MATHEMATICS FOR TTCs

TUTOR'S BOOK

YEAR

1

OPTION:

Science and Mathematics Education (SME)

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FOREWORD

Dear Tutor,

Rwanda Education Board is honoured to present the tutor's guide for Mathematics in the option of SME which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of the mathematics content. The Rwandan educational philosophy is to ensure that learners achieve full potential at every level of education which will prepare them to be well integrated in society and exploit employment opportunities.

Specifically, TTC curriculum was reviewed to train quality student-teachers who will confidently and efficiently implement the Competence Based Curriculum in pre-primary and primary education. The rationale of the changes is to ensure that TTC leavers are qualified for job opportunities and further studies in Higher Education in different programs under education career advancement.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate their learning process. Many factors influence what they learn, how well they learn and the competences they acquire. Those factors include the relevance of the specific content, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials.

The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market has necessitated the shift to a competence-based curriculum. After a successful shift from knowledge to a competence based curriculum in general education, TTC textbooks also were elaborated to align them to the new curriculum.

The book provides active teaching and learning techniques that engage student teachers to develop competences. In view of this, your role is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize group discussions for students considering the importance of social constructivism suggesting that learning occurs more effectively when the student-teachers works collaboratively with more knowledgeable and experienced people.
- Engage student-teachers through active learning methods such as inquiry methods, group discussions, research, investigative activities and group and individual work activities.

- Provide supervised opportunities for student-teachers to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation.
- Support and facilitate the learning process by valuing students' contributions in the class activities.
- Guide student-teachers towards the harmonization of their findings.
- Encourage individual, peer and group evaluation of the work done in the classroom and use appropriate competence-based assessment approaches and methods.

To facilitate you in your teaching activities, the content of this book is self explanatory so that you can easily use it. It is divided in 2 parts:

The part 1 explains the structure of this book and gives you the methodological guidance;

The part 2 details the teaching guidance for each concept given in the student book.

Even though this Teacher's guide contains the guidance on solutions for all activities given in the learner's book, you are requested to work through each question before judging student's findings.

I wish to sincerely express my appreciation to the people who contributed towards the development of this book, particularly, REB staff, TTC Tutors, Teachers from general education for their technical support. A word of gratitude goes also to the Head Teachers and TTCs principals who availed their staff for various activities.

Dr. NDAYAMBAJE Irénée

Director General, REB

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I wish to express my appreciation to the people who played a major role in the development of this tutor`s guide for Mathematics in the option of SME. It would not have been successful without active participation of different education stakeholders.

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I wish to extend my sincere gratitude to lecturers and teachers whose efforts during writing exercise of this tutor`s guide was very much valuable.

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Joan MURUNGI

Head of CTLR Department

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PART I. GENERAL INTRODUCTION

1.1 The structure of the guide

The tutor's guide of Mathematics is composed of two parts:

The Part 1 concerns general introduction that discusses methodological guidance on how best to teach and learn Mathematics, developing competences in teaching and learning, addressing cross-cutting issues in teaching and learning and Guidance on assessment.

The Part 2 is about the structure of a unit and the structure of a lesson. This includes information related to the different components of the unit and these components are the same for all units. This part provides information and guidelines on how to facilitate student teachers while working on learning activities. More other, all application activities from the textbook have answers in this part.

1.2 Methodological guidance

1.2.1 Developing competences

Since 2015 Rwanda shifted from a knowledge based to a competency-based curriculum for pre-primary, primary, secondary education and recently the TTC curriculum. This called for changing the way of learning by shifting from teacher centred to a learner centred approach. Teachers are not only responsible for knowledge transfer but also for fostering learners' learning achievement and creating safe and supportive learning environment. It implies also that learners have to demonstrate what they are able to transfer the acquired knowledge, skills, values and attitude to new situations.

The competence-based curriculum employs an approach of teaching and learning based on discrete skills rather than dwelling on only knowledge or the cognitive domain of learning. It focuses on what learner can do rather than what learner knows. Learners develop competences through subject unit with specific learning objectives broken down into knowledge, skills and attitudes through learning activities.

In addition to the competences related to Mathematics, student teachers also develop generic competences which should promote the development of the higher order thinking skills and professional skills in Mathematics teaching. Generic competences are developed throughout all units of Mathematics as follows:

Generic competences	Ways of developing generic competences
Critical thinking	All activities that require learners to calculate, convert, interpret, analyse, compare and contrast, etc have a common factor of developing critical thinking into learners
Creativity and innovation	All activities that require learners to plot a graph of a given algebraic data, to organize and interpret statistical data collected and to apply skills in solving problems of economics have a common character of developing creativity into learners
Research and problem solving	All activities that require learners to make a research and apply their knowledge to solve problems from the real-life situation have a character of developing research and problem solving into learners.
Communication	During Mathematics class, all activities that require learners to discuss either in groups or in the whole class, present findings, debate …have a common character of developing communication skills into learners.
Co-operation, interpersonal relations and life skills	All activities that require learners to work in pairs or in groups have character of developing cooperation and life skills among learners.
Lifelong learning	All activities that are connected with research have a common character of developing into learners a curiosity of applying the knowledge learnt in a range of situations. The purpose of such kind of activities is for enabling learners to become life-long learners who can adapt to the fast-changing world and the uncertain future by taking initiative to update knowledge and skills with minimum external support.
Professional skills	Specific instructional activities and procedures that a teacher may use in the class room to facilitate, directly or indirectly, students to be engaged in learning activities. These include a range of teaching skills: the skill of questioning, reinforcement, probing, explaining, stimulus variation, introducing a lesson; illustrating with examples, using blackboard, silence and non verbal cues, using audio – visual aids, recognizing attending behaviour and the skill of achieving closure.

The generic competences help learners deepen their understanding of Mathematics and apply their knowledge in a range of situations. As students develop generic competences they also acquire the set of skills that employers look for in their employees, and so the generic competences prepare students for the world of work.

1.2.2 Addressing cross cutting issues

Among the changes brought by the competence-based curriculum is the integration of cross cutting issues as an integral part of the teaching learning process-as they relate to and must be considered within all subjects to be appropriately addressed. The eight cross cutting issues identified in the national curriculum framework are: *Comprehensive Sexuality Education, Environment and Sustainability, Financial Education, Genocide studies, Gender, Inclusive Education, Peace and Values Education, and Standardization Culture.*

Some cross-cutting issues may seem specific to particular learning areas/subjects but the teacher need to address all of them whenever an opportunity arises. In addition, learners should always be given an opportunity during the learning process to address these cross-cutting issues both within and out of the classroom.

Below are examples of how cross-cutting issues can be addressed:

Cross-Cutting Issue	Ways of addressing cross-cutting issues
Comprehensive Sexuality Education: The primary goal of introducing Comprehensive Sexuality Education program in schools is to equip children, adolescents, and young people with knowledge, skills and values in an age appropriate and culturally gender sensitive manner so as to enable them to make responsible choices about their sexual and social relationships, explain and clarify feelings, values and attitudes, and promote and sustain risk reducing behaviour.	Using different charts and their interpretation, Mathematics tutor should lead students to discuss the following situations: "Alcohol abuse and unwanted pregnancies" and advise student teachers on how they can instil learners to fight those abuses. Some examples in powers and statistics should be used to well address the cross-cutting issue

Environment and Sustainability: Integration of Environment, Climate Change and Sustainability in the curriculum focuses on and advocates for the need to balance economic growth, society well-being and ecological systems. Learners need basic knowledge from the natural sciences, social sciences, and humanities to understand to interpret principles of sustainability.	Using Real life models or students' experience, Mathematics Tutor should lead student teachers to illustrate the situation of "population growth" and discuss its effects on the environment and sustainability. Some examples in proportional change, logarithms, and polynomial should be used to well address the cross-cutting issue
Financial Education: The integration of Financial Education into the curriculum is aimed at a comprehensive Financial Education program as a precondition for achieving financial inclusion targets and improving the financial capability of Rwandans so that they can make appropriate financial decisions that best fit the circumstances of one's life.	Through different examples and calculations on interest rate problems, total revenue and total cost, Mathematics Tutor can lead student teachers to discuss how to make appropriate financial decisions. Some examples in ratios and proportions, statistics and equations or polynomial functions should be used to well address the cross-cutting issue
Gender: At school, gender will be understood as family complementarities, gender roles and responsibilities, the need for gender equality and equity, gender stereotypes, gender sensitivity, etc.	Mathematics Tutor should address gender as cross-cutting issue through assigning leading roles in the management of groups to both girls and boys and providing equal opportunity in the lesson participation and avoid any gender stereotype in the whole teaching and learning process.
Inclusive Education: Inclusion is based on the right of all learners to a quality and equitable education that meets their basic learning needs and understands the diversity of backgrounds and abilities as a learning opportunity.	Firstly, Mathematics Tutors need to identify/recognize students with special needs. Then by using adapted teaching and learning resources while conducting a lesson and setting appropriate tasks to the level of students, they can cater for students with special education needs. They must create opportunity where student teachers can discuss how to cater for learners with special educational needs.

Peace and Values Education: Peace and Values Education (PVE) is defined as education that promotes social cohesion, positive values, including pluralism and personal responsibility, empathy, critical thinking and action in order to build a more peaceful society.	Through a given lesson, a tutor should: • Set a learning objective which is addressing positive attitudes and values, • Encourage students to develop the culture of tolerance during discussion and to be able to instil it in colleagues and cohabitants; • Encourage students to respect ideas from others.
Standardization Culture: Standardization Culture in Rwanda will be promoted through formal education and plays a vital role in terms of health improvement, economic growth, industrialization, trade and general welfare of the people through the effective implementation of Standardization, Quality Assurance, Metrology and Testing.	With different word problems related to the effective implementation of Standardization, Quality Assurance, Metrology and Testing, students can be motivated to be aware of health improvement, economic growth, industrialization, trade and general welfare of the people.

1.2.3 Guidance on how to help student-teachers with special education needs in classroom

In the classroom, student-teachers learn in different way depending to their learning pace, needs or any other special problem they might have. However, the tutor has the responsibility to know how to adopt his/her methodologies and approaches in order to meet the learning need of each student in the classroom. Also tutors need to understand that student-teachers with special needs, need to be taught differently or need some accommodations to enhance the learning environment. This will be done depending to the subject and the nature of the lesson.

In order to create a well-rounded learning atmosphere, teachers need to:

- Remember that learners learn in different ways so they have to offer a variety of activities (e.g. role-play, music and singing, word games and quizzes, and outdoor activities);
- Maintain an organized classroom and limits distraction. This will help learners with special needs to stay on track during lesson and

follow instruction easily;

- Vary the pace of teaching to meet the needs of each child. Some learners process information and learn more slowly than others;
- Break down instructions into smaller, manageable tasks. Learners with special needs often have difficulty understanding long-winded or several instructions at once. It is better to use simple, concrete sentences in order to facilitate them understand what you are asking.
- Use clear consistent language to explain the meaning (and demonstrate or show pictures) if you introduce new words or concepts;
- Make full use of facial expressions, gestures and body language;
- Pair a learner who has a disability with a friend. Let them do things together and learn from each other. Make sure the friend is not over protective and does not do everything for the one with disability. Both learners will benefit from this strategy;
- Use multi-sensory strategies. As all learners learn in different ways, it is important to make every lesson as multi-sensory as possible. Learners with learning disabilities might have difficulty in one area, while they might excel in another. For example, use both visual and auditory cues.

Below are general strategies related to each main category of disabilities and how to deal with every situation that may arise in the classroom. However, the list is not exhaustive because each child is unique with different needs and that should be handled differently.

Strategy to help learners with developmental impairment:

- Use simple words and sentences when giving instructions;
- Use real objects that learners can feel and handle. Rather than just working abstractly with pen and paper;
- Break a task down into small steps or learning objectives. The learner should start with an activity that she/he can do already before moving on to something that is more difficult;
- Gradually give the learner less help;
- Let the learner with disability work in the same group with those without disability.

Strategy to help learners with visual impairment:

- Help learners to use their other senses (hearing, touch, smell and taste) and carry out activities that will promote their learning and development;
- Use simple, clear and consistent language;
- Use tactile objects to help explain a concept;
- If the learner has some sight, ask him/her what he/she can see;
- Make sure the learner has a group of friends who are helpful and who allow him/her to be as independent as possible;
- Plan activities so that learners work in pairs or groups whenever possible;

Strategy to help learners with hearing disabilities or communication difficulties

- Always get the learner's attention before you begin to speak;
- Encourage the learner to look at your face;
- Use gestures, body language and facial expressions; Use pictures and objects as much as possible.
- Keep background noise to a minimum.

Strategies to help learners with physical disabilities or mobility difficulties:

- Adapt activities so that learners who use wheelchairs or other mobility aids, can participate.
- Ask parents/caregivers to assist with adapting furniture e.g. the height of a table may need to be changed to make it easier for a learner to reach it or fit their legs or wheelchair under;
- Encourage peer support when needed;
- Get advice from parents or a health professional about assistive devices if the learner has one.

Adaptation of assessment strategies:

At the end of each unit, the tutor is advised to provide additional activities to help students achieve the key unit competence. These assessment activities are for remedial, consolidation and extension designed to cater for the needs of all categories of student-teachers; slow, average and gifted student-teachers respectively. Therefore, the tutor is expected to do assessment that fits individual student.

Remedial activities	After evaluation, slow students are provided with lower order thinking activities related to the concepts learnt to facilitate them in their learning. These activities can also be given to assist deepening knowledge acquired through the learning activities for slow students.
Consolidation activities	After introduction of any concept, a range number of activities can be provided to all students to enhance/ reinforce learning.
Extended activities	After evaluation, gifted and talented students can be provided with high order thinking activities related to the concepts learnt to make them think deeply and critically. These activities can be assigned to gifted and talented student-teachers to keep them working while other student-teachers are getting up to required level of knowledge through the learning activity.

1.2.4. Guidance on assessment

Assessment is an integral part of teaching and learning process. The main purpose of assessment is for improvement of learning outcomes. Assessment for learning/ Continuous/ formative assessment intends to improve students' learning and tutor's teaching whereas assessment of learning/summative assessment intends to improve the entire school's performance and education system in general.

Continuous/ formative assessment

It is an on-going process that arises during the teaching and learning process. It includes lesson evaluation and end of sub unit assessment. This formative assessment should play a big role in teaching and learning process. The teacher should encourage individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods.

Formative assessment is used to:

- Determine the extent to which learning objectives are being achieved and competences are being acquired and to identify which students need remedial interventions, reinforcement as well as extended activities. The application activities are developed in the learner book and they are designed to be given as remedial, reinforcement, end lesson assessment, homework or assignment

- Motivate students to learn and succeed by encouraging students to read, or learn more, revise, etc.
- Check effectiveness of teaching methods in terms of variety, appropriateness, relevance, or need for new approaches and strategies. Mathematics tutors need to consider various aspects of the instructional process including appropriate language levels, meaningful examples, suitable methods and teaching aids/ materials, etc.
- Help student-teachers to take control of their own learning.

In teaching Mathematics, formative or continuous assessment should compare performance against instructional objectives. Formative assessment should measure the student's ability with respect to a criterion or standard. For this reason, it is used to determine what students can do, rather than how much they know.

Summative assessment

The assessment can serve as summative and informative depending to its purpose. The end unit assessment will be considered summative when it is done at end of unit and want to start a new one.

It will be formative assessment, when it is done in order to give information on the progress of learners and from there decide what adjustments need to be done.

The assessment done at the end of the term, end of year, is considered as summative assessment so that the tutor, school and parents are informed of the achievement of educational objective and think of improvement strategies. There is also end of level/ cycle assessment in form of national examinations.

When carrying out assessment?

Assessment should be clearly visible in lesson, unit, term and yearly plans.

- Before learning (diagnostic): At the beginning of a new unit or a section of work; assessment can be organized to find out what student-teachers already know/ can do, and to check whether the students are at the same level.

- During learning (formative/continuous): When students appear to be having difficulty with some of the work, by using on-going assessment (continuous). The assessment aims at giving students support and feedback.
- After learning (summative): At the end of a section of work or a learning unit, the Mathematics Tutor has to assess after the learning. This is also known as Assessment of Learning to establish and record overall progress of students towards full achievement. Summative assessment in Rwandan schools mainly takes the form of written tests at the end of a learning unit or end of the month, and examinations at the end of a term, school year or cycle.

Instruments used in assessment.

- **Observation:** This is where the Mathematics tutor gathers information by watching student-teachers interacting, conversing, working, playing, etc. A tutor can use observations to collect data on behaviours that are difficult to assess by other methods such as attitudes, values, and generic competences and intellectual skills. It is very important because it is used before the lesson begins and throughout the lesson since the tutor has to continue observing each and every activity.

- **Questioning**

(a) Oral questioning: a process which requires a student to respond verbally to questions

(b) Class activities/ exercise: tasks that are given during the learning/ teaching process

(c) Short and informal questions usually asked during a lesson

(d) Homework and assignments: tasks assigned to students by their tutors to be completed outside of class.

Homework assignments, portfolio, project work, interview, debate, science fair, Mathematics projects and Mathematics competitions are also the different forms/ instruments of assessment.

1.2.5. Teaching methods and techniques that promote active learning

The different learning styles for students can be catered for, if the teacher uses active learning whereby learners are really engaged in the learning process.

The main teaching methods used in mathematics are the following:

- **Dogmatic method** (the teacher tells the students what to do, What to observe, How to attempt, How to conclude)
- **Inductive-deductive method:** Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples.
- **Analytic-synthetic method:** Analytic method proceeds from unknown to known, 'Analysis' means 'breaking up' of the problem in hand so that it ultimately gets connected with something obvious or already known. Synthetic method is the opposite of the analytic method. Here one proceeds from known to unknown.
- **Skill laboratory method:** Skill Laboratory method is based on the maxim "learning by doing." It is a procedure for stimulating the activities of the students and to encourage them to make discoveries through practical activities.
- Problem solving method, Project method and Seminar Method.

The following are some active techniques to be used in Mathematics:

- Group work
- Research
- Probing questions
- Practical activities (drawing, plotting, interpreting graphs)
- Modelling
- Brainstorming
- Quiz Technique
- Discussion Technique
- Scenario building Technique

What is Active learning?

Active learning is a pedagogical approach that engages learners in doing things and thinking about the things they are doing. Learners play the key role in the active learning process. They are not empty vessels to fill but people with ideas, capacity and skills to build on for effective learning. Thus, in active learning, learners are encouraged to bring their own experience and knowledge into the learning process.

The role of the teacher in active learning	The role of learners in active learning
• The tutor engages learners through active learning methods such as inquiry methods, group discussions, research, investigative activities, group and individual work activities. • He/she encourages individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods. • He provides supervised opportunities for learners to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation. • Teacher supports and facilitates the learning process by valuing learners' contributions in the class activities.	A learner engaged in active learning: • Communicates and shares relevant information with peers through presentations, discussions, group work and other learner-centred activities (role play, case studies, project work, research and investigation); • Actively participates and takes responsibility for his/her own learning; • Develops knowledge and skills in active ways; • Carries out research/investigation by consulting print/online documents and resourceful people, and presents their findings; • Ensures the effective contribution of each group member in assigned tasks through clear explanation and arguments, critical thinking, responsibility and confidence in public speaking • Draws conclusions based on the findings from the learning activities.

Main steps for a lesson in active learning approach

All the principles and characteristics of the active learning process highlighted above are reflected in steps of a lesson as displayed below. Generally, the lesson is divided into three main parts whereby each one is divided into smaller steps to make sure that learners are involved in the learning process. Below are those main part and their small steps:

1) Introduction

Introduction is a part where the teacher makes connection between the current and previous lesson through appropriate technique. The teacher opens short discussions to encourage learners to think about the previous learning experience and connect it with the current instructional objective. The teacher reviews the prior knowledge, skills and attitudes which have a link with the new concepts to create good foundation and logical sequencing.

2) Development of the new lesson

The development of a lesson that introduces a new concept will go through the following small steps: discovery activities, presentation of learners' findings, exploitation, synthesis/summary and exercises/application activities.

❖ **Discovery activity**

Step 1

- The tutor discusses convincingly with learners to take responsibility of their learning
- He/she distributes the task/activity and gives instructions related to the tasks (working in groups, pairs, or individual to instigate collaborative learning, to discover knowledge to be learned)

Step 2

- The tutor let learners work collaboratively on the task;
- During this period the teacher refrains to intervene directly on the knowledge;
- He/she then monitors how the learners are progressing towards the knowledge to be learned and boosts those who are still behind (but without communicating to them the knowledge).

❖ **Presentation of learners' findings/productions**

- In this episode, the tutor invites representatives of groups to present their productions/findings.
- After three/four or an acceptable number of presentations, the tutor decides to engage the class into exploitation of learners' productions.

❖ **Exploitation of learner's findings/ productions**

- The tutor asks learners to evaluate the productions: which ones are correct, incomplete or false
- Then the tutor judges the logic of the learners' products, corrects those which are false, completes those which are incomplete, and confirms those which are correct.

❖ **Institutionalization or harmonization (summary/conclusion/ and examples)**

- The tutor summarizes the learned knowledge and gives examples which illustrate the learned content.

❖ **Application activities**

- Exercises of applying processes and products/objects related to learned unit/sub-unit
- Exercises in real life contexts
- Tutor guides learners to make the connection of what they learnt to real life situations. At this level, the role of teacher is to monitor the fixation of process and product/object being learned.

3) Assessment

In this step the teacher asks some questions to assess achievement of instructional objective. During assessment activity, learners work individually on the task/activity. The tutor avoids intervening directly. In fact, results from this assessment inform the teacher on next steps for the whole class and individuals. In some cases, the teacher can end with a homework/ assignment. Doing this will allow learners to relay their understanding on the concepts covered that day. Tutor leads them not to wait until the last minute for doing the homework as this often results in an incomplete homework set and/or an incomplete understanding of the concept.

PART II: SAMPLE LESSON PLAN

School:..........................

Tutor's Name:...

Term	Date	Subject	Class	Unit Nº	Lesson Nº	Duration	Class size
------	----/------ /-----	MATHEMATICS	Year one	6	1of 6	80minutes	40 student-teachers

Type of Special Educational Needs to be catered for in this lesson and number of learners in each category	2 student-teachers with hearing impairment will be seat near the tutor and the use of gestures will be improved in the lesson

Unit title	PROBLEMS ON POWERS, INDICES, RADICAL S AND COMMON LOGALITHMS
Key Unit Competence:	Solve problems related to powers, indices, radical and common logarithms
Title of the lesson	Definition of powers/ indices and radicals
Instructional Objective	Through the given activities, student-teachers should be able to solve problems involving powers accurately using properties of powers which are written on flash cards.
Plan for this Class (location: in / outside)	Inside the classroom
Learning Materials (for all learners)	Flash Cards, papers, Pens, Exercise Books, other supporting teaching aids such as Chalks and Chalkboard, etc...
References	Student-teachers book of mathematics year one and Tutor's guide year one

Steps and Timing	Description of teaching and learning activities		Competences and Cross-Cutting Issues to be addressed
	Student-Teachers are organized into groups to discuss the activity 6.1.1 and the examples, the reporter from one group, presents the findings and the Student-Teachers interact. The tutor facilitates Student-Teachers to capture the key concepts of the lesson through harmonization. -Finally, the Student-Teachers are assigned to individual tasks, and the correction is done on the chalk board.		
	Tutors activities	**Learners activities**	
Introduction **10mins**	**Powers and related problems** Tutor distributes flash cards to student-teachers in their small group discussions and invite them to brainstorm on the activity 6.1.1 Tutor moves around to help those who are struggling and guides them in finding of definition and properties of powers. Tutor invites student-teachers to present their findings. Tutor harmonizes the answers from presentation	Student-teachers receive flashcards, discuss and brainstorm on the activity 6.1.1 They guess the definition of power and properties of powers. **Group representative present their findings from groups and other participate actively in the presentation by asking questions**	Cooperation Improves team working spirit and developed through working together in small group discussions. Communication skills are developed through small group discussions

Development of the lesson: 20mins	Tutor assigns to student-teachers in their small group discussions the examples 6.1.1 (1 and 2) and invite them to brainstorm on and gives instructions related to this activity. Tutor moves around in order to help struggling student-teachers Tutor invites student-teachers to present their findings.	In their respective groups, Student-teachers discuss and brainstorm on examples 6.1.1 (1 and 2) Student-teachers present their findings.	**Critical thinking**, **problem solving skills** and **Finance Education** are developed through analysing and solving real life Mathematical problem. **Cooperation** and **communication** are developed during presentations and group discussions **Inclusive education** is addressed by providing the remediation activities and tasks to struggling student-teachers.
Conclusion 10 min	**Summary:** Tutor guides student-teachers to summarize the lesson of the day.	Student-teachers summarize the lesson guided by the tutor.	Communication skills is developed through small group discussion and presentation.

	Assessment -Tutor asks learners to individually work out the application activity 6.1.1	Student-teachers work independently on the application activity 6.1.1	**Critical thinking** and **problem solving skills** are developed through analysing and solving real life Mathematical problem.
	Tutor give the homework to student-teachers	Write homework and ask more clarification on it.	**Critical thinking** and **problem solving skills** are developed through analysing and solving real life Mathematical problem.
Tutor self-evaluation	To be completed after receiving the feed-back from the Student-teachers (what did the Student-teachers liked, what challenged them,...)		

PART III: UNIT DEVELOPMENT

UNIT 1

SET OF NUMBERS

1.1 Key unit competence

Classify numbers into naturals, integers, rational and irrationals

1.2 Prerequisite

Student-teachers will perform well in this unit if they have a good background to:

- Correctly use language, vocabulary and symbols of mathematics learnt in Ordinary Level such as set and its elements, cardinality of sets, etc;
- Correctly carry out basic operations such as addition, subtraction, multiplication and division of different numbers;
- Interpret simple diagrams and recognize ways in which representations can be misleading

1.3 Cross-cutting issues to be addressed

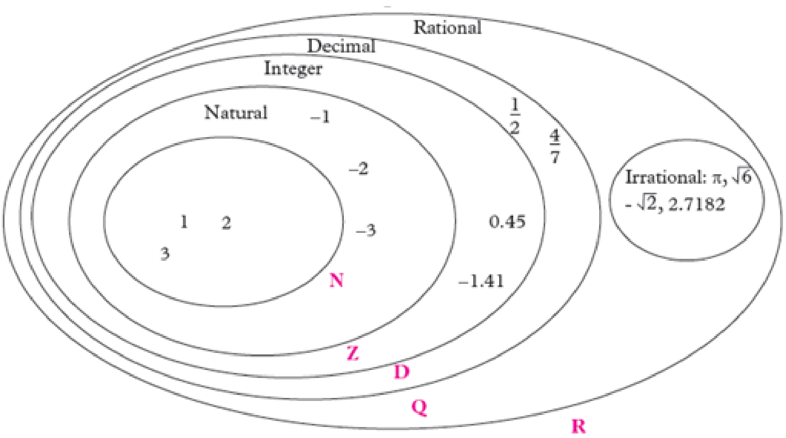
- **Inclusive education**: promote the participation of all student-teachers while teaching by providing remedial, consolidation and extended activities where needed and prepare enough and appropriate teaching and learning materials as well as providing special assistance to any case of student-teacher in need of special education needs.
- **Peace and value Education**: During group activities, tutor will encourage student-teachers to help each other and to respect opinions of colleagues.
- **Gender:** Give equal opportunities to all learners (girls and boys) to present their findings. Encourage them to participate actively in all learning and teaching activities from the beginning to the end of the lesson.

1.4 Guidance on introductory activity 1

- Form small groups of student-teachers and guide them to work on the introductory activity1.
- Provide learning materials accordingly to the given activities and give clear guidance and instructions to perform the activities.
- Give time to student-teachers to read and analyse the given activity and let them discuss about different possible solutions of the problem.
- Walk around in different groups to provide advice and facilitations where necessary and remind them to justify and support their answer / findings.
- Lead student-teachers to recognize that the given activity should get different answers depending on the considered set of number.
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for the introductory activity, use different questions to prompt them to give their predictions and ensure that you arouse their curiosity on what is going to be leant in the unit 1.
- After presenting their finding, tutor harmonize and guide class discussions and interventions.

Answer of introductory activity 1 (Answers may vary)

1. Lead student teacher to know that in the question1, set of numbers they already know from senior one (S1) in secondary schools, are: $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \ldots$.
2. The numbers we use in counting plus zero are called Natural numbers; integers are numbers which have either negative or positive sign and includes zero. The set of integers is represented by $\mathbb{Z}$; the set of rational numbers $\mathbb{Q}$ and the set of irrational numbers $\mathbb{I}$ form the set of real numbers. The set of real numbers is denoted by $\mathbb{R}$
3. Some examples of numbers in each set:



4. The relationship between set of numbers is as follows: Natural numbers are part of integers, integers are part of rational numbers, rational numbers and irrational numbers are pats of real numbers. Therefore, $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

1.5 List of lessons

No	Lesson title	Learning objectives	Number of periods
0	Introduction activity	Arise the curiosity of student teachers on the set of numbers	1
1	Definition of the set of natural numbers and its Subsets	Identify set of natural numbers, its subsets and establish the relationship between them	2
2	Operations and properties on natural numbers	Work systematically to carry out basic operations on natural numbers and deduce the related properties.	1
3	Definition and Subsets of integers	Identify set of integers and relationship between its subsets.	1
4	Operations and properties on integers	Work systematically to carry out basic operations on integers and deduce the related properties.	1

5	Definition and Subsets of rational numbers	Represent rational numbers as a fraction or a decimal which may terminate or occur and identify the relationship between its subsets	1
6	Operations and properties on rational numbers	Work systematically to carry out basic operations on rational numbers and deduce the related properties.	1
7	Definition of irrational numbers	Show that irrational numbers cannot be expressed exactly as decimal	1
8	Definition and Sub sets of real numbers	Work systematically to carry out basic operations on real numbers.	1
9	Operations and properties on real numbers	Work systematically to determine the operation properties on set of real numbers.	2
10	End unit assessment		2
Total number of periods in this unit.			**14**

Lesson 1: Definition of natural numbers and its sub sets

a) Learning objective: Identify set of natural numbers, its subsets and establish the relationship between them

b) Teaching resources: Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites / Revision / Introduction:

Student-teachers will perform well in this unit if they have a good background on how to use correctly language, vocabulary and mathematical symbols previously learnt. The mastery of different concepts related to set of numbers will be needed and these includes: elements of a set, sub-sets, types of set notation, number of elements in a set, set representation, etc.

d) Learning activities

- Remind all students that they are full of potentials to perform any given activity related to the set of numbers.
- Organize the students into small groups and help them to look for a group secretary, group representative, and timekeeper. Make sure boys and girls participate actively in their groups and have equal opportunities.
- Ask student-teachers to read and workout the **activity 1.1.1** from their book.
- Be around and facilitate student-teachers where possible by providing advice, prompting, and reminding them to give their own examples.
- After a given time, ask randomly some groups to present their findings to the whole class
- During the presentation, let students discover that the numbers used to register different assert are natural numbers.
- Guide student- teachers to work individually **application activity** 1.1.1, given in their book in order to better improve their knowledge and skills as well as develop competences.
- Let student-teachers in small groups workout the activity 1.1.2. from student-teacher's book.
- After a given time, ask randomly some groups to present their findings to the whole class and during presentation, lead student-teachers to discover that any given set of natural numbers can be subdivided into different subsets.
- Ask student-teachers to work individually the application activity 1.1.2, given in their book, for consolidation of the skills they have acquired.

e) Answers of activity 1.1.1

All of these numbers belong to the set of natural numbers.

For other elements of set of natural numbers, answers may vary but any value might be an element of $\mathbb{N}=\{0,1,2,3,\ldots\}$

f) Answers of application Activity 1.1.1

- First ten elements of natural numbers starting from zero are $0,1,2,3,4,5,6,7,8$ and 9.
- Natural numbers have a great importance in real life and they are mainly used in different domain of everyday life. The following are examples where natural numbers can be used:
- When people want to indicate time in hour, minute, second, they use natural numbers.
- When people count living and non-living things, they record the results using natural numbers.
- When people buy different items, they use money and calculations using natural numbers.
- Natural numbers can be used in counting the votes for and against something.
- Natural numbers are used in making different payments (school fees, renting house, traveling, paying bills, etc). All of those are "daily life" applications of natural numbers.

a) Answers of activity 1.1.2

a) • Even numbers are numbers which are divisible by 2 or numbers which are multiples of 2.

- Odd numbers are numbers which leave a remainder of 1 when divided by 2.
- Prime number is a number that has only two divisors; 1 and itself.

b) (i) Odd numbers between 0 and 20 are 1,3,5,7,9,11,13,15,17 and 19.

(ii) Even numbers between 0 and 20 are 2, 4, 6, 8, 10,12, 14, 16 and 18.

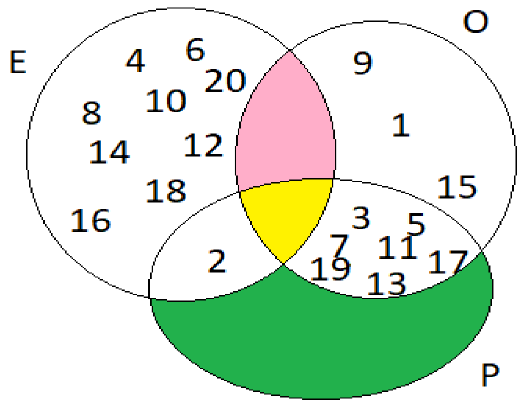
(iii) Prime numbers between 0 and 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

(iv) 2 is only one even number that is a prime number.

(v) Odd numbers between 0 and 20 which are prime numbers are 3, 5, 7, 11, 13, 17 and 19.

c) Let consider the following sets:

- E: set of odd numbers between 0 and 20
- O: set of even numbers between 0 and 20
- P: set of prime numbers between 0 and 20
- Then, their representation on Venn diagram is as follows



f) Answers of Application Activity 1.1.2

From E = {1, 4, 8, 11, 16, 25, 49, 53, 75},

(a) Even numbers are 4, 8 and 16.

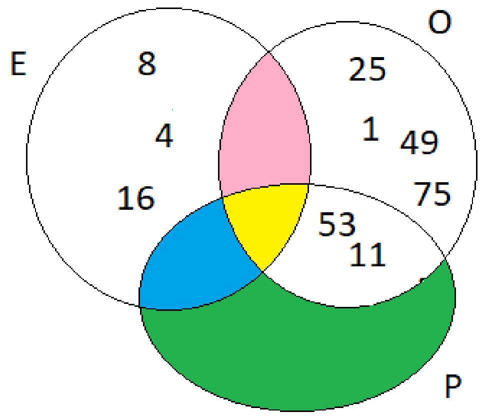
(b) Odd numbers are 1, 11, 25, 49, 53 and 75.

(c) Prime numbers are 11 and 53.

Let consider the following sets

- E: set of odd numbers between 0 and 20;
- O: set of even numbers between 0 and 20;
- P: set of prime numbers between 0 and 20.

Then, their representation on Venn diagram is as follows:



Lesson 2: Operations and properties on natural numbers

a) Learning objective: Work systematically to carry out basic operations on natural numbers and deduce the related properties.

b) Teaching resources: T-square, ruler, student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they have a good background on the following:

- Use correctly language, vocabulary and suitable mathematical symbols previously learnt;
- Carry out correctly numerical calculations
- Interpretation of simple diagrams and recognize ways in which representations can be misleading.

d) Learning activities

- Organize the students into small groups and make sure that the principal of inclusiveness is respected.
- Help student-teachers to look for a group secretary, group representative, and timekeeper. Make sure that both boys and girls participate actively in their groups and they are given equal opportunities during the lesson.
- Let them workout the **activity 1.1.2.** from student-teacher's book.

- After a given time, ask randomly some groups to present their findings to the whole class and during presentation, lead student-teachers to discover that any given set of natural numbers can be subdivided into different subsets.
- Ask student-teachers to work individually the **application activity 1.1.2**, given in their book**,** for consolidation of the skills they have acquired.

g) Answers of activity 1.1.3

1. $\forall a, b, c, d \in \mathbb{N}$

   a) $a+b=d \in \mathbb{N}$,

   b) Example: $2+3=5 \in \mathbb{N}$.

   c) $a+b=b+a=d \in \mathbb{N}$. Example: $2+3=3+2=5 \in \mathbb{N}$

   d) $(a+b)+c=a+(b+c)=d \qquad \forall d \in \mathbb{N}$

   Example: $(2+3)+1=2+(3+1)=6 \in \mathbb{N}$

2. $\forall a, b, c, d \in \mathbb{N}, \quad (a-b) \neq (b-a)$ and $(a-b)-c \neq a-(b-c)$

   (i) $(3-2)=1$ and $(2-3)=-1 \notin \mathbb{N}$

   (ii) $(7-4)-2=1$ and $7-(4-2)=5$

   $(7-4)-2 \neq 7-(4-2)=5$

   $1 \neq 5$

3. Given any three natural numbers a, b and c,

$(a \times b)=(b \times a)$ and $(a \times b) \times c=a \times(b \times c)$

   (i) $(3 \times 2)=6$ and $(2 \times 3)=6 \notin \mathbb{N}$

   (ii) $(3 \times 2) \times 4=24$ and $3 \times(2 \times 4)=24 \notin \mathbb{N}$

   (iii) $2 \times(3+4)=2 \times 7=14$ and $2 \times(3+4)=(2 \times 3)+(2 \times 4)=6+8=14$

   (iv) $2 \times(7-4)=2 \times 3=6$ and $2 \times(7-4)=(2 \times 7)-(2 \times 4)=14-8=6$

4. $\forall a, b, d \in \mathbb{N}$,$(a:b) \neq (b:a)$, Example: $10:5=2$ and $5:10=0.5 \notin \mathbb{N}$.

h) Answers of application Activity 1.1.3

Answers may vary; but for any natural number, it is easily verifiable that

1) $a+b \in \mathbb{N}$, since $\mathbb{N}$ is **closed** under addition

$a+b=b+a$, since addition is **commutative** in $\mathbb{N}$.

$a+(b+c)=(a+b)+c$ since addition is **associative** in $\mathbb{N}$.

2) $a \times b \in \mathbb{N}$, since $\mathbb{N}$ is **closed** under multiplication

$a \times b = b \times a$, since multiplication is **commutative** in $\mathbb{N}$.

$a \times (b \times c) = (a \times b) \times c$ since multiplication is **associative** in $\mathbb{N}$.

3) $a \times (b+c) = ab+ac$ since multiplication is distributive over addition.

Lesson 3: Definition and Subsets of integers

a) Learning objective: Identify set of integers and relationship between its subsets.

b) Teaching resources: T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction: Student-teachers will learn better this lesson if they have a good background on concept of natural numbers and number line.

a) Learning activities

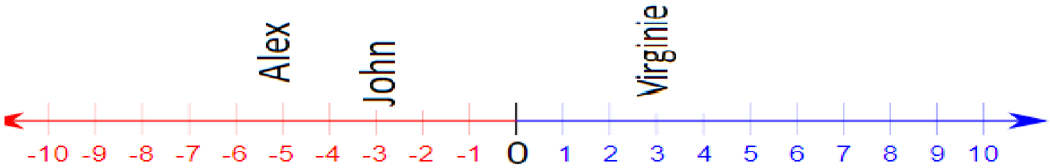
- Organize the student-teachers into small groups;
- Introduce the topic by having a review on natural numbers;
- Let them attempt the **Activity 1.2.1** in their groups and invite student-teachers to discuss and do all questions of the given activity and check if everybody is engaged;
- Invite group representative to present their findings, then harmonize their findings by helping all student-teachers to make content summary;
- Facilitate student-teachers to work individually the application **activities 1.2.1** for improving their skills and competences.

b) Answers of activity 1.2.1

(1) The tutor guides the student teacher to use dictionary and other resources like internet to search the definition of integer (negative numbers and positive numbers while natural numbers don't have negative side)

(2) $a)\ -50m,\quad b)\ +42^0C\quad ,c)\ -2m\quad ,d)\ +3m$

c) Answers of application Activity 1.2.1



d) Answers of activity 1.2.2

From integers between 12 and +20, form small sets which contain;

a) odd numbers are 13,15,17 and 19.

b) even numbers are 14,16 and 18

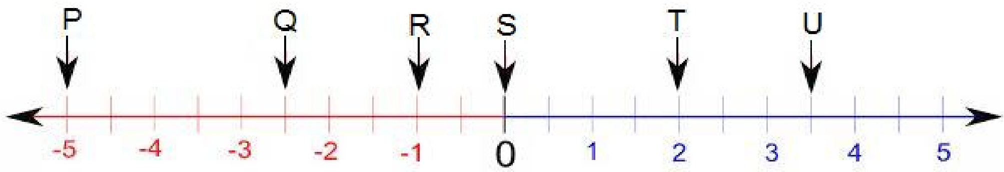
c) factor of 6 is 18

d) multiples of 3 are 15 and 18.

e) Answers of application Activity 1.2.2

(1) $\{\cdots,-5,-4,-3,-2,-1,0\}$ is the set of non-positive integers.

(2) From the following figure, the points P, Q, R, S, T and U are located.



Among them,

- Negative integers are P and R.
- Positive integer is T

- S is neither positive nor negative?
- Q and U are not integers?

Lesson 4: Operations and properties on integers

a) Learning objective: Work systematically to carry out basic operations on integers and deduce the related properties.

b) Teaching resources: Student-teacher's book, **Reference** textbooks to facilitate research

c) Prerequisites/Revision/Introduction: Student-teachers will learn better this lesson if they have a good background on basic concepts on natural numbers and perform different operations on natural numbers.

d) Learning activities

- Organize the student-teachers into groups and then give instructions related to the activity.
- Let them attempt **activity 1.2.3** and check whether everyone is engaged.
- Once the group discussion is over, ask a group, chosen randomly, to present his results while other learners are following attentively
- From different presentations, guide student-teachers to deduce the properties on integers for each of four mathematical operations.
- Lead them to read through the content summary given in Student-teacher's book and work individually **application activity 1.2.3** to consolidate their skills and competences.

e) Answers of activity 1.2.3

(1). Work out the following on a number line:

(a) (+3) + (+2)=5 (b) –(5) + - (3) =-8

(c) (+4) + (-3)=1

(d) When adding a negative number to a positive number, you move on left side of the number line starting from positive number.

e) You move on left side of the number line from minuend

(2) Work out the following and show your solutions on a number line.

(a) (-4)-(+3) = -7 (b) (+5) – (+3)=2 (c) (–6) – (-6) =0

(d) you move on left side of the number line from minuend

(e) you move on left side of the number line from minuend

(3) Work out the following:

(a) $(+5)\times(-6)=-30$ (b) $(+5)\times(+6)=30$ (c) $(-5)\times(+6)=-30$

(d) $(-5)\times(-6)=30$ (e) No

(4). Work out the following and show your solutions on a number line.

(a) $(-4)\div(+4)=-1$ (b) $(+4)\div(+4)=1$ (c) $(-4)\div(-4)=1$

(d) $(+4)\div(-4)=-1$. (e) No.

f) Answers of application Activity 1.2.3

(1) If the temperature drops 3 degrees from -23^0C outside, then, temperature is -26^0C.

(2) While doing practical activities in laboratories, in higher studies when you do experiments basically to deal with experimental data, people need the knowledge of integers. For better interpretation of graphs in scientific research, people need the knowledge of integers

Lesson 5: Definition and Subsets of rational numbers

a) Learning objective: Represent rational numbers as a fraction or a decimal which may terminate or occur and identify the relationship between its subsets

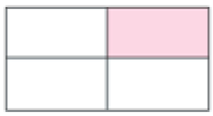
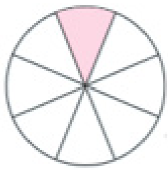
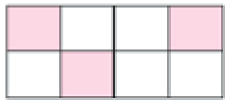
b) Teaching resources: T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction: Student-teachers will perform well in this lesson if they have a good background on concept of fractions.

d) Learning activities

- Organize the student-teachers into small groups;
- Introduce the activity by guiding student-teachers to define a fraction;
- Let them attempt the **Activity 1.3.1** from student-teacher's book;
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings
- Guide student-teachers to perform individually application activity 1.3.1 to assess their knowledge and skills.

e) Answers of activity 1.3.1

Fraction of			
Shaded parts	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$
Unshaded parts	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{5}{8}$

Fractions are elements of set of rational numbers.

Given integers, a *and* b with b a non-zero integer, $\frac{a}{b}$ is always a fraction and examples may vary ($\frac{2}{3}, \frac{7}{8}$,...are examples)

f) Answers of Application Activity 1.3.1

There are three type of fractions.

- Proper Fractions: the numerator is less than the denominator.

  Examples: $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

- Improper Fractions: the numerator is greater than (or equal to) the denominator.

  Examples: $\frac{3}{2}, \frac{5}{4}, \frac{11}{6}, \frac{17}{8}, \frac{19}{5}, \ldots$

- Mixed Fractions: a whole number and proper fraction together.

  Examples: $1\frac{1}{2}, 1\frac{1}{4}, 1\frac{5}{6}, 2\frac{1}{8}, 3\frac{4}{5}, \ldots$

g) Answers of Activity 1.3.2

(5) All integers are rational numbers.

This statement is true. Note that $\mathbb{Z} \subset \mathbb{Q}$

(6) No rational numbers are whole numbers.

This statement is false. Note that $\mathbb{N} \subset \mathbb{Q}$ or $\mathbb{Q} \supset \mathbb{N}$.

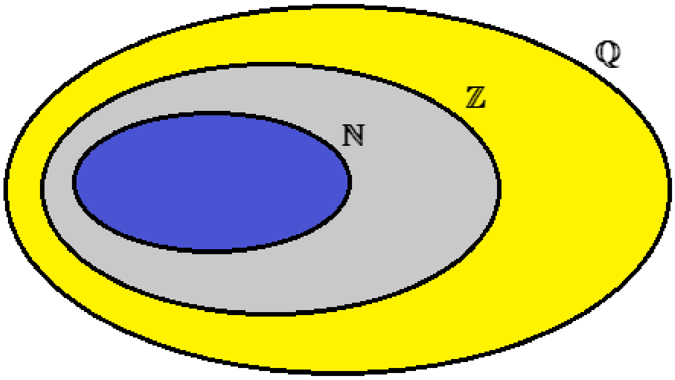
(7) All rational numbers are integers.

This statement is false. Note that $\mathbb{Q} \not\subset \mathbb{Z}$ but $\mathbb{Z} \subset \mathbb{Q}$.

(8) All whole numbers are rational numbers.

This statement is true. Note that $\mathbb{N} \subset \mathbb{Q}$ or $\mathbb{Q} \supset \mathbb{N}$.

h) Answers of Application Activity 1.3.2



$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

Lesson 6: Operations and properties on rational numbers

a) **Learning objective:** Work systematically to carry out basic operations on rational numbers and deduce the related properties.

b) **Teaching resources:** Student-teacher's book and other Reference textbooks to facilitate research

c) **Prerequisites/Revision/Introduction:** Student-teachers will perform well in this lesson if they have a good understanding on basic concepts of rational numbers as well as demonstrating the ability to perform addition, subtraction, multiplication and division of numbers.

d) **Learning activities**

- Organize the student-teachers into small groups and then give them instructions to follow while they workout the provided **activity 1.3.3**.
- Ask group representatives to share their finds with another group so that they identify the misconception
- Basing on their discussions and presentation guide student-teachers to deduce the properties on rational numbers for each of four mathematical operations.
- Lead them to read through the content summary given in Student-teacher's book and work individually **application activity 1.3.3** to reinforce their competences.

e) Answers of activity 1.3.3

1. $\forall a, b, c, d \in \mathbb{Q}$

a) $a+b=d \in \mathbb{Q}$,

Example: $\frac{2}{3}+\frac{3}{5}=\frac{19}{15} \in \mathbb{Q}$ $\left(\text{to calculate the common denominator, use LCM}\right)$.

b) $a+b=b+a=d \in \mathbb{Q}$.

Example:

$$\left(\frac{2}{3}+\frac{3}{5}\right)=\left(\frac{3}{5}+\frac{2}{3}\right)=\frac{19}{15} \in \mathbb{Q} \quad \left(\text{to calculate the common denominator, use LCM}\right)$$

c) $(a+b)+c=a+(b+c)=d \qquad \forall d \in \mathbb{Q}$

Example: $\left(\frac{2}{3}+\frac{3}{7}\right)+\frac{1}{2}=\frac{67}{42}$ and $\frac{2}{3}+\left(\frac{3}{7}+\frac{1}{2}\right)=\frac{67}{42} \in \mathbb{Q}$

2. $\forall a,b,c,d \in \mathbb{Q}, \qquad (a-b)-c \neq a-(b-c)$

i) $\left(\frac{3}{4}-\frac{2}{3}\right)=\frac{1}{12}$ and $\left(\frac{2}{3}-\frac{3}{4}\right)=\frac{-1}{12}$ $, thus\,(a-b)\neq(b-a)$

ii)

$\left[\left(\frac{7}{2}-\frac{4}{5}\right)-\frac{2}{3}\right]=\frac{61}{30}$ and $\left[\frac{7}{2}-\left(\frac{4}{5}-\frac{2}{3}\right)\right]=\frac{101}{30}$ **,** thus $(a-b)-c\neq a-(b-c)$

3. Given any three rational numbers a, b and c,

$(a\times b)=(b\times a)$ and $(a\times b)\times c=a\times(b\times c)$

(i) $\left(\frac{3}{4}\times\frac{1}{5}\right)=\frac{3}{20}$ and $\left(\frac{1}{5}\times\frac{3}{4}\right)=\frac{3}{20} \notin \mathbb{Q}$

(ii) $\left(\frac{7}{4}\times\frac{3}{5}\right)\times\frac{1}{6}=\frac{7}{40}$ and $\frac{7}{4}\times\left(\frac{3}{5}\times\frac{1}{6}\right)=\frac{7}{40} \in \mathbb{Q}$

(iii) $2\times(7-4)=2\times3=6$ and $2\times(7-4)=(2\times7)-(2\times4)=14-8=6$

4. $\forall a,b,d \in \mathbb{Q}$,

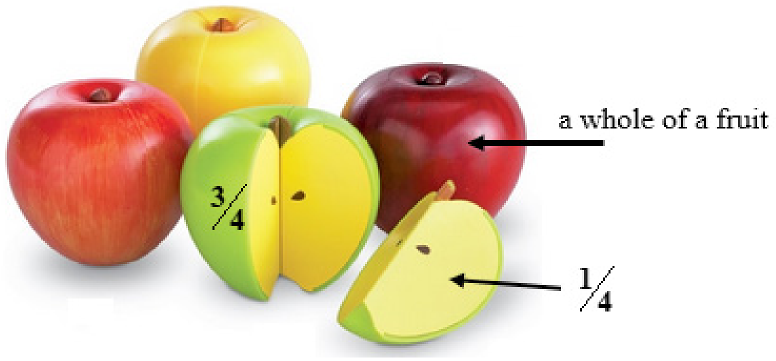
$\frac{10}{3}:\frac{5}{4}=\frac{10}{3}\times\frac{4}{5}=\frac{8}{3}$ and $\frac{5}{4}:\frac{10}{3}=\frac{5}{4}\times\frac{3}{10}=\frac{3}{8}\in\mathbb{Q}$ **so** $\left(\frac{10}{3}:\frac{5}{4}\right)\neq\left(\frac{5}{4}:\frac{10}{3}\right)$.

f) Answers of application Activity 1.3.3

Answers may vary; but for any rational numbers, you take it is applicable in everywhere in our daily life. As tutor, verify whether the student-teachers 'answers are correcting. For example, we use fractions every day without even knowing it. Here are a few examples you might be familiar with.

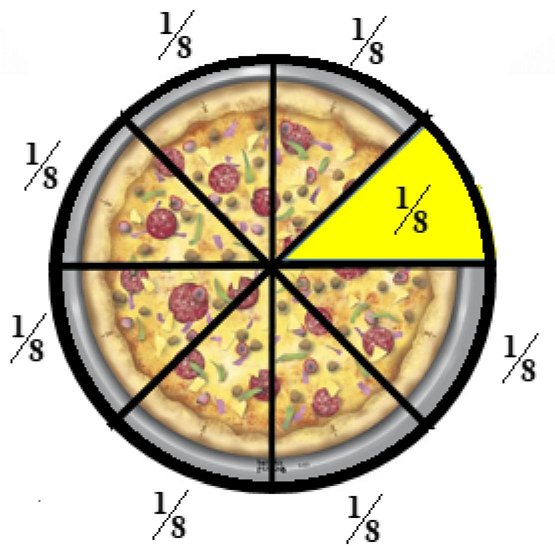
(1) **Sharing fruits**

Fruit is a great example. Every time we cut an apple, an orange, or any kind of fruit, we are taking a piece of the whole. We can represent those pieces as fractions. The picture below shows pictures of apples that can be used as manipulatives. The one that says 1/4 is already shown. For the other apples, the denominator tells us how many equal pieces those apples will be cut into.



(2) **Fractions in Everyday Life**

A pizza is a great example of fractions! Each piece represents a part of a whole. In the picture, below, the pizza is divided into 8 pieces. If I have one piece, the fraction of pizza I am eating is one-eighth. (1/8)



Lesson 7: Definition of irrational numbers

a) **Learning objective:** Show that irrational numbers cannot be expressed exactly as decimals

b) Teaching resources: Student-teacher's book, Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction: Student-teachers will perform well in this lesson if they have a strong understanding of conversion of fractions into decimal numbers and vice- versa.

d) Learning activities

- Introduce the task by making review on how to convert decimals to fractions
- Organize the learners into small groups and ask them to attempt **activity 1.3.4** in the student-teachers book.
- Check if everybody is engaged in the given activity
- As they are discussing, concentrate on slow learners for further explanation and provide assistance to groups where needed.
- Choose randomly one group to present their findings to the whole class and then harmonize their works and generate the lesson summary.
- Let student-teachers work out individually the **application activity 1.3.4** to check their understanding level and apply the acquired.

e) **Answers of activity 1.3.4**

1) Recurring $0.66666\ldots$ as a fraction gives $0.66666\ldots = \frac{6}{10-1} = \frac{6}{9} = \frac{2}{3}$.

Notice: $q = \frac{recuring\ digit}{n^{th}\ position\, of\ recuring\ decimal - 1}$

For our case, recurring digit is 6 and it is tenth.

2) By using calculator, carry out the following;

(a) $\sqrt{2} = 1.414213562\ldots$ (b) $\sqrt[3]{5} = 1.7099759467\ldots$

It is not possible to express these numbers as fractions because there are not terminate nor recurring decimals. $\sqrt{2}=1.414213562\ldots$ and $\sqrt[3]{5}=1.7099759467\ldots$ are elements of set of irrational numbers.

f) **Answers of activity 1.3.4**

1. The sum of rational number and irrational number is irrational.

   The given statement is always true. For example $\frac{1}{2}+\sqrt{2}=0.5+1.414213562\ldots=2.914213562\ldots$ is an irrational number.

2. The product of rational number and irrational number is irrational.

   The given statement is sometimes true.

   For example, $2\pi, \frac{3\pi}{4}, \frac{5}{7}\sqrt{11}, \ldots$ are irrational numbers.

   But, since 0 is rational number and any irrational number times 0 is 0, for this case, the product is not an irrational number!

3. The sum of two irrational numbers is irrational.

The given statement is sometimes true. For example $\sqrt{2}+\sqrt{3}$ is irrational number but $-\sqrt{3}+\sqrt{3}=0$ is the rational number..

4. The product of two irrational numbers is irrational.

The given statement is sometimes true.

For example, $\sqrt{3}\times\sqrt{5}=\sqrt{15}$ is an irrational number but $\sqrt{3}\times\sqrt{3}=3$ is a rational number!

5. Between two rational numbers, there is an irrational number.

   The given statement is always true.

6. If you divide an irrational number by another, the result is always an irrational number.

   The given statement is sometimes true.

For example, $\frac{\sqrt{3}}{\sqrt{6}}=\frac{1}{\sqrt{2}}$ is irrational number but $\frac{\sqrt{2}}{\sqrt{2}}=1$ is rational number.

Lesson 8: Definition and Sub sets of real numbers

a) **Learning objective:** Work systematically to carry out basic operations on real numbers.

b) **Teaching resources:** T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research

c) **Prerequisites/Revision/Introduction:** Student-teachers will learn better this lesson if they have a good understanding of set of rational numbers with its subsets and interpret simple mathematical diagrams.

d) **Learning activities**

- Organize the student-teachers into small groups
- Introduce the topic by making review on set of numbers and operations: natural numbers, integers, rational and irrational numbers and their subsets.
- Let them attempt the **activities 1.4.1 and 1.4.2** from student-teacher's book
- Move around to ensure that all student-teachers in groups are engaged.
- Invite group representatives to present their findings to whole class, then correct those which are false to harmonize their work
- Lead student-teachers to read through content summary and work individually **application activities 1.4.1**and **1.4.2** for assessing their competences.

e) **Answers of activity 1.4.1**

1. The set of counting numbers is a subset of natural numbers.

This statement is **True** since $\mathbb{N}^+ = \{1,2,3,\ldots\} \subset \mathbb{N} = \{0,1,2,3,\ldots\}$.

2. The intersection of set of integers and counting numbers is the set of natural numbers.

This statement is **False** since $\mathbb{Z} \cap \mathbb{N}^+ = \mathbb{N}^+ \neq \mathbb{N} = \{0,1,2,3,\ldots\}$.

3. The intersection of set of integers and natural numbers is the set of counting numbers.

This statement is **False** since $\mathbb{Z} \cap \mathbb{N} = \mathbb{N} \neq \mathbb{N}^{+} = \{1, 2, 3, \ldots\}$.

4. The union of set of natural numbers and counting numbers is the set of natural numbers.

This statement is **True** since $\mathbb{N} \cap \mathbb{N}^{+} = \mathbb{N} = \{0, 1, 2, 3, \ldots\}$.

5. The intersection of set of rational numbers and irrational numbers is the set of irrational numbers. This statement is **False** since $\mathbb{Q} \cap I = \{\ \} \neq I$.

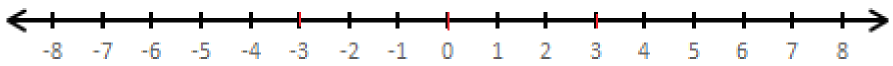
6. The union of set of rational numbers and irrational numbers is a set of irrational numbers.

This statement is **False** since $\mathbb{Q} \cup I = \mathbb{R} \neq I$.

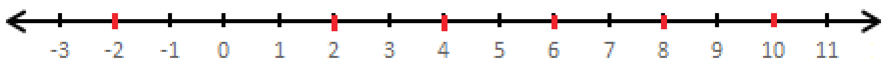
f) **Answers of application activity 1.4.1**

1) Appropriate scale and graph the following sets of real numbers on a number line

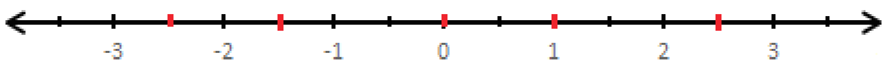
a) {−3, 0, 3} on a number line



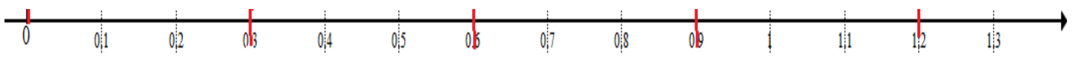
b) {−2, 2, 4, 6, 8, 10} on number line



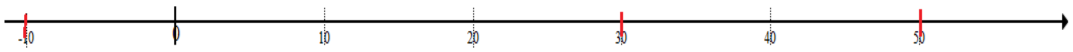
c) {−2.5, −1.5, 0, 1, 2.5}



d) {0, 0.3, 0.6, 0.9, 1.2}



e) {−10, 30, 50}



f) {−6, 0, 3, 9, 12}

2) Some examples of applications of real numbers in our daily life

We use real numbers to express almost all measurable quantities, e.g distance, time duration, weight, speed, in business, …

Most mechanical devices for measuring contain part of the real number line.

Some examples: (Marked) ruler, Protractor, Kitchen/bathroom scales, Micrometer (twice), Pressure gauge, Barometer, thermometer … You can read your altitude on the screen of a GPS unit - it will tell you how high or even how far below sea level.

a) **Answers of activity 1.4.2**

By understanding which sets are subsets of a set of numbers, we can verify whether statements about the relationships between sets are true or false. The picture given below clearly illustrates the subsets of real numbers.

Main subsets of real numbers are sets of natural numbers, integers, irrational numbers, rational numbers and its subsets.

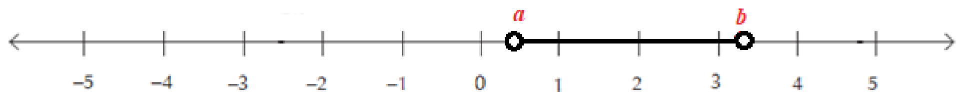
Subset of real numbers

Name	Description	Examples
Counting numbers	Numbers used for counting	$\{1,2,3,4,5,...\}$
Whole numbers or natural numbers	The counting numbers with 0	$\{0,1,2,3,4,5,...\}$
Integers	The whole numbers and the negative of the counting numbers	$\{...,-3,-2,-1,0,1,2,3,...\}$
Rational numbers	Fractions	$...,-17,\quad 3,\quad 0.4,\quad 5,\quad 0.666\quad ...$
Irrational numbers	Non-terminating, non-repeating decimals	$\sqrt{2},\quad \sqrt{11},\quad \pi,\quad 1.0010023,\quad ...$

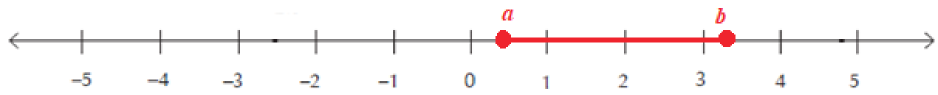
b) **Answers of application activity 1.4.2**

Certain subsets of real numbers are called **intervals**, they occur frequently in calculus and correspond geometrically to line segments.

- For example, if $a<b$, the **open interval** from a to b consists of all numbers between a and b and is denoted by the symbol (a,b) or $]a,b[$ and described as $(a,b)=\{x: a<x<b\}$.For this case, the end points of the interval namely a and b are excluded.



- The **closed interval** from a to b is the set described as $[a,b]=\{x: a\leq x\leq b\}$. Here the endpoints of the interval are included.



Notation	Set description	Geometrical representation
(a,b) or $]a,b[$	$\{x: a<x<b\}$	
$[a,b]$	$\{x: a\leq x\leq b\}$	
$[a,b)$ or $[a,b[$	$\{x: a\leq x<b\}$	
$(a,b]$ or $]a,b]$	$\{x: a<x\leq b\}$	
$(a,+\infty)$ or $]a,+\infty[$	$\{x: a>x\}$	
$[a,+\infty)$ or $[a,+\infty[$	$\{x: a\geq x\}$	
$(-\infty,b)$ or $]-\infty,b[$	$\{x: x<b\}$	
$(-\infty,b]$ or $]-\infty,b]$	$\{x: x\leq b\}$	
$(-\infty,+\infty)$ or $]-\infty,+\infty[$	$\mathbb{R}$ (Set of all real numbers)	

- Let student-teachers do by themselves the remaining representations of the given intervals for the application activity 1.4.2.

Lesson 9: Operations and properties on real numbers

a) Learning objective: Work systematically to determine the operation properties on set of real numbers.

b) Teaching resources: Student-teacher's book, Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction: Student-teachers will perform well in this lesson if they have a good understanding about basic concepts of numbers and operations properties in the set of natural numbers, integers, rational and irrational numbers.

d) Learning activities

- Organize the student-teachers into groups and then give instructions to follow for the provided activity.
- Let them attempt **activity 1.4.3** and check whether everyone is engaged.
- Check if the different groups are choosing different real numbers in performing the given task.
- Ask group representatives to present their finds to whole class and harmonize their work
- During the presentation, let student-teachers discover the properties on real numbers for each of four mathematical operations.
- Lead them to read through the content summary given in Student-teacher's book and work individually **application activity 1.4.3** to assess their competences.

e) Answers of activity 1.4.3

As tutor, you check on the following points, whether students emphasize on them while answering.

1. For any real numbers a,b and c, you verify easily that

a) $a+b$ is always a real number since $\mathbb{R}$ is **closed** under addition.

b) $a+b$ and $b+a$ always give the same answer since addition is **commutative** in $\mathbb{R}$.

c) $a+(b+c)$ and $(a+b)+c$ always give the same answer since addition is associative in $\mathbb{R}$.

2. For any three real numbers a,b and c, investigate the following operations : $a-b$, $a-b$ and $b-a$, $a-(b-c)$ and $(a-b)-c$

After substitution, you notice that

(1) $a-b$ is always a real number since $\mathbb{R}$ is **closed** under subtraction.

(2) $a-b \neq b-a$, which means that subtraction is not **commutative** in $\mathbb{R}$.

(3) $a-(b-c) \neq (a-b)-c$ which means that subtraction is not **associative** in $\mathbb{R}$.

For any three real numbers a,b and c, after substitution, you notice that

(1) $a \times b$ is always a real number since $\mathbb{R}$ is **closed** under multiplication.

(2) $a \times b = b \times a$, which means that multiplication is **commutative** in $\mathbb{R}$.

(3) $a \times (b \times c) = (a \times b) \times c$ which means that multiplication is **associative** in $\mathbb{R}$.

For any two real numbers a,b and c, investigate the following operations : $a \div b$ and $b \div a$, $a \div (b \div c)$ and $(a \div b) \div c$

a) Answers of application activity 1.4.3

1. For

a) $3(2x-5) = 6x-15$, distributivity of multiplication over addition has been expressed.

b) $(0.08+0.12)+\frac{1}{2} = 0.8+\left(0.12+\frac{1}{2}\right)$ associativity property under addition has been expressed.

c) $(3\times5)\times2 = 3\times(5\times2)$ but $(30\div5)\div2 \neq 30\div(5\div2)$ associativity property for multiplication has been expressed but it cannot be applied on division (since division is not associative).

d) $\pi - 2 \neq 2 - \pi$ but $\pi + 2 = 2 + \pi$ subtraction is not commutative but for addition, commutativity property has been applied.

2. Closure Property under division for real numbers is not satisfied. Note that 0 itself is a rational number (0 = 0/1).

So 3 ÷ 0 is a "rational being divided by a rational".

But the result violates the definition of rational form $\frac{p}{q}$, *where* $q \neq 0$.

3. A field having width of $60m$ out $160m$ of length has the area of $60m \times 160m = 96,000m^2$ while that one having the width of $100m$ as it is its length has the area of $100m \times 100m = 100,000m^2$. Thus, the biggest field is that one having the width of $100m$ as it is its length. Even if the field having width of $60m$ out $160m$ of length has larger perimeter than that one having the width of $100m$ as it is its length, the area of second field is the largest (Not confuse perimeter from area).

1.6. Unit Summary

(a) Natural numbers: are counting numbers and include zero. A set of natural numbers is denoted by $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$

a. Even numbers, odd numbers, prime numbers, are all subsets of natural numbers

b. Integers: Are whole numbers, negative whole numbers and zero. A set of integers is denoted by $\mathbb{Q}$ i.e. $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

(b) A set $\mathbb{N}$ of natural numbers is a subset of integers.

a. A set $\mathbb{Z}$ of integers is a subset of rational numbers.

(c) Rational numbers: are numbers that can be expressed as a fraction where the denominator and the numerator are integers.

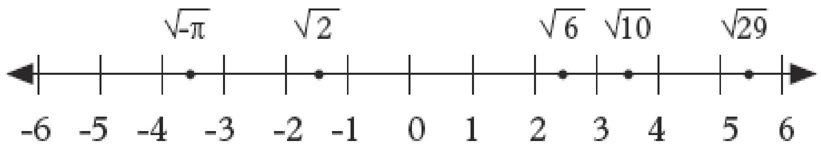
(d) A set $\mathbb{Q}$ of rational numbers is a subset of real number. Example:

$\mathbb{Q}$=$\{$ **...,** $3, 2, -1, 20, 0.143, 16, -18, -0.21, ...\}$

(e) A set of irrational number is a subset of real number. Irrational numbers: are numbers that cannot be expressed as a quotient of two integers such that the denominator is not zero i.e. cannot be expressed as a ratio. An irrational number is denoted by I e.g.

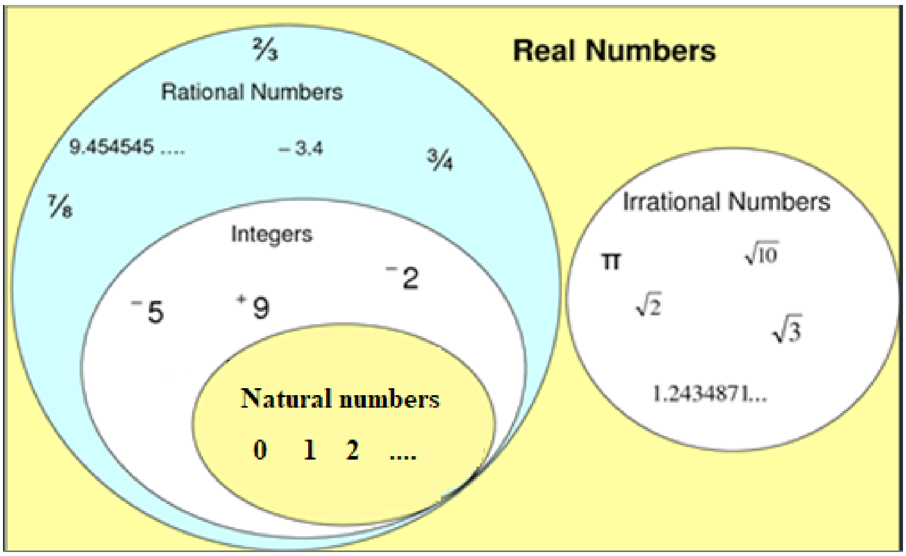
$I = \{...,3.14159, \ldots, 2, 2.71828, \ldots. 99,...\}$

(f) Set of real numbers is denoted by $\mathbb{R}$. Real numbers: are numbers that can be found on a number line.



They include both rational and irrational numbers. A set of real numbers is denoted by $\mathbb{R}$ e.g. $\mathbb{R} = \{..., 2, 1.5,...,-3.4,1,2,...\}$

Note: All these sets are subsets of real numbers that is natural numbers, integers, rational numbers, irrational numbers. This can be shown in the Venn diagram below.



1.7. Additional Information for teachers

By the end of this unit the tutor has to inform student teacher about operations on decimal numbers in order to help them to easily perform operations in $\mathbb{R}$

Examples: Express the following fractions into decimals.

(a) $\frac{3}{8} = 0.375$ ...(b) $\frac{8}{9} = 0.8888888888888888...$

1.8. End unit assessment

1. Example of three rational numbers between 0 and 1

Answers may vary as they are infinite rational numbers between 0 and 1.

Some are $\left\{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}\right\}$ or $\left\{\frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$ or $\left\{\frac{2}{3}, \frac{5}{7}, \frac{11}{13}\right\}$ and so on.

2. The sets to which each of the following numbers belong

	Number	Counting	Natural	Integers	Rational	Irrational	Real
1	-1			□	□		□
2	$\frac{1}{7}$				□		□
3	0		□	□	□		□
4	$-\sqrt{7}$					□	□
5	$\sqrt{0.03}$					□	□
6	$\frac{\sqrt{4}}{2}$	□	□	□	□		□
7	$\frac{1}{0}$						
8	$\frac{\pi}{2}$					□	□
9	$\frac{\sqrt{27}}{\sqrt{3}}$	□	□	□	□		□
10	$\sqrt[3]{-27}$			□	□		□

4. $altitude = 22,000 - 8,500 + 5,000 =$

5. $Width = 3 \times 1.5\,cm = 4.5\,cm$

6. $\frac{3}{4} of\ Frw 520,000 = \frac{3}{4} \times 520,000 = Frw 390,000$

7. Coordinate is (0,-3)

1.9. Additional activities

1.9.1. Remedial activities

Work out the following:

(a) (-28) ÷ (-7);

(b) (+55) ÷ (-11) ;

(c) (+19) ÷ (-3);

(d) (-48) ÷ (-6) ;

(e) (-36) ÷ (+9) ;

(f) (-100) ÷ (-10)

Answers: a) +4; b) -5; c) -6.333333333…; d) +8; e) -4; f) +10

1.9.2. Consolidation activities

Work out the following:

(a) (-10) × (-5) × (-6) :(-3) × (-2) ; (b) (-30) × (+2) × (-10): (-5) × (+2)

(c) Find the square root of each of the following.

(1) i) $\sqrt{6}$; (ii) $\sqrt{8}$;

(2) Find the values of;

(i) $\{9 + ((-2) \times (-15))\} \times (-2 + 7) \div 3$;

(ii) $-6 + (-5) + (8 \times (-2)) - 4 + (-2)$

(iii) $(-3 \times 23) + ((-5) \times (-1)) + ((-8) \times (-4))$;

(iv) $9 \times (-2) - 4 - (-2)$

Answers: 1. i) -50; ii) -60;

2. a) i) 2.449489743; ii) 2.828427125 ;

b) i) 65; ii) -33; iii) -32; iv) -20

1.9.3. Extended activities

Think about a real number multiply it by two the answer you obtain you add one the new answer you get divide it by three you get twenty-seven.

a) Now what is that number;

b) Think another similar example and you discuss with your neighbour.

Answer: a) let x be the number: $x \times 2 = 2x$

The answer found add one: $2x + 1 = z$

Again the answer found divide it by three to be twenty seven: $\frac{z}{3} = 27 \Rightarrow z = 81$

finally let us find the value of x: $2x + 1 = 81 \Rightarrow 2x = 80 \Rightarrow x = 40$, so the number was 40.

UNIT 2

SET THEORY

2.1 Key unit competence

Solve problems that involve Sets operations using Venn diagrams.

2.2 Prerequisites

Learners will perform better in this unit if they have background on: Expressing mathematical problem set using a Venn diagram. Representing a mathematical problem using a Venn diagram. Interpreting, modelling, and solving a mathematical problem set.

2.3 Cross-cutting issues to be addressed

- **Inclusive education**: promote the participation of all student-teachers while teaching
- **Peace and value Education**: During group activities, the teacher will encourage student-teachers to help each other and to respect opinions of colleagues.
- **Gender:** Give equal opportunities to all learners (girls and boys) to present their findings. Encourage them to participate actively in all learning and teaching activities from the beginning to the end of the teaching and learning process.

2.4 Guidance on introductory activity 2

- Form groups of student-teachers and invite them to work on introductory activity to understand the concept of set theory;
- Give time to student-teachers to analyse the activity and provide pieces of advice and adequate facilitations where necessary.
- Invite group to present their findings and try to harmonize them;
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, use different questions to arouse their curiosity on what is going to be leant in this unit.

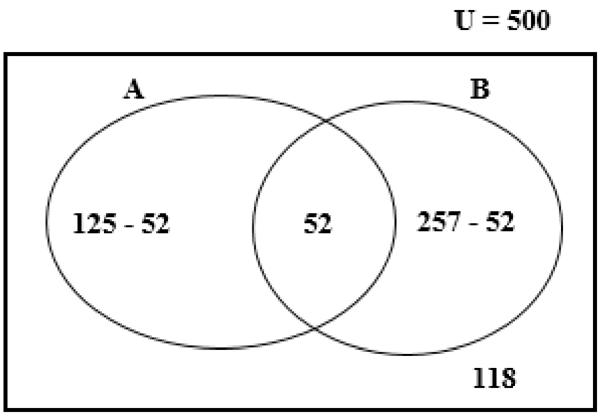
Answer to the introductory activity 2:

At a TTC school of 500 students- teachers, there are 125 students enrolled in Mathematics club, 257 students who play sports and 52 students that are enrolled in mathematics club and play sports.

Complete the following table

Symbol	Description	Value for this problem
$n(M)$	The number of elements in set M(Math club)	125
$n(S)$	The number of elements in set S(Sport club)	257
$n(M \cap S)$	The number of elements in the intersection of sets M and S (all the elements that are in both sets-the overlap)	52
$n(M \cup S)$	The number of elements in the union of sets M and S (all the elements that are in one or both of sets)	382

A Venn diagram to illustrate the information in the table above is the following:



2.5 List of lessons

No	Lesson title	Learning objectives	Number of periods
0	Introduction activity	To arouse the curiosity of student teachers on the content of unit 2	1
1	Sets and Venn diagram: Analysis and interpretation of a problem using set language (intersection, union…)	Interpret mathematical problem using a Venn diagram.	1
2	Operations on sets (Set difference, symmetric difference, complement of set, set union and intersection).	Perform operations on set (union, intersection, difference, and symmetrical difference on sets).	2
3	Representation of a problem using Venn diagram	Use Venn diagram to illustrate a mathematical problem.	3
4	Modelling and solving problems involving Venn diagrams	Interpret, model, and solve a mathematical problem on set using Venn diagrams.	3
5	End unit assessment		2
	Total number of periods		**12**

Lesson 1: Sets and Venn diagrams

a) Learning objective:

Interpret mathematical problem using a Venn diagram.

b) Teaching resources:

Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, textbooks and wall charts and wall maps, Mathematical models and Internet connection (where available).

c) Prerequisites / Revision / Introduction:

Student-teachers will perform well in this unit if they have a good background on:

Identifications of sets of numbers (natural, integers, rational and real) and relationship among them.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by asking them to use the student teacher's book to discuss the **activity 2.1**
- Move around to each group and ensure all student-teachers participate actively;
- Call upon group representatives to present their findings in a whole class discussion;
- Harmonize their findings insisting on how to illustrate a situation using Venn diagrams;
- Guide student teachers to perform individually the **application activity 2.1 and then,** assess the knowledge and skills acquired.

Answers for activities 2.1

Given the set $A=\{1,3,5,7,9\}$ and $B=\{1,2,3,4,5\}$

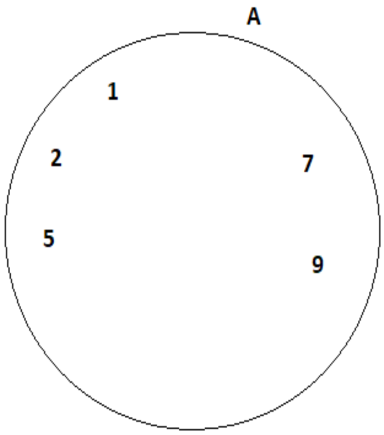
a) Common elements for both sets make $\{1,3,5\}$

b) Elements of set A which are not in set B make $\{7,9\}$

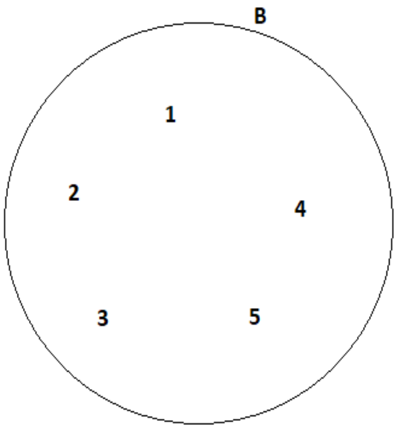
c) Elements of set B which are not in set A make $\{2,4\}$

d) All elements in set A and set B make $\{1,2,3,4,5,6,7\}$

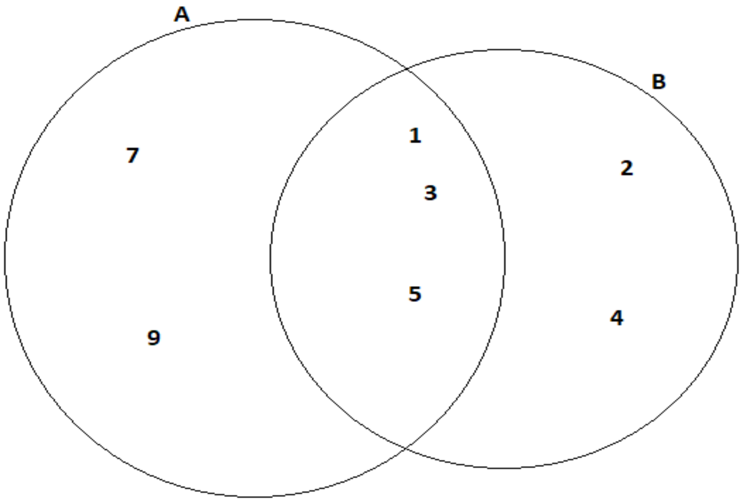
e) Set A in a Venn diagram



f) Set B in a Venn diagram:

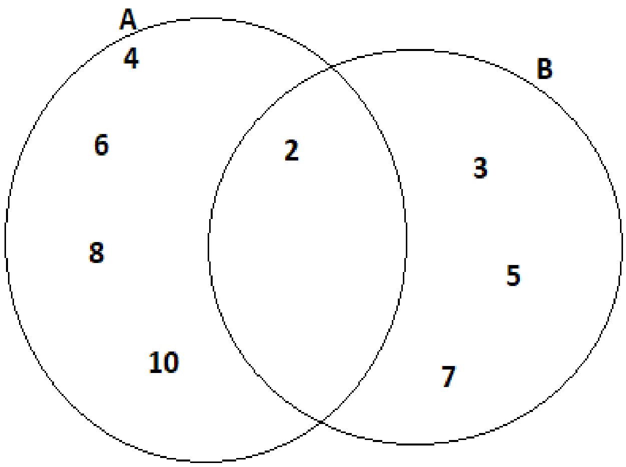


g) Sets A and B using one Venn diagram



e) Answers for application activity 2.1

$A = \{2,4,6,8,10\}$ *while* $B = \{2,3,5,7\}$



By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation of this lesson.

Lesson 2: Operations on sets and Venn diagrams

a) Learning objective:

Apply operation of set: Perform union, intersection, difference–and symmetrical difference on sets.

b) Prerequisites/Revision/Introduction:

Student-teachers will perform better in this lesson if they refer to: operation of sets learnt in S1.

c) Teaching resources:

They include: Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, wall charts and wall maps, Mathematical models and Internet connection where applicable.

d) Learning activities:

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Ask student-teachers to use the student teacher's book to discuss the **activity 2.2;**
- Move around to each group to ensure all student-teachers participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and guide student-teachers to highlight the union of sets, intersection on sets and their differences.
- Use different probing questions to guide student-teachers to explore examples and the content given in the student's book to the use of operation on sets: intersection, union and difference and guide them to highlight the corresponding properties;
- Guide all student-teachers to explore the symmetric difference and the complement of set;
- After this step, guide student-teachers to do the application activity **2.2** and assess their competences and evaluate whether lesson objectives were achieved.

Answers for activity 2.2

a) All students who were absent in the English class: A'

b) All students who were present in at least one of the two classes: $A \cup B$

c) All the students who were present for both English as well as History classes: $A \cap B$

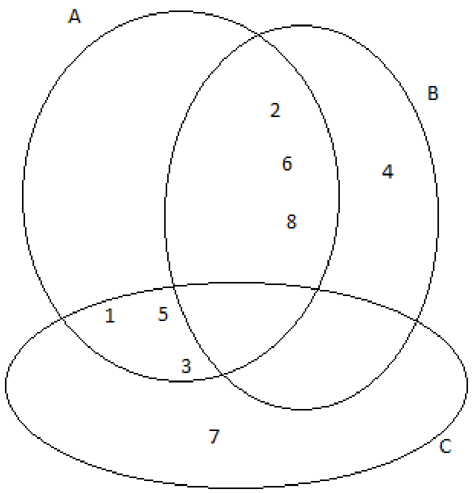
d) All the students who have attended only the English class and not the History class: $A - B$

The tutor helps or guides student teacher to discover their own content about classification of sets.

After the tutor tells the student teacher to continue the other examples found in the student teacher book entitled application activities 2.1.2.

e) Answers for application activity 2.2

1. $A = \{1,2,3,5,6,8\}$; $B = \{2,4,6,8\}$; $C = \{1,3,5,7\}$



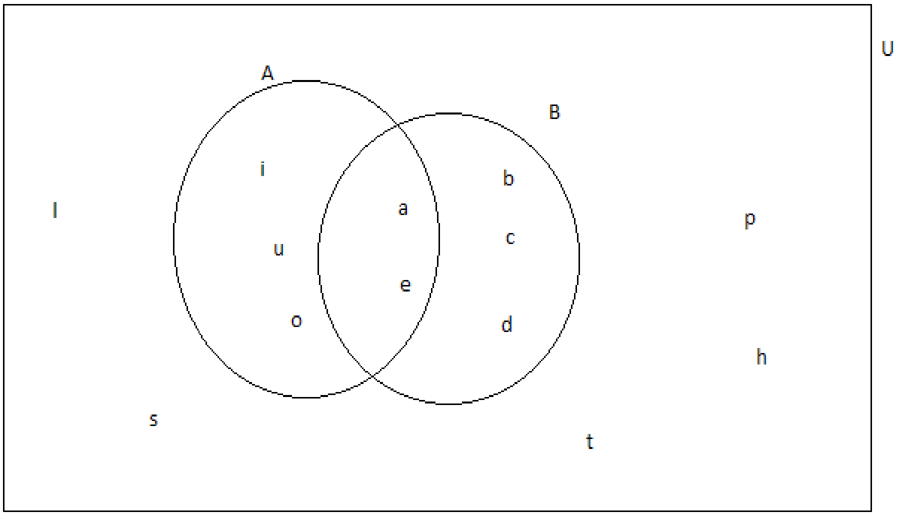
a) $B \cap C = \{\phi\}$; b) $A \cap C = \{1,3,5\}$; c) $A \cap B = \{2,6,8\}$;

d) $B \cup C = \{1,2,3,4,5,6,7,8\}$; e) $A \cup B = \{1,2,3,4,5,6,8\}$

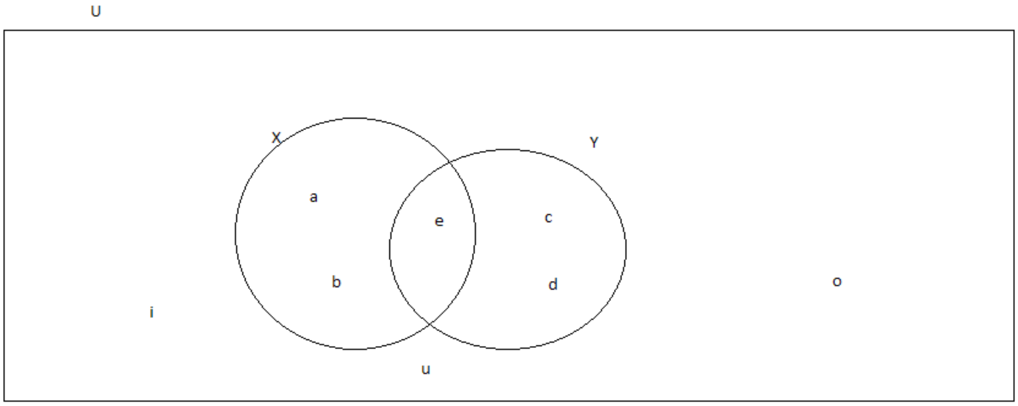
2. a) $A = \{i,u,o,a,e\}$; b) $B = \{a,b,c,d,e\}$; c) $A - B = \{i,u,o\}$;

d) $B - A = \{b,c,d\}$; e) $A \Delta B = \{i,u,o,b,c,d\}$;

f) $U=\{e,l,p,h,\text{a},\text{t},\text{i},\text{s}\}$



3. $U=\{a, e, i, o, u, c, d\}$; $X=\{a, b, e\}$ *and* $Y=\{c, d, e\}$



a) $X \cap Y=\{e\}$; b) $(\text{X}\cap \text{Y})'=\{a,b,c,d\}$;

c) $X \cup Y=\{a,b,\text{c},\text{d},e\}$; d) $(\text{X}\cup \text{Y})'=\{i,u,o\}$

At the end of this lesson, you can give other many possible exercises as remedial and consolidation activities of this lesson.

Lesson 3: Use Venn diagram to represent a mathematical problem

a) Learning objective

Represent problems using Venn diagrams

b) Teaching resources

Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, wall charts and wall maps.

c) Prerequisites / Revision / Introduction

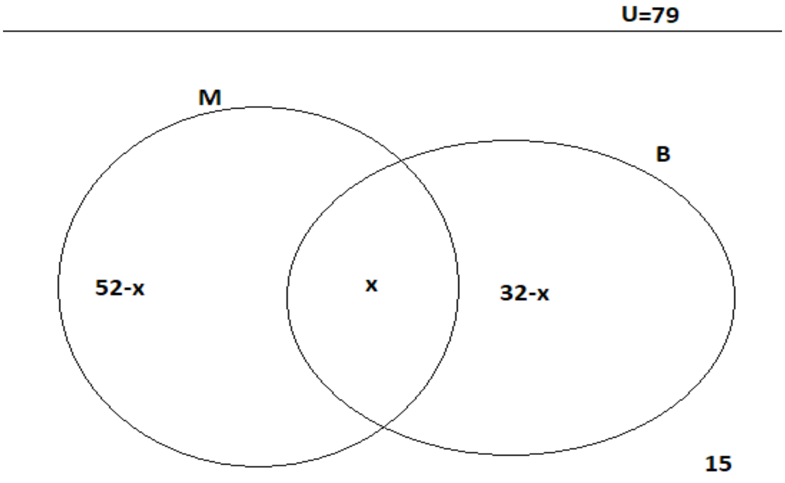
Student-teachers will perform well in this lesson if they revise the Representation of sets using Venn diagrams.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Ask student-teachers to use the student teacher's book to discuss all questions of the **activity 2.3;**
- Move around to each group to ensure all student-teachers participate actively;
- Call upon groups to present their findings and harmonize their answers;
- Use different probing questions to guide students to explore examples and the content given in the student-teacher's book on how to illustrate and solve some practical word problems by using Venn diagrams;
- After this step, guide student-teachers to do the application activity **2.3** and assess their competences and evaluate whether lesson objectives were achieved

Answers to activities 2.3

1. a) and b) let M represent those who bought milk and B represent those who bought bread: $n(\text{M}) = 52; \text{n}(\text{B} = 32)$; $\text{n}(\text{U}) = 79$ let those who bought both milk and bread be represented by x that is $x = n(\text{M} \cap \text{B})$;



$52-x+32-x+x+15=79$

$x=20$

(i) Those who bought milk and bread are 20

(ii) Those who bought bread only are= 32-20=12

(iii) Those who bought milk only are= 52-20=32

c) The easiest method is to use in (a) and (b) above, is the Venn diagram approaches to represent the situation, because it clarifies the situation in a simple way.

2. As 55 students like Mathematics; $x+x+5+20=55 \Rightarrow 2x=30 \Rightarrow x=15$

a) 15 students like all subjects

b)Thetotalnumberofsenioronestudentsis=$U=10+60+20+15+15+5=125$

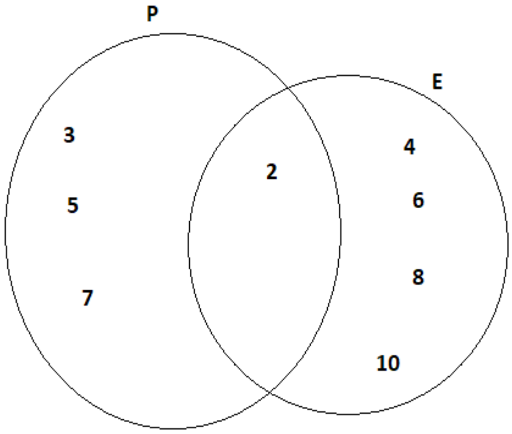
c) Students who like Physics and Kinyarwanda only are =0 (no one)

e) Answers of application activities 2.3

a) $P=\{2,3,5,7\}$ and $E=\{2,4,6,8,10\}$;

i) $P\cap E=\{2\}$; ii) $P\cup E=\{2,3,4,5,6,7,8,10\}$

b). The Venn diagram representing the situation is:



By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation activity in this lesson.

Lesson 4: Modelling and solving problems involving Venn diagrams

a) Learning objective:

Interpret, model, and solve a mathematical problem using set.

b) Teaching resources:

Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, the classroom, textbooks and wall charts and wall maps.

c) Prerequisites / Revision / Introduction:

Student-teachers will perform well in this lesson if they learnt well the previous lesson.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers to use the student teacher's book to discuss the **activity 2.4;**
- Move around to each group to ensure all student-teachers participate actively;

- Call upon groups to present their findings and harmonize their answers in a whole class discussion.
- Use different probing questions to guide students to explore examples and the content given in the student's book on how to model word problems by using Venn diagrams;
- Guide student-teachers to perform individually **application activity 2.4** to assess their knowledge and skills.

Answers to activities 2.4

(a) Let H be the number of people that ate at Hill top Hotel;
S be the number of people that ate at Serena Hotel; L be the number of people that ate at Lemigo Hotel

$H \cap S$ be the number of people that ate at Hill top Hotel and Serena Hotel;

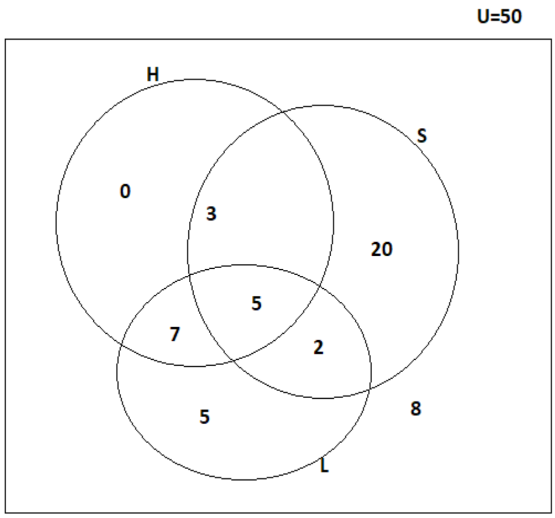
$S \cap L$ be the number of people that ate at Serena Hotel and Lemigo Hotel

$(S \cap L) \cap H$ be the number of people that ate at Serena Hotel, Lemigo Hotel and Hill top Hotel

$H = 15$; $S = 30$; $L = 19$;

$H \cap S = 8$; $H \cap L = 12$; $S \cap L = 7$;

$(S \cap L) \cap H = 5$



(b) The people ate at Hilltop = 0

(c) Hilltop and Serena but not at Lemigo:23

(d) People who did not eat from any of these three hotels are 8

e) Answers to application activities 2.4

a) Let x be the number of student - teacher. The student- teachers who like the three subjects=15

b) The total number of year one student- teachers in ECLPE= $60+10+15+20+20=125$

c) student-teachers who like Physics and Kinyarwanda only = 0.

By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation activity of this lesson**.**

2.6 Summary of the unit

(a) **Set:** It is a group of items with a common feature.

(b) **Member of a set:** It is an object or item in a set.

(c) **Subset:** It is a set which is formed by obtaining some elements in all the elements from a given set.

(d) **Venn diagram**: It is a circular or rectangular pattern used to represent sets and its elements.

(e) **Intersection of sets:** It is the set formed by common elements which appear in two or more sets.

(f) **Union of sets:** It is the set formed by putting together elements of two or more sets.

(g) **Complement of a set:** It is a set of all elements in the universal set that are not members of a given set.

(h) **Difference between sets A and B (A-B):** It is a set formed by the elements appearing in set A but not in set B.

2.7 Additional information for teacher

Here the tutor has to focus on general problems in sets using Venn diagrams.

2.8 End Unit assessment

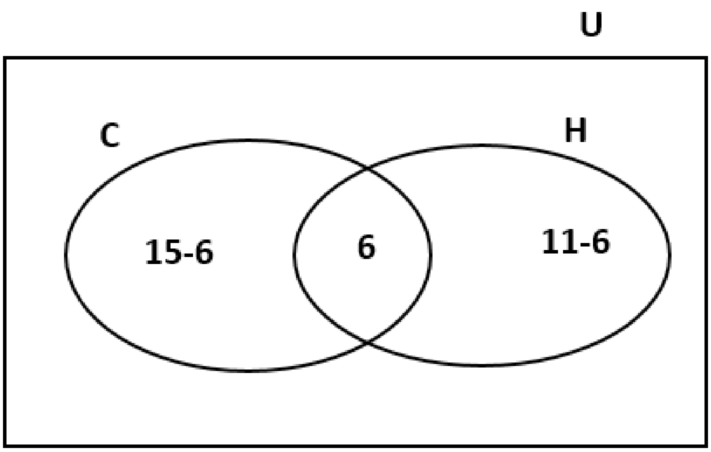
Student teacher should be able to solve all the exercises found in end assessment of unit 2.

Answers:

1. Let the total number of student teacher in that class be U

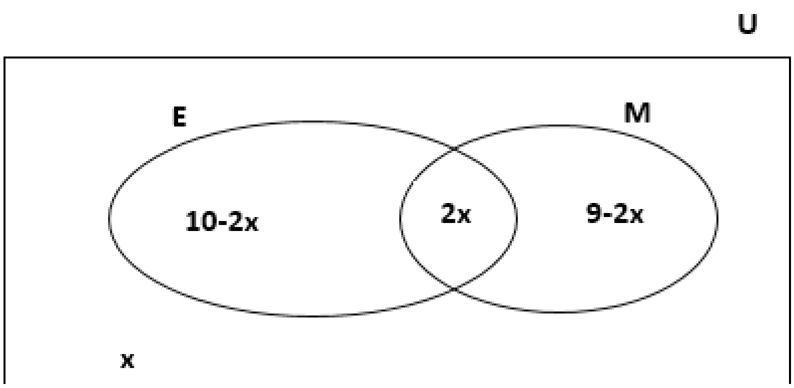
n(Cricket) = 15; n (Hockey) = 11; n($C \cup H$) = 6

The Venn diagram, is then:



$n(\mathrm{U}) = 15 - 6 + 6 + 11 - 6 = 20$ so Number of pupils in the class is 20.

2. Let x be the number of those who teach none

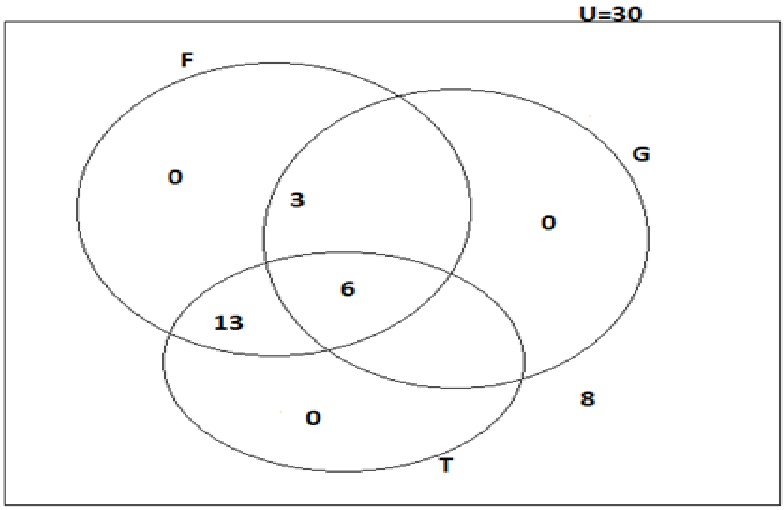


$n(U)=17; n(E)=10; n(M=9)$

From Venn diagram: $10-2x+2x+9-2x+x=17\ ; x=2$

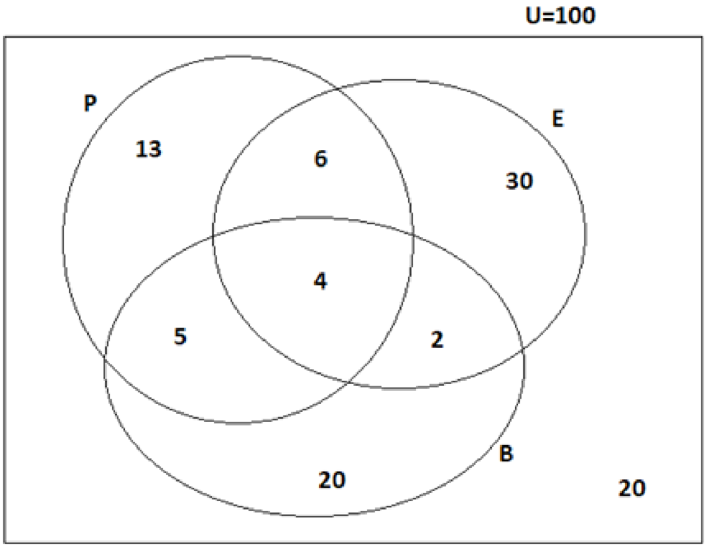
a).. $E\cap M\ =2\times 2=4$; b) none=2; c) only one Subject = $10-4=6$ (Economics); $9-4=5$ (Mathematics); total : $6+5=11$

3. Students did not participate in any of the sports.



The students did not participate in any of the sports=8

4.



a) Students took none of the three subjects: 20

b) Students took PE, but not BIO or ENG=13

c) Students took BIO and PE but not ENG=5

2.9 Additional activities

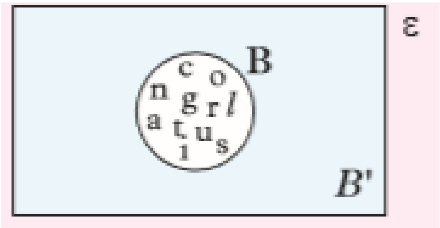
2.9.1 Remedial activities

1. Given that: ε = {a, b, c, d,…,z} ; and B = {letters in the word congratulations} Describe the set B′ and show set B and B′ in a Venn diagram.

2. Given that ε= {a, b, c, d, e} and A= {a, b}, find A′

Solutions:

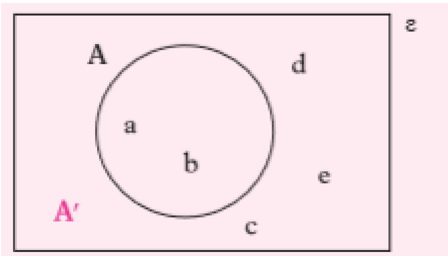
1. Draw a rectangle and label it with ε to show the universal set. Within the rectangle draw set B as a subset of ε



Shade the region within the universal set but outside set B and label it B′. B′ is the set of all the alphabets except those in set B

2. ε= {a, b, c, d, e} A= {a, b} thus,

A’ = {c, d, e}. This can be represented on a Venn diagram as shown.



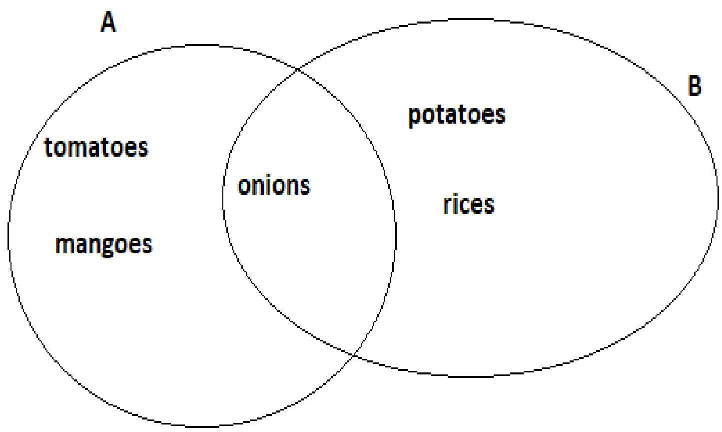
2.9.2 Consolidation activities

1. If set A = {tomatoes, onions, mangoes} and B = {onions, potatoes, rice}, by usingVenn diagram , find: (a) $A \cap B$; (b) $A \cup B$; (c) A – B ; (d) B – A ;(e) $A \Delta B$

2. Consider this Venn diagram, find the number of universe if: n(P)= 8; n(E)=8; n(B)=7

Answers:

1.



a) $A \cap B = \{onions\}$;

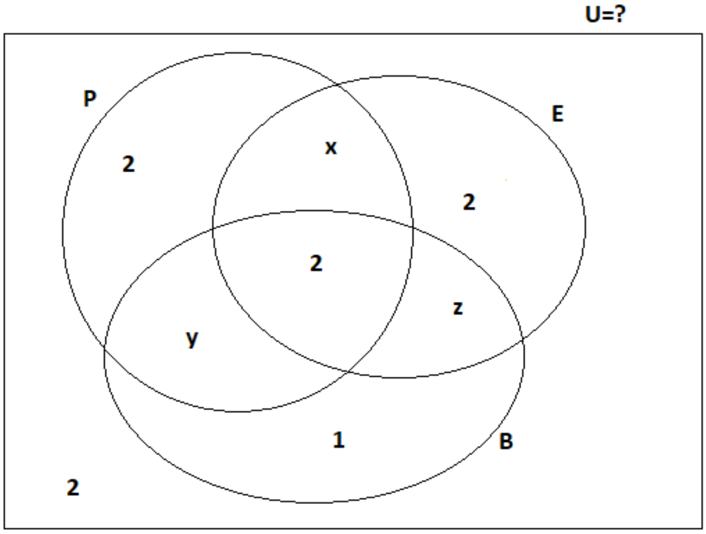
b) $A \cup B = \{tomatoes, mangoes, onions, potatoes, rices\}$;

c) $A - B = \{tomatoes, mangoes\}$;

d) $B - A = \{potatoes, rices\}$;

e) $A \Delta B = \{tomatoes, mangoes, potatoes, rices\}$

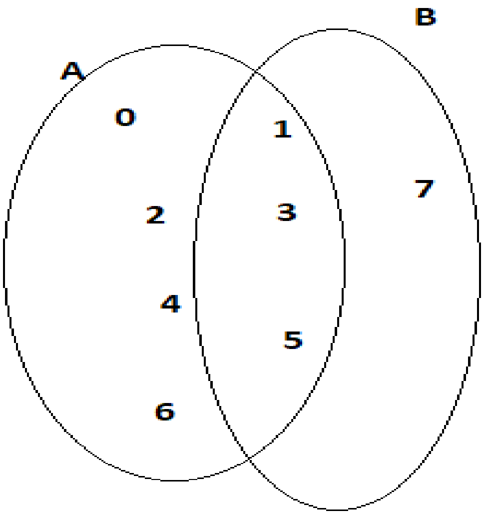
2.



3. $\begin{cases} 2+y+2+x=8 \\ x+2+z+2=8 \\ 2+y+1+z=7 \end{cases} \Rightarrow \begin{cases} x=2 \\ y=2 \\ z=2 \end{cases} \Rightarrow n(\text{U}) = 2+2+2+2+2+2+2+1=15$

2.9.3 Extended activities

1. $A=\{0,1,2,3,4,5,6\}$ *and* $B=\{1,3,5,7\}$



i) Intersection: $A \cap B = \{1,3,5\}$

ii) Union set : $A \cup B = \{0,1,2,3,4,5,6,7\}$

iii) Universal sets: $U = \{\{0,1,2,3,4,5,6\},\{1,3,5,7\}\}$

iv) Simple difference of sets: $A - B = \{0,2,4,6\}$; $B - A = \{7\}$

v) Symmetric difference of sets: $A \Delta B = \{(\mathrm{A} - \mathrm{B}) \cup (\mathrm{B} - \mathrm{A})\} = \{0,2,4,6,7\}$

vi) Complements of sets: $A' = \{7\}$; $B' = \{0,2,4,6\}$

UNIT 3

PROBLEM ON RATIOS AND PROPORTIONS

3.1 Key unit competence

Apply ratios, proportions and multiplier proportion change to solve real life related problems.

3.2 Prerequisites (Knowledge, Skills and Values)

Student teacher will perform well in this unit if they have a good background on:

- Ratio and proportion (Senior1: unit8),
- Multiplier for proportional change (S2 unit 4),
- Properties of algebraic fractions (Senior 3: unit 3).

3.3 Cross-Cutting issues to be addressed

- **Inclusive education**: promote the participation of all student-teachers while teaching;
- **Peace and value Education**: During group activities, the teacher will encourage student-teachers to help each other and to respect opinions of colleagues.
- **Gender:** Give equal opportunities to all students (girls and boys) to present their findings.
- Encourage them to participate actively in all learning and teaching activities from the beginning to the end of the teaching and learning process.
- **Financial education:** guide students to discuss how to invest money into a common project. This should be addressed via problems that imply the way of using money that encourage learner to deal with money financially.

3.4 Guidance on introductory activity

- Invite student-teachers to form groups as heterogeneous as possible and guide them to work on the introductory activity.

- Give time to student-teachers to analyse the activity;
- Invite group representatives to present findings in a whole class discussion;
- Harmonize students' answers enhancing that equal share can be done in different ways;
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, use different questions to arouse their curiosity on what is going to be leant in this unit.

Answer to the introductory activity 3:

A ratio is a comparison between quantities.

A ratio is an ordered pair of numbers written in the order $a : b$ where b cannot be zero (0).

Ratios may be used while calculating things which are compared proportionally.

(a) It depends on the total number of the student teachers that are studying in your class. For example: your class can contain 50 learners including 20 boys and 30 girls. In terms of ratio boys can be expressed as: 2:5 and girls can be expressed by 3:5

(b) Yes, it is possible to share equally a certain number of mathematics textbooks to different groups in your classroom and then figure out the ratio of Mathematics textbooks per learner. Here it depends on the number of textbooks of mathematics and the number of the student teachers' groups. For example, if we consider that there are 60 textbooks of mathematics and 50 learners grouped into 10 groups, we see that each group will have 6 textbooks. The ratio of mathematics textbooks per learner is 6:10

(c) In real life ratio are used for example in Restaurant, in schools, in business, in bank, (student teachers can give other applications)

3.5 List of lessons

No.	Lesson title	Learning objectives	Number of periods
0	Introduction activity	To arouse the curiosity of student teachers on the content of unit 3.	1
1.	Equal and unequal share, Ratio and proportion	Share quantities in a given ratio.	2
2.	Direct and indirect proportion	Compare two quantities by using a given proportion.	2
3.	Calculation of proportional change using multiplier and compound proportional change or continued proportions	Solve problems involving multiplier and compound proportional change;	3
4	Problems involving direct and indirect proportions	Solve problems involving direct and indirect proportions.	2
5	End assessment		2
	Total number of periods		**12**

Lesson 1: Equal and unequal share, Ratio and proportion

a) Learning objective:

Share quantities in a given ratio

b) Teaching resources:

Student-teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators, Internet connection where applicable.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better in this lesson if they have a good understanding on concepts of shares, ratio and proportion learnt in senior 1.

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask student-teachers to use their books to discuss the **activity 3.1.1** and motivate the**m** to determine how to share money respecting a given ratio . Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts of sharing quantities using ratios.
- Use different probing questions to guide students to explore examples and the content related to ratios given in the student's book;
- Guide student-teachers to perform individually **application activity 3.1.1** to assess their competences.

Answers to activities 3.1.1

1) i) Sharing would be done in the ratio of $2:3$

ii) 7000 FRW is to be shared in the ratio $2:3$.

It is spilt into 5 equal parts i.e 2+3=5 equal parts

The first old man gets $\frac{2}{5}\times 7000 = 2800\ FRW$

The second old man gets $\frac{3}{5}\times 7000 = 4200\ FRW$

2) i) A ratio is a mathematical statement which shows how two or more quantities or numbers are compared.

ii) a) Their contributions were in the ratio of 800:120

The simplest ratio is 20:3

b) First $7.5\ l = 7500\ ml$

Their milk sales are in ratio of $4500:7500$. The simplest ratio is $3:5$

e) Answers for application activity 3.1.1

Ingabire, Mugenzi and Gahima, after investing in buying and selling of shares in the Rwanda stock exchange market, they realised a gain of 1 080 000 Frw and intend to uniquely share it in the ratio $2:3:4$ respectively. The task is to find the share of Mugenzi, as follows:

Mugenzi's share

$$= \frac{3}{2+3+4} \text{ of } 1\ 080\ 000 \text{ FRW}$$

$$= \frac{3}{9} \times 1\ 080\ 000 \text{ FRW}$$

$$= 360\ 000 \text{ FRW}$$

Lesson 2: Direct and indirect proportion

a) Learning objective:

Compare two quantities by using a given proportion

b) Teaching resources:

Student teacher's book, Reference books; Ruler, T-square, Manila paper, Scientific calculators.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they make a short revision on concepts of shares, ratio and proportion learnt in senior 1.

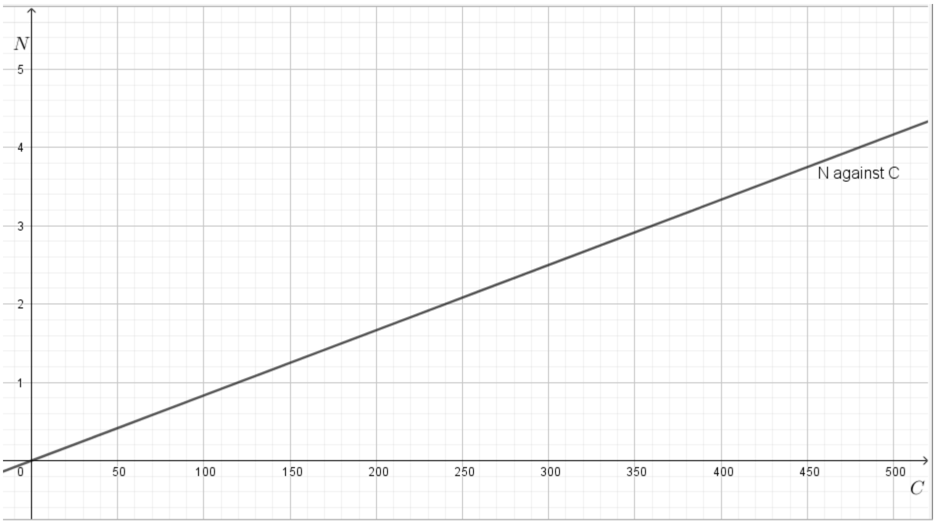
d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask student-teachers to use their books to discuss the **activity 3.1.2** and motivate them to differentiate direct from indirect proportions, determine how to distribute objects respecting the direct or indirect proportions . Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts of distributing objects using direct or indirect proportions;

- Use different probing questions to guide students to explore examples and the content related to direct or indirect proportions given in the student's book;
- Guide student-teachers to perform individually **application activity 3.1.2** to assess their competences.

Answers to activities 3.1.2

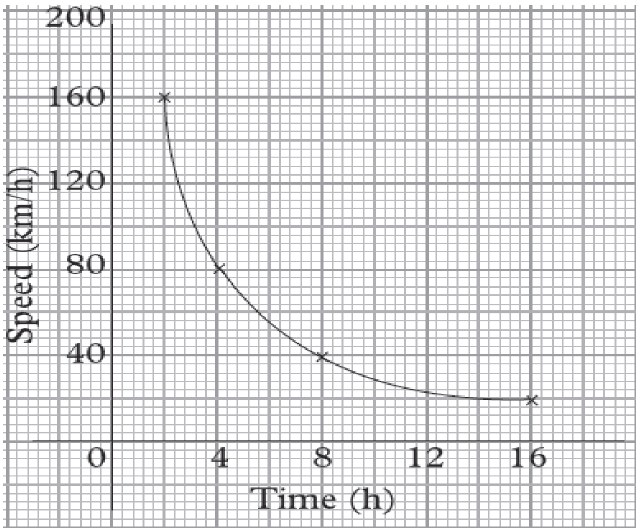
1) i) The graph of the number of pens (N) against cost (C)



ii) The graph of (N) against C is a straight line passing through the origin

2) i) We notice that when then the speed is doubled the time is decreasing

ii)



iii) From the above graph you notice that the speed increases in the ratio $1:2$, the time decreases in the ratio $2:1$ and vice versa. Thus the two quantities are said to be **inversely proportional**.

It means that the car is decelerating with acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{20-160}{16-2} = -10\ kmh^{-2}$$

e) Solution for application activity 3.1.2

1) Expand the products $(x+9)(y-2)=(x+3)(y-6)$

$$xy - 2x + 9y - 18 = xy - 6x + 3y - 18$$
$$9y - 2x = 3y - 6x$$
$$4x = -6y$$
$$\frac{4x}{4y} = \frac{-6y}{4y}$$
$$\frac{x}{y} = \frac{-3}{2} \quad or \quad x:y = -3:2$$

2)

a) $(ap+aq):(bp+bq)$

$$a(p+q):b(p+q)$$
$$a:\text{b} \quad \text{or} \quad \frac{a}{b}$$

b) $(p^2-q^2):(p+q)$

$$(p-q)(\text{p+ q}):\text{p+ q}$$
$$p-q:1 \quad or \quad \frac{p-q}{1}$$

3) $If \quad b \propto c^2 \ then\ b = k\,c^2$

$72 = k \times 12^2$

$72 = 144k$

$k = \frac{72}{144}$

$k = \frac{1}{2}$

4)

a) $mn = k \ or\ m = \frac{k}{n}$, where k is the constant of variation, $k \neq 0$

$25 = \frac{k}{2} \Rightarrow k = 50$

b) $p \times q = k \Rightarrow 16 \times 4 = k$

$k = 64$.

Lesson 3: Calculation of proportional change using multiplier and compound proportional change or continued proportions

a) Learning objective:

Solve problems involving multiplier and compound proportional change.

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding on concepts of shares, ratio, multiplier and proportional change learnt in senior 2, in senior 3 and 2 previous lessons;

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;

- Ask students to use their books to discuss the activity 3.2 and motivate them to determine the proportional change using multiplier. Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts of proportional change;
- Use different probing questions to guide students to explore examples and the content related to Proportional change using multiplier and compound proportional change or continued proportions given in the student's book;
- Guide student-teachers to perform individually **application activity 3.2** to assess their competences.

Solution to activity 3.2

(1)

- Multiplier is a quantity by which a given number is to be multiplied.
- If the shirt is at 20% discount, then the selling price is $100\% - 20\% = 80\%$ of the original price.
- The 80% converted to fraction gives $\frac{80}{100} = 0.80$
- 0.80 is the multiplier price of the shirt.

(2) It is evident that the shirt has been sold at a reduced price compared to the initial buying price. The marked price is reduced proportional by 10% which translates to 50FRW. Therefore the customer bought the shirt at 50 FRW less. It means that the new price was $500FRW - 50FRW = 450FRW$

(3) **1st alternative:**

If 3 people take 3 days to cultivate 2 acres, the number of the people is not increased but days are increased to 5 days.

So, ratio of acres to days is $\frac{2}{3}$ *or* $2:3$

$For\ 5\ days,\ we\ have\ \frac{2}{3}\times\ 5\ =\ 3.33\ acres$

If days are increased to 5, then 3.33 acres are cultivated.

2nd alternative:

$3\,people\ in\ 3days\ cultive\ 2acres$

$3\,people\ in\ 1day\ cultivate\ \frac{2}{3}acres$

$3\,people\ in\ 5days\ cultivate\ \frac{2}{3}\times 5acres = 3.33acres$

e) Solution for application activity 3.2

(1) A 45% decrease means the final percentage for the quantity will be $100\% - 45\% = 55\%$

55% as a decimal $\frac{55}{100} = 0.55$

0.55 is the multiplier.

(2) The salary increase was $\frac{20}{100}\ of\ 15000FRW$

$= \frac{20}{100}\times 15000$

$= 3000FRW$

The new salary

$=\ 15\ 000\ FRW\ +\ 3\ 000\ FRW$

$=\ 18\ 000\ FRW$

(3) Tonnes processed in $2004 = 800$

Percentage decreased $= 30\%$

$=\ 30\%\ of\ \ 800\ tonnes$

Amount decreased $= \frac{30}{100}\times 800$

$= 240\ tones$

Amount produced in 2005:

$= (800 - 240)\ tonnes$

$= 560\ tonnes$

(4) Five men will use $\frac{4\times10}{5} = 8\ days$

Lesson 4: Problems involving direct and indirect proportions

a) Learning objective

Solve problems involving direct and indirect proportions

b) Teaching resources

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

c) Prerequisites / Revision / Introduction

Student teachers will learn better this lesson if they have a good understanding on concepts of shares, ratio and proportion learnt in senior1, multiplier and proportional change learnt in senior 2 , senior 3 and the previous lessons.

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask students to use their books to discuss the activity 3.3 and motivate them to solve problems involving direct and indirect proportions. Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize how to solve a problem involving direct or indirect proportion;
- Use different probing questions to guide students to explore examples and the content related to how to solve a problem involving direct or indirect proportion given in the student's book;
- Guide student-teachers to perform individually **application activity 3.3** to assess their competences.

Solutions to activity 3.4

(1) $F = k \times x \quad \Rightarrow 6 = 4k$

$$\Rightarrow k = \frac{6}{4} \quad \Rightarrow F = \frac{6}{4} \times 5 \quad \Rightarrow F = \frac{30}{4}$$

(2) $A = k \times B^2 \quad \Rightarrow 10 = 4k$

$$\Rightarrow k = \frac{10}{4} = \frac{5}{2} \quad \Rightarrow A = \frac{5}{2} \times B^2 \quad \Rightarrow A = \frac{5}{2} \times 3^2 = \frac{45}{2}$$

The tutor harmonizes findings of learners and check if the following steps are included

i) Understand the problem: Think what information is given and what information is required

ii) Decide on a strategy: List the strategies with which you think the solution can be found

iii) Apply the strategy: Find the solution using the strategy you have chosen

iv) Look back:

- Have you verified your solution?
- Are there other solutions?
- Can you solve a simpler problem?

Have you answered the question as it was initially stated?

a) **Solution for application activity 3.2**

(1) $A = kP^2 \quad \Rightarrow \quad 4 = 8^2 k \quad \Rightarrow \quad k = \frac{4}{64} = \frac{1}{16}$

$$A = \frac{1}{16} P^2 \quad \Rightarrow \quad P = \sqrt{16A} = 4\sqrt{A}$$

(1) a)

$$D=\frac{k}{A}\Rightarrow 120=\frac{k}{40}\Rightarrow k=4800 \qquad \Rightarrow \quad A=\frac{k}{D}=\frac{4800}{D}$$

b) $A=\frac{4800}{150}=32cm^2$

c).. $D=\frac{k}{A}=\frac{4800}{60}=80\ cm^2$

3.6 Summary of the unit

- **A ratio**: It is a relation that compares two or more quantities of the same kind, such as lengths, using division giving one quantity as a fraction of another.
- **Simplifying ratios**: This is where two quantities of a ratio may be multiplied or divided by the same number without changing the value ratio.
- **Sharing** :To share a quantity into two parts in the ratio $a:b$ is where the quantity is split into $a+b$ equal parts and the required parts become $\frac{a}{a+b}$ and $\frac{b}{a+b}$ of the quantity
- If two ratios have the **same value** then they are **equivalent**, even though they may look different.
- **A proportion:** It is a mathematical statement of the equality of two ratios
- **A direct proportion**: It is the proportion in which two quantities are such that, when one quantity increases through a particular ratio, the other quantity increases in the same ratio and vice versa.
- **An inverse proportion**: It is the proportion in which two quantities are such that, when one quantity increases in the ratio $\frac{a}{b}$, the other quantity decreases in the ratio $\frac{b}{a}$
- **A decreasing multiplier** is a factor that reduces the proportion of a given quantity. To calculate the new price, we proceed as

$New\ price = initial\ price \times multiplier$, where,

$Multiplier = \frac{(100-x)}{100}$ and x is the percentage decrease on the cost price.

- **An increasing multiplier** is a factor that increases the proportion of a given quantity. To calculate the new price, we proceed as

New price = initial $\times$ *multiplier*, where

$Multiplier = \frac{(100+x)}{100}$ and x is the percentage increase on the cost price.

3.7 Additional information for tutors

It is important for the tutors to understand that student teachers have to be equipped with skills of sharing. For this reason, student teachers have to be engaged in many practical activities of sharing to understand these concepts very easily.

3.8 End unit assessment

1) a) $F = \frac{k}{d^2} \Rightarrow 0.006 = \frac{k}{2^2} \Rightarrow k = 0.05N$

$F = \frac{0.024}{d^2}$

b) $F = \frac{0.024}{(2.5)^2} = 0.00384N$

c) $d^2 = \frac{0.024}{0.001} \Rightarrow \quad d = 4.90m$

2) a) $2:5 = 4:x \quad i.e\ \frac{2}{5} = \frac{4}{x} \Rightarrow 2x = 20 \Rightarrow x = 10$

b). $8:4 = 20:4x \quad i.e\ \frac{8}{4} = \frac{20}{4x} \Rightarrow 32x = 80 \Rightarrow x = \frac{80}{32}$

c) $4:3x=45:63$ *i.e* $\frac{4}{3x}=\frac{45}{63} \Rightarrow 4\times 63=45\times 3x \Rightarrow 252=135x \Rightarrow x=\frac{252}{135}$

3) Express each of the following in lowest terms:

a) $-16xy:32y \Rightarrow -x:2$

b) $(8k-2k):2k \Rightarrow 6k:2k \Rightarrow 3k:k$

3.9 Additional activities

3.9.1. Remedial activities (Questions and answers)

1) Given the ratio $a:b=5:2$ and ratio $b:c=3:4$ find the ratio $a:b:c$

Solution

Rewrite the ratio $a:b:\text{c}$ as follows

$a:b:c$
$5:2$
$3:4$

Since b is a quantity in both ratios, we make the value of b the same in both ratios so as to join the two ratios as one.

We do this by multiplying the first ratio with the value of b in the second ratio and then we multiply the second ratio with the value of b in the first ratio

$a:b:c$
$(5:2)\times 3$
$2\times(3:4)$
$a:b:c$
$15:6$
$6:8$

Since b is the same, $\begin{matrix} a:b:c \\ 15:6:8 \end{matrix}$

3.9.2. Consolidation activities

1) 150 cm3 of water are contained in 15 litres of a chemical. Find the simplest ratio of water to the chemical.

Solution

Since quantities of a ratio must be of the same unit, we need to convert liters to cubic centimeters

$1 liter = 1000 cm^3$

$15 liters = 15 \times 1000 cm^3 = 15000 cm^3$

$$\text{Re} quired\ ratio = \frac{150 cm^3}{15000 cm^3} = \frac{1}{100}$$

The ratio is $1:100$

1.1.3. Extended activities

The following table shows the time taken to cover a distance of $120km$ at various speeds

Speed, $x(km/h)$	20	30	40	60	80	120
Time, t(h)	6	4	3	2	1.5	1.0

a) Draw a graph of speed against time taken

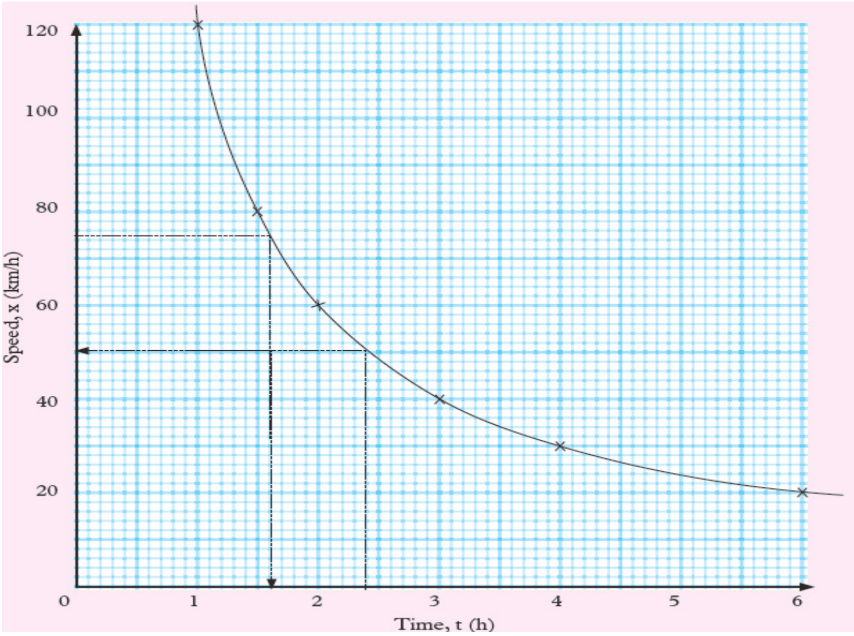
b) Use your graph to determine:

i) The time taken to cover the same distance at a speed of $75km/h$

ii) The speed required to cover the same distance in $2.4h$

Solution

a)



b) From the graph:

(i) The time taken to cover the same distance at a speed of 75 km/h is 1.6 h.

(ii) The speed required to cover the same distance in 2.4 h is 50 km/h.

UNIT 4

OPERATION ON POLYNOMIALS

4.1. Key unit competence

To perform operations, on polynomials and solve related problems.

4.2 Prerequisite

Student teachers will perform better in this unit if they have background on:

- Definition of polynomial ; linear equation (S1 unit 3);
- Perform operations on polynomials;
- Giving common factor of algebraic expressions;
- Applying operation properties to carry out given operation of polynomials

4.3 Cross-cutting issues to be addressed

- **Inclusive education**: promote the participation of all student-teachers while teaching
- **Peace and value Education**: During group activities, the teacher will encourage student-teachers to help each other and to respect opinions of colleagues.
- **Gender:** Give equal opportunities to all learners (girls and boys) to present their findings. Encourage them to participate actively in all learning and teaching activities from the beginning to the end of the teaching and learning process.

4.4 Guidance on introductory activity

- Invite student teacher to form groups and lead them to work on introductory activity to understand the concept of polynomials; linear equation and algebraic expressions
- Give clear guidance and instructions to perform the activities.
- Give time to students to analyse the activity and let them discuss about different it. Provide pieces of advice and adequate facilitations

where necessary.

- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, use different questions to facilitate them to do a good presentation that arouse their curiosity on what is going to be leant in this unit.
- After presenting their finding, you harmonize and guide class discussions and interventions.

Answer of introductory activity 4

1. Lead student teachers to know that in the question 1, from their own researches to **Polynomial** comes from **poly**- meaning "**many**" and -**nomial** meaning "**term**".

2. i) $P = 2(\text{L} + \text{W}) = 2(4x)\left(x^2 + 3x + 2\right) = 8x^3 + 24x^2 + 16x$;

ii) $A = L \times W = 4x(x^2 + 3x + 2) = 4x^3 + 12x^2 + 8x$

3. $A = 3x\left(2x + 3\right) = 6x^2 + 9x$ or you find the sums of the areas of two rectangles: $A_1 = 3x(2x) = 6x^2$ *and* $A_2 = 3x(3) = 9x$ and their sum become: $A = 6x^2 + 9x$

4.5. List of lessons

No.	Lesson title	Learning objectives	Number of periods
0	Introductory activity	To arise the curiosity of student teachers on the content of unit 4	1
1.	Defining and comparing polynomials	Define polynomial	3
2.	Operations on polynomials	Recognize operation properties on polynomials	3
3.	Factorization of polynomials	Factorize a given algebraic expression using appropriate methods	3
4.	Expansion of polynomials	Appreciate the role of numerical value of polynomial and algebraic identities in simplifying mathematical expressions.	3
5.	End assessment		2
Total periods			**15**

Notice:

For application of mathematics content to other subjects, the teacher will consider the prerequisite of learners in this domain then act accordingly; the time spent and importance given to application activities depending on the student teacher's level of knowledge.

Lesson 1: Defining and comparing polynomials

a) **Learning objective:**

Define and classify polynomials by degree and number of terms

b) **Teaching resources:**

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection
- **Note:** If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction:

- Student teachers will learn better this lesson if they have a good understanding on concepts of quadratic equation, linear equation studied in previous year.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 4.1.1.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.

- Harmonize their findings and facilitate them to summarize the concepts related to lesson.
- Guide student-teachers to perform individually application **activity 4.1.1** to assess their knowledge and skills.

e) Answers to activity 4.1.1

The tutor guides the student teachers to discuss about the classification of polynomials basing on the number of terms. The following table identifies the types of polynomial.

Number of terms	Type of polynomial	Example
One term	Monomial	$2x$
Two terms	Binomial	$5x-1$
Three terms	Trinomial	$3a+7b+c$
....	....	....
n terms	Polynomial	$a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$ where $a_n \neq 0$ and the degree $n \geq 0$

The **degree** of a polynomial with only one variable is the **highest exponent** of that variable. For example, in $4x^3 - x + 3$ the degree is **3** which is the highest exponent of *x*.

We are ready to answer the activity 5.1

1) a) 1; b) 2; c) 3; d) 4; e) 5

2) a) 1; b) 1; c) 2; d) 3; e) 4

3) Consider the above to classify the polynomials.

The tutor helps or guides student teacher to discover their own content about: definition of polynomial.

After the tutor tells the student teacher to continue the other examples found in the student teacher book entitled application activities 4.1.1.

f) Answer for application activities: 4.1.1

1. a) polynomial known as quadrinomial;

   b) binomial;

   c) monomial;

   d) binomial;

   e) binomial

2. As A polynomial containing two or more variables are said to be **homogenous** if every term is of the same degree.

For example, $xy^2 + x^2y + 3x^3$ *and* $3x + 2y - 4z$ are homogenous polynomials of degree 3 and 1 respectively.

a) Homogeneous degree 1 ; b) non homogeneous degree 2; c) Homogeneous degree 3; d) Homogeneous degree 2

Lesson 2: Operations on polynomials

a) Learning objective:

Recognize operations properties and perform them on polynomials

b) Teaching resources:

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection

c) Prerequisites / Revision / Introduction:

- Student teachers will learn better this lesson if they have a good understanding on operation on sets of numbers studied in previous years.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 4.2.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts related to lesson.
- Guide student-teachers to perform individually **application activity 4.2.** to assess their knowledge and skills.

e) Answers of activity 4.2.

1.

$$\begin{cases} 3x^3 \\ -13x^2 + x^2 + 5x^2 \\ 4x + 3x - 3x \\ -2 + 4 + 3 \end{cases} \Rightarrow 3x^3 - 7x^2 + 4x + 5$$

2.

If $x = 2 \text{ and } y = 3$

a) $2^2 + 3 + 1 = 8$; b) $3(2)^2 + 2(3) - 3 = 15$

3.

a) $4a + 5 + 3a = 7a + 5$; b) $4a - 5 + 3a = 7a - 5$; c) $4a - 10 - 6a = -2a - 10$; d) $4a + 6a + 10 = 10a + 10$

4.

$$\begin{array}{r} x+6 \\ x+3\overline{)\ x^2 + 9x + 18} \\ -\left(x^2 + 3x\right) \downarrow \\ \hline 6x + 18 \\ -(6x + 18) \\ \hline 0 \end{array}$$

The tutor helps or guides student teacher to discover their own content about: operation on polynomials.

After the tutor tells the student teacher to continue the other examples found in the student teacher book entitled application activities 4.2.

f) Solutions of application activities 4.2

1. **a)** $-9-6+12+16=+13$; **b)** $54+27-18-8=55$

2. **a)** $x^2-xy-2x+xy-y^2-2y \Rightarrow x^2-y^2-2x-2y$;

   b) $6x^3-6x^2+3x-4x^2+4x-2 \Rightarrow 6x^3-10x^2+7x-2$

3. After using long division we get: a) quotient: $3x^2+4x+8$;

   b) remainder: 12

By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation of this lesson.

Lesson 3: Factorization of polynomials

a) Learning objective:

Factorize a given algebraic expression using appropriate methods

b) Teaching resources:

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection

c) Prerequisites / Revision / Introduction:

- Student teachers will learn better this lesson if they refer to: Factorizing quadratic expressions (S2 unit 2); Quadratic equations by factorization method (S3 unit 5).

a) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 4.3.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts related to lesson.
- Guide student-teachers to perform individually **application activity 4.3.** to assess their knowledge and skills.

e) Answers to activity 4.3.

1. The tutor guides the student teacher to discuss on factorization of polynomial and the tutor chooses some groups to share with other by presenting their finding; some of them are: Like factorization of integers in arithmetic, we have factorization of polynomials into other irreducible polynomials in algebra.

For example, $x^2 + 2x$ is a polynomial. It can be factorized into $x \text{ and } (x + 2)$.

$x^2 + 2x = x\,(x + 2)$.

So, $x \text{ and } x + 2$ are two factors of $x^2 + 2x$. While x is a monomial factor, $x + 2$ is a binomial factor.

2. **a)** $2(a+b)$; **b)** $3r(1+2r)$; **c)** $xy(1+a)$;

d) $3xy(3x+5y)$

The tutor helps or guides student teacher to discover their own content about factorization of polynomials.

After the tutor tells the student teacher to continue the other examples found in the student book entitled application activities 4.3.

f) Answers to application activities 4.3

a) $2(ab+2c)$; **b)** $-3b(b+3)$; **c)** $3x(x^2+2x-3)$

By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation of this lesson.

Lesson 4: Expansion of polynomials

a) **Learning objective:**

Factorize a given algebraic expression using appropriate methods

b) **Teaching resources:**

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection

c) **Prerequisites / Revision / Introduction:**

- Student teachers will learn better this lesson if they refer to: Factorizing quadratic expressions (S2 unit 2); Quadratic equations by factorization method (S3 unit 5).

d) **Learning activities**

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 4.4.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts related to lesson.

- Guide student-teachers to perform individually **application activity 4.4.** to assess their knowledge and skills.

e) **Answers to activity 4.4.**

The tutor guides the student teacher to discuss on : expansion of polynomials

1. $x^2 - 2x + 4x - 8 = x^2 + 2x - 8$ now let us factorize this polynomial: $m + n = 2$ *and* $m \times n = -8$; let $m = 4$ *and* $n = -2$ so $(x+m)(x+n) = (x+4)(x-2) = x^2 + 2x - 8$

2. i) $x^2 + 8x + 16$;

   ii) $x^2 - 2x + 1$; we see that all the final polynomials both have 3 terms for each.

   And:

If a trinomial is a perfect square,

The first term must be a perfect square.

The last term must be a perfect square.

The middle term must be twice the product of numbers that were squared to give the first and last terms.

After the tutor tells the student teacher to continue the other examples found in the student book entitled application activities 4.4.

f) **Answers to application activities 4.4**

1. i) $x^2 + x + 3x + 3 \Rightarrow x^2 + 4x + 3$;

   ii) $6x + 3 - 2x^2 - x \Rightarrow -2x^2 + 5x + 3$;

   iii) $8a^2 - 12a - 6a + 9 \Rightarrow 8a^2 - 18a + 9$;

   iv) $4b^2 + 24b - b - 6 \Rightarrow 4b^2 + 23b - 6$;

   v) $12y - 3y^2 + 24 - 6y \Rightarrow -3y^2 + 6y + 24$

2. i) $\begin{cases} x^2 = x \times x & ; \\ 16 = 4 \times 4 & \Rightarrow (x+4)(x+4) \\ 8x = 2(4 \times x) \end{cases}$

ii) $\begin{cases} x^2 = x \times x \\ 36 = 6 \times 6 & \Rightarrow (x+6)(x+6) \\ 12x = 2(6 \times x) \end{cases}$

4.6. Summary of the unit

i) **Unlike terms:** These are terms which have different variable parts.

For example: $2x$ and $3y$ are unlike terms.

ii) **Like terms:** These are terms which have exactly the same variable(s) to the same power. For example: $4n$ and $2n$ are like terms.

iii) **Monomial:** A monomial is an algebraic expression which consists of only one term.

iv) **For example** : $2x$.

v) **Binomial:** A binomial is an algebraic expression which contain (or is made up of) two terms only. **For example:** $3x^2 - 4$.

vi) **Trinomial:** A trinomial is an algebraic expression which is made up of three terms. **For example** : $4xy - 3x + 8$.

vii) **Polynomial:** A polynomial is an algebraic expression containing more than two terms of different powers of the same variable or variables.

The general form of a polynomial is $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2}...$

viii) **Degree or order of the variable:** It is defined by the highest power of the variables in a polynomial

4.7. Additional information for tutor

By the end of this unit the tutor has to inform student teacher about factorization we generally use this formula: $a^2 + 2ab + b^2$; we see that:

$$\begin{cases} a^2 = a \times a \\ b^2 = b \times b \qquad \Rightarrow (a+b)(a+b) \\ 2ab = 2(a \times b) \end{cases}$$

a) The first term of the product is the square of the first term of the binomial, i.e. $(a)^2 = a^2$

b) The second term of the product is two times the product of the two terms of the binomial, i.e. $2 \times (a \times b) = 2ab$

c) The third term of the product is equal to the square of the second term of the binomial, i.e. $(b)^2 = b^2$.

Thus, $(a+b)^2 = a^2 + 2ab + b^2$ and not $a^2 + b^2$

d) This is a common error which must be avoided. Similarly, $(a-b)^2 = a^2 - 2ab + b^2$ and not $a^2 - b^2$.

4.8. End unit assessment

This part provides the answers of end unit assessment activities designed in integrative approach to assess the key unit competence with cross reference to the student teacher book.

At the end of this unit every student teacher is able to solve these exercises found in end activity of unit five:

Answers:

(1) $a)$ *binomial*; $b)$ *monomial*; $c)$ *trinomial*; $d)$ *trinomial*

(2).

$a)$ *homogeneous*; *degree* 1; $b)$ *homogeneous*, *degree* 1; $c)$ *homogeneous*; *degree* 3;

$d)$ *homogeneous*; *degree* 2; $e)$ *homogeneous*; *degree* 3

(3).

a) $(2x-y+3)(2x-y+3)=4x^2-2xy+6x-2xy+y^2-3y+6x-3y+9$

$\Rightarrow 4x^2+y^2+12x-6y-4xy+9$

b). $(a-2b)(3a^2-2ab+b^2)=3a^3-2a^2b+ab^2-6a^2b+4ab^2-2b^3$

$\Rightarrow 3a^3-2b^3-8a^2b+5ab^2$

c). $-x(2x-3x-1)=-2x^2+3x^2+x=x^2+x$

d). $(x-y-2)(2x-y+3)=2x^2-xy+3x-2xy+y^2-3y-4x+2y-6$

$\Rightarrow 2x^2+y^2-x-y-3xy-6$

(4). a) $a(x+y)$; b) $3(x+z)$; c) $3x(7y-2x)$; d) $3x(2x+5y)$;

e) $9x^2(1-5xy^2)$; f) $2x(2+7x)$

(5). i) $\begin{cases} 9p^2=3p\times 3p \\ 16q^2=4q\times 4q \\ 24pq=2\times(3p\times 4q) \end{cases} \Rightarrow (3p+4q)(3p+4q)$;

ii) $\begin{cases} 4x^2=2x\times 2x \\ 9=3\times 3 \\ 12x=2\times(2x\times 3) \end{cases} \Rightarrow (2x+3)(2x+3)$

4.9. Additional activities

4.9.1. Remedial activities:

Simplify:
$a)\ 2x-4y+5x-3y$
$b)\ x^2-3x-2+4x^2-2x+5$
$c)\ 3y^2-4y-6-3-2y-3y^2$

Answers:
$a)\ 7x-7y$
$b)\ 5x^2-5x+3$
$c)\ -6y-9$

4.9.2. Consolidation activities:

Remove brackets and simplify: $a)\ (2x^2-3x)+(5x-8)-(7x^2-4)$

$b)\ 2(3x-y)+4(x+2y)-3(2x-3y)$

$c)\ \{3y-(x-2y)\}-\{5x-(y+3x)\}$

Answers: $a)\ -5x^2+2x-4;\ b)\ 4x+15y;\ c)\ 6y-3x$

4.9.3. Extended activities:

Expand these expressions after you substitute in the final answers when $x=-2$ *and* $y=+2$:

1) $(2x^3-5y^3)(x^2+xy+y^2)$; ii) $(x-y)^3$

Solution: $i)\ 2x^5+2x^4y+2x^3y^2-5x^2y^3-5xy^4-5y^5$; when we replace x and y by their values we get: -224

ii) $x^3-3x^2y+3xy^2-y^3$; again we get : -64

UNIT 5

LINEAR AND QUADRATIC EQUATIONS / INEQUALITIES

5.1. Key unit competence

Solve algebraically or graphically linear, quadratic equations or inequalities

5.2. Prerequisite

In this lesson, Student-teachers must be skilled in **Unit 3** of **S1,** and **Unit 6** of **S3,** i.e:

- Solve problems related to linear equations, inequalities and represent the solutions graphically
- Solve problems involving linear or quadratic functions and interpret the graphs of quadratic functions

5.3. Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others view and thoughts during class discussions)
- Gender (equal opportunity of boys and girls in the lesson participation)
- Standardization culture (regulation of the current intensity)

5.4. Guidance on introductory activity 5

- In groups, facilitate student-teachers read and do the introductory activity from Student -teacher's book
- Facilitate discussions to avoid noise or other unnecessary conversation,
- Be aware of straggling Student –teacher.
- Call them to present their findings and promote gender into presentation.
- Through question-answer, facilitate Student-teachers to realize that introductory activity stimulates them to get idea on Unit 5.

- Through class discussions, let student-teachers think on different ways of getting solutions.

5.5. List of lessons

No	Lesson title	Learning objectives	Number of periods
0	Introductory activity	To arise the curiosity of student teachers on the content of unit 5	1
1	Linear and quadratic equations	Solve linear and quadratic equations or inequalities	3
2	Equations reducible to quadratic		3
3	Linear and quadratic inequalities (algebraically and graphically)		6
4	Solving word problems involving linear or quadratic equations	List and clarify the steps in modelling a problem by linear or quadratic equations and inequalities. Solve mathematical problems involving linear and quadratic equations	5
5	Solving and discussing parametric equations	Solve parametric equations and inequalities	4
6	End unit assessment		2
	Total periods		24

Lesson 1: Linear and quadratic equations

a) Learning objective:

Solve linear and quadratic equations or inequalities

Teaching resources:

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection
- **Note:** If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

b) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding on:

- Representation and interpretation of graphs of linear functions
- Solving linear equations and inequalities and checking / represent their solutions
- Solving problems involving linear or quadratics functions and interpretation of the graphs of quadratic functions.

c) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 5.1.1.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the

concepts related to lesson.

- Guide student-teachers to perform individually **application activity 5.1.1 and 5.1.2** to assess their knowledge and skills.

d) Answers to activity 5.1.1

The tutor guides the student teachers to discuss about polynomials and how to the classification of polynomials is based on the number of terms. The following table identifies the types of polynomial.

1) $x+1=5$ $x+1-1=5-1$ $x=4$	2) $2x-4=0$ $2x-4+4=0+4$ $2x=4$ $\frac{2x}{2}=\frac{4}{2}$ $x=2$	3) $2x+1=-5$ $x+1-1=-5-1$ $x=-6$	4) $x-4=10$ $x-4+4=10+4$ $x=14$

e) Answer of applications activity 5.1.1

(1) $x+5=9 \Rightarrow x+5-5=9-5 \Rightarrow x=4$

(2) $6x+5=5 \Rightarrow 6x+5-5=5-5 \Rightarrow 6x=0 \Rightarrow x=0$

(3) $x-2=3 \Rightarrow x-2+2=3+2 \Rightarrow x=5$

(4) $25=2x-5 \Rightarrow 30=2x \Rightarrow x=15$

(5) $-5=x-1 \Rightarrow -5+1=x \Rightarrow x=-4$

(6) $3x-4=2x+1 \Rightarrow x=5$

(7) $x+5=9x+1 \Rightarrow -8x=-4 \Rightarrow x=\frac{1}{2}$

(8) $-6x-5=9 \Rightarrow -6x=14 \Rightarrow x=\frac{-14}{6}=\frac{-7}{3}$

(9) $x+100=99 \Rightarrow x=99-100 \Rightarrow x=-1$

(10) $6x-51=9 \Rightarrow 6x=9+51 \Rightarrow 6x=60 \Rightarrow x=10$

f) Answer of activity 5.1.2

1)

$x^2 + 2x - 24 = 0$

$Sum = 2$ for $-4; 6$

$\text{Product} = -24$

$x^2 + 2x - 24 = 0$

$x^2 - 4x + 6x - 24 = 0$

$(x^2 - 4x) + (6x - 24) = 0$

$x(x-4) + 6(x-4) = 0$

$(x-4)(x+6) = 0$

$x = 4$ and $x = -6$

2) If the length is twice the width and suppose that width is x, then;

For $W = x$

$L = 2x$

$A = L \times W$

$900 = 2x \times x$

$900 = 2x^2$

$x = \pm\sqrt{\frac{900}{2}}$

$x = \pm\sqrt{450}$

$x = \pm\sqrt{225 \times 2} = \pm 15\sqrt{2}$

$\therefore x = +15\sqrt{2} = W; L = 15\sqrt{2} \times 2 = 30\sqrt{2}$

- Lead student-teachers to realize that the solutions contain some concepts of linear and quadratic equations.
- Lead them to know and use factorization method, discriminant method and completing the square method to solve quadratic equations.
- In small group, lead them to discuss on the examples in student book.
- Guide them to do **application activity 5.1.2** to master the content.

g) Answers of application activity 5.1.2

I. $1) S = \{-2, -4\}$

$2) S = \{3, -1\}$

II. $1) S = \{11, 1\}$

$2) S = \{-7, 5\}$

III. $1) S = \{-8, 3\}$

$2) S = \{9, 4\}$

Lesson 2: Equations reducible to quadratic

a) Learning objective:

Solve linear and quadratic equations or inequalities

b) Teaching resources:

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection
- **Note:** If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding on taught matter in Unit 1 &3 of S1, Unit 2 of S2. This means that they should to be skilled on:

- Representation and interpretation of graphs of linear functions
- Solving problems involving linear or quadratics functions
- Performing operations, factorizing polynomials

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 5.2.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts related to lesson.
- Guide student-teachers to perform individually **application activity 5.2** to assess their knowledge and skills.

e) Answers to activity 5.2.

1) For $u = x^2$; then $x^4 - 2x^2 + 4 = 0$ becomes $u^2 - 2u + 4 = 0$.

2) For $u = x^2$; then $6x^4 + 5x^2 + 1 = 0$ becomes $6u^2 + 5u + 1 = 0$.

- Let student-teachers know the way of reducing biquadratic equations to quadratic equations.
- Through examples in and Guide them to do **application activity 5.2** to master the content.

f) Application activity 5.2

1). $S = \{-3, -2, 2, 3\}$ 2). $S = \{-3, -1, 1, 3\}$ 3. $S = \{-6, -5, 5, 6\}$

Lesson 3: Linear and quadratic inequalities

a) **Learning objective:**

Solve linear and quadratic equations or inequalities

b)**Teaching resources:**

- Student teacher's book,
- Reference books.

- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection
- **Note:** If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) **Prerequisites / Revision / Introduction:**

Student teachers will learn better this lesson if they have a good understanding on taught matter in Unit 1 &3 of S1, Unit 2 of S2. This means that they should to be skilled on:

- Representation and interpretation of graphs of linear functions
- Solving problems involving linear or quadratics functions
- Performing operations, factorizing polynomials

d) **Learning activities**

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 5.3.1, 5.3.2 and 5.3.3**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts related to lesson.
- Guide student-teachers to perform individually **application activity 5.31, 5.3.2 and 5.3.3** to assess their knowledge and skills.

e) **Answers to activity 5.3.1**

1) $\mathbf{0, 1, 2, 3, 4}$ 2) $\mathbf{1, 2, 3, 4, 5}$ 3) $\mathbf{-4, -3, -2, -1, 0}$ 4) $\mathbf{-1, 0, 1, 2, 3}$

f) **Answers for activity 5.3.2**

Solve inequalities in the set of real numbers

1. Start by solving $(x+1)(x-1)=0$

$x+1=0$ $\quad$ $x-1=0$

$\Rightarrow x=-1$ or $\Rightarrow x=1$

The next is to find the sign table.

x	$-\infty$		-1		1		$+\infty$
$x+1$		-	0	+		+	
$x-1$		-		-	0	+	
$(x+1)(x-1)$		+	0	-	0	+	

Since the inequality is $(x+1)(x-1)<0$; we will take the interval where the product is negative. Thus, $S=]-1,1[$

2. Start by solving

$\frac{x+2}{x-1}=0; x-1\neq 0$ or $x-1=0$

$x+2=0$ $\quad$ $\Rightarrow x=1$

$\Rightarrow x=-2$

The next is to find the sign table.

x	$-\infty$		-2		1		$+\infty$
$x+2$		-	0	+		+	
$x-1$		-		-	0	+	
$\frac{x+2}{x-1}$		+	0	-	//	+	

Since the inequality is $\frac{x+2}{x-1}\leq 0$; we will take the interval where the quotient is negative. Thus, $S=[-2,1[$.

g) Answers for activity 5.3.3

$1)2\le x\le 6:\{2,3,4,5,6\}$

2)*none*

$3)1\le x<2:\left\{1,\frac{3}{2},\frac{5}{3},\frac{7}{4},\frac{9}{5}\right\}$

h) Answers for application activity 5.3.1

$1)S=]-\infty,9[$

$2)S=]-\infty,10[$

$3)S=]-\infty,5]$

$4)S=\left[\frac{26}{3},+\infty\right[$

i) **Answers for application activity 5.3.2**

$1)S=]-\infty,-3[\cup]3,+\infty[$

$2)S=]-\infty,-2]\cup[3,+\infty[$

j) Answers for application activity 5.3.3

1) $x^2-10x-20>0\Rightarrow x_1=5+3\sqrt{5};x_2=5-3\sqrt{5}$

$(x-5-3\sqrt{5})(x-5+3\sqrt{5})=0$

x	$-\infty$ $5-3\sqrt{5}$ $5+3\sqrt{5}$ $+\infty$
$(x-5+3\sqrt{5})$	- - - 0 + + + + + + +
$(x-5-3\sqrt{5})$	- - - - - - - - 0 + + + + ++ +
$(x-5+3\sqrt{5})(x-5-3\sqrt{5})$	+ + + + + 0 - - - 0 + + + + +

$s=]-\infty,5-3\sqrt{5}[\cup]5+3\sqrt{5},+\infty[$

2) $6x^2-5x+1<0 \Rightarrow x_1=\frac{1}{2}; x_2=\frac{1}{3}$

x	$-\infty$ $+\frac{1}{3}$ $+\frac{1}{2}$ $+\infty$
$(x-\frac{1}{2})$	- - - - - - 0 + + + + +
$(x-\frac{1}{3})$	- - - - 0 + + + + + + +
$(x-\frac{1}{2})(x-\frac{1}{3})$	+ + + + 0 - - - 0 + + + + +

$s=\left]\frac{1}{3},\frac{1}{2}\right[$

3) $x^2+2x+12>0;$ the solution set is the set of real $\mathbb{R}$**,** because $\Delta<0$ same as question

Lesson 4: Solving word problems involving linear or quadratic equations

a) **Learning objective:**

- List and clarify the steps in modelling a problem by linear or quadratic equations and inequalities.
- Solve mathematical problems involving linear and quadratic equations

b) **Teaching resources:**

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection

- **Note:** If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) **Prerequisites / Revision / Introduction:**

Student teachers will learn better this lesson if they have a good understanding on:

- Represent and interpret graphs of linear functions.
- Solve linear equations and inequalities, appreciate the importance of checking their solution, and represent the solution
- Solve problems involving linear or quadratics functions and interpret the graphs of quadratic functions.

d) **Learning activities**

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 5.4.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts related to lesson.
- Guide student-teachers to perform individually **application activity 5.4** to assess their knowledge and skills.

e) **Answers to activity 5.4**

The tutor guides the student teachers to discuss about polynomials and how to the classification of polynomials

1. The answer is obvious/optional.

2.

a) The steps to proceed:

Let the number be x *: then*

Two times the number: $2x$

Six less than two times a number: $2x-6$

Six less than two times a number is equal to nine: $2x-6=9$

b) The steps to proceed:

Let the cost of the shoes be y*; then*

The cost of clothes is $y+2100$

The total cost of both shoes and clothes $(y)+(y+2100)=22100$

Solving the equation:

$$(y)+(y+2,100)=22,100$$
$$\Rightarrow 2y=20,000$$
$$\Rightarrow y=10,000$$

Therefore, the cost of the shoes was 10,000Frw

3. The price of the demand drop to 1000 units: $P_2=23.69$

$$D=2000+100P-6P^2 \Rightarrow 1000=2000+100P-6P^2$$
$$-6P^2+100P+1000=0 \Rightarrow \Delta=10000+24000=34000$$
$$\sqrt{\Delta}=20\sqrt{85}$$

$$P_1=\frac{-100+20\sqrt{85}}{-12}=-7.03$$
$$P_2=\frac{-100-20\sqrt{85}}{-12}=23.69$$

As the price can't be negative, P_1 is rejected and we consider only $P_2=23.69$

Through examples given in student book , guide student-teachers to model problems involving either inequalities or equations.

Call student-teachers to do **application activity 5.4** to master the content.

f) Answers of application activity 5.4

1) $420-0.2q=60+0.4q \Rightarrow 0.6q=360 \Rightarrow q=600$

$420-0.2q=P \Rightarrow P=420-0.2(600)=300$ we see that the equilibrium quantity=600 and the equilibrium price=300 .

2) $h=\frac{4}{3}w$

$\Rightarrow \quad 192=\frac{4}{3}w^2$

$\Rightarrow \quad w^2=144$

$\Rightarrow \quad w=12m$

$\Rightarrow \quad h=\frac{4}{3}\times 12m=16m$

Lesson 5: Solving and discussing parametric equations

a) **Learning objective:**

Solve parametric equations and inequalities

b) **Teaching resources:**

- Student teacher's book,
- Reference books.
- Ruler, T-square, Manila paper
- Scientific calculators.
- Internet connection
- **Note:** If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) **Prerequisites / Revision / Introduction:**

Student teachers will learn better this lesson if they have a good understanding to:

- Perform operations on linear or quadratic polynomials
- Solve quadratics functions.

d) **Learning activities**

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Tutor asks learners to use the student teacher book to discuss on **activity 5.5.**
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts related to lesson.
- Guide student-teachers to perform individually **application activity 5.5** to assess their knowledge and skills.

e) Answer of activity 5.5

The tutor guides the student teacher to discuss on the given parameter λ in the equation $\lambda x^2+(\lambda-1)x+2=0$

$$x_1, x_2 = \frac{-(\lambda-1)\pm\sqrt{(\lambda-1)^2-8\lambda}}{2\lambda}$$

If $\lambda=0$, there is no root

If $(\lambda-1)^2=8\lambda$, there is one double root

If $(\lambda-1)^2<8\lambda$, there is no real root

If $(\lambda-1)^2>8\lambda$, there are two distinct real root

In small groups, facilitate student-teachers to do examples in the student book.

Call student-teachers to do **Application activity 5.5** to master the content.

f) **Answers of Application activity 5.5**

1. Summary table

λ	Δ	p	s	Conclusion
$\left]-\infty,-\frac{3}{2}\right[$	+	-	+	Two distinct real roots with different signs
$-\frac{3}{2}$	+	-	0	Two opposite real roots: $x_1=-2,\ x_2=2$
$\left]-\frac{3}{2},-1\right[$	+	-	-	Two distinct real roots with different signs
-1	+	| |	| |	Equation of first degree. $x=2$
$\left]-1,-\frac{1}{2}\right[$	+	+	+	Two distinct positive real roots
$-\frac{1}{2}$	0	+	+	One positive double root: $x=2$
$\left]-\frac{1}{2},+\infty\right[$	+	+	+	Two distinct positive real roots

2. Summary table

λ	Δ	p	s	Conclusion
$]-\infty,0[$	+	-	+	Two distinct real roots with different signs
0	+	0	+	Two distinct real roots: $x_1=0,\ x_2=\frac{3}{5}$
$]0,1[$	+	-	-	Two distinct positive real roots
1	0	+	+	One positive double root $x=\frac{1}{3}$
$]1,9[$	-	+	+	No real roots
9	0	+	+	One positive double root: $x=3$

$]9,10[$	+	+	+	Two distinct positive real roots
10	+	||	||	Equation of first degree. $x=\frac{5}{3}$
$]10,+\infty[$	+	-	-	Two distinct real roots with different signs

3. Summary table

a	Δ	p	s	Conclusion
$]-\infty,0[$	+	-	-	Two distinct real roots with different signs
0	+	-	0	Two opposite real roots: $x_1=-1,\ x_2=1$
$\left]0,\frac{1}{2}\right[$	+	-	+	Two distinct real roots with different signs
$\frac{1}{2}$	+	0	+	Two distinct real roots $x_1=0,\ x_2=1$
$\left]\frac{1}{2},1\right[$	+	+	+	Two distinct positive real roots
1	0	+	+	One positive double root: $x=1$
$]1,+\infty[$	+	+	+	Two distinct positive real roots

5.6. Summary of the unit

- In case certain coefficients of equations contain one or several letter variables, the equation is called parametric and the letters are called real parameters. In this case, we solve and discuss the equation (for parameters only).
- If at least one of the coefficients a, b and c depend on the real parameter which is not determined, the root of the parametric quadratic equation depends on the values attributed to that parameter.

5.7. Additional Information for Teachers

- Emphasize on the discriminant method (quadratic formula) in solving quadratic equation:
    - If $\Delta>0$, there are two real distinct roots.

- If $\Delta = 0$, double root.
- If $\Delta < 0$, there are no real root.

- Emphasize on how to write rational fraction, we always set condition/ restriction.

5.8 End unit assessment

This part provides the answers of end unit assessment activities designed in integrative approach to assess the key unit competence with cross reference to the student teacher book.

Answers:

1. a) $x = 13 \Rightarrow s = \{13\}$; b) $x \geq 4 \Rightarrow s = [4, +\infty[$; c) $s =]-3, 2[$;

d) $s = \left]-\frac{7}{3}, 2\right[$; e) $s = \left\{5 - 2\sqrt{6}, 5 + 2\sqrt{6}\right\}$; f) $s = \left[\frac{1}{3}, \frac{1}{2}\right]$;

g) $s = \{1\}$

5.9. Additional activities

5.9.1 Remedial activities

Find the solution of these inequalities:

1. $(x-2)(x+5) \geq 0$
2. $(-x+1)(x-3) \leq 0$

Solutions: (1) $s =]-\infty, -5] \cup [2, +\infty[$; (2). $s =]-\infty, 1] \cup [3, +\infty[$

5.9.2. Consolidation activities:

Solve in real set

1. $x^4 - 5x^2 + 4 = 0$
2. $x^4 - 25x^2 + 144 = 0$

Answers: 1.

$x^4-5x^2+4=0 \Rightarrow u^2-5u+4=0$

$\Delta=25-16=9 \Rightarrow \sqrt{\Delta}=\pm 3$

$u_1=\frac{5+3}{2}=4$

$u_2=\frac{5-3}{2}=1$

$u=x^2 \Rightarrow x^2=4 \Rightarrow x_1=2; x_2=-2$

$x^2=1 \Rightarrow x_3=1; x_4=-1$

$s=\{-2,-1,1,2\}$

2.

$x^4-25x^2+144=0 \Rightarrow u^2-25u+144=0$

$\Delta=625-576=49 \Rightarrow \sqrt{\Delta}=\pm 7$

$u_1=\frac{25+7}{2}=16$

$u_2=\frac{25-7}{2}=9$

$u=x^2 \Rightarrow x^2=16 \Rightarrow x_1=4; x_2=-4$

$x^2=9 \Rightarrow x_3=3; x_4=-3$

$s=\{-4,-3,3,4\}$

5.9.3 Extended activities:

Solve in set of real numbers

1. $2x^3-3x^2-3x+2=0$
2. $3x^3-13x^2+13x-3=0$
3. $6x^4-5x^3-38x^2-5x+6=0$

Solutions:

1) $s=\left\{-1,\frac{1}{2},2\right\}$; **2)** $s=\left\{\frac{1}{3},1,3\right\}$; **3)** $s=\left\{-2,-\frac{1}{2},\frac{1}{3},3\right\}$

UNIT 6

PROBLEMS ON POWERS, INDICES, RADICALS AND COMMON LOGALITHMS

6.1. Key unit competence

Solve problems related to powers, indices, radical and common logarithms

6.2. Prerequisite

Student teacher will perform well in this unit if they have a good background on

- Apply laws of indices and surds to simplify mathematical expressions as taught in S2 (Unit1).
- Represent very small numbers or large numbers in standard form.
- Use the conjugates of surds to compute rationalization of denominator on surds.
- Solve simple equations involving indices and surds.
- Appreciate the importance of indices and surds in solving mathematical problems.

6.3. Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others view and thoughts during class discussions)
- Gender (equal opportunity of boys and girls in the lesson participation)
- Standardization culture (regulation of the current intensity)

6.4. Guidance on introductory activity 6

- In groups, facilitate student-teachers read and do the introductory activity from Student -teacher's book
- Facilitate discussions to avoid noise or other unnecessary conversation,
- Be aware of straggling Student –teacher.

- Call them to present their findings and promote gender into presentation.
- Through question-answer, facilitate Student-teachers to realize that introductory activity stimulates them to get idea on Unit 6.
- Through class discussions, let student-teachers think on different ways of getting solutions.

Answer of introductory activity 6

Lead student-teachers to know that in the question1, the area of painted region is equal to the difference between outer rectangle of design and inner rectangle.

$Outer\ area = \sqrt{256} \times \sqrt{100}\ square\ length\ unit$

Lead student-teachers to simplify radicals

$Outer\ area = 16 \times 10\ square\ length\ unit = 160\ Osquare\ length\ unit$

Then, $Inner\ area = \sqrt{16} \times \sqrt{64}\ square\ length\ unit.$

Simplifying radicals;

$Inner\ area = 4 \times 8\ square\ length\ unit = 32 square\ length\ unit.$

Therefore: $painted\ region = (160 - 32) sq.leng.unit = 28 sq.lng.unit$

Lead student-teachers to do question2, and help them to provide properties of powers and radicals.

a) If $P = b^p$ and $Q = b^q$, then

i) $P \cdot Q = b^p \cdot b^q = b^{p+q}$

ii) $\frac{P}{Q} = \frac{b^p}{b^q} = b^{p-q}$

iii) $P^n = (b^p)^n = b^{pn}$

iv) $\sqrt[n]{P} = \sqrt[n]{b^p} = b^{\frac{p}{n}}$

b) Substituting the expression by $n = 3$, $b = 2$, $p = 3$ and $q = 7$

i) $b^{p+q} = 2^{3+7} = 2^{10} = 1024$

ii) $b^{p-q} = 2^{3-7} = 2^{-5} = \frac{1}{32}$

iii) $b^{pn} = 2^{3\times3} = 512$

iv) $b^{\frac{p}{n}} = 2^{\frac{3}{3}} = 2$

Through class discussions, let student-teachers think of different ways of application of powers and radicals

6.5. List of lessons

No	Lesson title	Learning objectives	Number of periods
0	Introductory activity	To arise the curiosity of student teachers on the content of unit 6	1
1	Powers and radicals Definition of powers/ indices and radicals.	Define powers/ exponents or indices, and radicals.	2
2	Properties of indices and radicals	Identify the properties of powers/ exponents or indices, radicals.	3
3	Operations on indices and radicals	Simplify indices in radicals and Perform operations on indices and radicals	2
4	Decimal logarithm Definition	Define decimal logarithms	1
5	Properties and operations on Decimal logarithms	-Identify the properties of decimal logarithms -Simplify and Perform operations logarithms -Transform a logarithmic expression to equivalent power or radical form and vice versa.	7
6	End unit assessment		2
Total periods			18

Lesson 1: Definition of powers/ indices and radicals

a) **Learning objective**

Define powers/ exponents or indices, and radicals.

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research.

c) **Prerequisites/Revision/Introduction**

Through examples, let student-teachers discuss how to simplify powers, and radicals in real life situations.

d) **Learning activities**

- Remind all student-teachers that they are full of potentials to perform the given activities.
- Organize the student-teachers into groups and make sure that they are inclusive..
- Help them know that they need a group secretary, representative, and timekeeper. Make sure that they all (boys and girls) participate actively in their groups
- Let them read through the **activity 6.1.1.** from student-teacher's book.
- Be around and facilitate where possible, and remind them to give their own examples at least 5 each group.
- After a given time, ask randomly some groups to present their findings to the whole class
- During the presentation, let student-teachers discover that they are using natural numbers.
- Guide student-teachers to work individually **application activity 6.1.1**, given in **Student-teacher's book,** to assess their competences.

e) **Answer of activity 6.1.1**

1. a) $5\times5\times5\times5\times5\times5\times5=5^7$

   b) $3\times3\times3\times3\times3\times t\times t\times s\times s\times s\times s\times s=3^5t^2s^5$

   c) $(x+2)(x+2)(x+2)=(x+2)^3$

2. a) $\sqrt{4}=\sqrt{2^2}=2$

   b) $\sqrt[3]{8}=\sqrt[3]{2^3}=2$

c) $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

d) $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$

e) $\sqrt[6]{64} = \sqrt[6]{2^6} = 2$

f) $\sqrt[7]{128} = \sqrt[7]{2^7} = 2$

Lead student-teachers to realize that the solutions involve the concept of powers and radicals.

- Let student-teachers discover that the product $a \times a \times a \times ... \times a = \boldsymbol{a^n}$ as the simplest form **powers/indices/ exponents** and when the index is **a positive integer n** then $\boldsymbol{a^n}$ is the **general form of powers/indices/ exponents**.
- Lead student-teachers to realize that when you simplify $\sqrt[n]{a^m}$, the radical is completely simplified if $n = m$. Otherwise $\sqrt[n]{a^m}$ becomes **a surd**.
- Let student-teachers discover that for n is positive integer and a is a real number, then the symbol $\sqrt[n]{a}$ denotes **n^{th} roots of a .**
- Lead them to do example 1-3 to emphasize the skills he/she got.
- Lead them to do **Application activity 6.1.1,** to master the content he/she got.

f) **Answers to application activity**

1. a) $10\sqrt{7}$ b) $2\sqrt[3]{3}$ c) $-2\sqrt[5]{2}$ d) $6\sqrt{2}$ e) $\frac{5}{12}$

   f) $\frac{\sqrt{10}}{4}$

2. $side = 150cm$

Lesson 2: Properties of indices and radicals

a) **Learning objective**

Identify the properties of powers/ exponents or indices, radicals.

b) **Teaching resources**

Learner's book and other Reference textbooks to facilitate research.

c) **Prerequisites/Revision/Introduction**

Through examples, let learners discuss how they should obtain the general form of powers and how the radicals are simplified.

d) **Learning activities**

- Remind all student-teachers that they are full of potentials to perform the given activities.
- Organize the student-teachers into groups and make sure that they are inclusive..
- Help them know that they need a group secretary, representative, and timekeeper. Make sure that they all (boys and girls) participate actively in their groups
- Let them read through the **activity 6.1.2.** from student-teacher's book.
- Be around and facilitate where possible, and remind them to give their own examples at least 5 each group.
- After a given time, ask randomly some groups to present their findings to the whole class
- During the presentation, let student-teachers discover that they are using natural numbers.
- Guide student-teachers to work individually **application activity 6.1.2**, given in **Student-teacher's book,** to assess their competences.

e) **Answer of activity 6.1.2**

1.

a) $10^2 \cdot 10^3 = 10^{2+3} = 10^5$ or $10^2 \cdot 10^3 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

b) $(2^5)^2 = 2^{5\times 2} = 2^{10}$ or $(2^5)\cdot(2^5) = 2^{5+5} = 2^{10}$

c) i) $\frac{5^4}{5^3} = \frac{5\times5\times5\times5}{5\times5\times5} = 5$ or $5^{4-3} = 5$

ii) $\frac{3^6}{3^8} = \frac{6\times6\times6\times6\times6\times6}{6\times6\times6\times6\times6\times6\times6\times6} = \frac{1}{6^2}$

iii) The answer is 1.

$\frac{3^6}{3^8} = \frac{6\times6\times6\times6\times6\times6}{6\times6\times6\times6\times6\times6\times6\times6} = \frac{1}{6^2}$

2.

a. $\sqrt{36\cdot 81} = \sqrt{36}\cdot\sqrt{81}$ this is true because $\sqrt{36} = 6$; $\sqrt{81}$=9 then $9 \times 6 = 54$ while $36 \times 81 = 2916$ then $\sqrt{2916} = 54$.

b. $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$ the statement is true.

- Lead student-teachers to realize that the solutions involve powers properties.
- Through guidance on examples in student-teacher's book, let student-teachers develop power properties and radical properties such as:

Properties of indices/powers/exponents.

- **Multiplying powers with the same base**

To multiply numbers or variables with the same base, add their powers.

$a^m \cdot a^n = a^{m+n}$; for positive integers m and n.

- **Finding a power of a power**

To find a power of a power, multiply those powers.

$(a^m)^n = a^{m\cdot n}$; for positive integers m and n.

- **Raising a product to a power**

For every non-zero number a and b and positive integer n then,

$$(ab)^n = a^n b^n.$$

- **Dividing powers with the same bases**

To divide numbers or variables with the same base, subtract their powers.

$\frac{a^m}{a^n} = a^{m-n}$; for $a \neq 0$ and positive integers m and n.

- **Raising a ratio to a power**

For every non-zero number a and b and positive integer n then,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

- **Zero as a power**

If a power of a non-zero number or variable is zero, then $a^0 = 1$; for $a \neq 0$.

- **Negative powers**

$a^{-n} = \frac{1}{a^n}$; for $a \neq 0$.

- **Fractional index/power property and its converse**

For every non-zero number a and positive integers m and n

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \quad \text{and} \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a} \qquad \text{and} \qquad \sqrt[n]{a} = a^{\frac{1}{n}}$$

Properties of n^{th} roots.

- **Multiplication property of radicals and its converse**

For every number $a \geq 0$ and $b \geq 0$ and positive integer n

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \text{ and } \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

- **Division property of radicals and its converse**

For every number $a \geq 0$ and $b > 0$ and positive integer n

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ **and** $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

- **A radical of a radical property and its converse**

a) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = a^{\frac{1}{nm}}$

In fact,

$$\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{m}\times\frac{1}{m}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$$

- But it must be carefully noted that

For non-zero a and b: $\sqrt[n]{a \pm b} \neq \sqrt[n]{a} \pm \sqrt[n]{b}$

f) **Answers to application activity 6.1.2**

1.a) $ab\sqrt[10]{a^5}$ or $ab\sqrt{a}$ b) $ab^2c\sqrt{abc}$ c) $\frac{s}{3}$

2. The error of student-teachers is that he/she confused $(-5)^0 and\ -5^0$.

   Since we have $(-5)^0 = 1$ while $-5^0 = -1$.

3. The answer is negative, because $-3^{-2} = -\left(\frac{1}{3^2}\right) = -\frac{1}{9}$

4. Volume of a cube is s^3, so $V = 3^{12} \cdot 2^{18} m^3$

5. $(u^3)^4 = u^{3\times4} = u^{12}$ while $u^{3^4} = u^{3\times3\times3\times3} = u^{81}$, so the expressions are totally different.

6. a) 2^{13} can be written as $2^6 \times 2^7$, $2^9 \times 2^4$, $2^1 \times 2^{12}$, $2^3 \times 2^4 \times 2^5 \times 2^1$,...

   b)Because $a^m \cdot a^n = a^{m+n}$

7. a) No, because $-(2^3)^2 = -2^6$ while $(-2^3)^2 = 2^6$

   b) No, because $(3y)^7 = (3)^7(y)^7$ while $3(y)^7$

8. 2^{30} because $2^{15} \times 2^{15}$ or $(2^{15})^2$

9. 2^{20} can be written as $2^{10} \cdot 2^{10}$, $(2^{10})^2$, $(2^4)^5$, $2^{11} \cdot 2^5 \cdot 2^{16} \cdot 2^{-12}$, ...

10. a) $A = 16c^2 square\ unit$ b) $A = 25x^4 cm^2$

Lesson 3: Operations on indices and radicals

a) **Learning objective**

Simplify indices in radicals and perform operations on indices and radicals.

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research.

c) **Prerequisites/Revision/Introduction**

Through examples, let student-teachers to recall the way of evaluating and simplifying powers and radicals using properties.

d) **Learning activities**

- Remind all student-teachers that they are full of potentials to perform the given activities.
- Organize the student-teachers into groups and make sure that they are inclusive.
- Help them know that they need a group secretary, representative, and timekeeper. Make sure that they all (boys and girls) participate actively in their groups
- Let them read through the **activity 6.2.** from student-teacher's book.
- Be around and facilitate where possible, and remind them to give their own examples at least 5 each group.
- After a given time, ask randomly some groups to present their findings to the whole class
- During the presentation, let student-teachers discover that they are using natural numbers.

- Guide student-teachers to work individually **application activity 6.2**, given in **Student-teacher's book,** to assess their competences.

e) **Answers to activity 6.2.**

1) **a)** $x^3x^2 = x^{2+3} = x^5$ **b)** $\frac{6xy^2}{3xy} = 2y$ **c)** $\frac{xy}{4yx} = \frac{1}{4}$

2) 1. $\sqrt{18} + \sqrt{2} = \sqrt{9 \times 2} + \sqrt{2} = 3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$

2. $\sqrt{12} - 3\sqrt{3} = \sqrt{4 \times 3} - 3\sqrt{3} = 2\sqrt{3} - 3\sqrt{3} = -\sqrt{3}$

3. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

4. $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$

- Lead student-teacher's to realize that the solutions involve some operation on powers and radicals.
- Through given examples in **student-teacher's book,** Let student-teachers discover that:
  - When **adding or subtracting the indices**, we may need to combine like terms to make one.
  - When **adding or subtracting the radicals** we may need to simplify if we have similar radicals (Similar radicals are the radicals with the same indices and same bases).
  - When **multiplying or dividing the indices**, we may need to apply the rules

a) $a^m \cdot a^n = a^{m+n}$

b) $(a^m)^n = a^{mn}$

c) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

d) $\frac{1}{b^m} = b^{-m}$

e) $\frac{a^m}{a^n} = a^{m-n}$

f) $(ab)^m = a^m b^m$

These rules help us to simplify some powers.

- When multiplying or dividing the **radicals**, we may need to apply the rules

$\forall n \in \mathbb{N} \setminus \{1\}, m \in \mathbb{R}$

f) Answers to application activity 6.2

1) **a)** $5x^2y^6$ **b)** 0

2) **a)** $3\sqrt{5}$ **b)** $3\sqrt{7}$ **c)** $2\sqrt{3}$ **d)** 12 **e)** $13\sqrt{5}$

3) **a)** $\frac{\sqrt{2}}{2}$ **b)** $\frac{2\sqrt{5}-\sqrt{15}}{10}$ **c)** $\frac{-2-2\sqrt{6}}{5}$ **d)** $\frac{-3-\sqrt{6}+\sqrt{10}+\sqrt{15}}{2}$

Lesson 4: Definition of Decimal logarithm

a) Learning objective

Define decimal logarithms.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction

Through examples, let student-teachers share information about the use of power properties especially **standard form notation(** $e.g: 10{,}000{,}000 = 10^7$**)**.

d)Learning activities

- Remind all student-teachers that they are full of potentials to perform the given activities.
- Organize the student-teachers into groups and make sure that they are inclusive.

- Help them know that they need a group secretary, representative, and timekeeper. Make sure that they all (boys and girls) participate actively in their groups
- Let them read through the **activity 6.3.1.** from student-teacher's book.
- Be around and facilitate where possible, and remind them to give their own examples at least 5 each group.
- After a given time, ask randomly some groups to present their findings to the whole class
- During the presentation, let students discover that they are using natural numbers.
- Guide student-teachers to work individually **application activity 6.3.1**, given in **Student-teacher's book,** to assess their competences.

e) Answers of activity 6.3.1

1) 0 because $10^0 = 1$

2) 1 because $10^1 = 10$

3) 2 because $10^2 = 100$

4) 3 because $10^3 = 1000$

5) 4 because $10^4 = 10000$

6) 5 because $10^5 = 100000$

Let learners discover that the **decimal logarithm** of a positive real number x is defined to be a real number y for which 10 must be raised to obtain x means $y = 10^x$. Hence, $\forall x > 0$, $y = \log x$ **or** $y = \log_{10} x$

In general notation we do not write this base for decimal logarithm.

In the notation $y = \log x$, x is said to be the **antilogarithm of** y.

Therefore $y = \log x$ means $10^y = x$

Co-logarithm

Co-logarithm of a number is the logarithm of the reciprocal of that number, equal to the negative of the logarithm of the number itself,

$$\operatorname{colog} x = \log\left(\frac{1}{x}\right) = -\log x$$

f) Answers to application activity 6.3.1

1.a) $\log_7 49 = 2$ b) $\log_6 \frac{1}{6} = -1$ c) $\log_0 1 = -1$ d) $\log_8 2 = \frac{1}{3}$

2.a) -2 b) -1.6 c) -1.17

3.a) $3^4 = 81$ b) $10^1 = 10$ c) $5^{-4} = \frac{1}{625}$ d) $9^{\frac{3}{2}} = 27$

4. a) -3 b) -3 c) $\frac{1}{3}$ d) -2
e) 5 f) 36 g) -5

Lesson 5: Properties and operations on decimal logarithms

a) Learning objective

- Identify the properties of decimal logarithms
- Simplify and Perform operations logarithms
- Transform a logarithmic expression to equivalent power or radical form and vice versa.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction

Through examples, let student-teacher's share information about how they can identify the properties and simplify logarithms

d) Learning activities

- Remind all student-teachers that they are full of potentials to perform the given activities.

- Organize the student-teachers into groups and make sure that they are inclusive.
- Help them know that they need a group secretary, representative, and timekeeper. Make sure that they all (boys and girls) participate actively in their groups
- Let them read through the **activity 6.3.2.** from student-teacher's book.
- Be around and facilitate where possible, and remind them to give their own examples at least 5 each group.
- After a given time, ask randomly some groups to present their findings to the whole class
- During the presentation, let student-teachers discover that they are using natural numbers.
- Guide student-teachers to work individually **application activity 6.3.2**, given in **Student-teacher's book,** to assess their competences.

e) Answer to activity 6.3.2

1. Calculating logarithms:

a) $\log 100 = \log 10^2 = 2\log 10 = 2$ b) $\log 1000 = \log 10^3 = 3\log 10$ 17^2

2. Calculating logarithms one can find:

$$\begin{aligned}\log(100)\times(1000) &= \log 100 + \log 1000 \\ &= \log 10^2 + \log 10^3 \\ &= 2\log 10 + 3\log 10 \\ &= 2\log 10 + 3\log 10 \\ &= 2 + 3 = 5\end{aligned}$$

3. Comparing the results in 2 and the sum $\log 100 + \log 1000$. One can notice that the result is 5 and from this, one can deduce that

   $\log(a \times b) = \log a + \log b$, when a *and* b are positive real numbers

- Let student-teachers discover that:
  - *Logarithm of the product of two or more positive numbers is the sum of logarithm of those numbers*

a) $\log ab = \log a + \log b$

- *Logarithm of the quotient of two positive numbers is the difference of logarithm of those numbers*

b) $\log \frac{a}{b} = \log a - \log b$

- *Logarithm of the power of a positive number is the product of the exponent with the logarithm of that number.*

c) $\log a^n = n \log a$

- *Others properties*

  - $\log \frac{1}{b} = -\log b$
  - $\log_a a = 1$
  - $\log_a 1 = 0$

- $\log_a \sqrt[n]{a} = \frac{1}{n}$ and $\log_a \sqrt[n]{b} = \frac{1}{n}\log_a b$
- $\log_a \sqrt[n]{a^m} = \frac{m}{n}$ and $\log_a \sqrt[n]{b^m} = \frac{m}{n}\log_a b$

f) Answers to application activity 6.3.2

1. a) $a > b$ b) $a = b$ c) $a < b$
2. a) 2.17 b) 0.66 c) 0.30

6.6. Summary of the unit

1. We call n^{th} power of a real number a that we note a^n, the product of n factors of a. that is

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}} \quad \begin{cases} n \text{ is an exponent} \\ a \text{ is the base} \end{cases}$$

2. The n^{th} root of a real number is $\frac{1}{n}$ power of that real number. It is noted by $\sqrt[n]{b}$, $b \in \mathbb{R}, n \in \mathbb{N} \setminus \{1\}$. $\forall a,b \in \mathbb{R}, \sqrt[n]{b} = a \Leftrightarrow b^{\frac{1}{n}} = a \Leftrightarrow b = a^n$

$$\begin{cases} n \text{ is called the index} \\ b \text{ is called the base or radicand} \\ \sqrt[n]{\ } \text{ is called the radical sign} \end{cases}$$

3. Rationalizing is to convert a fraction with an irrational denominator to a fraction with rational denominator. To do this, if the denominator involves radicals we multiply the numerator and denominator by the conjugate of the denominator.

4. The decimal logarithm of a positive real number x is defined to be a real number y for which 10 must be raised to obtain x. We write $\forall x > 0,\quad y = \log x$

6.7. Additional Information for tutor

1. $\log_a x \pm y \neq \log_a(x \pm y)$ because $\log_a x \pm y = (\log_a x) \pm y$

2. $\log_a(x \pm y) \neq \log_a(x) \pm \log_a(y)$

3. $\log_a x^{-1} \neq \frac{1}{\log_a x}$ because $\log_a x^{-1} = \log_a \frac{1}{x}$ while $(\log_a x)^{-1} = \frac{1}{\log_a x}$

4. $(\log_a x)^n \neq \log_a x^n$ because $(\log_a x)^n = \log_a x \times \log_a x \times \log_a x \times \ldots \times \log_a x$ while

   $\log_a x^n = n \log_a x$

5. $\sqrt{x \pm y} \neq \sqrt{x} \pm \sqrt{y}$

6.8. End unit assessment

Answers of End unit assessment

1. a) ab^2c b) abc c) $\frac{2}{3}$ d) x

   e) $\frac{xy^4}{2}$

2. a) $\log_b X + \log_b Y - \log_b Z$ b) $\log_b X - \log_b Y - \log_b Z$

c) $2\log_b P + \frac{1}{3}\log_b Q$ d) $\frac{1}{2}\log_b P + \frac{3}{2}\log_b Q - \frac{1}{4}\log_b R - \frac{1}{2}\log_b S$

3. a) $\log_b \frac{xy}{z^2}$ b) $\log_b 2\pi\sqrt{\frac{l}{g}}$

4. a) $c = 10cm$ b) $b = 4cm$ c) $a = 5cm$

6.9. Additional activities

6.9.1. Remedial activities

1. Simplify

a) $(x-1)(x-1)$ b) $x.x.x.x.x$

c) $\frac{x^{-3}y^5}{y^{-2}x^3}$ d) $\sqrt[3]{\sqrt{\frac{x^6y^{12}}{z^3}}}$

2. Expand and simplify the following logarithms in terms of their composites.

a) $\log_2 \frac{1}{2}$ b) $\log_3 3xyz$ c) $\log_p P^2$ d) $\log_b \sqrt{\frac{Q^3}{S}}$

3. Express each expression below as simple logarithm

a) $\log_a b + 2\log_a a - \log_a c$ ***b)*** $\log_b 2 + \log_b \pi$

6.9.2. Consolidation activities

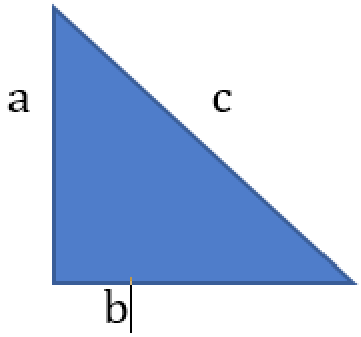
1. Expand the following logarithms in terms of their composites.

a) $\log_b \frac{bt}{z}$ b) $\log_b \frac{x}{yb}$ c) $\log_b b^2 \cdot \sqrt[3]{b}$ d) $\log_b \sqrt{\frac{b^3}{b^{\frac{1}{2}}}}$

2. Express each expression below as simple logarithm

a) $\log_b xy^3 - 2\log_b xy + \log_b z$ b) $\log_b 2 + \log_b \pi + \frac{4}{5}\log_b k - \frac{2}{5}\log_b gh$

3. Use the right triangle below and find the missing length (keep your answer in cm)



a) a=12cm, b=8cm

b) a=0.36dm, c=4.4cm

6.9.3 Extended activities

1. Expand and simplify the following logarithms in terms of their composites.

a) $\log_b \frac{\sqrt[3]{xy^6z^{10}}}{\sqrt[3]{zx^4y^3}}$ b) $\log_b \sqrt{\left(\frac{P}{Q}\right)^3} \cdot \sqrt[3]{\frac{P}{Q}}$ c) $\log_b \sqrt[3]{\frac{P \cdot Q^3}{R^{\frac{1}{2}}S}}$

2. Evaluate and simplify

a) $\sqrt{3}\log_b b^2 - 2(\log_b b^5 + \log_z z^{\sqrt{3}})$ b) $\sqrt{3}\log_b b^2 - \frac{1}{\sqrt{3}}(\log_b b^5 + \log_z z^{\sqrt{3}})$

3. Express each of the expressions in simplest form.

a. $4\sqrt{2} + 3\sqrt{2} - 2\sqrt{2}$

b. $6\sqrt{3} - \sqrt{27}$

c. $\sqrt{3} \cdot \sqrt{15}$

d. $2\sqrt[3]{5} - \sqrt[3]{135} + 4\sqrt[6]{25}$

e. $\sqrt[3]{18} \cdot \sqrt[3]{5}$

f. $\sqrt[4]{64} - 5 \cdot \sqrt[6]{\frac{1}{8}}$

g. $2\frac{1}{\sqrt{7}} + 3\sqrt{28} - \sqrt{63}$

4. Write each answer as a power of 2.

a) Computer capacity is often measured in bits and bytes. A bit is the smallest unit, a 1 or 0 in the computer's memory. A byte is 2^3 bits. A megabyte (MB) is 2^{20} bytes. How many bits are in a megabyte?

b) A gigabyte (GB) is 2^{10} megabytes. How many bytes are there in a gigabyte? How many bits are there in a gigabyte?

5. Explain why $x^8 \cdot x^2$ has the same value as $x^5 \cdot x^5$.

6. Mugisha thinks that $e^3 + e^3$ simplifies to $2e^3$. Rukundo thinks that $e^3 + e^3$simplifies to e^6. Which result is correct? Explain

7. Explain why $(-xy)^2 = (xy)^2$.

8. Rationalize the denominator

1. $\frac{5}{\sqrt{7}}$ 2. $\frac{3-2\sqrt{2}}{1-\sqrt{2}}$ 3. $\frac{2\sqrt{2}}{4+3\sqrt{3}}$

4. $\frac{a-\sqrt{b}}{\sqrt{d}}$ 5. $\frac{3\sqrt{3}+2\sqrt{2}}{1+2\sqrt{2}}$

9. Let $P = b^p$ and $Q = b^q$, then $\log_b P = p$ and $\log_b Q = q$. Show that:

1) $P \cdot Q = b^{p+q}$ and $P \cdot Q = \log_b P + \log_b Q$

2) $\frac{P}{Q} = b^{p-q}$ and $\frac{P}{Q} = \log_b P - \log_b Q$

3) $P^n = b^{np}$ and $P^n = n \log_b P$

4) $\sqrt[n]{P} = b^{\frac{p}{n}}$ and $\sqrt[n]{P} = \frac{1}{n}\log_b P$

UNIT 7

PROPOSITIONAL AND PREDICATE LOGIC

7.1 Key unit competence

Use Mathematical logic as a tool of reason and argumentation in daily situation

7.2 Prerequisite

Student-teachers will perform well in this unit if they have a good background on types of sentences as learnt in English grammar; and Set theory as in unit two.

7.3 Cross-cutting issues to be addressed

- **Inclusive education**: promote the participation of all student-teachers while teaching)
- **Peace and value Education**: During group activities, the teacher will encourage student-teachers to help each other and to respect opinions of colleagues.
- **Gender:** During group activities try to form heterogeneous groups (with boys and girls) or when student-teachers start to present their findings encourage both (boys and girls) to present.

7.4 Guidance on introductory activity 7

- Form groups of student-teachers that are as heterogeneous as possible and guide them to work on the introductory activity.
- Walk around all groups to provide pieces of advice where necessary.
- After a given time invite student-teachers to present their findings and harmonize them.

Answer of introductory activity 7

Lead student-teachers to know that in the given activity, you can get different answers depending on the given conditions.

Through class discussions, let student-teachers think of different possible solutions and justify their validity.

During the presentation, let student-teachers discover the concept of logic.

Answers may vary; some of them are as follows:

1. If you give a child an orange and another an orange, they will have two oranges as $1 orange + 1 orange = 2 oranges$.
2. If you give a child an orange and another an orange, they will have none as they will have eaten them.

From different given and justified solutions, deduce that logic is a process of constructing arguments by careful deduction.

7.5 List of lessons

#	Lesson title	Learning objectives	Number of periods
0	**Introduction activity**	To arise the curiosity of student teachers on the content of unit 8	1
1	Simple statement and compound statements	• Give example of a logical statement • Convert into logical formula composite propositions and vice versa.	1
2	Truth tables	• Draw the truth table of a proposition • Draw the truth table of a composite proposition.	2
3	Negation	Use correctly negation of logical statements in daily life	1
4	Conjunction	Use correctly conjunction of logical statements in daily life	2
5	Disjunction	Use correctly disjunction in logical statements of daily life	1

6	Conditional statement	Use correctly conditional statements in daily life	2
7	Bi-conditional statements	Use correctly bi-conditional statements in daily life statements	1
8	Tautologies and Contradictions	Show that a given logic statement is tautology or a contradiction	2
9	Predicates	Use correctly logical statements, propositions, connectives and quantifiers in daily life	1
10	Quantifiers	Use correctly logical statements, propositions, connectives and quantifiers in daily life	1
11	Negation of quantifiers	Use correctly negation of quantifiers in logical statements of real life	1
End unit assessment			2

Lesson 1: Simple statement and compound statements

a) Learning objective:

Give example of a logical statement or proposition and Convert into logical formula composite propositions and vice versa

b) Teaching resources:

- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have back ground on the forms of sentences with examples as learnt in English.

d) Learning activities

- **Guidance**
  - Organize the student-teachers into groups
  - Introduce the topic by asking them the forms of sentences and give examples for each.
  - Let them attempt the **Activity 7.1.1** from student-teacher's book.
  - Ask randomly some groups to present their findings to the whole class
  - During the presentation, let student-teachers discover that only declarative (affirmative or negative) sentence is a statement while interrogative, imperative (command) or exclamative sentences could not be statements.
  - Lead them to define a compound statement and give examples
  - Guide student-teachers to work individually Application activity 7.1.1, given in **Student-teacher's book,** to assess their competences.
- **Answers of Activity 7.1.1**
  1. T
  2. F
  3. Neither true nor false
  4. F
  5. Neither true nor false
  6. Neither true nor false
  7. T

e) Answers of Application Activity 7.1.1

1. Answers of question 1:

   a) A statement, whose true value is true. A statement, whose true value is true.

   b) Not a statement. It is an imperative sentence.

   c) A statement, whose true value is false

   d) A statement, whose true value is true.

   e) Not a statement. It is an exclamative sentence.

2. Answers of question 2:

a) F

b) T

c) T

d) F

e) F

Lesson 2: Truth tables

a) Learning objective: Draw the truth table of a proposition or a composite proposition.

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of logical statement as learnt in 1st lesson of this unit.

d) Learning activities

- **Guidance**

- Organize the student-teachers into groups and ask them to attempt the **Activity 7.1.2** from student-teacher's book and introduce the concept logical statement.
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings
- Lead students to work on example 7.1 and 7.2 by letting them work individually application activity 7.1.2 for checking the skills they have acquired.

- **Answers of Activity 7.1.2**

  a) 4 possibilities for the truth-values of p and of q?

  b) Using a table,

i.

p	q
T	T
T	F
F	T
F	F

$\{(T,T),(T,F),\ (F,T),\ (F,F)\}$

ii.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

There are 8 possibilities, for the triples of truth-values of three statements:

$\{(T,T,T),(T,T,F),(T,F,T),(T,F,F),(F,T,T),(F,T,F),(F,F,T),(F,F,F)\}$.

f) **Answers of Application Activity 7.1.2**

1.

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F

F	T	T
F	T	F
F	F	T
F	F	F

2.

p	q	r	s
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	T	T
T	F	T	F
T	F	F	T
T	F	F	F
F	T	T	T
F	T	T	F
F	T	F	T
F	T	F	F
F	F	T	T
F	F	T	F
F	F	F	T
F	F	F	F

Lesson 3: Negation

a) Learning objective: Use correctly negation of logical statements in daily life

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good back ground of negative sentence, the concept of logical statement and truth table as learnt in the previous lessons of this unit.

d) Learning activities

- **Guidance**
  - Make different pair groups of students
  - Introduce the topic by asking them some examples of negative sentences
  - Let them attempt the **Activity 7.2.1** from student-teacher's book
  - Invite randomly group representatives to present their findings, then help all student-teachers to conclude
  - Let them work on example 7.3 and 7.4 and then, work individually application activities 7.2.1 to check the skills they have acquired.
- **Answers of Activity 7.2.1**
  1. It is not raining
  2. Uganda is not African country
  3. London is not in France
  4. Kagabo does not read Newsweek

f) **Answers of Application Activity 7.2.1**

1.

   a) *Today is not raining.*

   b) *Sky is not blue*

   c) *My native country is not Rwanda.*

   d) *Benimana is not smart and not health.*

2.

p	q	r	$\neg p$	$\neg q$	$\neg r$
T	T	T	F	F	F
T	T	F	F	F	T
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	T
F	F	F	T	T	F
F	F	F	T	T	T

Lesson 4: Conjunction

a) Learning objective: Use correctly conjuction logical statements and connectives in daily life

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of compound statement, truth values and truth table as learnt in the 1st and 2nd lessons of this unit.

d) Learning activities

- **Guidance**

- Organize the student-teachers into groups
- Introduce the topic by having a review on compound statement and truth value
- Let them attempt the **Activity 7.2.2** from student-teacher's book
- Through group discussions invite student-teachers to do all questions of the given activity and check if everybody is engaged
- Invite group representatives to present their findings, then help all student-teachers to construct the truth table of a compound

statement connected with conjunction **"and"**

- Facilitate student-teacher to do the provided example 7.5; 7.6 and work individually application activities 7.2.2 for assessing their competences.

- **Answers of Activity 7.2.2**
    1. True
    2. False
    3. False
    4. False.
    5. True
    6. False
    7. False (weight is different from mass)

e) Answers of Application Activity 7.2.2

1. If p stands for the statement "*It is cold*" and q stands for the statement "*It is raining*", then what does $\neg q \wedge \neg q$ stands for "*It is not cold and it is not raining*"

Truth table of $\neg q \wedge \neg q$

p	q	$\neg p$	$\neg q$	$\neg q \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2.

				1	2	3	4
p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge q$	$p \wedge \neg q$	$\neg(p \wedge q)$
T	T	F	F	T	F	F	F
T	F	F	T	F	F	T	T
F	T	T	F	F	T	F	T
F	F	T	T	F	F	F	T

Lesson 5: Disjunction

a) Learning objective: Use correctly logical statements and connectives in daily life

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good background on concept of compound statement, truth values and truth table as learnt in the 1st and 2nd lessons of this unit.

d) Learning activities

- **Guidance**
  - Through group discussions invite student-teachers to do the **Activity 7.2.3** and check if everybody is engaged
  - Invite group representatives to present their findings, then help all student-teachers to construct the truth table of a compound statement connected with disjunction "**or**"
  - Facilitate student-teachers to do example 7.7, 7.8 and let them work individually application activities 7.2.3 for assessing their competences.
- **Answers of Activity 7.2.3**

1. True
2. False
3. True
4. True
5. True
6. True

e) Answers of Application Activity 7.2.3

1. *Translation in symbolic form*

i) *Let p: Bwenge reads News Paper; q: Bwenge reads Mathematics book, Bwenge reads News Paper or Mathematics book is translated symbolically as* $p \vee q$

ii) *Let p: Rwema is a student-teacher, q: Rwema is a book seller, thus Rwema is a student-teacher or not a book seller is translated symbolically as* $p \vee \neg q$

2. *If* p *is a false statement, and* q *is a true statement.*

a) *The truth-value of the compound statement* $\neg p \vee q$ *is* ***true***

b) *The truth-value of the compound statement* $p \vee \neg q$ *is* ***false***

c) *The truth-value of the compound statement* $p \vee q$ *is* ***true****; the truth-value of the compound statement* $\neg p \vee \neg q$ *is* ***true****.*

				1	2	3	4	5
p	q	$\neg p$	$\neg q$	$p \vee q$	$p \vee \neg q$	$p \wedge (p \vee \neg q)$	$\neg(p \vee q) \wedge (\neg p \vee \neg q)$	$(\neg q \vee p) \wedge (\neg p \vee q)$
T	T	F	F	T	T	T	F	T
T	F	F	T	T	T	T	F	F
F	T	T	F	T	F	F	F	F
F	F	T	T	F	T	F	T	T

Lesson 6: Conditional statement

a) Learning objective: Use correctly logical statements and connectives in daily life

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of truth values, truth table and negation, conjunction, disjunction connectives as learnt in the previous lessons of this unit.

d) Learning activities

- **Guidance**
  - Organize the student-teachers into groups and ask them to attempt the **Activity 7.2.4** from student-teacher's book
  - Invite group representatives to present their findings, then help all student-teachers to construct the truth table of a compound statement connected with conjunction **"implication"**
  - Facilitate them to perform example 7.9, 7.10, 7.11 given in **Student-teacher's book** and work individually application activity 7.2.4 to check the skills they have acquired.
- **Answers of Activity 7.2.4**

There several answers. Some of them are:

1. I will go to school whenever you buy me a pen
2. The earth is flat implies that the mars is flat
3. You will be a member of our volleyball team if you are tall
4. You buy me these shoes unless I will not go with you.
5. Whenever you pay school fees, you will not get your school report.

e) Answers of Application Activity 7.2.4

1. Let p : Mico is fat

   q : Mico is happy

   a) If Mico is fat then she is happy is written in symbolic form as $p \Rightarrow q$

   b) Mico is unhappy implies that Mico is thin is written in symbolic form as $\neg q \Rightarrow \neg p$.

2.

a) Let p be "n is prime", q be "n is odd" and r be "n is 2". We have $p \to (q \vee r)$

b) Let p be "x is non negative", q be "x is positive" and r be "x is 0". We have $p \to (q \vee r)$

c) Let p be "Tom is Ann's father", q be "Jim is her uncle" and r be "Sue is her aunt". We have

Lesson 7: Bi-conditional statement

a) Learning objective: Use correctly logical statements and connectives in daily life

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they refer to the concepts of conjunction connective and conditional statement as learnt in the 4th to 7th lesson of this unit.

d) Learning activities

- **Guidance**
  - Organize the student-teachers into groups
  - Introduce the activity by guiding student-teachers to remember how to construct truth table of conditional statement
  - Let them attempt the **Activity 7.2.5** from student-teacher's book
  - Move around to ensure all student-teachers in groups participate actively.
  - Call upon groups to present their findings.
  - Harmonize their findings

- Guide student-teachers to work through example 7.12 and 7.13, then perform individually application activity 7.2.5 to increase their knowledge and skills.

- **Answers of Activity 7.2.5**

1. a) $p \Rightarrow q$ is False b) $q \Rightarrow p$ is True c) $(p \Rightarrow q) \wedge (q \Rightarrow p)$ is False

2. a) $r \Rightarrow s$ is True b) $s \Rightarrow r$ is True c) $(r \Rightarrow s) \wedge (s \Rightarrow r)$ is True

3.

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Answers of Application Activity 7.2.5

1. If r is a false statement, s a true statement, then

   a) the truth-value of the compound statement $(\neg r) \Leftrightarrow s$ is **True**

   b) the truth-value of the compound statement $r \Leftrightarrow (\neg s)$ is True

   c) the truth-value of the compound statement $r \Leftrightarrow s$ is False

   d) the truth-value of the compound statement $\neg(r \Leftrightarrow (\neg s))$ is **False**

2. Construct the truth table for

   a) $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$p \leftrightarrow q$ and $(\neg p \vee q) \wedge (\neg q \vee p)$

p	q	$p \Leftrightarrow q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \wedge (\neg q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$\neg(p \leftrightarrow q)$ and $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$\neg(p \Leftrightarrow q)$	$p \vee q$	$\neg(q \wedge p)$	$(p \vee q) \wedge \neg(q \wedge p)$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	T	T	T	T
F	F	F	F	T	F

$\neg(p \leftrightarrow q)$ and $(p \wedge \neg q) \vee (\neg p \wedge q)$

p	q	$\neg(p \Leftrightarrow q)$	$p \wedge \neg q$	$(\neg p \wedge q)$	$(p \wedge \neg q) \vee (\neg p \wedge q)$
T	T	F	F	F	F
T	F	T	T	F	T
F	T	T	F	T	T
F	F	F	F	F	F

From these results, we note that

a) $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are equivalent

b) $p \leftrightarrow q$ and $(\neg p \vee q) \wedge (\neg q \vee p)$ are equivalent

c) $\neg(p \leftrightarrow q)$ and $(p \vee q) \wedge \neg(p \wedge q)$ are equivalent

d) $\neg(p \leftrightarrow q)$ and $(p \wedge \neg q) \vee (\neg p \wedge q)$ are equivalent

Lesson 8: Tautologies and Contradictions

a) Learning objective: Show that a given logic statement is tautology or a contradiction

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of all logical connectives learnt in the previous lessons.

d) Learning activities

- **Guidance**
  - Organize the student-teachers into groups
  - Introduce the topic by making synthesis of truth values of logical connectives
  - Let them to attempt the **Activity 7.2.6** from student-teacher's book
  - Ask groups to present their findings to the whole class and then harmonize their works to provide the lesson summary and deduce that tautology and contradiction are applications of logic connectives
- Let them work on activities in example 7.14, and then, individually attempt perfectly application activity 7.2.6 to assess their competences.

- **Answers of Activity 7.2.6**

1.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

2.

p	$\neg p$	$p \wedge \neg p$
T	F	T
F	T	T

3.

p	q	$\neg p$	$p \wedge q$	$\neg p \wedge (p \wedge q)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

4.

p	q	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \wedge q) \vee (p \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	T

e) **Answers of Application Activity 7.2.6**

1.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \wedge \neg(p \wedge q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	F
F	F	F	T	F

$p \wedge \neg(p \wedge q)$ is neither tautology nor contradiction

2.

q	r	$\neg q$	$q \wedge r$	$\neg q \wedge (q \wedge r)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

$\neg q \wedge (q \wedge r)$ is a contradiction

3.

p	q	$p \wedge q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	F
F	F	F	T	F

$(p \wedge q) \wedge \neg(p \vee q)$ is neither tautology nor contradiction

4.

p	r	$p \vee r$	$\neg r$	$(p \vee r) \vee \neg r$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

$(p \vee r) \vee \neg r$ is a tautology

Lesson 9: Predicates

a) Learning objective: Use correctly logical statements, propositions, connectives and quantifiers in daily life

b) Teaching resources:

- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they have a good back ground on concept of logical proposition and truth values learnt in the 1st lesson of this unit.

d) Learning activities

- **Guidance**
  - Organize the student-teachers into groups
  - Explain instructions related to the task to be done in the **Activity 7.3.1**.
  - Monitor how students are performing the task and provide support where necessary to guide them
  - Invite representatives of groups to present their findings.
  - Judge the logic of the students' findings, correct those which are false
  - Give the summary of expected feedback based on students' answers.
  - Guide them while doing examples 7.15-7.17 given in **Student-teacher's book** and work individually application activity 7.3.1 for checking the skills they have acquired.
- **Answers of Activity 7.3.1**
  1. Neither
  2. Neither
  3. True
  4. Neither
  5. False
  6. Neither
  7. False
  8. False

Answers of Application Activity 7.3.1

1. Truth set is $\{2,4,6,8,10\}$

2. a) true b) false

3. The truth value of $q(3,4,5)$ is true. The truth value of $q(2,2,3)$ is false. There are infinite values of (x,y,z) .

Lesson 10: Quantifiers

a) Learning objective: Use correctly logical statements, propositions, connectives and quantifiers in daily life

b) Teaching resources:

- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they have a good concept of predicates as learnt in the previous lesson of this unit.

d) Learning activities

- **Guidance**
  - Invite student-teachers to work out the activity 7.3.2 in groups
  - Check if everybody is engaged in the given activity
  - Choose randomly one group to present their findings to the whole class and then harmonize their works to provide the lesson summary.
  - Let student-teachers read through the example 7.18-7.21 and motivate them to work out individually the application activity 7.3.2 to check the skills they have acquired.
- **Answers of Activity 7.3.2**

1. False. For example $\frac{1}{2}$ is a real number but $\frac{1}{\frac{1}{2}}=2>\frac{1}{2}$

2. True

3. False. For example Ostrich is a bird but it cannot fly.

4. True

5. False. February has either 28 or 29 days

6. False. For example 1 and 2 are natural numbers and $1+1>1,\ 2+1>2$

b) **Answers of Application Activity 7.3.2**

1. a) $(\exists x)[p(x)\wedge q(x)]$, where $p(x)$ - x cried out for help and $q(x)$ -x called the police

   b) $(\forall x)[\neg p(x)]$, where $p(x)$ - x can ignore her.

2. a) True, for example for $x=9$, $r(9)=9-7=2$

   b) False, counter example: $y=10$

3. a) First assume that $\forall x[p(x)\wedge q(x)]$ is true. So for all *x*, $p(x)$ is true and $q(x)$ is true. Therefore $\forall xp(x)$ is true and $\forall xq(x)$ is true. Therefore $\forall xp(x)\wedge\forall xq(x)$ is true. Now assume that $\forall xp(x)\wedge\forall xq(x)$ is true. So $\forall xp(x)$ is true and $\forall xq(x)$ is true. So for all x, $p(x)$ is true and for all x, $q(x)$ is true. Therefore, for all x, $p(x)\wedge q(x)$ is true. Therefore $\forall x(p(x)\wedge q(x))\equiv\forall xp(x)\wedge\forall xq(x)$

   b) From a) we can write

   $\forall x[\neg r(x)\wedge\neg s(x)]\equiv\forall x\neg r(x)\wedge\forall x\neg s(x)$.

   Negate both side we have

   $\neg(\forall x[\neg r(x)\wedge\neg s(x)])\equiv\neg(\forall x\neg r(x)\wedge\forall x\neg s(x))$

   or

   $\exists x[r(x)\vee s(x)]\equiv\exists xr(x)\vee\exists xs(x)$

We can write

$$\exists x\left(p(x)\vee q(x)\right)\equiv \exists xp(x)\vee \exists xq(x)$$

Lesson 11: Negation of quantifiers

a) Learning objective: Use correctly negation of quantifiers in logical statements of real life

b) Teaching resources:

- T-square, ruler
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they have a good concept of quantifiers as learnt in the previous lesson of this unit

d) Learning activities

- **Guidance**
  - Organize the student-teachers into groups
  - Introduce the topic by making review on negation connective
  - Let them attempt the **Activity 7.3.3** from student-teacher's book
  - Check if everybody is engaged
  - Invite group representatives to present their findings to whole class, then correct those which are false to harmonize their work
  - Lead student-teachers to read through example 7.22 and work individually application activities 7.2.3 for assessing their competences.
- **Answers of Activity 7.3.3**
  1. Some grapefruit are not pink
  2. No celebrities are modest.
  3. Some people weigh more than five hundred kg.
  4. No one is more than ten metres tall.
  5. Some snakes are not poisonous.

6. No mammals can stay under water for two days without surfacing for air.

7. Some birds cannot fly

Note: There are a great variety of different ways of writing the negation.

Answers of Application Activity 7.3.3

1. No student is mathematics major (or All students are not mathematics major)

   $\forall x\, p(x)$, where $p(x) \sim x$ is not mathematics major

2. Some real numbers are not positive, negative or zero

   $\exists x\, p(x)$, where $p(x) \sim x$ is not positive, negative or zero

3. Some good boys does not do fine

   $\exists x\, p(x)$, where $p(x) \sim x$ is does not do fine

4. All desk in our classroom are not broken (or No desk in our classroom is broken)

   $\forall x\, p(x)$, where $p(x) \sim x$ is not broken

5. Some lockers must not be turned in by the last day of class

   $\exists x\, p(x)$, where $p(x) \sim x$ is not be turned in by the last day of class

6. Some haste do not make waste

   $\exists x\, p(x)$, where $p(x) \sim x$ does not make waste.

7.6. **Summary of the unit**

A proposition (statement or verbal assertion) is a sentence which is either true or false but not both.

We can always summarize the truth values of compound statement in a table called truth table. If the compound statement contains n distinct components, we need to consider 2^n possible combinations of truth values in order to obtain the truth table.

Given statements p and q, we can combine them with various connectives. The most five useful logical connectives are negation, conjunction, disjunction, conditional and biconditional.

The negation of a statement by introducing the word "not" denoted by prefixing the statement has opposite truth value from the statement. It is denoted by $\neg p$ or $\overline{p}$ or $\sim p$.

The conjunction (AND) of two statements p and q is denoted $p \wedge q$. It has the truth value true whenever both p and q have the truth value true; otherwise it has the truth value false.

The disjunction (OR) of two statements p and q is denoted $p \vee q$. It has the truth value false only when p and q have truth value false, otherwise it has the truth value true.

The conditional statement $p \Rightarrow q$ (read "p *implies* q") has the truth value false when q has the truth value false while p has truth value true, otherwise it has the truth value true.

The biconditional statement $p \Leftrightarrow q$, which we read "p if and only if q" or "p is equivalent to q" is true if both p and q have the same truth values and false if p and q have opposite truth values.

A tautology is a statement formula that is always true regardless of the truth values of the individual statements substituted for its statement variables.

A contradiction is a statement formula that is always false regardless of the truth values of the individual statements substituted for its statement variables.

A predicate or a declarative sentence is an open statement if:

It contains one or more variables, and

It is not a statement, but

It becomes a statement when the variables in it are replaced by certain allowable choices.

The quantifiers help decide the frequencies with which a predicate becomes true, whether it is satisfied by no element of a domain, or one element, or some elements, or all elements.

There are two basic quantifiers:

The existential quantifiers $\exists$ ("there exist"), and

The universal quantifier $\forall$ ("for all")

These quantifiers are negated as follows

$$\neg\left[\forall x p(x)\right] \equiv \exists x\left[\neg p(x)\right]$$

$$\neg\left[\exists x p(x)\right] \equiv \forall x\left[\neg p(x)\right]$$

Additional Information for tutors

Tutor may facilitate student-teacher by providing examples related to real life;

For example,

on the conjunction connective, they can use the following statement and find out the rule of conjunction "and";

"Abjuru and Bwenge are going to the market" $(p \wedge q)$.

They find that it is true that Abijuru is going and true that Bwenge is going, then they would know this statement to be true. But if Abijuru is going, Bwenge is not, then the above statement will be false, because the statement claims both are going, and in this case only Abijuru is going. They deduce also that the above statement will be false in the case Abijuru is not going, even though Bwenge is, and in the case in which neither Abijuru nor Bwenge are going.

This, then, will be the rule for conjunction connective $(p \wedge q)$that it is true when all the components are true, and false otherwise.

On the disjunction connective, consider the following example when someone says "Abijuru or Bwenge is going to the market". If only one of them is going, the statement would still be true. The only time the disjunction would be false, would be when neither are going. This, then, will be our rule for disjunction. An disjunction statement $(p \vee q)$is true when either one or both of the simple statement parts are true, and false only when both parts are false.

We defined $p \Rightarrow q$ and $q \Rightarrow p$ according to the $(\Rightarrow)$ rule and then put them together by the $(\wedge)$ rule. This then will be our rule for $(\equiv)$: A biconditional statement $(p \equiv q)$ is true only when both components have the same truth value and false otherwise.

At this point, a little common-sense reflection shows why this is so. When a mother tells her son that he will go out if and only if he cleans his room, the mother's statement would be true in only two cases:

when the son cleans his room and the mother fulfills her promise by letting the son go out , and when the son does not clean his room and suffers the consequence of not going out.

In the other two cases—when the son does not clean his room and the mother lets him go out anyway , and when the mother does not let her son go out even after he has cleaned his room the mother's statement would be false.

One can show that two logical expressions are equivalent by showing they have the same truth values in all circumstances. For example, it is not necessary to have the connective ⇒ because $p \Rightarrow q$ is equivalent to $\neg p \vee q$, as can be seen in the following truth table

p	q	$\neg p$	$p \Rightarrow q$	$\neg p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

An equivalent way of writing p implies q is "If p, then q." the form usually used in English writing, and is called an implication. There are three statements that are gotten from the implication:

- Converse: If q, then p.
- Inverse: If $\neg p$, then $\neg q$.
- Contrapositive: If $\neg q$, then $\neg p$.

We note that the implication and the contrapositive are equivalent because both are equivalent to $\neg p \vee q$. Similarly, the inverse and the converse are equivalent. But the converse and inverse are not equivalent to the implication and the contrapositive. A simple example is the implication "If there is light coming through the window, then there is light in the room." The converse is "If there is light in the room, then there is light coming through the window." a false statement at night or in a room without windows. So be careful with your implications but remember that you can prove the contrapositive and still have proven the implication.

The existential quantifier (there exists) has many forms in English: " some or for some or there is $a(n)$". The universal quantifier (for all) may be expressed as – for each or for any or any or every or for every. These are used in sentences with variables in them.

7.7. End unit assessment

1. a) proposition

   b) not a proposition

   c) proposition

   d) proposition

   e) not a proposition

2. The negation is "Today is not Monday".

3. The conjunction of these propositions is the proposition "Today is Sunday and the moon is made of cheese"

4. The disjunction of these propositions is the proposition "Today is Sunday or the moon is made of cheese".

5. For a compound statement made from n statements, 2^n rows, not counting the top one, are needed to construct the truth table.

6.

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

Make a truth table for this proposition:

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

7. The proposition is TRUE, since it is composed of two propositions each of which is FALSE.

8. The truth table is

p	q	r	$\neg p \to q$	$(\neg p \to q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

9. a) $(p \wedge \neg q) \wedge r$ b) $(p \wedge q) \to r$

10. To show that $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

Therefore, $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology.

11. To show that $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

Therefore, $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology.

12. $p(1,2)$ is the proposition " 2 = 1 + 3" which is false. The statement $p(0,3)$ is the proposition " 3 = 0 + 3" which is true.

13. Let $q(x)$ be the predicate " x has seen a computer". Then the statement "Every student in this class has seen a computer" can be written as $\forall x\, q(x)$, where the universe of discourse consists of all students in this class. Also, this statement can be expressed as $\forall x(p(x) \to q(x))$, where $p(x)$ is the predicate " x is in this class" and the universe of discourse consists of all students.

14. $p(x)$ is not true for all real numbers x ; for instance, $p\left(\frac{1}{2}\right)$ is false. Thus, the proposition $\forall x\, p(x)$ is false.

15. a) The quantification $\forall x \forall y\, p(x, y)$ denotes the proposition "For every pair x, y $p(x, y)$ is TRUE". Clearly, this proposition is FALSE.

b) The quantification $\forall x \exists y\, p(x, y)$ denotes the proposition "For every x there is an y such that $p(x, y)$ is TRUE". Given a real number x, there is a real number y such that $x + y = 3$, namely $y = 3 - x$. Hence, the proposition $\forall x \exists y\, p(x, y)$ is TRUE.

16. a) Let $l(x)$ be the predicate " x likes mathematics", where the universe of discourse is the set of students in this class.

The original statement is $\forall x\, l(x)$ and its negation is $\exists x \neg l(x)$. In English, it reads "Some student in this class does not like mathematics".

b) Consider the predicates $p(z,y)$: "room z is in building y" and $q(x,z)$: "student x has been in room z ". Then the original statement is $\exists x \forall y \exists z\left(p(z,y) \wedge q(x,z)\right)$. To form the negation, we change all the quantifiers and put the negation on the inside, then apply De Morgan's law. The negation is therefore $\forall x \exists y \forall z\left(\neg p(z,y) \vee \neg q(x,z)\right)$, which is also equivalent to $\forall x \exists y \forall z\left(p(z,y) \rightarrow \neg q(x,z)\right)$. In English, this could be read "For every student there is a building on the campus such that for every room in that building, the student has not been in that room".

7.8. Additional activities

7.8.1. Remedial activities

1. Find out which of the following sentences are statements and which are not.

i. The moon revolves around the sun.

ii. A triangle has four sides.

iii. $\sqrt{3}$ is an irrational number.

iv. What is your age?

v. Africa is a continent.

vi. $x+7=10$.

vii. The earth is a planet.

viii. $7+9>12$.

2. Write down the truth value of the following statements.

i. 1 is a prime number.

ii. Every square is a rectangle.

iii. All real numbers are rational.

iv. Karongi is in western province.

v. Uganda is north-west of Rwanda.

Answers:

1. To find out which of the following sentences are statements and which are not.

i. "The moon revolves around the sun". This is false and so it is a statement.

ii. A triangle has four sides. This is false and so it is a statement.

iii. $\sqrt{3}$ is an irrational number. This is true and so it is a statement.

iv. What is your age? This is not a statement as it is an interrogative sentence.

v. Africa is a continent. This is true and so it is a statement

vi. The earth is a planet. This is true and so it is a statement

vii. $7+9>12$. This is true and so it is a statement.

2. The truth values

vi. The statement has truth value False

vii. True

viii. False.

ix. True.

x. False.

7.8.2 Consolidation activities

If the statements p, q, r all have truth value "True" and u, v, w are false statements, which of the following are true and which are false?

1.	$(r \vee w) \wedge (v \vee q)$	13.	$u \Rightarrow (v \Rightarrow r)$
2.	$(p \wedge q) \vee (u \wedge v)$	14.	$(p \Rightarrow q) \Rightarrow w$
3.	$\neg(q \vee u) \wedge \neg(v \vee w)$	15.	$[(u \Rightarrow v) \Rightarrow q] \Rightarrow w$
4.	$\neg(r \vee q) \vee \neg(\neg u \wedge v)$	16.	$[(q \Rightarrow w) \Rightarrow q] \Rightarrow w$
5.	$\neg q \vee r$	17.	$u \Rightarrow (q \Rightarrow w)$
6.	$\neg(q \vee u)$	18.	$[(u \Rightarrow p) \Rightarrow u] \Rightarrow u$
7.	$\neg u \vee p$	19.	$[u \Rightarrow (v \Rightarrow w)] \Rightarrow [(u \Rightarrow v) \Rightarrow w]$
8.	$\neg(u \vee v)$	20.	$\{p \Rightarrow (q \Rightarrow r) \equiv \neg u\} \Rightarrow \{x \Rightarrow [(p \wedge q) \Rightarrow r]\}$
9.	$\neg[(q \vee p) \vee (\neg p \vee q)]$	21.	$\{[u \Rightarrow (v \Rightarrow w)] \Rightarrow [(u \wedge v) \Rightarrow w]\} \equiv [(u \Rightarrow p) \Rightarrow (q \Rightarrow v)]$
10.	$\neg[(\neg v \vee w) \vee (\neg w \vee v)]$	22.	$(q \Rightarrow p) \equiv (\neg p \Rightarrow \neg q)$
11.	$[p \wedge (q \vee r)] \wedge [(p \wedge q) \vee (p \wedge r)]$	23.	$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$
12.	$\neg[u \wedge (\neg p \vee w)] \vee [(u \wedge \neg p) \vee (u \wedge w)]$	24.	$[p \Rightarrow (q \Rightarrow r)] \equiv [(p \wedge q) \Rightarrow r]$

Extended activities

Determine which of the pairs of statements in the following are logically equivalent

a) $\neg[(p \wedge q) \wedge r]$ and $\neg[p \wedge (q \wedge r)]$

b) $(p \vee q) \vee r$ and $p \vee (q \wedge r)$

c) $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$ and $(p \vee q) \wedge c$, where c is a contradictory

d) $[(\neg p \vee q)] \wedge [p \wedge (\neg q)]$ and $(p \wedge q) \vee t$, where t is a tautology

e) $(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$ and $[(\neg q) \wedge q] \vee t$, where t is a tautology

Answers

a) Truth table of $\neg[(p \wedge q) \wedge r]$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$\neg[(p \wedge q) \wedge r]$
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

Truth table of $\neg[p \wedge (q \wedge r)]$

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$\neg[p \wedge (q \wedge r)]$
T	T	T	T	T	F
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	F	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

As the truth values in the last column of both tables are the same, the statement $\neg[(p \wedge q) \wedge r]$ is logically equivalent to $\neg[p \wedge (q \wedge r)]$

b)Truth table of $(p \vee q) \vee r$

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T

F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Truth table of $p \vee (q \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Since the truth values in the last column of both tables are not the same, the statement $(p \vee q) \vee r$ is not logically equivalent to $p \vee (q \wedge r)$.

c) Truth table of $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$

p	q	$\neg p$	$\neg q$	$\neg q \wedge p$	$p \wedge (\neg p)$	$[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

Truth table of $(p \vee q) \wedge c$, where c is a contradictory

p	q	c	$p \vee q$	$(p \vee q) \wedge c$
T	T	F	T	F
T	F	F	T	F
F	T	F	T	F
F	F	F	F	F

Getting that the truth values in the last column of both tables are the same, then statement $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$ is logically equivalent to $(p \vee q) \wedge c$, where c is a contradictory.

d) Truth table of $[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \wedge (\neg q)$	$[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

Truth table of $(p \wedge q) \vee t$, where t is a tautology

p	q	t	$p \wedge q$	$(p \vee q) \wedge c$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

As the truth values in the last column of both tables are not the same, the statement $[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$ is not logically equivalent to $(p \wedge q) \vee t$, where t is a tautology.

e) Truth table of $(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \wedge (\neg q)$	$(\neg p) \vee [p \wedge (\neg q)]$	$(p \wedge q) \vee$ $\{(\neg p) \vee [p \wedge (\neg q)]\}$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T

Truth table of $[(\neg p) \wedge q] \vee t$, where t is a tautology

p	q	t	$\neg p$	$(\neg p)\wedge q$	$[(\neg p)\wedge q]\vee t$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	T	T	F	T

As the truth values in the last column of both tables are the same, the statement $(p\wedge q)\vee\{(\neg p)\vee[p\wedge(\neg q)]\}$ is logically equivalent to $[(\neg p)\wedge q]\vee t$, where t is a tautology.

UNIT 8

APPLICATION OF TRIGONOMETRIC CONCEPTS IN SOLVING PROBLEMS

8.1. **Key unit competence**

Apply trigonometric concepts in solving problems on triangles and real-life situation.

8.2. **Prerequisite**

Student-teachers will easily learn this unit, if he/she has a good back ground on concept of

- Vector representation learnt from S2 in Unit 7;
- Right-angled learnt from S3 ;
- Set of numbers learnt in unit 1;
- Algebraic fractions learnt from Senior 3, in unit 3;
- Isometries learnt from S2, unit 9.

8.3. **Cross-cutting issues to be addressed**

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others view and thoughts during class discussions)
- Gender (equal opportunity of boys and girls in the lesson participation and making groups)

8.4. **Guidance on introductory activity 8**

Student-teachers work on the introductory activity to understand the use of trigonometry.

Let them read and do the introductory activity 8 in the Student-teacher's book.

Make sure that all student-teachers are activating and performing well.

Through class discussions, let student-teachers think of different ways of application of trigonometry.

Through different examples, help student-teachers to understand the importance of trigonometry by showing their application in real life for example in construction, satellite systems and astronomy, naval and aviation industries, land surveying and in cartography (creation of maps) and so on.

Answer of introductory activity 8

Pythagoras theorem is not enough for finding the height of the given cathedral. By using sine rule, the required height can be determined as

$$\frac{\sin 60^0}{h}=\frac{\sin 30^0}{280}\Leftrightarrow h\sin 30^0=280\sin 60^0\Leftrightarrow \frac{h}{2}=\frac{280\sqrt{3}}{2}\Leftrightarrow h=280\sqrt{3}\simeq 484.97m$$

8.5. **List of lessons**

#	Lesson title	Learning objectives	Number of periods
0	**Introduction activity**	To arise the curiosity of student teachers on the content of unit 8	1
1	Angles measurement	Appreciate the use of trigonometry in daily life situation	1
2	Units conversion	Convert radians to degree and vice versa	2
3	Unit circle	Appreciate the relationship between the trigonometric values for different angles	1
4	Definitions of trigonometric ratios	Define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle	2
5	Trigonometric ratio of special angles $30^\circ, 45^\circ, 60^\circ$	Know trigonometric ratios of special angles $\left(30^0, 45^0, 60^0\right)$.	5
6	Evaluating Trigonometric Functions with a Calculator	Appreciate the relationship between the trigonometric values for different angles	2
7	Trigonometric identities	Appreciate the relationship between the trigonometric values for different angles	3

8	Reduction of Equivalent angles (co-terminal angles) to functions of positive acute angles	Appreciate the relationship between the trigonometric values for different angles	2
9	Reduction of Negative angle or opposite angle to functions of positive acute angles	Appreciate the relationship between the trigonometric values for different angles	2
10	Reduction of Complementary angles to functions of positive acute angles	Differentiate between complementary angles and co-terminal angles	2
11	Reduction of Supplementary angles to functions of positive acute angles	Differentiate between complementary angles, supplementary angles and co-terminal angles	2
12	Sine law	Use trigonometry, including the sine rule to solve problems involving triangles	3
13	Cosine law	Use trigonometry, including the sine and cosine rules, to solve problems involving triangles	2
14	Applications	Appreciate the use of trigonometry in daily life situation	4
Assessment			2

Lesson 1: Angles measurement

a) Learning objective: Appreciate the use of trigonometry in daily life situation

b) Teaching resources:

- T-square, ruler, protractor, compass, calculator, if possible, Math draw software as GeoGebra, math lab….
- Student-teacher's book and other Reference textbooks to facilitate research

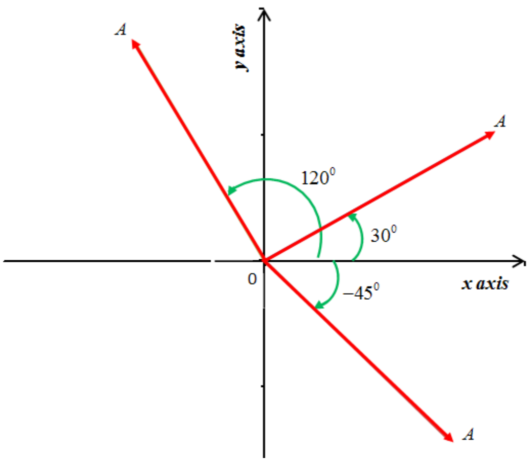
c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of

- a vector learnt from S2 in Unit 7 ;
- Right-angled learnt from S3 in Unit 8.

d) Learning activities

- **Guidance**
  - Organize the student-teachers into groups and ask them to attempt the **Activity 8.1.1.1** from student-teacher's book and introduce the concept of angles
  - Introduce the topic by giving some examples of angles and their measurements
  - Discuss with student-teachers on how a given angle is measured.
  - Guide student-teachers in drawing the given angles.
  - Invite one group for presentation of its work to other groups.
  - Facilitate them to do the provided examples given in **Student-teacher's book** and work individually application activity 8.1.1.1 to check the skills they have acquired.
- **Answers of Activity 8.1.1.1**



e) Answers of Application Activity 8.1.1.1

1. **a)** $10\pi\, cm$ b) $9.2\pi\, cm$

2. **a)** $\frac{7}{\pi} cm$ b) $\frac{8}{\pi} cm$

3. The angle in degrees that describes the compass bearing

   a) SSW (south-southwest) is 202.5°

   b) WNW (west-northwest) is 292.5°

   c) NNW (north-northwest) is 337.5° 41.

   d) ESE is closest at 112.5°.

   e) SW is closest at 225°.

   f) In 8h, the hour hand of a clock rotates $\frac{4\pi}{3}$. In 1 week, the hour hand of a clock rotates 28π.

4. At 4:00, the hour hand of a clock has rotated $\frac{2\pi}{3}$. At 2:30, the hour hand of a clock has rotated $\frac{5\pi}{12}$. At 10:12, the hour hand of a clock has rotated $\frac{17\pi}{10}$.

Lesson 2 : Units conversion

a) Learning objective: Appreciate the relationship between the trigonometric values for different angles

b) Teaching resources:

- Piece of paper, strings, scissors or laser blades, pins, a compass.
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of

- Set of numbers learnt in unit 1
- Algebraic fractions learnt from Senior 3, in unit 3

d) Learning activities

► Guidance

- Let Student-teachers attempt the **Activity 8.1.1.2** in Student-teacher's book and guide them
- Make sure that everybody is engaged/ involved.
- Facilitate student-teachers on cutting a piece of thread or string with the same size as the radius
- Invite one group for presentation of its work to other groups.
- Guide student-teachers in defining a radian
- As they are discussing, concentrate on slow student-teachers for further explanation and provide assistance to groups in need.
- Facilitate them to do the provided examples given in **Student-teacher's book** and work individually application activity 8.1.1.2 to check the skills they have acquired.

Answers of Activity 8.1.1.2

Show how a piece of string and graph paper may be used to demonstrate radian measure. Measure a radius-length of string and then show, by wrapping the string around the circle, that it takes a little more than 6 radius-lengths to go all the way around the circle

1) $2\pi r$ 2) a little more than 6 radians 3) the change in radius affects change of circumference but does not for radians (Answer may vary) 4) π radians

e) Answers of Application Activity 8.1.1.2

1. **Without using a calculator,**

a) $\frac{\pi}{4} = 45^0$ b) $\frac{2\pi}{3} = 120^0$ c) $\frac{\pi}{6} = 30^0$ d) $\frac{\pi}{8} = 22.5^0$ e) $\frac{5\pi}{12} = 75^o$

f) $\frac{3\pi}{8} = 67.5$ g) $\frac{2\pi}{5} = 72^0$ h) $\frac{5\pi}{4} = 225$

2. Leaving the result in terms of π (without using a calculator),

a) $150^0 = \frac{5\pi}{6}$ b) $225^0 = \frac{5\pi}{4}$ c) $45^0 = \frac{\pi}{4}$ d) $90^0 = \frac{\pi}{2}$ e) $30^0 = \frac{\pi}{6}$

f) $270^0 = \frac{3\pi}{2}$ g) $60^0 = \frac{\pi}{3}$ h) $135^0 = \frac{3\pi}{4}$ i) $345^0 = \frac{5\pi}{4}$

3. $\frac{17\pi}{30}$

4. a) $h = 16\pi\, cm \simeq 50.265\, cm$ b) $h = 2\pi \simeq 6.283\, m$

5. a) $2.61 cm$ b) $20.1 cm$

6. a) $1.80 m$ b) $0.419 m$ c) $10.2 m$

7. $0.76 m$

8. 4 *cm*

Lesson 3: Unit circle

a) Learning objective: Appreciate the relationship between the trigonometric values for different angles

b) Teaching resources:

- Compass, T-square, ruler, protractor
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of elements of a circle learnt from S3 in Unit 9.

d) Learning activities

- **Guidance**
  - Let Student-teachers attempt the **Activity 8.1.1.3 (in Student-**

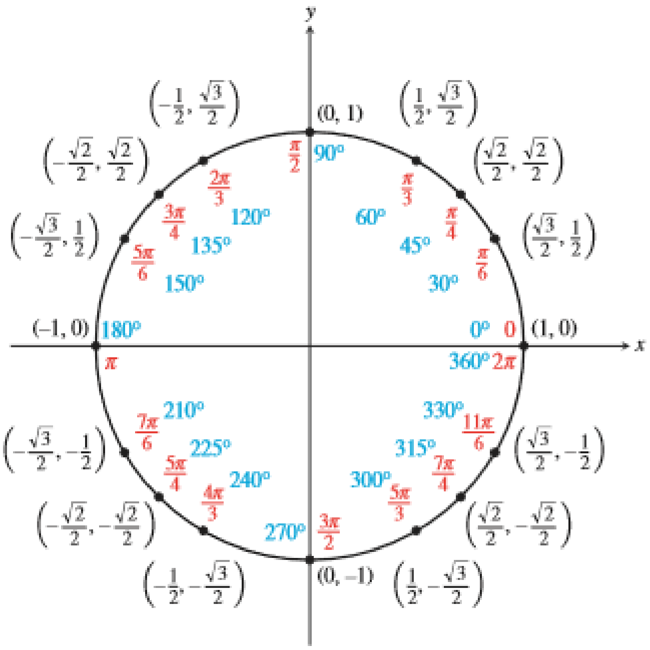
teacher's book) and make sure that everybody is engaged/ involved.

- Discuss with student-teachers on how to construct a unit circle.
- Guide student-teachers in drawing the given angles.
- Invite one group for presentation of its work to other groups.
- Facilitate them to do the provide examples given in **Student-teacher's book** to emphasize the skills, he/she has got.

▪ **Answers of Activity 8.1.1.3**

While, the wheel turns, how height of the point from the centre remains constant. The graph may vary.

Answers of Application Activity 8.1.1.3



Lesson 4: Definitions of trigonometric ratios

a) Learning objective: Appreciate the relationship between the trigonometric values for different angles

b) Teaching resources:

- Ruler, T-square, Compass , protractor and calculator
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good back ground on concept of right-angled triangle learnt from S3, in unit 8.

d) Learning activities

- **Guidance**
  - Arrange the student-teachers into groups for the discussion of **Activity 8.1.2.1 (in Student-teacher's book)** and make sure that everybody is engaged/ involved.
  - Facilitate student-teachers in plotting the similar figures and choices of $P(x, y)$
  - Once the group discussion is over, ask a group, chosen randomly, to present its results while other student-teachers are following attentively.
  - Facilitate them to do the provide examples given in **Student-teacher's book** to emphasize the skills, he/she has got.
- **Answers of Activity 8.1.2.1**

The figures may vary. For any point $P(x, y)$, $\frac{x}{1} = \cos\theta$ and $\frac{y}{1} = \sin\theta$.

$\left(\frac{x}{1}\right)^2 + \left(\frac{y}{1}\right)^2 = 1$, we deduce that $(\cos\theta)^2 + (\sin\theta)^2 = 1$ for any value of θ.

e) Answers of Application Activity 8.1.2.1

1. a) Pairs: sine and cosecant, cosine and secant, tangent and cotangent

   b) $\tan\theta$ c) $\sec\theta$ d) 1 e) $\sin\theta$ and $\cos\theta$

2. From $x-axis$:5; from $y-axis$:3; from origin $\sqrt{34}$.

3. $\csc\theta = 1{,}000$ and θ is in II^{nd} quadrant

4. $\frac{5\sqrt{61}}{61}, \frac{6\sqrt{61}}{61}, \frac{5}{6}, \frac{6}{5}, \frac{\sqrt{61}}{6}, \frac{\sqrt{61}}{5}$ and θ is in I^{st} quadrant

Lesson 5: Trigonometric ratio of special angles $30^\circ, 45^\circ, 60^\circ$

a) Learning objective: Know trigonometric ratios of special angles $(30^0, 45^0, 60^0)$.

b) Teaching resources:

- Ruler, T-square, Compass
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of

- Definition of trigonometric ratios learnt in previous lesson
- Pythagoras theorem learnt from S2, in unit 6.

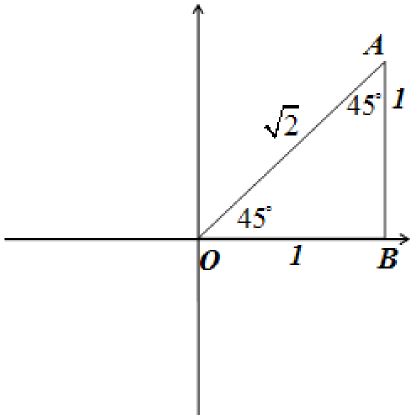
d) Learning activities

- **Guidance**
  - Let Student-teachers attempt the **Activity 8.1.2.2 (Student-teacher's book)** and motivate them so that everybody is engaged/ involved.
  - Guide student-teachers to get trigonometric ratios of the given angle from definition.
  - Use triangles to assist student-teachers define trigonometric ratios related to 30°, 45° and 60°.
  - Invite one group for presentation of its findings to other groups.
  - Facilitate them to do the provide examples given in **Student-teacher's book** to emphasize the skills, he/she has got.

► Answers of Activity 8.1.2.2

Answer of question 1:

$$\sin 45^\circ, \cos 45^\circ \textit{ and } \tan 45^\circ$$

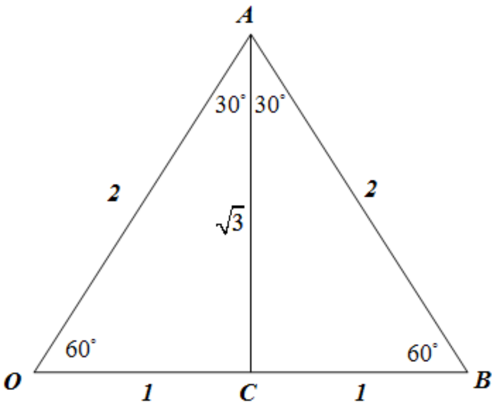


From the diagram

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}} \qquad \tan 45^\circ = \frac{1}{1} = 1$$

Answer of question 2:

$$\textit{The six trigonometric values of } 60^\circ \textit{ and } 30^\circ$$



From ΔOAC,

$$\sin 60^0 = \frac{\sqrt{3}}{2}, \cos 60^0 = \frac{1}{2}, \tan 60^0 = \sqrt{3}, \csc 60^0 = \frac{2\sqrt{3}}{3}, \sec 60^0 = 2, \cot 60^0 = \frac{\sqrt{3}}{3}$$

From ΔOAC,

$$\sin 30^0 = \frac{1}{2}, \cos 30^0 = \frac{\sqrt{3}}{2}, \tan 30^0 = \frac{\sqrt{3}}{3}, \csc 30^0 = 2, \sec 30^0 = \frac{2\sqrt{3}}{3}, \cot 30^0 = \sqrt{3}.$$

e) Answers of Application Activity 8.1.2.2

1.

θ	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\nexists$

a) $\frac{\sqrt{3}-1}{2}$ b) 0

3. a)1 b) $\sqrt{3}$ c) $\frac{\sqrt{3}}{3}$ d) 0 e) does not exist f) 0

g) does not exist h) does not exist i)0 j) does not exist k) 0

Lesson 6: Trigonometric Functions with a Calculator

a) Learning objective: Appreciate the relationship between the trigonometric values for different angles

b) Teaching resources:

- Scientific calculators
- Student-teacher’s book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

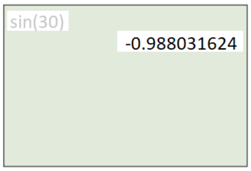
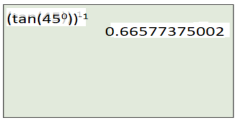
Student-teachers will learn better this lesson if they have a good understanding on concept of Set of numbers learnt in 1st unit

d) Learning activities

► Guidance

- Let Student-teachers attempt the **Activity 8.1.2.3 (Student-teacher's book)** and introduce the concept of set of numbers
- Discuss with student-teachers on how to use a calculator
- Let them to perform different calculations related to trigonometric ratios
- Check how adequately the student-teachers are using calculators and how each member of the group is contributing to the discussion.
- Facilitate them to do the provided examples given in **Student-teacher's book** to emphasize the skills, they have got.

▪ **Answers of Activity 8.1.1.3**

	This answer is wrong since the sine of an angle lying in second quadrant must be positive.
	This answer is wrong since $\left(\tan\left(45\right)^{0}\right)^{-1}=\frac{1}{\tan\left(45^{0}\right)}=\frac{1}{1}=1$.

e) Answers of Application Activity 8.1.1.3

Answers of question 1

a) $cos\left(\frac{3,333,333\pi}{2}\right)=0$

b) $\sin\left(\frac{\pi}{6}+47,000\pi\right)=\frac{1}{2}$

c) $\tan\left(1,234,567\pi\right)-\tan\left(7,654,321\pi\right)=0$

d) $\tan\left(\frac{3\pi - 70,000\pi}{2}\right)$ does not exist

Answers of question 2

a) $\sin 74^0 = 0.96$ b) $\tan 8^0 = 0.14$ c) $\cos 19^0 23' = 0.94$ d) $\tan 23^0 42' = 0.44$

e) $\tan\left(\frac{\pi}{12}\right) = 0.27$ f) $\sin\left(\frac{\pi}{15}\right) = 0.21$ g) $\sec 49^0 = 1.52$ h) $\csc 19^0 = 3.07$

i) $\cot\left(\frac{\pi}{8}\right) = 2.41$ j) $\csc\left(\frac{\pi}{10}\right) = 3.24$

Lesson 7: Trigonometric identities

a) Learning objective:

Apply operation of set: Perform union, intersection, difference–and symmetrical difference on sets.

b) Prerequisites/Revision/Introduction:

Student-teachers will perform better in this lesson if they refer to: operation of sets learnt in S1.

c) Teaching resources:

They include: Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, wall charts and wall maps, Mathematical models and Internet connection where applicable.

d) Learning activities:

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Ask students to use the student teacher's book to discuss the **activity 2.2;**

- Move around to each group to ensure all student-teachers participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and guide students to highlight the union of sets, intersection on sets and their differences.
- Use different probing questions to guide students to explore examples and the content given in the student's book to the use of operation on sets: intersection, union and difference and guide them to highlight the corresponding properties;
- Guide all students to explore the symmetric difference and the complement of set;
- After this step, guide students to do the application activity **2.2** and assess their competences and evaluate whether lesson objectives were achieved.

a) **Learning objective**: Appreciate the relationship between the trigonometric values for different angles

b) Teaching resources:

- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on

- Definition of trigonometric ratios learnt from previous lessons
- Pythagoras theorem learnt from S2, unit 6.

d) Learning activities

- **Guidance**

- Let Student-teachers attempt the **Activity 8.1.3 (Student-teacher's book)** and introduce the concept of trigonometric ratios
- Guide student-teachers in constructing right-angled triangle and labelling it
- Facilitate student-teachers in applying Pythagoras theorem
- Verify whether every student-teacher is engaged
- Facilitate them to do the provided examples given in **Student-**

teacher's book to emphasize the skills, he/she has got.

- **Answers of Activity 8.1.3**

In this triangle,

$$\sin\theta=\frac{y}{r},\ \cos\theta=\frac{x}{r},\ \sin\alpha=\frac{x}{r},\ \cos\alpha=\frac{y}{r}$$

$$\left(\frac{x}{r}\right)^2+\left(\frac{y}{r}\right)^2=(\sin\theta)^2+(\cos\theta)^2=\sin^2\theta+\cos^2\theta$$ and then

$$\sin^2\theta+\cos^2\theta=1$$

$$\sin^2\alpha+\cos^2\alpha=\left(\frac{x}{r}\right)^2+\left(\frac{y}{r}\right)^2=1$$

e) **Answers of Application Activity 8.1.3**

1. To prove the given trigonometric identities

a) $\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}=\frac{1}{\cos\theta\sin\theta}=\sec\theta\cos ec\theta$

b) $\sin^2\theta-4\cos^2\theta+1=\sin^2\theta-4\cos^2\theta+\sin^2\theta+\cos^2\theta$

$$=2\sin^2\theta-3\cos^2\theta=\left(2\sin^2\theta-2\cos^2\theta\right)-\cos^2\theta$$

$$=\left(2\sin^2\theta-2\cos^2\theta\right)-\left(1-\sin^2\theta\right)=3\sin^2\theta-2\cos^2\theta-1$$

c) $\frac{\cos^2 A}{1+\tan^2 A}-\frac{\sin^2 A}{1+\tan^2 A}=\frac{\cos^2 A}{\sec^2 A}-\frac{\sin^2 A}{\sec^2 A}=\frac{1-\sin^2 A-\sin^2 A}{\sec^2 A}=\frac{1-2\sin^2 A}{\sec^2 A}$

2. To simplify

a) $\sec^4\theta\left(1-\sin^4\theta\right)-2\tan^2\theta=\sec^4\theta\left(1-\sin^2\theta\right)\left(1+\sin^2\theta\right)-2\tan^2\theta$

$$=\sec^4\theta\cos^2\theta\left(1+\sin^2\theta\right)-2\tan^2\theta$$

$$=\sec^2\theta\left(1+\sin^2\theta\right)-2\tan^2\theta$$

$$=\sec^2\theta+\sec^2\theta\sin^2\theta-2\tan^2\theta$$

$$=\sec^2\theta+\frac{\sin^2\theta}{\cos^2\theta}-2\tan^2\theta$$

$$= \sec^2\theta + \tan^2\theta - 2\tan^2\theta$$

$$= \sec^2\theta - \tan^2\theta$$

$$= 1 + \tan^2\theta - \tan^2\theta = 1$$

b) $\cot^2 A - \cot^2 B + \dfrac{\sin^2 A - \sin^2 B}{\sin^2 A \sin^2 B} = \dfrac{\cos^2 A}{\sin^2 A} - \dfrac{\cos^2 B}{\sin^2 B} + \dfrac{\sin^2 A - \sin^2 B}{\sin^2 A \sin^2 B}$

$$= \frac{\cos^2 A \sin^2 B - \cos^2 B \sin^2 A}{\sin^2 A \sin^2 B} + \frac{\sin^2 A - \sin^2 B}{\sin^2 A \sin^2 B}$$

$$= \frac{\left(\cos^2 A \sin^2 B - \sin^2 B\right) - \left(\cos^2 B \sin^2 A - \sin^2 A\right)}{\sin^2 A \sin^2 B}$$

$$= \frac{\sin^2 B\left(\cos^2 A - 1\right) - \sin^2 A\left(\cos^2 B - 1\right)}{\sin^2 A \sin^2 B}$$

$$= \frac{-\sin^2 B \sin^2 A + \sin^2 A \sin^2 B}{\sin^2 A \sin^2 B} = 0$$

$$\frac{\cos^3\theta + \cos\theta\sin^2\theta + \sin\theta}{3\cos^3\theta + 3\cos\theta\sin^2\theta - \sin\theta} = \frac{\cos\theta\left(\cos^2\theta + \sin^2\theta\right) + \sin\theta}{3\cos\theta\left(\cos^2\theta + \sin^2\theta\right) - \sin\theta} = \frac{\cos\theta + \sin\theta}{3\cos\theta - \sin\theta}$$

Lesson 8: Reduction of Equivalent angles (co-terminal angles) to functions of positive acute angles

a) Learning objective: Appreciate the relationship between the trigonometric values for different angle

b) Teaching resources:

- Ruler, T-square, compass, protractor
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of

- Unit circle learnt in lesson 3 of this unit.
- Isometries learnt from S2, unit 9.

d) Learning activities

► Guidance

- Let Student-teachers attempt the **Activity 8.2.1 (Student-teacher's book)** and introduce the concept of isometry.
- Discuss with student-teachers on how a given angle is measured.
- Guide student-teachers in rotating vectors
- Facilitate them to do the provided examples given in **Student-teacher's book** to emphasize the skills, he/she has got.

► Answers of Activity 8.2.1

1. -240^0 and 840^0
2. There are many answers. Some of them

   a) -180^0 and 540^0 $\left(\textit{Generally } 180^0+360^0k, k\in\mathbb{Z}\right)$

   b) 219^0 and -501^0 $\left(\textit{Generally } -141^0+360^0k, k\in\mathbb{Z}\right)$

   c) -231^0 and 489^0 $\left(\textit{Generally } 129^0+360^0k, k\in\mathbb{Z}\right)$

   d) -360^0 and 360^0 $\left(\textit{Generally } 0^0+360^0k, k\in\mathbb{Z}\right)$

3) The trigonometric values of two co-terminal angles are equal.

e) **Answers of Application Activity 8.2.1**

1) A positive angle coterminal with a 55° angle is $415^0+k360^0, k\in\mathbb{Z}\cdot$

A negative angle coterminal with a 55° angle is $-205^0+k360^0, k\in\mathbb{Z}\cdot$

2) A positive angle coterminal with a $\frac{\pi}{3}$ angle is $\frac{7\pi}{3}+2k\pi, k\in\mathbb{Z}$.

A negative angle coterminal with a $\frac{\pi}{3}$ angle $-\frac{5\pi}{3}+2k\pi, k\in\mathbb{Z}\cdot$

Lesson 9: Reduction of Negative angle or opposite angle to functions of positive acute angles

a) **Learning objective:** Appreciate the relationship between the trigonometric values for different angles

b) Teaching resources:

- Ruler, T-square, compass, protractor
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

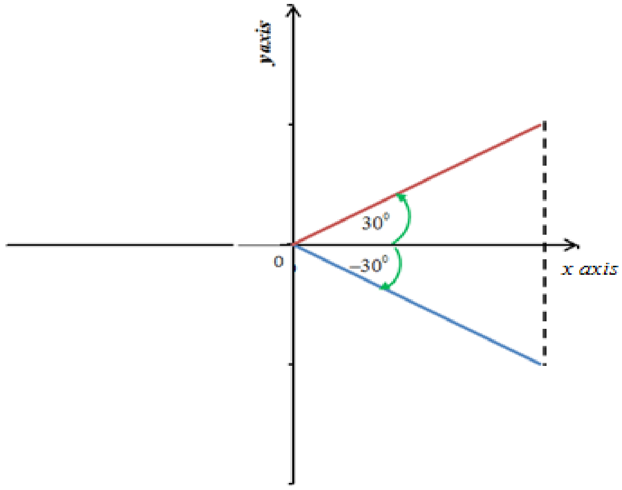
Student-teachers will learn better this lesson if they have a good understanding on concept of

- Unit circle learnt from lesson 3 of this unit.
- Opposite angles learnt from S1, Unit 6.
- Isometries learnt in S2, unit 9.

d) Learning activities

- **Guidance**
  - Let Student-teachers attempt the **Activity 8.2.2 (Student-teacher's book)** and introduce the concept of isometry and opposite angle.
  - Guide student-teachers in rotating vectors for the given direction and angle.
  - Facilitate the student-teachers while comparing the trigonometric ratios for of opposite angles
  - Facilitate them to do the provided examples given in **Student-teacher's book** to emphasize the skills, he/she has got.

- **Answers of Activity 8.2.2**



	The given angle(in degree)		New angle(in degree) after reflection		Observations from trigonometric values of both angles
	Figure 1	Figure 2	Figure 1	Figure 2	
sine	$\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	Opposite
cosine	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	Equal
tangent	1	$-\frac{\sqrt{3}}{3}$	-1	$\frac{\sqrt{3}}{3}$	Opposite

e) Answers of Application Activity 8.2.2

1. If $\sin\theta = -0.1903$, $\sin(-\theta) = 0.1903$

2. If $\cos(\theta) = 0.0133$, $\cos(-\theta) = 0.0133$

3. If $\sec(\theta) = -1.753$, $\cos(-\theta) = 0.570451$

4. If $\csc(-\theta) = \sqrt{3}$, $\sin\theta = \frac{\sqrt{3}}{3}$

5. If $\cos(-\theta)=\frac{1}{7}$, $\sec\theta=7$

6. If $\cot(\theta)=-5.4219$, $\tan(-\theta)=0.184437$

Lesson 10: Reduction of Complementary angles to functions of positive acute angles

a) Learning objective: Differentiate between complementary angles and co-terminal angles

b) Teaching resources:

- Ruler, T-square, protractor and compass
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of

- Unit circle learnt in lesson 3 of this unit.
- Isometries learnt in S2, unit 9.

d) Learning activities

- **Guidance**

- Let Student-teachers attempt the **Activity 8.2.3 (Student-teacher's book)** and introduce the concept of complementary angles
- Guide student-teachers in rotating vectors for the given direction and angle.
- Facilitate the student-teachers while comparing the trigonometric ratios of complementary angles
- Invite one group for presentation of its work to other groups.
- Make sure that everybody is engaged/ involved.
- Facilitate them to do the provided examples given in **Student-teacher's book** to emphasize the skills, they have got.

- **Answers of Activity 8.2.3**

Examples of pairs angles whose sum is right angle:

$(30^0,60^0),(45^0,45^0)\ and\ (0^0,90^0)$

After presenting them on circle, we find that

$$\begin{cases}\sin 30^0=\cos 60^0=\frac{1}{2}\\ \sin 45^0=\cos 45^0=\frac{\sqrt{2}}{2}\\ \sin 0^0=\cos 90^0=0\end{cases}$$

Conclusion : If $\theta+\alpha=90^0=\frac{\pi}{2}, then\ \sin\theta=\cos\alpha\ and$ vice versa

e) Answers of Application Activity 8.2.3

1. If $\sin\theta=0.00213$, $\cos\left(\frac{\pi}{2}-\theta\right)=0.00213$
2. If $\tan\left(\frac{\pi}{2}-\theta\right)=-0.11221$, $\cot\theta=-0.11221$
3. If $\cot(-\theta)=1.1482$, $\tan\left(\theta-\frac{\pi}{2}\right)=-1.1482$
4. If $\cos(\theta)=0.5$, $\csc\left(\theta-\frac{\pi}{2}\right)=-2$
5. If $\sin\left(\frac{\pi}{2}-\theta\right)=\frac{1}{2}$, $\sec\theta=2$
6. If $\sec\left(\theta-\frac{\pi}{2}\right)=7$, $\csc\theta=7$

Lesson 11: Reduction of Supplementary angles to functions of positive acute angles

a) Learning objective: Differentiate between complementary angles, supplementary angles and co-terminal angles

b) Teaching resources:

- Ruler, T-square, protractor and compass
- Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of

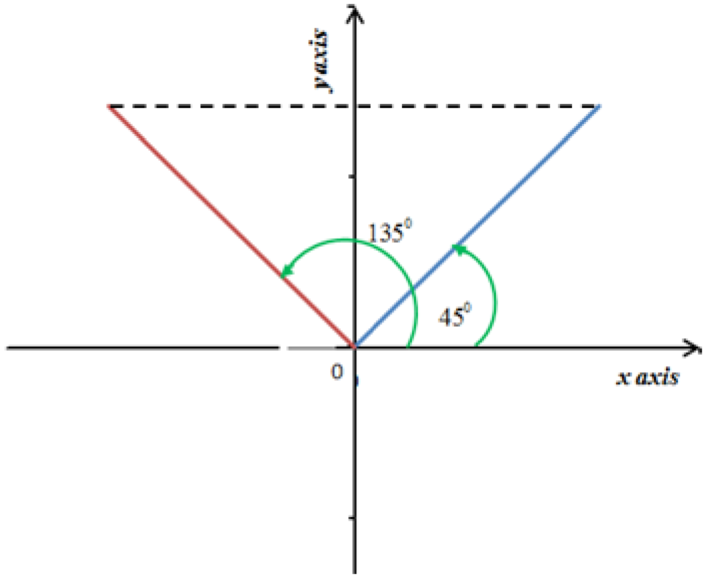
- Unit circle learnt in lesson 3 of this unit.
- Isometries learnt in S2, unit 9.

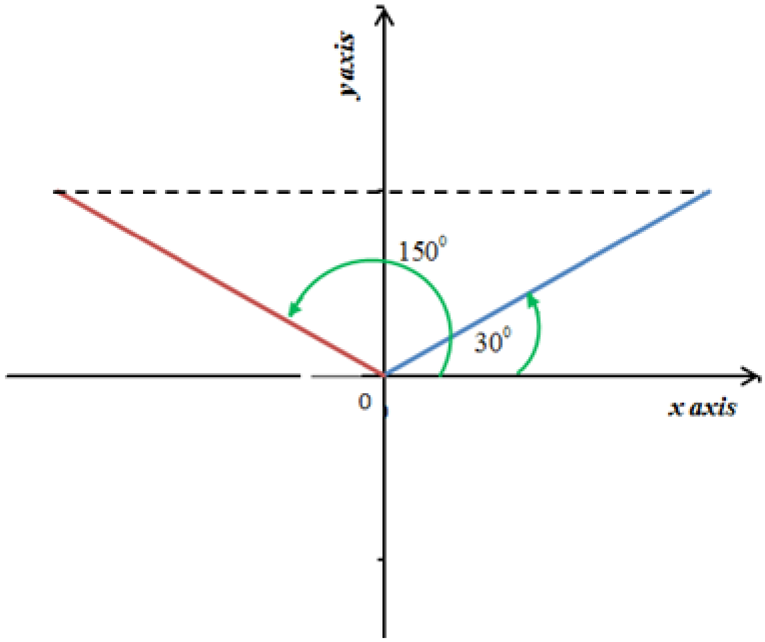
d) Learning activities

► Guidance

- Let Student-teachers attempt the **Activity 8.2.4(Student-teacher's book)** and introduce the concept of supplementary angles
- Guide student-teachers in rotating vectors for the given direction and angle.
- Facilitate the student-teachers while comparing the trigonometric ratios for of supplementary angles
- Invite one group for presentation of its work to other groups.
- Make sure that everybody is engaged/ involved.
- Facilitate them to do the provide examples given in **Student-teacher's book** to emphasize the skills, they have got.

e) **Answers of Activity 8.2.4**





Angle	The given angle(in degree)		New angle(in degree) after reflection		Observations from trigonometric values of both angles
	Figure 1	Figure 2	Figure 1	Figure 2	
sine	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	Equal
cosine	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	Opposite
tangent	1	$\frac{\sqrt{3}}{3}$	-1	$-\frac{\sqrt{3}}{3}$	Opposite

f) **Answers of Application Activity 8.2.4**

1. If $\sin\theta = 0.0312$, $\sin(\pi-\theta) = \sin\theta = 0.0312$
2. If $\tan(\pi-\theta) = -0.11221$, $\tan\theta = -0.11221$
3. If $\cot(\theta) = -2.5148$, $\cot(\theta-\pi) = -2.5148$
4. If $\sec(\theta) = 0.5$, $\cos(\theta-\pi) = 2$
5. If $\csc(\pi-\theta) = \frac{1}{2}$, $\csc\theta = \frac{1}{2}$
6. If $\sec(\theta-\pi) = 3$, $\sec\theta = 3$

Lesson 12: Sine law

a) Learning objective: Use trigonometry, including the sine rule to solve problems involving triangles

b) Teaching resources:

g) Ruler, T-square, protractor

h) Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of

- Definition of trigonometric ratios
- Pythagoras theorem

d) Learning activities

▶ i) **Guidance**

- Let Student-teachers attempt the **Activity 8.2.1 (Student-teacher's book)** by introducing the concept of right-angled triangle.
- Facilitate the student-teachers while comparing the sides of the given triangle
- Invite one group for presentation of its work to other groups.
- Make sure that everybody is engaged/ involved.
- Facilitate them to do the provided examples given in **Student-teacher's book** to emphasize the skills, they have got.

f) Answers of Activity 8.2.1

1. $\sin B = \frac{h}{a}$, $\sin A = \frac{h}{b}$ .2

$h = a\sin B$ and $b\sin A = h$, then $a\sin B = b\sin A$ or $\frac{a}{\sin A} = \frac{b}{\sin B}$

2. $\sin A = \frac{k}{c}$, $\sin C = \frac{k}{a}$.

$k = c\sin A$ and $k = a\sin C$, then $c\sin A = a\sin C$ or $\frac{c}{\sin C} = \frac{a}{\sin A}$

3. Now, $\frac{a}{\sin A} = \frac{b}{\sin B}$ and $\frac{c}{\sin C} = \frac{a}{\sin A}$. This gives $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

f) Answers of Application Activity 8.3.1

1. The height of the hill is 763.94 m.

2. The length of the field is 2460.36 m.

3. Height of the tree is 34.64 m and the breadth of the river is 20 m.

4. 308.11 km

Lesson 13 : Cosine law

a) Learning objective: Use trigonometry, including the sine and cosine rules, to solve problems involving triangles

b) Teaching resources:

a) Ruler, T-square, protractor and compass

b) Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of

- Definition of trigonometric ratios
- Pythagoras theorem

d) Learning activities

► Guidance

- Let Student-teachers attempt the **Activity 8.3.1 (Student-teacher's book)** by introducing the concept of right-angled triangle.
- Facilitate the student-teachers while comparing the sides of the given triangle
- Make sure that everybody is engaged/ involved
- Invite one group for presentation of its work to other groups.
- Facilitate them to do the provide examples given in **Student-teacher's book** to emphasize the skills, he/she has got.

f) **Answers of Activity 8.3.1**

1. $\cos A = \frac{AX}{b}$

2. $b^2 = h^2 + (AX)^2 \Rightarrow h^2 = b^2 - (AX)^2$

3. $a^2 = h^2 + (XB)^2 \Rightarrow h^2 = a^2 - (XB)^2$

4. $h^2 = b^2 - (AX)^2$ and $h^2 = a^2 - (XB)^2$ gives $b^2 - (AX)^2 = a^2 - (XB)^2$

But $XB = c - AX$, then

$b^2 - (AX)^2 = a^2 - (c - AX)^2$

$\Leftrightarrow b^2 - (AX)^2 = a^2 - \left(c^2 - 2cAX + (AX)^2\right)$

$\Leftrightarrow b^2 - (AX)^2 = a^2 - c^2 + 2cAX - (AX)^2$

$\Leftrightarrow b^2 + c^2 - 2cAX = a^2$

But $\cos A = \frac{AX}{b} \Rightarrow AX = b\cos A$. Then

$\Leftrightarrow b^2 + c^2 - 2cb\cos A = a^2$

$\Leftrightarrow a^2 = b^2 + c^2 - 2bc\cos A$

e) Answers of Application Activity 8.3.1b

1. $9.43\,cm$
2. $c = 21.7\,cm$
3. 130.42 m
4. 0.6km

Lesson 14: Applications

a) Learning objective: Use trigonometry, including the sine and cosine rules, to solve problems involving triangles and appreciate the use of trigonometry in daily life situation

b) Teaching resources:

g) Ruler, T-square, protractor, calculator

h) Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of

- Trigonometric ratios learnt from previous lessons of this unit
- Sine law learnt in 12^{th} lesson of this unit
- Cosine law learnt in 13^{th} lesson of this unit

d) Learning activities

► i) **Guidance**

- Let Student-teachers attempt the **Activity 8.3.2 (Student-teacher's book)** and make a review on sine and cosine laws
- Guide student-teachers on how to solve triangle.
- Facilitate them while solving the given activity by checking if everybody is engaged
- Make sure that everybody is engaged/ involved.
- Help the students to use trigonometry, including the sine and cosine rules, to solve problems involving triangles.
- Ask them to state where we can apply trigonometric ratios.
- Let one group member to present their findings
- Facilitate them to do the provided examples given in **Student-teacher's book** to emphasize the skills, they have got.

e) Answers of Activity 8.3.2

Strip B width $\simeq 3.244\,m$, Strip C width $\simeq 2.168\,m$

f) Answers of Application Activity 8.3.2

1. The tree was *8.91 m* tall.
2. The distance is *369.15 m.*
3. The distance is *45.7 m*
4. The bearing of B from A is S54°10'W and the distance of B from A is *22.2 km*.
5. *20 m*
6. *24.3m*

8.6. Summary of the unit

1. A **rotation angle** is formed by rotating an **initial side** through an angle, about a fixed point called **vertex**, to terminal position called **terminal side**. Angle is positive if rotated in a counter clockwise direction and negative when rotated clockwise.

2. The amount we rotate the angle is called the measure of the angle and is measured in: **degree** or **radian**.

3. To convert degree measure to radian measure, multiply by $\frac{\pi\, radian}{180^0}$.

4. To convert radian measure to degree measure, multiply by $\frac{180^0}{\pi\, radian}$.

5. In a triangle whose hypotenuse r, the adjacent side x and the opposite side y:

$$\sin\alpha = \frac{opposite\ side}{hypotenuse} = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}} \qquad \cos\alpha = \frac{adjacent\ side}{hypotenuse} = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\tan\alpha = \frac{opposite\ side}{adjacent\ side} = \frac{y}{x} \qquad \cot\alpha = \frac{adjacent\ side}{opposite\ side} = \frac{x}{y}$$

$$\csc\alpha = \frac{hypotenuse}{opposite} = \frac{1}{\sin\alpha}, sec\alpha = \frac{hypotenuse}{adjascent} = \frac{1}{\cos\alpha}\ and\ \cot\alpha = \frac{adjacent}{opposite} = \frac{1}{\tan\alpha}$$

6. The table trigonometric number of remarkable angles

α	0°	30°	45°	60°	90°	180°	270°	360°
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Does not exist	0	Does not exist	0
$\cot\alpha$	Does not exist	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Does not exist	0	Does not exist

7. Trigonometric identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

8. Two angles are **equivalent** if their difference is $2k\pi,\ k \in \mathbb{Z}$ (or $360^\circ k,\ k \in \mathbb{Z}$). This means that the angle α and $\alpha + 2k\pi$ are equivalent angles.

$$\left.\begin{aligned} \sin\left(\alpha + 360^\circ k\right) &= \sin\alpha \\ \cos\left(\alpha + 360^\circ k\right) &= \cos\alpha \\ \tan\left(\alpha + 360^\circ k\right) &= \tan\alpha \\ \cot\left(\alpha + 360^\circ k\right) &= \cot\alpha \end{aligned}\right\} k \in \mathbb{Z}$$

9. Angle $-\alpha$ is **opposite** of the angle α

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$
$$\tan(-\alpha) = -\tan\alpha$$
$$\cot(-\alpha) = -\cot\alpha$$

10. Two angles are said to be **complementary** if their sum is 90° (or $\frac{\pi}{2}$). Note that α and $90^\circ - \alpha$ are complementary.

$$\sin\left(90^\circ - \alpha\right) = \cos\alpha$$
$$\cos\left(90^\circ - \alpha\right) = \sin\alpha$$
$$\tan\left(90^\circ - \alpha\right) = \cot\alpha$$
$$\cot\left(90^\circ - \alpha\right) = \tan\alpha$$

11. Two angles are said to be **supplementary** if their sum is 180° (or π). It is easy to discover that α and $180^\circ - \alpha$ are supplementary.

$$\sin\left(180^0-\alpha\right)=\sin\alpha$$
$$\cos\left(180^0-\alpha\right)=-\cos\alpha$$
$$\tan\left(180^0-\alpha\right)=-\tan\alpha$$
$$\cot\left(180^0-\alpha\right)=-\cot\alpha$$

12. If a,b,c are the lengths of the sides of a triangle and A,B,C are the opposite angles respectively, then cosine law says that

$$\begin{cases} a^2=b^2+c^2-2bc\cos\widehat{A} \\ b^2=a^2+c^2-2ac\cos\widehat{B} \\ c^2=a^2+b^2-2ab\cos\widehat{C} \end{cases}$$

13. If a,b,c are the lengths of the sides of a triangle and A,B,C are the opposite angles respectively, then the sine law is

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

14. Applications

Many real situations involve right triangle. Using angles and trigonometric functions, we can solve problems involving right triangle like:

- Bearings and air navigation
- Angles of elevation and angle of depression
- Inclined plane

8.7. Additional Information for tutor

Notice on reduction to a positive angle (reference angle)

The values of the circular functions of an angle, if they exist, are the same, up to a sign, of the corresponding circular functions of its reference angle. More specifically, if α is the reference angle for θ, then:

$\sin\theta=\pm\sin\alpha$	$\cos\theta=\pm\cos\alpha$	$\tan\theta=\pm\tan\alpha$
$\csc\theta=\pm\csc\alpha$	$\sec\theta=\pm\sec\alpha$	$\cot\theta=\pm\cot\alpha$

The choice of the (±) depends on the quadrant in which the terminal side of θ lies.

For example, 150^0 is a II quadrant angle, the reference angle is 30^0.

We find out that

$\sin 150^0 = \sin 30^0$ as the sine of any II quadrant angle is positive

$\cos 150^0 = -\cos 30^0$ as the cosine of any II quadrant angle is negative.

$\tan 150^0 = -\tan 30^0$ as the tangent of any II quadrant angle is negative.

Notice on identities

► **The Pythagorean Identities:**

1. $\cos^2\theta + \sin^2\theta = 1$ **Fundamental formula**

Alternative Forms:

• $1-\sin^2\theta = \cos^2\theta$	• $1-\cos^2\theta = \sin^2\theta$

2. $1+\tan^2\theta = \sec^2\theta$, provided $\cos\theta \neq 0$.

Alternative Forms:

• $\sec^2\theta - \tan^2\theta = 1$	• $\sec^2\theta - 1 = \tan^2\theta$

3. $1+\cot^2\theta = \csc^2\theta$, provided $\sin\theta \neq 0$.

Alternative Forms:

• $\csc^2\theta - \cot^2\theta = 1$	• $\csc^2\theta - \cot^2\theta = 1$

Pythagorean conjugates are useful in proving trigonometric identities:

- $(1+\cos\theta)(1-\cos\theta)=1-\cos^2\theta=\sin^2\theta$
- $(1+\sin\theta)(1-\sin\theta)=1-\sin^2\theta=\cos^2\theta$
- $(\sec\theta+1)(\sec\theta-1)=\sec^2\theta-1=\tan^2\theta$
- $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=\sec^2\theta-\tan^2\theta=1$
- $(\csc\theta+1)(\csc\theta-1)=\csc^2\theta-1=\cot^2\theta$
- $(\csc\theta+\cot\theta)(\csc\theta-\cot\theta)=\csc^2\theta-\cot^2\theta=1$

1.8. **End unit assessment**

1. a. $\dfrac{\sin a(1+\cos a)}{(1-\cos a)(1+\cos a)}=\dfrac{\sin a(1+\cos a)}{\sin^2 a}$

$=\dfrac{1+\cos a}{\sin a}$

b. $\dfrac{1}{\cos^2 a}+\dfrac{1}{\sin^2 a}=\dfrac{\sin^2 a+\cos^2 a}{\cos^2 a\sin^2 a}$

$=\dfrac{1}{\cos^2 a}\dfrac{1}{\sin^2 a}$

$=\sec^2 a\csc^2 a$

c. $(\sec^2 a+\tan^2 a)(\sec^2 a-\tan^2 a)=(\sec^2 a+\tan^2 a)\left(\dfrac{1}{\cos^2 a}-\dfrac{\sin^2 a}{\cos^2 a}\right)$

$=(\sec^2 a+\tan^2 a)\left(\dfrac{1-\sin^2 a}{\cos^2 a}\right)$

$=(\sec^2 a+\tan^2 a)\left(\dfrac{\cos^2 a}{\cos^2 a}\right)$

$=\sec^2 a+\tan^2 a$

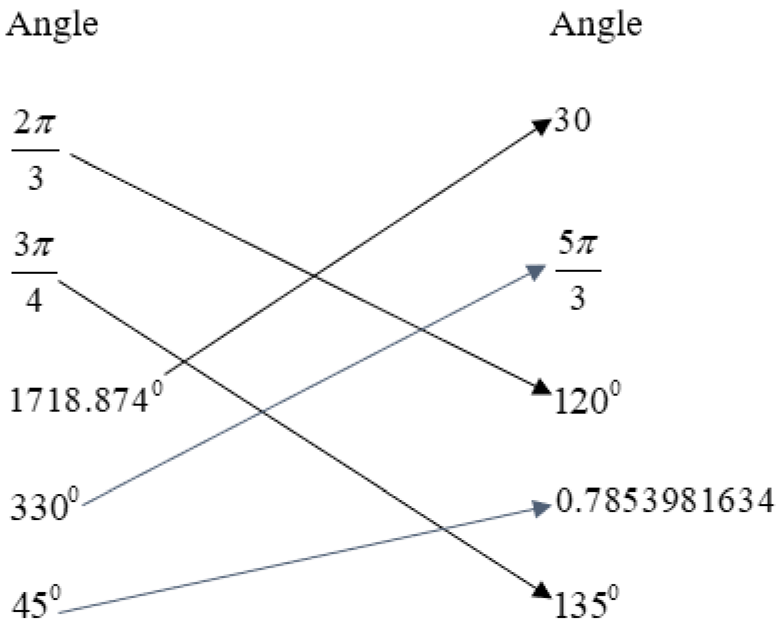
d. $\sqrt{\frac{(1-\cos a)(1-\cos a)}{(1+\cos a)(1-\cos a)}} = \sqrt{\frac{(1-\cos a)^2}{\sin^2 a}} = \frac{1-\cos a}{\sin a}$

2. $\tan\theta = 3.18$

3. $\cos\theta = -0.8;\ \tan\theta = -0.75$

4. $\cos 14^0 = \sqrt{1-\sin^2 14^0}$ *or* $\cos 14^0 = \sin\left(90^0 - 14^0\right) = \sin 76^0$

5.



6. Angles which have the same sine as

a) 40^0 are 140^0 or 400^0 i.e. its supplementary or equivalent angles.

b) 56^0 are 124^0 or 416^0 i.e. its supplementary or equivalent angles.

c) 130^0 are 50^0 or 490^0 i.e. its supplementary or equivalent angles.

d) 240^0 or 660^0 i.e. its supplementary or equivalent angles.

7. Pairs of equal value are

$\left(\sin 90^0, \cos 360^0\right);\ \left(\cos 180^0, \sin 270^0\right);\ \left(\tan 195^0, \tan 15^0\right);\ \left(\tan 150^0, \tan 330^0\right)$

8. 30^0, 100^0

9. 160^0

10. 85^0

11. 327^0

12. 235^0

13. 255^0

14. 315^0

15. The height of the cliff is *107.96 m*.

16. The distance between the bill-boards is (a) *6.14 m* (b) *49.82 m*

17. *11. 342 m*

18. 12. 384 km per hour

19. *205.26m*

20. *849.77 m*

8.9. Additional activities

8.9.1. Remedial activities

1. In the following Exercises, convert the angle from degree measure into radian measure, giving the exact value in terms of π.

a) 30^0 b) 240^0 c) 135^0 d) -270^0 e) -315^0 f) 150^0 g) 45^0 h) 225^0.

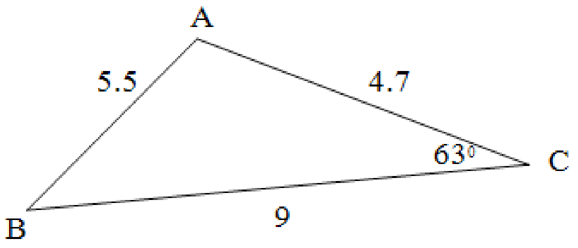
2. In the following Exercises, convert the angle from radian measure into degree measure.

a) $\frac{2\pi}{3}$ b) $-\frac{7\pi}{6}$ c) $\frac{11\pi}{6}$ d) $\frac{\pi}{3}$ e) $-\frac{5\pi}{3}$ f) $\frac{\pi}{6}$ g) $-\frac{\pi}{2}$.

3. Find the cosine and sine of the following angles.

a) $\theta = 270^\circ$ b) $\theta = -\pi$ c) $\theta = 45^\circ$ d) $\theta = \frac{\pi}{6}$ **e) $\theta = 60^\circ$**

4. Using sine rule , find out the angle B



Answer:

1.

Degrees	30^0	240^0	135^0	-270^0	-315^0	150^0	45^0	225^0
radians	$\frac{\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	$-\frac{3\pi}{2}$	$-\frac{7\pi}{4}$	$\frac{5\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{4}$

radians	$\frac{2\pi}{3}$	$-\frac{7\pi}{6}$	$\frac{11\pi}{6}$	$\frac{\pi}{3}$	$-\frac{5\pi}{3}$	$\frac{\pi}{6}$	$-\frac{\pi}{2}$
Degrees	120	–210	330	60	–300	30	–90

3.

Angle	$\theta = 270^\circ$	$\theta = -\pi$	$\theta = 45^\circ$	$\theta = \frac{\pi}{6}$	$\theta = 60^\circ$
Cosine	0	–1	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
sine	–1	0	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

4. 49.6^0

8.9. 2. Consolidation activities

1. Using the given information about θ, find the indicated value

   a) If θ lies in 2nd quadrant with $\sin\theta = \frac{3}{5}$, find $\cos\theta$.

   b) If $\pi < \theta < \frac{3\pi}{2}$, with $\cos\theta = -\frac{\sqrt{5}}{5}$, find $\sin\theta$.

   c) If $\sin\theta = 1$, find $\cos\theta$.

2. Before calculators became common classroom tools, they used trigonometric tables to find trigonometric ratios. Below is a simplified trigonometric table for angles between 40° and 56°. Without using a calculator, can you determine which column gives sine values, which gives cosine values, and which gives tangent values?

Degrees	?	?	?
40	0.83909963	0.64278761	0.76604444
42	0.90040404	0.66913061	0.74314483
44	0.96568877	0.69465837	0.71933980
46	1.03553031	0.71933980	0.69465837
48	1.11061251	0.74314483	0.66913061
50	1.19175359	0.76604444	0.64278761
52	1.27994163	0.78801075	0.61566148
54	1.37638192	0.80901699	0.58778525
56	1.48256097	0.82903757	0.55919290

3. In order to determine the height of a tree in Nyungwe forest, two sightings from the ground, one 200m directly behind the other, are made. If the angles of inclination were 45^{000} and 30^0, respectively, how tall is the tree to the nearest metre?

4. In an airport control tower A, 2 planes B and C are located at the same altitude on a radar screen. The range finder determines one plane to bear N60^0E at 100km while the other bears S50^0E at 150km. How far apart are the planes from each other?

5. Multiple choice

i). Which of the following trigonometric ratios could not be π?

(A) $\tan\theta$ (B) $\sec\theta$ (C) $\cot\theta$ (D) $\cos\theta$ (E) $\csc\theta$

ii). If a nonhorizontal line has slope $\sin\theta$, it will be perpendicular to a line with slope

(A) $\cos\theta$ (B) $-\cos\theta$ (C) $\csc\theta$ (D) $-\csc\theta$ (E) $-\sin\theta$

iii). If θ is the smallest angle in a $3-4-5$ right triangle, then $\sin\theta =$

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{5}{4}$ (D) $\frac{3}{4}$ (E) $\frac{5}{3}$

iv). Which of the following expressions does not represent a real number?

(A) sin 30° (B) tan 45° (C) cos 90° (D) csc 90° (E) sec 90°$0^{0000000000}$

v). A central angle in a circle of radius r has a measure of θ radians. If the same central angle were drawn in a circle of radius 2r, its radian measure would be

(A) $\frac{\theta}{2}$ (B) $\frac{\theta}{2r}$ (C) 2θ (D) θ (E) $2r\theta$

vi). If the perimeter of a sector is 4 times its radius, then the radian measure of the central angle of the sector is

(A) 2 (B) 4 (C) $\frac{4}{\pi}$ (D) $\frac{2}{\pi}$ (E) impossible to determine without knowing the radius.

vii). What is the radian measure of an angle of x degrees?

(A) $\frac{x}{180}$ (B) πx (C) $\frac{\pi x}{180}$ (D) $\frac{180}{x\pi}$ (E) $\frac{180x}{\pi}$

Answers:

1. a) From the Pythagorean identity, we get $\cos\theta = \pm\frac{4}{5}$, since θ is a Quadrant II angle, thus $\cos\theta = -\frac{4}{5}$.

b) From the Pythagorean identity, we get $\sin\theta = \pm\frac{2\sqrt{5}}{5}$, since we are given that $\pi < \theta < \frac{3\pi}{2}$, we note that θ is a Quadrant III angle, thus $\sin\theta = -\frac{2\sqrt{5}}{5}$.

c) When we substitute $\sin\theta = 1$ into $\cos^2\theta + \sin^2\theta = 1$, we get $\cos\theta = 0$.

2.

Degrees	tangent	Sine	cosine

3. The tree is approximately 273 m tall.

4. i) D ii) E iii) B iv) E v) D vi) A vii) C

5. 157km

8.9.3. Extended activities

1. Trigonometric Tables below is a simplified trigonometric table for angles between 32° and 42°. Without using a calculator, can you determine which column gives cotangent values, which gives secant values, and which gives cosecant values?

Degrees	?	?	?
32	1.17917840	1.60033453	1.88707991
34	1.20621795	1.48256097	1.78829165
36	1.23606798	1.37638192	1.70130162
38	1.26901822	1.27994163	1.62426925
40	1.30540729	1.19175359	1.55572383
42	1.34563273	1.11061251	1.49447655

2. A 100-degree arc of a circle has a length of 7 cm. To the nearest centimeter, what is the radius of the circle?

3. When I stand 30 m away from a tree at home, the angle of elevation to the top of the tree is 50◦ and the angle of depression to the base of the tree is 10◦. What is the height of the tree? Round your answer to the nearest metre.

4. From the observation deck of the lighthouse at Rubavu Point 50 m above the surface of Lake Kivu, a lifeguard spots a boat out on the lake sailing directly toward the lighthouse. The first sighting had an angle of depression of 8.2^o and the second sighting had an angle of depression of 25.9^0. How far had the boat traveled between the sightings?

5. The broadcast tower for radio station has two enormous flashing red lights on it: one at the very top and one a few metres below the top. From a point 5000 m away from the base of the tower on level ground the angle of elevation to the top light is 7.970^0 and to the second light is 7.1250^0. Find the distance between the lights to the nearest metres.

6. The lengths of the sides of a triangle are $x, y, \sqrt{x^2+y^2+xy}$. Find the measure of the greatest angle.

7.InatriangleABC,determinetheangleAforwhich $(a+b+c)(b+c-a)=3bc$.

8. David is a cross-country skier and skis 10km in a direction N40^{0}E of the ski lodge. At this point she turns and skis S10^{0}E for 4km and arrives at a chalet. How far is David from the lodge?

Answers:

1.

Degrees	secant	cotangent	cosecant

2. 4 cm

3. The tree is about 41m tall.

4. The boat has travelled about 244 m.

5. The lights are about 75 m apart.

6. 120^0

7. $A=60^0$

8. 8 km

UNIT 9

POINTS, STRAIGHT LINES AND CIRCLE IN 2D

9.1. Key unit competence

Determine an equation of a line and a circle.

9.2. Prerequisite

Student-teachers will easily learn this unit, if he/she is able to

- Define the coordinates of a point in 2D as it is taught in S1 (UNIT3),
- Calculate the distance between two points and mid-Point of a segment in 2D as seen in S2(UNIT7)
- Determine equations of a straight line. S3(UNIT6),
- Appreciate the importance of a point and a line in a plane. (help to draw shapes)
- Be accurate in plotting/graphing and calculations.

9.3. Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others view and thoughts during class discussions)
- Gender (equal opportunity of boys and girls in the lesson participation)
- Standardization culture (regulation of the current intensity)

9.4. Guidance on introductory activity 9

a) Student-teachers work on the introductory activity to be aware of this unit.

b) Let student-teachers read and do the introductory activity in the student-teacher's book by stating location of some places and use direction to know the place with respect to another place.

Answers of introductory activity 9

a) At $A3$ there is Change rooms.

b) The Main pool covers 3squares.

c) i) Diving area is at $F2$.

ii) Canteen is at $A1$.

d) i) Water slides is East from the Kids'pool.

ii) Change rooms is North from Canteen.

iii) Main pool is West from Diving.

c) Through question-answer, facilitate student-teachers to discover the importance of points and direction in daily activities.

d) Ask them to give more examples where the point and direction are important.

e) Through class discussions, let student-teachers think of different ways of getting solutions

9.5. List of lessons

#	Lesson title	Learning objectives	Number of periods
1	Introductory activity	To arouse the curiosity of student teachers on the content of unit 9.	2
2	9.1. Location of a point in 2D.	Define and plot the coordinates of a point in 2D.	2
3	9.2. Mid-point and Distance between two points in 2D	Define a straight line Calculate the distance between two points and mid-Point of a segment in 2D.	3

4	9.3. Determination of equation of a straight line 9.3.1. Vector equation, parametric equation, Cartesian equation given: 2 points, Direction vector	Determine equations of a straight line.	4
5	Determination of equation of a straight line : vector equation, parametric equation, Cartesian equation given Gradient		4
6	Determination of equation of a circle: Cartesian equation of a circle	• Define a circle. • Establish the equation of circle from centre and radius or two points of circle • Find the centre, radius, and diameter from the equation of a circle.	4
7	Determination of equation of a circle: parametric equations of a circle		3
8	Assessment	Determine an equation of a line and a circle.	2

Lesson 1: Location of a point in 2D

a) Learning objective

Define and plot the coordinates of a point in 2D.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils,…

c) Prerequisites/Revision/Introduction

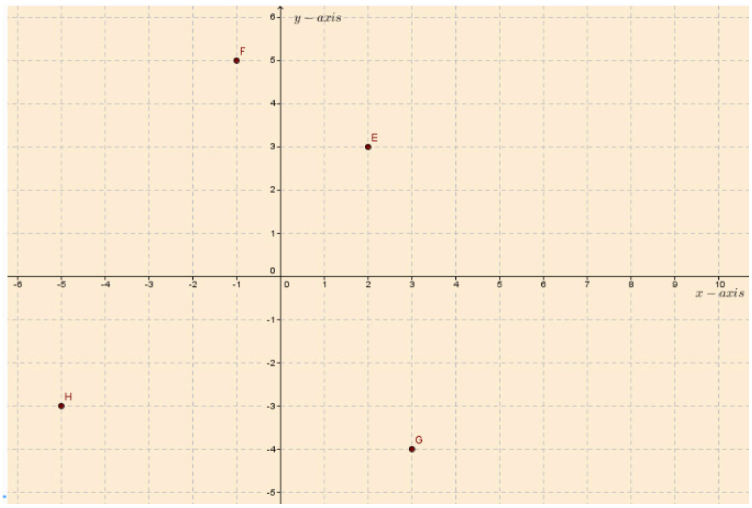
Student-teachers should have knowledge and skills on how to draw axes of coordinates in cartesian plane and how to plot points in 2D.

c) Learning activities

- Ask student-teachers in small groups to read and discuss on the **activity 9.1** to plot points and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling student-teachers.
- Call student-teachers to present the findings, and help them to harmonize the answer.

Answer of activity 9.1

1.



2. $AB = \sqrt{(1-3)^2 + (-1-5)^2} = 2\sqrt{10}$

$BC = \sqrt{(-4-1)^2 + (-16+1)^2} = 5\sqrt{10}$

$AC = \sqrt{(-4-3)^2 + (-16-5)^2} = 7\sqrt{10}$

$\therefore AB + BC = 2\sqrt{10} + 5\sqrt{10} = 7\sqrt{10} = AC$

Or

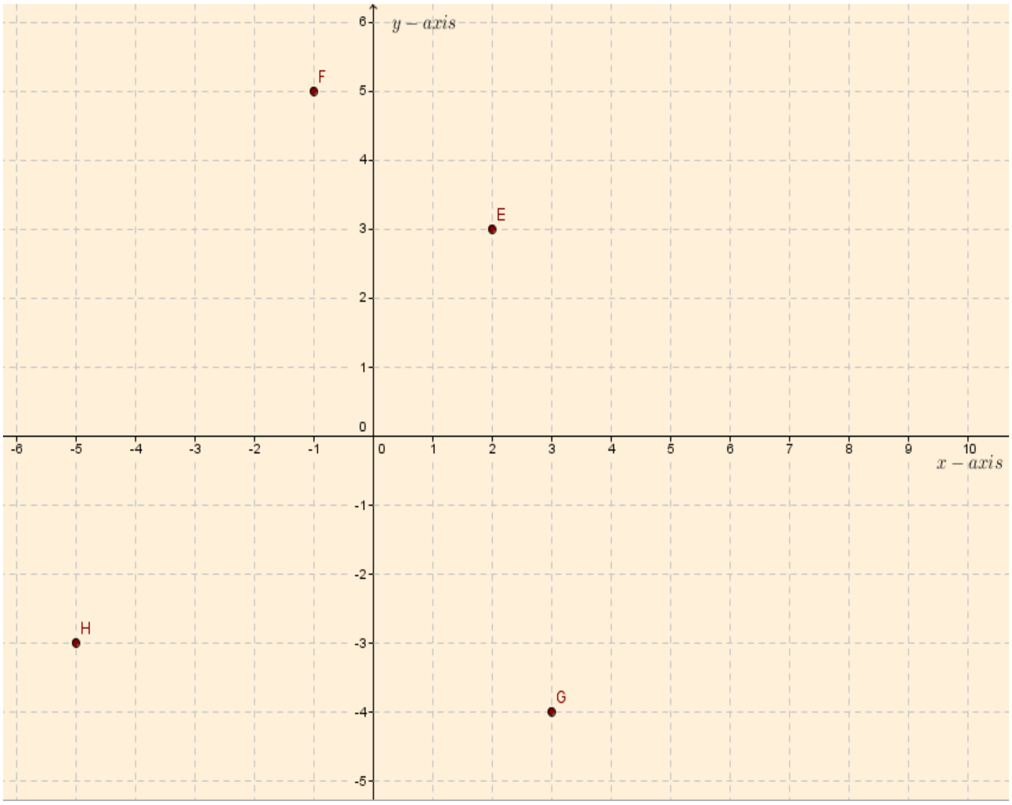
Lead the student-teachers to plot points and join them respectively, then use a ruler to measure and record the distance of each side. Hence facilitate them to use the result from measuring to verify that $AB + BC = AC$.

- Lead student-teachers to realize that the solutions contain the concept of points and their uses in calculations.
- Let student-teachers discover that in 2D :
  - a point has paired coordinates (x, y),
  - The first number x is called the x-coordinate (or x-component), as it is the signed distance from the origin in the direction along the x-axis.
- Similarly, the second number y is called the y -coordinate (or y -component), as it is the signed distance from the origin in the direction along the y -axis.
- The intersection point $(0,0)$ of the two axes is called **the origin.**
- We denote points using capital letters such as $A, B, C, D, \ldots$
- Lead them to do example 1 to emphasize the skills for accurate graphs.
- Guide them to do **application activity 9.1** to master the content.

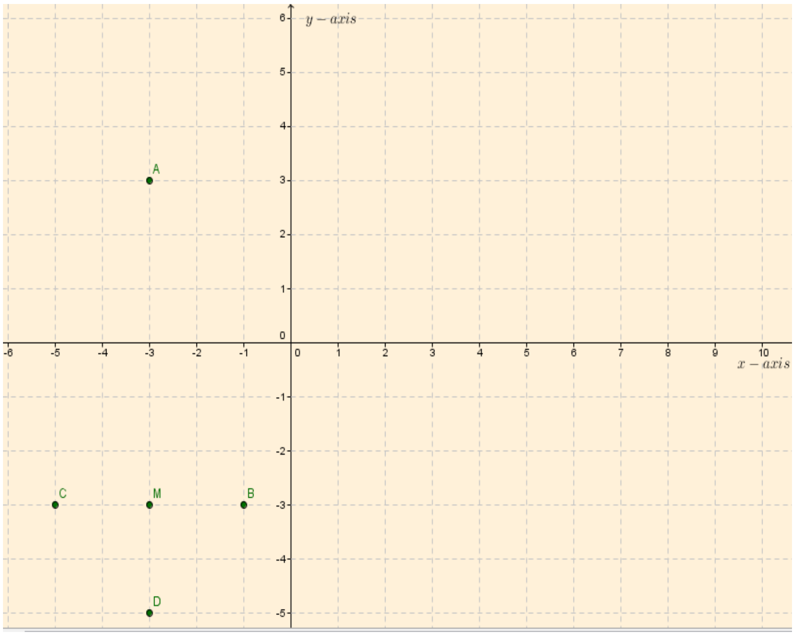
d) Application activity 9.1

Answers of Application activity 9.1

1.

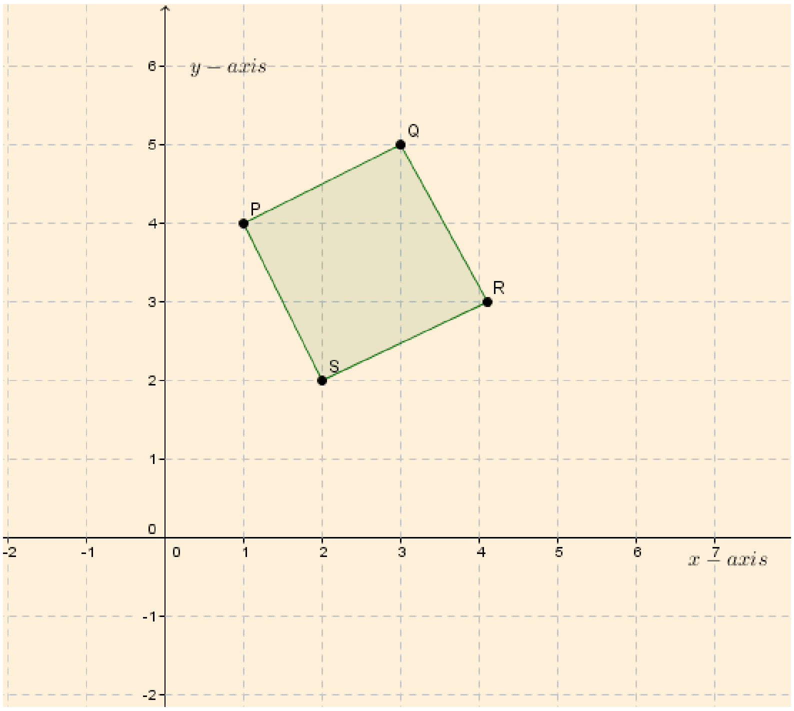


2. a) $M(-3, -3)$



b) $B(-1, -3)$ is East of M

3. a)



b) the shape is a square

4. a) i) Peter   ii) Sue   iii) Amber

b)Tennis Club $\to D7$ Pool $\to C2$

c) $F5 \to$ School $H6 \to$ Theatre

d) Robert $\to B6$

Lesson 2: Mid-point and Distance between two points in 2D

a) **Learning objective**

Define and calculate the distance between two points and mid-Point of a segment in 2D.

b) **Teaching resources**

Student-teachers 'book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

Student-teachers should have knowledge and skills on how to draw the axes of coordinates in cartesian plane and how to plot shapes in 2D.

d) **Learning activities**

- Ask student-teachers in pairs to read and discuss on the **activity 9.2** to plot points and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling learners.
- Call student-teachers to present the findings, and help them to harmonise the answer.

Answer of activity 9.2

- Guide student-teachers to name M as midpoint and d as distance

a) $M\left(\frac{3+7}{2},\frac{4+10}{2}\right)=(5,7)$

$d=\sqrt{(7-3)^2+(10-4)^2}=\sqrt{52}=2\sqrt{13}\ l.u$ (l.u: length unit)

$M\left(\frac{2+10}{2},\frac{8+4}{2}\right)=(6,6)$, $d=\sqrt{(10-2)^2+(4-8)^2}=\sqrt{80}=4\sqrt{5}\ l.u$

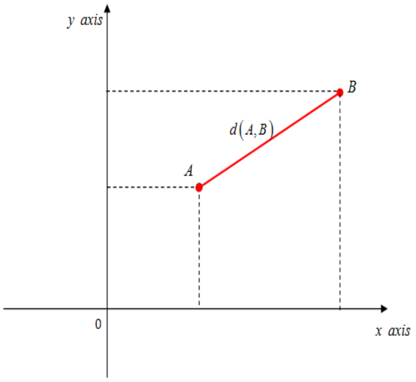
b) $M\left(\frac{0+6}{2},\frac{5+3}{2}\right)=(3,4)$, $d=\sqrt{(6-0)^2+(3-5)^2}=\sqrt{40}=2\sqrt{10}$ l.u

c) $M\left(\frac{4+1}{2},\frac{3+5}{2}\right)=(2.5,4)$, $d=\sqrt{(1-4)^2+(5-3)^2}=\sqrt{13}$ l.u

- Student-teachers discover that in 2D:

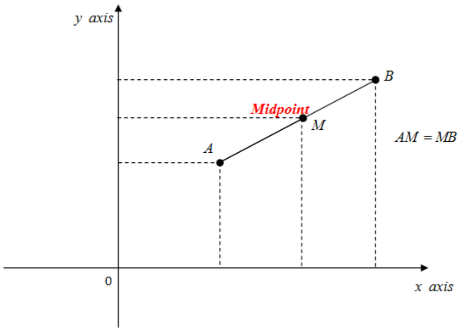
If $A(a_1,a_2)$ and $B(b_1,b_2)$ are two points in plane, then the distance between these two points denoted $d(A,B)$ is given by

$$d(A,B)=\left\|\overrightarrow{AB}\right\|=\sqrt{(b_1-a_1)^2+(b_2-a_2)^2}$$



- Let them also discover that in 2D, the midpoint $M(x_M,y_M)$ of segment $[AB]$ with $A(x_A,y_A)$ and $B(x_B,y_B)$ is given by

$$M=\frac{1}{2}(A+B) \text{ or } M=\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2}\right); \text{ With } \begin{cases} x_M=\frac{x_A+x_B}{2} \\ y_M=\frac{y_A+y_B}{2} \end{cases}$$



- Lead student-teachers to do example 1 to emphasize the skills for accurate graphs.
- Guide them to do **application activity 9.2** to master the content.

d) Application activity 9.2

Answers of application activity 9.2

1) $AB = \sqrt{10}\ l.u$

2) $k = 5$ or $k = -3$

3) a) $M(4,8)$ b) $M(1.5,7)$

4) $B(4,2)$

Lesson 3: Vector equation, parametric equation, Cartesian equation of a straight line ggiven 2 points and Direction vector

a) **Learning objective**

Determine equations of a straight line.

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

Student-teachers should have knowledge and skills on how to draw axes, graphs in cartesian plane and how to plot shapes in 2D.

d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 9.3.1** to plot points and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling learners.
- Call student-teachers to present the findings, and help them to harmonize the answer.

Answer of activity 9.3.1

Direction vector $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}3\\-2\end{pmatrix}-\begin{pmatrix}1\\4\end{pmatrix}=\begin{pmatrix}2\\-6\end{pmatrix}$

$\vec{V}_1=2\begin{pmatrix}2\\-6\end{pmatrix}=\begin{pmatrix}4\\-12\end{pmatrix}$ $\vec{V}_2=-3\begin{pmatrix}2\\-6\end{pmatrix}=\begin{pmatrix}-6\\18\end{pmatrix}$ $\vec{V}_3=\frac{1}{2}\begin{pmatrix}2\\-6\end{pmatrix}=\begin{pmatrix}1\\-3\end{pmatrix}$

- Lead student-teachers to realize that the solution involves concept of vectors.
- Help student-teachers discover that in 2D, the equations of the line passing through two points $A(x_A, y_A)$ *and* $B(x_B, y_B)$ and the direction vector can be defined in three forms:

Vector equation

$(x,y)=\overrightarrow{0A}+r\overrightarrow{AB}$ or $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x_A\\y_A\end{pmatrix}+r\begin{pmatrix}x_B-x_A\\y_B-y_A\end{pmatrix}$, where $\overrightarrow{AB}$ is the direction vector.

Parametric equations

$$\begin{cases}x=x_A+r(x_B-x_A)\\y=y_A+r(y_B-y_A)\end{cases}$$

Cartesian equation

$$\frac{x-x_A}{x_B-x_A}=\frac{y-y_A}{y_B-y_A}$$

- Facilitate student-teachers to determine the equation of the line passing through the point $P(x_0, y_0)$ and any other point $Q(x,y)$ on the line with direction vector $\vec{v}=(a,b)$ as:

$$(x,y)=(x_0,y_0)+r(a,b) \qquad \text{or} \qquad \binom{x}{y}=\binom{x_0}{y_0}+r\binom{a}{b}$$

- Lead them to know that the **parametric equations** of the line that is parallel to the vector $\vec{v}=(a,b)$ and which passes through the point P with position vector $\overrightarrow{0P}=(x_0,y_0)$ are given by $\begin{cases} x=x_0+ra \\ y=y_0+rb \end{cases}$
- And let them know the **symmetric equation** or **Cartesian equation that is** found after eliminating parameter **r**. Then the equation is $\frac{x-x_0}{a}=\frac{y-y_0}{b}$
- Lead them to do examples to emphasize the skills for finding equations of a line.
- Call student-teachers to do Application activity 9.3.1 to master the content.

d) Answer for application activity 9.3.1

1. Vector equation: $(x,y)=(2,-3)+r(1,6)$

Parametric equations:

$$\begin{cases} x=2+r \\ y=-3+6r \end{cases}$$

Cartesian equation:

$$x-2=\frac{y+3}{6}$$

2. **Parametric equations**

$$\begin{cases} x=3-2r \\ y=4-2r \end{cases}$$

3. **Vector equation:** $x\vec{i}+y\vec{j}=r(3\vec{i}-2\vec{j})$

Parametric equations:

$$\begin{cases} x = 3r \\ y = -2r \end{cases}$$

Cartesian equation:

$$\frac{x}{3} = \frac{-y}{2}$$

- Lead them to do examples to emphasize the skills for finding equations of a line.
- Call student-teachers to do Application activity 9.3.1 to master the content.

Lesson 4: Vector equation, parametric equation, Cartesian equation given gradient

a) Learning objective

Determine equations of a straight line.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) Prerequisites/Revision/Introduction

Student-teachers should have knowledge and skills on how to draw axes, graphs in cartesian plane and how to plot shapes in 2D.

d) Learning activities

- Ask learners in small groups to read and discuss on the **activity 9.3.2** to plot points and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling learners.
- Call student-teachers to present the findings, and help them to harmonize the answer.

Answer of activity 9.3.2

Using **slope-intercept form** of equation of line $y = mx + c$, then

a) $y = \frac{3}{2}x - 2 \rightarrow slope = \frac{3}{2}$ and y - intercept $= -2$

b) $y = -3x + \frac{5}{2} \rightarrow slope = -3$ and y - intercept $= \frac{5}{2}$

- Let student-teachers discover that in 2D, the equation of the line in **point-slope form** is defined by

$$y - y_1 = m(x - x_1).$$

From the equation above, we take any other point $P_2(x_2, y_2)$ lies on this line, then

Vector equation

$$\overrightarrow{OP} = \overrightarrow{OP_1} + r\overrightarrow{P_1P_2} \text{ or } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + r\begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix},$$

$\overrightarrow{P_1P_2}$ is the direction vector.

Parametric equations

$$\begin{cases} x = x_1 + r(x_2 - x_1) \\ y = y_2 + r(y_2 - y_1) \end{cases}$$

Cartesian equation

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

- Lead them to do examples to emphasize the skills for finding equations of a line.
- Call student-teachers to do Application activity 9.3.2 to master the content.

a) **Application activity 9.3.2**

Answers of Application activity 9.3.2

1. a) $y+4=6(x-3)$ b) $y+7=-\frac{3}{2}(x+2)$ c) $y-2=0(x+5)$

d) $y-2=-\frac{5}{3}(x-4)$ e) $y-0=1(x-4)$ f) $y+8=-\frac{1}{5}(x-1)$

2. Yes, because the point verifies the equation.

3. $y=7x+16$

4. a) $y+6=m(x+4)$ b) it has infinity number of equations because m is a variable.

5. Perpendicular lines: a) f) c)

Parallel lines: b) e) g)

Lesson 5: Cartesian equation of a circle

a) **Learning objective**

Find the centre, radius, and diameter from the equation of a circle.

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

Student-teachers should have known how to draw axes , graph in cartesian plane and how to plot shapes in 2D.

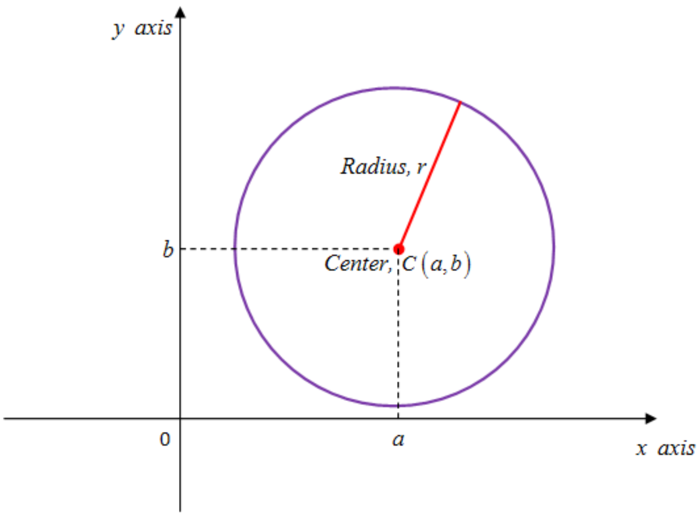
d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 9.4.1** to plot points and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling learners.

- Call student-teachers to present the findings, and help them to harmonize the answer.

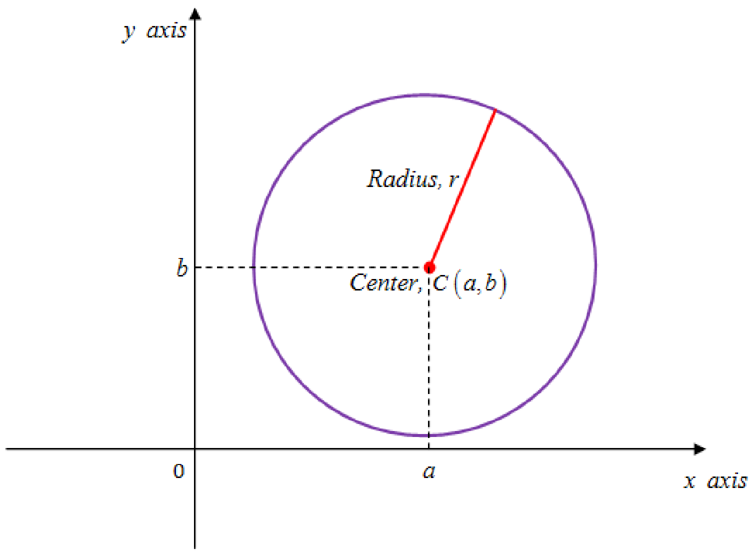
Answer of activity 9.4.1

Answers



Square length of line segment CP is given by: $(x-a)^2+(y-b)^2=r^2$

- Let student-teachers discover that in 2D, the circle is the locus of all points (x,y) which are equidistant from a fixed given point and the fixed point is called the **centre** of the circle while the distance from the centre to that point is called the **radius** of the circle.



- Help student-teachers to know that Cartesian equation of the circle with centre $C(a,b)$ and radius r is given by $(x-a)^2+(y-b)^2=r^2$
- Lead them to expand this equation to get this form $x^2+y^2-2ax-2by+a^2+b^2=r^2$.
- Help student-teachers to know that Cartesian equation of the circle with centre $C(a,b)$ and radius r is C $x^2+y^2+kx+ly+m=0$ for $k=-2ak=-2a$, $l=-2bl=-2b$ and $m=a^2+b^2-r^2$ $m=a^2+b^2-r^2$.

Noted: In this case the centre is given by $C=\left(-\frac{k}{2},-\frac{l}{2}\right)$ and the radius is given by $r=\frac{1}{2}\sqrt{k^2+l^2-4m}$.

- Help student-teachers to know that, if the coordinates of the point CC are $(0,0)(0,0)$, then the centre is at **origin.** In this case the equation of the circle is given by $x^2+y^2=r^2x^2+y^2=r^2$.
- Lead them to do the given examples to emphasize the skills for finding equation of a circle.
- Call student-teachers to do Application activity 9.4.1 to master the content.

e) **Application activity 9.4.1**

Answers of application activity 9.4.1

1. $x^2+y^2-6x-4y-3=0$
2. $x^2+y^2+4x-4y+4=0$

   Or $x^2+y^2-6x+4y-51=0$

3. Centre: $(2,-4)$, radius: $4\sqrt{2}$

4. Centre: $(3,2)$, radius: $\sqrt{22}$

Lesson 6: Parametric equations of a circle

a) **Learning objective**

Find the centre, radius, and diameter from the equation of a circle.

b) **Teaching resources**

Student-teachers's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

Student-teachers should have already know how to draw axes, graphs and to plot shapes in cartesian plane.

d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 9.4.2** to plot points and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling learners.
- Call student-teachers to present the findings, and help them to harmonize the answer.

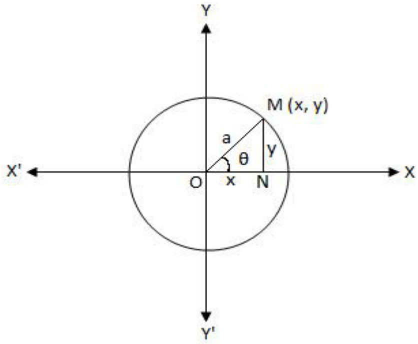
Answer of activity 9.4.2

$x^2+y^2=19$

a) $(x-0)^2+(y-0)^2=\left(\sqrt{19}\right)^2 \rightarrow r=\sqrt{19}$ and $C(0,0)$

b) $\begin{cases} x=r\cos\theta \\ y=r\sin\theta \end{cases} \rightarrow \begin{cases} x=\sqrt{19}\cos\theta \\ y=\sqrt{19}\sin\theta \end{cases}$ for $0\leq\theta\leq 2\pi$

- Let student-teachers discover that in 2D, a circle with **centre at the origin** and radius a, the equation of a circle is given $x^2 + y^2 = a^2$ or $x^2 + y^2 = r^2$; $(r = a)$

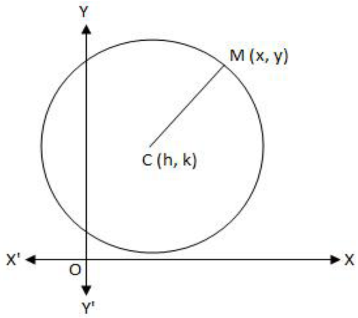


Expressing the equation of a circle using a parameter θ, we write parametric equations

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \text{ for } 0 \leq \theta \leq 2\pi$$

For a circle with centre at the point (h, k) and radius r, the equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$



Hence, we write parametric equations

$$\begin{cases} x = h + r\cos\theta \\ y = k + r\sin\theta \end{cases} \text{ for } 0 \leq \theta \leq 2\pi$$

- Lead them to do provided examples to emphasize the skills for finding equation of a circle.
- Call learners to do Application activity 9.4.2 to master the content.

e) **Application activity 9.4.2**

Answers of application activity 9.4.2

1. a) $x^2 + y^2 - 25 = 0$

$\begin{cases} x = 5\cos\theta \\ y = 5\sin\theta \end{cases}$ for $0 \leq \theta \leq 2\pi$

b) $x^2 + y^2 - 8x + 4y - 44 = 0$

$\begin{cases} x = 4 + 8\cos\theta \\ y = -2 + 8\sin\theta \end{cases}$ for $0 \leq \theta \leq 2\pi$

c) $r = \sqrt{(1+4)^2 + (3+2)^2} = 5\sqrt{2}$ then $x^2 + y^2 + 8x + 2y - 30 = 0$

$\begin{cases} x = 4 + 5\sqrt{2}\cos\theta \\ y = 2 + 5\sqrt{2}\sin\theta \end{cases}$ for $0 \leq \theta \leq 2\pi$

2. (1) a) $(x-5)^2 + (y-4)^2 = 6^2$

b) $C(5,4)$ c) $r = 6$

d) $\begin{cases} x = 5 + 6\cos\theta \\ y = 4 + 6\sin\theta \end{cases}$ for $0 \leq \theta \leq 2\pi$

(2) a) $(x-3)^2 + (y-4)^2 = 0^2$

b) $C(3,4)$ c) $r = 0$

d) no parametric equations

9.6. Summary of the unit

- In 2D, a point has paired coordinates (x, y) and in terms of vector, P has position vector $\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix}$. The first number x is called the x-coordinate (or x-component), while the second number y is called the y-coordinate (or y-component. The intersection point $(0,0)$ of the two axes is called **the origin.**

- If A and B are two points we can form a vector $\overrightarrow{AB}$ and the distance between these two points denoted $d(A,B)$ is given by $\|\overrightarrow{AB}\|$. Thus, if $A(a_1,a_2)$ and $B(b_1,b_2)$ are points of plane then $d(A,B)=\|\overrightarrow{AB}\|=\sqrt{(b_1-a_1)^2+(b_2-a_2)^2}$

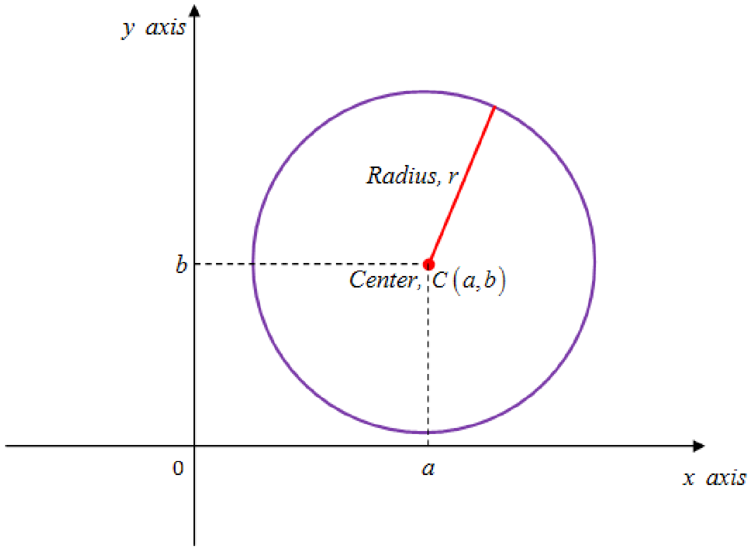
- In 2D the vector equation of the line passing through two points $A(x_A,y_A)$ *and* $B(x_B,y_B)$ and the direction vector can be defined as: $(x,y)=\overrightarrow{0A}+r\overrightarrow{AB}$ or $\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x_A \\ y_A \end{pmatrix}+r\begin{pmatrix} x_B-x_A \\ y_B-y_A \end{pmatrix}$, where $\overrightarrow{AB}$ is the direction vector.

- In 2D the parametric equation of the line passing through two points $A(x_A,y_A)$ *and* $B(x_B,y_B)$ and the direction vector can be defined as: $\begin{cases} x=x_A+r(x_B-x_A) \\ y=y_A+r(y_B-y_A) \end{cases}$

- In 2D the Cartesian equation of the line passing through two points $A(x_A,y_A)$ *and* $B(x_B,y_B)$ and the direction vector can be defined as: $\frac{x-x_A}{x_B-x_A}=\frac{y-y_A}{y_B-y_A}$

- Consider a line having gradient m and passing through the point $P_1(x_1,y_1)$. Suppose that the point $P(x,y)$ is an other point on the line. Then the gradient of the line is defined by the change in y to the change in x and $m=\frac{y-y_1}{x-x_1}$. Thus, the equation of the line in point-slope form is defined by $y-y_1=m(x-x_1)$

- The circle is the locus of all points (x,y) which are equidistant from a fixed given point. The fixed point is called the **centre** of the circle and the distance from the centre to ant point is called the **radius** of the circle.



- The equation of the circle with centre $C(a,b)$ and radius r is given by $(x-a)^2+(y-b)^2=r^2$
- If the coordinates of the point C are $(0,0)$, then the centre is at origin. In this case the equation of the circle is given by $x^2+y^2=r^2$.
- Expressing the equation of a circle using a parameter θ, we write parametric equations

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \text{for} \quad 0 \le \theta \le 2\pi.$$

- For a circle whith centre at the point (h,k) and radius r, the equation of a circle

$(x-h)^2+(y-k)^2=r^2$.Hence, we write parametric equations

$\begin{cases} x = h + r\cos\theta \\ y = k + r\sin\theta \end{cases}$ for $0 \le \theta \le 2\pi$. Noted: the angle θ is given by

$\begin{cases} \cos\theta = \frac{x-h}{r} \\ \sin\theta = \frac{y-k}{r} \end{cases}$ with $C(h,k)$ and r.

$$\begin{cases}\cos\theta = \frac{x}{r}\\ \sin\theta = \frac{y}{r}\end{cases}$$ with $C(0,0)$ and r.

9.7. Additional Information for Tutors

- Using the canonical basis $\{\vec{i}=(1,0), \vec{j}=(0,1)\}$ the equation of the line can be rewritten as $x\vec{i}+y\vec{j}=x_0\vec{i}+y_0\vec{j}+r(a\vec{i}+b\vec{j})$
- $\{\vec{i}=(1,0), \vec{j}=(0,1)\}$ are unit vectors and form a basis of IR^2.
- $\vec{i}$ is on $x-axis$ while $\vec{j}$ is on $y-axis$.
- The parameter $\theta\theta$ in parametric equations of a circle is given by

v $\begin{cases}\cos\theta = \frac{x-h}{r}\\ \sin\theta = \frac{y-k}{r}\end{cases}\begin{cases}\cos\theta = \frac{x-h}{r}\\ \sin\theta = \frac{y-k}{r}\end{cases}$ with $C(h,k)C(h,k)$ and rr.

v $\begin{cases}\cos\theta = \frac{x}{r}\\ \sin\theta = \frac{y}{r}\end{cases}\begin{cases}\cos\theta = \frac{x}{r}\\ \sin\theta = \frac{y}{r}\end{cases}$ with $C(0,0)C(0,0)$ and rr.

- A circle that has radius $r=0$, is called **point circle.**
- A circle that has radius $r=1$, is called **unit circle.**

9.8. End unit assessment

Answers of End unit assessment

1. a) $6\sqrt{2}$ b) 6 c) $\sqrt{41}$ d) $\sqrt{x^2+y^2}$

2. a) $C(0,0);\mathrm{r}=\sqrt{10}$ b) $(3,0);r=5$ c) $(2,-1);r=3\sqrt{2}$ d) $(-1,1);r=0$

4. $(2,11);r=\sqrt{10}$ then equation of the line $(x-2)^2+(y-11)^2=(\sqrt{10})^2$

9.9. Additional activities

9.9.1. Remedial activities

1. State the equation of the circle that can satisfy the given conditions. Hence, write parametric equations for each.

    a. $C(2,0); r = 5$

    b. $C(-1,-2); r = 7$

    c. $C(4,2)$and passes through $P(1,3)$

2. Write the equation in point-slope form for the line through the given point that has the given slope

    a) $(0,2); m = \frac{4}{5}$

    b) $(5,-8); m = -3$

    c) $(-6,1); m = \frac{2}{3}$

    solution

    a) $y-2=\frac{4}{5}(x-0)$ b) $y+8=-3(x-5)$ c) $y-1=\frac{2}{3}(x+6)$

3. Given the equations of a circle

    (1) : $x^2 + y^2 = 100$

    (2) $x^2 + 2x + y^2 = 4$

    a) Express each equation below in the form

    $(x-a)^2 + (y-b)^2 = r^2$.

    b) Write the coordinates of centre of the circle.

    c) What is the radius of the circle?

    d) Find parametric equations.

4. State which of the following are equations of circles. Justify

    (a) $x^2 + y^2 = 49$

(b) $x^2 + y^2 = 9$ (d) $2x^2 + 3yx - y^2 + 2y = 16$

(c) $x^2 + 3xy - y^2 = 6$ (e) $x^2 + y^2 - 2x + 2y = 3$

9.9.2. Consolidation activities

1. Determine the centre and radius of the following circles. Write parametric equations for $4x^2 + 4y^2 + 4x = 99$

2. Find the vector equations of the lines whose Cartesian equations are given below

a) $\frac{x-5}{1} = \frac{y+3}{-4}$

b) $3y = 2x - 4$

9.9.3. Extended activities

1. The points A, B and C have coordinates $(2,1),(7,3)$, *and* $(5,k)$ respectively. If AB and BC are of equal length, find the possible values of k.

Solution

$k = 8$ or $k = -2$

2. Determine the centre and radius of the following circles. Write parametric equations for. $2x^2 + 2y^2 - 2x + 2y = 1$

1. Find the Cartesian equations of the lines whose vector equations are given below. Give your answers in the form $y = mx + c$

a. $(x, y) = (3,2) + r(1,4)$ $x\vec{i} + y\vec{j} = 4\vec{i} - 5\vec{j} + r(2\vec{i} + 3\vec{j})$

UNIT 10

CENTRAL TENDENCES AND DISPERSION

10.1. Key unit competence

Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to the standard deviation.

10.2. Prerequisite

Student-teachers will easily learn this unit, if they remember the notions of statistics seen in ordinary level

10.3. Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others view and thoughts during class discussions)
- Gender (equal opportunity of boys and girls in the lesson participation)
- Standardization culture (regulation of the current intensity)

10.4. Guidance on introductory activity

- In small groups or pairs, let student-teachers read and do the introductory activity in the student-teachers' book.
- Facilitate Student-teachers to think on different ways of getting solutions.
- Through question-answer, facilitate student-teachers to understand how statistics is important /used in everyday life.
- After a given time invite student-teachers to present their findings and harmonize them.
- From presentations, the tutor decides to engage the class into discussions that help to the introduction of the unit.

Answers for the introductory activity:

The table below shows the types and the number of sold fruits in one week.

Type of fruit	A (Banana)	B (Orange)	C (Pinneple)	D (Avocado)	E (Mango)	F (apple)
Number of fruits sold	1100	962	1080	1200	884	900

The highest number of fruits sold is 1200 (Avocadoes)

b) The least number of fruits sold is 884 (mangoes)

c) The total number of fruits sold during the week is 6126 fruits

d) The average number of fruits sold per day is $\frac{6126}{6} = 1021$

2.

a) The mean mark of the class is $\frac{3+5+6+3+8+7+8+4+8+6}{10} = \frac{58}{10} = 5.8$.

b) The mark that was obtained by many students is 8

c) Comparing the mean mark of the class and the mark for every student-teacher, one can find that 4 students have the marks (3 , 4 and 5) below the mean, 2 students scored the mark near the mean while 4 students have scored higher marks than the mean. Mathematics tutor should prepare remedial activities for students whom their marks are below and near the mean.

10.5. List of lessons

#	Lesson title	Learning objectives	Number of periods
1	Introductory activity	To arouse the curiosity of student teachers on the content of unit 10	1
2	Collection and presentation of ungrouped data	Calculate and interpret Measures of central tendency	2
3	Collection and presentation of grouped		2

3	Central tendencies (mean, median, mode)	Determine the measures of dispersion of a given statistical series.	3
4	Graphical representation of grouped and ungrouped data	Interpret critically data and infer conclusions.	4
5	Measure of dispersion (range, variance, Standard Deviation and coefficient of variation)	• Define the variance, standard deviation and the coefficient of variation. • Express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.	6
6	Application of statistics in daily life	• Apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.	4
7	End unit assessment.		2

Lesson 1: Collection and presentation of ungrouped data

a) Learning objective

Calculate and interpret measures of central tendency.

b)Teaching resources

Student book's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils,

c) Prerequisites/Revision/Introduction

Student-teachers should have knowledge and skills on how to:

- Define quantitative data and qualitative data.
- Differentiate discrete and continuous data.
- Present data on a frequency distribution.

- Apply data collection to carry out a certain research.

d) Learning activities

- Ask student-teachers in small groups to read and discuss on the **activity 10.1.1** to complete frequency distribution table
- Facilitate working, especially the special need education.
- Call student-teachers to present the findings, and help them to harmonise the answer.

1. **Answer of activity 10.1.1**

Age	15	16	17	18	19	20	21	22	23	24
Number of student-teachers	2	3	1	2	1	1	2	2	0	1

1.

Cathegorical data	Numerical data
Arm span Dominant hand reaction time Favourite sport Language mostly spoken at home Opinions on environmental conservation State/Territory live in Travel method to school Year level	Amount of money earned last week Birthdate Height Hours slept per night Foot length School post code Travel time to school

- Lead student-teachers to realize that the solutions contain the concept of statistical data.
- Let student-teachers discover that everyday life, people come across a wide variety of information in form of facts or **categorical data** and **numerical data** in form of tables.
- Lead them to know definitions of some terms those we are familial with in statistics such as:

Statistics: the branch of mathematics that deals with the collection, presentation, interpretation and analysis of data.

Qualitative data

Qualitative data is a categorical measurement expressed not in terms of numbers, but rather by means of a natural language description.

Example of qualitative data	Possible categories variable
• Marital status	• Single, married, divorced
• Gender	• Male, Female
• Pain level	• None, moderated, severe
• Colour	• Red, black, green, yellow

Quantitative data

Quantitative data is a numerical measurement expressed not by means of a natural language description, but rather in terms of numbers.

Discrete data represent items that can be counted; they take on possible values that can be listed out.

Continuous data represent measurements; their possible values cannot be counted and can only be described using intervals on the real number line.

Raw data

Data which have been collected in **original form**, they are called **raw data**

Frequency distribution

A frequency distribution is a table showing how often each value (or set of values) of the collected data occurs in a data set. A frequency table is used to summarize categorical or numerical data. Data presented in the form of a frequency distribution are called grouped data.

Cumulative frequency

The cumulative frequency corresponding to a particular value is the sum of all frequencies up to the last value including the first value. Cumulative frequency can also have defined as the sum of all previous frequencies up to the current point.

STEM AND LEAF DISPLAYS

Is a plot where each data value is split into a leaf usually the last digit and a stem the other digit. The stem values are listed down, and the leaf values are listed next to them.

- Facilitate them to organise data in frequency distribution table.
- Lead them in small groups to do the provided examples in student-book.
- Guide them to do **application activity 10.1.1** ,to master the content.

d) Application activity 10.1.1

Answer of Application activity 10.1.1

1.Below is frequency distribution table

Age	8	9	10	11	12	13	14	15	16	17	18
Number of student-teachers	2	9	11	8	2	2	1	3	4	4	4

2. Below is the table that shows some examples of qualitative and quantitative data

Qualitative data	Quantitative data
• Product rating, • basketball team classification.	• Number of student-teachers in the classroom, • Weight, • Age, • Number of rooms in a house, • Number of teachers in school.

Lesson 2: Collection and presentation of grouped data

a)**Learning objective**

Calculate and interpret Measures of central tendency

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils,…

c) **Prerequisites/Revision/Introduction**

Student-teachers should have knowledge and skills on how to:

- Define quantitative data and qualitative data.
- Differentiate discrete and continuous data.
- Apply data collection to carry out a certain research.
- Represent statistical information using frequency distribution tables,
- Interpret correctly graphs involving statistical data
- Develop research and creativity.

d) **Learning activities**

- Ask student-teachers in small groups/individual to read and discuss on the **activity 10.1.2** and make sure that everybody is engaged/ involved.
- Facilitate working, especially the special need education student-teachers.
- Call student-teachers to present the findings, and help them to harmonise the answer.

Answer of activity 10.1.2

The table below shows a frequency distribution table of mass of tomatoes.

Mass(g)	84.5-89.5	89.5-94.5	94.5-99.5	99.5-104.5	104.5-109.5	109.5-114.5	114.5-119.5
Number of tomatoes	4	7	6	13	10	5	5

- Let student-teachers discover that when the range of data is large, the data must be grouped into classes that are more than one unit in width. In this case a grouped frequency distribution is used. Data in this case are grouped in a frequency distribution using groups **or classes.**

  Class limits: The class limits are the lower and upper values of the class

  Lower class limit: Lower class limit represents the smallest data value that can be included in the class.

  Upper class limit: Upper class limit represents the largest data value that can be included in the class.

  $$\textit{class midpoint} = \frac{\textit{lower class limit} + \textit{upper class limt}}{2}$$

  Class boundaries: Class boundaries are the midpoints between the **upper class limit** of a class and **the lower class limit of the next class.** Therefore, each class has a lower and an upper class boundary.

- Through examples in student-teacher's book, lead student-teachers to organize data in **a grouped frequency distribution table.**
- Guide them to do **application activity 10.1.2** to master the content.

d) Application activity 10.1.2

Answers of application activity 10.1.2

The table below shows a frequency distribution table of distances are between department store and employees 'homes.

Distance(km)	1-3	4-6	7-9	10-12	13-15	16-18
Number of tomatoes	10	14	10	6	5	5

Lesson 3: Central tendencies : mean, median, mode

a) Learning objective

Determine the measures of dispersion of a given statistical series.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) Prerequisites/Revision/Introduction

Student-teachers should have knowledge and skills on how to

- Present data on a frequency distribution.
- Determine the mode, mean, and median of statistical data.
- Define grouped data and represent grouped data on a frequency distribution.
- Identify mode, middle class, modal class, and median of given grouped statistical data.
- Read diagram of grouped statistical data.

Learning activities

- Ask student-teachers in small groups to read and discuss on the **activity 10.2**
- Facilitate working, especially straggling student-teachers.
- Call student-teachers to present their findings, and help them to harmonize the answer.

1. **Answer of activity 10.2**

Marks (x)	Frequency (f)	f x
3	2	6
4	3	12
5	2	10
6	1	6
7	1	7
10	1	10
Total	10	41

i) $Mean = \frac{\sum xf}{\sum f} = \frac{41}{10} = 4.1$

ii) $Median = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2} = \frac{x_5 + x_6}{2} = \frac{4+5}{2} = 4.5$

iii) $Mode = 4$

iv) $Range = 10 - 3 = 7$

1. **a)** 3*marks* obtained by 29 student-teachers**.**

b) Mean marks is given by the sum of product of xf divided by sum of frequencies.

$$Mean = \frac{\sum xf}{\sum f}$$

- Lead student-teachers to realize that the solution of an activity help them to reach easily to the content.
- Help student-teachers discover that for **ungrouped data**;
  - **The mean** is given by $\bar{x} = \frac{1}{n}\sum xfi$,
  - **The mode** is the number that appears the most often from the set of data
  - **The median:** The middle number in arranged data from the smallest to the largest.

When $\sum fi = n$ is odd the median is given by

$$Me \to \left(\frac{n+1}{2}\right)^{th} \text{ or } Me = x_{\frac{n+1}{2}}$$

When n is even, the median is given by

$$Me = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right] \text{ or } Median = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$$

- Help student-teachers discover that for **grouped data**:

    - The mean is given by $\bar{x} = \frac{1}{n}\sum xfi$,
    - The mode is given by

    $$Mode = L_m + c\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)$$

    - **The median is given by**

    $$Median = L_m + c\left(\frac{\frac{n}{2} - CF_b}{f_m}\right)$$

- Facilitate them to determine the mean, mode, and median in both ungrouped and grouped data trough given examples in student-teacher book.
- Call student-teachers to do Application activity 10.2 to master the content.

d) Application activity 10.2

Answers Application activity 10.2

1. $Mean = 3.48$ $Mode = 4$ $Median = 3.5$

2. $Mean = 8.2$ $Mode = 10$ $Median = 4$

- Lead them to do the provided examples in student-teacher's book to emphasize the skills for finding equations of a line.

- Call student-teachers to do Application activity 10.2 to master the content.

Lesson 4: Graphical representation of grouped and ungrouped data

a) **Learning objective**

Interpret critically data and infer conclusions.

b) **Teaching resources**

Learner's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

Student-teachers should have knowledge and skills on how to

- Apply data collection to carry out a certain research.
- Represent statistical ungrouped and grouped information using frequency distribution tables,
- Read diagram of grouped statistical data,

d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 10.3**
- make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling student-teachers.
- Call student-teachers to present the findings, and help them to harmonise the answer.

Answer of activity 10.3

1. *the number of students with small is 4*

   the number of students with medium is 5

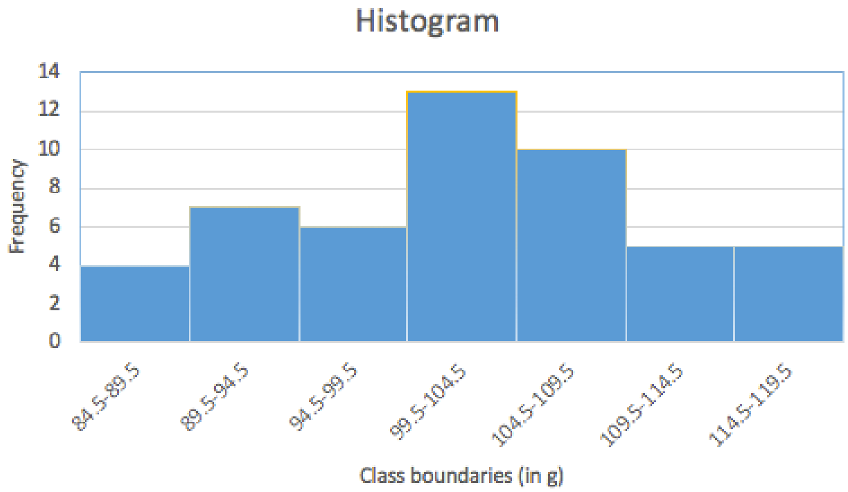
   the number of students with large is 8

   the number of students with extra-large is 13.
2. *the graph is a Bar chart*

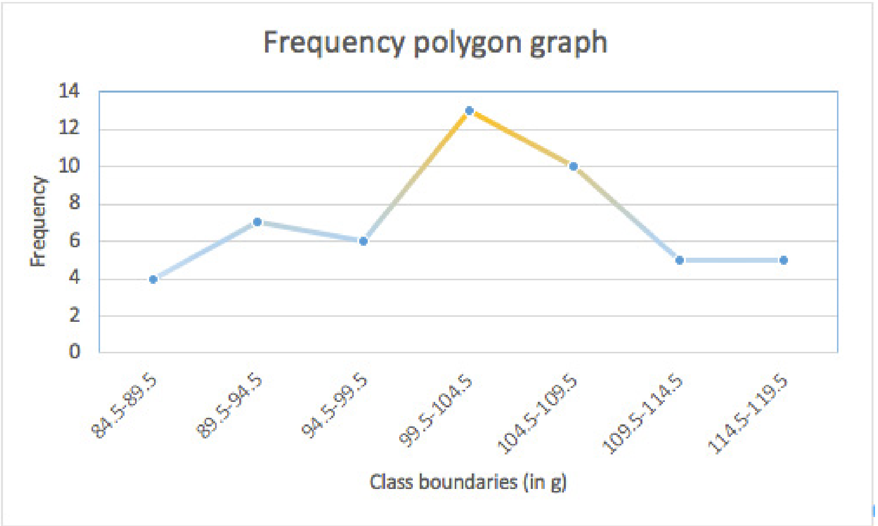
a) The table below shows cumulative frequency.

Class boundaries (in g)	84.5-89.5	89.5-94.5	94.5-99.5	99.5-104.5	104.5-109.5	109.5-114.5	114.5-119.5
Frequency	4	7	6	13	10	5	5
Cumulative frequency	4	11	17	30	40	45	50

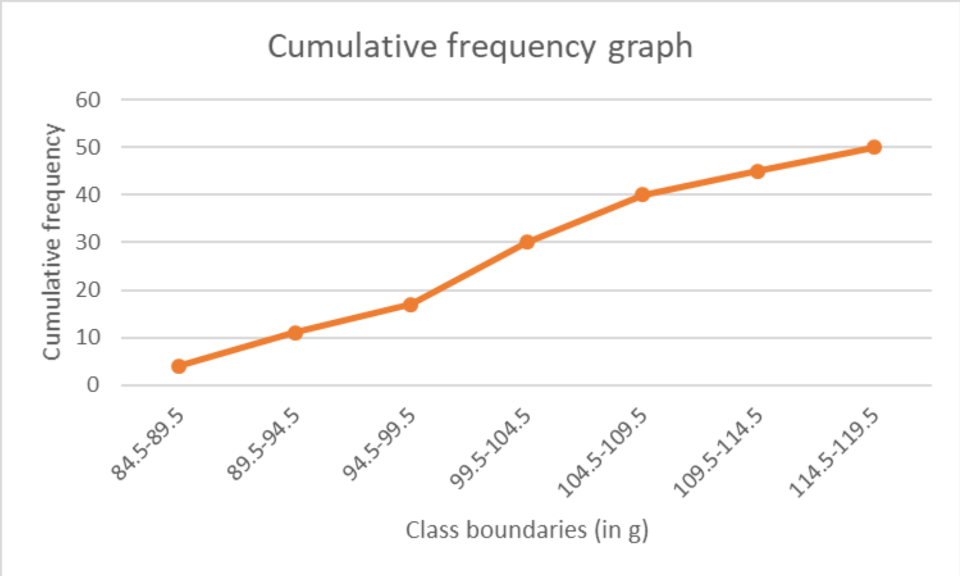
b) i) Histogram (graph)



ii)Frequency polygon (graph)



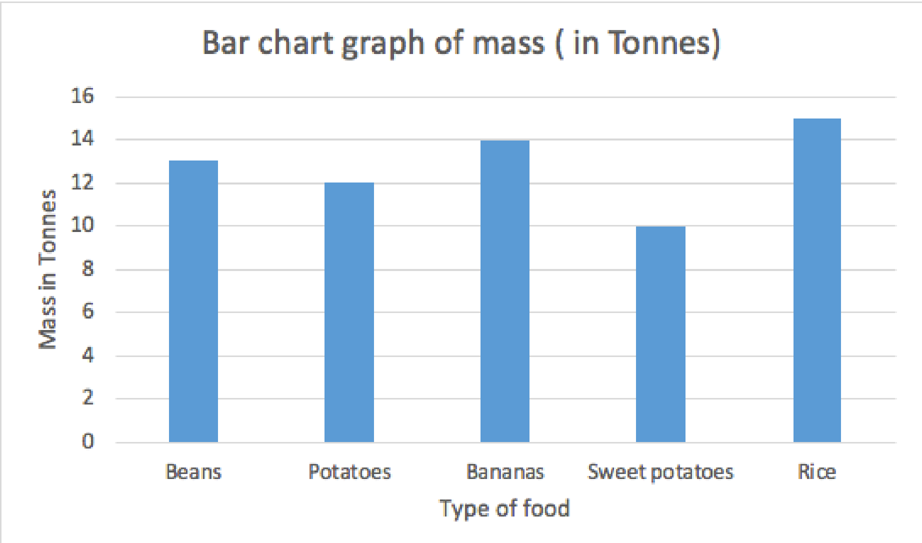
iii) Cumulative frequency polygon (graph)



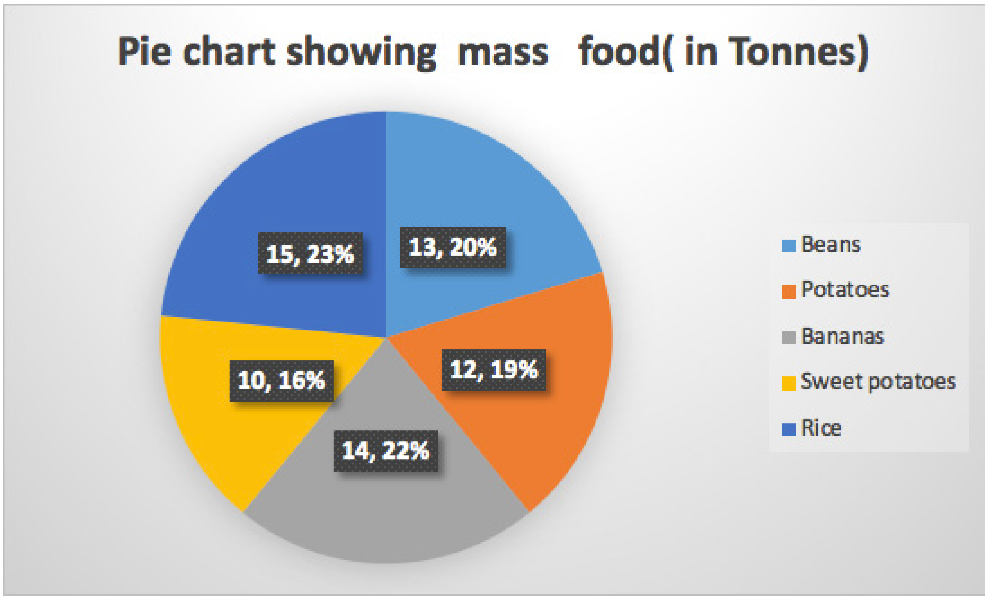
3.Frequency distribution table

Type of food	Beans	Potatoes	Bananas	Sweet potatoes	Rice
Mass (Tonnes)	13	12	14	10	15

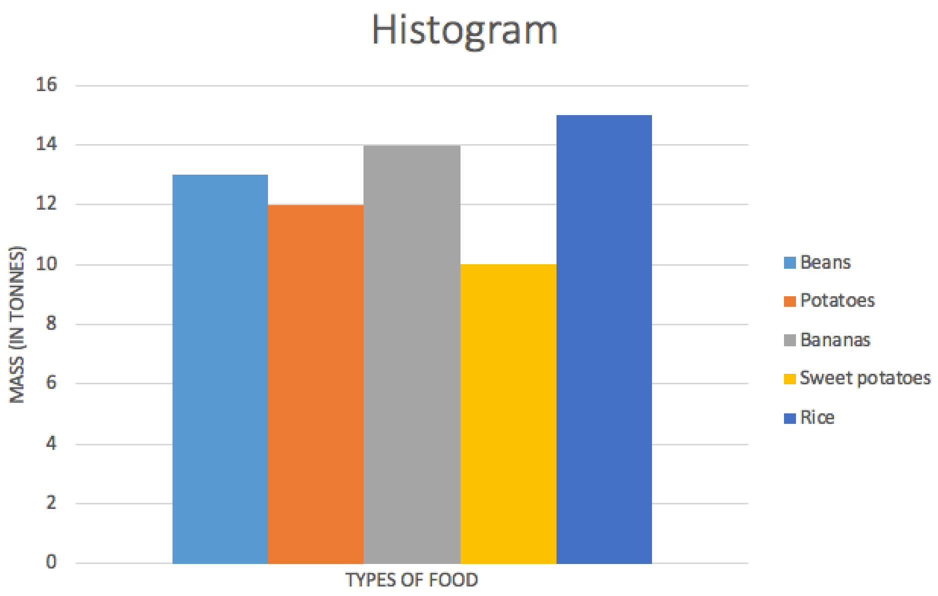
- Through examples given in student-teacher's book, guide student-teachers to know how accurately draw
- Bar chart,



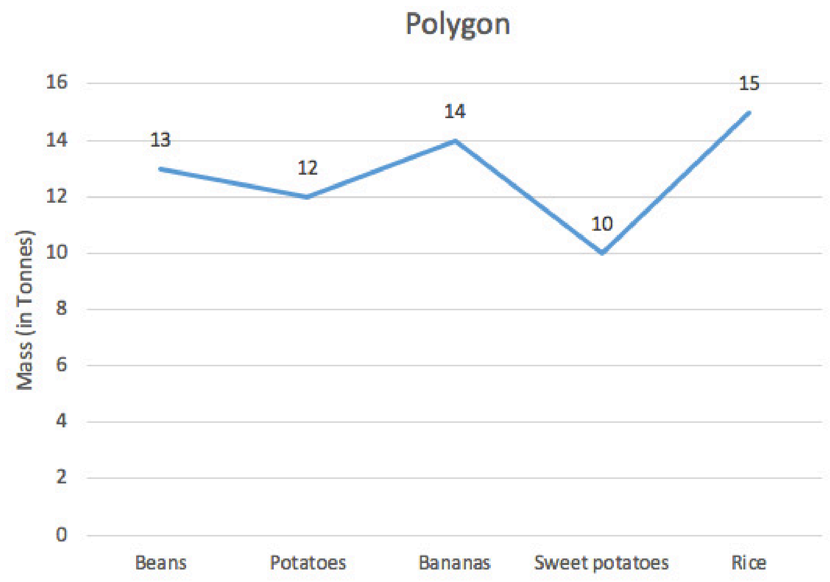
- Pie chart,



- Histogram,



- Polygon,

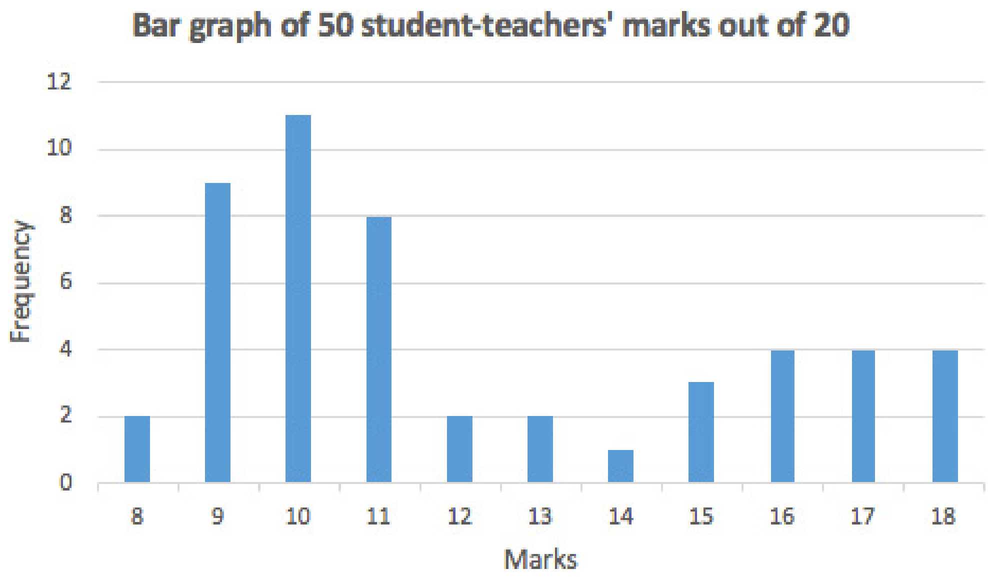


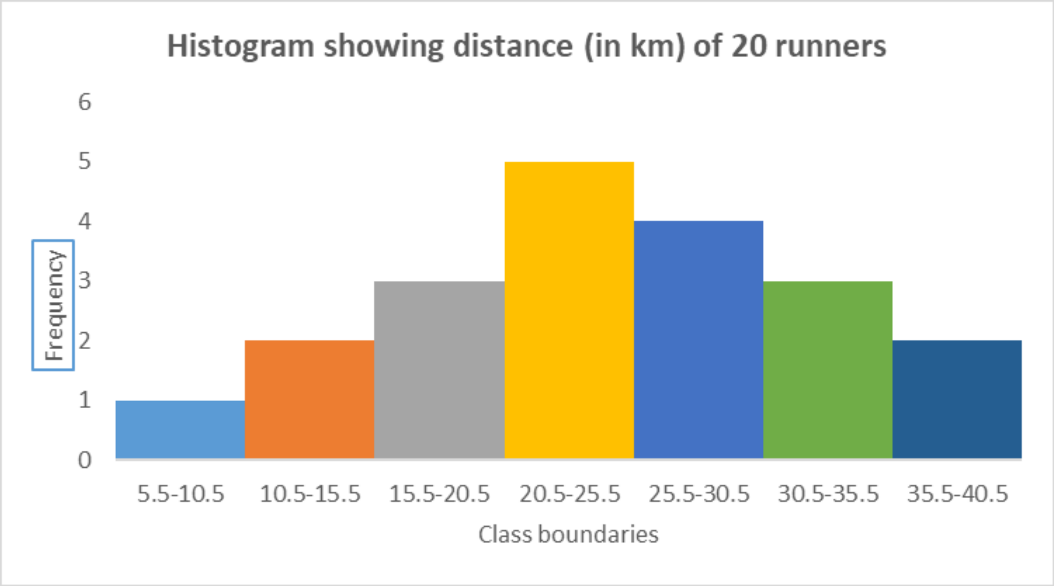
- Call student-teachers to do Application activity 10.3 to master the content.

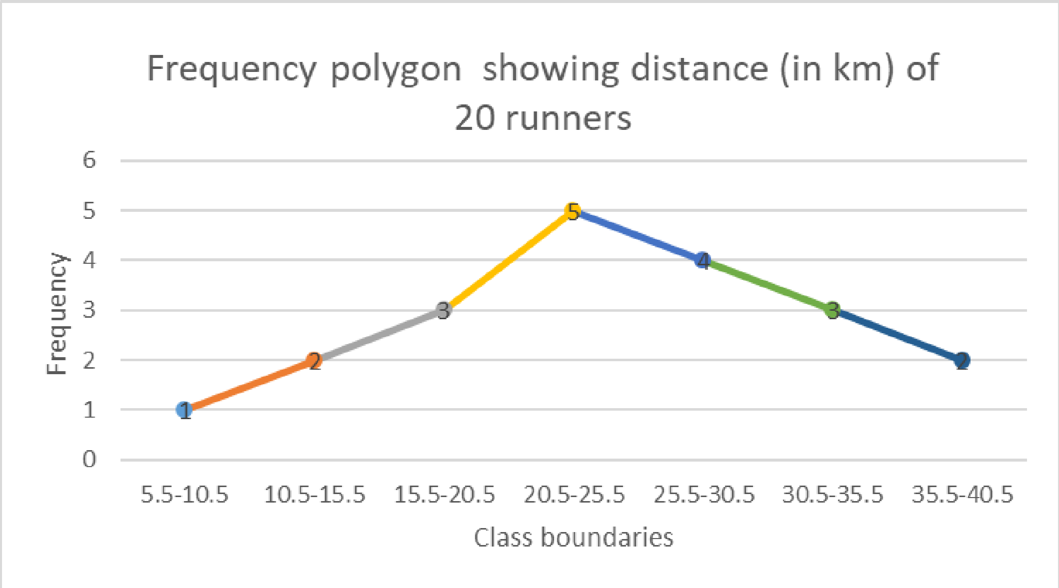
a) **Application activity 10.3**

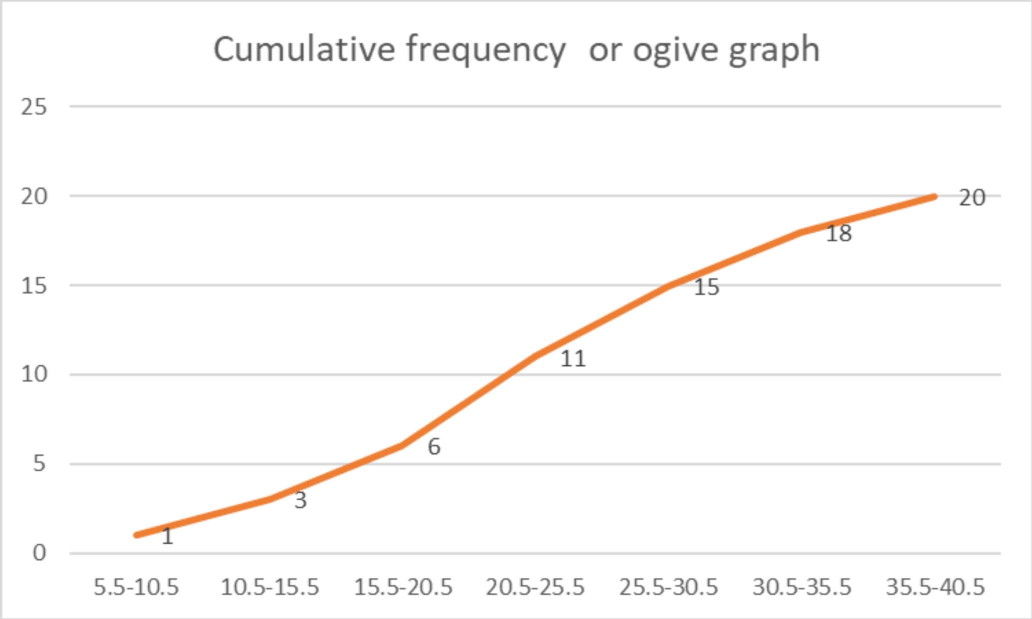
Answers of Application activity 10.3

1. **Bar graph of 50 student-teachers' marks out of 20**




Histogram showing distance (in km) of 20 runners
Frequency
0
1
2
3
4
5
6
5.5-10.5
10.5-15.5
15.5-20.5
20.5-25.5
25.5-30.5
30.5-35.5
35.5-40.5
Class boundaries


Frequency polygon showing distance (in km) of 20 runners
Frequency
0
1
2
3
4
5
6
5.5-10.5
10.5-15.5
15.5-20.5
20.5-25.5
25.5-30.5
30.5-35.5
35.5-40.5
Class boundaries



Lesson 5: Measure of dispersion: range, variance, Standard Deviation and coefficient of variation

a) Learning objective

- Define the variance, standard deviation and the coefficient of variation.
- Express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.

b) Teaching resources

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) Prerequisites/Revision/Introduction

Student-teachers should have knowledge and skills on how to

- Represent statistical information using frequency distribution tables,
- Determine the mode, mean, and median of statistical data.
- Define ungrouped data and represent ungrouped data on a frequency distribution.

d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 10.4 given in** Student-teacher's book and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling student-teachers.
- Call student-teachers to present the findings, and
- Help them to harmonize their answers.

Answer of activity 10.4

For $\bar{x} = 16.875$

x	f	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$	$f\left(x-\bar{x}\right)^2$
12	4	−4.875	23.765625	95.0625
13	2	−3.875	15.015625	30.03125
15	1	−1.875	3.515625	3.515625
19	4	2.125	4.515625	18.0625
21	5	4.125	17.015625	85.078125
	$\sum f = 16$			$\sum f\left(x-\bar{x}\right)^2 = 231.75$

- Help student-teachers how to calculate quartiles

$$Q_1 = \frac{1}{4}(n+1)^{th} \text{ or } Q_1 = x_{\frac{n+1}{4}}. \qquad Q_2 = \frac{1}{2}(n+1)^{th} \text{ or } Q_2 = x_{\frac{n+1}{2}} = Me$$

$$Q_3 = \frac{3}{4}(n+1)^{th} \text{ or } Q_3 = x_{\frac{3(n+1)}{4}}$$

The inter-quartile range is given by the difference between the third quartile and the first quartile.

- The variance is denoted and defined by

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$$

Developing this formula, we have

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i^2 - 2x_i\bar{x} + \left(\bar{x}\right)^2\right)}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n}2\bar{x}\sum_{i=1}^{n} x_i + \frac{1}{n}\left(\bar{x}\right)^2\sum_{i=1}^{n}1$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\bar{x}\bar{x} + \left(\bar{x}\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2$$

Thus, the variance is also defined by

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2$$

Recall that the mean of the set of n values $x_1, x_2, x_3, ..., x_n$ is denoted and defined by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

$$= \sum_{i=1}^{n}\frac{x_i}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i$$

- The standard deviation is denoted and defined by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}} \text{ or } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2}$$

Coefficient of variation

The coefficient of variation is a calculation built on other calculations: the standard deviation and the mean as follows:

$$Cv = \frac{\sigma}{\bar{x}} \times 100$$

Where:

- σ is the standard deviation
- $\bar{x}$ is the mean.

Range

In the case of ungrouped data, the range of a set of observations is the difference in values between the largest and the smallest observations in the set.

In the case of grouped data, the range is defined as the difference between the upper limit of the highest class and the lower limit of the smallest class.

- Lead them to do the provided examples in student-teacher's book.
- Call student-teachers to do Application activity 10.4 to master the content.

e) **Application activity 10.4**

Answers of **Application activity 10.4**

1. $Me = \frac{64+49}{2} = 56.5$

2. For $\overline{X} = \frac{54+55+55+56+57+58+59}{7} = 56.285$

x	f	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
54	1	−2.285	5.221225	5.221225
55	2	−1.285	1.651225	3.30245
56	1	−0.285	0.081225	0.081225
57	1	0.715	0.511225	0.511225
58	1	1.715	2.941255	2.941255
59	1	2.715	7.371225	7.371225
	$\sum f = 7$			$\sum f(x-\bar{x})^2 = 19.428575$

Therefore, standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n}}$$

$$\sigma = \sqrt{\frac{19.428575}{7}} \simeq 1.67$$

3.

a) $\overline{X} = 5.8$ $Mode = 5$ $Median = 5.5$

b) $Q_1 = 5$ $Q_2 = 5.5$ $Q_3 = 8.25$

c) $Variance = 3.76$ $\sigma = 1.94$ $C.v = 33.45$

Lesson 6: Application of statistics in daily life

a) **Learning objective**

Apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.

b) **Teaching resources**

Learner's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

Student-teachers should have knowledge and skills on how to

- Apply data collection to carry out a certain research.
- Represent statistical information using frequency distribution tables, bar charts, histograms, polygons, pie charts, or pictogram.
- Determine the mode, mean, and median of statistical data.
- Interpret correctly graphs involving statistical data
- Develop research and creativity.
- Read diagram of grouped statistical data

d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 10.5** to plot points and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling student-teachers.
- Call student-teachers to present the findings, and help them to harmonize the answer.

Answer of activity 10.5

- Let student-teachers make their own research and discover that there are different applications of statistics in real life.
- Let student-teachers do activity 10.5 to find out the usefulness of statistical terms like frequency distribution, Bar graph, Histogram, Frequency polygon, Mean.
- Let student-teachers do application activity 10.5 to find out the usefulness of statistical terms like frequency distribution, Pie chart, cumulative frequency polygon or ogive , Mode and median, Standard deviation .

1. **Importance of Mean, Mode, Median, Variance , standard deviation**

A measure of central tendency is a single value that attempts to describe a set of data by identifying the central position within that set of data. The mean, median and mode are all valid measures of central tendency, but under different conditions, some measures of central tendency become more appropriate to use than others. The mean is essentially a model of your data set. It is the value that is most common. You will notice, however, that the mean is not often one of the actual values that you have observed in your data set. However, one of its important properties is that it minimizes error in the prediction of any one value in your data set. That is, it is the value that produces the lowest amount of error from all other values in the data set. An important property of the mean is that it includes every value in your data set as part of the calculation. In addition, the mean is the only measure of central tendency where the sum of the deviations of each value from the mean is always zero

Standard Deviation is a statistical term used to measure the amount of variability or dispersion around an average. Technically it is a measure of volatility. Dispersion is the difference between the actual and the average value. The larger this dispersion or variability is, the higher is the standard deviation.

Finance and banking is all about measuring and managing risk and standard deviation measures risk (Volatility). Standard deviation is used by all portfolio managers to measure and track risk.

The standard deviation tells those interpreting the data, how reliable the data is or how much difference there is between the pieces of data by showing how close to the average all of the data is.

- *A low standard deviation means that the data is very closely related to the average, thus very reliable.*
- *A high standard deviation means that there is a large variance between the data and the statistical average, thus not as reliable.*
- *Standard deviation and variance* may be basic mathematical concepts, but they play important roles throughout the financial sector, including the areas of accounting, economics, and investing. In the latter, for example, a firm grasp of the calculation and interpretation of these two measurements is crucial for the creation of an effective *trading strategy*.
- *Standard deviation and variance are both determined by using the mean of the group of numbers in question. The mean is the average of a group of numbers, and the variance measures the average degree to which each number is different from the mean. The extent of the variance correlates to the size of the overall range of numbers—meaning the variance is greater when there is a wider range of numbers in the group, and the variance is lesser when there is a narrower range of numbers.*

  Some examples of situations in which standard deviation might help to understand the value of the data:

- *A class of students took a math test. Their teacher found that the mean score on the test was an 85%. She then calculated the standard deviation of the other test scores and found a very small standard deviation which suggested that most students scored very close to 85%.*
- *A class of students took a test in Language Arts. The teacher determines that the mean grade on the exam is a 65%. She is concerned that this is very low, so she determines the standard deviation to see if it seems that most students scored close to the mean, or not. The teacher finds that the standard deviation is high. After closely examining all of the tests, the teacher is able to determine that several students with very low scores were the outliers that pulled down the mean of the*

entire class's scores.

- *An employer wants to determine if the salaries in one department seem fair for all employees, or if there is a great disparity. He finds the average of the salaries in that department and then calculates the variance, and then the standard deviation. The employer finds that the standard deviation is slightly higher than he expected, so he examines the data further and finds that while most employees fall within a similar pay bracket, three loyal employees who have been in the department for 20 years or more, far longer than the others, are making far more due to their longevity with the company. Doing the analysis helped the employer to understand the range of salaries of the people in the department.*

2. **Importance of statistical graphs**

One goal of statistics is to present data in a meaningful way. That is where graphs can be invaluable, allowing statisticians to provide a visual interpretation of complex numerical stories.

Good graphs convey information quickly and easily to the user. Graphs highlight the salient features of the data. They can show relationships that are not obvious from studying a list of numbers. They can also provide a convenient way to compare different sets of data.

Different situations call for different types of graphs, and it helps to have a good knowledge of what types are available. The type of data often determines what graph is appropriate to use. Qualitative data, quantitative data, and paired data each use different types of graphs.

Some examples of graphs are the following:

- *Bar graph is a way to visually represent qualitative data. Data is displayed either horizontally or vertically and allows viewers to compare items, such as amounts, characteristics, times, and frequency.*
- *Pie chart is helpful when graphing qualitative data, where the information describes a trait or attribute and is not numerical. Each slice of pie represents a different category, and each trait corresponds to a different slice of the pie; some slices usually noticeably larger than others. By looking at all of the pie pieces, you can compare how much of the data fits in each category, or slice.*
- *A histogram* in another kind of graph that uses bars in its display. This type of graph is used with quantitative data. Ranges of values,

called classes, are listed at the bottom, and the classes with greater frequencies have taller bars.

- *A stem and leaf plot* breaks each value of a quantitative data set into two pieces: a stem, typically for the highest place value, and a leaf for the other place values. It provides a way to list all data values in a compact form.

Answer of application activity 10.5

1. Answers may vary with the group, as a tutor, try to harmonize them and provide feedback.

Examples: To analyse the scores of athletes in a given competition.

These scores may be represented using the frequency distribution table, the mean, median and standard deviation of the scores can be calculated respectively. One can explain the role of these measures

2. Answers will vary from group to another. Try to organize a session where every group will have time to present its findings and others will ask questions and provide constructive feedbacks for learning purpose.

10.6. Summary of the unit

- **Statistics** is a blanch of mathematics concerned with scientific method for collecting and presenting, organizing and summarizing and analyzing data as well as deriving valid conclusions and making reasonable decisions on the basis of this analysis.
- A sequence of observations, made on a set of objects included in the sample drawn from population, is known as **statistical data.** There are qualitative and quantitative or numerical data
- Numerical data can be further broken into discrete or continuous data.
- **Raw data:** Data which have been arranged in a systematic order are called raw data or ungrouped data.
- **Frequency distribution: A** frequency distribution is a table showing how often each value (or set of values) of the collected data occurs in a data set. A frequency table is used to summarize categorical or numerical data. Data presented in the form of a frequency distribution are called **grouped data**.
- **Cumulative frequency:** Cumulative frequency can also defined

as the sum of all previous frequencies up to the current point.

- **Stem and leaf displays:** Is a plot where each data value is split into a leaf usually the last digit and a stem the other digit. The stem values are listed down, and the leaf values are listed next to them.
- In case, the range of data is large, the data must be grouped into groups **or classes.**
- **Mean, mode, median** are the measures of central tendency

  The median: middle data/item of arranged or ordered data. If total observation ($\sum fi = n$) then $Me = \left(\frac{n+1}{2}\right)^{th}$ or $Me = x_{\frac{n+1}{2}}$ and read the number which located on this position. On the other side when n is even, $Me = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right]$ or $Median = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$ then the median is a half of the sum of number located on those two positions.
- **The mean formula of ungrouped data is the following:**

  $\bar{x} = \frac{1}{n}\sum xfi$
- **The mode**: The mode is the number that appears the most often from the set of data
- **The range:** In the case of ungrouped data, the range of a set of observations is the difference in values between the largest and the smallest observations in the set.
- **Formula for Mean, mode, median and range of grouped data are as follow:**

3.**Mean** is given by $\bar{x} = \frac{1}{n}\sum xfi$, where $n = \sum f$: the sum of frequencies of data.

4. $Mode = L_m + c\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)$, with L_m is lower boundary of modal class,

C is class width: the difference between upper and lower boundary of modal class ($C = U_m - L_m$), f_m : Frequency of modal class, $\Delta_1 = f_m - f_b$, $\Delta_2 = f_m - f_a$

f_b is frequency followed by f_m and f_a is frequency follows f_m.

5. $Median = L_m + C\left(\frac{\frac{n}{2} - CF_b}{f_m}\right)$, $n = \sum f$: the sum of frequencies of data,

CF_b: cumulative frequency preceded by cumulative frequency of modal class (cumulative frequency before modal class)

- The most commonly used graphs are: Bar graph, Pie chart, Histogram, Frequency polygon, Cumulative frequency graph or Ogive.
- **Some measures of dispersion** are Quartiles, variance, Range, standard deviation, coefficient of variation

$Q_1 = \frac{1}{4}(n+1)^{th}$ or $Q_1 = x_{\frac{n+1}{4}}$. $Q_2 = \frac{1}{2}(n+1)^{th}$ or $Q_2 = x_{\frac{n+1}{2}} = Me$

$Q_3 = \frac{3}{4}(n+1)^{th}$ or $Q_3 = x_{\frac{3(n+1)}{4}}$

- **The inter-quartile range**: is given by the difference between third quartile and the first quartile.
- Variance: Variance measures how far a set of numbers is spread out. A variance of zero indicates that all the values are identical. Variance is always non-negative: a small variance indicates that the data points tend to be very close to the mean and hence to each other, while a high variance indicates that the data points are very spread out around the mean and from each other. The variance is given by : $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$ or $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2$
- The standard deviation has the same dimension as the data, and hence is comparable to deviations from the mean. We define the **standard deviation** to be the square root of the variance. Thus, the standard deviation is denoted and defined by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}} \text{ or } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\bar{x}\right)^2}$$

- The coefficient of variation measures variability in relation to the mean (or average) and is used to compare the relative dispersion in one type of data with the relative dispersion in another type of data. It allows us to compare the dispersions of two different distributions if their means are positive. The greater dispersion corresponds to the value of the coefficient of greater variation. The coefficient of variation is given by: $Cv = \frac{\sigma}{\overline{x}} \times 100$ where: σ is the standard deviation and $\overline{x}$ s the mean.

10.7. Additional Information for Tutors

- Emphasize on the following results follow directly from the definitions of mean and standard deviation:

  When all the data values are multiplied by a constant *a*, the new mean and new standard deviation are equal to *a* times the original mean and standard deviation. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $a\left(\overline{x}\right)$ and the standard deviation is $a\sigma$.

  When a constant value, *b*,is added to all data values, then new mean is increased by *b*. However standard deviation does not change. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $\overline{x} + b$ and the standard deviation is σ.

- Emphasize on calculating median of ungrouped data. We need to clarify formula

for $n - odd$ and for $n - even$ as highlighted in the student-teachers book and paying attention on notation.

When n is odd, the median is given by

$Me \to \left(\frac{n+1}{2}\right)^{th}$ or $Me = x_{\frac{n+1}{2}}$ we don't write $Me = \left(\frac{n+1}{2}\right)^{th}$

When n is even, the median is given by

$$Me = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right] \text{ or } Median = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$$

- Emphasize on calculating mode and median of grouped data.

$$Mode = L_m + c\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)$$

$$Median = L_m + c\left(\frac{\frac{n}{2} - CF_b}{f_m}\right)$$

- Emphasize on formulae of quartiles for $n - odd$ and for $n - even$

$Q_1 \rightarrow \frac{1}{4}(n+1)^{th} \; or Q_1 = x_{\frac{n+1}{4}}$. We don't write $Q_1 = \frac{1}{4}(n+1)^{th}$

$Q_2 \rightarrow \frac{1}{2}(n+1)^{th} \; or Q_2 = x_{\frac{n+1}{2}} = Me$ we don't write $Q_2 = \frac{1}{2}(n+1)^{th}$

$Q_3 \rightarrow \frac{3}{4}(n+1)^{th} \; or Q_3 = x_{\frac{3(n+1)}{4}}$ we don't write $Q_3 = \frac{3}{4}(n+1)^{th}$

10.8. End unit assessment

1)a) **Mode is 70,000 Frw**

f) **Range is 60,000 Frw**

g)

Monthly wage(Frw)	40,000	50,000	60,000	70,000	80,000	90,000	100,000
Number of workers	4	10	12	24	12	4	18

2) a) $Mean = 6.6$ $Q_1 = 6$ $Q_2 = 6.5$ $Q_3 = 8$

$interquartile\ range = 2$

b) $Variance = 2.04$ $\quad Standard\ deviation \approx 1.43$

c) $Coeff.of\ variance \approx 21.7$

10.9. Additional activities

10.9.1. Remedial activities

In test of French, 17 student-teachers got the following marks out of 17:

6,7,6,7,8,9,5,4, 8, 5, 7, 6, 6, 9, 4, 8,8.

a. Calculate the mean, mode, median, range and quartiles and interquartile range

b. Calculate the variance and standard deviation

c. Calculate the coefficient of variation.

10.9.2. Consolidation activities

The six runners in a 200 meters race clocked times (in seconds) of 24.2, 23.7, 25.0, 23.7, 24.0,4.6

a) Find the mean and standard deviation of these times.

b) These readings were found to be 10% too low due to faulty timekeeping. Write down the new mean and standard deviation.

c) Draw bar graph of the above information

Solution

a) $\bar{x} = \frac{24.2+23.7+25.0+23.7+24.0+24.6}{6} = 24.2 \text{ seconds}$

$$\sigma = \sqrt{\frac{(24.2-24.2)^2+(23.7-24.2)^2+(25.0-24.2)^2+(23.7-24.2)^2+(24.0-24.2)^2+(24.6-24.2)^2}{6}}$$

$= 0.473$ seconds

b) We must divide each term 0.9 to find the correct time. The new mean is $\bar{x} = \frac{24.2}{0.9} = 26.9$ sec. The new standard deviation is

$$\sigma = \frac{0.4726}{0.9} = 0.525 \text{ sec}$$

10.9.3. Extended activities

Hospital records indicated that maternity patients stayed in the hospital for the number of days shown in the distribution:

Number of days stayed	Frequency
3	15
4	32
5	56
6	19
7	5
Total	127

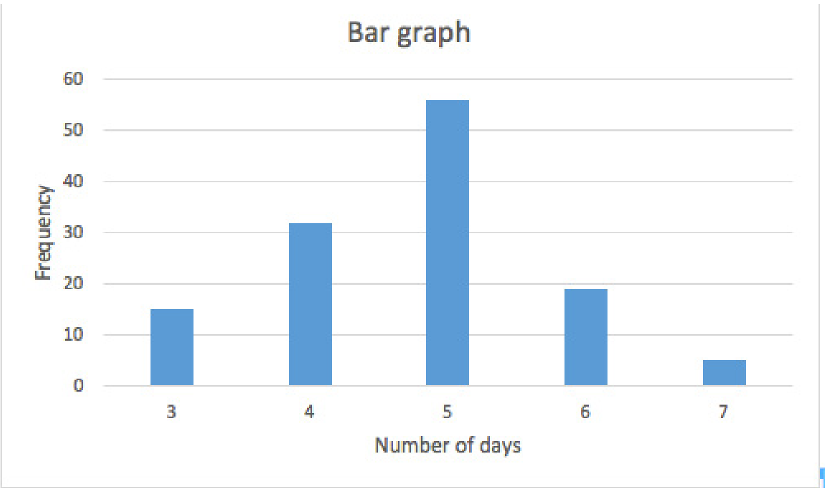
Construct

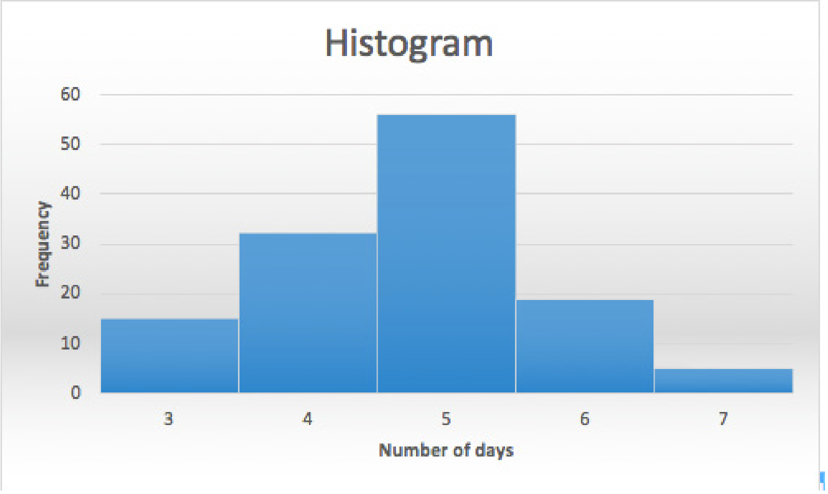
i) Bar graph

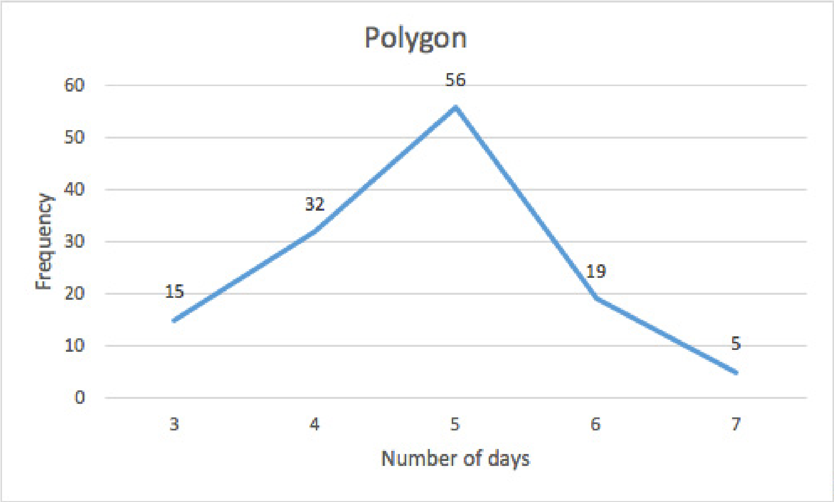
ii) Histogram

iii) Polygon

Solutions




Histogram
60
50
40
30
20
10
0
Frequency
3
4
5
6
7
Number of days


Polygon
60
50
40
30
20
10
0
Frequency
15
32
56
19
5
3
4
5
6
7
Number of days

UNIT 11

POLYNOMIAL FUNCTIONS

11.1. Key unit competence

Use concepts and definitions of functions to determine the domain of polynomial functions, solve related problems and represent them graphically in simple cases (plotting linear and quadratic functions)

11.2. Prerequisite

In this lesson, Student-teachers must be skilled in **Unit 1 &3** of **S1, Unit 2** of **S2** and **Unit 6** of **S3.**

11.3. Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others view and thoughts during class discussions)
- Gender (equal opportunity of boys and girls in the lesson participation)
- Standardization culture (regulation of the current intensity)

11.4. Guidance on introductory activity

- In groups, facilitate student-teachers read and do the introductory activity from Student -teacher's book
- Facilitate discussions to avoid noise or other unnecessary conversation,
- Be aware of straggling Student –teacher.
- Call them to present their findings and promote gender into presentation.
- Through question-answer, facilitate Student-teachers to realize that introductory activity stimulates them to get idea on Unit 11.
- Through class discussions, let student-teachers think on different ways of getting solutions.

11.5. **List of lessons**

#	Lesson title	Learning objectives	Number of periods
0	Introductory activity	To arouse the curiosity of student teacher on the content of unit 11	1
1	Definition of a polynomial function	Define a polynomial function.	2
2	Domain and range of a polynomial function	Determine the domain and range of a function.	4
3	Parity(even , odd) of a polynomial function	Find whether a function is even, odd, or neither.	5
4	Plotting linear and quadratic functions.	Plot linear and quadratic functions Interpret graphs of functions (linear and quadratic) related to practical context and make conclusions	6
5	Solve problems related to linear and quadratic functions.	Analyse, model and solve problems involving linear or quadratic functions and interpret the results. Show concern of using graphs of linear and quadratic functions in solving mathematical problems	4
6	End unit assessment		2

Lesson 1: Definition of a polynomial function

a) **Learning objective**

Define a polynomial function.

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

In this lesson, Student-teachers must be skilled in **Unit 1 &3** of **S1, Unit 2** of **S2** and **Unit 6** of **S3.**

d) Learning activities

- Ask student-teachers in pairs to read and discuss on the **activity 11.1** to classify polynomials
- Facilitate working especially straggling student-teachers.
- Call student-teachers to present the findings, and help them to harmonise the answer.

Answer of activity 11.1

1.

2	x	$5x^3$	$2x+5$	x^2-x	$x-5$	x^2+5x+6	x^5-3x+8	$2x^2+5x+6$
One Term	one term	One Term	Two terms	two terms	Two terms	three terms	Three terms	Three terms

2.

Degree 0	Degree 1	Degree 2	Degree 3	Degree 5
2	x $2x+5$ $x-5$	x^2-x x^2+5x+6 $2x^2+5x+6$	$5x^3$	x^5-3x+8

- Lead student-teachers to realize that the solutions contain some concepts of polynomial functions.
- Lead them to know definitions of some terms those we are familial with in polynomial such as "monomial, binomial, trinomial, linear, quadratic, ..."

- **Guide them to classify polynomial functions either by number of terms or by degrees.**

Degree 1	Degree2	Degree3	Degree4	etc.
Linear polynomial function/1st degree polynomial function.	quadratic polynomial function/ 2nd degree polynomial function.	Cubic polynomial function/3rd degree polynomial function.	4th degree polynomials (bi-quadratic polynomial functions).	

One term	Two terms	Three terms	Four terms	Etc
Monomial function	Binomial function	Trinomial function	Qua-trinomial function	

- **Let them know the general form of a polynomial function.**

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2 x^2 + a_1 x^1 + a_0 x^0$$

- In small group, lead them to discuss on the examples
- Guide them to do **application activity 11.1** to master the content.

d) Application activity 11.1

Answer of Application activity 11.1

1. The table below shows the names of the polynomials according to their degrees.

Polynomial function in general form	Degree	Name of function
$y = ax + b$	1	1st degree polynomial/ linear polynomial.
$y = ax^2 + bx + c$	2	2nd degree polynomial/ quadratic polynomial.
$y = ax^3 + bx^2 + cx + d$	3	3rd degree polynomial/ cubic polynomial.
$y = ax^4 + bx^3 + cx^2 + dx + e$	4	4th degree polynomial/ biquadratic Polynomial.

2. From the table above, the general form is deduced as

$$f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+...+a_2x^2+a_1x^1+a_0x^0$$

Lesson 2: Domain and range of a polynomial function

a) **Learning objective**

Determine the domain and range of a function.

b) **Teaching resources**

Studen-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

In this lesson, Student-teachers must be skilled in **Unit 1 &3** of **S1, Unit 2** of **S2** .

d) **Learning activities**

- Ask student-teachers in small groups/individual to read and discuss on the **activity 11.2** and make sure that everybody is engaged/ involved.
- Facilitate working, especially for straggling student-teachers.
- Call student-teachers to present the findings, and promote gender in this presentation.
- help them to harmonise the answers.

Answer of activity 11.2

2 is mapped to 3

3 is mapped to 1

4 is mapped to 2

5 is mapped to 4 and 5

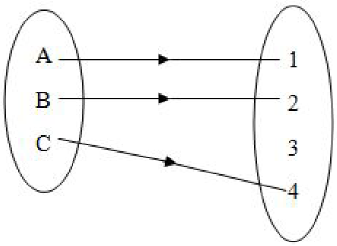
Let student-teachers know difference between a polynomial functions and mappings.

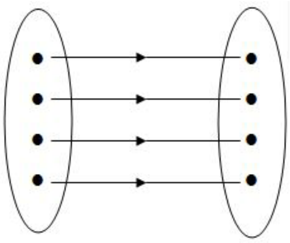
- Lead them to know definitions of some terms those we are familial with in polynomial such as “a **co-domain**, **domain, and range.**
- Through examples in , Lead student-teachers to find range and codomain for a given domain.
- Guide them to do **application activity 10.1.2** to master the content.

d) Application activity 11.2

Answers of application activity 11.2

1. The 1st and the 2nd arrow diagrams are functions.





2. $Dom=\{a,b,c,d,e\}$ $co-domain=\{1,2,3,4,5,6\}$ $Range=\{1,2,3,4\}$

3.

$a) f(2)=8$

$b) f(-2)=0$

$c) f(d)=2d+8$

$d) f(a)=a$

$2a+8=a$

$a=-8$

4. a) $\mathbb{R}$ b) $\mathbb{R}$ c) $\mathbb{R}$

Lesson 3: Parity of a polynomial function

a) **Learning objective**

Find whether a function is even, odd, or neither.

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

Student-teachers in this lesson, Student-teachers must be skilled in **Unit 1 &3** of **S1, Unit 2** of **S2**.

d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 11.3**
- Facilitate working, especially straggling student-teachers.
- Call student-teachers to present their findings,
- Promote gender while presenting findings, and help them to harmonize the answer.

Answer of activity 11.3

1. $f(x)=x^2+2x+3$

$\circ f(-x)=(-x)^2+2(-x)+3=x^2-2x+3$

$\circ -f(x)=-(x^2+2x+3)=-x^2-2x-3$

$\therefore f(-x)\neq -f(x)$

2.

$f(x)=2x+5$

$\circ f(-x)=2(-x)+5=-2x+5$

$\circ -f(x)=-(2x+5)=-2x-5$

$\therefore f(-x)\neq -f(x)$

3.

$f(x) = x^5 - 3x + 8$

$\circ f(-x) = (-x)^5 - 3(-x) + 8 = -x^5 + 3x + 8$

$\circ - f(x) = -\left(x^5 - 3x + 8\right) = -x^5 + 3x - 8$

$\therefore f(-x) \neq -f(x)$

4.

$f(x) = x^4 - 3x^2$

$\circ f(-x) = (-x)^4 - 3(-x)^2 = x^4 - 3x^2$

$\circ - f(x) = -\left(x^4 - 3x^2\right) = -x^4 + 3x^2$

$\therefore f(-x) \neq -f(x)$

5.

$f(x) = x^5 + 3x$

$\circ f(-x) = (-x)^5 + 3(-x) = -x^5 - 3x$

$\circ - f(x) = -\left(x^5 + 3x\right) = -x^5 - 3x$

$\therefore f(-x) = -f(x)$

- Help student-teachers discover that

A function $f(x)$ is said to be **even** if the following conditions are satisfied

$\forall x \in Domf, -x \in Domf$

$f(-x) = f(x)$

A function $f(x)$ is said to be **odd** if the following conditions are satisfied

$\forall x \in Domf, -x \in Domf$

$f(-x) = -f(x)$

- In small groups, lead them to do examples
- Call student-teachers to do Application activity 11.3 to master the content.

Answers Application activity 11.3

1.

$f(x) = 2x^2 + 2x - 3$

$\circ f(-x) = 2(-x)^2 + 2(-x) - 3 = 2x^2 - 2x - 3$

$\circ - f(x) = -(2x^2 + 2x - 3) = -2x^2 - 2x + 3 \quad \therefore f(x) = 2x^2 + 2x - 3$

$f(-x) \neq -f(x)\, and$

$f(-x) \neq -f(x)$

is neither odd nor even.

2.

$g(x) = x^3 - x$

$\circ g(-x) = (-x)^3 - (-x) = -x^3 + x$

$\circ - g(x) = -(x^3 - x) = -x^3 + x$

$g(-x) = -g(x) \qquad \therefore g(x) = x^3 - x$ is odd.

3.

$g(x) = x\left(x^2 + x\right) = x^3 + x^2$

$\circ g(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2$

$\circ - g(x) = -(x^3 + x^2) = -x^3 - x^2 \quad \therefore g(x) = x\left(x^2 + x\right)$

$g(-x) \neq -g(x)\, and$

$g(-x) \neq -g(x)$

is neither odd nor even.

Lesson 4: Plotting linear and quadratic functions

a) **Learning objective**

Interpret graphs of functions (linear and quadratic) related to practical context and make conclusions.

b) **Teaching resources**

Student-teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

In this lesson, Student-teachers must be skilled in **Unit 3** of **S1** and **Unit 6** of **S3.**

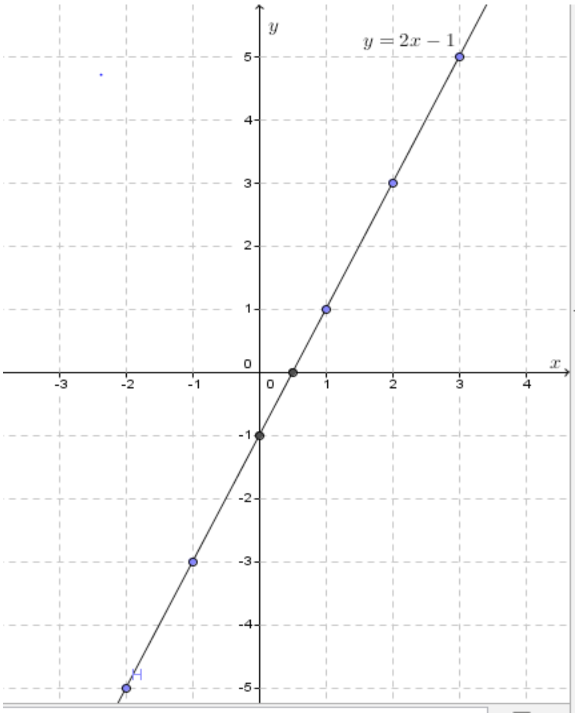
d) **Learning activities**

- Ask student-teachers in pairs to read and discuss on the **activity 11.4**
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling student-teachers.
- Facilitate the use of geometric materials to make accurate graphs.
- Make sure that the notebooks of student-teachers include squared papers/graph papers.
- Call student-teachers to present the findings and promote gender where possible.
- Help them to harmonise the answers.

Answer of activity 11.4

1. $y = 2x - 1$ for $-3 \leq x \leq 3$

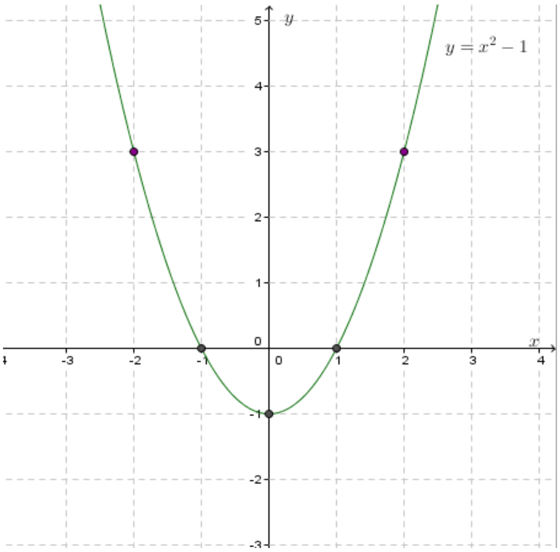
x	-3	-2	-1	0	1	2	3
$y = 2x - 1$	-7	-5	-3	-1	1	3	5



This is linear function

2. $y = x^2 - 1$ for $-3 \leq x \leq 3$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 1$	8	3	0	-1	0	3	8



This is quadratic function

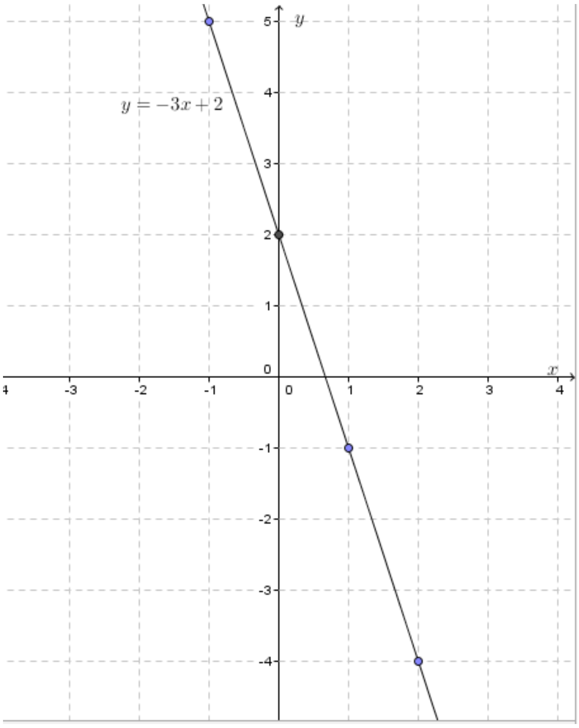
- Through examples given in student-teacher's book, guide student-teachers to know how accurately draw
    - Linear function
    - Quadratic function
- Call student-teachers to do Application activity 11.4 in student-teacher's book to master the content.

e) **Application activity 11.4**

Answers of Application activity 11.4

1. a) $y = -3x + 2$

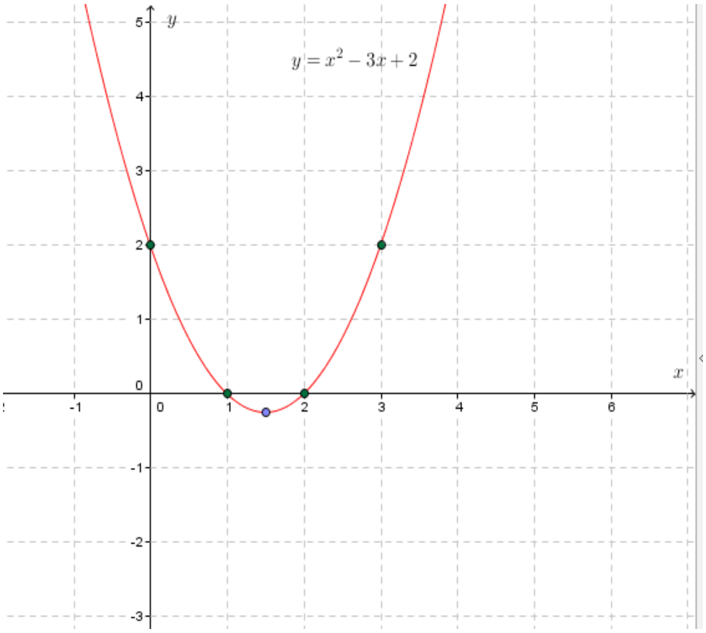
x	-1	0	1	2
$y = -3x + 2$	5	-2	-1	-4



b) $y = x^2 - 3x + 2$

x	-1	0	1	2	3

$y = x^2 - 3x + 2$	6	2	0	0	2



2. (a) $y = 2x^2 + 5x - 1$

vertex $v(h,k)$ with $h = -\frac{b}{2a} = -\frac{5}{4}$ *and* $k=2\times\left(-\frac{5}{4}\right)^2 + 5\times\left(-\frac{5}{4}\right) - 1 = -\frac{33}{8}$

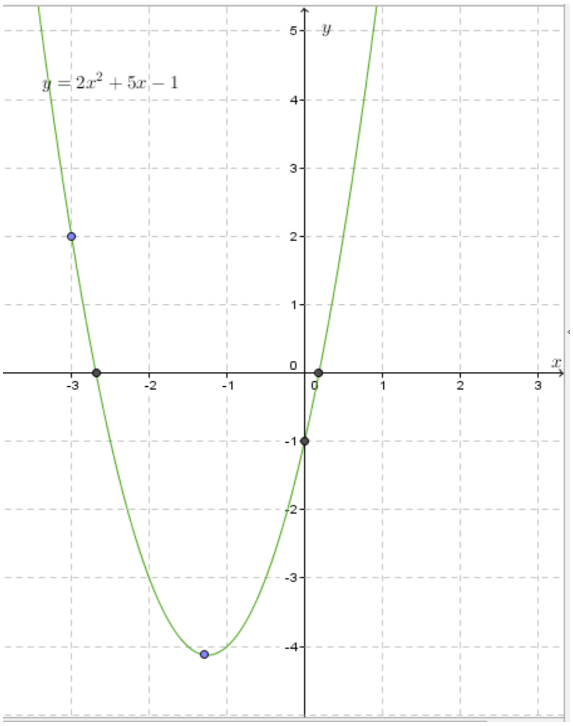
$\Rightarrow v = \left(-\frac{5}{4}, -\frac{33}{8}\right)$

axis of symmetry $x = -\frac{5}{4}$

if $x = 0 \Rightarrow y = -1$ and y-intercept is $(0,-1)$

if

$y = 0 \Rightarrow x = \frac{-5+\sqrt{33}}{4} \simeq 0.186$ or $x=\frac{-5-\sqrt{33}}{4} \simeq -2.68$ then x-intercept is $(0.18, 0)$ *or* $(-2.68, 0)$



(b) $y = 3x^2 + 8x - 6$

vertex $v(h,k)$ with $h = -\frac{b}{2a} = -\frac{8}{6} = -\frac{4}{3}$ and $k = 3\times\left(-\frac{4}{3}\right)^2 + 8\times\left(-\frac{4}{3}\right) - 6 = -\frac{34}{3}$

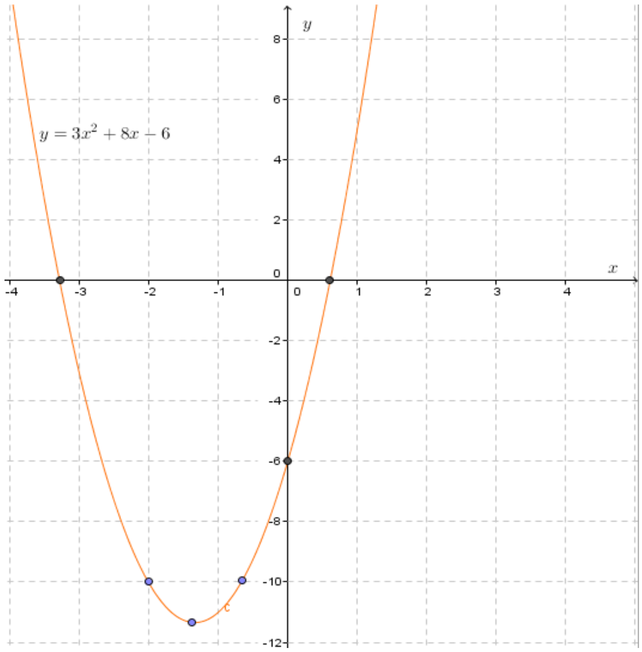
$\Rightarrow v = \left(-\frac{4}{3}, -\frac{34}{3}\right)$

axis of symmetry $x = -\frac{4}{3}$

if $x = 0 \Rightarrow y = -6$ and y-intercept is $(0,-6)$

if

$y = 0 \Rightarrow x = \frac{-8+\sqrt{136}}{6} \simeq 0.61$ or x=$\frac{-8-\sqrt{136}}{6} \simeq -3.27$ then x-intercept is $(0.61, 0)$ *or* $(-3.27, 0)$



Lesson 5: Solve problem related to linear and quadratic functions

a) **Learning objective**

Analyse, model and solve problems involving linear or quadratic functions and interpret the results.

b) **Teaching resources**

Student -teacher's book and other Reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) **Prerequisites/Revision/Introduction**

In this lesson, Student-teachers must be skilled in **previous lessons of year 1.**

d) **Learning activities**

- Ask student-teachers in small groups to read and discuss on the **activity 11.5** and make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling student-teachers.
- Call student-teachers to present the findings, and
- Help them to harmonise their answers.

Answer of activity 11.5

a) $\mathbf{C(10) = 80(10) + 150};\ x = 10$

$\qquad = \mathbf{800 + 150}$

$\qquad = \mathbf{950}$

Therefore, the cost of 10 products is 950 FRW.

b) $C(x) = 15{,}000$

$15{,}000 = \mathbf{80}x + \mathbf{150}$

$$x = \frac{15{,}000 - 150}{80}$$

$x = 185.625$

Since, the company can produce 185 products for 15,000 FRW.

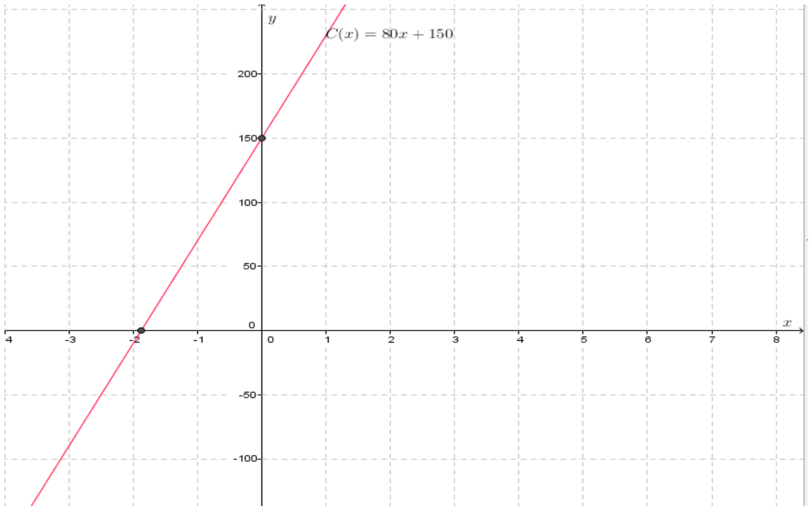
c) The restriction on the domain $x \geq 0$ is necessary because it makes no sense when the number of products produced is negative.

d) $x = 0$

$f(0) = 80(0) + 150 = 150$

It has no meaning to pay zero product. But this amount of 150FRW is considered as the fixed or start-up of the venture.

e) We recognize that the slope is defined by $m = \dfrac{\Delta y}{\Delta x} = \dfrac{\Delta c}{\Delta x} = \dfrac{80}{1} = 80$



- In small groups, facilitate student-teachers to do examples given in student-teacher
- Call student-teachers to do Application activity 11.5 to master the content.

e) **Application activity 11.5**

Answers of Application activity 11.5

1.

a. Consider two points formed by corresponding price and products. These points are respectively $(20,220)\, and\, (40,190)$.

Use the point-slope form to find equation of the linear function, where

$$m=\frac{190-220}{40-20}=-\frac{3}{2}$$

Then, use $(20,220)$ and $-\frac{3}{2}$;

$$y-220=-\frac{3}{2}(x-20)$$

$$y=-\frac{3}{2}x+250$$

$\therefore \quad P(x)=-\frac{3}{2}x+250$ this is the linear function.

b. To determine the applied domain, we look at the physical constraints of the products. Certainly, we can't sell negative number of products, so $x\geq 0$.

c. Since the slope is negative ($m=-\frac{3}{2}=-1.5$), we have that the price is decreasing at a rate of \$1.5 per a product sold (said differently, we can sell one more product for every \$1.5 drop in price)

d. $P(150)=-\frac{3}{2}(150)+250=25$ Therefore, the price of 150 products is \$25.

e. When the price \$150, then

$$150 = -\frac{3}{2}x + 250$$

$$300 = -3x + 500$$

$$x = \frac{500-300}{3} = \frac{200}{3} = 66.6$$

This means we would be able to sell 66 products a week, if the price were \$150 per unit or item.

2. Average rate is given by

$$\frac{\Delta C}{\Delta x} = \frac{C(5000)-C(3000)}{5000-3000}$$

$$\bullet C(5000) = (5000)^2 - 10(5000) + 27 = 24,950,027$$

$$\bullet C(3000) = (3000)^2 - 10(3000) + 27 = 8,970,027$$

$$\therefore Rate = \frac{24,950,027-8,970,027}{5000-3000} = \frac{15980000}{2000} = 7990$$

A hundred is cost 7990FRW, it means one pen costing 79.9FRW.

3. Solution is optional.

11.6. Summary of the unit

A monomial is a variable, a real number, or a multiplication of one or more variables and a real number with whole-number exponents

A function that is expressible as the sum of finitely many monomials in x is called **polynomial in *x*.**

A function is a rule that assigns to each element in a set A one and only one element in set B. We can even define a function as any relationship which takes one element of one set and assigns to it one and only one element of second set. The second set is called a **co-domain**. The set A is called the **domain,** denoted by ***Domf or Df.***

Functions for which each element of the domain is associated onto a different element of the range are said to be **one-to-one.**

Relationships which are **one-to-many** can occur, but from our preceding definition, they are **not functions.**

A function $f(x)$ is said to be **even** iff $f(-x) = f(x)$ for all

$x \in Domf \ and -x \in Domf$

A function $f(x)$ is said to be **odd** iff $f(-x)=-f(x)$ for all $x \in Domf \ and -x \in Domf$

A linear function is any function of the form $f(x)=mx+b$ with $m \neq 0$. The ***domain*** of this function is the set of all real numbers. The ***range*** of f is the set of all real numbers. The graph of f is a line with slope m and y intercept b.

A function $f(x)=b$, where b is a constant is called a constant function. Its graph is a horizontal line at $y=b$.

The quadratic function written as $f(x)=ax^2+bx+c$, has: **a vertex** $v\left(-\frac{b}{2a}, f(-\frac{b}{2a})\right)$, axis of symmetry $x=-\frac{b}{2a}$

There are two intercepts : x-intercept. (o,c) and y-intercept $(x_1,0)$ and $(x_2,0)$ where x_1 and x_2 *are the roots of* $ax^2+bx+c=0$ with $a \neq 0$

x-intercept for any quadratic function is calculated by letting y = 0 and y- intercept is calculated by letting x = 0

Graph of a quadratic function

The graph of a quadratic function can be sketched using the table of values of x and y or by the vertex, x-intercepts and y-intercept

Some applications of polynomial functions:

- Polynomial functions can also be used to model different situations, like in the stock market to see how prices will vary over time.
- Business people also use polynomial functions to model markets, as in to see how raising the price of a good will affect its sales.
- Polynomial functions are important in calculating medicine, building structures (houses, businesses…), vehicle design, designing games, to build computers (formulas that are used to plug to computer programs), knowing how much change you should receive when making a purchase, driving (amount of gas needed for travel)

11.7. **Additional Information for Tutors**

- Emphasize on the graph paper/squared paper while student-teachers draw the graphs.
- Emphasize and facilitate the use of geometric materials to ameliorate the quality of graphs.
- Guide the student-teachers in scaling axes.
- Remind them to name axes (x-axis and y-axis).
- Recall them to mention/highlight the origin/intersection point of axes by 0.

11.8. **End unit assessment**

Answers of End unit assessment

1) **a)** $\mathbb{R}$ **.** **b)** $\mathbb{R}$ **.** **c)** $\mathbb{R}$ **.**

2) **a) None.** **b) None.**

3) $a) Range = \{3,4,5,6,7\}$ **this is one to one.**

$b) Range = \{9,4,1,0\}$ this is many to one.

$c) Range = \mathbb{R}$ this is many to one.

4) $f(x)$ **is neither odd nor even.**

5) **a)** $T(4)=72$ $T(8)=128$ $T(12)=200$

b) $rate = \frac{T(8)-T(4)}{8-4} = \frac{128-72}{4} = 14$

c) $rate = \frac{T(12)-T(8)}{12-8} = \frac{200-128}{4} = 18$

11.9. **Additional activities**

11.9.1. **Remedial activities**

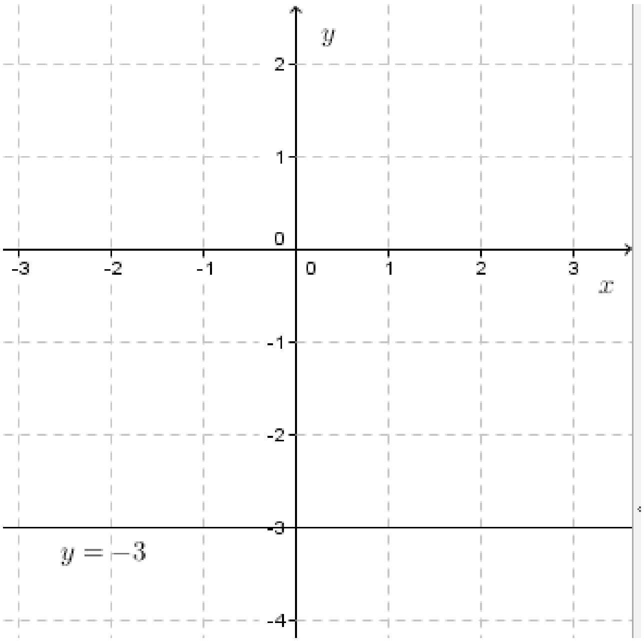
1. Draw the graph of a linear function and determine the properties of a function : (domain of a function, range of a function, function is/is not one-to-one function, even/odd function, coordinates of intersections

with the x-axis and with the y-axis, intervals of monotonicity - increasing/decreasing function)

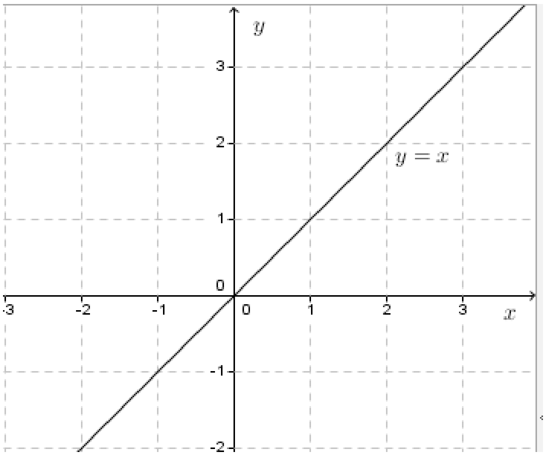
$a) y = -3$ $b) y = x$ $c) y = 4x + 2$ $d) 2x + 4y - 6 = 0$

Solutions

$a) D(f):]-\infty, +\infty[$ the function $y = -3$ is not one-to-one and it is an even
$R(f): \{-3\}$
function, X-axis intercept: $\varnothing$, y-axis intercept: $(0, -3)$, increasing : $\varnothing$, decreasing: $\varnothing$

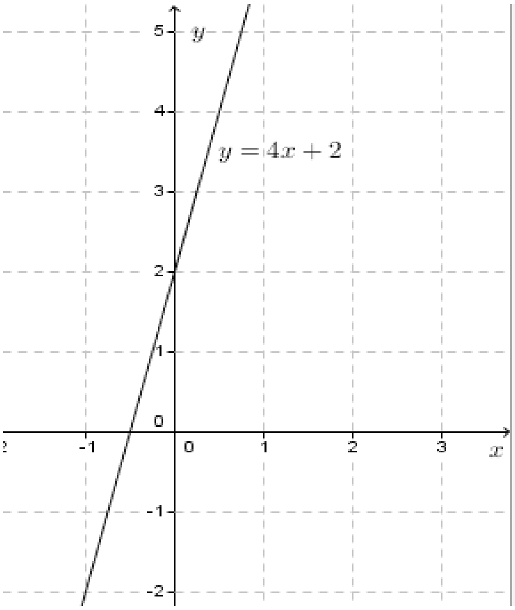


$b) D(f):]-\infty, +\infty[$
$R(f): [-\infty, +\infty[$ the function $y = x$ is one-to-one and it is an odd function, X-axis intercept: $(0,0)$, y-axis intercept: $(0,0)$, increasing : $]-\infty, +\infty[$, decreasing: $\varnothing$



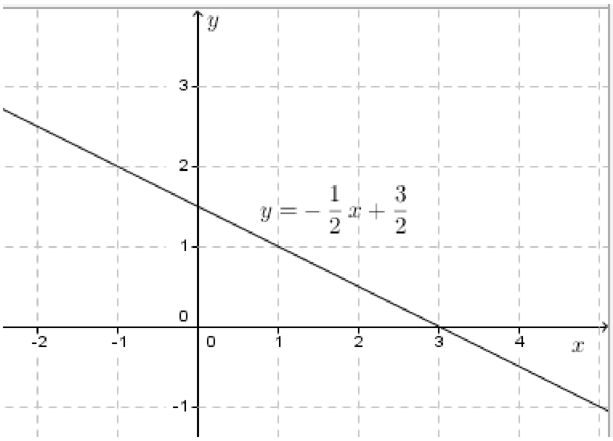
$c) D(f):]-\infty, +\infty[$

$R(f): [-\infty, +\infty[$ the function $y = 4x + 2$ is one-to-one and it is neither even nor odd function, X-axis intercept: $\left(-\frac{1}{2}, 0\right)$, y-axis intercept: $(0, 2)$, increasing : $]-\infty, +\infty[$, decreasing: $\varnothing$



$d) D(f):]-\infty, +\infty[$

$R(f): [-\infty, +\infty[$ the function $2x + 4y - 6 = 0$ is one-to-one and it is neither even nor odd function, X-axis intercept: $(3, 0)$, y-axis intercept: $\left(0, \frac{3}{2}\right)$, increasing : $\varnothing$, decreasing: $]-\infty, +\infty[$

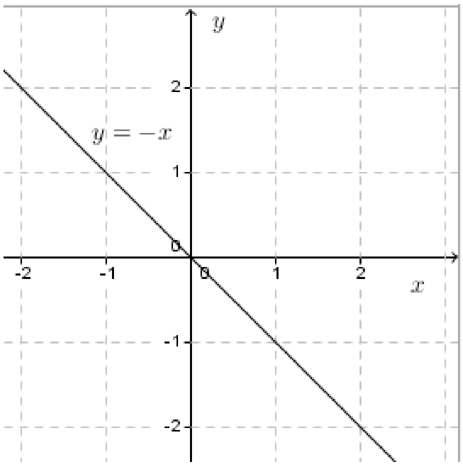


2. Find the equation of a linear function and draw the graph of a function that passes through the points:

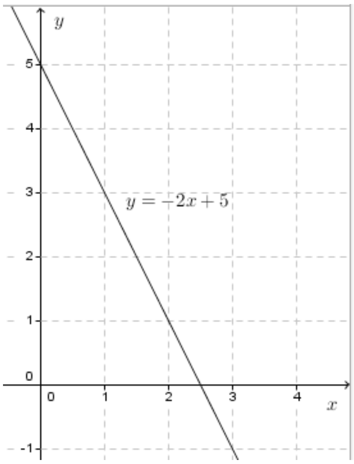
$a)A(0,0) \quad and \quad B(4,-4)$
$b)C(1,3) \quad and \quad D(3,-1)$

Solutions

$a)y=-x$



$b)y=-2x+5$



11.9.2. **Consolidation activities**

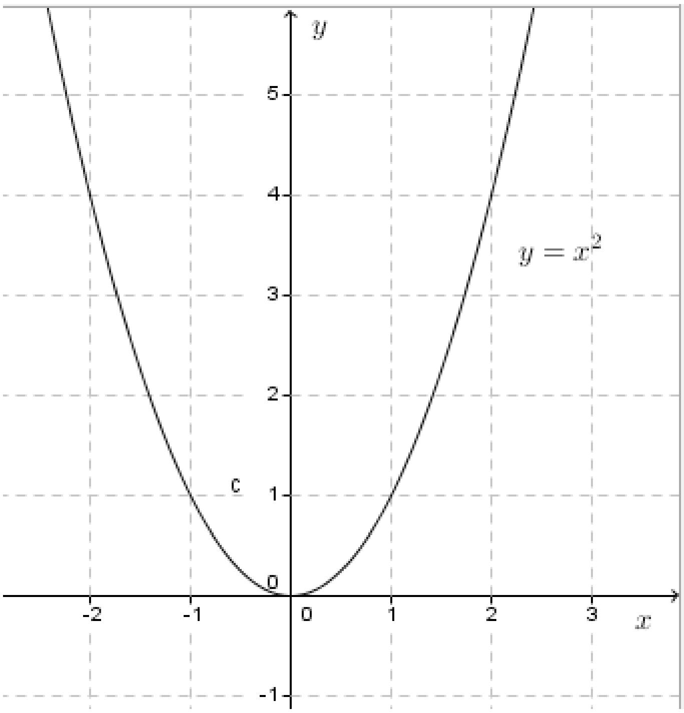
1. Draw the graph of a quadratic function and determine the properties of a function: (domain of a function, range of a function, function is/ is not one-to-one function, even/odd function, vertex of a parabola, coordinates of intersections with the *x*-axis and with the *y*-axis, local extrema - local minimum & local maximum, intervals of monotonicity

- increasing/decreasing function)

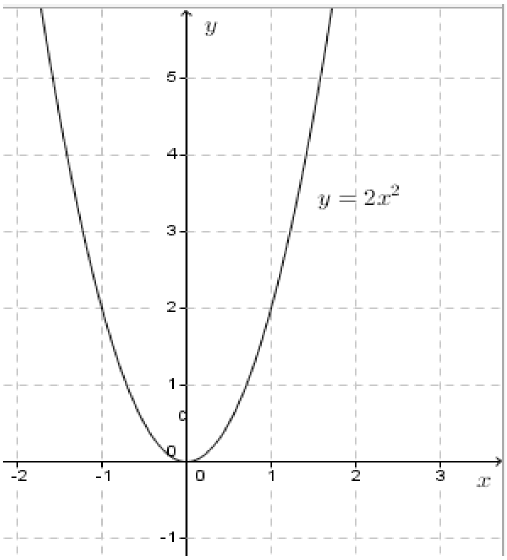
$a) y = x^2$ $b) y = 2x^2$ $c) y = \frac{1}{4}x^2$

Solutions

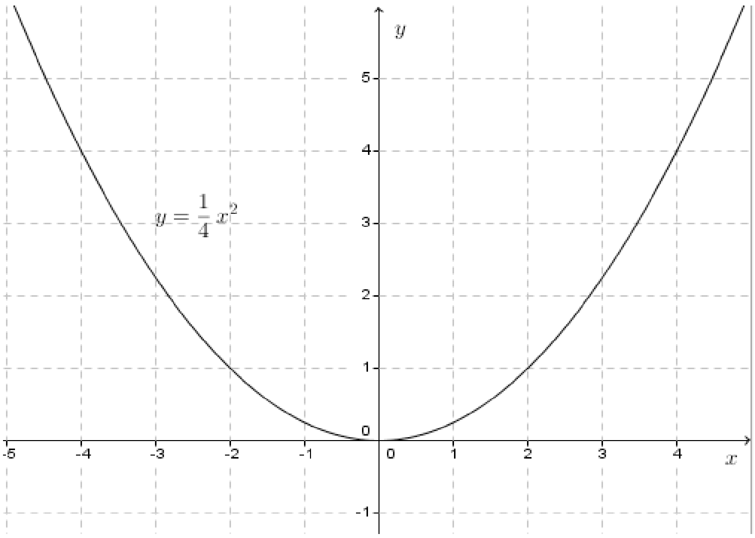
$a) D(f):]-\infty, +\infty[$
$R(f): [0, +\infty[$ the function $y = x^2$ is not one-to-one and it is an even function, vertex : $(0,0)$, X-axis intercept: $(0,0)$, y-axis intercept: $(0,0)$, local minimum: $(0,0)$, local maximum : $\varnothing$, increasing : $]0, +\infty[$, decreasing: $]-\infty, 0[$



$b) D(f):]-\infty, +\infty[$
$R(f): [0, +\infty[$ the function $y = 2x^2$ is not one-to-one and it is an even function, vertex : $(0,0)$, X-axis intercept: $(0,0)$, y-axis intercept: $(0,0)$, local minimum: $(0,0)$, local maximum : $\varnothing$, increasing : $]0, +\infty[$, decreasing: $]-\infty, 0[$



$c) D(f):]-\infty, +\infty[$ the function $y = \frac{1}{4}x^2$ is not one-to-one and it is an
$R(f): [0, +\infty[$
even function, vertex : $(0,0)$, X-axis intercept: $(0,0)$, y-axis intercept: $(0,0)$, local minimum: $(0,0)$, local maximum : $\varnothing$, increasing : $]0, +\infty[$, decreasing: $]-\infty, 0[$



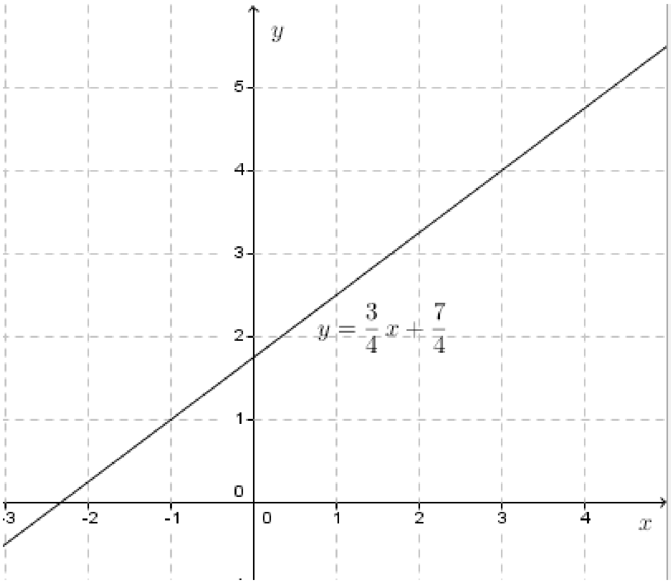
2. Find the equation of a linear function and draw the graph of a function that passes through the points:

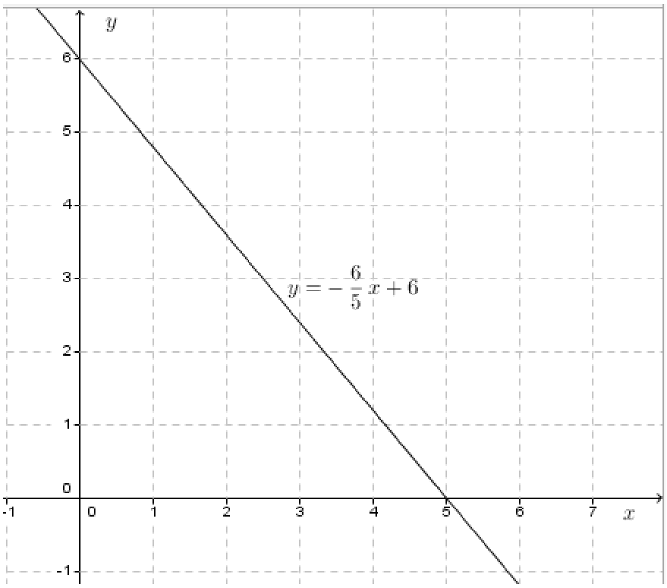
$a) A(7,7)$ *and* $B(-1,1)$
$b) C(5,0)$ *and* $D(0,6)$

Solutions

$a) y=\frac{3}{4}x+\frac{7}{4}$

$b) y=-\frac{6}{5}x+6$





11.9.3. **Extended activities**

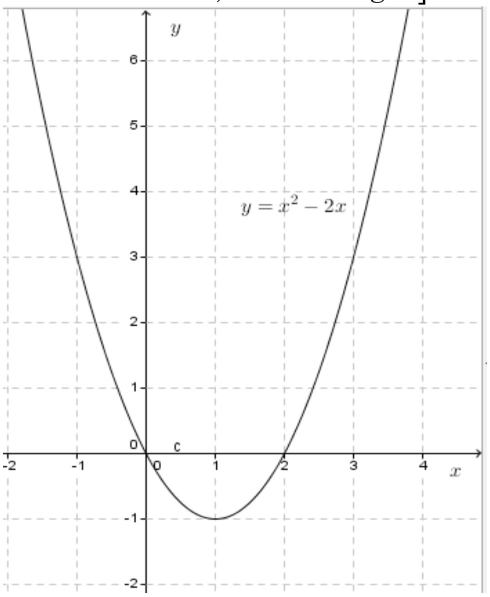
1. Draw the graph of a quadratic function and determine the properties of a function: (domain of a function, range of a function, function is/is not one-to-one function, even/odd function, vertex of a parabola, coordinates of intersections with the *x*-axis and with the *y*-axis, local

extrema - local minimum & local maximum, intervals of monotonicity - increasing/decreasing function)

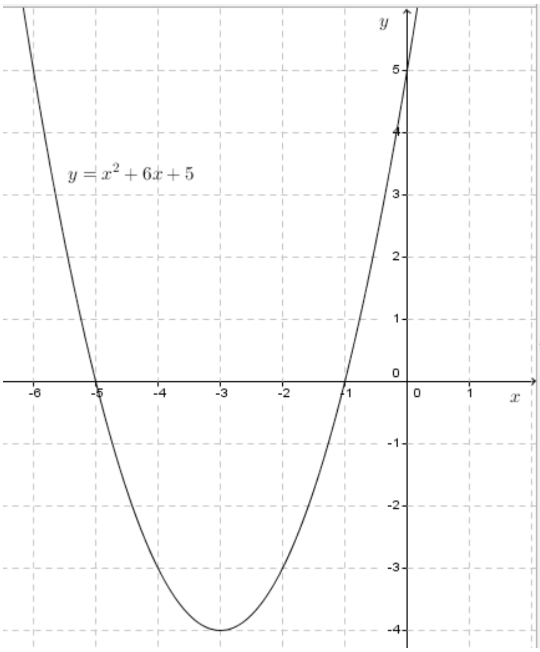
$a) y = x^2 - 2x$ $\qquad$ $b) y = x^2 + 6x + 5$ $\qquad$ $c) y = -x^2 - 1$

Solutions

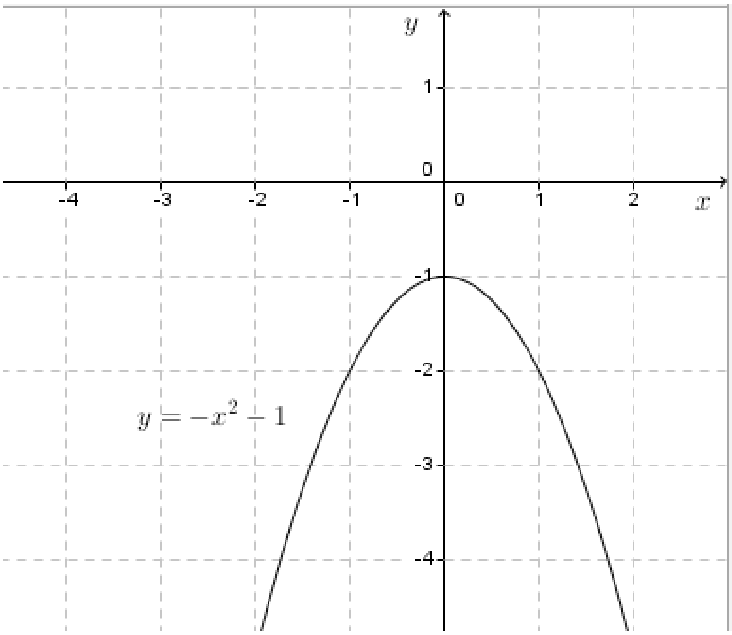
$a) D(f):]-\infty, +\infty[$
$R(f): [-1, +\infty[$ the function $y = x^2 - 2x$ is not one-to-one and it is neither even nor odd function, vertex : $(1, -1)$, X-axis intercept: $(0,0)$ *and* $(2,0)$, y-axis intercept: $(0,0)$, local minimum: $(1,-1)$, local maximum : $\varnothing$, increasing : $]1, +\infty[$, decreasing: $]-\infty, 1[$



$b) D(f):]-\infty, +\infty[$
$R(f): [-1, +\infty[$ the function $y = x^2 + 6x + 5$ is not one-to-one and it is neither even nor odd function, vertex : $(-3, -4)$, X-axis intercept: $(-5,0)$ *and* $(-1,0)$, y-axis intercept: $(0,5)$, local minimum: $(-3,-4)$, local maximum : $\varnothing$, increasing : $]-3, +\infty[$, decreasing: $]-\infty, -3[$



$c) D(f):]-\infty, +\infty[$ the function $y = -x^2 - 1$ is not one-to-one and it is an
$R(f):]-\infty, -1]$
even function, vertex : $(0,-1)$, X-axis intercept: $\varnothing$, y-axis intercept: $(0,-1)$, local minimum: $\varnothing$, local maximum : $(0,-1)$, increasing : $]-\infty, 0[$, decreasing: $]0, +\infty[$



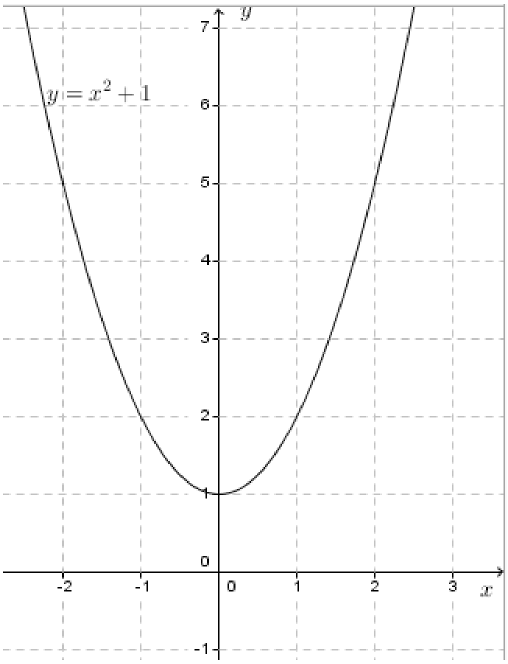
2. Find the equation of a quadratic function and draw the graph of a function that passes through the points:

$a) A(0,1), B(1,2) \text{ and } C(2,5)$ $b) C(0,1), D(1,1) \text{ and } E(2,1)$

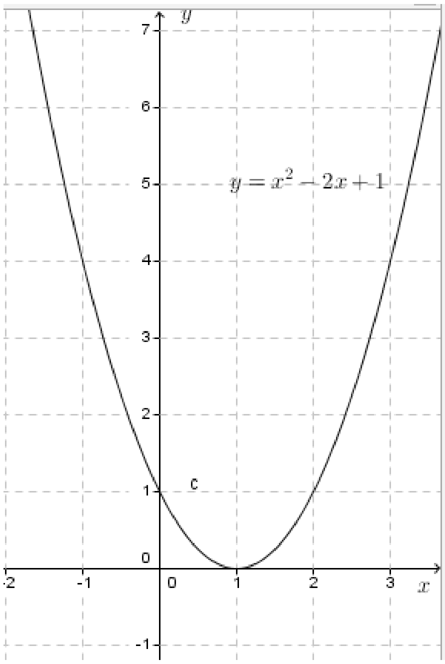
$c) A(0,7), B(1,4) \text{ and } C(2,-9)$ $d) C(0,-1), D(1,2) \text{ and } E(2,7)$

Solutions

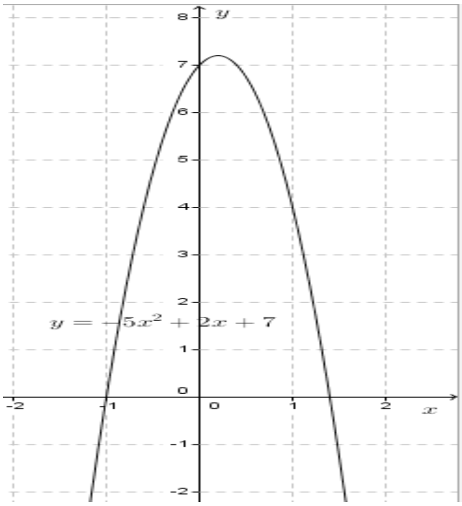
$a) y = x^2 + 1$



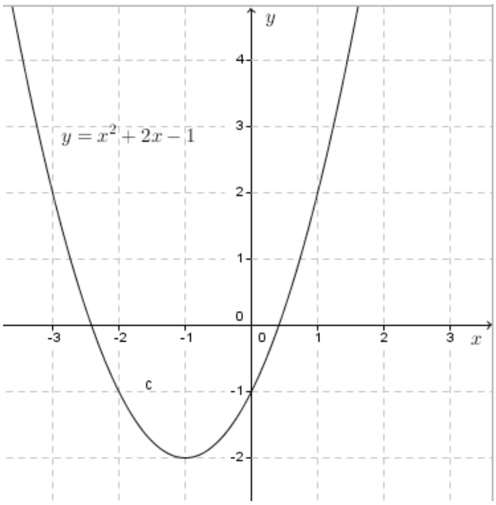
$b) y = x^2 - 2x + 1$



$c) y = -5x^2 + 2x + 7$



$d) y = x^2 + 2x - 1$



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