charts, illustrations, maps, flash cards, posters, graphs, flannel board, bulletin boards, chalkboard, overhead projector, slides etc. Out of these, the chalkboard and chalks are the commonest ones. It includes also the community resources: field trips, resource person such as a carpenter or tailor
- **Audio - visual aids:** The aids which involve the sense of vision as well as hearing for example film projector, videos of Mathematics lessons or illustration of a concept (simulations), computers, Mathematics software and applications, etc.

Locally-made materials: With the improvisation, the available locally materials are used to make teaching and learning aids for facilitating Mathematics lesson (e.g wooden materials, and plastic materials like counters, 2D shapes, 3D solids, etc).

2. Production and utilization of Mathematics locally made resources

School environment has been described as an organization where resources are produced, managed and organized in such a way that enables the learners to acquire desirable learning competences. Mathematics is a subject which aims at equipping learners with appropriate scientific attitude, competences and ability to apply scientific knowledge to every challenges of life.

It has been observed that Mathematics education is a veritable tool for scientific and technological advancement of any nation. This is why Rwanda has decided to include Mathematics among other subjects that are taught to all learners in basic education to equip the learners to live effectively in the modern age of science and technology.

To achieve this, resources for the teaching and learning of Mathematics, sciences and technology must be adequately produced and utilized judiciously in our schools. It is on this basis that this unit tends to discuss the utilization and production of Mathematics teaching resources.

Improvisation of Mathematics resources in schools

It has been observed in recent times that both the rural and urban primary schools are increasing in population and the problem is that of acquiring adequate materials or facilities for effective teaching and learning of Mathematics as the subject taught in all levels of basic education. The effective teaching and learning of Mathematics requires material resources.

The material resources in the teaching and learning include all the materials that are needed for the effective teaching and learning of Mathematics. These resources include the following: meter ruler, stop watch, math-set, hand-lens, gas cylinder, mathematical charts, beakers, school garden, models, measuring cylinders, filter funnel, hydrometer, weighing balance, rain gauge, thermometer, computer, charts, anemometer, etc.

The complete provision and utilization of these materials resources will help make the teaching and learning of Mathematics interesting and effective. One possible solution to this problem of inadequate material resources according to Adejoh (2001) is the improvisation.

Improvisation is the act of construction of instructional materials from locally available materials that can adequately replace or function in place of the original material which otherwise may be very expensive or in short supply or unavailable (Eriba, 2011). Improvisation therefore can be a state of mind and it is a skill that lies at the heart of good mathematics teaching.

It is also defined as an art of identifying, developing and using suitable materials in the absence of the real one for effective teaching and learning process. It could mean making of instructional material or teaching aid by Mathematics teachers where the original materials are not available or where there are available but not functional.

However, it has been observed that most Mathematics teachers are not creative enough to carry out the process of improvisation. The availability of resources and adequacy of these facilities promote effective teaching and learning activities in schools while inadequacy affects the learners' performance negatively. In TTCs the TRCs (Teacher Resource Centers) should serve as a good area for student-teachers to make materials or improvise Mathematics teaching aids.

Need for improvisation of Mathematics resources

In an ideal world, all Mathematics learners would be taught in non-overcrowded but well equipped classes. In absence of those well-equipped classrooms, the time for engaging every learner with manipulative or practical activities cannot be emphasized, yet those materials required for the teaching and learning of Mathematics are in short supply. There is a total or partial absence or inadequacy of the Mathematics teaching resources and inadequate finances for purchasing of Mathematics equipment, poor culture of some school heads to maintain Mathematics teaching and learning materials available at school.

With all these above mentioned problems, it seems that the best option is the improvisation of Mathematics teaching materials in the classroom by teachers and even learners. The Rwandan school system today is experiencing limited resources; the teacher initiative to guide the production of teaching resources in local materials becomes a task for learning to be more effective and productive.

Factors affecting the production of Mathematics Materials Resource for primary schools

Technical factor: This relates to the question of degree of accuracy and precision that is possible with improvised equipment. Depending on the age of learners, some materials they produce under the guidance of teachers are not accurate. In addition, there is a lack of guiding document on how to produce locally made Mathematics teaching aids.

Human factor: The lack of skills and adequate professional training in developing the resources is a major problem militating against the effective use of local resources for Mathematics teaching. When the teacher has not such skills, he/she cannot guide learners to make or bring from home their own learning materials.

Therefore, there is a need for every Mathematics teacher to have regular self-study on how to make improvisation.

Influence of improvisation of resource materials in the teaching and learning of Mathematics

Some influences that improvised materials would have on Mathematics teaching and learning are the following:

- Improvised materials provide a cognitive bridge between abstraction and reality to learners. It is not easy to teach learners to count objects without bringing examples of those objects and use them when counting. Example: beans, maize, etc.

- Improvisation saves cost and in addition the teacher and the learners make positive effort towards effective instruction. Instead of paying money to buy counting rods, one can use beans.
- Improvisation undertaken by teacher enables him/ her to think and research for cheaper, better and faster methods for making the teaching and learning process easier to learners. This makes the teacher innovative and a lifelong learner.

Guide on improvisation of Mathematics resources

Improvised materials should possess certain qualities and these are the following:

- Appropriateness of teaching aids to the age of the learners.
- Its clarity in illustrations and simplification of concepts.
- Its adequacy in size.
- Its relevance to the lesson they are meant for.
- It should be interesting to the learners and durable.

Basic consideration in the improvisation of Mathematics resources

On embarking on any improvisation in the teaching and learning of Mathematics certain pedagogical consideration is necessary. Some of this consideration includes:

1. What is to be taught?
2. What is the activity to be done?
3. Objectives of the lesson.
4. The background knowledge of the learner
5. The durability of the improvised materials.
6. The cost advantage of improvisation materials.
7. Individual differences of learners (special educational needs).
8. Learning environment.

Utilization of Mathematics resources in primary schools

It has been found out that learners learn better through practical approach with the use of appropriate manipulatives. The process of managing and organizing those resources is called resource utilization. The utilization of resources in teaching brings about fruitful learning as such it stimulates learners' senses as well as motivating them. During the learning activity, resources should be provided in quality and quantity in mathematics classroom for effective teaching-learning process.

Importance of effective utilization of improvised Mathematics resources

Teaching aids play a very important role in Teaching- Learning of Mathematics.

- **Motivation:** Teaching aids motivate the learners when they are in mathematics lesson so that they can learn better and develop new competence.
- **Clarification:** Through teaching aids, the teacher clarifies the subject matter more easily.
- **Discouragement of cramming:** Teaching aids can facilitate the proper understanding of Mathematics concepts to the learners and discourage the act of cramming.
- **Arouse learners' curiosity on the new concept:** Teaching aids helps to guess the new concept to be introduced.
- **Saves time:** the use of teaching aids saves time that teacher may take during explanations for learners understanding.
- **Avoids dullness:** This means the class becomes more active, lively and participatory
- **Direct experience:** Teaching aids provide direct experience to the learners which make them learn easily.

Care and maintenance of instructional resources

To make teaching and learning resources useful in Mathematics lessons, they should be maintained carefully to ensure that they are in good condition. It is important that the teachers know all resources that are available in their school and where they are kept. The teacher should catalogue and store the resources in a special place from where other teachers can borrow the resources for use and then return them. The storage can be done in a room, a box, or cupboard. New items should be collected or made while old, torn or spoilt ones replaced.

The following are general ways of taking care of and maintaining instructional materials:

- Storing materials in a flat position to avoid folding and creasing (e.g. charts on manila papers or rice sacks)
- Mounting materials/aids correctly and firmly/ secured
- Storing materials in a clean, dry place.
- Avoiding folding the materials to prevent wear and tear at the fold
- Keeping electronic equipment in bags to protect them from dust.
- Edges binding using sell tape, polythene paper or commercial edge binders.

- Using synthetic spray, polythene paper or varnish to protect the surface of picture or maps

Resources, when appropriately utilized in teaching and learning of Mathematics, make learning more concrete, real, immediate and permanent. In this context resources were conceptualized as devices used to provide the richest possible learning environment which helps the teachers and learners to achieve specific objective in the shortest possible time.

The teaching and learning of Mathematics involving learners' interaction with resources will no doubt enhance student's acquisition of much desired process skills of self-reliance which most nations of the world including Rwanda are yearning for.

It is a noted fact that some primary Mathematics teachers do not put the available resources in their school into use. They need to cultivate into themselves the culture of utilizing available resources to enhance effective teaching because the effective teaching and learning to a large extend is dependent on a combination of factors which include personality of teachers, learner's characteristics, instructional support, availability and utilization of instructional resources.

On the other hand, when instructional resources are misused, Mathematics teaching and learning process may be adversely affected.

Instructional resources are misused when they are not effectively put into use to achieve predetermined objectives. Misused instructional resources as observed could lead to misconception of ideas, create confusion and make a learner hate the subject because he/she did not find the motivation that arouse his interest on the topic. Misused instructional resources emanating from lack of knowledge could result to the failure of Mathematics teaching and learning.

Application activity 2.2

1. Refer to a Mathematics lesson on addition of numbers in lower or upper primary and suggest how to make related teaching materials.
2. Given that many Mathematics teachers for primary schools are not used to produce teaching and learning materials. What can you recommend to them in order to produce relevant, age appropriate and low cost teaching and learning materials?

End unit assessment 2

Read carefully the below text, observe a suggested list of resources that can be used in Mathematics lessons and answer the related questions.

"To develop mathematical concepts, learners need to interact with concrete and semi-concrete materials before they interact with symbolic presentation. This helps them realize that hand-on and minds-on tasks are fundamental to the development of mathematics understanding. Learners at all levels can benefit from practical work if this is appropriately selected, well planned, and relevant to the concepts. The teacher's role is significant in selecting appropriate resources and giving clear instructions to the learners. If a teacher does not select appropriate resources to the given activities, this can hinder the learning process instead of benefiting learners. The use of mathematical software and online activities can also play an important role in developing conceptual understanding in Mathematics".

A list of materials which can be used in Mathematics lessons:

1. Place value blocks	12. Physical balance
2. Number cards	13. Pair of scissors
3. Multi-link cubes or different-coloured counters	14. Masking tape
4. Fraction blocks	15. Weight scale
5. Tangram	16. Measuring tape or scale
6. Abacus model	17. Measuring cylinders
7. Beakers or graduated cups	18. Geoboard
8. 2D shapes or 3D objects	19. Board geometry box
9. Dotted papers	20. Graph board
10. Square papers	21. Rope or thread
11. Number grid	22. Paper and graph paper

a. Make a research, identify and classify all materials into concrete and semi- concrete materials,

b. Select among the list, all materials that can be used to teach and learn topics related to numbers and operations. Support your answer with examples of concepts that can be developed using those materials.

c. Select among the list, all materials that can be used to teach and learn topics related to geometry. Support your answer with examples of concepts that can be developed using those materials.

d. Select among the list, all materials that can be used to teach and learn topics related to measurements. Support your answer

Unit 3 GENERAL PRINCIPLES AND METHODS OF TEACHING MATHEMATICS

Key unit competence: Use a combination of teaching and assessment principles, methods and techniques to make mathematics lessons successful

Introductory activity 3

1. Read carefully the following statements and choose the ones you find the best in teaching and learning Mathematics in lower or upper primary schools. Justify your choice.

- Teaching Mathematics in primary schools is for learning; learning is for understanding; understanding is for reasoning, applying and ultimately problem solving
- Teaching Mathematics in primary should build on learner's knowledge, take cognizance of learner's interests and experiences; and engage them in active and reflective learning.
- Teaching should connect learning to the real world, harness ICT tools and emphasize 21st century competences.
- While teaching Mathematics in primary it is very important to consider the progression of concepts and the topics
- Teaching Mathematics should always be starting on a learning activity which can allows learners to give their predictions toward a given lesson and then use a variety of activities though individualization and cooperative learning.
- Effective teaching of Mathematics always starts with something tangible, existing, and concrete.
- Effective teaching of Mathematics in primary schools should move from the specific to the general (from concrete to abstract)
- Effective teaching of Mathematics in primary schools should begin with general and end with specific.
- After teaching a mathematical concept, learners should use what they learned in many different situations.

2. Read carefully the following statements and explain why every statement is important in setting appropriate Mathematics activities in lower or upper primary.

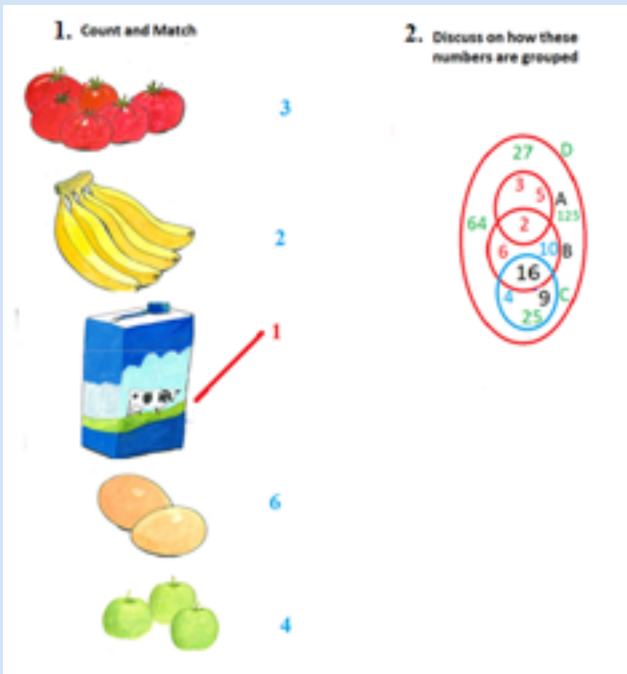
- Activities and tasks in Mathematics assessment should be age appropriate

- Assessment should reflect the Mathematics that is most important to learners.
- Assessment should enhance Mathematical learning and support good instructional practice.
- Assessment should support every learner's opportunity to learn important mathematical concepts

3.1. General principles of teaching Mathematics

Activity 3.1

1. Observe and analyse the diagrams below and explain how this type of activities can individually or in groups motivate and engage learners in a Mathematics lesson.



2. In teaching and learning Mathematics, the following key elements are important: contextualization, concretization, progression and generalization. Read carefully the following activities and explain how each of the elements is expressed or taken into consideration (see activities on a worksheet and the 1st activity above).

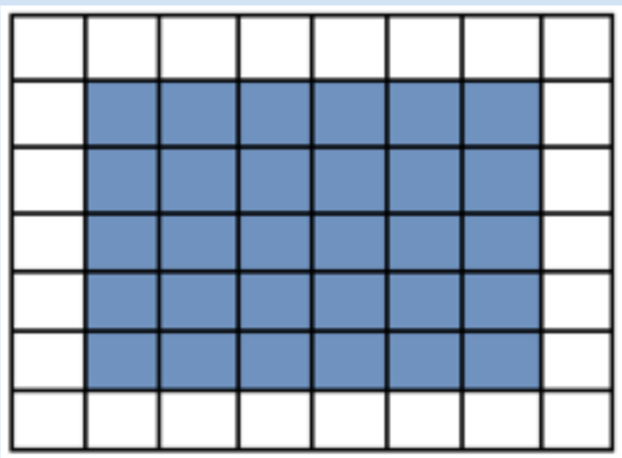
Mathematics worksheet

Activity 1: Use sticks to form the following pattern. Build the next two shapes.

No. of rectangles	1	2	3	4	5	6	7	10	13
No. of sticks	4	7							

- Fill in the grid based on the models developed with matchsticks, only up to the fifth model.
- Predict the number of sticks needed to make 6 rectangles. Give a reason for your response.
- How many sticks are needed for 10 rectangles?
- Is there a possibility of having 13 sticks in any model? Give a reason for your response.
- Is there a possibility of having 100 sticks in any model? Give a reason for your response.

Activity 2: observe the following grid showing the bottom plan of a doghouse. In the grid, the side of each small square is 1 cm2 in reality.



- What is the area of the doghouse in square centimetres?
- Figure out the side lengths in cm.

Activity 3: observe the following table showing the term school fees for 5 different schools and answer the questions.

Schools	School fees per term in Rwandan Francs (Frw)
School 1	57 000
School 2	45 450
School 3	69 000
School 4	50 000
School 5	39 000

- Write the school fees of all schools in place value table.
- Order the numbers in ascending order.
- Explain the process you used to order the fees from the cheapest to the most expensive school.
- How might parents/ guardians/ families save money so that they can easily pay school fees?

Content summary

The following are the principles of teaching Mathematics lessons.

3.1.1. Motivation

Teaching and learning Mathematics in primary should build on learner's knowledge, interests and experiences. It should also connect learning to the real world and use ICT tools and facilities.

Motivated learners develop positive attitudes toward Mathematics as follow:

- Learners are interested in what have been taught
- They are constantly attentive
- They put more personal effort
- They do not get tired and they never get discouraged
- They are interested in their progress and achievements
- They learn more quickly and understand better.

Effective Mathematics teaching requires an understanding of what learners know and need to learn and then challenging and supporting them to learn it well. Learners can learn Mathematics more effectively with the appropriate use of technology. When technological tools are available, learners can focus on decision-making, reflection, reasoning, and problem-solving

3.1.2. Engaging activity

A learner learns better Mathematics if he/she is completely involved in an action. In active method, the teacher creates a learning environment, where the learners take part and communicate among themselves in the group. The learner's activity is a central concept in most explanatory theories of development and learning. The purpose of engaging activity is to give the learner more autonomy, more initiative, more personal motivation and to develop his/her creativity. The learners' intellectual activity must always be based on the achievement of accordingly defined tasks where learner's participation, reflection and discovery are encouraged.

3.1.3. Concretisation

Effective teaching always starts with something tangible, existing, concrete, that is the teaching focuses on a real context. Reality gives meaning to the context and allows the learner to understand any context, referring it to a well-known and familial context.

There are two classical approaches of a Mathematics lesson: induction (to move from the specific to the general or from concrete to abstract) and the deductive argument (begin with general and end with specific).

The two approaches have their proponents. Even if until now research has not yet stated the best approach for young learners, experienced teachers support the inductive argument because a learner can integrate a new argument only if it is, in one way or another, a solution to his or her problem.

3.1.4. Progression

While teaching Mathematics in primary it is very important to consider the progression of concepts and topics. The teaching must take place gradually; the learner must be ready to make readjustments, revision, summaries, repetitions... For instance, the teacher will have to avoid mechanical application of the procedure or rules while dealing with mental calculation. Teaching Mathematics should always be starting on a learning activity which can allows learners to give their predictions toward a given lesson and then use a variety of activities though individualization or group works.

Students must learn Mathematics with understanding, actively building new knowledge from experience and prior knowledge.

3.1.5. Co-operation

This is a method of teaching which uses the teamwork; all the learners work together and take part in the implementation of a Mathematics task.

Cooperation is based on the following elements:

- Interdependence
- Sense of responsibility
- Cooperation skills
- Evaluation of group work

In a context of cooperation, a group is involved in carrying out a given task. Here competition and individualism are not allowed. Here the involvement of the group is vital, because all the members must learn and they must teach each other. Each and every student can learn Mathematics.

3.1.6. Individualisation

Individualisation refers to an education system that allows the student to learn on his/her own pace and possibly using diversified ways.

- The process of learning and teaching must take into account the pace of the student,
- Respect individual rhythm according to special education needs of students,
- Help individual to seek self-fulfilment.
- Assessment should support students' learning and provide useful information to both teachers and students.

3.1.7. Transfer (knowledge, skills, attitudes and values)

The goal of education demand more than the classroom mastery of content. If students can learn and retain what they have learnt, then they must take their new knowledge with them when they leave school.

Therefore, after teaching a Mathematics concept, student should use what they learned in many different situations.

In the psychology of learning, transfer of learning is the application of skills, habits, attitudes or other responses from one situation in which they were initially acquired to some other situations where they are applicable. For the transfer to take place there must be something in common between the earlier learning and the situation in which is to be made.

From the activity 3.1, one can observe that most of principles of teaching mathematics are taken into consideration. In the diagram, the count and match activity, is considering the following principles:

- **Concretization :** students are required to match real or known objects to numbers (from known to abstract) ,
- **Engaging activity:** student is required to work individually and has more autonomy, more initiative, more personal motivation to develop his/her creativity

In the activity on **discuss on how number are grouped,** students are required to discuss in groups and come up with characteristics of different groups of numbers. The principles of cooperation, engaging activity and transfer of knowledge are taken into consideration.

Activities 1 and 2 in the Mathematics worksheet shows that progression of concepts, engagement of students , concretization of content, and motivating activities are taken into account. The activity 3 shows that the following principles are considered: concretization, engaging activity, transfer of knowledge, skills, attitudes and values.

3.1.8. Contextualization

Contextualization is based on a constructivist theory of teaching and learning; Learning takes place when teachers are able to present information in such a way that learners are able to construct meaning based on their own experiences.

Learners need to understand the concepts as they relate to the school environment, workplace and to the larger society in which they live and work.

In such an environment, learners discover meaningful relationships between abstract ideas and practical applications in the context of the real world; concepts are internalized through the process of discovering, reinforcing, and relating.

Application activity 3.1

Take the unit 1 of P2 or P4 Mathematics textbook, analyse some activities or tasks and find out if the following components of teaching mathematics are presented:

- Progression of concepts,
- Engaging activities,
- Concretization of content,
- Motivating activities,
- Individualization or cooperative learning,
- Transfer of knowledge, skills, attitudes and values

Discuss with a partner, prepare a presentation justifying your position.

3.2. Assessment principles in Mathematics lessons

Activity 3.2

Suppose you are a P2 or P6 primary teacher and you are required to set assessment task during your Mathematics lesson. You get chance to be exposed with two scenarios to be used as your reference. Which one can inspire you the most and why?

Scenario 1:

A Mathematics teacher is preparing assessment tasks on data collection, data presentation using a pie chart and data interpretation. To make this assessment more relevant and useful, he/ she considered the following:

- Setting tasks which are age appropriate,
- Setting tasks basing on all content in the unit of statistics and elementary probability,
- Setting tasks that make students more engaged and motivated,
- Setting tasks that cater for all ability of students (lower, medium, talented and gifted learners or any other category of learners with special education needs).
- Setting task that connect the learnt knowledge and skills to real life practices.

Scenario 2:

A Mathematics teacher is preparing assessment tasks on data collection, data presentation using a pie chart and data interpretation. To make this assessment more relevant and useful, he/ she considered the following:

- Setting tasks basing on all content in the unit of statistics and elementary probability,
- Setting tasks which are appropriate to the level of learners' understanding
- Setting task that are considering the mathematical concepts only with no emphasis on connecting them to real life practices.

Content summary

Assessment should be seen as an integral part of the learning process rather than something which is just 'tacked on' at the end in order to get the achievement of learners. It should therefore be seen as a vital part of the initial design of Mathematics lesson. The following assessment principles:

The content principle: assessment should reflect the mathematics that is most important for students to learn. All students are engaged in assessment activities that address the content described in the Mathematics syllabus.

The learning principle: assessment should enhance Mathematics learning and support good instructional practice. All students are engaged in assessment activities that function primarily to improve learning. They are also engaged in assessment activities based upon rich, challenging problems for Mathematics and other disciplines.

The equity principle: assessment of Mathematics content and concepts should support every student to learn important Mathematics. Multiple approaches should be considered while setting the mathematics assessment.

Relevance and transferability: Whatever the outcome of that, the transfer of skills is certainly more likely to be successful when the contexts in which they are developed and used are similar. When devising an assessment task it is important that it both addresses the skills you want the student to develop and that as much as possible it puts them into a recognisable context with a sense of 'real purpose' behind why the task would be undertaken and a sense of a 'real audience', beyond the tutor, for whom the task would be done.

Application activity 3.2

Take the unit 1 of P3 or unit10 of P5 mathematics textbook, analyse any activity or task and do the following:

- Find out the Mathematics concept that is being assessed,
- Explain how the activity is supporting learning,
- Discuss if the knowledge and skills expressed in the activity are transferable

3.3.Methods and techniques in teaching and learning Mathematics in primary schools

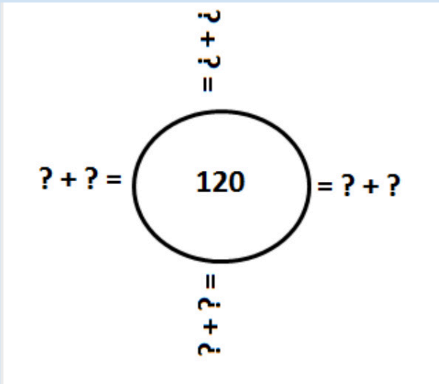
Activity 3.3

Suppose you are a primary school Mathematics teacher. Analyse the activities on handout, find out and explain techniques and strategies used in teaching, counting, grouping and addition of numbers.

Hand-out

Activity 1: Using MINGO game, let us study numbers and grouping, numbers and addition. If the teacher says MINGO, MINGO, students move very fast around the circle, if the teacher says a number (e.g.: two, four, six, eight, ten), students make groups according to that number. Finally, a number of group members is counted and recorded.

Activity 2: Brainstorm and find out the two numbers added up to get 120.



Activity 3: Observe the following number grid. Different mathematical concepts can be explored using the grid. Find out Even and odd numbers, Prime and composite numbers. Discuss and share the common characteristics for each category of numbers.

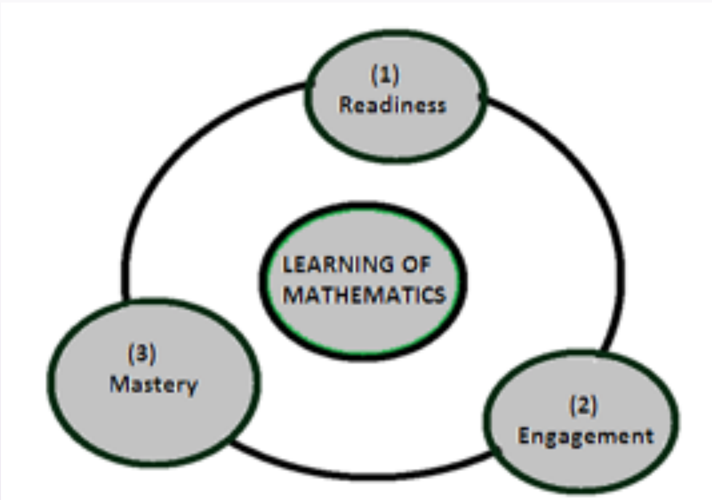
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Content summary

Methods and techniques of effective teaching and learning of Mathematics in primary schools depend on the lesson and phase of learning. The main teaching methods and strategies are the following:

- **Dogmatic method:** The teacher tells the students what to do? What to observe? How to attempt? How to conclude?
- **Inductive-deductive method:** Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples.
- **Analytic-synthetic method:** Analytic method proceeds from unknown to known, 'Analysis' means 'breaking up' of the problem in hand so that it ultimately gets connected with something obvious or already known. Synthetic method is the opposite of the analytic method. Here one proceeds from known to unknown.
- **Laboratory method:** Laboratory method is based on the maxim "learning by doing." It is a procedure for stimulating the activities of the students and to encourage them to make discoveries through practical activities.

Effective teaching of Mathematics involves three phases of learning: Readiness, Engagement, and Mastery. Every phase of learning should use more than one method.



Readiness phase

In the readiness phase of learning, Mathematics teachers prepare students so that they are ready to learn. This requires considerations of prior knowledge and experience of students as well as motivating contexts.

- **Prior knowledge and experiences of students**

For learners to be ready to learn, teachers need to know students' prior knowledge in relation to the new learning. This requires knowing whether learners have the pre-requisite concepts and skills. Some form of diagnostic assessment is necessary to check that learner is ready to learn. At this level, brainstorming and questioning techniques should be used.

- **Motivating contexts**

For learners to be read to learn, teachers need to provide motivating contexts for learning. These contexts should be developmentally appropriate. For example, lower primary learners may like contexts such as stories and songs, and play-based activities such as games, whereas upper primary students may appreciate contexts related to everyday life so that they can see the relevance and meaningfulness of mathematics. Real objects for manipulation, pictures or diagrams, graphs interpretation and short videos are also very important. Brainstorming and discussion techniques should be used.

Engagement phase

This is the main phase of learning where teachers use a variety of methods and techniques to engage learners in learning new concepts and skills. Activity-based learning, teacher- directed inquiry and direct instruction are three techniques that support most of the mathematics teaching and learning in the classroom. They are not mutually exclusive and could be used in different parts of a lesson. For example, the lesson could start with an activity, followed by teacher- led inquiry and end with direct instruction.

1. Activity- based learning: This approach or technique is about learning by doing. It is particularly effective for teaching mathematics concepts and skills at primary level. It can be done in groups or individually. They could use of manipulation or other resources to construct meanings and understanding.
2. Teacher-directed inquiry: This approach or technique is about learning through guided inquiry, instead of giving the answers, teacher leads learners to explore, investigate and find answers on their own. Learners learn to focus on specific questions and ideas and are engaged in communication, explaining and reflecting on their answers.
3. Direct instruction: This approach or technique is about explicit teaching. Teachers introduce, explain and demonstrated new concepts and skills. Direct instruction is most effective when learner is told what they will be learning and what they are expected to be

able to do. This helps them focus on the learning goals. Teacher draw conclusion, pose questions, emphasize key concepts, and role model thinking.

Mastery phase

This is the final phase of learning where teachers help learners consolidate and extend their learning. The mastery approaches include:

1. Motivated practice: Learners need practice to achieve mastery. Practice can be motivating and fun. Practice must include repetition and variation to achieve proficiency and flexibility. Structuring practice in the form of games is one good strategy to make practice motivating and fun, while allowing for repetition and variation. There should be a range of activities, from simple recall of facts to application of concepts.
2. Reflective review: It is important that learners consolidate and deepen their learning through tasks that allow them to reflect on their learning. This is good habit that needs to be cultivated from an early age. Summarizing their learning using concepts maps and writing the more important should be encouraged
3. Extended learning: Learners who are mathematically gifted and talented should have opportunities to extend learning. These can be in the form of more challenging tasks that stretch their thinking and deepen their understanding.

Application activity 3.3

Using examples of mathematical concepts, explain why in teaching and learning Mathematics in primary, the following are important:

- Building on prior knowledge and experience of students;
- Using motivating contexts
- Activity – based learning,
- Teacher directed inquiry ,
- Extended learning.

Note: use any primary mathematics textbook to perform this application activity.

End unit assessment 3

Refer to the learning theories and teaching approaches, recall and discuss some of the teaching strategies your Mathematics teachers used while teaching. Are there any advantages or disadvantage of the used teaching methods or strategies? Explain.

Unit 4 LESSONS ON NUMBERS AND OPERATIONS

Key unit competence: **Prepare lessons on numbers and operations**

Introductory activity 4

In teaching and learning Mathematics, the acquisition of mathematical concepts and skills is very crucial for continuous learning and supporting other subjects as well as developing thinking, reasoning, communication, application and meta-cognitive skills through a mathematical approach to problem solving with operations. Take a syllabus and textbooks for lower or upper primary Mathematics and identify all the topics related to numbers and operations to be studied?

For each topic, try to find out appropriate resources for facilitating related lessons using low cost materials and suggest how to produce them.

Are teaching techniques to facilitate lessons on the addition of numbers the same as those to be used in the lessons related to multiplication of numbers? Explain your answers.

4.1. Recall on concepts related to numbers and operations

Activity 4.1

Mathematical concepts refer to the basic mathematical content categories that are needed for solving mathematical problems. These concepts include: *numerical, geometrical, algebraic, statistical and probabilistic* concepts.

Use primary Mathematics book of lower or upper primary and do the following:

a. Select different concepts related to numbers and operations (numerical concepts).
b. Among the selected topics which do you find challenging to you?
c. Make a self study on the challenging topics. You can ask for a help from your colleague or your tutor

Content summary

Mathematical concepts refer to the basic mathematical concepts that are needed for solving mathematical problems. These concepts include: *numerical, geometrical, algebraic, statistical and probabilistic* concepts. These concepts are connected and interdependent.

Numerical concepts development

The concept of number is the biggest of the "big ideas" of Mathematics. Having a sense of number concept and operations is an important building block for all primary school learners, which serves as the foundation for higher Mathematics learned in middle and high school and for the workplace of the future.

Activities in number and operations are derived from applying of problem solving, reasoning, communicating, connecting, and representing to content objectives.

In this concept, learners have to develop:

- Understanding numbers (counting object), ways of representing numbers (writing numbers), relationships among numbers (comparing objects and numbers), and number systems (types of numbers).
- Understanding meanings of operations and how they relate to one another (addition, subtraction, multiplication and division).
- Computational skills and solving mathematical words problems in context.

Application activity 4.1

Use a primary Mathematics syllabus and the book, select one lesson on the addition of numbers, one lesson on the subtraction of numbers, one lesson on multiplication of numbers, and one lesson on the division of numbers.

For each lesson, highlight the skills to be acquired by students

4.2 Production and organization of appropriate resources for the lessons related to numbers and operations using low cost materials

Activity 4.2

By the use of the Mathematics syllabus and textbook for lower or upper primary choose one lesson on numbers and operations (e.g. lesson on multiplication of numbers).

Organize appropriate teaching resources for your choice and explain how those resources can be used while teaching and learning the selected topic (e.g lesson on multiplication of numbers).

Content summary

Teacher should continuously develop, design and exchange teaching and learning materials. If they are well constructed, organized and presented, successful teaching and learning can be achieved.

Teaching and learning resources can be acquired by producing or constructing them, by collecting them from the environment, by asking learners to prepare them, by requesting the school administration or parent to buy them, by borrowing or hiring them or through improvisation.

Many materials found in the environment can be used when teaching and learning numbers and operations. Some items that can be collected and used for improvisation on lessons related to number and operations include:

a. **Clay soil or modelling clay**: Clay soil is a low cost material that can be used in **forming** operation signs (+, -, x).

b. **Bottle tops:** In teaching and learning counting, learners can count bottle tops

c. **Plastic bottles:** with the use of plastic bottles, learners can group them when learning the addition, subtraction or the multiplication of numbers.

d. **Bean seeds:** use beans seeds to count or to form groups in mathematical operations for facilitating the learning on addition, subtraction, multiplication, division of numbers.

e. Number grid on Manila paper to teach the concept of multiplication of two numbers.

Materials that can be bought are for example: counters, rods, marbles, abacus, etc.

Application activity 4.2

In TRC, use low cost materials (rice sac, Manila paper…) and produce materials related to the teaching and learning of numbers and their operations (addition, subtraction, multiplication and division) and explain how they can be used in lower or upper primary Mathematics lessons.

4.3. Active teaching techniques and strategies for facilitating lessons related to numbers and operations

Activity 4.3

In teaching Mathematics lesson one of the challenge is to make the lesson enjoyable for learners. If learners are involved in the teaching and learning process, the lesson become more active and participative.

1. What can you do to make the lesson on "counting numbers" be enjoyable?
2. Discuss active teaching techniques and strategies you think are the best for facilitating lessons related to:

   a. Addition of numbers in P2 or P5?

   b. Multiplication of numbers in P1 or P4

   c. Division of numbers in P3 or P6

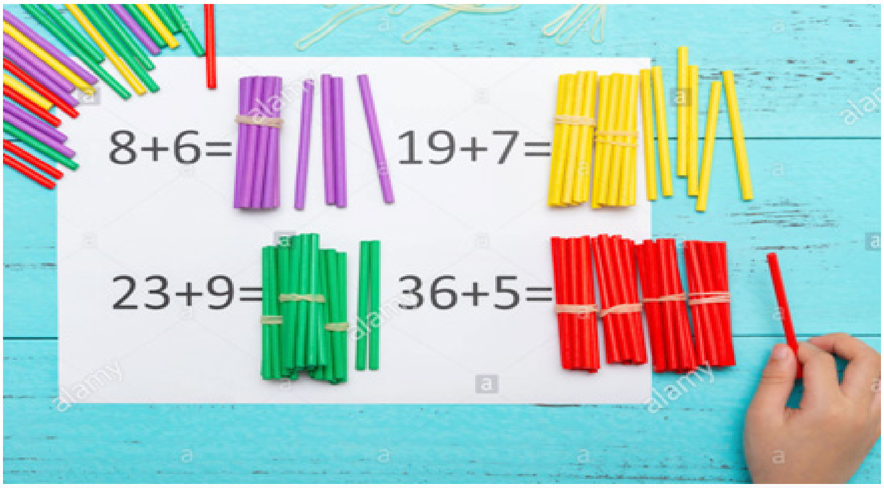
Content summary

Concept development

To "ensure learners develop a tangible understanding of the Mathematics concepts and skills, it is necessary to use the concrete level of understanding, so that learners are able to later use this foundation and link their conceptual understanding to abstract. In this regard, learners become capable of building number relationship and expand arithmetic because they discover that numbers are the repeated addition of one.

There are three main steps for teaching and learning numbers and operations that when combined they create intellectual competence in logical reasoning, spatial visualization, analysis and abstract thought in the learner.

Concrete stage: In this stage, the teacher begins the lesson by modelling each mathematical concept with concrete materials. In other words, this stage is the "doing" stage, using concrete objects to model problems. Those materials are real objects that learners manipulate and discuss how to use them for better learning. With manipulations, learners study by counting and grouping. With real materials like beans, small stones, chalks and bottle tops, learners form numbers, count them or group them and the teacher harmonize the work done by the learners. Learners can write and read the numbers of such objects individually or in groups.



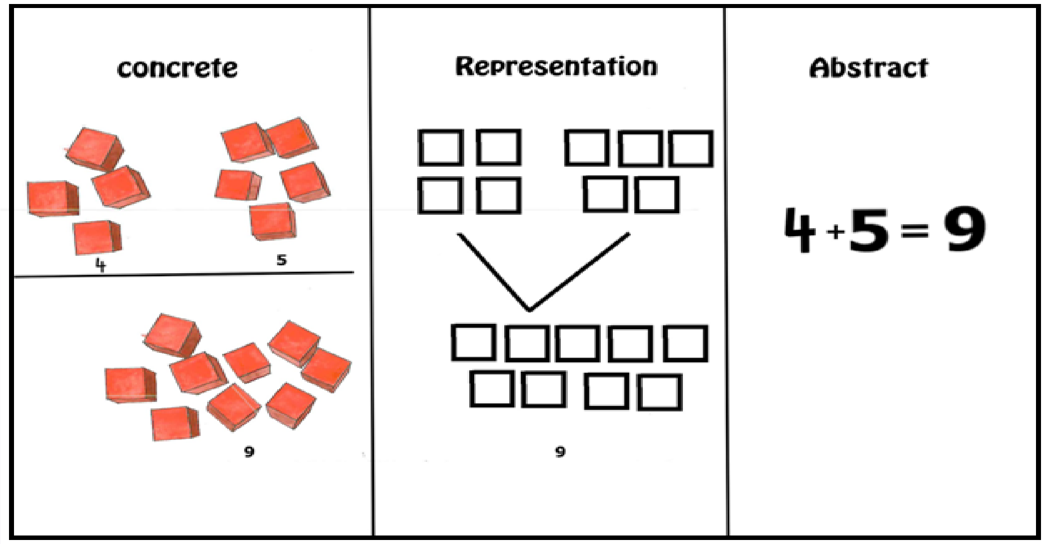
Semi- concrete stage, visualization or representation: In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawings or pictures; using circles, dots, and tallies; or using pictures for counting. In other words, this is the "seeing" stage, using representations of the objects to mode problems. A research showed that during the learning, we remember 20% of what we hear, 40% of what we see and 80% of what we discover for ourselves (Department of education, Papua New Guinea, 2004).

Abstract stage: *In* this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, –, x, :) to indicate addition, multiplication, or division. This is the "symbolic" stage, where students are able to use abstract symbols to model problems.

Therefore, visual aids can produce a very stimulating and exciting environment in the mathematics classroom. In the majority of cases, visual aids are used to offer concrete examples of a process before asking the learners to move to the abstract notion of working from memory or a textbook. Young children's learning starts out with visual, tangible, and kinesthetic experiences to establish basic understanding, and then learners are able to extend their knowledge through pictorial representations (drawings, diagrams, or sketches) and then finally are able to move to the abstract level of thinking, where learners are exclusively using mathematical symbols to represent and model problems.

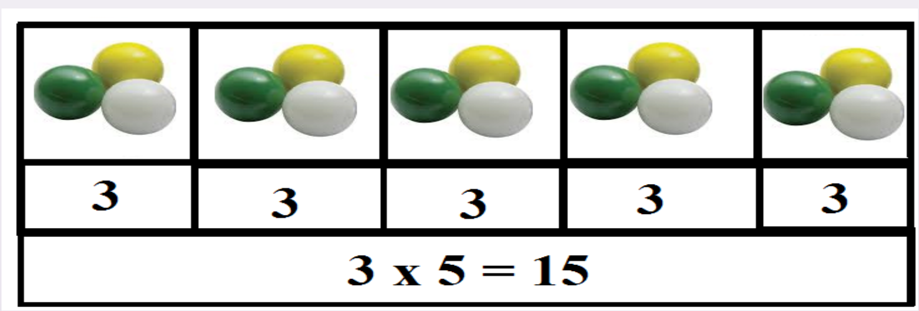
This requires extensive use of visual aids such as counters, building blocks or unfixed cubes, particularly when learning the basic operations of addition, subtraction, multiplication and division.

Example 1: Show how you can illustrate the addition of two numbers 4 and 5



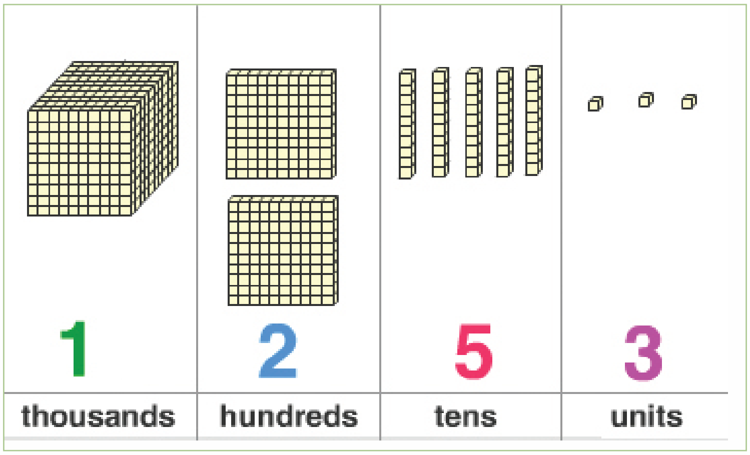
Example 2: show you can illustrate multiplication of two numbers 3 and 5

- By being able to physically add to or remove objects from a list or group of objects, the learners can act out the process of addition or subtraction.
- Similarly, five piles of three objects allow the learners to visualize the mathematical operation 3×5. By counting the total number of objects the learners will be able to establish 3×5=15.

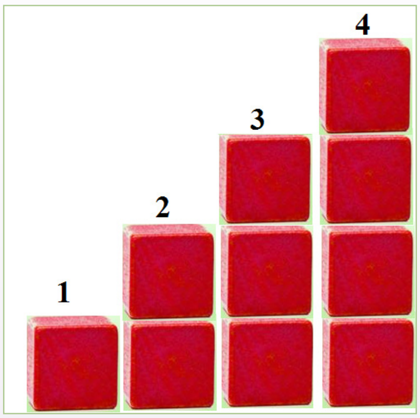


- The reverse action can be demonstrated when illustrating the physical process of dividing a group of 15 objects into groups of 3 objects.

Example 3: An example to teach the place value concept using place value blocks is shown in the following grid.



Example 4: show steps of counting using number blocks



Learners need to develop their ability in counting skills focusing on 3 key concepts to enable meaningful calculation.

One to one: each object should have one unique number word.

Stable order: order of number word should be maintained each time counting process is made such as one, two, three, and four…when counting concrete objects, pictorial objects and finally counting in abstract.

Cardinality: The last number word counted represents the total of objects in the set.

Example 5: To verify that addition of two whole numbers is commutative

Materials required: Chart paper, sheets of paper, scissors, ruler, glue, pen/pencil, colors.

Procedure:

1. Take a Chart Paper and cut two strips each of length a = 5 units (say) and breadth 1 unit.
2. Divide each strip into five equal parts to form unit squares and color them green as shown in Fig. 1 (a)

Fig. 1 (a)

3. Similarly, make two strips each containing b, say 3 unit squares and color them pink, as shown in Fig. 1 (b)

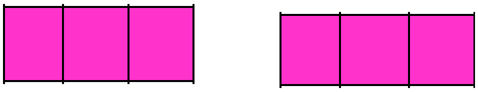


Fig. 1 (b)

4. Draw two lines l_1 and l_2 on a sheet of paper as shown in Fig. 2.

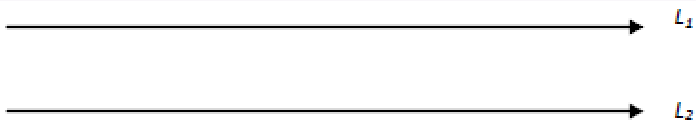


Fig.2

5. Now arrange/paste the green and pink strips side by side on lines l_1 and l_2, as shown in Fig. 3

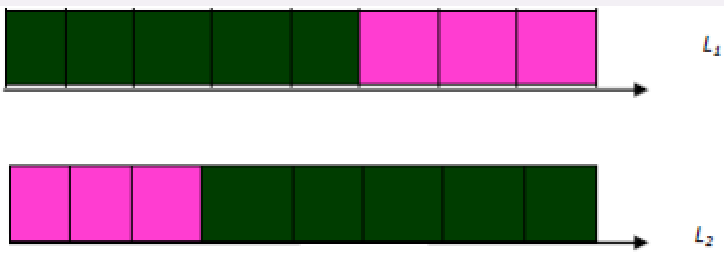


Fig.3

Observation: From Fig. 3,

a. The length of the combined strips on line l_1 = 5+3 =8

b. The length of the combined strips on line l_2 = 3+5 =8

c. One can see that the length of combined strips on l_1 is the same as the length of the combined strips on l_2.

d. So, 5 + 3 = 3 + 5 i.e., addition of 5 and 3 is commutative

One can repeat this activity by taking different pairs of numbers like 2, 5; 4, 6; 5, 7 and strips corresponding to these pairs.

2. Teaching and learning strategies related to numbers and operations

In the teaching and learning of numbers and operations, teachers use a repertoire of teaching strategies as there is no single suitable method to teach mathematics to emphasize mastery of knowledge and skills, critical and creative thinking, communication and problem solving. In the mathematics lesson on numbers and operations, teachers adopt a combination of different teaching approaches to engage learners in learning. These approaches will help learners talk more openly, think more creatively and ultimately become more engaged in the process of learning lessons related to numbers and operations.

Some of the common combinations of strategies are:

- Teacher exposition followed by learners practice
- Investigational activity followed by conjecturing or whole class discussion: learners may work independently, in pairs or in groups; learners may use technology or manipulatives to carry out their investigations.
- Practical activity followed by whole class discussion: learners could work independently, in pairs or in groups; learners may play mathematical games or do outdoor activities.
- Problem solving or mathematical modeling followed by presentation of work by learners in groups.
- Independent learning during e-learning: learners may do on-line tutorials at home and submit the work to teacher electronically.
- Peer teaching activities: peer instruction encompasses a range of scenarios where learners instruct skills or explain concepts to classmates.
- Outdoor activities: learners go outsides of the classroom to search

materials like small stones, beans, bottle tops, or sticks and come back in the classroom for counting, arranging, grouping these materials, and make different operations using those materials.

Application activity 4.3

Take a syllabus, textbooks and teachers guide and organize the teaching techniques you can use to make the lesson on subtraction of fractions in P3 or P5 more active and participative.

4.4. Activities for developing generic competences and integration of cross-cutting issues in the lessons related to numbers and operations

Activity4.4

In the P2 or P5 Mathematics textbooks a Mathematics activity was provided to learners. Read the activity carefully, analyse it and answer the questions.

Activity: Using the number cards with digits 1, 2, 3, 6 and 7

a. Form the largest number and form the smallest number

b. Add vertically the obtained numbers

c. Explain clearly your working step to the class members.

d. Suppose that the smallest and the largest numbers represent money in Rwandan francs. Calculate the difference and make a list of priorities of what you can buy using that money (difference). Explain your list of priorities.

Questions

1. Identify and explain the competences to be developed while performing the activity in (a), (b) and (c)?
2. Use the syllabus and find out the class or level the above activity can be given to.
3. Is there any cross-cutting issue that has been addressed? Explain your answer.

Content summary

Basing on the activity 4.4, the following Generic Competences can be developed:

1. **Communication skills:** through presentations, reading, writing numbers and the process used to order numbers
2. **Cooperation skills:** through working in pairs and groups, learners develop the spirit of working together and sharing ideas.
3. **Critical thinking skills:** in solving problems related to numbers and operations, the application of knowledge gained to real life situations as well as through predicting. For example figuring out how 5 or 6 digit number might be read and represented on an abacus.
4. **Problem solving skills:** through applying the concepts learnt to the solution of different problems of the real life of the learner, there is a development of solving different problems related to putting things together, sharing things to different people, forming groups of the same number of members, etc.

In addition, Creativity, innovation, research and Lifelong learning can be developed during the lessons on numbers and operations through different activities.

Basing on the activity 4.4, the following **cross- cutting issues can be addressed:**

1. **Financial education**: through counting money, lessons on buying, selling, filling cheques and bills, and interpreting them, learners find the use of money, value, importance of money and how to save it.
2. **Gender** addressed through group activity of where boys and girls take similar responsibilities,
3. **Environment and sustainability** integrated through a number of problems and discussions related to tree planting, maintain hygiene of their working places, etc.
4. **Inclusive education** addressed through: activities that should cater for different ability of learners, teaching and learning materials that are adapted to different special education needs, teaching techniques that are adapted to different ability of learners.

In addition, other cross-cutting issues such as environment and sustainability, peace and value education and standardization culture can be dressed through different activities.

Application activity 4.4

As a student-teacher, set up an activity on numbers and operations that helps primary learners to develop generic competences and address crosscutting issues. Explain how your activity develops generic competences and addresses crosscutting issues.

4.5. Assessment tasks in the lessons related to numbers and operations

Activity 4.5

Explore a Mathematics textbook of P2, P3, P4, P5 or P6 and identify any activity related to the topic on Numbers and operations (place value, addition, subtraction or multiplication of numbers). Analyze the activity and explain whether the action verbs used develop low, medium or high order thinking skills.

Content summary

The teaching and learning of mathematics has known two main problems:

- A majority of children have a sense of fear and failure regarding Mathematics. Hence, they give up early on, and drop out mathematics.
- Lack of effective teacher preparation and support in the teaching of Mathematics.

One of the solutions is to enrich teachers with a variety of mathematical resources including appropriate activities/ tasks that are age appropriate and which intend to develop high order thinking skills. Such materials should highlight that mathematics books must be activity-oriented with tasks appropriate to the age of learners.

How mathematics activities/ tasks are developed

Use observable action verbs consistent with the level of learning expected, considering all the low, medium and higher order thinking skills and competences.

Examples of action verbs used in setting tasks and criteria:

- **Low order** (knowledge and understanding): define odd number, name the types of numbers, list, identify, label, match the set and the number of elements, outline **etc.**
- **Medium order:** explain the prime number, describe, examine, classify, express, summarize, calculate, relate, show, solve, use, etc

- **Higher order:** compare, analyse, illustrate, differentiate, compose, construct, design, formulate, evaluate, justify, form a number and interpret.

Example: The table below shows example of how tasks on numbers and operations should be formulated.

<table>
<tr><th>Assessment task</th><th>Topic</th><th>Sample activity</th><th>Example</th></tr>
<tr><td>Practical and oral task</td><td>counting</td><td>Work in group and observe a group of beans, estimate their number before counting,

Make groups of tens and the remaining beans that are less than ten and then, count tens and ones to tell the number that is less than 50.</td><td>Counting the number less than 50 such as 45.

Use 45 beans: there are 4 groups of ten beans and 5 beans apart.

There are 40 beans and 5 beans. This means forty five beans.</td></tr>
<tr><td>Visualization and representation tasks</td><td>Addition or subtraction</td><td>Make addition and subtraction stories using concrete objects or pictures and write an addition or subtraction equality for each story.</td><td>Mary has ten notebooks. The teacher gives her other 5 notebooks. How many notebooks she has now?

Use numbers to show how you find the total number of those notebooks.</td></tr>
<tr><td></td><td></td><td>Use the base ten set or money to illustrate the standard algorithms for addition and subtraction up to 3 digits.</td><td>Show how you can find

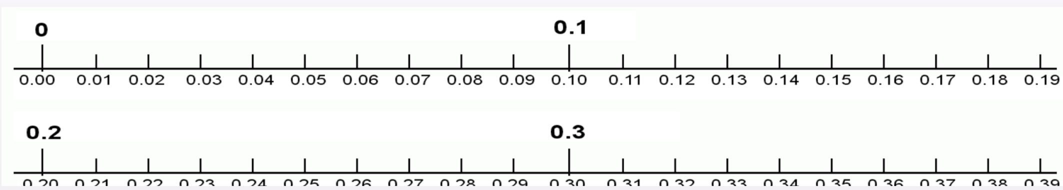
234 + 145 =
<table><tr><td></td><td>2</td><td>3</td><td>4</td></tr><tr><td>+</td><td>1</td><td>4</td><td>5</td></tr><tr><td></td><td>3</td><td>7</td><td>9</td></tr></table>Using the place value of numbers.</td></tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td>Achieve a mastery of addition and subtraction up to 4 digits by using applets (abacus) or playing digital games.

Solve a variety of problems: 1-step word problems 2-part word problems (with on step for each part), 2-step word problems and non-routine problems to become familiar with the problem solving process.</td>
<td>One step problem: David has 45 marbles. His sister gave him 32 more. How many marbles does he have altogether?

2 step problem: A sports club had 2564 members. 130 members left the club after a year. How many new members should the club recruit in order to have 3000 members?</td>
</tr>
<tr>
<td></td>
<td>Multiplication</td>
<td>Make equal groups using concrete objects and count the total number of objects in the groups by repeated addition using language such as “3 groups of 5 make 15 and “3 fives” make 15.</td>
<td>Complete the table for the multiples of 5 using marbles or other available counters.
<table><tr><td>5</td><td>10</td><td>15</td><td>…</td><td>…</td></tr></table></td>
</tr>
<tr>
<td></td>
<td>Division</td>
<td>- Share a given number of concrete objects/ picture cut-outs and explain how the sharing is done and whether the objects can be shared equally.

- Divide a set of concrete objects into equal groups, and discuss the grouping and sharing concepts of division.</td>
<td>The following are 21 banana, share them to Keza, Erica and Peter so that they all have equal number of bananas. How many bananas will have each one?
<table><tr><td colspan="3">Given 21 bananas</td></tr><tr><td>Mary</td><td>Keza</td><td>Erica</td></tr><tr><td>?</td><td>?</td><td>?</td></tr></table></td>
</tr>
</table>

		Divide a number of concrete objects in equal groups to discover that sometimes objects left over as remainder and write the answer as quotient and remainder.	
	Comparing numbers	Use the place value cards to compare numbers digit by digit from left to right, and use language such as "*greater than*, *greatest*, *smaller than*, *smallest*, and *the same as* or *is equal to*" for describing the comparison.	Given the following numbers: 35493 and 36514. Show the place values for each number. Compare those numbers using the place value table.
	Rounding decimal to the nearest whole number, 1 decimal place, 2 decimal places.	Place a given decimal on a number line between two consecutive whole numbers/ tenths/hundredths, and determine which whole number/tenth/ hundredth is nearer to the given decimal	Use a number line and show that the number 0.12 is near to 0.1 instead of being near to 0.2
 			
	Problem solving	Work in groups to create 3-step word problems involving the 4 operations based on everyday experiences and give it to other groups to solve.	**3 step world problem**: Hirwa and Jerome discussed the number of mangoes they eat over the weekend. Hirwa eats three times as far as Jerome. If Hirwa eats 6 mangoes, how many more mangoes did Jerome eat?

Application activity 4.5

Refer to the activities in the Mathematics textbook of P2, P3, P4, P5 or P6 and set 3 activities related to place value.

- In the first activity use action verb that can develop low order thinking skills,
- In the second activity use action verb that can develop medium order thinking skills,
- In the third activity use action verb that can develop high order thinking skills.

4.6. Lesson plan on number and operations

Activity 4.6

You are supposed to teach the lesson on the meaning and classification of odd, even and prime numbers to P3 or P4 learners. Refer to the syllabus, appropriate books, and the lesson planning procedures and format learnt in Foundation of education subject and does the following:

a. Set the instructional objectives (with its 5 main components) of the lesson

b. Organize teaching materials to be used

c. Formulate teacher's activities and learners' activities depending on the step of the lesson

d. Devise generic competences to be developed and crosscutting issues to be addressed depending on the step of the lesson

e. Summarize the teaching and learning techniques to be followed in this lesson.

f. Make a lesson plan on the meaning and classification of odd, even and prime numbers.

Content Summary

Before teaching any mathematics lesson, the teacher should devise a lesson plan which is a tool showing him/her the step-by-step procedure for delivering a lesson before being situated in that lesson.

The lesson planning has many aspects including:

- Identifying lesson objectives,
- Selecting appropriate teaching methods and aids to achieve the stated objectives,
- Allocating time to various lesson activities,
- Producing a summary of the content (if appropriate)
- Identifying appropriate methods for evaluating whether learning has taken place, i.e. whether the lesson objectives have been achieved.

Lesson objectives: The objectives to be achieved depend on the topic, the learners to teach, the class environment, etc. This is why Mathematics teacher must devise the **instructional objectives.**

An instructional objective is to be used for **one lesson** (with single or double periods). It should have 5 components (it is better to respect the order in which the components are listed below):

1. Reflect on the CONDITIONS, or how the learners will accomplish the task;
2. Determine WHO you're talking about;
3. Note the BEHAVIOUR/ACTION/COMPETENCE you're looking for - evidence of learners' action (choose from the list of verbs in the tips and aim for higher levels of ability);
4. Include the CONTENT you want the learner to learn;
5. Have a STANDARD OF PERFORMANCE - criteria for acceptable performance.

Example 1: Given a sheet of paper, a pencil, a pair of compasses and a protractor (**condition/situation**), the learner should be able to (**learner**) construct (**action**) a right- angled triangle (**content**/subject matter) accurately (**criterion**) using problem solving skills.

Example 2: Using the rulers, learners will be able to measure the exterior perimeter of their classroom and justify their findings correctly.

Example 3*:* By the use of a one litre bottle, learners will be able to determine how many litres that can fill a jerry-can of $1dal$.

Generic competences: They are transferable competences throughout a range of learning areas and situations in life. The generic competences to be developed in each step of the lesson must be highlighted.

In addition, the teacher thinks about the **crosscutting issues** to be addressed in each lesson and highlight them where appropriate.

Teaching techniques and strategies: As mathematics lessons should be activity oriented, the teacher must prepare the teaching techniques and strategies that are related to the topic, its objective, and set activities to be carried out by learners. In addition, these teaching techniques depend on the guidance of the teacher from teaching instructions to the guidance on how learners will be working and how to assess the development of competences.

The format showing the components for a developed lesson was learnt in The foundation of education.

Example of planned lessons on numerical concepts

School Name: X Primary School ....**Teacher's name**: ....

Term	**Date**	**Subject**	**Class**	**Unit Nº**	**Lesson Nº**	**Duration**	**Class size**
1	27 /05/ 2019	Mathematics	P4	3	1 of 5	7h20-8h00	42 learners

Type of Special Educational Needs to be catered for in this lesson and number of learners in each category	One learner with hearing impairment will sit near the chalkboard
Unit title	Classifying numbers by their properties
Key Unit Competence:	Be able to classify numbers flexibly, seeing them as belonging to various families.
Title of the lesson	Meaning and classification of odd, even and prime numbers.
Instructional Objective	Given a real life situation and list of numbers on flash cards or number cards, Learners will be able to classify odd, even and prime numbers correctly, confidently in a given time
Plan for this Class (location: in / outside)	In the classroom, arrange desks so that learners can work individually and in small groups
Learning Materials (for all learners)	Flash cards with numbers from 1 to 20. The number of flashcards depends on the number of learners in a class.
References	Mwungeri E.et al.2008. Mathematics: Learners Book, P.103-106

Example of planned lessons on numerical concepts

School Name: X Primary School............... **Teacher's name**: ....

<table>
<tr><td rowspan="2">Steps and Timing</td><td colspan="2">Description of teaching and learning activity

In groups, learners work out activities to classify numbers into: odd, even or prime numbers using flash cards or number cards. Learners make presentation, teacher harmonize their findings and facilitate them to synthesize. Learners do application activity in pairs and individually the do assessment tasks. Finally, the correction of application activity and assessment tasks is done on the chalk board.</td><td rowspan="2">Competences and Cross-Cutting Issues to be addressed</td></tr>
<tr><td>Teacher's activities</td><td>Learners activities</td></tr>
<tr><td>Review (5 minutes)</td><td>Facilitate learners to chant.</td><td>Chant in 2's ,3's ,4's,5's ,......from 0 to 40 (forward and backward)</td><td>When learners are singing a song related to counting numbers they develop communication skills.</td></tr>
</table>

Development of the lesson (25 minutes) **Discovery activities:** **Presentation of findings**	Distribute flashcards to learners so that every learner gets one flash card.	**Activity1: classification of odd, even, prime numbers from 1-10** Individually learners divide their number by 2.then do it in groups as they come up with summary. Learners with numbers which are divisible by 2 put them together in one group, learners with numbers which are not divisible by 2 make another group then learners with numbers which are divisible only by one and by themselves make their group. Learners from each group make presentation of their findings and class take conclusion if the said number is in the right group.	Cooperation developed through working together in group. Communication developed through the presentation of their findings. Peace and value addressed when all learners share ideas in a peaceful way with respect of others views. Gender addressed when both girls and boys working together in groups. Inclusive education addressed in classroom by encouraging all learners to work together including those learners with special education needs.

	Put learners in groups and assign them the work of classifying and describing the properties of odd, even and prime numbers from 10-20	Conclusion made by the whole class: Numbers which are divisible by 2 are: 2, 4, 6, 8 ,10 Number which are not divisible by 2 are: 1, 3, 5, 7, 9	Peace and value education developed through consensus leading to conclusion
Synthesis and summary		Numbers which are divided by one and themselves are:1, 3, 5, 7 **Activity2: classifying and describing the properties of odd, even and prime numbers from 10-20** Learners work in groups, divide by 2 all the numbers between 10-20 and classify them as follows: Put together all numbers divisible by 2 Put together all numbers which are not divisible by 2 Put together all numbers which divisible by themselves	

Application		Observations from groups: those divisible by 2 are: 10,12,14,16,18 not divisible by 2 are: 11, 13, 15, 17, 19 those are divisible by one and themselves are: 11,13,17,19 Learners conclude that all numbers in a) are even, all numbers in group b) are odd and all numbers in c) are prime numbers.	
	Facilitate learners in brainstorming the definition of even number, odd number and prime number and give examples Write the definition and share with learners	An even number is a number that can be divided exactly by 2. For example the numbers 2, 4, 6, 8, 10 are all even numbers An odd number is a number that cannot be divided by 2. For example 1, 3,5,7,9,15 and 21 A prime number is a number which is divided by 2 numbers only 1 and itself. For example 3, 5, 7,11, …. 1 is not a prime number because it is divided by only one.	With high level of summarizing the lesson learners develop critical thinking

<table>
<tr>
<td></td>
<td>Guide learners in pairs to do given activity.</td>
<td>The following are numbers written on T-shirt of 5 students in class analyze them and answer the questions that follows:
<table>
<tr><td>student</td><td>numbers</td></tr>
<tr><td>Student 1</td><td>6</td></tr>
<tr><td>Student 2</td><td>8</td></tr>
<tr><td>Student 3</td><td>5</td></tr>
<tr><td>Student 4</td><td>3</td></tr>
<tr><td>Student 5</td><td>1</td></tr>
</table>
Write all even numbers from the class

Write down all odd numbers written on those T-shirts

Write down prime numbers written on those T-shirts</td>
<td>When learners apply their knowledge in producing a new knowledge, they develop problem solving skills.

Peace and value education developed through consensus leading to conclusion</td>
</tr>
<tr>
<td>Conclusion

(10 minutes)</td>
<td>Assessment

Write assessment activities on the chalkboard.

Ask learners to do the assessment.</td>
<td>Write down all the odd numbers between 12 and 26, 20 and 40

September is one month of the year, select the days in that month that correspond with even, odd and prime numbers(days)</td>
<td>Learners develop a high level of thinking as they work on activities for assessment individually.

Research and problem-solving developed though the activity to be done at home with support of parents and other family members.</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>Homework

Write homework questions on the chalkboard</td>
<td>The table below shows different number of kilograms for each item needed by the school in 1st term of some food at market.

<table>
<tr><th>Item</th><th>No of kilogram</th></tr>
<tr><td>Irish potatoes</td><td>255</td></tr>
<tr><td>banana</td><td>180</td></tr>
<tr><td>Oranges</td><td>200</td></tr>
<tr><td>salt</td><td>5</td></tr>
<tr><td>vegetables</td><td>191</td></tr>
</table>

Answer the following:

Write down all the odd numbers from the above table

Write down all even and prime numbers from the table.

Why do we need to buy the items said above?</td>
<td></td>
</tr>
<tr>
<td>Teacher self-evaluation</td>
<td colspan="3">All learners have actively and anticipatively engaged in the lesson. The lesson objectives have been achieved but more activities for reinforcement are needed.</td>
</tr>
</table>

Application Activity 4.6

Refer to the syllabus and appropriate books and prepare a lesson plan on reading and writing numbers from 500 to 999 in P3 or numbers of 4 digits in P4.

End unit assessment 4

The additive inverse of numbers is a lesson to be taught in P5 while subtraction of two numbers is a lesson to be taught in P1, P2 or P3. Refer to the syllabus and appropriate books, organize teaching and learning materials to be used and prepare a related lesson to be taught to 46 students including one low vision student.

Unit 5 LESSONS RELATED TO FRACTIONS, DECIMALS AND PROPORTIONAL REASONING

Key unit competence: **Prepare active lessons related to fractions, decimals and proportional reasoning.**

Introductory activity 5

Fractions, decimal and proportional reasoning are ones of the most important topics primary learners need in order to be successful in algebra and beyond. Therefore, it is absolutely critical that you teach fractions, decimal and proportional reasoning well, present fractions, decimal and proportional reasoning as interesting and important, and commit to helping students understand the big ideas

- Take a syllabus and textbooks for P2 or P5 Mathematics and identify all topics related to fractions, decimal numbers and proportional reasoning
- For each topic, try to find out appropriate resources or materials for facilitating related lessons using low cost materials.
- Discuss on how you should facilitate lesson on fraction and lesson on proportional reasoning. Are teaching and assessment techniques for the lesson on fraction the same as the ones to be used in the lesson on proportions? Explain your answers.

5.1.Recall on concepts related to lessons on fraction, decimals and proportional reasoning

Activity 5.1

Use primary Mathematics syllabus and textbooks of lower or upper primary and do the following:

a. Select different concepts related to fractions, decimals and proportional reasoning.

b. Among the selected topics which do you find challenging to you?

c. Make a self-study on the challenging topics. You can ask for a help from your colleague or your tutor

d. Discuss the importance of studying fractions, decimal numbers and proportional reasoning in primary schools.

Content summary

In the topics related to fractions, decimals and proportional reasoning, learners have to develop:

- Greater understanding of fractions (type of fractions, representing, comparing fractions, writing fractions in figure or in words, modeling mathematical word problems involving fractions…
- Understanding meanings of operations and later using equivalent fractions to simplify the task including addition, subtraction, division and multiplying fractions, and how they relate to one another
- Understand and analyze word problems involving fractions in order to find adequate solutions working step by step.
- Decimal notation to represent numbers that have fractional parts and learn to compute with decimals to solve word problems that require operations.
- Understanding of fractions and decimals to situations involving division of these values. They increasingly work with ratio and proportion, particularly, in percentage situations
- Applying concepts of fractions, decimal and proportional reasoning to the real life situations.

Application activity 5.1

Use a primary Mathematics syllabus and the book, select one lesson on fraction (Addition of fractions with the same denominator), one lesson on decimal (Comparing decimal numbers), and one lesson on proportional reasoning (Direct proportion in contexts), highlight the skills to be acquired by learners in one of these lesson.

5.2. Materials to be used in lessons on fraction, decimals and proportional reasoning

Activity 5.2

By the use of the P3 or P4 Mathematics syllabus and P3 or P4 Mathematics textbook, identify all materials you may need to make the teaching and learning of division of a fraction by a whole number in P4 or a fraction of a whole number in P3 easier and more meaningful.

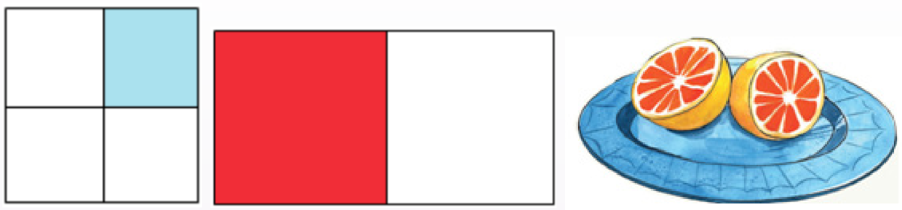
Explain how those identified resources can make your Mathematics lesson easier and more meaningful.

Content summary

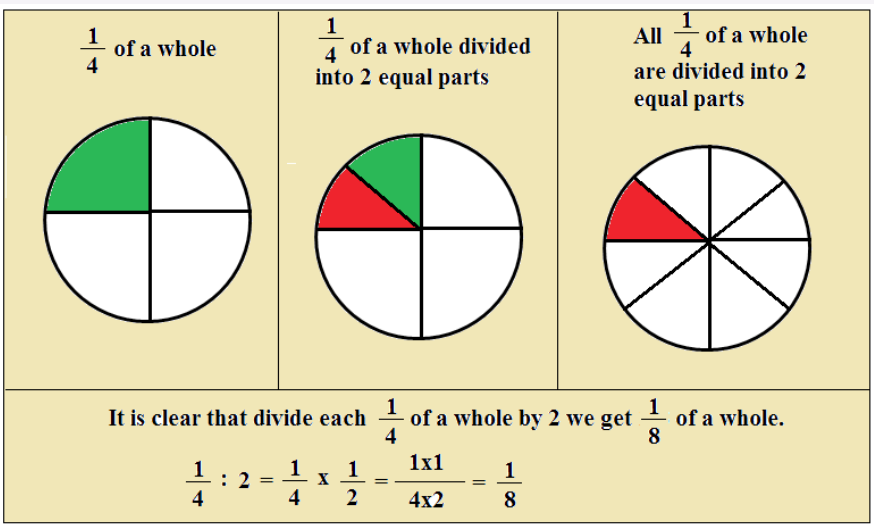
Teaching and learning resources can be acquired by producing or constructing them, by collecting them from the environment, by asking learners to prepare them, by requesting the school administration or parent to buy them, by borrowing or hiring them or through improvisation. Teaching and learning materials should be used to introduce new concept or reinforce learning of a concept related to fractions, decimals and proportional reasoning.

Example1: concrete materials

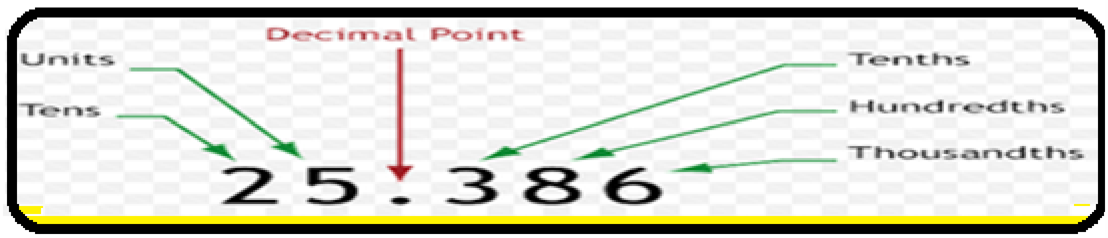
Folding or shading a sheet of paper, cutting an orange, tomatoes or any other real object to show the following fractions: ½, or ¼



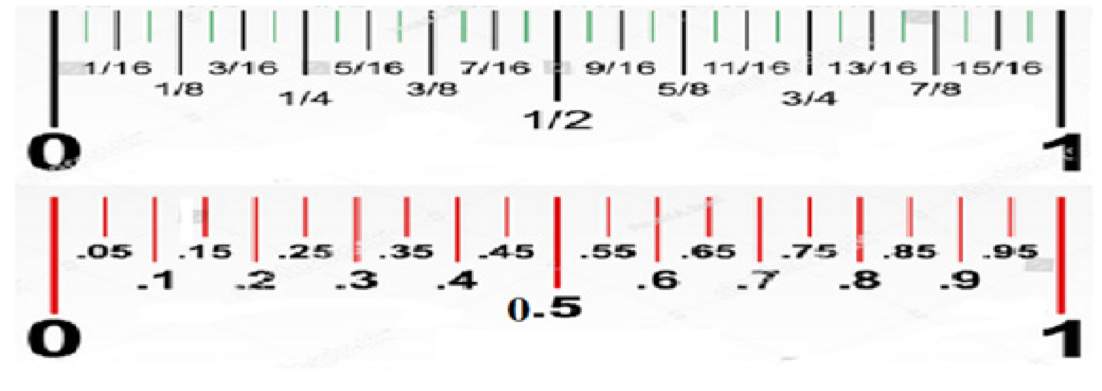
Example2: semi-concrete material on Manila paper or a rice sack to illustrate division of a fraction by a whole number. If this material is on manila papers or cartons and students use scissors for cutting, the materials became concrete ones.



Example3: semi-concrete material on Manila paper or a rice sack to illustrate different parts of a decimal number.



Example4: semi-concrete material on Manila paper or a rice sack to illustrate relationship between fractions and decimal numbers on a number line.



Application activity 5.2

In TRC, use low cost materials (rice sack, Manila paper, chalk box, carton, used papers…) and produce materials related to the teaching and learning of fractions, decimal and proportional reasoning including their operations (addition, subtraction, multiplication and division) and explain how they can be used in lower or upper primary Mathematics lessons.

5.3.Active teaching techniques and strategies for facilitating lessons related to fraction, decimals and proportional reasoning.

Activity 5.3

The teacher of P3 or P5 primary school decided to make learners do the following tasks, when he/she is going to teach fractions:

- Write down in their notebook what they know and what they want to know on fractions
- Use papers or manila papers and cut them into some representation of fractions (e.g: ½, 2/3, 1/3,…) using scissors,
- Cut again the above representation of fractions into 2 or 3 equal parts
- Make analysis on the findings after cutting, and compare the final ones with the preceding, using some operations like addition, subtraction and division to find for example ½ +1/4=.
- Making presentation of your findings

As student-teacher

1. Use primary textbook to highlight other resources they can use apart from what they used, for studying well fractions
2. Discuss on different best active teaching techniques and strategies they think that teacher need to use for next lessons related to fraction, decimal numbers or proportional reasoning

Content summary

1. **Concept development**

A basic way to develop a concept of **fraction** (common **fraction**) is to use cuts out or dividing a whole object into several equal parts. The denominator of a fraction tells how many equal parts the whole is subdivided into while the numerator tells the number of those parts that are under consideration

To ensure students develop understanding of fraction, decimal fractional and proportional concepts, it is necessary to use the concrete, semi- concrete and abstract level of understanding.

- **Concrete stage:** In this stage, the teacher begins the lesson by modelling each mathematical concept with concrete materials like oranges, paw paws, sugar canes, papers, manila papers, cartons knives, scissors… to allow students making cut-outs and exploring different parts of a whole. Visual aids are used to offer concrete examples.

Here the student may use papers and fold into different parts of the fraction and though that they may learn more aspect of fractions like equivalent fractions, adding, subtracting and division.

- **Semi- concrete stage, visualization or representation:** In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawings or pictures; flash cards, wall charts, rice sacks that includes fractional, decimal or proportional concepts.
- **Abstract stage:** In this stage, the teacher models the fractional, decimal or proportions using symbols and numbers notation. This is the "symbolic" stage, where students are able to use abstract symbols referring to rules and properties.

Five instructional strategies for developing proportional reasoning

Helping learners develop the ability to reason proportionally requires diligence and patience. It is important to note that learners do not develop proportional reasoning in one or two lessons, a unit, or even a year. It is a process that begins in late primary school and continues into secondary school. As a result, instruction should nurture and provide numerous opportunities for this type of thinking slowly and overtime. Expecting learners to develop a solid understanding of proportionality too quickly is counterproductive. There are several research-based instructional strategies that can help learners develop proportional reasoning.

1. Use a variety of proportion-type problems and sequence accordingly

There are four different proportion problem types

- **Rate problems** involve well-known measurements such as speed or cost per item.
- **Part-part-whole** problems involve a subset of a whole as it is compared to its complement, such as boys with girls or the number of boys as compared to the number of learners in the whole class.
- **Associated set problems** pertain to quantities specific to a given situation such as pencils and learners or people and candy.
- **Growth problems**, otherwise known as "stretcher" and "shirker" problems, require scaling up or scaling down and involve a relationship between two linear quantities such as height, length, or width. Scaling up and scaling down also cause changes in the area of plane figures or volume of solid figures when the dimensions are changed. Instruction should include a balance of all four semantic types. Textbooks are often weighted heavily in favour of only one or two problem types instead of a balance of all four; consequently, supplemental problems may need to be used. Early in their development of proportional reasoning, learners perform best on associated set problems because they can use pre-

existing knowledge of patterns, counting, and matching techniques. Learners usually find growth problems the most difficult because of a tendency to apply additive strategies rather than multiplicative ones. As a result, sequencing instruction should begin with associated sets or part-part-whole type of questions before moving on to growth problems.

2. Choose tasks that have multiple solution strategies and a variety of contexts

Using a variety of contexts gives learners exposure to the variety of situations and the types of scenarios that apply to multiplicative relationships. While learners may not be able to recognize a multiplicative relationship in one situation, they might be able to in a different situation. Using a variety of tasks also elicits different solution strategies. Learners can respond to the questions in different ways depending on their level of understanding.

3. Build upon learners' intuitive knowledge

Through experience in daily life and exposure to ideas through their school work, learners possess prior knowledge that can be used while solving proportion problems. Learning that allows learners to use their prior knowledge and intuition is important because it allows for the development of personal sense-making strategies. Learners need time to develop strategies on their own. High-level cognitive tasks require time for learners to grapple and explore their ideas. When teachers jump in too early and begin providing assistance by offering shortcuts or procedures, the complexity of the task is greatly reduced. Learners have remarkable meta-cognitive abilities to monitor and judge the reliability of their thinking without direct instruction; therefore, instruction should be designed to take advantage of learners' invented strategies. When rules and procedures are not learned with connections and meaning, learners will forget or will not understand when or why to use them.

4 Utilize multiple representations to develop fluency in proportional reasoning

Manipulatives, pictures, and diagrams are important tools that help represent proportional situations. The availability of manipulatives, especially early on, helps with sense-making and encourages informal problem solving strategies. Cubes assist with regrouping quantities or in unitizing, especially in associated-sets and part-part-whole problems. Ratio tables are a record-keeping tool that helps display the building up or scaling down of quantities in proportional situations. Building ratio tables provide learners with opportunities to discuss and present construction strategies.

Early on, problems should be situational and solved using objects and pictures. Gradually instruction can build up to more complex problems and methods of solving.

5. Informal strategies before cross-multiplication procedures

Many traditional mathematics curricula focus on a limited number of proportion-type problems and use the cross-products procedure for solving proportions, without ever helping learners develop a reason for why the strategy works. Instruction should not promote specific strategies. Learners who are encouraged to use their own strategy rather than a single algorithm are generally more successful in developing proportional reasoning. As a result, symbolic algebra and the cross-products method should only be introduced after learners have had an opportunity to develop their informal strategies.

Teaching and learning materials should be used to introduce new concept or reinforce learning of a concept related to fractions, decimals and proportional reasoning.

2. Teaching and learning strategies related to fractions, decimal numbers and proportional reasoning

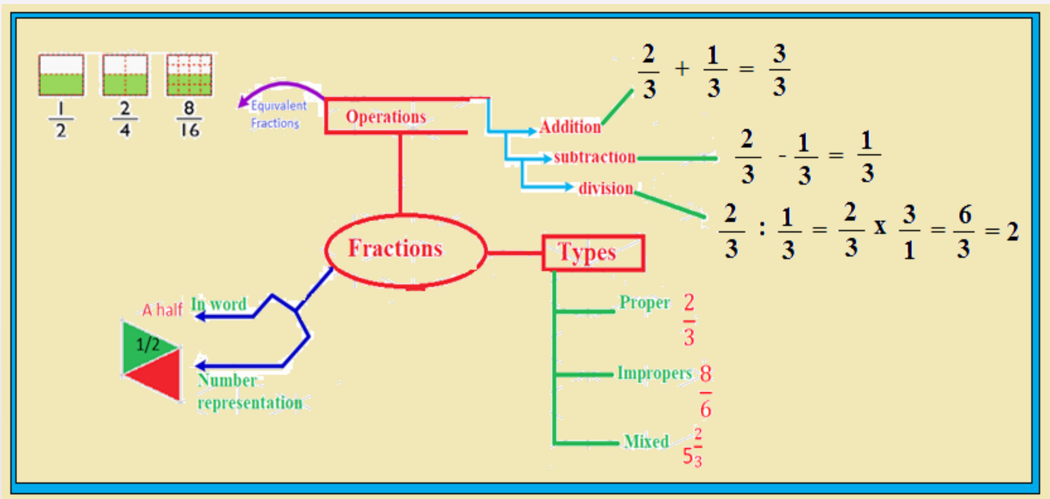
In the teaching and learning of fractions, decimal numbers or direct proportion, teachers use a repertoire of teaching approaches as there is no single suitable method to teach mathematics to emphasize mastery of knowledge and skills, critical and creative thinking, communication and problem solving. In teaching of Mathematics, it is advised to adopt a combination of different teaching approaches to engage learners in learning. These approaches will help students talk more openly, think more creatively and ultimately become more engaged in the process of learning lessons related to fractions, decimal numbers or direct proportion.

Some of the common combinations of approaches are:

- Teacher exposition followed by student practice
- Investigational activity followed by conjecturing or whole class discussion: students may work independently, in pairs or in groups; students may use technology or manipulatives to carry out their investigations.
- Practical activity followed by whole class discussion: students could work independently, in pairs or in groups for deeper understanding of phenomena; observation and reporting through presentation.
- Problem solving or mathematical modeling followed by presentation of work by learners in groups.

- Peer teaching activities: peer instruction encompasses a range of scenarios where learners instruct skills or explain concepts to classmates.
- **Research work** may be used to discover the rules related to operations (addition, subtraction, multiplication, division and comparison) on fractions and decimal numbers.
- **Think-pair-share** should be used to allow students exchange their knowledge about fractions, decimal numbers or direct proportions.
- **Demonstration** at the time of presentation of students' findings, this technique should be used. It may also being used with the teacher to demonstrate new concept using a variety of teaching and learning materials (Videos, pictures or diagrams).
- **Matching** should be used while comparing fractions using numbers and drawings
- **KWL technique** (know, want to know, learned) should be used from the beginning of the lesson to the end of the learning, where the teacher facilitate the students to write what they know on fractions and what they want to know, and end up on presenting what they learnt from each other.
- **Mind mapping:** is a technique with a **visual form of note taking** that offers an overview of a topic and its complex information, allowing students to comprehend, generate new ideas and build connections. This technique can be used at the beginning of a unit (introductory activity) and at the end of unit (end unit assessment). It also enhances peer support and fosters learner-centred practices that can promote quality teaching and learning.

Example of mind mapping diagram on fractions



- **Game:** Design a play activity that allows learners to review content in a fun, enjoyable manner (i.e. pick a card, memory, imitations, and concept ball). Games help learners learn faster and better, and in enjoyable manner. According to this teaching strategy the teacher prepares the flash cards with fractional representation in numbers, which the group members will use to pick two cards and justify if all are equivalent fractions or not if yes, he/she preserve and continue to pick others. If not, he/she returns to its initial position. At this step all next students must be attentive. This strategy helps to create a classroom environment that actively engages learners. They develop communication and other important skills such as **social skills**, **critical thinking**, **problem-solving**, **numeracy and literacy** skills.
- **Learning corners or Gallery walk:** students should work in groups on different activities of fractions, decimal numbers or direct proportion. During the plenary session, each group hang their finding on the wall of the classroom and the representative of the group explains to other groups. They do the same for other groups until the last presentation. Variety of questions can be asked by learners for more understanding. For closing the lesson, some learners can summaries the work done; the teacher can correct the misconceptions of the learners.

Application activity 5.3

Use a Mathematics syllabus, textbooks and teachers guide and organize the teaching techniques you can use to make the lesson on application of direct proportion in P5 or the lesson on finding the missing number in addition in P3 more active and participative.

5.4. Activities for developing generic competences and integrating crosscutting issues in lessons on fraction, decimals and proportional reasoning;

Activity 5.4

In the P5 Mathematics textbook a Mathematics activity was provided to learners. Read the activity carefully, analyse it and answer the questions. Activity: A certain farmer has goats and chickens in her farm. The ratio of goats to chickens is 3:5. The total number of chickens and goats is 320.

a. How many chickens are there in her farm?
b. Calculate the number of goats in her farm. Explain your steps.
c. The farmer sold 20 goats and 80 chickens so as to get money for school fees. Find the ratio of goats to chickens after selling her animals. Why is it important to educate children?

Questions

1. Identify and explain the competences to be developed while performing the activity
2. Is there any cross-cutting issue that has been addressed? Explain your answer.

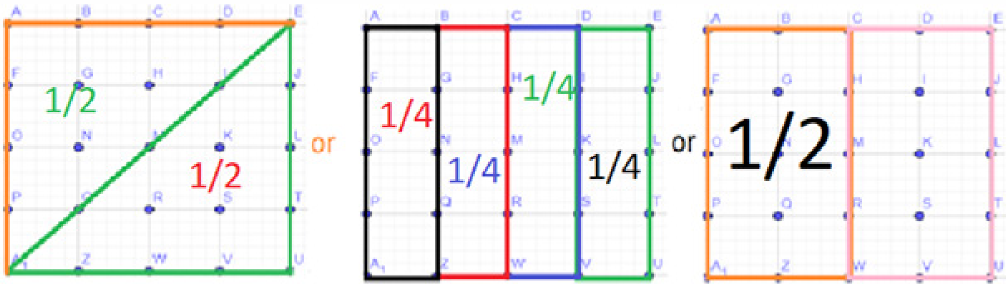
Content summary

Different generic competences can be developed in lessons on fraction, decimals and proportional reasoning; Basing on the activity 5.4, the following can be developed:

1. Critical thinking through imagination on how they can represent the activity using any pictorial representation of fractions
2. Creativity and innovation through getting new idea such as getting shapes of figures or any real life material.
3. Communication through presentations and sharing information about fractions, direct proportions and ratios.
4. Cooperation and interpersonal through working together in order to achieve the target.
5. **Problem solving**: through applying the concepts learnt to the solution of different problems of the real life of the learner, there is a development of solving different problems related to fractions, decimal and proportional reasoning.

Example of how students can develop competences through an activity.

Students work in groups using the papers, cartons and then draw their designs on dot paper or grid paper and write the appropriate fractions for the pictorial representations.



Different **cross- cutting issues** can be addressed in lessons on fraction, decimals and proportional reasoning.

However, basing on the activity 5.4, the following cross- cutting issues can be addressed:

1. **Gender** addressed through group activity of which boys and girls take similar responsibilities,
2. **Inclusive education** addressed through activities that should cater for different ability of students, teaching and learning materials that are adapted to different special education needs, teaching techniques that are adapted to different ability of students (braille & tactile)
3. **Peace and value education** addressed through personal respect while exchanging ideas and thoughts.
4. **Financial education** addressed through the culture of saving and investing in education.

Application activity 5.4

As a student-teacher, set up an activity on fractions, decimals and proportions that helps primary learners to develop generic competences and address crosscutting issues. Explain how your activity develops generic competences and addresses crosscutting issues.

5.5. Assessment tasks in the lessons related to fraction, decimals and proportional reasoning

Activity 5.5

Explore a Mathematics textbook of P3 (unit 4), P4 (unit 5), P5 (unit 6) or P6 (unit 6) and identify any activity related to the topic on fractions, decimal numbers or direct proportions (place value of decimals up to 2 decimal places, direct proportion in context, percentages and ratios). Analyse the activity and explain whether the action verbs used develop low, medium or high order thinking skills.

Content summary

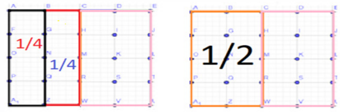
The lack of well learning activities and assessment tasks that are well to support the effective learning of Mathematics is one of the causes of failure and negative attitudes towards mathematics subject for some learners.

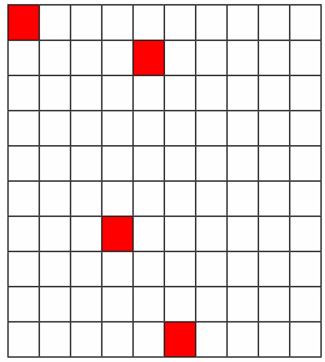
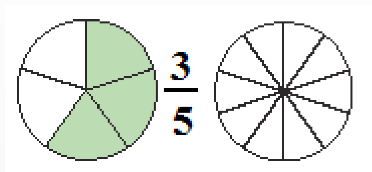
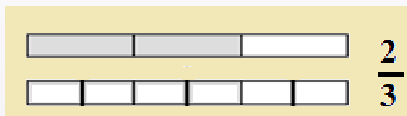
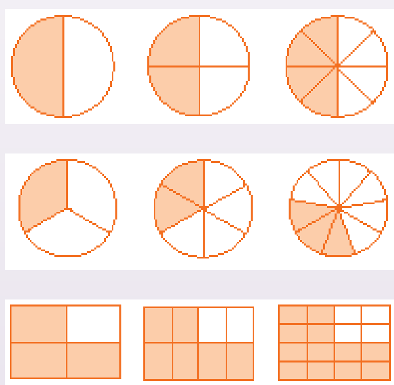
One of the solutions is to enrich teachers with a variety of appropriate activities/ tasks that are age appropriate and which intend to develop high order thinking skills. Such tasks are formulated by the use of action verbs that are appropriate to the learning expected by considering the low, medium and higher order thinking skills and competences.

Examples of action verbs used in setting tasks related to fractions, decimal numbers, and direct proportions:

- **Low order** (knowledge and understanding): define, name, list, identify, match etc.
- **Medium Order:** explain, arrange , describe, examine, classify, carry out, calculate, solve, use, etc
- **Higher order:** compare, shade, illustrate, differentiate, compose, construct, design, formulate, evaluate, justify, generate and interpret.

The table below shows examples of how tasks on metric measurements should be formulated.

Assessment task	**Topic**	**Sample activity**	**Example**
Practical and oral task	Equivalent fractions	Use cut outs of rectangular shapes or circle on manila papers to show equivalence of fractions (example ½ and 2/4)	Draw a square of 4 cm a side on a paper, shade 2/4 of it and use a scissor to cut out 2/4 of the square. Draw a square of 4cm a side on other paper, shade ½ of it and use a scissor to cut out ½ of the square. Compare the two parts and talk about their size.  Draw and shade 2/3 in A. Shade the equivalent fraction in B. Make paper cutouts and compare their sizes. 
	Decimal fractions	In groups learners should follow steps to perform a given task	**Materials**: a pair of scissors, manila paper, a ruler, a pencil **Task and Steps:** (a) Draw a square of 10 cm. Do it on manila paper. (b)Draw smaller squares each measuring 1 cm inside the bigger square. (c)Count the number of small squares. (d) Shade four of the small squares. (e) What fraction have you shaded?

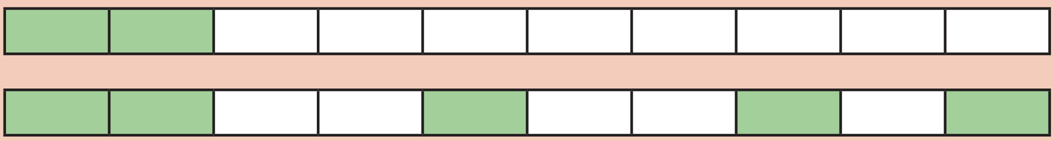
			(f) Write this fraction in words.  (g) Present and explain your work.
Visualization and representation tasks	Equivalent fractions	Work in groups to discuss how to get equivalent fractions that are drawn and written on flash cards or manila papers.	Shade the equivalent fraction of each of the fraction given below.   Write the shaded fractions. In each case, are they equivalent? Explain your answer. 

	Subtraction of fractions	Use mathematical word problems in the real context of learners and let them work in groups to discuss subtraction of fractions.	A storage tank weighed 3/4 tones when full of water. After five days, the family had used water from the tank in washing clothes and cleaning utensils. The weight of the water in the tank became 2/5 tones. Calculate the weight of water used by the family in five days. Explain your steps.

Application activity 5.5

You are a Mathematics teacher of primary three or five, analyse these activities and explain if the used action verbs develop low, medium or higher order thinking skills.

1. From each diagram below, write the fraction and decimal fraction (P3 and P5)



2. Read and write the following decimal fractions in words. Present your answer (P5 only).

   (a) 0.256 (b) 2.513 (c) 436.2 (d) 196.261 (e) 0.75 (f) 0.4

3. A farmer collected 10 eggs and harvested 10 apples. The mass of each egg was 23 g while each apple was 25 g.

   (a) Find the mass of the eggs in kilograms. (b) Find the mass of the apples in kilograms. (c) Compare the total mass of eggs and apples using the symbol (>).

   Which items had smaller mass? Explain. (P5 only)

5.6. Writing lesson plan for lessons on fractions

Activity 5.6

A good lesson planning is essential to the process of teaching and learning. A lesson plan is a teacher's detailed description of the course of instruction or 'learning trajectory' for a lesson. A daily lesson plan is developed by a teacher to guide class learning. Details will vary depending on the preference of the teacher, subject being covered, and the needs of the learners.

Suppose you are a P3 or P5 Mathematics teacher and you are supposed to teach the lesson on **multiplication of a fraction by a whole number in P3 or equivalent fractions** in P5 to 46 learners. Analyse the example of a lesson provided bellow and do the following:

a. Verify if the instructional objective has all 5 main components(condition/situation , who, action/ competence, content, standard of performance)

b. Find out all teaching materials used and propose the other ones you find relevant to make the lesson more engaging and practical.

c. Identify all teaching and learning techniques that have been used and suggest the new ones to make the lesson more active and learner-centered.

d. Identify which generic competences are developed and crosscutting issues which have been addressed. What can you do to cater for slow learners and learners with low vision?

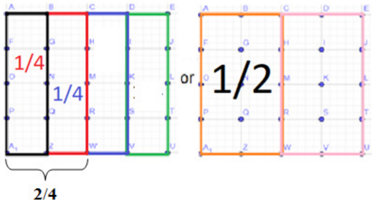
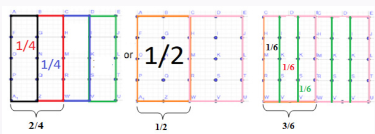
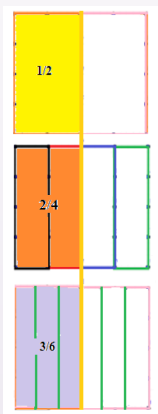
Example of a lesson plan on equivalent fractions

School Name:............................ Teacher's names ....

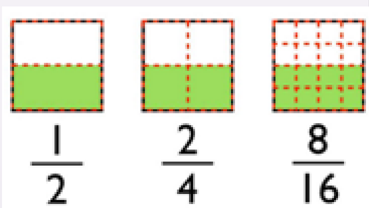
Term	**Date**	**Subject**	**Class**	**Unit Nº**	**Lesson Nº**	**Duration**	**Class size**
....	....	**Mathematics**	P_5	4	......	40 min	50

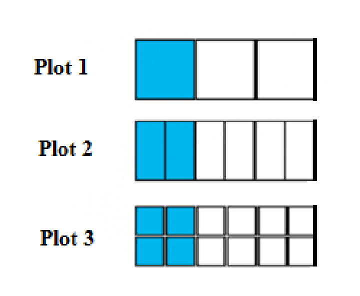
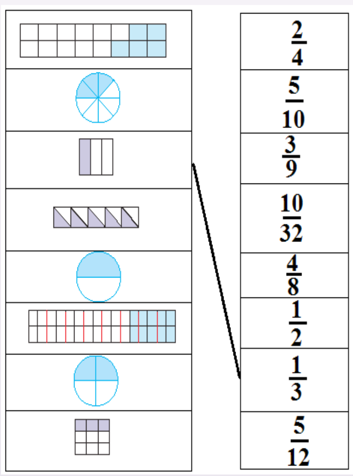
Type of Special Educational Needs to be catered for in this lesson and number of learners in each category	2 Learners with low vision or visual difficulties. 1 slow learner
Unit title	Equivalent fractions and operations.
Key Unit Competence	To be able to add, subtract and find equivalent fractions.
Title of the lesson	Arithmetic sequence
Instructional Objective	Using paper, manila papers, dot paper or grid paper and cartons, learners will be able to cut them into fractional representations and match dot to form any figure representing a portion of an equivalent fractions, correctly, precisely and confidently in a given time.
Plan for this Class (location: in / outside)	Arrange desks in class in order to allow learners to work individually, in pair or in groups
Learning Materials **(for all learners)**	Manila papers, dot paper, grid paper, cartons, scissors, pencil or pens to join dots on paper and grid paper..
References	Primary learners Mathematics P4, Kigali January 2019; Upper primary mathematics syllabus Kigali 2015.

Timing for each step	Description of teaching and learning activities	**Generic competences** and **Cross cutting issues** to be addressed **+ a short explanation**
	Learners start on making review on what they know about fractions, and write down what they would like to know after the lesson, in groups they do activities related to equivalent fractions by cutting papers into fractional representations and matching the equivalent fractions. Learners do group presentations and harmonization of the results under the facilitation of the teacher. Finally, learners are assigned individual tasks for applications and assessment and the correction is done on the chalk board in plenary session under the guidance of the teacher.	

	Teacher activities	Learner activities	
Introduction **(5min)** **development of the lesson** **Practical activities** (10**min**)	- Using a concrete/ tactile dot paper invite learners to show different fractional portion they can form and compare them	Learners show and compare different fractional portion on dot papers. e.g.: ½; 2/4; 3/6,….  Activity 1: Cutting into equivalent fractions In groups, using same size papers, same size manila paper, razors, scissors, pen, pencils, rulers draw, cut and compare the fractional portions of the following fractions 3/6, 2/4, ½  	Communication: through answering questions and group discussion. -Cooperation, interpersonal management and life skills: learners share their ideas in groups and in presentation of the work. -Critical thinking skills are developed through observation/ touching, feeling, matching and cutting dot on the papers to form fractional portion which are equivalent. -Gender equality is addressed through assigning both to girls and boys the same tasks in the whole lesson.

Presentation of learners' findings and exploitation: (10 min) **Summary (5 min)**	Organizes learners into groups and ask them to do activities 1 and 2 and gives instructions related to activity	**Activity 2: using dot papers, grid papers.** In pairs, learners observe, touch the dot papers and grid papers and form by matching, joining dot or numbers and shading in order to get portions of the following fractions: 2/3; 4/6; 8/12 , compare them and conclude if they are equivalent or not. Through gallery walk, the reporters (representative of the group) present their findings on behalf of the groups while other learners follow the presentations and ask questions for better understanding.	-Inclusive education is addressed through using dot papers, cut-outs, and different colours to facilitate 2 low vision students while 1 slow learner is given more simple and practical activities on equivalent fractions . -Communication skills are developed through presentations and answering questions.

Application (5 min)	Invites groups to post their findings in different corners of the classroom and ask them to present their work. Takes notes on key points from learners' presentations and build on them to harmonize their works.		
	Facilitates learners to elaborate the summary from the presentation. Asks learners to individually work out the application activity.	Learners write down the main points in their notebooks. In math, **equivalent fractions** can be defined as **fractions** with different numerators and denominators that represent the same value or proportion of the whole. Here's an example of **equivalent fraction.** 	Critical thinking skills are developed through analysis of different fractions and deducing that they have the same value

		Individually learners work out the application activity and finally, they make a correction on the chalk board. **Application activity:** The three diagrams bellow show rectangular plots of land. Observe the shaded parts and write down the represented fractions for Plot 1, Plot 2 and Plot 3. Compare the three fractions and explain how you find their size. 	Problem solving skills are developed through application of equivalent fractions in real life situations.
Conclusion **(5min)**	Write assessment questions on the chalk board and ask learners to work out.	Individually learners do the assessment activity. **Activity:** analyze this diagram and, match the drawings with the right fraction. Identify and write down equivalent fractions among the list. 	Inclusive education developed through using flash cards or drawings to help low vision learners

Teacher self-evaluation	All learners have actively and inclusively participated in the lesson. The objectives were total achieved. Reinforcement activities are needed for learners to enhance learning.

Application activity 5.6

As a student-teacher, re-examine the lesson plan above and make it more engaging, more active and inclusive.

End unit assessment 5

As a P3 or P5 Mathematics teacher, you are supposed to teach the lesson on length measurements in P3 or application of direct proportion in P5.

Refer to the syllabus, appropriate Mathematics books for P5 and do the following:

a. Set the instructional objectives of the lesson;

b. Organize teaching materials to be used;

c. Determine the generic competences to be developed in each step of the lesson and highlight crosscutting issues to be addressed in this lesson.

d. Using answers in a, b and c, prepare a lesson plan illustrating step-by-step all procedures and components of a competent mathematics lesson.

Unit 6 LESSONS RELATED TO METRIC MEASUREMENT

Key unit competence: Prepare active lessons related to metric measurement

Introductory activity 6

With the help of the syllabus and textbooks for lower or upper primary Mathematics, identify all topics related to metric measurement to be studied?

For each topic, try to find out appropriate resources for facilitating related lessons using low cost materials and suggest how to produce them.

Are teaching techniques to facilitate lessons on operations of metric measurement the same as those to be used in the lessons related to converting metric measurement units? Explain your answers.

6.1. Recall on concepts related to metric measurement

Activity 6.1

Mathematical concepts refer to the basic mathematical content categories that are needed for solving mathematical problems and they include geometrical concepts.

Use primary Mathematics book of lower or upper primary and do the following:

a. Select different concepts related to metric measurement

b. Among the selected topics which do you find are challenging to you?

c. Make a self-study on the challenging topics. You can ask for a help from your colleague or your tutor.

Content summary

Concepts related to metric measurement

In primary school, concepts related to metric measurement are the following: Length measurements, Capacity measurements, Mass measurements, Area and land measurements, Time, Money and its financial application, volume, Speed, distance, and temperature.

The concept underlying measurement is the **comparison** of one thing with another according to a specified attribute. Attributes can be **spatial**: length, area and volume; **physical**: mass (weight) and temperature; or have no obvious physical connection with objects: time. Some measurement attributes seem to be easier for learners than others. For example, length and area appear to be easier than volume due to the complexity of spatial visualization and/or quantifying involved. These difficulties can be reduced if explicit links are made between the one-dimensional, two-dimensional and three-dimensional aspects of these attributes. Many children take long time and many experiences to gain complete understanding of notions such as area, volume and mass. Before learners can compare or measure an attribute (e.g. mass), they must be aware of what the attribute is. Teachers need to help learners to perceive measurable attributes and identification of these in the environment is helpful.

In the early years of schooling, measurement activities that focus on comparisons help learners understand the idea that they are measuring specific attributes of objects. Such comparisons should proceed from direct to indirect.

Direct comparisons involve directly aligning the attributes to be compared. Activities should include comparing two similar objects (e.g. two sticks of different heights) and different objects (e.g. the length of a pair of scissors and a pencil case). Initial activities should involve comparisons of two objects. Comparing and ordering three or more objects is difficult as learners are required to identify an object as being both bigger than and smaller than other objects at the same time. Such comparisons lead to the development of transitive reasoning, which is crucial for measurement.

Transitive reasoning involves thinking that follows the line that, for example, if A is longer than B, and B is longer than C, then A is longer than C.

Indirect comparison is the process of comparing two objects that cannot be directly aligned, such as the length of a desk and the height of a doorway. An intermediary device, such as a length of paper or a piece of string, is required to assist with the comparison. Subsequently, the notion of using a unit to compare measurements is introduced through non-standard units and followed by standard units.

Non-standard units include things such as body parts (e.g. hand spans, stride lengths) and classroom or kitchen materials (e.g. pencils, paper clips, cups, spoons). Non-standard units are practical, personal and familiar, and are used in real-life situations. With use, some may become handy personal referents for estimation. For example, an individual may know that their hand span is approximately 20 centimeters, that 14 of their normal paces will take them approximately 10 meters, or that a cup holds approximately 250 milliliters. The early use of non-standard units in measurement activities may strengthen links between measurement and number. This is achieved through the development of counting, understandings of the quantity of numbers, and of the position of a number relative to other numbers in the measurement context.

The use of different referents by individuals and variations arising from the use of the same referent to obtain measurements can lead learners to see to the need for **standard units**. For example, where learners are using paces to measure the distance between two objects, the leg length of learners may result in different totals. Standard units enable more accurate and consistent measuring in different places by different people and facilitate communication that yields the same understanding about measurements.

The standardized system of measures used in Rwanda is metric. Understanding that the prefix "*milli*" denotes one thousandth of a unit (e.g. a milligram is one thousandth of a gram), "*centi*" denotes one hundredth of a unit (e.g. centimeter), and *kilo* denotes $(10)^3$ or 1000 times a given unit (e.g. a kilometer is one thousand meters) will assist learners to understand the relationship between units of measure.

Experience with both non-standard and standard units contributes to the **development of estimation skills**. Thinking, reasoning and working mathematically in terms of measurement requires the ability to **estimate** in terms of units. Students should be provided with many opportunities to practice estimation of length, mass, area and volume using a variety of senses. For example, sight may be used to estimate length, area and volume, and touch (hefting) to estimate mass. Estimation helps students recognize when a measurement is reasonable and is a lifelong skill used in many professions. Students should be encouraged to estimate and make an informed judgment before they measure.

There are at least three strategies for estimation that learners need to learn. These are: **use of referents, chunking and unitizing**. Examples of the use of referents include individuals using knowledge of their own height or the height of a meter to estimate the height of a window, or counting 'one cat and dog', 'two cats and dogs', three cats and dogs' … (for seconds) to estimate durations of time. Chunking involves breaking the measurable attribute into 'chunks' then adding the estimations.

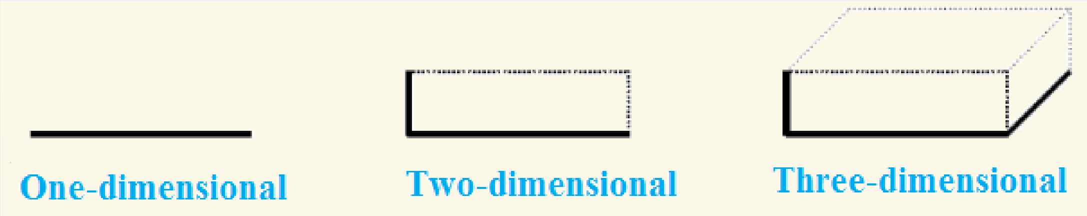
For example, the length of a room could be estimated using the positions of windows and furniture to divide it into chunks: chunk one being from the corner to the window; chunk two from the window to the table; and chunk three from the table to the corner. Unitizing involves dividing the measurable attribute into equal parts of a familiar length. For example, the length of a swimming pool could be estimated by visualizing its length divided into a number of equal parts each of a metre. When these strategies are used for estimation, learners realize that estimating is a thinking exercise rather than a guessing activity.

Mass poses particular problems for some learners because they confuse it with volume, size or quantity. **A common misconception** is that the larger the size of an object, the greater the mass. This occurs when the judgment made is based on sight (a transfer from learning about length) rather than on feel. For example, learners may think that a tennis ball is heavier than a golf ball because it is larger. In relation to quantity, learners may think that three foam balls are heavier than a soccer ball because there are more of them.

Relationships: Learners need to understand the inverse relationship between the size of a unit and the number of units in the measure. That is, the larger the unit, the fewer required to measure and vice versa.

Learners need extensive experiences with different attributes and units of measure to develop their conceptual understandings before embarking on the use of formulas. They should be provided with opportunities to investigate areas and volumes, and to generalize about how and when various attributes are related. For example, from investigations of the areas of rectangles, learners could generalize about the relationship between area, length and width.

The relationship between length, area and volume is one-dimensional, two-dimensional and three dimensional respectively.



Investigations of the volumes of different 3-D shapes will help learners understand how the volume relates to the height and the area of the base of the prism. The **introduction of formulas** before learners have a sound understanding of the relationships between the different dimensions has been recognized as a cause of many difficulties learners experience with area and volume.

Therefore, when teaching and learning, measurement skills provide lots of opportunities for learners to sort objects by size, observe, measure, reading and writing measurement and manipulate them by using everyday activities and experiments with real objects.

Application activity 6.1

Use a primary Mathematics syllabus and the textbook, select one lesson on addition and subtraction of capacity measurements.

For each lesson, highlight the skills to be acquired by learners.

6.2. Production and organization of appropriate resources for the lessons related to metric measurement

Activity 6.2

By the use of the syllabus and textbook for lower or upper primary Mathematics choose one lesson on metric measurement (e.g. addition and subtraction of capacity measurements).

Organize appropriate teaching resources for your choice and explain how those resources can be used while teaching and learning the selected topic (addition and subtraction of capacity measurements).

Content summary

The teaching methods of metric measurement in primary schools mostly refer to concrete and inductive methods. Mathematics learners arrive at abstract postulations, generalizations by **observing and manipulating concrete objects** and concrete examples and **through inductive conclusions**. This is familiar and appropriate for learners of that age of primary level. The **inductive procedure** is made up of a chain of inductive steps which lead to the understanding of the general. It takes many “hands-on” experiences using manipulative materials before conservation of quantity is fully comprehended. Indeed, it is generally agreed that understanding this concept involves several stages of intellectual development. This requires a classroom teacher to continuously develop, design and exchange teaching and learning materials. If they are well constructed, organized and presented, successful teaching and learning can be achieved.

Collection of resources can be used to enrich classroom and reinforce learners learning of metric measurement. These resources are helpful to learners as they become familiar with the metric system and learn more about international system basics.

1. Use small real objects like sticks, pens and tables: learners measure the length of sticks, pens in different units of length measurement.
2. Use some containers whose capacity is graduated like cup, syringes and empty bottle of medicine.
3. Jerrycans: learners measure how many liters can fill the jerrycan by putting water in jerry can and counting the numbers of liters.
4. Clocks: make a well drawn clock on manila papers which is to be used in teaching and learning time measurement.
5. Metric Pyramid: Using card paper, print out the easy to construct three-dimensional. The pyramid has a list of the metric symbols, and conversion factors for area, time, length, and mass.
6. Ruler and other measuring devices to measure lengths in metres, centimetres and millimetres and record lengths in decimal notation
7. Use protractor, T-square and related improvised materials to measure angles.
8. XO laptop and related mathematics software to calculate volume, area, etc.

Application activity 6.2

By the use of the syllabus and textbook for lower or upper primary Mathematics choose one lesson on metric measurement (e.g. addition and subtraction of capacity measurements).

Organize appropriate teaching resources for your choice and explain how those resources can be used while teaching and learning the selected topic (addition and subtraction of capacity measurements).

6.3. Active teaching techniques and strategies for facilitating lessons related to metric measurements

Activity 6.3

In teaching and learning different Mathematics lessons one of the challenges is to make the lesson enjoyable for learners. If learners are involved in teaching and learning, the lesson becomes more active and participative.

1. What can you do to make the lesson on “Conversion of capacity measurements” be enjoyable?
2. Discuss active teaching techniques and strategies you think are the best for facilitating lessons related to:

   a. Conversion of capacity measurements in P3 or P4?

   b. Reading and writing area measurement in P3 or P4

Content summary

The concept of length should be introduced first, because other measurement concepts, especially in the metric system, are related to it. Learners begin with the development of pre-measurement skills and progress to nonstandard and standard measurement learning levels. This must take many “hands-on” experiences using manipulative materials before conservation of quantity is fully comprehended. Indeed, it is generally agreed that understanding this concept involves several stages of intellectual development.

6.3.1 Skill development

Teaching Mathematics at primary schools requires the teacher to use various techniques and strategies for facilitating metric measurement lessons where learners use manipulative materials and are engaged in the process of teaching and learning.

Procedure of introducing metric measurement in primary school

1. Through activities which engage learners to make practice (e.g. measuring the classroom length, school ground), briefly explain what the metric system is and how it came about from the performed activity.
2. Use real materials (e.g. a metric ruler and a meter stick for illustration) show how meters, centimetres, and millimetres can be converted using powers of 10. Use a graduated cup to show learners how many millilitres, centilitres, litres, etc…

3. Have leaners measure various items such as pencils, books, desks, and other learners and decide which unit is most appropriate.
4. Learners read and write the measurement units obtained after measuring such materials.
5. Homework might be to find out and measure at least two items appropriate to each unit of measurement (e.g. meters, centimetres, and millimetres, litres, kilograms, etc…)

Basic procedure can be used to introduce each of the units of measurement and the process for converting units.

- Find out the function of the specific unit. For example, metres tell us how long something is. We use millimetres to measure objects that are not very big, centimetres to measure things that are medium sized and metres to measure objects which are big.
- Use concrete demonstrations of equivalencies in capacity, length and mass to illustrate each unit.
- Demonstrate how to use measuring tools, measuring to the nearest whole unit.
- Provide exercises which require students to determine the appropriate tool to use when measuring an object. Ask questions such as: *What unit would we use to tell how long a pencil is? What unit would we use to measure how much juice we should give to the child?*
- Present equivalency facts, such as 10 millimetres equal one centimetre. This step should make links between units from all types of measures.

The three phases for teaching measurement

In teaching and learning measurement there are 3 main phases to be used for better helping learners about them.

Phase 1: Identifying the attribute

As metric measurements have different characteristics, the purpose is to develop the concept of the characteristic, distinguish it from other characteristics and gain intuitive understanding of properties.

Phase 2: Learning to measure

The Purpose of learning to measure is to learn to measure the quantity, use formal units and Estimate measurements.

Phase 3: Learning to calculate

Learning to calculate measurements is to convert from one unit to another, calculate, instead of direct measure.

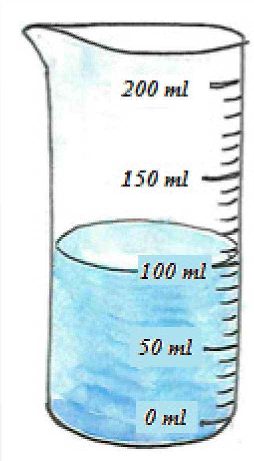
There are three main steps for teaching and learning metric measurement that when combined they create intellectual competence in logical reasoning, spatial visualization, analysis and abstract thought in the learner

Concrete stage: With manipulations of real objects such as (rulers, graduated cup, balance and other materials related) learners study by measuring the length, capacity, weight, area and volume).

Semi- concrete stage, visualization or representation: In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawings or pictures.

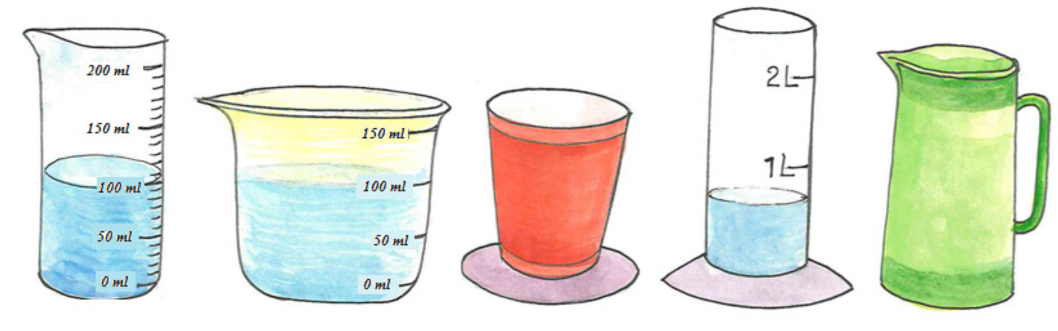
Abstract stage: *In* this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of measured units.

Example 1: learners use graduated cup to measure the quantity of milk in millilitres



A measuring cup showing 200ml

Example 2: The teacher provides learners with different pictures of measuring containers holding an amount of liquid. Each should have a measuring scale (up to three litres). Learners interpret the scale and estimate how much liquid is in each container



Discuss the graduations on each container. How is the scale recorded?

Example3. The teacher gives learners a set of containers. Learners estimate the capacity of each container and record their estimates in a table. After estimating the capacity of each container, learners use the measuring jug to find the actual capacity of each container and record it. They can convert the units to other units.

6.3.2 Teaching and learning strategies related to metric measurement

In the teaching and learning of metric measurement, teachers use a repertoire of teaching approaches as there is no single suitable method to teach mathematics to emphasize mastery of knowledge and skills, critical and creative thinking, communication and problem solving. In the mathematics lesson on metric measurement, teachers adopt a combination of different teaching approaches to engage learners in learning. These approaches will help learners talk more openly, think more creatively and ultimately become more engaged in the process of learning lessons related to metric measurement.

Some of the common combinations of approaches are:

- Investigational activity followed by conjecturing or whole class discussion: learners may work independently, in pairs or in groups; learners may use technology or manipulatives to carry out their investigations.
- Practical activity followed by whole class discussion: learners could work independently, in pairs or in groups; and present their findings. Learners may play mathematical games or do outdoor activities for better learn metric measurement.
- Problem solving or mathematical modeling done in groups followed by presentation of work by learners to the whole class.
- Independent learning during e-learning: learners may do on-line tutorials at home and submit the work to teacher electronically.

- Peer teaching activities: peer instruction encompasses a range of scenarios where learners instruct skills or explain concepts to classmates.
- Questioning and monitoring: As the learners are exploring and figuring out how to solve the task at hand, the teacher will go around and listen to their discussions, ask questions to assess the learners' understanding, and to identify possible misconceptions the learners may have about the mathematical concept related to metric measurement being explored and provide clarifications where necessary.
- **Outdoor activities:** Learners go outside of the classroom to measure the perimeter of the school compound, length of the ground, measure the area of a tiled floor by counting the number of tiles it is made up or they can also measure the area of gridded paper by counting the number of squares it is made of before being given the formula for multiplying the number of tiles found on the length and those on the width.

Application activity 6.3

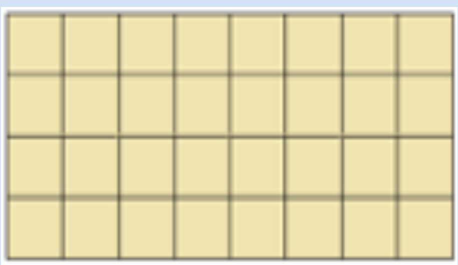
Take a syllabus, textbooks and teachers' guides and organize the teaching techniques you can use to make the lesson on measuring the length and perimeter of various objects in different units more active and participative.

6.4. Activities for developing generic competences and integration of cross-cutting issues in the lessons related to metric measurement

Activity 6.4

Read the activity bellow carefully, analyse it and answer the questions that follows.

Let have square line papers.



- Count the number of squares on horizontal line and vertical line.
- Multiply the squares on horizontal and vertical line, then guess the unit to express your answer.

- Count the all the squares on paper.

Suppose the paper represents the form of a squared tiled floor with the side L in a house. Questions:

1. Explain how you can find the needed number of tiles?
2. Identify and explain the competences to be developed while performing the above activity
3. Use the syllabus and find out the class or level the above activity can be given to.
4. Is there any cross-cutting issue that has been addressed? Explain your answer.

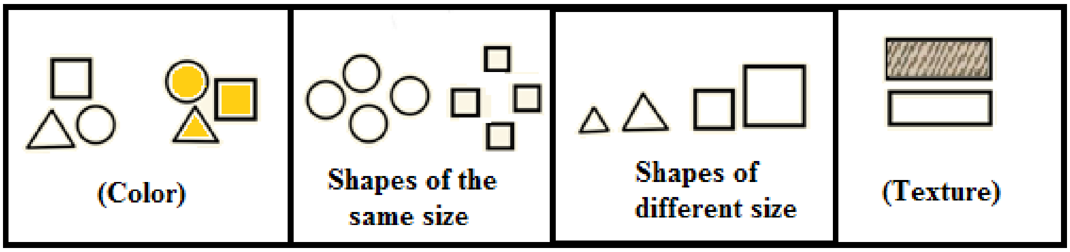
Content Summary

In lessons related to metric measurements, different generic competences can be developed. However, the following generic competences can be developed in the activity 6.4:

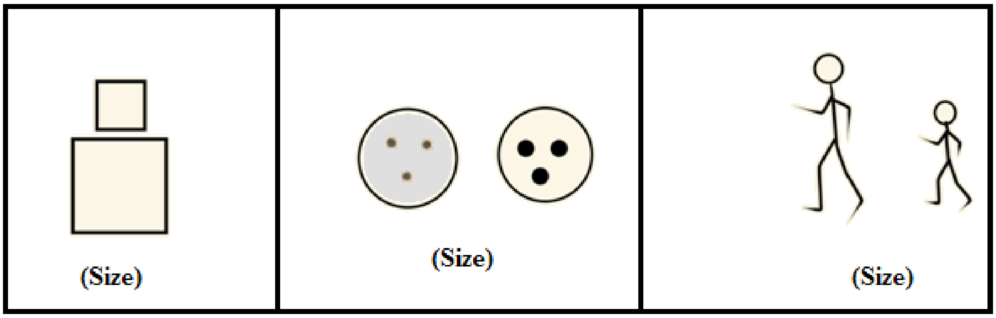
1. **Critical thinking**: in solving problems, measuring objects related to metric measurement and recording data, problem solving skill can be developed.
2. **Communication**: through answering and discussing while measuring.
3. **Cooperation**: through working in pairs and groups, learners develop the spirit of working together and sharing ideas.
4. **Problem solving**: through applying the concepts learnt to the solution of different problems of the real life of the learner, there is a development of solving different problems related to use the acquired concept in solving related problems for example knowing the number of tiles and perimeter help in calculating needed cost.

The following are the other types of activities that can be assigned to learners to help them develop above mentioned competences:

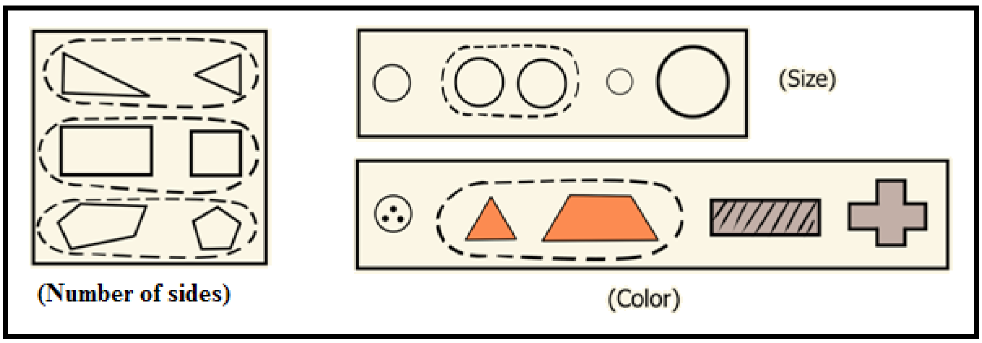
- **Compare**: Simple comparison begins at a very early age and grows increasingly complex as learners develop skill in this area. Children should be encouraged to estimate relationships and then verify them by making actual comparisons of concrete objects. Penalties for poor estimates should be avoided as this will discourage learners from trying.
- **Qualitative Comparison**: identifying objects which are similar in one or more properties, for instance, color, shape, texture, prettiness, hardness, flexibility.



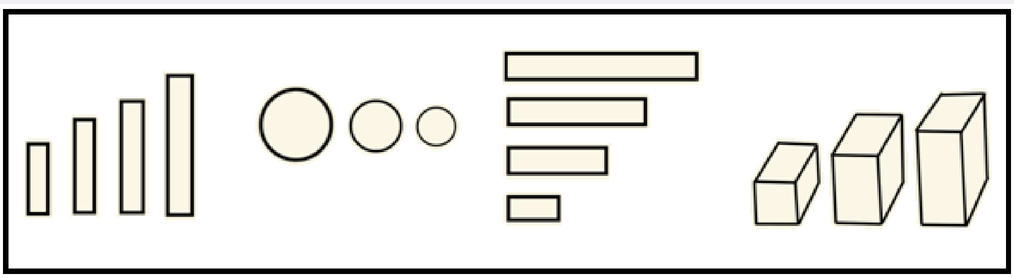
- **Size Comparison**: identifying objects that are larger, smaller, shorter, longer, etc.



- **Matching**: recognizing objects that are the same in number, color, size, shape, texture, weight, or other characteristics.



- **Ordering**: arranging various sized objects in a progression, for example, from the shortest to longest, largest to smallest, lightest to heaviest (give your own examples).



Measuring:

Learners measure objects with a variety of nonstandard units. A list of such units might include paper clips, pencils, books, sticks, straws, floor tiles, fingers and hands. Teachers should encourage learners to pick their own units and discover advantages and disadvantages for their choices. All measurement learning should originate from "hands-on" activities and experiences.

- **Instrument development:** Experiences with a variety of measuring instruments (nonstandard and standard) will demonstrate that:
    1. Objects can be measured in a variety of ways.
    2. Efficiency in measuring comes from choosing an instrument appropriate to the object to be measured.
    3. Some instruments provide a more accurate measure than others.
    4. Units of various sizes are needed. (Paper clip units are satisfactory for measuring short distances, but a broom handle is better for measuring longer distances).
- **Use of metric chart**: Learners are guided to be familiar of the use of metric chart to show the logic of the metric system, to introduce the metric prefixes and symbols and to emphasize the units that are commonly used and how to convert units using such chart.
- **Activity for symbol usage:** The metric symbols are used internationally and learners should be given various activities to let them become familiar that rules for using the symbols of the International System of Units (SI) must be respected.

In lessons related to metric measurements, different cross- cutting issues can be addressed.

However, the following cross- cutting issues can be addressed in the activity 6.4:

1. **Financial education**: through calculation of area for lands, the tiles needed for the house construction, length for materials used in tailoring and their cost, capacity for different oil containers, one can help learners to realize the value of money, their use and savings applying metric measurements on buying and selling.
2. **Gender** developed through assigning the same roles for all males and females.
3. **Inclusive education** developed through: activities that should cater for different ability of students, teaching and learning materials that are adapted to different special education needs, teaching techniques that are adapted to different ability of students.

4. **Environment and sustainability:** This is addressed when learners take care of the environment protection; as they are measuring the area of land, they are committed to take care of different plantations of the field.

Example: Observe the following photo:

	1. For which purpose are children planting trees? 2. If they plant trees on lines for a ***rectangular*** land where they plant 10 trees on the length and 6 trees on the land's width. How many trees did they plant in that land? 3. Calculate the area of the surface covered by all trees planted by children if those trees are equidistant maintaining 1 m between two consecutive trees.

Application activity 6.4

As a student-teacher, set up an activity on metric measurements that helps primary learners to develop generic competences and address crosscutting issues. Explain how your activity develops generic competences and addresses crosscutting issues.

6.5. Assessment tasks in the lessons related to metric measurements

Activity 6.5

Use a Mathematics textbook of P1, P3 or P6 and identify any activity related to the addition of metric measurement. Analyse the activity and explain whether the action verbs used develop low, medium or high order thinking skills.

Content summary

The lack of good learning activities and assessment tasks to support the effective learning of Mathematics is one of the causes of failure and negative attitudes towards mathematics subject for some learners.

One of the solutions is to enrich teachers with a variety of mathematical resources including appropriate activities/ tasks that are age appropriate and which intend to develop high order thinking skills.

Such tasks are formulated by the use observable action verbs that are appropriate to the learning expected by considering low, medium and higher order thinking skills and competences.

The table below shows examples of how tasks on metric measurements should be formulated.

Assessment task	Topic	Sample activity
Practical and oral task	Counting amount of money	– Match a coin/note of one denomination to an equivalent set of coins/notes of another denomination using play money, and realize that a greater number of coins/ notes is not necessarily a greater amount of money; – Use play money to add, subtract and make change during shopping activities. – Use play money to make up a given amount of money in different ways (For example 100Frw is made up two coins of 50 Frw or 5 coins of 20Frw, etc).
Visualization and representation tasks	Use units of measurement	Use the arm span to show 1m and estimate the length in meters.
	Measuring length, capacity or mass.	Work in groups to measure the length (or capacity) of different lengths (or sized containers) using measuring tools such as ruler (jars and beakers).

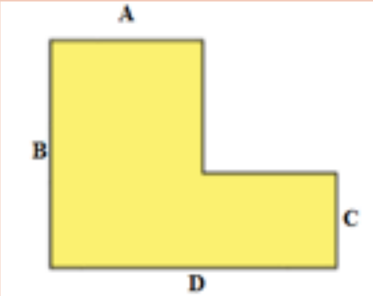
		Work in groups to measure length/mass using appropriate units and explain their choices of units and how the measurement is done: for example measure the length of a longer simple iron cable in meters and the mass of a heavier stone in kg). – Make a composite figure using cut-outs of rectangles and squares or draw the figure on the squared grid (or tiled floor) and calculate its area and perimeter.
	Telling time	– Tell the time fixed by a colleague on the geared clock and fix the time on the clock for others to read; – Work in groups to create problems involving time in hours and minutes for other groups to solve. – Describe every day events using 24-hours clock, including starting time, finishing time and time duration and use this concept to solve problems.
	Determine the area	– Find the area of square and rectangle drawn on square grid by counting and by formula to compare answers. – Estimate the area of a rectangular or squared land and compare the sizes of rectangles /squares using area.
	Connecting area and volume	– Apply multiplication and division concepts to find one dimension of a rectangle given its area/perimeter and the other dimension. – Make connection between $1dm^2$ and $1dm^3$ using sheets of paper by making a square of $1dm^2$ and a cube of $1dm^3$.

Application activity 6.5

Consider the following activity provided by a primary Mathematics teacher to one group of learners made by boys and girls.

Question:

Take instruments of length measurements and go outside to measure the length of the sides for a ground earlier well designed by the teacher. The ground has the following form as you are going to see it:



You are asked to:

1. Measure the length for all sides: A, B, C and D.
2. Determine the Area for the ground after measuring the lengths for its sides.

Describe the skills to be developed by learners in such an activity is this activity appropriate to P3 or P4 learners in Rwanda? Explain your answer.

6.6. Lesson plan on metric measurements

Activity 6.6

You are supposed to teach the lesson on the conversion of mass measurements to 46 P3 or P4 learners. Refer to the syllabus, appropriate books, and the lesson planning procedures learnt in Foundation of education subject and do the following:

a. Set the instructional objectives (with its 5 main components) of the lesson

b. Organize teaching materials to be used

c. Formulate teacher's activities and learners' activities depending on the step of the lesson

d. Devise generic competences to be developed and crosscutting issues to be addressed depending on the step of the lesson

e. Summarize the teaching and learning techniques to be followed in this lesson.

Example of a lesson plan on measurements

School Name: X Primary School............... **Teacher's name**: ....

Term	Date	Subject	Class	Unit Nº	Lesson Nº	Duration	Class size
1	27 /05/ 2019	Mathematics	P4	6	1 of 6	8h00-8h40	40 learners
Type of Special Educational Needs to be catered for in this lesson and number of learners in each category				One learner with hearing impairment will sit near the chalkboard			
Unit title	Length measurements						
Key Unit Competence:	Convert between units of length and apply them in solving mathematical problems related to daily life situations including perimeter						
Title of the lesson	Concepts of length measurements						
Instructional Objective	Given small distances and different objects, learners will be able to make appropriate estimations of length with justifications as well as measuring them.						

Plan for this Class (location: in / outside)	Outside of the classroom where learners will measure the length of different objects
Learning Materials (for all learners)	Meter, decameter, ruler, sticks, rope, play ground
References	REB. (2019). Primary learner's Mathematics book, P4. Kigali Rwanda.

Steps and Timing	Description of teaching and learning activity Through observation of the lengths of different objects and distances, individually learners make estimations. In groups, learners use length measurement instruments (ruler, decametres, 1 meter or 10 meter rope and measure different lengths or distances.		**Competences and Cross-Cutting Issues to be addressed**
	Teacher's activities	**Learners activities**	
Introduction (5 minutes)	Ask learners to do exercises on previous content with the use of math game	Discuss what they learnt in previous lesson. In small groups, play the game: which decimal comes before or after? One learner in group say decimal number for example 0.5 and the other learners one by one, say the next.	Communication developed through sharing the acquired previous content by singing and playing mathematics game.

Development of the lesson (25 minutes) Discovery activities:	Allocate tasks to different groups Facilitate learners to measure objects using different tools.	**Activity1: Estimation of length** In small groups, learners estimate the length of different objects and distances through observations and using their prior knowledge on length measurements from lower primary. Groups present their findings with justifications to the whole class. **Activity2: measuring using different tools** In groups, learners measure the following using different tools:	Through presentation, reading, writing and speaking), learners develop communication skills. Cooperation developed through working together in groups. Peace and value education is addressed when all learners share ideas in a peaceful way.

	Allocate tasks to different groups	Measure the distance between two walls of the classroom by meter ruler for one group and another group use a measured stick Learners compare their findings by using ruler meter and by using sticks	Gender addressed when both girls and boys are working together in groups. Inclusive education addressed in classroom by encouraging all learners to participate including those with special education needs.
	Facilitate learners to measure objects using different tools.	Find out that meter is the standard unit to measure length. **Activity3:selection of measurement tools** Learners are invited to select tools for measuring different objects or distance: small objects(pens, pencil, crayons)	
Synthesis and summary:		Learners measure small distance using ruler or small ruler of 30cm **Activity4: Measuring an object in different length units**	
Application:		In small groups, 1^{st} group measure length of stick in meter, 2^{nd} group measure length of stick in decimeter and 3^{rd} group measure the stick in centimeter.	

		Learners discover that to measure the length of small objects you can consider different unit measurements: m(standard unit), dm, cm and mm 1m= 10dm= 100c= 1000mm	
	Facilitate learners to draw conclusion on length measurements.	It is very important in our life to make appropriate estimation of length. The standard unit of measurement of length is meter(m) The length of small distance and objects is measured in meter, decimeter, centimeter and millimeter To have exact length measurements we use standard tools such as meter ruler, a ruler, etc… The relationship between units of lengths is as follow: 1m= 10dm= 100cm= 1000mm	With high level of summarizing the lesson learners develop critical thinking

	Put learners in different groups and give them objects to use such as ruler, 1meter string and ask them to measure length of: wall, teacher's table, their desk, chalkboard, etc...	Learners avail their rulers in their groups They estimate the length of objects they are going to measure and write their estimations in their notes books They then measure the objects they estimated and write their findings in their notes books.	
Conclusion (10 minutes) **Assessment** **Homework**	Facilitate learners to measure different objects. Show learners the side of the class to be estimated and measured	A learner from each group write their estimation and findings on the chalkboard and others look the estimations with actual measurements and choose which group had well estimated. Learners measure the length of the following objects individually using the ruler Their Mathematics book Their pencil or pens Through observation learners estimate the length of their classroom door and window.	Through taking initiatives to continue update the knowledge on length measurement learners develop a lifelong learning skills.

<table>
<tr><td></td><td>Ask learners to estimate and measure distances of different objects</td><td>Estimate the length of the door at your home
Complete the following table.<table><tr><td>item</td><td>Estimated length</td><td>Actual length</td></tr><tr><td>fork</td><td></td><td></td></tr><tr><td>Door</td><td></td><td></td></tr><tr><td>Chair seat</td><td></td><td></td></tr></table></td><td></td></tr>
<tr><td>Teacher self-evaluation</td><td colspan="3">Most of the learners have understood the lesson but learners with learning impairment need more exercises for measuring lengths of different objects</td></tr>
</table>

Application Activity 6.6

Refer to the syllabus units of P3 or P 4 Mathematics and appropriate books and prepare a lesson plan on the conversion of mass measurements.

End unit assessment 6

You are supposed to teach the lesson on solving real life problems related to the time for everyday events using 24-hours clock (including starting time, finishing time and time duration).

Refer to the syllabus, appropriate books and do the following:

f. Set the instructional objectives of the lesson;

g. Organize teaching materials to be used;

h. Determine the generic competences to be developed in each step of the lesson and highlight crosscutting issues to be addressed in this lesson.

i. From your findings in a, b and c, deduce a lesson plan illustrating the step-by-step procedures for the lesson.

Unit 7 LESSONS RELATED TO ALGEBRA

Key unity competence: **Prepare active lessons related to algebra.**

Introductory activity 7

In teaching and learning algebra in primary schools, the acquisition of mathematical concepts and skills such as number patterns, missing number in a sequence, algebraic expressions, algebraic equations and inequalities is very crucial for continuous learning and developing thinking, reasoning, communication and application skills.

- Take Mathematics syllabus and Mathematics textbooks for upper primary and identify all the units related to algebra.
- Are those units on algebra having anything in common? Explain.
- Suppose you are requested to teach a lesson from one of algebraic units, make a list of materials which can support you to make your Mathematics lesson more practical.
- Identify and explain some teaching techniques or strategies you may use to facilitate any lesson on algebraic topics.

7.1. Recall of concepts related to algebra

Activity 7.1

Using lower or upper primary Mathematics syllabus and textbooks:

a. Select different concepts related to algebraic units in P2, P3, P4, P5 or P6,

b. Indicate which concepts may be challenging to you and through a self study, try to overcome those challenges. You can ask for a help from your colleague or your tutor

Content summary

Algebraic concepts are very important in Mathematics. Having a sense of number patterns, number sequence, algebraic expressions, equations and inequalities is an important building block for all primary school learners, which serves as the foundation for higher Mathematics and for the workplace in the future.

In all units related to algebra, students have to develop:

- Understanding on how to describe and generate number patterns following a rule,
- Ability to solve missing number problems involving addition and subtraction,
- Skills on how to write sequences of whole numbers, fractions and decimals,
- Meaning of algebraic expressions,
- Skills on how to form and solve simple algebraic equations and inequalities.

Application activity 7.1

Use P2, P3 and P5 Mathematics syllabus and textbook, select one lesson in the unit 13 (in P2 or P3) unit10 (in P5) and highlight the skills to be acquired by learners. Extend your work to P4 and P6 units related to algebra.

7.2.Production and organization of appropriate teaching and learning resources to be used in algebraic lessons

Activity 7.2

- Observe the following teaching and learning material made in Manila paper or rice sack to facilitate lessons on algebra. Use a Mathematics syllabus and textbook for lower or upper primary mathematics, Highlight all lessons related to algebraic units in P4, P5 or P6 you can teach using this material.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Using a Mathematics syllabus and textbook for upper primary choose any lesson on number patterns in the unit 12 , P4 or any lesson on filling in missing numbers and suggest appropriate teaching resources for your choice and explain how those resources can be used while teaching and learning the selected topic (e.g lesson on arithmetic progression).

Content summary

Teaching and learning resources can be acquired by producing or constructing them, by collecting them from the environment, by asking learners to prepare them, by requesting the school administration or parent to buy them, by borrowing or hiring them or through improvisation. Concrete materials such as manipulative (plastic bottles, bottle tops), flash cards, number blocs, can be used when teaching and learning algebraic concepts like number patterns, number sequence while semi- concrete materials such as pictures, patterns on charts, number- line, short videos,…should be relevant to teach and learn some concepts like algebraic expressions, algebraic equations and inequalities.

Examples:

1. Plastic bottles can be used in teaching and learning arithmetic or geometric progressions.

2. Flash cards can be used to facilitate quick memorization of number sequences, provide opportunities for learners to interact each other and allow individualization learning.

Application activity 7.2

In TRC, use low cost materials (rice sac, Manila paper…) and produce materials related to the teaching and learning of number patterns; number sequences; algebraic expressions, equations or inequalities. Explain how they can be useful in teaching and learning algebra in lower or upper primary classes

7.3.Techniques and strategies for facilitating lessons related to algebra

Activity 7.3

As student –teacher, read carefully the following scenarios, identify different active teaching and learning techniques used by Mathematics teacher A and B. Which scenario do you find more inspiring to you and why?

Scenario 1: In Mathematics class, a teacher A has a lesson about arithmetic progression. He/she came in class with different materials (plastic bottles with number sequences, colored number grid on a rice sack, flash cards containing number sequences) and a lesson plan. He/ she forms groups, asks students to explore materials, analyse the activity, find out different number sequences and make a group presentation on their findings by using gallery walk. During and after students' presentations, teacher A harmonizes their works and helps students to brainstorm in order to facilitate them to generate the summary.

Scenario 2: A teacher B has a lesson about arithmetic progression. She/he gives students the definition, properties and examples of arithmetic progression on the chalkboard and students copy notes. After explanations from the teacher B, students are given opportunities to ask questions for better understanding the concept of arithmetic progression.

Content summary

Concept development

To "ensure learners develop understanding of algebraic concepts , concrete and semi concrete materials are very important in order to help learners to later use this foundation and link their conceptual understanding to abstract.

Concrete stage: In this stage, the teacher begins the lesson by modelling each mathematical concept with concrete materials. These materials are real objects that learners manipulate and discuss how to use them for better learning. Through manipulations, explorations, investigations and computing learners make patterns and number sequences using real materials like bottle tops, plastic bottles, flash cards, number grid on a rice sack or Manila paper...

Semi- concrete stage, visualization or representation: In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawings or pictures; charts…

Using representations, students are expected to explore patterns, investigate number sequences and compute to make new terms of number sequences.

Abstract stage: *In* this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number patterns and sequences.

Example 1: Pattern-seeking task – a real experience of Gauss

A Mathematics teacher gave the class the task to sum up the integers 1 to 100. Gauss, a mathematician, completed the task within a few seconds and said that the sum of the integers is 5050. How did Gauss get the answer in a few seconds?

Hint: Gauss wrote down 1 to 10 and paired the first and last number, that is, 1 + 10 = 11. There were five pairs, so 11 times 5 is 55. This means the sum of 1 to 10 integers is 55. Similarly, 1 + 100 = 101, 2 + 99 = 101, 3 + 98 = 101, ..., 50 + 51 = 101 was written. Since there are 50 pairs of numbers, each of which adds up to 101, the sum of all the numbers must be 50 × 101 = 5050. This technique provides another way of deriving the formula, namely 1 + 2 + 3 + ... + n = (n(n+1))/2 for the sum of the first 'n' positive integers. You need only display the consecutive integers 1 through n in two rows as follows:

1	2	3 ...	n-1	n
n	n-1	n-2 ...	2	1

Addition of the vertical columns produces n terms, each of which is equal to n+1; when these terms are added, we get the value n(n+1). Because the same sum is obtained on adding the two rows horizontally, what occurs is the formula n(n+1) = 2(1 + 2 + 3 + ... + n). Then dividing by 2 gives the actual formula to calculate the sum of integers which is $\frac{n(n+1)}{2}$.

Example 2: To verify that addition of two whole numbers is commutative and generate an algebraic expression

Using this representation of green and pink strips side by side on lines l_1 and l_2, as shown in Fig. 3, commutative property of addition is demonstrated and an algebraic expression is generated.

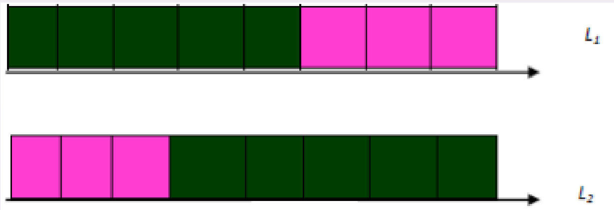


Fig.3

Observation:

From Fig. 3,

a. The length of the combined strips on line $l_1 = 5+3 = 8$

b. The length of the combined strips on line $l_2 = 3+5 = 8$

c. One can see that the length of combined strips on l_1 is the same as the length of the combined strips on l_2.

d. So, 5 + 3 = 3 + 5 i.e., addition of 5 and 3 is commutative

One can repeat this activity by taking different pairs of numbers like 2, 5; 4, 6; 5, 7 and strips corresponding to these pairs.

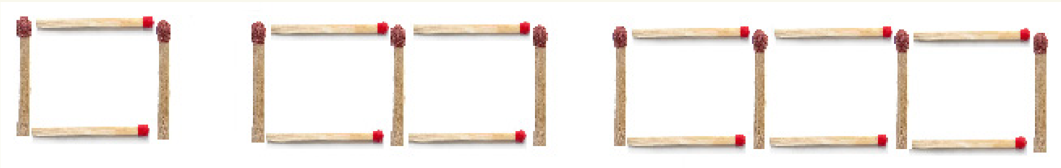
Conclusion: Addition of whole numbers is commutative, i.e., if $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two whole numbers, then $\boldsymbol{a + b = b + a}$.

Example 3: using patterns to generate terms of number sequences

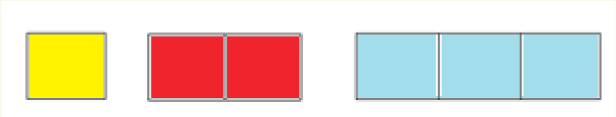
Use sticks to form the following pattern. Build the next two shapes, and generate the next two numbers of sticks

- Fill in the grid based on the models developed with matchsticks, only up to the fifth model.
- Predict the number of sticks needed to make 6 squares. Give a reason for your response.
- How many sticks are needed for 10 squares?
- Is there a possibility of having 13 sticks in any model? Give a reason for your response.
- Is there a possibility of having 100 sticks in any model? Give a reason for your response.

Concrete: real objects



Semi-concrete: drawings



Abstract: numbers									
No. of squares	1	2	3	4	5	6	7	10	13
No. of sticks	**4**	**7**	**10**	…..	……				

2. Teaching and learning strategies related to Algebraic concepts

In the teaching and learning of algebraic concepts, teachers use a repertoire of teaching approaches as there is no single suitable method to teach mathematics to emphasize mastery of knowledge and skills, critical and creative thinking, communication and problem solving. Mathematics teachers should adopt a combination of different teaching approaches to engage learners in learning. These approaches will help learners talk more openly, think more creatively and ultimately become more engaged in the process of learning.

Some of the common combinations of approaches are:

- Teacher exposition followed by learner practice
- Exploration and investigational activity followed by conjecturing or whole class discussion: learners may work independently, in pairs or in groups; learners may use technology or manipulatives to carry out their investigations.
- Practical activity followed by presentations and whole class discussion: learners could work independently, in pairs or in groups; learners may make a gallery walk presentation.
- Problem solving or mathematical modeling followed by presentation of the learners' work in groups.
- Brainstorming, questioning and answers followed by harmonization and making summary.

Application activity 7.3

Use the syllabus, textbooks and teachers guide of Mathematics for lower or upper primary and organize the teaching techniques you can use to make more active and participative the lesson on solving simple equations with 1 unknown in P3 or P6.

7.4. Activities for developing generic competences and activities that helps to integrate crosscutting issues in algebra lessons

Activity 7.4

Use the syllabus, textbooks and teachers guide of Mathematics for upper primary and organize the teaching techniques you can use to make more active and participative the lesson on solving simple equations with 1 unknown in P6.

Content summary

In lessons related to Algebra, different generic competences can be developed. However, the following generic competences can be developed in the activity 7.4:

1. **Critical thinking skills** are developed in solving problems that involving simple equations or inequalities, analysing and find out patterns and missing numbers in a number sequence.
2. **Communication skills** are developed through group discussions while working on number patterns, number sequences, solving problems that involving simple equations or inequalities.
3. **Cooperation skills** are developed through teamwork, working in pairs or in groups help learners to develop the spirit of working together and sharing ideas.
4. **Problem solving skills** are developed through applying the concepts learnt to the solution of different problems in real life.

In lessons related to algebra, different cross- cutting issues can be addressed.

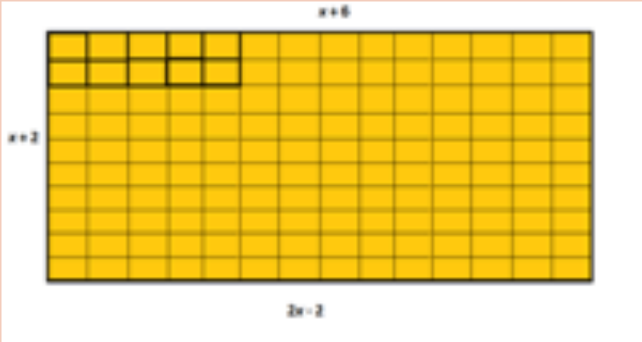
However, the following cross- cutting issues can be addressed in the activity 7.4:

- **Gender** is addressed through assigning the same roles for all males and females learners in teaching and learning process.
- **Inclusive education** is addressed through activities that should cater for different ability of learners, teaching and learning materials that are adapted to different special education needs, teaching techniques that are adapted to different ability of learners.
- **Peace and value education** is addressed through teaching techniques and strategies that are adapted to different attitudes of learners, activities that should develop positive values among learners.

Application activity 7.4

As student- teacher analyse the following activity and respond to the related questions.

Activity: In groups observe the diagram bellow and do the following questions:



a. Use the properties of rectangle to find the value of x.

b. Suppose that the 1st small square is made by 4 sticks, complete the following table:

No. of small squares	1	2	3	4	5	6	7	10
No. of sticks	4	7	10	…..	……			

Questions related to the activity

1. Identify and explain the competences to be developed while performing the above activity
2. Use the syllabus and find out the class or level the above activity can be given to.
3. Is there any cross-cutting issue that has been addressed? Explain your answer.

7.5. Assessment tasks related to lessons of algebra

Activity 7.5

As a student-teacher, set up an activity on algebra that helps primary learners to develop generic competences and address crosscutting issues. Explain how your activity develops generic competences and addresses crosscutting issues.

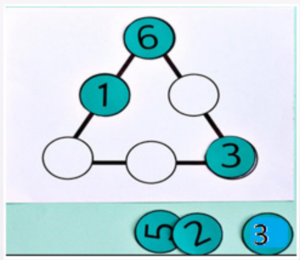
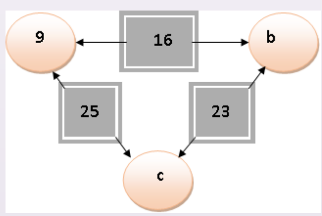
Content summary

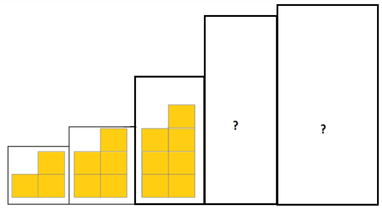
In the lesson related to algebra, Mathematics activities or tasks are developed or set using observable action verbs consistent with the level of learning expected, considering all the low, medium and higher order thinking skills and competences.

Examples of verbs used in setting tasks:

- **Low order** (knowledge and understanding): define, name, list, identify, label, match etc.
- **Medium Order:** explain, arrange , describe, examine, classify, express, summarize, calculate, relate, show, solve, use, etc
- **Higher order: c**ompare, analyse, illustrate, differentiate, compose, construct, design, formulate, evaluate, justify, generate and interpret.

Example: The table below shows example of how tasks on algebraic concepts should be formulated.

Assessment task	**Topic**	**Sample activity**	**Example**
Practical and oral task	Number patterns	Work in group and observe patterns before relating with corresponding numbers. Learners can find the missing number in an arithmetic or geometric progression and formulate their own examples They should also discuss about arithmetic or geometric progressions on flash cards and discover the clue/pattern. Learners may discuss and discover the missing numbers in an arithmagon(a polygon with numbers at its vertices which determine the numbers written on its edges) such that when they	1. Complete the magic triangle bellow using the numbers 2, 3 and 5. *Pattern to follow "subtracting two numbers on the vertices of a triangle you get a number in the middle".*  2. Find the value of b and c and explain your working steps 

		add, subtract, multiply or divide the numbers on 2 vertices the sum will be on the edge between those two vertices.	3. Discuss the following sequence: 25, 28, 31, 34, 37, ___. Discover the pattern used and find the next number. Form your own sequences with constant differences. Then make a presentation to the class.
Visualization and representation tasks	Sequences that include whole Numbers, fractions or decimals.	In group work, learners should discuss and discover the pattern used in sequences which involve whole numbers, fractions or decimals numbers using representations on pictures or numbers on flash cards. Learners should use a number line or patterns to determine the sequence of numbers.	Observe the picture and do the related questions.  Considering the number of squares, deduce the pattern and the number sequence from the picture. Generate the 2 next numbers Considering the sides of squares, deduce the pattern and the number sequence from the picture. Generate the 2 next numbers

Application activity 7.5

Refer to the activities in the Mathematics textbook of P2, P3, P4, P5 or P6 and set 3 activities related to missing number in addition or in a number sequence.

- In the first activity use action verb that can develop low order thinking skills,
- In the second activity use action verb that can develop medium order thinking skills,
- In the third activity use action verb that can develop high order thinking skills.

7.6. Writing a lesson plan on algebraic topics

Activity 7.6

As a P4 Mathematics teacher, you are supposed to teach the lesson on arithmetic sequences to 46 students. Examine the example of the lesson below, and do the following:

a. Analyze the instructional objective by showing its 5 main components,

b. Select all teaching materials to be used in the lesson,

c. Explain if the suggested learners' activities are engaging, practical and can easily develop the concept of arithmetic progression.

d. Identify generic competences to be developed and crosscutting issues to be addressed depending on the set activity and the step of the lesson.

e. Summarize the teaching and learning techniques to be followed in this lesson.

Example of a lesson plan on arithmetic and geometric sequence

School Name: .... **Teacher's names:.....................**

Term	Date	Subject	Class	Unit Nº	**Lesson Nº**	**Duration**	**Class size**
....	....	Mathematics	P4	12	....	40 mins	46
Type of Special Educational Needs to be catered for in this lesson and number of learners in each category				2 learners with visual difficulties (low vision).			

Unit title	**Number patterns**
Key Unit Competence	**To be able to describe and generate number patterns following a rule**
Title of the lesson	**Arithmetic sequence**
Instructional Objective	Given numbers on flashcards, learners will be able to order them following arithmetic pattern or rule correctly and confidently in a given time
Plan for this Class (location: in / outside)	Arrange desks in class in order to allow learners to work individually, in pair or in groups
Learning Materials **(for all learners)**	Manila papers, markers or pens and scissors to prepare number cards, flash cards.
References	Primary learners Mathematics P4, Kigali January 2019

<table>
<tr><td rowspan="2">Timing for each step</td><td colspan="2">Description of teaching and learning activities

Learners do an introductory activity individually. They discuss in pairs or in small groups the discovery/ presentation activities, followed by the group presentation through gallery walk, interaction of learners and harmonization of the results under the facilitation of the teacher.

Finally, learners are assigned individual tasks for applications and assessment and the correction is done on the chalk board, in pairs and in small groups and then the correction is done on the chalk under the guidance of the teacher.</td><td rowspan="2">Generic competences

and Cross cutting issues to be addressed + a short explanation</td></tr>
<tr><td>Teacher activities</td><td>Learner activities</td></tr>
<tr><td>Introduction

(5min)</td><td>Facilitate learners to do a mental Math on counting in fifties</td><td>Individually learners count in fifties up to 500 and try to find out the difference between two consecutive numbers:
<table><tr><td>50</td><td>100</td><td>150</td><td>200</td><td>250</td></tr><tr><td>300</td><td>350</td><td>400</td><td>450</td><td>500</td></tr></table></td><td>Communication skills are developed through counting and pronunciation of numbers in words.</td></tr>
<tr><td>development of the lesson

Discovery activities

(10min)</td><td>Organizes learners into groups and ask them to do activity 1 and gives instructions related to activity</td><td>Activity 1: concept of arithmetic progression or sequence

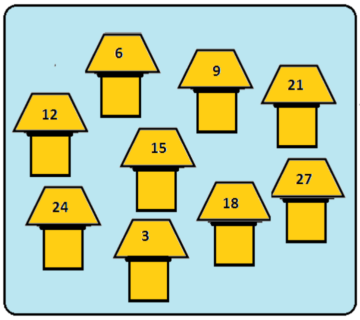
In groups, arrange plastic bottles in a consecutive order by adding two bottles (starting by 1) until you get the number 7

Learners observe that the number of plastic bottles increase of 2 bottles from one to another and the number sequence is as follow: 1 3 , 5, 7.
Activity 2:concept of pattern In pairs, learners observe the sequence (1, 3, 5, 7) from activity 1 and deduce the pattern used for getting the next number and predict the next 2 numbers after 7.</td><td>Cooperation and peace education values are developed through working in groups and respecting different views form colleagues

Critical thinking skills are developed through analysis of different terms of an arithmetic progression, finding the pattern and the next numbers of the sequence.</td></tr>
</table>

Presentation of students' findings and exploitation: (10 min) Summary (5 min) Application (5 min)		Learners use the pattern "increase **of 2 bottles**" and find out that the 2 next numbers after 7 are 9 and 11and then the new number sequence is 1, 3, 5, 7, 9, 11.	Gender equality is addressed through assigning both to girls and boys the same tasks in the whole lesson. Inclusive education is addressed through using plastic bottles, number cards or flash cards to facilitate low vision learners
	Invites groups to post their findings in different corners of the classroom and ask them to present. Takes notes on key points from learners' presentations	The reporters present their findings on behalf of the groups while other learners follow the presentations and ask questions for better understanding.	Communication skills are developed through presentations and answering questions.
		.	

	Facilitates learners to elaborate the summary from the presentation Asks learners to individually work out the application	Learners write down the main points in their notebooks An arithmetic progression/ sequence is a number sequence in which the difference between every two consecutive numbers in the sequence is the same. That difference is called "a pattern". e.g. for the sequence 2, 6, 10, 14, the pattern is 4 since 6-2=4, 10-6=4, and 14-10=4 Individually learners work out the application activity and finally, they make a correction on the chalk board	Critical thinking skills are developed through analysis of different terms of a progression, finding the pattern and define an arithmetic progression. Problem solving skills are developed through
		Activities: A display of cans on a grocery shelf consists of 20 cans on the bottom, 18 cans in the next row, and so on in an arithmetic progression until the top row has 4 cans. Find out all terms of the sequence and write the sequence in ascending order. How many cans in total are in the display? Find the total price of all cans if the unit price is 600 Frw.	application of arithmetic progression in real life situations. Financial education is addressed through business: calculating the total price of all cans

Conclusion (5min)	Write assessment questions on the chalk board and ask learners to do them	Individually learners do the assessment activity. **Activity:** Order the following numbers on flash cards to make an arithmetic progression.  Find out the pattern and the 4 next numbers after 27.	Inclusive education developed through using flash cards to help low vision students
Teacher self-evaluation	To be completed after receiving the feed-back from the learners (what did the learners liked, what challenged them...)		

Application activity 7.6

As a student-teacher, refer to the lesson plan above and prepare another one with different title, more engaging, more active and inclusive.

End unit assessment 7

As a P6 Mathematics teacher, you are supposed to teach the lesson on simple algebraic inequalities with one unknown.

Refer to the syllabus, appropriate Mathematics books for P6 and do the following:

a. Set the instructional objectives of the lesson;

b. Organize teaching materials to be used;

c. Determine the generic competences to be developed in each step of the lesson and highlight crosscutting issues to be addressed in this lesson.

d. Using answers in a, b and c, prepare a lesson plan illustrating step-by-step all procedures and components of a competent mathematics lesson.

Unit 8 LESSONS RELATED TO GEOMETRY

Key unit competence: **Prepare active lessons related to geometry**

Introductory activity 8

In primary school, the understanding of geometric concepts means to develop some skills of observation and description of the geometric figures and solids, namely to form at the young children some active representations of imaginative bases. This becomes the key of using critical thinking in helping the young children to create, reproduce, exercise the ability to think and to use various ways of transforming learning in a useful and enjoyable process.

Refer to the syllabus and textbooks for lower or upper primary mathematics and point out geometry related concepts to be studied.

For each topic, try to brainstorm and summarize how to produce appropriate resources for facilitating lessons using low cost materials.

Explain whether teaching techniques to facilitate lessons on the types of lines are the same as those to be used in the lessons related to the types of angles.

8.1.Recall on concepts related to geometry taught in primary levels

Activity 8.1

Use Mathematics syllabus, textbooks and the teacher's guides for lower or upper primary and do the following:

a. Select different concepts related to geometry (geometrical concepts);

b. Among the selected topics which do you find are challenging to you?

c. Make a self-study on the challenging topics. You can ask for a help from your colleague or your tutor.

Content summary

The fundamental geometrical concepts taught in primary school depend on 4 basic concepts: point, line, plane and space. However, these terms will not be precisely defined; their meanings will be explained through examples of their elements or subsets. This is the reason why learners will have to develop:

- Understanding of the types of lines and angles and use a protractor to measure angles (classify lines, angles)
- Procedural skills for spatial visualisation of angles, lines, 2D geometrical shapes and area,3D geometrical shapes and the volume of cuboids;
- Estimation of the measurement of angle before measuring by the use geometrical tools;
- Confidently use of technologies such as calculator, XO Laptop in drawing lines, angles, geometric shapes for exploration and problem solving.

Application activity 8.1

Use a primary Mathematics syllabus and books, select one lesson on the types of lines, one lesson on angles formed by intersecting lines and one lesson on the determination of the volume for a cylinder.

For each lesson, highlight the skills to be acquired by learners.

8.2. Production and organization of appropriate resources for the lessons related geometry

Activity 8.2

The use of manipulatives (concrete materials), practical work and use of technological aids should be part of the learning experiences of the learners in geometry.

By the use of the syllabus and textbook for lower or upper primary mathematics choose one lesson on geometry (e.g. lesson on types of angles).

1. Organize appropriate teaching resources for your choice and explain how those resources can be used while teaching and learning the selected topic (e.g. lesson on types of angles).
2. Some teaching resources of the geometric concepts can be produced using local materials, explain how you can produce them.

Content summary

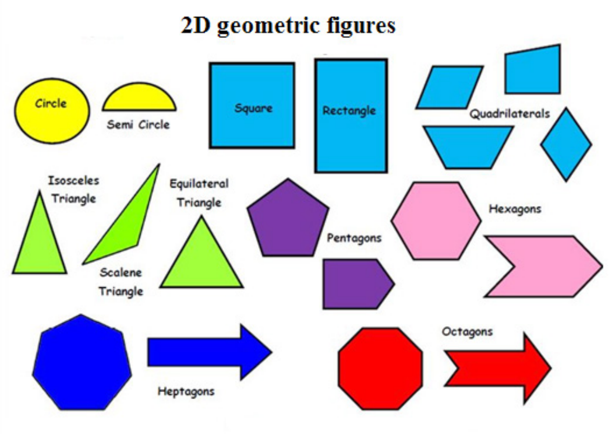
Teaching and learning resources for geometry lessons can be acquired by producing or constructing them, by collecting them from the environment, by asking learners to prepare them, by requesting the school administration or parent to buy them, by borrowing or hiring them or through improvisation. But also ICT Materials can be used for enhancing the teaching and learning lessons related to geometry.

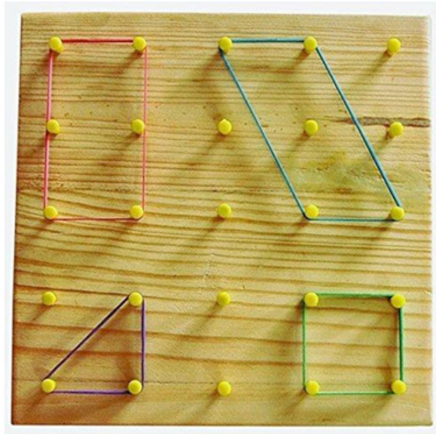
Some local materials can be collected and used for improvisation on lessons related to geometry:

- **a. Sticks**: when constructing geometric shapes like triangles, squares, rectangle, pentagon, hexagon, etc… sticks used by joining them according to the shape needed then used in teaching and learning geometric shapes. Fold the stick for having a form of circle.
- b. Use of **scissors** and sheets of **paper** to build **boxes** of 3D shapes and calculate the area of formed shape.
- **c. Plastic bottles:** with the use of plastic bottles, learners cut them and form different shapes like cone, cylinder
- **d. Manila paper or rice sack: D**raw different shapes with their formulas and properties on manila paper which used during teaching and learning
- **e. Sheet of Paper:** by folding paper and cut edges, continue folding paper in many times by cutting the edges you will get the polygon of n sides which tends to be a circle. This is a good kind of low cost materials for teaching different polygons.
- f. Wood, cartons, plastic papers can be used to demonstrate properties of 2D shapes or 3D solids. Nets of solids are very important tools for calculating surface area of solids.

Examples of teaching aids:

1. Wall charts of 2D geometric figures can be bought by the school or made by the teacher:





2. Wooden 3-D geometric Solids

ICT Tools in teaching Geometry

With computers, we can create not only narrative animations but also interactive animations, allowing the viewer to make her own explorations. Even at the level of 2D geometric diagrams, modern interactive packages have led to new insights into elementary Euclidean geometry. Drawing accurate images of 3D geometric objects by hand requires care and training in perspective. But these are easy to create with computer graphics, and it is trivial even to render a pair of images for stereoscopic viewing with the left and right eye.

The following ICT tools are very essential in the process of teaching and learning geometry and help in practical work with manipulations in primary schools:

1. **XO Laptop programs:** the XO laptops has different programs like scratch activities where learners draw different geometric forms. In Turtle activities leaners draw different geometric forms

like rectangle, circle, triangle and then they can calculate the area, perimeter etc…

2. **Computers** : are used for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangle, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.by the use of different application software also leaners learn how to solve word problems involving geometry with the application of those software and computer.
3. **Calculator:** in calculation both teacher and learners need to calculate solution in numerical and give answers with the use of calculator.
4. **Different mathematics software** such as Geogebra can be used to construct geometric figures.

Application activity 8.2

In the teacher resource center (TRC), use low cost materials (rice sack, Manila paper, plastic bottles) and produce materials related to the teaching and learning of surface area of cuboids and explain how they can be used in lower or upper primary Mathematics lessons.

8.3. Active teaching techniques and strategies for facilitating lessons related to geometry

Activity 8.3

One of the problems most of teachers encountered in facilitating lessons related to geometry is to make the lesson fun for learners so that they participate actively in teaching and learning.

1. What can you do to make the lesson in P3on "Types of angles" enjoyable?
2. Discuss active teaching techniques and strategies you think are the best for facilitating these lessons: the lesson on "area of a triangle" in P4 and the lesson on "angle sum of the triangle" in P5.

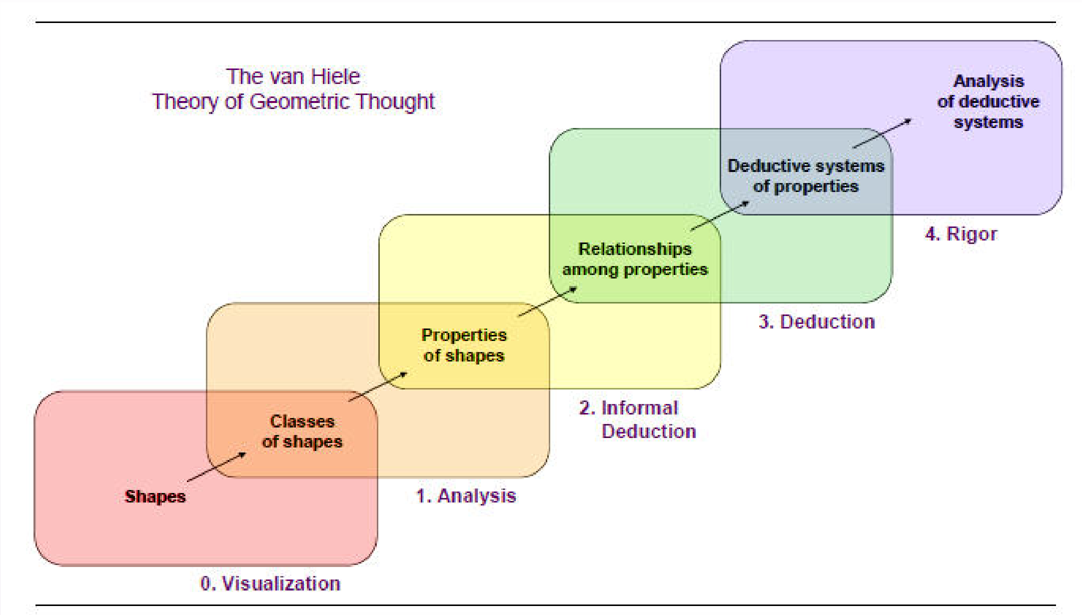
Content summary

1. Geometric concept development

To "ensure learners develop competences (tangible understanding of geometrical concepts and skills), it is necessary to use the *concrete level of understanding* of concepts and skills, so that learners are able to later *use this foundation and link their conceptual understanding to abstract*. After this, learners are exposed to geometry related problems of their real life experience. Note that children need to have many experiences in which they are actively involved in exploring and communicating about their observations of shapes, properties and relationships.

The van Hiele levels of geometric thoughts

In Mathematics Education, the Van Hiele model is a theory developed by Van Hiele in 1986 that describes how generally learners learn geometry. There are five levels, which are sequential and hierarchical. They are: Visualization, Analysis, Informal Deduction, Deduction, and Rigor levels



Source: *Van Hiele, P. M (1986)*

Level	Description
0-Visualisation	Children recognize shapes by their global, holistic appearance

1-Analysis	Children observe the component parts of figures (e.g. a parallelogram has opposite sides that are parallel) but are unable to explain the relationships between properties within a shape or among shapes
2-Informal deduction	Children deduce properties of figures and express interrelationships both within and between figures
3-Formal deduction	Children create formal deductive proofs
4-Rigor	Children rigorously compare different axiomatic systems

To move from one level to the next, children need to have many experiences in which they are actively involved in exploring and communicating about their observations of shapes, properties and relationships.

Particularly, the teaching and learning of geometry in primary school respects the three main steps to be used to create intellectual competence in logical reasoning, spatial visualization, analysis and abstract cognitive skills.

Concrete stage: Teacher begins the lesson helping learners to model each geometrical concept with concrete materials. The use of concrete objects helps learners identify shapes and figures according to their concrete examples at 'visual level for learners. The students manipulate and discuss how to use them for better learning. With real materials like rulers, T-square for a carpenter, sticks, bottles and boxes, learners can construct geometric shapes or figures themselves and the teacher harmonize their work.

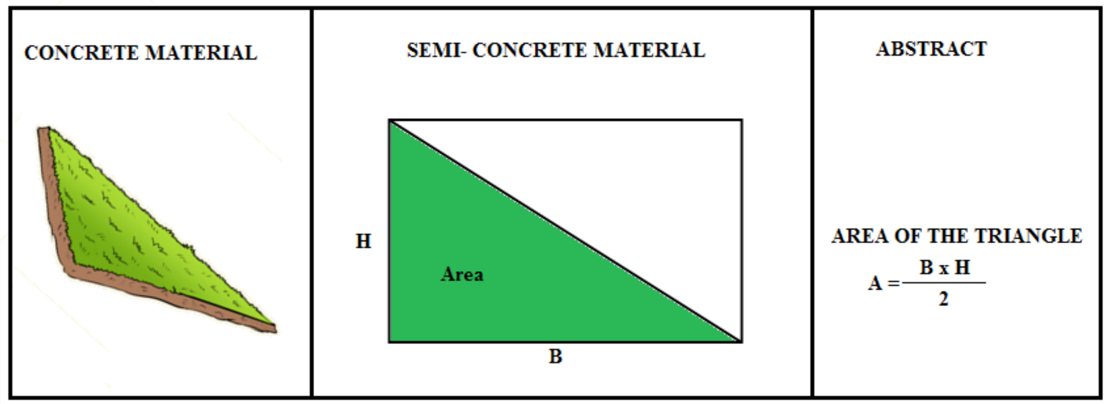
Semi- concrete stage, visualization or representation: Students transforms the concrete model into a representational (semi-concrete) level guided by the teacher. This may involve drawings; where students have to represent the geometric shapes by drawing with using lines. This corresponds to the descriptive level where learners identify shapes according to their properties and can for example describe a rhombus as a figure with four equal sides.

Abstract stage: *T*he teacher guide learners to model the geometric concept at a symbolic level, using only numbers, notation, and mathematical symbols where they can write the formula of perimeter, area and volume. They can identify relationships between classes of figures for example that a square is a special form of rectangle and can discover properties of classes of figures by simple logical deduction.

In advanced classes, there is the fourth or formal level, where learners can produce a short sequence of statements to logically justify a conclusion and can understand that deduction is the method of establishing geometric truth. For example, an understanding develops about interaction of necessary and sufficient conditions. As learners are used to geometry, they can understand and visualize geometry in abstract and can also compare systems based on different axioms and work with them.

As cognitive science suggests, we learn to see; we create from what we see; visual reasoning or 'seeing to think' is learned, it can also be taught and it is important to teach it. Teachers who have learned and became skillful in the use of visualization and 'seeing to think' would be able to reinforce mathematical concepts and improve the learning process in the classroom.

Example 1: Illustration of area of the triangle



Teaching and learning strategies related to geometry

Teachers use a variety of teaching approaches in teaching lessons related to geometry to emphasize mastery of knowledge and skills, critical and creative thinking, communication and problem solving. In the geometry lessons, use strategies that engage learners in learning. These strategies will help learners talk more openly, do more practical work, think more creatively and ultimately become more engaged in the process of learning lessons related to geometry.

Some common approaches of teaching geometry in primary are the following:

- **Practical work** to get learners excited about geometry and significantly to enhance their learning by making more practice with manipulations, evaluating formulas, etc. Continue assess learners throughout different activities in class or outside of the classroom.
- **Innovative strategies** that will have a positive influence on learners' attitudes about learning geometry.

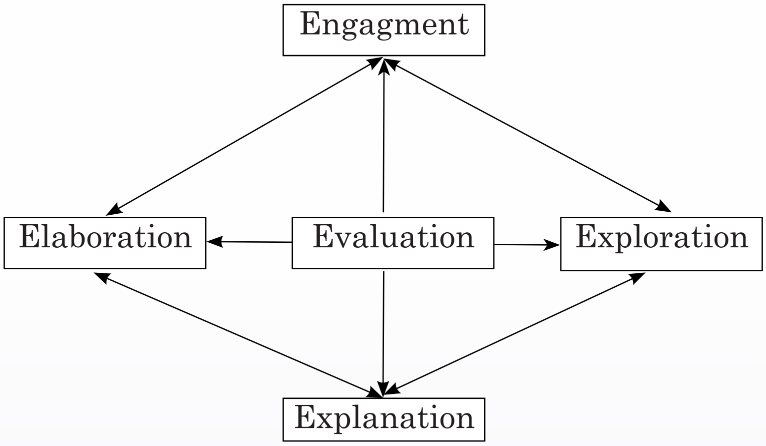
- **Investigational activity** followed by conjecturing or whole class discussion: learners may work independently, in pairs or in groups; learners may use technology or manipulatives to carry out their investigations.
- **Through outdoors activities** learners involved in making more practice related to the lesson on geometry by drawing, measuring the area, perimeter and volume of such given objects or materials. Learners go outsides of the classroom to search materials like sticks, objects in different forms and come back in the classroom for using them to construct or form geometric shapes form those materials.
- **Problem solving or mathematical modeling** followed by presentation of work by learners in groups.
- **Independent learning during e-learning**: learners may do on-line tutorials at home and submit the work to teacher electronically.
- **Peer teaching activities**: peer instruction encompasses a range of scenarios where learners instruct skills or explain concepts to classmates.
- **Reciprocal questioning:** Use **reciprocal questioning (Question and answer)** to encourage an open discussion in which learners take on the role of the teacher and createtheir own questions about a topic related to geometry and find out the answers together. You can divide the class into pairs or small groups and have learners come up with a few questions related to geometry for discussion with the rest of the class.

Abraham's (1997) learning cycle developed for Science and Mathematics teaching is found useful in investigative teaching of concepts in geometry and indeed in other areas of Mathematics.

The approach, which derives from the cognitivist and constructivist position on teaching and knowledge-seeking, divides instruction into four progressive stages. Step one is engage, where the teacher creates an enabling environment to engage learners in activities that generate curiosity and interest in the planned topic of the day. Usually, an inquiry question is presented to the learners at this stage.

The second stage is explore, where learners explore the question(s) raised at the engage stage and generate answers. At this stage, the learners are placed in groups, and the teacher acts as a facilitator and usually asks further questions to guide learners' explorations, and provides hints about how to proceed, without showing learners "exactly how to go about solving the problem" Usually, this stage is characterized by a series of questions and introductory activities that are similar to the topic presented in their worksheets.

Then comes the third stage, explain, where opportunities are provided for learner groups to present solutions or answers to the inquiry question(s), giving justifications and explanations for their claims. And then comes the last stage, extend, where learners extend their concepts and skills to other situations by applying what they have learned in the explain stage. Usually, and particularly so in mathematics, further tasks in which these skills can be exhibited are provided for the learners. It could be in groups, pairs, or individually. At each of these stages, and indeed at the end, evaluation of the process would go on simultaneously.



This emphasis represents an important merger between mathematics as an investigation and mathematics as a body of competences, where learners develop competences through investigation and experimentation in order to facilitate verbalizing, understanding and applying principles in the real world".

Application activity 8.3

Use the syllabus, textbooks and teachers guide of Mathematics for lower or upper primary and organize the teaching techniques you can use to make more active and participative the lesson on area of a triangle in P3 or P4.

8.4. Activities for developing generic competences and integration of cross-cutting issues in the lessons related to geometry

Activity 8.4

Read the following activity carefully, analyse it and answer the questions.

Activity: Use a protractor and a ruler, draw a right angle.

a. Divide the angle into two angles and explain clearly your working step to the class members.

b. Write down the size of each angle.

c. Find out the sum of the two angles.

d. The angle 1 and 2 are complementary angles, deduce the definition of complementary angles.

1. If you have one protractor in class. Explain how you will handle the problem.
2. Identify and explain the competences to be developed while performing the activity in (a), (b), (c) and (d)?
3. Is there any cross-cutting issue that has been addressed? Explain your answer.

Content summary

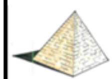
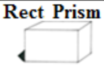
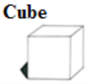
In lessons related to geometry, different generic competences can be developed. However, the following generic competences can be developed in the activity 8.4:

1. **Communication**: through presentations and the explanation made in the process of constructing figures, drawings and comparing the size of shapes, learners develop the communication skills.
2. **Cooperation**: through working in pairs and groups and sharing ideas when they are constructing geometric shapes learners develop the spirit of working together.
3. **Critical thinking**: through comparing the types of lines, quadrilaterals, angles as well as their properties, learners develop the critical thinking skills.

4. **Problem solving**: through applying the skills learnt in the concepts of geometry in the process of solving different problems involving geometric concepts in the real life of the learner, there is a development of the problem solving skills.

The following are examples of activities for developing competences:

- **Categorizing**: Analyze the geometric figure and determine the number of edges, vertices, and number of sides, number of angle, the area and the volume.

	EDGES	VERTICES	FACES	LOOKS LIKE
Cone 	0	1	1	
Cylinder 	0	0	2	
Pyramid 	5	8	5	
Rect Prism 	12	8	6	
Cube 	12	8	6	
Sphere 	0	0	0	

- **Exploring**: Each group of learners will draw any triangle; they use protractors to measure the amplitude for each angle of their triangle. They are asked to make the sum for the measures they find in their respective groups. Then all groups discuss the total sum of three angles for their triangle and they will deduce the rule that the sum of three angles in each triangle always add up to 180 degrees.

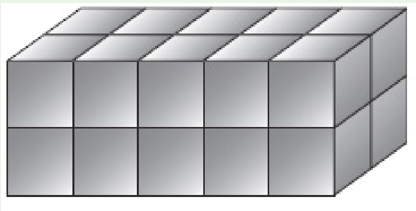
In lessons related to geometry, different cross- cutting issues can be addressed. However, the following cross- cutting issues can be addressed in the activity 8.4:

1. **Peace and value education:** through the good positive attitude of sharing geometric materials in the classroom, learners show trust, empathy and honest to each other while solving problems.
2. **Gender** is addressed through group activity where boys and girls take similar responsibilities in working

3. **Inclusive education** addressed through activities that should cater for different ability of learners, teaching and learning materials that are adapted to meet special educational needs and teaching techniques that are adapted to different ability of learners.
4. Standardization culture: By searching materials for facilitating lessons related to geometry, printed or non-printed, visual or audio-visuals, learners can be sensitized to try to make adequate materials that meet high standards.
5. Financial education: through different problems for calculating the price for the lands basing on their area or the price for a liquid in the container basing on the volume of that container, learners will calculate cost and realize the value of money, their use and saving.

Example of activity developing competence that address a financial education as a cross-cutting issue:

Activity: Work in group and do the following activity:



Observe this figure that represents a big transparent carton containing other small boxes with pens.

a. How many boxes do you have at the base of the big carton?

b. How many boxes are on one vertical edge?

c. Count the number of all small boxes in the big catron.

d. Calculate the number of all boxes (the volume) of the carton using the findings in (a) and (b).

e. If one box contains 50 pens, how many pens are contained in the carton?

f. Considering that one pen costs 50 Frw, what is the money the teacher should have to buy all those pens for your classroom? What can do a P4 learner to have such amount of money? What can you do with such amount of money?

Application activity 8.4

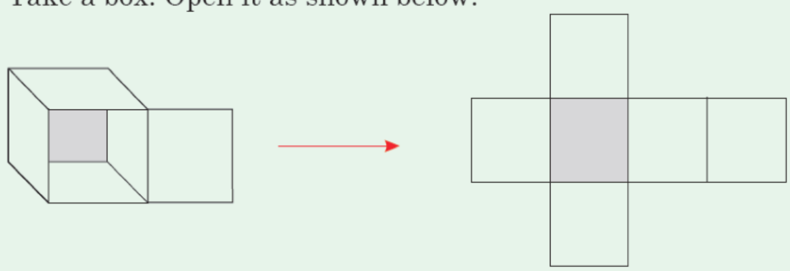
As a student-teacher, set up an activity on geometry that helps primary learners to develop generic competences and address crosscutting issues. Explain how your activity develops generic competences and addresses crosscutting issues.

8.5. Assessment tasks related to lessons of geometry

Activity 8.5

This activity is from one P5 Mathematics book for learners:

- Take a box. Open it as shown below.



- The completely opened shape of the box is called a net. To make a cube or a cuboid, we must first prepare a '**net**'.
- Open other cubes and cuboids to form different nets.
- How many faces are in each net?
- Fold a net to form a cube or cuboid.
- Fold nets to form a cube or cuboid. Present your model.

Analyse the activity and explain whether the action verbs used develop low, medium or high order thinking skills.

Content Summary

When learning activities and assessment tasks are well structured, learning of lessons related to geometry become more effective and learners participate actively in the process of teaching and learning. One of the solutions of avoiding failure in Mathematics lessons especially lessons related to geometry is to enhance appropriate activities/ tasks that are age appropriate and which intend to develop high order thinking skills and problem solving. Activities/ tasks must be set reference to action verbs by considering the low, medium and higher order thinking skills and competences.

The table below shows examples of how tasks on geometry should be formulated

Assessment task	Topic	Sample activity/ task
Practical and task	Types of lines and angles.	– Work in pairs to explore how to construct perpendicular and parallel lines using a T-square and protractor, and to check whether two given straight lines are perpendicular or parallel, and explore how those lines can be drawn on in their notebooks with square grids.
Visualization and representation tasks	Measuring angle	– Find angles in the environment and use a paper right angle to identify right angles, angles greater than a right angle and angles smaller than a right angle. – Estimate before measuring angles and use a protractor and a ruler to draw angles, and divide the angle into two angles and Write down the size of each angle. – In group, Measure the sides of the given triangles, Use the protractor to measure the angles of these triangles, Name these triangles, Each group present the work to the whole class. – Associate the amount of a turning (rotation), clockwise or anticlockwise, with an angle measured in degrees. – Sketch and draw different triangles according to given angles and lengths using ruler, protractor and T-square. – Each group of learners will draw any triangle; they use protractors to measure the amplitude for each angle of their triangle. They make the sum for the measures they find in their respective groups, All groups discuss the total sum of three angles for their triangle and they will deduce the rule that the sum of three angles in each triangle always add up to 180 degrees. – Make collection of quadrilaterals(4-sided figures) from pictures and photographs, and identify the various special quadrilaterals besides squares and rectangles;

		– Sketch and draw special quadrilaterals according to given angles and lengths using geometric materials.
	Area of 2D shapes	– Describe properties of rectangles and squares in terms of perpendicular and parallel lines, and discuss the relationship between a square and a rectangle; – Sketch and draw rectangles and squares according to given lengths using a ruler, protractor and a T-square. – Draw cutout squares of different sizes and commit to memory the areas of the squares, – Make composite figure using cutouts of rectangle and squares or draw the figure on a square grid paper and calculate its area and perimeter. – Fold any rectangle in half along its diagonal for having a triangle, measure the lengths of triangle for calculating its area; – Calculate the area of square and rectangle drawn on square grid by counting and by formula to compare answers. – Estimate the area of a rectangular or squared land and compare the sizes of rectangles / squares using area. – Apply multiplication and division concepts to find one dimension of a rectangle given its area/ perimeter and the other dimension; – Visualize how a L-shaped figure can be partitioned into rectangles and squares or be formed by removing a rectangle/square from a bigger rectangle/square, and calculate the area and perimeter from given lengths; – Work in groups to determine the basic shapes (rectangles squares and triangles) that make up composite figure, or use basic shape cutouts to form different composite figures.

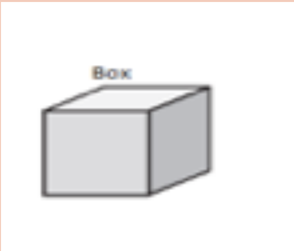
	Volume of cuboids	– Use unit cubes to build different solids (3D figure) and express their volumes in cubic units; – Compare sizes of solids in terms of their volumes, make connection between unit squares and unit cubes, – Work in groups to draw a cube or a cuboids taking into consideration size and orientation.

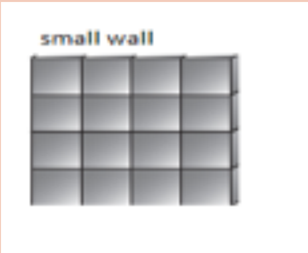
Application activity 8.5

The following activity is given by primary Mathematics teacher for learners. Analyse it and answer to the questions that follows:

Activity:

Take a protractor, metre and measure the four angles of the box and its surface area of one side then after go outside of the classroom to measure the angle and surface area of the small wall constructed with blocks.





You are asked to:

a. Measure the angles of box and 2 angles of wall

b. Determine the Area of one side of box and area of small wall.

Questions:

1. Describe the skills to be developed by learners in that activity
2. Is this activity appropriate to P3 or P4 learners in Rwanda in the lessons related to geometry? Explain your answer.

8.6. Lesson plan on a concept related to geometry

Activity 8.6

Suppose that you are Mathematics teacher of 45 learners in primary school and you are going to teach the lesson on “Perimeter or area of a rectangle”. Refer to the syllabus of Mathematics of lower primary P3 or upper primary P4, appropriate books, and the lesson planning procedures learnt in Foundation of education subject and do the following:

c. Set the instructional objectives (with its 5 main components) of the lesson

d. Organize teaching materials to be used on the lesson (Area of rectangle)

e. Formulate teacher’s activities and learners’ activities depending on the step of the lesson

f. Devise generic competences to be developed and crosscutting issues to be addressed depending on the step of the lesson

g. Summarize the teaching and learning techniques to be followed in this lesson.

Application activity 8.6

Read carefully the lesson plan on “concept of triangles and quadrilaterals”, analyze and improve it in order to make the lesson plan a competent based one.

Look for the suggested teaching and learning materials, active techniques, and how crosscutting issues are addressed and improve them to make the lesson more engaging, practical and enjoyable.

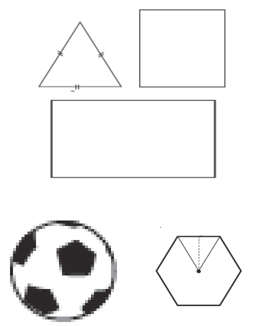
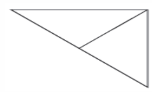
Example of a lesson plan on geometry

School Name: X Primary School.............. **Teacher's name**: ....

<table>
<tr><th>Term</th><th>Date</th><th>Subject</th><th>Class</th><th>Unit N°</th><th>Lesson N°</th><th>Duration</th><th>Class size</th></tr>
<tr><td>1</td><td>27 /05/ 2019</td><td>Mathematics</td><td>P4</td><td>6</td><td>1 of 6</td><td>8h00-8h40</td><td>40 learners</td></tr>
<tr><td colspan="4">Type of Special Educational Needs to be catered for in this lesson and number of learners in each category</td><td colspan="4">One learner with hearing impairment will sit near the chalkboard</td></tr>
<tr><td>Unit title</td><td colspan="7">2D Shapes and properties</td></tr>
<tr><td>Key Unit Competence:</td><td colspan="7">To be able to use geometric properties, including symmetry and to sort shapes</td></tr>
<tr><td>Title of the lesson</td><td colspan="7">Concept of triangles and quadrilaterals</td></tr>
<tr><td>Instructional Objective</td><td colspan="7">Given different shapes drawn on manila paper, learners will be able to classify each shape into the appropriate classification as triangle or quadrilaterals confidently and accurately in a given time.</td></tr>
<tr><td>Plan for this Class (location: in / outside)</td><td colspan="7">In the classroom, arrange the desks so that learners can work individually, in pairs or in small groups.</td></tr>
<tr><td>Learning Materials (for all learners)</td><td colspan="7">Manila paper on which different triangle and quadrilaterals shapes are drawn, ruler, protractor, cards shapes, computer, scissors and papers for making shapes</td></tr>
<tr><td>References</td><td colspan="7">Mwungeri E. and al.2008. Mathematics learner's book. Primary four. NCDC Rwanda.</td></tr>
</table>

Steps and Timing	Description of teaching and learning activity In groups learners use different shapes and do classification basing on the number of sides and angles.		Competences and Cross-Cutting Issues to be addressed
	Teacher's activities	**Learners activities**	
Review (5 minutes)	Ask learners to use their arms or legs to make different angles and they said the formed angle.	Learners make an angle by their arms or legs Learners say the name of the made.	Through making an angle by using arms or legs, creativity developed and learners show the curiosity of innovation.
Development of the lesson **Discovery activities:** **(15 minutes)** **Synthesis and summary:** **(5 minutes)**	Distribute materials to learners including papers and rulers. Ask learners to fold the paper then cut into n-times for getting different shapes and ask them the types of shapes they got	**Activity1: classification of shapes according to their edges/sides** In pairs, learners work on the task given by the teacher: After folding and cutting the paper into shapes they have to classify the geometric shapes according to the number of edges or sides they have. Learners conclude that the geometric shapes which have 3 sides are triangles while the geometric with 4 sides are quadrilaterals **Activity2: classification of shapes according to the number of sides and angles.**	Learners develop communication through working in pairs where they discuss on given task. Cooperation developed through working together in groups. Peace and value addressed when all learners share ideas in a peaceful way. Gender addressed when both girls and boys working together in groups. Inclusive education addressed in classroom by encouraging all learners to work together including those with special education needs.

		In small group learners classify the shapes according to the number of sides and angles.	
		Learners conclude that the geometric shapes which have 3 sides and 3 anglesare triangles while the geometric shapes which has 4 sides and 4 angles are quadrilaterals.	
Application: (5 minutes)	Facilitate learners to draw conclusion on triangles and quadrilaterals	The triangle is a three-sided figure with three angles in its interior or inside	With high level of summarizing the lesson learners develop critical thinking
		A quadrilateral is a four sided polygon with four angles. There are many kinds of quadrilaterals like squares where all sides are equal, rectangles where two parallel sides are equal and four equal angles	Through making summary and synthesis learners show the curiosity of problem solving and continue research.
	In pairs, learners are given materials to use such as ruler and papers. Ask them to draw geometric shapes.	In pairs, Learners use different materials such as pens, pencils, papers, rulers draw three triangles and two quadrilaterals of their choice.	When learners apply their knowledge in producing a new knowledge, they develop research and problem solving skills.

Conclusion (10 minutes)	**Assessment** Write assessment questions on chalkboard or distributes worksheets Ask learners to answer individually the questions.	Classify the following shapes as triangle or quadrilaterals: 	In the classification of triangles and quadrilaterals, learners develop a high level of critical thinking. Through taking initiatives to continue update the knowledge on triangles and quadrilaterals they develop research and problem solving skills.
		 In the classroom identify objects which are in the shapes of triangles and quadrilaterals.	
	Homework Write homework questions on the chalkboard	Select objects in the form of triangles and quadrilaterals then draw them on piece f paper. At home learners identify the objects which are triangles or quadrilaterals shaped in their home or garden and draw them on paper	
Teacher self-evaluation	The objective of the lesson has been achieved and learners have been interested by the use of coloured shapes, shapes with different size in classification of quadrilaterals and triangles.		

End unit assessment 8

Suppose that you are P3 or P6 Mathematics teacher and you are going to teach the lesson on “the use of cuts out to determine the perimeter /or the area of a composite figure”.

Refer to the syllabus of Mathematics of lower/or upper primary, appropriate books and the lesson planning procedures learnt in Foundation of education subject and do the following:

a. Set the instructional objectives (with its 5 main components) of the lesson;

b. Organize teaching materials to be used on the lesson (perimeter or area of a composite figure);

c. Formulate teacher’s activities and learners’ activities depending on the step of the lesson;

d. Devise a lesson plan that can facilitate a P3 or P6 class teacher to teach such a lesson.

Unit 9 LESSONS RELATED TO STATISTICS AND ELEMENTARY PROBABILITY (FOR STUDENT –TEACHERS IN SME ONLY)

Key unit competence: **Prepare active lessons related to statistics and elementary probability**

Introductory activity 9

With the help of the syllabus and textbooks for upper primary Mathematics:

a. Identify all topics related to statistics and elementary probability to be studied in P4, P5 and P6? What is the importance of studying statistics in primary schools?

b. For each topic, try to find out appropriate resources for facilitating related lessons using low cost materials and suggest how to produce them.

c. Are teaching techniques to facilitate lessons on statistics the same as those to be used in the lessons related to elementary probability? Explain your answers.

9.1.Recall on concepts related to statistics and elementary probability

Activity 9.1

Use upper primary Mathematics syllabus and textbook and do the following:

a. Select different concepts related to statistics and elementary probability.

b. Among the selected topics which do you find are challenging to you?

c. Make a self-study on the challenging topics. You can ask for a help from your colleague or your tutor.

Content summary

Statistics is the study of the collection, organization, analysis, interpretation and presentation of data. It deals with all aspects of data, including the planning of data collection in terms of the design of surveys and experiments. It also includes describing mathematical relationships between variables and presenting these to an audience in a way that best conveys meaning.

Descriptive statistics for primary schools are extremely important in our everyday life. Statistics are the method of conducting a study about a particular topic by collecting, organizing, interpreting, and finally presenting data. Descriptive statistics include both numerical (e.g. mean, mode, variance…) and graphical tools (e.g. bar charts, polygons, histograms, boxplot…) which allow summarizing a set of data and extracting important information such as central tendencies and dispersion. For summarizing information, statisticians can use many ways of presenting the finding (data) in form of different graphs including: bar graph, pie charts, scatter diagram, histogram, Line graphs, Scatter plot, Stem and plot, frequency polygon and frequency curve.

The elementary probability to be studied in primary schools includes the probability language, basic terminologies used to describe how the event is look like and simple experiments helping to determine the probability of an event. The following are some language of probability:

Sample space: The sample space is the set of all possible outcomes.

Event: This is any possible outcome from the sample space. An event is also a subset of a sample space.

More likely: an event that has more chances of happening (has more chance of occurring).

Equally likely: two events that have the same chance of occurring.

Less likely: an event that can rarely happen.

Impossible: event that cannot happen.

Certain: an event that will definitely happen.

The following experiment of tossing a coin can help us to understand the difference between the above terminologies, and generating other one which are impossible, equally likely, less likely and certain.

In tossing a coin, either head or tail face up.

i. It is sure or certain to see either head or tail in a toss.

ii. It is not possible to have both head and tail face up at once in a toss.

iii. It is impossible for a coin to face up and give outcome of both head and tail at once.

iv. It is equally likely to see either tail or head in a toss. Both head and tail have equal chance.

v. When tossing a coin several times, it is unlikely to observe heads only or tails only. It is likely you will observe heads and tails.

Application activity 9.1

Use upper primary Mathematics syllabus and textbooks, select one lesson on statistics and elementary probability in P4, P5 and P6.

For each lesson, highlight the skills to be acquired by learners.

9.2. Production and organization of appropriate resources to be used in the teaching and learning of statistics and elementary probability

Activity 9.2

By the use of upper primary Mathematics syllabus and Mathematics textbooks for P4, P5 and P6, choose one lesson on statistics or elementary probability (e.g. Data collection using tables, play games of chance and decide whether or not they are fair. interpret bar charts and line graphs to draw a conclusion…)

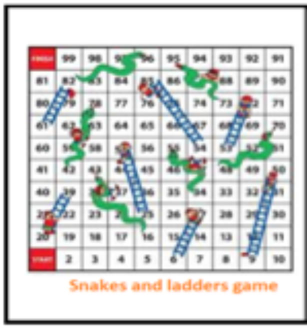
Identify all materials you may need to make more active and practical the teaching and learning of the selected topic

Explain how those resources can be used while teaching and learning the identified topics related to statistics and probability.

Content Summary

Teaching and learning resources for statistics and elementary probability can be acquired by producing or constructing them, by collecting them from the environment, by asking learners to prepare them, by requesting the school administration or parent to buy them, by borrowing or hiring them or through improvisation. But also ICT Materials can be used for enhancing the teaching and learning lessons related to statistics and elementary probability.

- Data collection is important; it can make statistics more engaging and allow learners to make more meaningful connections, the teacher must prepare manila papers, markers for writing and recording data.
- Snakes and ladders game made in rice sacks can be used to clarify the way of using dice, where learners can run some terminologies like chance, rolling. The bottle tops can be used to toss when the coins are not available



- The illustration of some statistical graphs (bar graph, pie charts, scatter diagram, histogram, Line graphs, Scatter plot, Stem and plot, frequency polygon and frequency curve) can be used as semi-concrete resources. They can be hanging on the wall of the classroom or can be made into manila papers, and rice sacks.
- Calculators and spreadsheet of XO Laptops can be used for simple calculations in statistics and elementary probability.

Application activity 9.2

Use a primary Mathematics syllabus and P6 primary book, pick on lesson related to statistics or elementary probability, Highlight learning materials to be used and describe how you will use them in teaching and learning process.

9.3. Techniques and strategies for facilitating lessons related to statistics and elementary probability

Activity 9.3

Teaching any lesson require to use an appropriate technique, or combination of more than one technique across all parts of the lesson.

Observe the list of techniques below and discuss active teaching techniques and strategies you think are the best for facilitating the lesson on representation of data using a bar chart or simple pie chart.

a. Brainstorming

b. Group work

c. Practical work and research activities

d. Demonstration

e. Concept mapping

f. Expository

Content summary

1. Concept development

To "ensure learners develop a tangible understanding of statistics and elementary probability concepts , concrete and semi concrete materials are very important in order to help learners to later use this foundation and link their conceptual understanding to abstract.

Concrete stage: Through explorations and investigations of real life situations, learners collect data; present those data using tables or statistical graphs and make interpretations of those data.

Semi- concrete stage, visualization or representation: In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve tables and graphs on a manila papers or rice sacks, … .Using table or graph representations, learners are expected to explore, investigate, interpret and draw conclusions.

Abstract stage: In this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols and formulas to compute mean, mode, median …

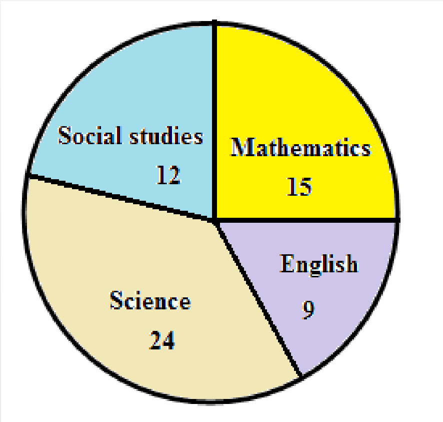
Example 1: using table to establish relationship between statistical data

The table below shows the marks obtained by 60 P6 learners in a Mathematics exam by percentage in a school.

Marks	Tally	Frequency (number of learners)
96	||||	4
95	~~||||~~	5
93	~~||||~~ ||	7
84	~~||||~~ ~~||||~~ ||	12
81	~~||||~~ ~~||||~~ ~~||||~~ |||	18
72	~~||||~~	5
70	|||	3
68	~~||||~~ |	6
Total		60

Example 2: using graph to represent statistical data

In a P. 6 class, 15 learners like Mathematics,12 learners like Social Studies, 24 learners like Science and 9 learners like English. Represent the information in a pie chart.

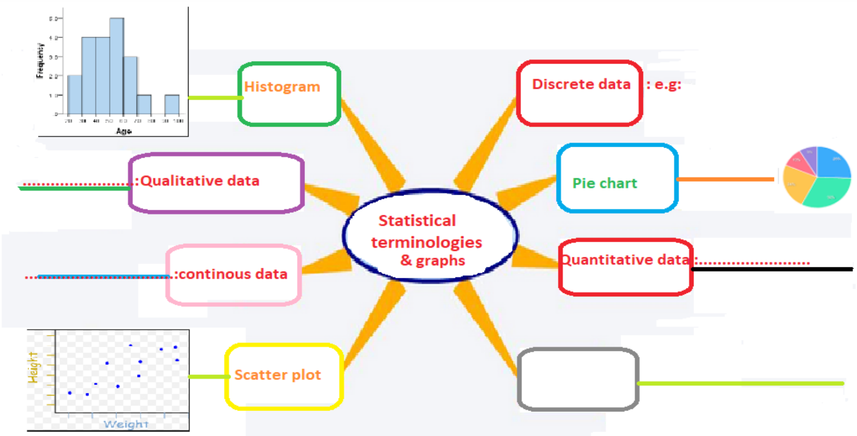


2. Teaching and learning strategies related to statistics and elementary probability.

In the teaching and learning of statistics and elementary probability concepts, teachers use a repertoire of teaching approaches as there is no single suitable method to teach mathematics to emphasize mastery of knowledge and skills, critical and creative thinking, communication and problem solving. Mathematics teachers should adopt a combination of different teaching approaches to engage learners in learning. These approaches will help learners talk more openly, think more creatively and ultimately become more engaged in the process of learning.

Some of the approaches or techniques are:

- **Brainstorming:** this technique can be used in the introduction part, and in plenary session for checking the learners' understanding. Teacher should ask learners to explain some terminologies and vocabularies used in statistics and elementary probability such as impossible, certain, equally likely event…
- **Collaborative learning** or **group work activities** for the purpose of sharing materials (Mathematics textbooks, XO Laptops,..) , discussing and solving mathematical problems, sharing ideas and understanding.…
- **Practical work and research activities** for construction of knowledge, reflection and exploration of statistical concepts... Through this technique, learners should be able to explain and interpret statistical concepts and results. Such activities encourage deeper understanding of concepts or phenomena; develop into learners the observation, practical work, planning, and reporting skills.
- **Demonstration**, using this technique the teacher should explain and demonstrate how some formulas are used, how to navigate spreadsheet for graphing pie chart, histogram, tables using ICT tools.
- **Expository method,** this can be used while defining a mathematical concept, generating formulas and statistical graphs like pie chart, bar graph and histogram in case learners fail to define or generate them.
- **Concept mapping**: this technique should be used to prompt learners to give the meaning of any mathematical concept you want them to learn. The teacher or learners themselves should write any associations they have with that concept. This technique can be used in introduction or revision part, to facilitate learners show their views and thoughts. This technique helps both teacher and learner to make content larger (extension).



Application activity 9.3

Use the syllabus, textbooks and teachers guide of Mathematics for upper primary and organize the teaching techniques you can use to make more active and participative the lesson on recording data in tables and representing them using a bar chart in P5.

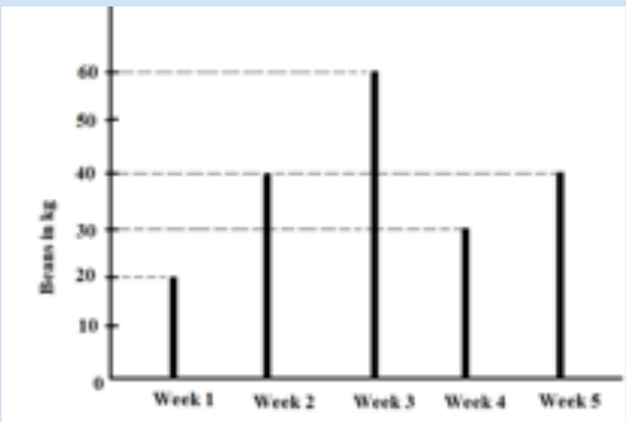
9.4. Activities for developing generic competences and integrating crosscutting issues in lessons related to statistics and elementary probability

Activity 9.4

Read carefully the activity for P5 learners, analyse it and answer the questions 1 and 2.

Activity: Study the graph and answer the questions that follow:

Beans in kg sold at a shop in 5 weeks



a. In which weeks did the shopkeeper sell the same quantities of beans?

b. In which week did the shopkeeper sell the highest quantity of beans?

c. Find the total kilogram of beans sold in the 5 weeks

d. If each kg of beans costs 300 Frw, how much did the shopkeeper get from the sale of beans in the five weeks?

1. Identify and explain the competences to be developed while performing the activity
2. Is there any cross-cutting issue that has been addressed? Explain your answer.

Content summary

Activities related to analysing and integrating statistical graphs, discussion and exchanging coherent constructive content should develop competences among learners. From activity 9.4, the following are some of generic competences that can be developed:

1. **Communication**: through presentations and the explanation made in the process of interpretation of graph and drawing conclusions learners develop the communication skills.
2. **Cooperation**: through working in pairs and groups and sharing ideas when they are integrating graph and computing learners develop the spirit of working together.
3. **Critical thinking**: through analysis and interpretation of information given by the graph learners develop the critical thinking skills.
4. **Problem solving**: through applying the skills learnt in the process of solving different problems involving statistics or elementary probability in the real life of the learner, there is a development of the problem solving skills.

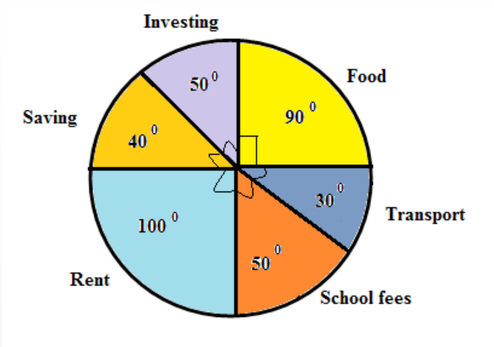
From activity 9.4, the following are some of cross- cutting issues that can be addressed.

1. **Peace and value education:** through the positive attitude in mathematics class , learners show trust, empathy and honest to each other while solving problems.

2. **Gender** is addressed through group activity where boys and girls take similar responsibilities in working
3. **Inclusive education** addressed through activities that should cater for different ability of learners, teaching and learning materials that are adapted to meet special educational needs and teaching techniques that are adapted to different ability of learners.
4. **Financial education:** through different problems for calculating the price for the beans , learners will calculate cost and realize the value of money, their use and saving.

Example of activity in statistics that develop competences and address financial education as cross- cutting issues.

The pie-chart represents the monthly expenditure of Mrs. Muziranenge's salary. Study it and answer the questions that follow.



If 240,000 Frw is spent on transport, how much does she earn?

- How much more money is spent on rent than on savings?
- Present your working to the class

Application activity 9.4

As a student-teacher, set up an activity on statistics or probability that helps primary learners to develop generic competences and address crosscutting issues. Explain how your activity develops generic competences and addresses crosscutting issues.

9.5. Assessment tasks in the lessons related to statistics and elementary probability

Activity 9.5

The below activities are from P5 Mathematics book for learners:

Activity 1: In 2016, some people were asked to predict the national team which would win the World Cup in football. Their predictions were as follow:

National team	Number of predictions
96	13
95	9
93	8
84	12
81	10
72	10
Total	8

Represent the above information in a pie graph or chart and in a bar graph.

Activity 2: Discuss the likelihood of the following events:

a. Getting a head when you toss a coin.

b. It will rain in Kigali this year.

c. A woman will give birth to a boy.

d. The sun will rise tomorrow.

e. You will get a six when you throw a die.

f. I was born yesterday.

Analyse the activity and explain whether the action verbs used develop low, medium or high order thinking skills.

Content summary

When learning activities and assessment tasks are well structured, learning of lessons related to statistics and elementary probability become more effective and learners participate actively in the process of teaching and learning. One of the solutions of avoiding failure in statistics and elementary probability is to set appropriate activities/ tasks that are age appropriate and which develop high order thinking and problem solving skills. Activities/ tasks must be set taking into consideration different ability of learners and by using action verbs which intend to develop both low, medium and higher order thinking skills and competences.

The table below shows examples of how tasks on statistics and elementary probability should be formulated

<table>
<tr><th>Assessment task</th><th>Topic</th><th>Sample activity/ task</th></tr>
<tr><td>Practical and oral task</td><td>Data collection and data interpretation</td><td>Work in groups to investigate a characteristic of continuous data (height, distance to school…) or discrete data (shoes size, number of brothers and sisters…).
Work in groups to conduct an experiment to investigate whether it is easier to get certain outcomes when two coins are tossed rather than other outcomes. Toss the coins 60 times. Use a tally to collect the data.
<table>
<tr><td>No of heads</td><td>Tally</td><td>Frequency</td></tr>
<tr><td>0</td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td></tr>
<tr><td colspan="2">Total frequency</td><td>60</td></tr>
</table>
Work in groups to analyse and interpret information of statistical data presented on a graph: bar chart, pie chart, histogram…</td></tr>
</table>

	Vocabularies of Probability and simple experiments	Warm up debate activity: for example, discuss the role of tossing a coin by the referee before starting a football match. Work in pairs toss a coin 20 times and record the outcomes As a class play Bingo! Learners are asked to write down their choice of six numbers between 1 and 12. Teacher throws two dice and tells the class the total. Learners strike out the number if it is in their list. Continue until the first learner has struck out all their numbers – that learner should call out, "Bingo!"
Visualization and representation tasks	Data presentation	Play the game several times. Are some numbers easier to get than others? Discuss possible reasons why – use the language of chance. Present statistical data in a frequency table, on a graph (bar chart, pie chart, histogram…).

Application activity 9.5

Refer to the activities in P5 Mathematics textbook and set 3 activities related to statistics (Data collection, data presentation and statistical graph interpretation).

- In the first activity use action verb that can develop low order thinking skills,
- In the second activity use action verb that can develop medium order thinking skills,
- In the third activity use action verb that can develop high order thinking skills.

9.6. Lesson plan of statistics and elementary probability

Activity 9.6

Read carefully the following lesson plan on qualitative and quantitative statistical data.

Is it a complete lesson plan that can help the teacher in the effective teaching and learning activities?

If no, try to modify it and make it an effective lesson plan which can be used to develop competences.

School Name: ....Teacher's names: ....

Term	Date	Subject	Class	Unit Nº	Lesson Nº	Duration	Class size
....	....	MATHEMATICS	P4	17	**2**	40 mins	50

Type of Special Educational Needs to be catered for in this lesson and number of learners in each category	1 slow learner
Unit title	Elementary statistics.
Key Unit Competence	To be able to collect, represent and interpret data.
Title of the lesson	Collecting quantitative and qualitative data
Instructional Objective	Using table and own examples, learners will be able to collect and differentiate quantitative with quantitative data correctly, confidently and in a given time.
Plan for this Class (location: in / outside)	In classroom
Learning Materials (for all learners)	Dictionaries, Reference Mathematics books and textbooks hand out…
References	REB (2019) Learners 'book Primary 4 Mathematics, page 176-177, teacher's guide, page 149-150

<table>
<tr><td rowspan="3">Timing for each step</td><td colspan="2">Description of teaching and learning activities</td><td rowspan="3">Generic competences

and Cross cutting issues to be addressed + a short explanation</td></tr>
<tr><td colspan="2">In groups learners work out on different activities on data collection, differentiate qualitative data from quantitative data. Students will give individually examples of qualitative and quantitative data. Students will present their findings and do application activity individually. Through a story or a game learners will record number and then arrange them using a table.</td></tr>
<tr><td>Teacher activities</td><td>Learner activities</td></tr>
</table>

Introduction **(5min)**	Facilitate learners to play game by choosing the number of learners who play the game and those who record the failed learners at each step.	**Game: up down game** Learners are asked to stand up or sit down by saying up or down. Learners who do the opposite of what the teacher said fail and removed from the group. The winner is the last learner. Suppose that 35 learners are chosen to play the game and 5 students are recording the number of students who fail(zero fail is not considered). Suppose that after 12 steps, we get the winner and the following numbers are recorded: 2, 5, 3, 2, 2, 2, 4, 1, 5, 6, 1, 1, 2.	**Generic competences** **Critical thinking** will be developed when following the game instructions and recoding data. **Peace and values education** addressed through respecting each other and obeying the game rules. • **Cooperation** developed through team work while playing a game and recording data.

<table>
<tr>
<td>Development of the lesson

Discovery activities

(10 min)</td>
<td>Ask questions about the recorded numbers

Facilitate learners to discover how to collect different types of data in the given activities</td>
<td>Activity 1: concept of data

Consider the recorded numbers in the game above (2, 5, 3, 2, 2, 2, 4, 1, 5, 6, 1, 1, 2) and in pair or small groups students represent the numbers in a table by showing the number of times each number is repeated.

<table>
<tr><td>Recorded numbbers</td><td>Number of times each number is repeated</td></tr>
<tr><td></td><td></td></tr>
</table>

Activity 2: collecting and arranging quantitative data.

In groups, learners discuss and present the information about age of students in a P6 class on the table using tally marks.

10, 11, 11, 12, 10, 10, 12, 11, 11, 12, 12, 12, 10, 10, 11, 11, 10, 11, 11, 10.

<table>
<tr><th>Age of P6 students</th><th>Tallies</th><th>Number of students</th></tr>
<tr><td>10 years</td><td></td><td></td></tr>
<tr><td>11 years</td><td></td><td></td></tr>
<tr><td>12 years</td><td></td><td></td></tr>
</table>
</td>
<td>Generic competences

Critical thinking will be developed when analysing data and present them in a frequency table.

Communication will be developed when discussing in groups.

Cooperation developed through team work

Cross cutting issues:

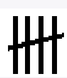
Gender addressed through assigning tasks to boys and girls.</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td>
Activity 3: collecting and arranging qualitative data.

In groups, learners are asked to analyse data from the given table and differentiate between the data in the table from activity 2 and the data in a table from activity 3.

In a certain school, the teacher has asked 9 students how often they bring their lunch to school. See their results.
<table>
<tr><td>Kamana</td><td>Always</td></tr>
<tr><td>Kamariza</td><td>Sometimes</td></tr>
<tr><td>Kamanzi</td><td>Never</td></tr>
<tr><td>Uwera</td><td>Sometimes</td></tr>
<tr><td>Mutoni</td><td>Sometimes</td></tr>
<tr><td>Umulisa</td><td>Always</td></tr>
<tr><td>Umurerwa</td><td>Sometimes</td></tr>
<tr><td>Umwari</td><td>Sometimes</td></tr>
<tr><td>Uwase</td><td>Always</td></tr>
</table>
</td>
<td>
Inclusive education addressed when all learners are participating in the lesson. 1 slow learner is given remedial activities.

Peace and values education addressed through respecting the views of every student in group discussion
</td>
</tr>
</table>

<table>
<tr>
<td>Presentation of learners’ findings and exploitation: (10 min)</td>
<td>Facilitate learners to present their findings from activity 1, 2 and 3.</td>
<td>Learners present findings from activity 1
<table>
<tr><th>Recorded numbbers</th><th>Number of times each number is repeated</th></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>1</td></tr>
<tr><td>4</td><td>1</td></tr>
<tr><td>5</td><td>2</td></tr>
<tr><td>6</td><td>1</td></tr>
<tr><td>Total</td><td>12</td></tr>
</table>
Learners present findings from activity 2
<table>
<tr><th>Age of P6 students</th><th>Tallies</th><th>Number of students</th></tr>
<tr><td>10 yaers</td><td></td><td>7</td></tr>
<tr><td>11 years</td><td></td><td>8</td></tr>
<tr><td>12 years</td><td></td><td>5</td></tr>
</table>
</td>
<td>Communication skills are developed through presentation of findings and answering questions from different groups</td>
</tr>
</table>

Summary (5 min)	Harmonize learners' findings and based on them, facilitate learners to write a summary from activity 1, 2 and 3.	**Qualitative and quantitative data** Data are collected through asking questions and reccording answers. Data can be represented in form of tallies or numbers. When tallies are used, the number of items is grouped in fives. When the collected data are words (cannot be expressed in numbers), they called qualitative data. Quantitative data are any information that can be expressed as a number. The number of repetition of any number in data is called frequency	**Critical thinking skills** are developed through making a summary of concepts learnt.
Application (5 min)	Ask learners to represent data in a frequency distribution table.	**Application activity:** A weather station recorded the daily rainfall during a rain season in *mm*. The following are the data recorded in 2 weeks: 77, 20, 20, 69, 36, 67, 57, 77, 36, 69, 36, 20, 77, 64. Present the data using frequance distribution table.	**Critical thinking skills** are developed while analysing data and representing them using frequency distribution table

Conclusion **(5min)**	Write assessment questions on the chalk board and ask learners to do them . Facilitate them to make corrections in pairs and then on the chalk board.	Individually, learners do the following tasks : 1. The following numbers represent the age of 18 students of a football team at school.: 14, 12,12, 11,13, 12,10, 12, 14, 10, 12, 13, 14, 10, 12, 11, 12, 13. Represent these data in a frequancy distribution table. Are this data qualitative or quantitative? Explain your answer. 2. During break Mwiza counted the different colored cars that passed by/ near the school gate in five minutes and she got the following data: **Red, black, green, red, white, blue, yellow, red, blue, white.** Represent these data in a frequancy distribution table. Are this data qualitative or quantitative? Explain your answer.	**Critical thinking skills** are developed while analysing data and representing them using frequency distribution table

Teacher self-evaluation	All learners have actively and inclusively participated in the lesson. The objectives were total achieved. Reinforcement activities are needed for learners to enhance learning.

Application activity 9.6

Basing on the above lesson re-examined, plan another lesson with different title, more engaging, more active and inclusive.

End unit assessment 9

Suppose that you are P5 Mathematics teacher and you are going to teach the lesson on presenting statistical graphs, refer to the syllabus of Mathematics of upper primary, appropriate books and the lesson planning procedures learnt in Foundation of education subject and do the following:

c. Set the good instructional objectives of the lesson “Qualitative and Quantitative data)

d. Organize teaching materials to be used on the lesson. (Qualitative and Quantitative data)

e. Formulate teacher’s activities and learners’ activities depending on the steps of the lesson;

f. Make a detailed lesson plan that can facilitate a P4 class teacher to teach such a lesson.

g. Describe the facts which make you teaching and learning process successful.

Unit 10 LESSONS RELATED TO WORD PROBLEMS

Key unit competence: **Prepare active lessons related to word problems**

Introductory activity 10

A Mathematics word problem is a real-world context in which mathematical quantities are given, values of one or more quantities are known, values of one or more quantities are unknown, relationships between or among quantities are described, a question is implied or stated asking one to find the value of one or more unknown quantities, and one or more of the operations addition, subtraction, multiplication, and division can be used to find the value of the unknown quantity or quantities.

Depending on the type of the problem to be solved, problem solving is a function of several interdependent factors such as knowledge acquisition and utilization, control, beliefs, affects, and various representational modes. The solution process often requires many steps back and forth until the learner is able to unravel the complexity of the problem situation.

Therefore, teachers are required to teach the type of word problems that are related to the age of learners.

1. What type of word problems to be solved by lower or upper primary learners?
2. Summarize appropriate resources for facilitating the lesson on words problems using low cost materials and suggest how to produce them.
3. Speculate on the role of visual representations and recommend other teachers the appropriate teaching techniques for facilitating a lesson on “solving word problems” to lower or upper primary learners.

10.1. Concepts on word problems in primary schools

Activity 10.1

Word problems in Mathematics deal with the application of acquired numerical, geometrical, algebraic, statistical and probabilistic concepts to solve real fife word problems. Students should develop and explore the mathematics ideas in deep, and see that Mathematics is an integrated whole, not merely isolated piece of knowledge.

Use the syllabus and Mathematics book of lower or upper primary and do the following:

a. Select topics that cover words problems in the syllabus and classify them according to the types of problem.

b. Among the selected topics and word problems which do you find challenging to you in solving?

c. Make a self-study on the challenging word problems. You can ask for a help from your colleague or your tutor

Content summary

Word problems are seen as very important lessons to be taught in all levels of basic education especially in f primary school. Children who understand operation meanings and can associate relationships between quantities given in word problems with those operation meanings are better problem solvers. So, the challenge for teaching word problems is how to help learners to use quantitative reasoning; that is, use reasoning to identify the relationships between the quantities in the problem and connect those relationships to appropriate operations.

At primary level, word problems can be examined on three levels:

- The verbal formulation
- The underlying mathematical relations
- The symbolic mathematical expressions.

Word problems support learners in explaining themselves with mathematical language (vocabulary like less, altogether, difference, more, share, multiply, subtract, equal, reduced, add, multiply etc…). Teachers tend to try and include word problems in their mathematics lessons for encouraging learners to apply the acquired knowledge in solving real life word problems.

In regard of this there are three main types of word problems appropriate to primary level learners:

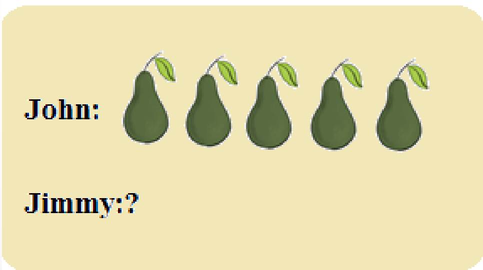
1. One-step word problems

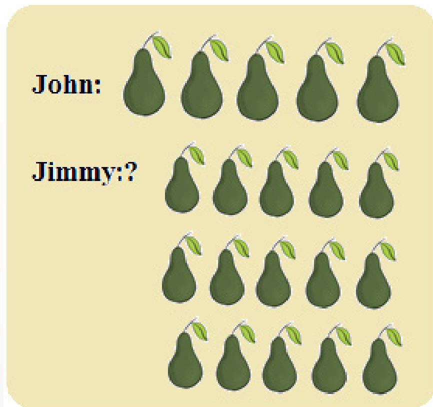
It is a word problem for which the solving requires only one operation

Example 1:

John has 5 avocados. Jimmy has triple this number of avocados. How many avocados does Jimmy have?

At the early age, children are given apparatus to help them with a problem solving: counters, plastic coins, number cards, number lines or picture cards etc.





Generally, learners are guided on how to use *bar diagrams as visual representations* to solve such problems:

Example: solution of question (a) the use of bar diagrams:

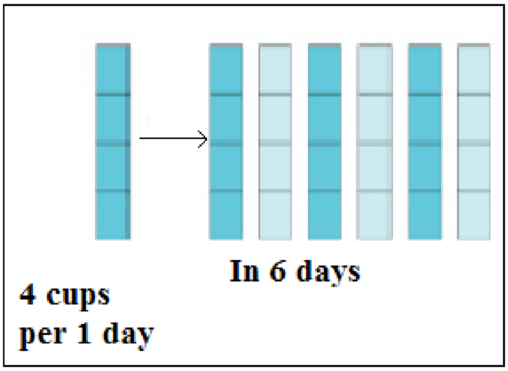
John:	5		
Jimmy	5	5	5

Therefore, jimmy has 5+5+5=15 avocados

Or Jimmy has 5x3 =15 avocados

It uses one step calculation.

Example 2: Manasseh drinks 4 cups of water per day. How many cups of water he will drink in 6 days?



2. Two-steps word problems

In two-steps word problems, learners work out problems where the finding out the answer requires **two separate operations.**

Children should feel confident in an efficient written method for each operation at this stage, they will continue to be given a variety of problems and have to work out which operation and method is appropriate for each.

Example:

The mother has 34 000 Frw, her husband gives her other 26 000 Frw. If she divides this money equally into 3 different bank accounts for 3 schools of her 3 children. How much money will she put in each bank account?

- **Step one:**

The total	34 000	26 000

To find the total, we use the addition: 34 000+26 000 = 60 000Frw

- ***Step 2****: The 3 bank account should have the same amount of money:*

?	?	?

3. Multi-step word problems

This type includes word problems that involve more than 2 operations. For **example**: addition, subtraction, multiplication, or division.

Examples:

Gerard earns a salary of 150 000 Frw in a month. He spends his money as follows:

Rent a house: 30 000 Frw, Food: 25 000 Frw, Transport: 15 000 Frw and pays the the monthly school fees for his 3 children. After this, he realizes a saving of 20 000 Frw.

i. If children pay the same amount of money on school fees, how much does each one pay per month?

ii. Why do you think it is important for Gerard to save money?

Common strategies for solving word problems

The "Steps" approach to teaching problem solving gives students a sequenced set of actions to follow to solve a problem instead of memorizing a solved model question to be imitated when solving the similar problem.

Step 1: Understand the problem: this requires learners to **read the problem** arefully for understanding. Not go straight to the conclusion without understand the problem.

Step 2: Plan or identify facts: Collect all needed information given in the word problem that help you in solving problem and organize them on sheet of paper: *organize given data and what question or problem need to be answered.*

In this strategy, you can eliminate information that is not necessary in finding the needed answers.

In addition, units within the problem must be considered when listing information because answer also will have units. For example, if dimensions are given in kilograms and you are asked to find out answer in grams, you need to convert the given units in grams. In conversion the table of conversions can be used, calculator used in operations, or you can use your own memory to get the conversion factors.

Step 3: Draw a diagram as visual representations

In some cases there is a use of diagram, a picture, a list/table or the use of bars to visualize the listed information in the problem. For example, drawing out a square when the problem is asking you to solve the unknown length of one of the sides or the length between opposite corners allows you to visualize what you are solving and how to go about it. Write other known information about the object, such as the width of the square, beside the object to better understand what is being asked of you and what information you already have.

Step 4: Develop a formula and solve the problem:

When the given word problem needs a formula to facilitate in finding solution, maybe there is a common Mathematics formula to use for calculation. Depending on the type of the problem, different Mathematics formulae can be used many times for helping you to get answer.

Solve the problem by plugging in the values and **solving for the unknown** variable. Double-check your calculations along the way to prevent any mistakes. Multiply, divide and subtract in the correct order using the order of operations. Exponents and roots come first, then multiplication and division, and finally addition and subtraction.

Step 5: Verify the answer and consult a reference

Check if your **answer makes sense** with what you know. Using common sense, estimate an answer and see if you come close to what you expected. If the answer seems absurdly large or too small, search through the problem to find where you went wrong.

Example:

Aline bought some oranges. She ate two of them and gave half of the remaining to Bosco. After that, she had 6 oranges. How many oranges did Aline buy?

- **Step one**: Read the problem
- **Step 2**: Plan or identify facts:

Given: Aline buy oranges, oranges eaten are 2, $\frac{1}{2}$ of the rest is given to Bosco, remaining oranges are 6.

- **Step 3:**

- **Step 4:**

Aline gave to Bosco the half of the rest: this means that the rest was 6 x 2=12

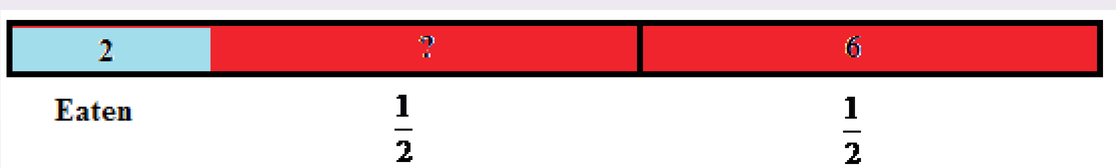
After eating 2 oranges she has 12 oranges.

Therefore, Aline has bought (12+2)= 14 oranges

In another way: If x is the number of all oranges she bought, $\frac{x-2}{2}=6$.

This means that $x-2=6.(2) \Leftrightarrow x=12+2$. **Finally** $x=14$

- **Step 5:**



Aline bought 14 oranges, she ate 2 and remains with 12; Then, she gave the half of 12 to Bosco which is 6 oranges and she remains with other 6 oranges.

Application activity10.1

Use a primary Mathematics syllabus and the book, select one lesson on "word problems involving addition or subtraction of numbers". Classify them and for each lesson, highlight the skills to be acquired by learners.

10.2. Appropriate resources for visualization in solving word problems

Activity 10.2

Use the syllabus and textbook for upper primary Mathematics choose a lesson on word problems involving multiplication and division of fractions.

Organize durable and appropriate teaching resources to facilitate that lesson and explain how those resources can be used in the lesson. Those materials will be kept in the Teacher Resources Centre (TRC).

Content summary

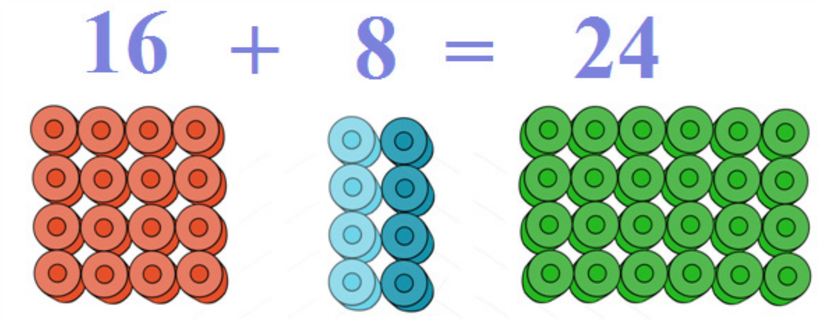
During the process of teaching and learning Mathematics, teaching and learning materials play a major role.

Resources for teaching word problems can be acquired by producing or constructing them, by collecting them from the environment, by asking learners to prepare them, by requesting the school administration or parents to buy them, by borrowing or hiring them or through improvisation according to the materials needed in that word problem.

Materials for facilitating word problems are identified according to the type and materials needed for that word problem. In lower primary, the teaching and learning word problems both teacher and learners use manipulative materials, improved materials, printable or non-printable materials. However, in upper primary, the teaching and learning resources of word problems depend on the content of the problems: many of them can be illustrated using diagrams or lines for visualizing the problem.

The following are example of materials that can be used:

Manipulative materials: counters, stones, bottle tops, sticks, flash cards, etc.



Graduated jugs or cups: in teaching and learning word problems learners use clearly a labelled measuring jug or cup and explains the measuring scales used to determine capacity (e.g. litre, 1 litre, and 2 litres) when they are solving word problems related to capacity measurement.

Pattern blocks: They can be used when solving word problems related to fractions, Geometry, Addition, Multiplication and Ratios.

Sticks: in teaching and learning word problems, sticks are used when counting, adding, subtracting on materials obtained within the problem.

Plastic bottles: with the use of plastic bottles, learners can group them when solving word problem on addition, subtraction or the multiplication**.**

Pictorials: all the above mentioned materials can be drawn on the chalk board or in the notebook to represent facts for the problem.

ICT tools also are very essential in the process of teaching and learning word problem:

1. **XO Laptop programs:** With turtle art activity learners solve simple word problems;
2. **Computer**: this can help for solving word problems involving the calculation of perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangle, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders by the use of different application software also learners learn how to solve word problems.
3. **Calculator:** in calculation, both teacher and learners need to calculate solution in numerical and give answers with the use of calculator when solving word problems.

Application activity 10.2

Use the syllabus and textbook for upper primary Mathematics choose a lesson on word problems involving multiplication and division of fractions.

Organize durable and appropriate teaching resources to facilitate that lesson and explain how those resources can be used in the lesson. Those materials will be kept in the Teacher Resources Centre (TRC).

10.3. Techniques and strategies for facilitating lessons related to word problems

Activity 10.3:

Use the syllabus and textbook for upper primary Mathematics choose a lesson on word problems involving multiplication and division of fractions.

Organize durable and appropriate teaching resources to facilitate that lesson and explain how those resources can be used in the lesson. Those materials will be kept in the Teacher Resources Centre (TRC).

Content summary

The challenge for teaching word problems is how to help learners to use quantitative reasoning to identify the relationships between the quantities in the problem and connecting those relationships to appropriate operations.

This is a good way to develop learners' quantitative reasoning abilities that can facilitate them to solve problems:

- Encourage learners to understand and meaningfully represent mathematical word problems rather than directly translate the elements of the problems into corresponding mathematical operations so that they may more successfully solve these problems and better comprehend the mathematical concepts embedded within them. (Pape, S.J., 2004)
- Improve the problem-solving performance by training children in the process of using diagrams to [meaningfully represent and] solve [mathematical word] problems than training them in any other strategy. (Yancey, Thompson, and Yancey, 1989)
- Emphasize the representation of the problem structure and de-emphasize the representation of surface features." (Diezmann and English, 2001).

This consistency in visual representations helps students see not only the connections between the diagrams but also connections between and among operations. This is the reason why Diezmann (2001) suggested how bar diagrams can be an integral part of teaching and learning mathematics:

- Model bar diagrams on a regular basis; not just in special lessons but frequently when word problems are encountered.
- Discuss the structure of bar diagrams and connect them to quantities in the word problem and to operation meanings.
- Use bar diagrams to focus on the structure of a word problem, not surface features like key words.
- Encourage learners to use bar diagrams to help them understand and solve problems.

During the lessons on solving word problems, the following activities may be used:

Peer teaching and learning activities: through peer teaching learners will learn *key vocabulary* used in Mathematics like *total, altogether, combined, additional, quarter*, etc…in encompassing a range of lessons related to word problems. *Difference, fewer, decreased and remains* are expressions which mean that subtraction of numbers needs to occur. Expression such as "*at the rate"* shows the multiplication, "*ratio"* means a division. In peer teaching, a learner develop a list of key vocabulary for mathematics problems which allow him or her in taking steps to really investigate what the problem is asking for and what mathematical operation will need to be engaged in that word problem.

Research work: learners are asked to read the problem and find out the needed process for helping him or her to find solution. Learners can visit library or internet for further research on word problem solving strategies and steps.

The **teacher** has to make sure if learners read and understand the problem and *clarify unfamiliar words* for better comprehend how to solve problem. If necessary, *rephrase the problem* with similar but simpler language and help learners to solve multi-steps word problems.

Round table: Each group members sit on one round table, the teacher reads a word problem to learners. Each learner in group writes the answer on a sheet of paper and passes it to the next member who adds or improves the colleague's activity. The process continues until each paper comes back to the owner. Each group chooses a member to present their improved work to the whole class. Learners share the important information for solving problems quickly guided by the teacher or themselves.

Use a diagram: visual representation of information from the problem can be a powerful tool to assist in mathematics. Drawing diagrams help learners to visualize the answers to the asked problems, especially with part-to-whole comparisons; can be of great help when lots of words can confuse learners.

Group work: in groups learners are given a series of word problems to be solved. Learners of the same group share ideas and feel free to explain to each other using their common language.

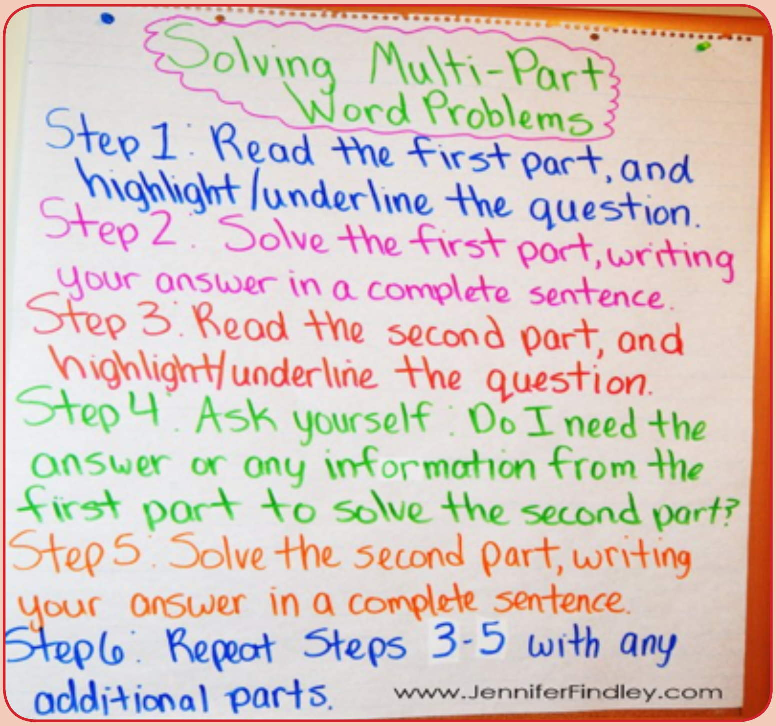
Questions in corners: Teacher form different corners and search word problems to be shared in all corners. One corner takes word problem, another problem goes to other corner, etc.. As each corner gets the series of word problem, they solve them and then after they share idea as a whole class.

Think, write, pair, and share (TWPS): Give learners the time to think independently about the problem and write their ideas before they share the ideas with their classmates and then with the entire group. As a teacher, identify the reflective problems that learners naturally met to help their own thinking and provide assistance.

Note: Learners use different ways to solve problems: Although most problems in mathematics are viewed as having only one answer, there may be many ways to get to that answer. Learning math is more than finding the correct answer; it is also a process of solving problems and applying what one has learnt to new problems. The teacher should take this as important and avoid imposing one process of solving a problem but give them the freedom for creativity and innovation.

Application activity 10.3

When teaching a multi-step word problem solving, one teacher designed the steps to be followed as below:



Read the following problem and design the similar process indicating the number of parts to be involved when solving it and how you can guide learners in solving such problems that require many operations.

Problem: One day a shopkeeper Peter went to market for buying a dozen boxes of markers, each box has 24 markers inside costing 4000Frw for each box. He repacked five of these boxes into packages of six markers each, and sold them for 2400Frw per package. He sold the rest of the markers separately at the rate of three markers for 1600Frw. How much profit did he make?

10.4. Activities for developing generic competences and integrating crosscutting issues in lessons on word problems

Activity 10.4:

In teaching lessons related to word problems, one of the challenges for the teacher is to make lesson pleasurable for learners who think that word problem is complicated. On the other hand, when learners are given opportunities to be engaged in manipulating concrete or pictorial objects in classroom activities, the lesson become more active and participative. Answer the following:

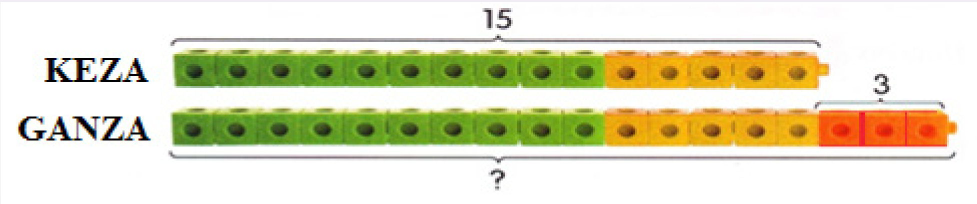
1. What can you do to make the lesson on "Problems involving multiplication and division of fractions." enjoyable?
2. Discuss active teaching techniques and strategies you think are the best for facilitating lessons related to real life problems that involve finding time intervals.

Content summary

In teaching and learning lessons related to word problems teacher can use a variety of activities to guide learners on how to solve real life problems with the application of mathematical concepts. Below are the examples of activities that help to develop competences and address cross-cutting issues.

Use manipulative materials: Children learn through concrete (manipulative) materials before proceeding to pictorial and abstract. By the use of available objects, sticks, papers, blocks, learners can create patterns or solve problems individually or in groups. Learners develop research and problem solving skills through the use of manipulative materials. They can proceed to deep research on how these materials are used for solving word problems.

Example: KEZA has 15 mangoes, GANZA has 3 more mangoes than KEZA. How many mongoes does GANZAhave? Learners can proceed to the use of counters and find the answer:



Use Patterns: when the problem is related to real life word problem it is possible to teach learners to refer to patterns to discover what is happening in that word problem. Through the exploration and the use of patterns, learners can develop lifelong learning skills.

Brainstorming: use of brainstorming is a fun activity in any word problem solving. Learners are given the chance to make a list of their ideas which conduct them to find out answer of given word problem. Through brainstorming activities learners develop critical thinking as they reason broadly and logically by making coherences. Gender and peace education can be addressed as both boys and girls participate in delivering ideas.

Create visual materials: In teaching and learning word problems, learners develop creativity and innovation as generic competences through making their own materials to be used**.** Cooperation also is developed through the discussion made on the use of these materials for finding possible solutions to the problem. By creating these materials, learners become fully involved in teaching and learning lessons related to word problems.

Example:

Read carefully the activity bellow and observe attentively the photo and answer the questions that follows: 25 learners in a school are given milk. Milk is in 500 milliliter packets. Learners take a packet of milk every day. one packet of milk cost 300Frw and they take them in a whole month of 30 days. 	**Questions:** How many packets are required in 7 weeks? How much money do these packets require in 7 weeks? Why is it important to drink milk? Before distributing milk to learners, the teacher verifies the parking date for each packet. Why is it necessary?
What are the competences developed and cross-cutting issues addressed in this problem?	

Application activity 10.4

As a student-teacher, set up an activity on word problem that helps primary learners to develop generic competences and address crosscutting issues. Explain how your activity develops generic competences and addresses crosscutting issues.

10.5. Tasks for assessing the problem solving skills

Activity 10.5

Read the activities bellow carefully, analyse them and answer the questions that follow:

1. A school farm collected 99 eggs each day. Each egg was sold at 110Frw. Calculate the money the school got in 5 days.
2. Students of primary 5 were standing on a line before doing sport. The teacher gave them the numbers from number 4 to number 44. Students work sport together and they can exchange ideas when they are doing sport. How many students were there on the line?

Questions

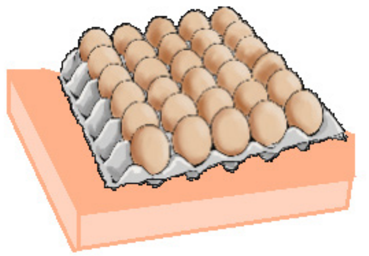
1. Identify and explain the competences to be developed while performing above activities.
2. Is there any cross-cutting issue that has been addressed? Explain your answer.

Content summary

Learners learn well and benefit from well-structured learning activities and assessment tasks formulated with reference to action verbs. These action verbs should be at deferent levels to help learners develop the low, medium and higher order thinking skills and competences.

The table below shows examples of how tasks on word problems should be formulated

Assessment task	**Topic**	– Sample activity/ task
Practical and task Or modeling a context mathematically Problems on division		– In pairs, explore how to construct a rectangle of 20cm and 6cm of width, and divide the rectangle into to 30 small squares within that rectangle. – Divide a set of concrete objects into equal groups, and discuss the grouping and sharing concepts of division. – Work in group to create 2-step word problems involving addition and subtraction up to 4-digits (or involving addition and the multiplication) for other group to solve. – Work in groups to create word problems (with pictorial representation if necessary) involving multiplication and division for other group to solve. – Discuss real world examples of data presented in bar graphs. – In a peace rally, 3 speakers talk to people in 7 districts. How many speakers are needed for 42 districts? Discuss the importance of peace in our country. For the word problem given: – draw an illustration for the problem – identify the unknowns in the problems – identify what you know – find the relationship between the unknowns and known – represent the relationship mathematically.

Visualization and representation tasks	Problems on division and multiplication	– Use the part-whole and comparison model to illustrate the concepts of addition and subtraction and use the models to determine which operation (addition or subtraction) to use when
		– solving 1-tep word problems. – Use the part-whole and comparison models to represent and solve word problems involving 4 operations. – In pairs draw rectangle tray with 30 eggs and find the cost of 180 eggs if one cost 70Frw 

Application activity 10.5:

The following activity is given by primary Mathematics teacher for learners. Analyse it and answer to the questions.

Activity:

Peter has farm in a rectangular form, one day he wanted to fence his farm with poles, he calls person with mathematics concepts for helping him to measure the perimeter and surface area and he found that the length is 30 metres and the width is 20 metres.

1. Determine the area of the farm
2. If 1 meter square need 10 poles, how many poles needed by Peter for fencing the whole farm?
3. If between two poles there is 2 metres on circumference. Calculate the number of poles on circumference.

Questions:

1. Describe the skills to be developed by learners in doing that activity.
2. Is this activity appropriate to P3 or P5 learners in Rwanda? Explain your answer.

10.6. Lesson plan on word problems

Activity 10.6:

Suppose you are Mathematics teacher of 46 students in primary school and you are going to teach the lesson on word problems involving addition or subtraction of whole numbers in P3 or integers in P5.

Refer to the syllabus of Mathematics of lower primary P2 or upper primary P5, appropriate books, the lesson planning procedures learnt in Foundation of education subject and do the following:

a. Set the instructional objectives (with its 5 main components) of the lesson,

b. Organize teaching materials to be used on the lesson that lesson,

c. Formulate teacher's activities and learners' activities depending on the step of the lesson;

d. select generic competences to be developed and crosscutting issues to be addressed depending on the step of the lesson;

e. Summarize the teaching and learning techniques to be followed in this lesson.

Content summary

Instructional objective, teaching and learning materials, strategies for helping learners to develop competences are very essential points to focus on when preparing and delivering the lessons related to word problems.

Example of instructional objective of the lesson related to word problems:

Given the geometric materials, learners will be able to use bar diagrams to solve correctly the multi step-word problems.

Application Activity 10.6:

Read carefully the following lesson plan on word problem.

Is it a complete lesson plan that can help the teacher in the effective teaching and learning process?

If no, try to modify it and make it a lesson plan.

Make a correction of the following lesson plan on word problem

School Name: X Primary School .... **Teacher's name**: ....

Term	**Date**	**Subject**	**Class**	**Unit Nº**	**Lesson Nº**	**Duration**	**Class size**
1	27 /05/ 2019	Mathematics	P4	3	1 of 5	7h20-8h00	4 2 learners
Type of Special Educational Needs to be catered for in this lesson and number of learners in each category				One learner with hearing impairment will sit near the chalkboard			

Unit title	Mathematical operations on whole numbers up to 100,000
Key Unit Competence:	To be able to read, write, compare and calculate whole numbers up to 100,000
Title of the lesson	Solve mathematical problems involving addition, subtraction, multiplication or division of numbers.
Instructional Objective	Using flash cards own examples, students will be able to solve word problems
Plan for this Class (location: in / outside)	In classroom
Learning Materials (for all learners)	Mathematics books and textbooks, sticks, sheet of papers
References	REB (2019) Learners 'book P4

Steps and Timing	Description of teaching and learning activity In groups learners work out on different activities on word problem. Students will give individually examples of word problem.		Competences and Cross-Cutting Issues to be addressed
	Teacher's activities	**Learners activities**	
Revision: (5 minutes)	Invite learners to follow and understand the scenario. Help learners to answer the questions	**Scenario:** One day john went to market and he bought 45 pineapples, when he came back at home he met with Mary and she told him that she want only 3 pineapple but John refusing saying that he gives her only one. Mary said thank you my friend John that is enough. So give me one pineapple. How many left pineapples to John? If one pineapple cost 50Frw, how much did he paid at market? Answer the questions guided by the teacher Answers: He left with (45-1) pineapples= 44 pineaples.	
Development of the lesson(25 minutes) Discovery activities:	Tell types of word problems to learners Give an example of word problem and solve it.	**Activity 1: types of word problems** There are three main types of word problems such as one-step, two-steps and multi-steps word problems. **Activity 2: solving an example of word problems**. Peter has 12 marbles. His friend John has 9 marbles. How many marbles do they both have altogether?	Gender addressed when both girls and boys working together in groups. Peace and value education developed through consensus leading to conclusion

Presentation of findings **Synthesis, summary and Application**	Ask learners to solve given word problem and present their findings	Answers: 12 marbles for peter and 9 marbles for John They all have 12marbles+9marbles=21marbles **Activity 3: ask learners to solve word problem**. In pairs, learners are asked to solve the following word problem: I have 34 000Frw. I am given another 26 000Frw. I divide this money equally into four different bank accounts. How much money do I put in each bank account? **Answers:** First read question The first step would be to add 34 000Frw and 26 000Frw to make 60 000Frw. The second step would be to divide 6 000Frw by 4 as there are four bank to make 15 000Frw So, I will put 15 000Frw for each bank.	
	Ask learners to sole independently word problem.	Activity4: Application activity on word problem John saved 68 000Frw at a bank on the other day peter saved amount of money to the account of john, 52 000Frw. John want to change an account and save that account of money on the other bank. Help him to divide that amount of money equally into four different account in the same bank. How much money do he will put in each bank account? **Answers:** - read question first	

		- make sum all money he get: 68 000Frw+52000Frw=120000Frw - other step is to find amount of money will deposit to each account by making division Each account He will save 30000Frw for each account in bank.	
Conclusion **(10 minutes)**	**Assessment** Write assessment activities on the chalkboard. Ask learners to do the assessment.	Mary went in Simba Supermarket and she have seen the same T-shirt in two different sales where the first sale is on the original price of 4000Frw, but has been reduced by a third. In the second sale, the T-shirt was priced at 4500Frw, but now has 40% off. How much does each T-shirt cost and which one is the cheapest?	Learners develop a high level of thinking as they work on activities for assessment individually.
Teacher self-evaluation	The objective of the lesson was achieved.		

End unit assessment 10

Let suppose that you will teach the lesson on word problems involving multiplication of integers.

Use lower or upper primary syllabus, appropriate books and do the following:

a. Set the instructional objectives of the lesson;

b. Organize teaching materials to be used;

c. Determine the generic competences to be developed in each step of the lesson and highlight crosscutting issues to be addressed in this lesson.

d. From your findings in a, b and c, deduce a lesson plan by illustrating the step-by-step procedures for the lesson of word problem.

Unit 11 MATHEMATICS LESSON OBSERVATION

Key unit competence: **Evaluate adequacy of teaching and learning strategies used during mathematics lessons.**

Introductory activity 11

Suppose that you are observing a lesson taught by another teacher. Describe items you can observe to ensure that learners are engaged in the learning process and the teacher is the one who guides all classroom activities and management.

11.1. Mathematics classroom observation for practice

Activity 11.1

Take the textbook for Foundation of education, refer to the lesson observation learnt in it and discuss the activities for a lesson observer and the aspects (elements) to consider when observing a Mathematics lesson.

Content summary

Evaluating and improving the teaching practice is done through the collection of the right information, analyzing it and providing feedback. In school, classroom observation can be used to characterize and identify both effective teaching and effective teachers.

The classroom observation to be done in TTC is designed to focus on improving teaching activities rather than evaluating teachers.

(a) Before lesson observation

First and foremost, student-teacher doing lesson observation must read the observation guide and be conversant with it. Apart from the lesson observation form, the observer must be equipped with a **notebook** to take down some important events (actions, gestures, teacher's or learners' attitudes, etc) that occurred during the lesson, which are likely to influence the teaching/learning process or the observer's appreciation. In addition, the observer should be equipped with subject knowledge so that feedback will be based on correct information related to the subject observed.

(b) During the lesson observation

The student-teacher observing the lesson progressively completes the form and takes notes on events as they occur in his/her notebook. Those notes will be a referential basis for the appreciation on different aspects of the lesson.

There are different categories of aspects to be focused on during the Mathematics lesson observation.

The first category refers to **the mathematical foci**: Conceptual (the conceptual development of his or her learners), derivational (the process of developing new mathematical entities from existing knowledge), structural (the links or connections between different mathematical entities, concepts, properties, etc), procedural (the acquisition of skills, procedures, techniques or algorithms), efficiency (learners' understanding or acquisition of processes or techniques that develop flexibility, elegance or critical comparison of working), problem solving (learners' engagement with the solution of non-trivial or non-routine tasks) and reasoning (learners' development and articulation of justification and argumentation).

The second category for observation focuses on the **contexts in which the teachers posed the tasks.** It has two dimensions: (1) whether the context was related to the real world or not and (2) whether the data or information used was genuine or invented by the teachers.

The third and final category concerns **teacher strategies** or "mathematical didactics" that might be used to facilitate learners' learning of Mathematics.

The teaching strategies to verify are such as: Activating prior knowledge, Exercising prior knowledge, Explaining, Sharing, Exploring, coaching, Assessing or evaluating, Motivating, Questioning, and the Differentiation in which the teacher should attempt to treat learners differently in terms of the kind of activities performed, materials provided and/or the expected outcome to make instruction optimally adapted to the learners' characteristics and needs.

(c) Feedback after the observation

The student-teacher doing lesson observation re-writes events from the notebook as comments in corresponding rows of the lesson evaluation form. *He/she gives time to the teacher to give his/her point of view on the delivered lesson,* the observer listens to teacher whose lesson was observed and tutor's feedback, and then takes time to ask questions related to his/her comments to the teacher whose lesson was observed and the tutor.

The lesson observer dedicates some minutes to the conversation with the teacher and tutor ; the latter expresses him/her self about some facts that characterized the lesson. The teacher will have ample time to express his/her intentions/motivation about his/her choice of lesson objectives, activities envisaged for learners, methods used, organizational and evaluation styles, processes and results.

Eventually, the lesson observer makes his/her general appreciation on each item or aspect observed by ticking the box matching with the suitable letter in the lesson evaluation form. He/she writes (if necessary) on the observation slip relevant recommendations towards the teacher and/or the school. The lesson observer counts down ticks in each box to formulate the general appreciation.

The observer's general appreciation, expressed with verbal scale is then calculated on the basis of appreciations on all aspects of the lesson evaluation form.

Six tips for improving lesson observation feedback

(i). Capture your lesson

Whether you are being observed or observing someone else, video can be an objective tool for you both, allowing you to have a two-way discussion rather than just a feedback session. Reviewing the video together means you can actively discuss, pause, rewind, fast-forward and allow your colleague to come to their own conclusions, making it more meaningful for you both.

(ii). Use exploratory questioning

This type of questioning naturally opens up dialogue, giving you time to think and draw conclusions. For example, try saying: "What do you think you could do to tackle the problem of…" rather than "What you need to do is…"

(iii). Make feedback constructive

Even if a lesson does not go to plan, the situation can be turned into a positive one if you give the teacher constructive feedback and help them to learn from the situation. Constructive feedback helps identify solutions for areas of weakness by looking at what can be improved rather than focusing on what went wrong.

(iv). Relate back to previous objectives set

Always keep a clear focus in mind. Relate your discussion to the targets already set, if new topics arise; set these as development targets for your next session. Staying focused will allow you to give some 'easy wins'; developing a feeling of immediate progression. Be clear in your own mind about why you are feeding back. What exactly do you want your colleague to achieve with what you are telling them?

(v). Be patient

Give you and your colleague time to draw your own conclusions and then explore them together without cutting across one another. You will develop a stronger, more professional relationship.

(vi). Do it again

Whether giving or receiving lesson observation feedback, request you do it again and regularly. Only by continually discussing and breaking down what you saw can help you understand progression.

11.2. Components of a Mathematics lesson observation sheet

The lesson observation sheet contains items to be verified but those items are given in a general way to help different subject teachers or observers to use it.

The following are the main components of a lesson observation sheet at the international level (Education Development Trust, 2018):

	Item	Indicator (detail)
1	Assessment and evaluation	Assessment is aligned with goals and instructional objectives
		The teacher gives explicit, detailed and constructive feedback

2	Differentiation and inclusion	The teacher creates an environment in which all learners are involved
		The teacher takes full account of learner differences
3	Clarity of instruction	The teacher shows good communication skills
		There is clear explanation of purpose
		Lessons are well structured
4	Instructional skills	The teacher is able to engage learners
		The teacher possesses good questioning skills
		The teacher uses various teaching methods and strategies
5	Promoting active learning and developing meta-cognitive skills	The teacher helps learners develop problem-solving and meta-cognitive strategies
		The teacher gives learners opportunities to be active learners
		The teacher fosters critical thinking in learners
		The teacher connects material to learners' real-world experiences
6	Classroom climate	All learners are valued
		The teacher initiates active interaction and participation
		The teacher interacts with all learners
		The teacher communicates high expectations
7	Classroom management	Learning time is maximized
		Clear rules are evident
		Misbehavior and disruptions are effectively dealt with

Source: *Education Development Trust, (2018).*

It is better to note that even though all of the items of the observation are desired qualities of a Mathematics classroom, not all of them are expected to be observed during a single lesson. Some items can be highlighted depending on the instructional objectives of a specified lesson observation. It is expected that all items can be considered in a formative manner.

The following items can be highlighted in Mathematics lesson observation:

1. Learners engaged in exploration/investigation/problem solving

Score	Description
3	Learners regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the learners engaged in exploration/investigation/problem solving.
2	Learners sometimes engaged in exploration, investigation, or problem solving. Several learners engaged in problem solving, but not the majority of the class.
1	Learners not often engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few learners engaged in problem solving while other learners watched but did not actively participate.
0	Learners did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.

2. Learners used a variety of materials or means (models, drawings, graphs, concrete materials, manipulative, etc) to represent mathematical concepts

Score	Description
3	The learners manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or learners, as appropriate.
2	The learners manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were not explicitly discussed by the teacher or learners.
1	The learners manipulated or generated one representation of a concept.
0	There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the learners only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.

3. Learners were engaged in mathematical activities

Score	Description
3	Most of the learners spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the learners are filling in the notes and interacting with the lesson mathematically.)
2	Most of the learners spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.
1	Most of the learners spend less than one-quarter of the lesson engaged in appropriate level mathematical activity. There is at least one instance of learners' mathematical engagement.
0	Most of the learners are not engaged in appropriate level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical – such as a coloring activity.

4. Learners critically assessed mathematical strategies

Score	Description
3	More than half of the learners critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
2	At least two but less than half of the learners critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
1	An individual learner critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a learner making a comment during direct instruction or individually to the teacher. The critical assessment was limited to one learner.

0	Learners did not critically assess mathematical strategies. This could happen for one of three reasons: 1) No strategies were used during the lesson; 2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally; 3) Strategies were discussed critically by the teacher but this amounted to the teacher telling the learners about the strategy (ies), and learners did not actively participate.

5. Learners persevered in problem solving

Score	Description
3	Learners exhibited a strong amount of perseverance in problem solving. The majority of learners looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of learners continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
2	Learners exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
1	Learners exhibited minimal perseverance in problem solving. At least one learner but less than half of learners looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one learner but less than half of learners continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a road block to score above a 0.
0	Learners did not persevere in problem solving. This could be because there was no learner problem solving in the lesson, or because when presented with a problem solving situation no learners persevered. That is to say, all learners either could not figure out how to get started on a problem, or when they confronted an obstacle in their strategy they stopped working.

6. The lesson promoted modelling with Mathematics

Score	Description
3	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with learners engaged in the modeling cycle.
2	Modeling is a major component, but the modeling has been turned into a procedure (i.e. a group of word problems that all follow the same form and the teacher has guided the learners to find the key pieces of information and how to plug them into a procedure.); **or** modeling is not a major component, but the students engage in a modeling activity that fits within the corresponding standard of mathematical practice.
1	The teacher describes some type of mathematical model to describe real-world situations, but the learners do not engage in activities related to using mathematical models.
0	The lesson does not include any modeling with Mathematics.

7. The lesson provided opportunities to examine mathematical structure. (Symbolic notation, patterns, generalizations, conjectures, etc)

Score	Description
3	The learners have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.
2	Learners are given some time to examine mathematical structure, but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.
1	Learners are shown generalizations involving mathematical structure, but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.
0	Learners are given no opportunities to explore or understand the mathematical structure of a situation.

8. The lesson included tasks that have multiple paths to a solution or multiple solutions

Score	Description
3	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different learners.
2	Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; **or** more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
1	Multiple solutions and/or multiple paths minimally occur, and are not explicitly encouraged; **or** a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
0	A lesson which focuses on a single procedure to solve certain types of problems and/or strongly discourages learners from trying different techniques.

9. The teacher's talk encouraged learner's thinking

Score	Description
3	The teacher's talk focused on **high levels** of mathematical thinking. The teacher may ask lower level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. ***Analysis***: examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. **Synthesis**: requires original, creative thinking. **Evaluation**: makes a judgment of good or bad, right or wrong, according to the standards he/she values.
2	The teacher's talk focused on **mid-levels** of mathematical thinking. **Interpretation**: discovers relationships among facts, generalizations, definitions, values and skills. **Application**: requires identification and selection and use of appropriate generalizations and skills
1	Teacher talk consists of **"lower order"** knowledge based questions and responses focusing on recall of facts. **Memory**: recalls or memorizes information. **Translation**: changes information into a different symbolic form or situation.
0	Any questions/ responses of the teacher related to mathematical ideas were rhetorical in that there was no expectation of a response from the students.

10. There were a high proportion of learners talking related to Mathematics

Score	Description
3	More than three quarters of the learners were talking related to the Mathematics of the lesson at some points during the lesson.
2	More than half, but less than three quarters of the learners were talking related to the Mathematics of the lesson at some points during the lesson.
1	Less than half of the learners were talking related to the Mathematics of the lesson.
0	No learners talked related to the Mathematics of the lesson.

11. There was a climate of respect for what others had to say

Score	Description
3	**Many** learners are sharing, questioning, and commenting during the lesson, including their struggles. Learners are also listening (active), clarifying, and recognizing the ideas of others.
2	The environment is such that **some** learners are sharing, questioning, and commenting during the lesson, including their struggles. Most learners listen.
1	Only a **few** share as called on by the teacher. The climate supports those who understand or who behave appropriately. **Or** Some learners are sharing, questioning, or commenting during the lesson, but most learners are actively listening to the communication.
0	No learners shared ideas.

12. In general, the teacher provided wait-time

Score	Description
3	The teacher **frequently** provided a sufficient amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a learner.
2	The teacher **sometimes** provided a sufficient amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a learner.
1	The teacher **rarely** provided a sufficient amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a learner.
0	The teacher **never** provided a sufficient amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a learner.

13. Learners were involved in the communication of their ideas to others (peer-to-peer)

Score	Description
3	**Considerable** time **(more than half)** was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.
2	**Some** class time **(less than half, but more than just a few minutes)** was devoted to peer to peer (pairs, groups, whole class) conversations related to the Mathematics.
1	The lesson was primarily teacher directed and little opportunities were available for peer to peer (pairs, groups, whole class) conversations. A few instances developed where this occurred during the lesson but only lasted less than 5 minutes.
0	No peer to peer (pairs, groups, whole class) conversations occurred during the lesson.

14. The teacher uses learner questions or comments to enhance conceptual mathematical understanding

Score	Description
3	The teacher **frequently** uses learner questions or comments to coach learners, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the learner responses that will be displayed in an intentional order, and/or connects different learners' responses to key mathematical ideas.
2	The teacher **sometimes** uses learner questions/ comments to enhance conceptual understanding.
1	The teacher **rarely** uses learner questions/ comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task verses conceptual knowledge of the content.
0	The teacher **never** uses learner questions/ comments to enhance conceptual mathematical understanding.

Application activity 11.1

Explain what you can do when the teacher whose lesson you observed refuses the feedback you are giving

End unit assessment 11

1. Explain why a teacher should be smart in terms of dressing.
2. Discuss the documents a teacher should carry during teaching and learning process.

Unit 12 MICRO-TEACHING

Key unit competence: Facilitate Mathematics lessons to peers in a simulated context

Introductory activity 12

Observe the figure illustrating one type of micro-teaching where one student-teacher is teaching his/her colleagues while the tutor is making an observation. Propose and justify the purpose of micro-teaching.

12.1. Micro-teaching

Activity 12.1

Take observation sheet and fill it while observing your neighbour teaching.

Content summary

Micro-teaching is a method which enables teacher trainees to practice a skill by teaching a short lesson to a small number of student-teachers. A tutor, using an appraisal guide, usually rates the lesson and then discusses it with the teacher trainee, where closed circuit television (CCTV) to take videos is available the appraisal guide may be redundant. The teacher trainee may alter his/her approach if necessary and later re-teaches the lesson to another group of student-teachers. This lesson is also rated by the supervisor and then analyzed and discussed with the teacher- trainee. The major premise underlying the concept of microteaching is that the complex teaching act can be split into component skills; each simple, well – defined and limited. These skills can be identified, practiced, evaluated, controlled and acquired through training. Micro-teaching implies micro-element that systematically attempts to simplify the complexities of the teaching process.

Micro-teaching is one of the most recent innovations in teacher training program which is used as a professional developmental tool in pre-service or in-service teacher training programs.

Micro-teaching helps student- teachers to better understand the processes of teaching and learning and provide the opportunity to learn teaching skills, to study their own teaching, and to study the teaching of others.

In Rwandan context, Micro-teaching is an organized, scaled-down teacher training program where a trainee teacher plans a short lesson, teaches it to a reduced group of student-teachers (Three to ten) or all his/her colleagues in a limited time (20 to 30 minutes) lesson, and then reflects on their teaching afterwards. The lesson is video recorded for either individual or peer review. The trainee teacher's micro-lesson is reviewed, discussed, analyzed, and evaluated to give a feedback. Based on this feedback, the trainee teacher re-teaches the micro-lesson, incorporating those points raised during the discussion and analysis. This micro-teaching has the potential to improve the student-teachers' pedagogic skills, competencies, self-confidence, beliefs, and attitudes with minimum available facilities and to provide student-teachers with valuable teaching experiences and make them aware of the benefits and relationships between theory and practice.

Core skills applicable in micro-teaching

The teaching activity as a whole is divided into its individual component skills.

1. **Lesson planning** with clear-cut objectives and an appropriate planned sequence. The content should be concise, appropriate, relevant, and could cover the specified duration.
2. **Introduction skill (Set induction)** - The process of gaining learner's attention at the beginning of the class by establishing rapport with learners, promoting their attentions, exposing them to essential contents, and linking their previous knowledge with the topic.
3. **Presentation and explanation skills** – Teacher enthusiasm, explanation, narration, giving appropriate illustrations and examples, planned repetition, and encouraging group discussion wherever necessary. The trainee teacher should be able to rightly explain the

concept by simple, relevant, and interesting examples to increase learners' understanding.

4. **Skill of stimulus variation –** Securing and sustaining the attention of the learner is imperative for a good teacher. The effective components of the skill are Gestures (Hand, facial, body),

   Change in the speech pattern, Voice variation and modulation (Pitch, volume, speed), Change in the interaction pattern, Focusing, Pausing movement, Emphasis on significant points.

5. **Proper use of audio-visual aids** – The increased awareness of the audiovisual aids and other equipment is important for this skill. Neatness, readability, adequate spacing, distinct size, proper spacing between words and lines, and use of relevant words or phrases are the key components for this skill.
6. **Skill of black-board writing:** Legibility (Easy to read, Size and alignment (In a straight line) , Highlighting main points , Utilization of the space ,Black-board summary ,Correctness ,Position of the teacher , Contact with the pupils
7. **Reinforcement** - Recognizing pupil's difficulties, listening, encouraging pupil participation and response. The use of positive verbal and non-verbal cues would be the key components for this skill.
8. **Skill of probing questions** - Probing questions are those which help the pupils to think in depth about the various aspects of the problem enabling pupils to understand the subject deeply. It is important to allow and encourage the fellow trainee teachers to ask structured questions and clarify doubts. Redirection, refocusing, and increasing critical awareness are important components of this skill.
9. **Silence and nonverbal cues** (Body language)
10. **Classroom management** – Providing proper instructions, restricting inappropriate behavior, and calling the pupils by name are essentials of this skill.
11. **Skill of achieving closure** - Method of concluding a teaching session so as to bring out the relevance of what has been learnt, its connection with past learning and its application to future learning. Questions and statements by the teacher by consolidation of the major points covered during the lesson and ability for applying the knowledge gained by pupils during the lesson to new situations. Closure should be timely! Prepare to start and end in time.

Benefits and roles of micro-teaching

- It provides the student teachers with real teaching situations
- It provides opportunity for practice and mastery of selected skills
- It offers feedback from tutor and peer that help to improve teaching skills,
- Student teachers get the opportunity to reflect and improve teaching skills.
- Boost the confidence of student teacher
- Allows student teachers to correct any mistake that may arises in scheming and lesson planning

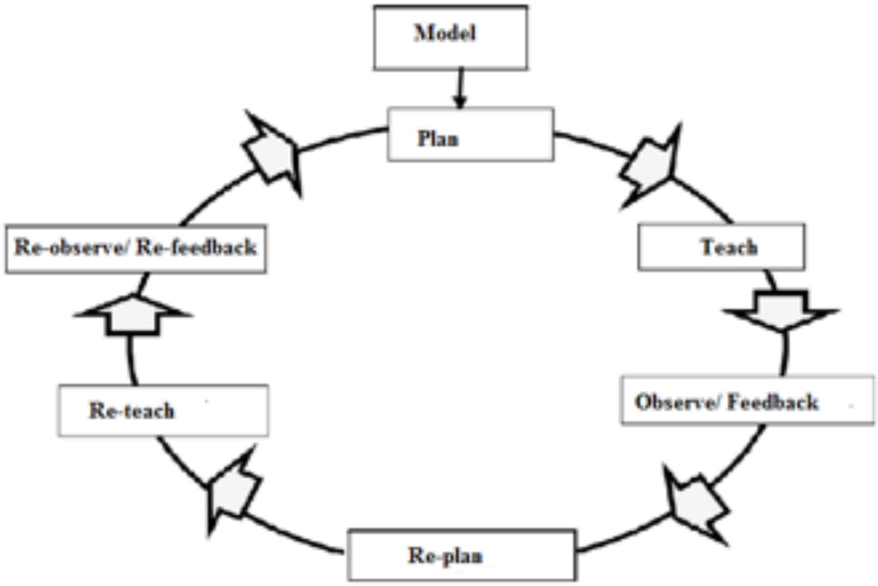
12.2. Steps in micro-teaching

1. **Orientation of the student teachers**: It involves providing necessary information and theoretical background about micro teaching on the following aspects:
   - Concept of micro teaching.
   - Significance of using micro teaching.
   - Procedures of micro teaching.
   - Requirements and setting for adopting micro teaching technique.
2. **Discussion of teaching skills**: In this step the concept of teaching skill is clarified to the teacher trainee so that they develop knowledge and understanding about :
   - Analysis of teaching into different component teaching skills.
   - Significance of these skills in classroom teaching.
   - Component teaching behaviours of different teaching skills.
3. **Selection of a particular teaching skill**: The teacher trainee selects a particular teaching skill for practice.
4. **Presentation of a model demonstration lesson**: A demonstration lesson in that particular teaching skill is presented to the teacher trainees. This stage is known as modelling. Demonstration can be given in a number of ways.
   - By exhibiting a film or a video tape.
   - By making them to listen to an audiotape.
   - By arranging a demonstration lesson from a live model i.e by the teacher educator or some expert.
   - By providing written material such as hand book, guide etc.
5. **Observation of the model lesson and criticism**: An observation schedule is designed for the observation of the lesson and is distributed to the teacher trainee. A critical appraisal of the model lesson is made by the student teachers on the basis of the observation and analysis.

6. **Preparation of micro lesson plan**: For practicing the demonstrated teaching skill, the student teacher prepares a micro lesson plan. For this he may take guidance and help from the teacher educator, books etc.
7. **Creation of micro-teaching setting**: The model of micro-teaching gives the following setting:
   - Number of students/pupils to tech
   - Type of pupils: real pupils or preferably peers
   - Type of supervisors - teacher educators or peers.
   - Time duration for micro-teaching lesson
   - Time duration for micro-teaching cycle (plan, teaching, feedback, re-plan, re-teach, re-feedback).
8. **Practice of the Skill**: Under this step the student teacher teaches a micro-lesson to a micro- class. This lesson is observed by the tutor and the peer group with the help of the appropriate observation schedule. The lesson can be recorded using an audiotape or video tape.
9. **Feedback:** Immediate feedback is given by the teacher educator and the peer group.
10. **Re-planning**: On the basis of the feedback the student teacher re-plans the lesson.
11. **Re-teaching**: The session where the student teacher re-teaches his/her micro-lesson on the basis of his/her re-planned lesson.
12. **Re-feedback**: The student teacher is provided re-feed back on the re-taught micro-lesson.
13. **Integration of the skills**: This is the last step and is concerned with the task of integrating several skills individually mastered by the student teacher. It is helpful in bridging a gap between training in isolated teaching skills and the real teaching situation faced by a teacher.

To sum up, during microteaching process, the "teach-re-teach" cycle may be repeated several times till desired level of skill or adequate mastery is achieved. Such repeated cycles of teaching, feedback and re-teaching help the teacher to improve his teaching skills one at a time.

Micro-teaching cycle (The cycle continues up to the extend when a trainee will be able to master a specific skill).



Source: Reddy KR, 2019; adapted by the content provider

Application activity 12.1

1. Consider the definition of micro-teaching and suppose that through in-service, teachers adopt this process at their school to improve the teaching practice where a group of teachers observe their colleague when teaching and give him/her feed back after that lesson and ask him/her to improve that lesson in a parallel class. This method is called a lesson study. Do research on internet and compile a document on the lesson study and highlighting the definition, its process, advantages and how it can be adapted in Rwandan context at your school.
2. In the teaching practice, summarize the following: who is supposed to give feedback, to whom, when and why to give feedback; the process of giving feedback and receiving feedback: readiness to accept and to improve.

End unit assessment 12

1. Discuss the skills a student teacher gains from micro-teaching.
2. Compare the microteaching and the lesson study.

Unit 13 CLASSROOM TEACHING PRACTICE

Note: This unit is supposed to be taught in year 2 of TTC

Key unit competence: Facilitate various mathematics lessons in demonstration schools

Introductory activity 13

Chose a lesson plan for primary Mathematics to be taught in demonstration school and analyse its implementation, step by step.

13.1. Teaching practice

Activity 13.1

Discuss the responsibilities of a mathematics teacher in the classroom.

Content summary

Teaching practice is a period that a student **teacher** spends **teaching** at a school as part of his or her training.

It can be also: an organized activity that helps the student teacher to real teaching and learning situation with an aim of helping him/her to gain confidence and experience in teaching.

It aims at preparing the student teacher for a teaching career at the primary school: the classroom organization and management and the mathematics teaching skills.

Examples of mathematics teaching skills: *The skill of questioning, the skill of reinforcement, the skill of probing, the skill of explaining, the skill of stimulus variation, the skill of introducing a lesson, manipulative skill, the skill of illustrating with examples, Skill of demonstration , the skill of using blackboard, the skill of silence and non verbal cues, the skill of using audio – visual aids, the skill of recognizing attending behavior, the skill of achieving closure.*

Purpose of teaching practice

- It provides the opportunity to develop positive approach and attitudes to the school and school community which facilitate to grow in profession awareness and development of the student teacher.
- It provides the opportunities to establish relationship with learners, teachers and other stakeholder involved in education.
- It provides opportunity to discover and develop one's ability as a teacher,
- It gives the opportunity to interact and understand learners and attempt to cater for their individual need;
- It provides the opportunities to have classroom experiences and apply the theoretical skills learnt in college;
- It gives the opportunity to develop qualities crucial to teaching such as humility, interpersonal skills, openness and patience.

Application activity 13.1

Discuss the teaching skills to be possessed by a mathematics teacher of primary school.

End unit assessment 13

Explain what you can do to verify learners' readiness when you start teaching mathematics for the first time in P2 or P5 classroom.

Note:The guidance on the school attachment is the same as the guidance given for school practice except that the teaching related activities are done in a school where the student teacher takes all responsibilities of a teacher in the school.

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