Subsidiary Mathematics

Teacher's Guide Book 4

Contents

Preface

To help the teacher to use the Subsidiary Mathematics-Learner's Book S4, that book also has a Teacher's Guide to go with it.

It is for this reason that we developed this teacher's guide to give pieces of advice about teaching particular units. This teacher's guide does not try to explain how to teach the whole subject and also does not turn into a training course for teachers. It gives the general introduction to the new curriculum the differences between the competence-based and knowledge-based approach. It contains guidance for the teacher about teaching the subject and on assessment both formative and summative. This teacher's guide contains some examples of lesson plan and scheme of work.

We hope that this teacher's guide for Subsidiary Mathematics-S4 will help teachers about teaching Subsidiary Mathematics-S4.

General introduction

1 General introduction to the new

The curriculum for Rwandan schools at primary and secondary levels has been changed from knowledge and content bases to competence based (CBE). CBE is of great importance in aligning Rwanda's education to the social and economic demands of society. It also presents answers to concerns about the capability and employability of school graduates.

2 General guidance to teachers

In Subsidiary mathematics-Learner's Book S.4, there are ten units. There are many activities to be done by learners before a new lesson. This will help learners to understand well the lesson.

Teacher must help learners to do those activities. Form groups of atleast six learners and let them do the activities in those groups.

At the end of the lesson, there is a series of exercises which summarise the lesson taught. As a teacher, let the learners do those exercises and correct them on a chalkboard.

Also at the end of each unit, there is a series of exercises which summarise the whole unit.

3 List of equipments needed for the subject

Learners will need geometric instruments for sketching curves and scientific calculators for some calculations.

4 General guidance on assessment both formative and summative

Assessment is the use of a variety of procedures to collect information about learning and instruction. Formative assessment is commonly referred to as assessment for learning, in which the focus is on monitoring a learner's response to and progress with instruction. Summative assessment provides immediate feedback

to both the teacher and learner regarding the learning process. Formative and summative assessments contribute in different ways to the larger goals of the assessment process.

The teacher must provide oral or written feedback to learners' discussion or work. For example, a teacher responds orally to a question asked in class; provides a written comment in a response or reflective journal; or provides feedback on learner's work.

At the end of each lesson, the teacher must give to the learners a small evaluation to see if they understood the lesson. Also at the end of the units, the teacher must give the general test which summarises the whole unit. When assessments reflect the stated learning objectives, a well-designed end of unit test provides teachers with information about individual learners (identifying any learner who failed to meet objectives), as well as provides an overall indication of classroom instruction.

Although formative and summative assessments serve different purposes, they should be used ultimately within an integrated system of assessment, curriculum, and instruction. To be effective in informing the learning process, assessments must be directly integrated with theories about the content, instruction, and the learning process and must be valid and reliable for the purposes for which they are used.

5 Guidance on grading and reporting

Academic achievement may be measured in a variety of ways, including compositions, presentations, oral discussion, learner work samples, observations, tests, and the products of project-based learning activities. Teachers should use the most current summative assessment data when determining achievement marks for the progress report. When determining what marks to use on daily and weekly assignments, remember that these marks should not conflict with the grades on the progress report. Teachers should not use letter grades when marking papers.

Some options may include:

- Raw scores or ratios (11/12 correct).
- Written feedback.
- Rubric scores (if using 4, 3, 2, or 1 on papers, there should be guidance as to what these marks mean).

Keep in mind that work that is sent home provides parents with a general impression of how learners are achieving in school but does not provide a complete picture. Other assessment data are collected that encompasses the progress report grade and some of these assessments are not sent home. Communication regarding progress should be ongoing.

Homework can be considered as part of the effort grade, but would not be used to grade academic achievement in elementary school since the function of Homework is to provide practice in skill areas.

Achievement marks will be reported on a 4-point scale and cannot be equated to former guidelines for letter grades. A grade of "4" indicates a high level of achievement; it communicates that a learner has a strong understanding of all the concepts and skills taught for that standard during the quarter and can demonstrate understanding independently and with very few errors. When determining grades for learners, teachers should consider the most current assessment data as evidence of learning. Earlier assessments may no longer be relevant if learners have demonstrated further progress.

Content map

The following table summarises every unit in Learner's Book.

	Unit 1 **Fundamentals of trigonometry**	**Unit 2** **Set IR of real numbers**
Number of periods	15 + Homework	12 + Homework
Introduction	Trigonometry studies relationship involving lengths and angle of a triangle. Trigonometry is really relevant in our day to day activities. In this unit, we will see how we can use this to resolve problems we might encounter.	Powers are a convenient way of writing multiplications that have many repeated factors. An expression that has a square root, cube root, ... etc is called a radical expression. A logarithm is the power to which a number must be raised in order to get some other number. In finance and business logarithms can be useful for calculating compound interest.
Classroom organisation	Whole class orientation; then working in groups.	Whole class orientation; then working in groups.
Equipment required	• Note books and pens • Manila papers and markers • Instruments of geometry • Scientific calculator	• Note books and pens • Manila papers and markers • Instruments of geometry • Scientific calculator
Activities	Group work, Practical, home work and research	Group discussion, pairing and home work
Competences practiced	• Team work. • Creativity. • Research.	Team work, data analysis.
Language practice	Discussion in group, presentation of findings.	Presenting result obtained in activities.
Vocabulary acquisition	• Trigonometry. • Radian. • Grade.	Logarithm.
Study skills	Descripture, drawing	• Writing a power in factor form. • Find the absolute value of a real number.
Revision	Revision exercises provided.	Revision exercises provided.
Assessments	• Formative assessments. • Summative assessments.	• Formative assessments. • Summative assessments.

	Unit 1 Fundamentals of trigonometry	Unit 2 Set IR of real numbers
Learning outcomes	• Define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle – know special values, • Convert radians to degree and vice versa. • Differentiate between complementary angles, supplementary angles and co-terminal angles. • Use trigonometric identities. • Apply trigonometric formulae in real world problems.	• Define a power, an exponential, a radical, a logarithm, the absolute value of a real number. • Illustrate each property of a power, an exponential, a radical, a logarithm, the absolute value of a real number. • Use logarithm and exponentials to model simple problems about growth, decay, compound interest, magnitude of an earthquake ...

	Unit 3 Linear, quadratic equations and inequalities	Unit 4 Polynomial, rational and irrational functions
Number of periods	12 + Homework	9+ Homework
Introduction	An equation is statement that the values of two mathematical expressions are equal while an inequality is a statement that the values of two mathematical expressions are not equal. A quadratic equation is an equation of degree 2, means that the highest exponent is 2. There are many financial decisions we make everyday based on how much we earn, less all our expenses, then we say we can spend up to x on a new purchase or you put x in a savings account, or something else. These are all inequalities. Quadratics equations and inequalities are useful in calculating areas, figuring out profits, finding speeds, athletics...	A polynomial is an expression that can have constants, variables and exponents that can be combined using addition, subtraction, multiplication and division. Polynomials are used to describe curves of various types; people use them in the real world to draw graph curves.

	Unit 3 **Linear, quadratic equations and inequalities**	**Unit 4** **Polynomial, rational and irrational functions**
Classroom organisation	Whole class orientation; then working in groups.	Whole class orientation; then working in groups.
Equipment required	• Note books and pens • Manila papers and markers • Instruments of geometry • Scientific calculator	• Note books and pens • Instruments of geometry • Scientific calculator
Activities	Group discussion, home work and research	Group discussion, class or home work and research
Competences practiced	Calculation and creativity.	Creativity, data analysis, communication skills.
Language practice	Communication skills.	Presenting result obtained in activities.
Vocabulary acquisition	Parameter, roots, discriminent.	
Study skills	Calculation, solving, discussion, explanation,….	Solving, modelling and creativity.
Revision	Revision exercises provided.	Revision exercises provided.
Assessments	• Formative assessments. • Summative assessments.	• Formative assessments. • Summative assessments.
	• Solve linear or quadratic equation. • Discuss the solution of parametric equation of the first degree or second degree. • Solve linear or quadratic inequality. • Discuss the solution of parametric inequality of the first degree. • Solve a system of linear or quadratic equations • Use equations and inequalities to solve word problems. • Apply equations and inequalities in real life problems.	• Factorisation of polynomials. • Generalities on numerical functions. • Application of rational and irrational functions.

	Unit 5 **Limits of polynomial, rational and irrational functions**	**Unit 6** **Differentiation of polynomials, rational and irrational functions**
Number of periods	9 + Homework	9+ Homework
Introduction	The limit of a function is a fundamental concept in calculus and analysis concerning the behaviour of that function near a particular point. Limits are also used to find the velocity and acceleration of a moving particle.	Calculus is concerned with things that do not change at a constant rate. The values of the function called the derivative will be that varying rate of change. Derivatives are met in many engineering and science problems, especially when modeling the behaviour of moving objects.
Classroom organisation	Whole class orientation; then working in groups.	Whole class orientation; then working in groups.
Equipment required	• Note books and pens • Manila paper and markers • Instruments of geometry • Scientific calculator	• Note books and pens • Manila paper and markers • Instruments of geometry • Scientific calculator
Activities	Group discussion, homework and research	Group discussion, practical, homework and research
Competences practiced	Use limits of a function in daily life.	Use derivative in daily life.
Language practice	Presenting result obtained in activities.	Presenting result obtained in activities.
Vocabulary acquisition	Limit. Asymptote.	Derivative.
Study skills	Analysis, explanation, discussion.	Finding limit of a function, explanation, calculation, creativity.
Revision	Revision exercises provided.	Revision exercises provided.
Assessments	• Formative assessments. • Summative assessments.	• Formative assessments. • Summative assessments.
Learning outcomes	• Concepts of limits. • Indeterminate cases. • Applications.	• Concepts of derivative of a function. • Rules of differentiation. • Apply the concepts of and techniques of differentiation to model, analyze and solve rates or optimization problems in different situations.

	Unit 7 **Vectors space of real numbers**	**Unit 8** **Matrices and determinants of order 2**
Number of periods	6+ Homework	9+ Homework
Introduction	A vector space (also called a linear space) is a collection of objects called vectors, which may be **added** together and **multiplied** by numbers, called **scalars** in this context. To put it really simple, vectors are basically all about directions and magnitudes. These are critical in basically all situations.	A matrix is a rectangular arrangement of numbers, expressions, symbols which are arranged in rows and columns. Matrices play a vital role in the projection of a three dimensional image into a two dimensional image. Matrices are used for taking seismic surveys.
Classroom organisation	Whole class orientation; then working in groups.	Whole class orientation; then working in groups.
Equipment required	• Note books and pens • Manila paper and markers • Instruments of geometry • Scientific calculator	• Note books and pens • Scientific calculator
Activities	Group discussions, practical, class or home work and research	Group discussions, homework and research
Competences practiced	Vector space in daily life.	Use matrices in daily life.
Language practice	Presenting result obtained in activities.	Presenting result obtained in activities.
Vocabulary acquisition		Logarithm.
Study skills	Calculation, analysis.	Calculus, analysis and creativity.
Revision	Revision exercises provided.	Revision exercises provided.
Assessments	• Formative assessments. • Summative assessments.	• Formative assessments. • Summative assessments.

	Unit 7 Vectors space of real numbers	Unit 8 Matrices and determinants of order 2
Learning outcomes	• Define and apply different operations on vectors. • Define linear combination of vectors. • Find the norm of a vector. • Calculate the scalar product of two vectors. • Calculate the angle between two vectors. • Apply and transfer the skills of vectors to other area of knowledge.	• Concepts of matrices. • The inverse of a matrix of order 2. • Operations on matrices of order 2. • Determine the matrix of a linear transformation in 2D. • Construct the matrix of the inverse of an isomorphism of IR2. • The matrix of geometric transformation in 2D.

	Unit 9 Measures of dispersion	Unity 10 Elementary probability
Number of periods	7 + Homework	21+ Homework
Introduction	The word dispersion has a technical meaning in statistics. The average measures the centre of the data. It is one aspect observations. The study of dispersion is very important in statistical data.	Probability is a common sense for scholars and people in modern days. It is the chance that something will happen-how likely it is that some event will happen. Some applications of the probability theory are character recognition, speech recognition, opinion survey, missile control, seismic analysis...
Classroom organisation	Whole class orientation; then working in groups.	Whole class orientation; then working in groups.
Equipment required	• Note books and pens • Manila paper and markers • Instruments of geometry specifically a ruler • Scientific calculator	• Note books and pens • Instruments of geometry ruler • Scientific calculator
Activities	Group discussions, class or home work and research	Group discussions, practical, class or home work and research
Competences practiced	Use measures of dispersion when interpreting data.	Use combinatorial analysis in daily life. Use probability in game theory.

	Unit 9 **Measures of dispersion**	**Unity 10** **Elementary probability**
Language practice	Presenting result obtained in activities.	Presenting result obtained in activities.
Vocabulary acquisition	Variance. Standard deviation. Coefficient of variation.	• Combinatorial • Arrangement • Permutation • Combination • Probability • Sample space • Event
Study skills	Calculation, analysis and interpretation of result.	Counting, critical thinking, explanation, creativity and interpretation.
Revision	Revision exercises provided.	Revision exercises provided.
Assessments	• Formative assessments. • Summative assessments.	• Formative assessments. • Summative assessments.
Learning outcomes	• Determine measures of dispersion. • Variance. • Standard deviation (including combined set of data). • Coefficient of variation. • Apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.	• Construct Pascal's triangle. • Distinguish between permutations and combinations. • Determine the number of permutations and combinations of "n" items, "r" taken at a time. • Determine the sample space of an experiment. • Explain different concept relating to events. • Find probability of different events. • Determine probability of an event with an equiprobable sample space.

Guidance about how to use the Learner's Book (Scheme of work)

Unit plan/ scheme of work 1

Academic year: …….. Term: One

Subject: MATHEMATICS Teacher's name: …….....

Dates & number of lessons (periods) in a week		Units + Key Unit competences	Lessons + evaluation
1	From .../...(Mon) to .../...(Friday) 3 periods	**Unit 1** Fundamentals of trigonometry **No. of lessons:15**	Angle and its measurements Unit circle
2	From .../...(Mon) to .../...(Friday) 3 periods	Key unit competence: Use the trigonometric concepts and formulas to solve related problems in Physics, Air navigation, Water navigation, bearings, Surveying, ...	Trigonometric ratios Evaluation 1
3	From .../...(Mon) to .../...(Friday) 3 periods	Learner's Book	Trigonometric identities Evaluation 2
4	From .../...(Mon) to .../...(Friday) 3 periods		Triangles and applications: • Bearing
5	From .../...(Mon) to .../...(Friday) 3 periods		• Air • Navigation • Inclined plane...
6	From .../...(Mon) to .../...(Friday) 3 periods	**Unit 2** Set of real numbers **No. of lessons:12**	Absolute value and its properties Evaluation 1

School: ……

Class + Combination: S.4: .....

	Learning objectives	Teaching methods & techniques + Evaluation procedures	Resources & references	Observa-tions
	• Convert radians to degree and vice versa, • Represent graphically sine, cosine and tangent, functions and, together with the unit circle.	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - Charts - ICT Tools	
	• Define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle and know special values.			
	• Use trigonometric identities			
	• Use trigonometry, including the sine and cosine rules, to solve problems involving triangles			
	Determine the restrictions on the variables in rational and irrational expressions	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - ICT Tools	

7	From .../...(Mon) to .../...(Friday) **3 periods**	**Key unit competence:** Think critically to understand and perform operations on the set of real numbers	Powers and radicals Evaluation 2
8	From .../...(Mon) to .../...(Friday) **3 periods**	**Learner's Book**	Decimal Logarithms Evaluation 3
9	From .../...(Mon) to .../...(Friday) **3 periods**		Properties and applications.
10	From .../...(Mon) to .../...(Friday) **3 periods**	**Unit 3** **Linear, quadratic equations and inequalities**	Equations and inequalities in one unknown Evaluation 1
11	From .../...(Mon) to .../...(Friday) **3 periods**	**No. of lessons:12**	Simultaneous equations in two unknowns ... Evaluation 2
12	From .../...(Mon) to .../...(Friday) **3 periods**	**Key unit competence:** Model and solve algebraically or graphically daily life problems using linear equations or inequalities **Learner's Book**	Applications: • Economics (Problems about supply and demand analysis, ...)
			SUMMATIVE EVALUATION

	• Illustrate each property of a power, an exponential, a radical, a logarithm, the absolute value of a real number • Use logarithm and exponentials to model simple problems about growth, decay, compound interest, magnitude of an earthquake ...			
	• List and clarify the steps in modeling a problem by linear equations and inequalities.	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - Charts - ICT Tools	
	• Solve linear equations and **simultaneous** equations on a graph paper.			
	• Appreciate, value and care for situations involving to linear, quadratic equations and inequalities in daily life situation.			

Unit plan/ scheme of work 2

Academic year: ..... Term: TWO

Subject: MATHEMATICS Teacher's name: .....

Dates & number of lessons (periods) in a week		Units + Key unit competences	Lessons + Evaluation
1	From .../...(Mon) to .../...(Friday) 3 periods	**Unit 3** **Linear, quadratic equations and inequalities(cont.)**	Applications: • Physics (Linear motions, electric circuits, projectile motions, ...). • Chemistry (Balancing equations,...) • Masonry (Arched shape...)
2	From .../...(Mon) to .../...(Friday) **3 periods**	**Unit 4** **Polynomial, rational and irrational functions** **No. of lessons:9**	Generalities on numerical functions: • Definitions • Domain and range of a function. Evaluation 1
3	From .../...(Mon) to .../...(Friday) **3 periods**	**Key unit competence:** Use concepts and definitions of polynomial, rational and irrational functions to determine the domain of polynomial, rational and irrational functions and represent them graphically in simple cases...	Parity of a function (odd or even). Evaluation 2
4	From .../...(Mon) to .../...(Friday) 3 periods	**Learner's Book**	Graphical representation of linear and quadratic functions and their use and interpretation in Economics, Physics... Evaluation 3
5	From .../...(Mon) to .../...(Friday) 3 periods	**Unit 5** **Limits of polynomial, rational and irrational functions** **No. of lessons:9**	Concepts of limits Evaluation 1

School: .....

Class + Combination: S4: .....

	Learning objectives	Teaching methods & techniques + evaluation procedures	Resources & references	Observations
	• Appreciate, value and care for situations involving to linear, quadratic equations and inequalities in daily life situation.	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - Charts - ICT Tools	
	Determine the domain and range of a function.	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - Charts - ICT Tools	
	Find whether a function is even, odd, or neither.			
	Analyse, model and solve problems involving linear or quadratic functions and interpret the results.			
	• Calculate limits of certain elementary functions. • Removing indeterminate forms.	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - Charts - ICT Tools	

6	From .../...(Mon) to .../...(Friday) **3 periods**	**Key unit competence:** Evaluate correctly limits of functions and apply them to solve related problems	Indeterminate cases: $\frac{\infty}{\infty}, \frac{0}{0}, \infty - \infty, 0 \cdot \infty$ Evaluation 2
7	From .../...(Mon) to .../...(Friday) **3 periods**	**Learner's Book**	Applications: • Continuity of a function at a point or on interval I • Asymptotes Evaluation 3
8	From .../...(Mon) to .../...(Friday) **3 periods**	**Unit 6** **Differentiation of polynomials, rational and irrational functions and their applications**	Concepts of derivative of a function Evaluation 1
9	From .../...(Mon) to .../...(Friday) **3 periods**	**No. of lessons:9**	Rules of differentiation Evaluation 2
10	From .../...(Mon) to .../...(Friday) **3 periods**	**Key unit competence:** Use the gradient of a straight line as a measure of rate of change and apply this to line tangent and normal of curves in various. **Learner's Book**	Applications of differentiation Evaluation 3
11	From .../...(Mon) to .../...(Friday) **3 periods**	**Unit 7** **Vectors space of real numbers** **No. of lessons:6**	• Dot product and properties • Modulus or Magnitude of vectors Evaluation 1
12		**Key unit competence:** Use concepts of vectors in 2D to solve related problems such as distance, angles... **Learner's Book**	• Angle between two vectors Applications
			SUMMATIVE EVALUATION

	• Solve problems involving continuity			
	• Use the concepts of limits to calculate the asymptotes to the rational functions.			
	• Evaluate derivatives of functions using the definition of derivative.	- Group discussions - Questioning - Research - Team work	-Text books - Internet -Local environment - Charts - ICT Tools	
	• Evaluate derivative using rules of derivative.			
	Use first principles to determine the gradient of a straight line at a point.			
	• Calculate the scalar product of two vectors. • Find the magnitude of a vector.	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - Charts - ICT Tools	
	• Determine the angle between two vectors • Apply and transfer the skills of vectors to other area of knowledge.			

Unit plan/ scheme of work 3

Academic year: ……. Term: Three

Subject: Mathematics Teacher's name: ……....

Dates & number of lessons (periods) in a week		Units + Key unit competences	Lessons + evaluation
1	From .../...(Mon) to .../...(Friday) 3 periods	**Unit 8** **Matrices of and determinants of order 2** **No. of lessons:9** **Key unit competence:** Use matrices and determinants of order 2 to solve other related problems such as organisation of data in a shopping, in Cryptography, in Physics (problems about quantum or circuits), … **Learner's Book**	Definition of matrix Operations on matrix of order 2: – Equality of matrices – Addition – Multiplication by a scalar – Multiplication of matrices – Transpose of a matrix Evaluation 1
2	From .../...(Mon) to .../...(Friday) 3 periods		**Determinant and inverse of a square matrix** • Determinant and properties Evaluation 2
3	From .../...(Mon) to .../...(Friday) 3 periods		• Inverse of a square matrix **Applications** Evaluation 3
4	From .../...(Mon) to .../...(Friday) 3 periods	**Unit 9** **Measures of dispersion** **No. of lessons: 6**	• Variance • Standard deviation Evaluation 1

School: .....

Class + Combination: S4: .....

	Learning objectives	Teaching methods & techniques + evaluation procedures	Resources & references	Observations
	• Define the order of a matrix. • Define operations on matrices of order 2. • Perform operations on matrices of order 2.	- Group discussions - Questioning - Research - Team work	- Text books - Internet -Local environment - ICT Tools	
	Show that a square matrix of order 2 is invertible or not.			
	Determine the inverse of a matrix of order 2 Use matrices to solve problems.			
	• Determine variance • Determine standard deviation	- Group discussions - Questioning - Research - Team work	-Text books -Internet -Local environment - ICT Tools	

5	From .../...(Mon) to .../...(Friday) 3 periods	**Key unit competence:** Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation. **Learner's Book**	• Coefficient of variation • Applications Evaluation 2
6	From .../...(Mon) to .../...(Friday) 3 periods	**Unit 10** **Elementary probability** **No. of lessons: 21**	Counting techniques: • Venn diagram • Tree diagrams • Contingency table Evaluation 1
7	From .../...(Mon) to .../...(Friday) 3 periods	**Key unit competence:** Use combinations and permutations to determine probabilities of occurrence of an event.	• Multiplication principles Arrangement and Permutations Evaluation 2
8	From .../...(Mon) to .../...(Friday) 3 periods	**Learner's Book pages**	Combinations Evaluation 3
9	From .../...(Mon) to .../...(Friday) 3 periods		
10	From .../...(Mon) to .../...(Friday) 3 periods		Concepts of probability Evaluation 4
11	From .../...(Mon) to .../...(Friday) 3 periods		Properties and formulae Evaluation 5
12	From .../...(Mon) to .../...(Friday) 3 periods		**SUMMATIVE EVALUATION**

	• Determine coefficient of variation • Use measures of dispersion in data analysis.			
	Use counting techniques to solve related problems. solve problems involving factorial notation.	- Group discussions - Questioning - Research - Team work	- Text books - Internet - Local environment - ICT Tools	
	Determine the number of permutations and combinations of “n” items, “r” taken at a time. Use properties of combinations for finding coefficients in Pascal’s triangle.			
	Use and apply elementary properties of probability to calculate the number possible outcomes of occurring event under equally likely assumptions.			

Examples of lesson plan

Example 1

Academic year: …….. Term: TWO

School: …… Subject: Mathematics

Teacher's name: …….... Class + Combination: S4.....

Term	Date	Subject	Class	Unit N°	Lesson N°	Duration	Class size
2	.....	Mathematics	.....	4	.....	40 minutes	.....

Type of special educational needs and number of learners 4 low vision learners. To avail big printed documents and facilitate these learners. Avoid making a group of low vision only otherwise it can be considered as segregation. 5 Gifted learners: to encourage them to explain, to each other and help their classmates.	
Topic area:	ANALYSIS
Sub-topic area:	FUNCTIONS
Unit title	POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS
Key Unit Competence:	Use concepts and definitions of polynomial, rational and irrational functions to determine the domain of polynomial, rational and irrational functions and represent them graphically in simple cases...
Title of the lesson	Domain of definition of a polynomial and a rational function.
Instructional objective	**Given polynomial function or irrational function, the learner should be able to determine their domain of definition accurately.**
Plan for this Class	Location: Classroom Learners are organized into groups of 6 and they have to do activity 3 in their groups.
Learning Materials	Calculator
References	Learner's Book

Description of teaching and learning activity

In groups of 6, learners will do activity 3 and each group will report the result to the class. At the end of the lesson, an assignment to be discussed as an activity of the next lesson "Finding domain of definition of irrational function" will be gven to the learners.

Timing for each step	Teacher's activities	Learners' activities	Competences and cross cutting issues to be addressed
Introduction **5 minutes**	Ask questions on previous lesson. **Question:** Give three examples of; 1. polynomial function 2. rational function	Respond to questions on the chalkboard **Answers:** Answers may vary. **Ex:** 1. $f(x)=x+1$ $g(x)=x^2+3x-3$ $g(x)=x^2+3x-3$ 2. $f(x)=\frac{x+1}{3x+1}$ $g(x)=\frac{x^2+3x-3}{x-2}$ $h(x)=\frac{(x-4)^4}{x}$	Students are developing communication skills when they are explaining and sharing ideas
Body of the lesson **10 minutes**	**Step 1:** Form groups of at least 6 learners each Request the learners to do activity 3 in their groups Goes round to check the progress of the discussion, and intervenes where necessary.	In their groups, learners will do activity 3	Cooperation and interpersonal management developed through working in groups Communication: learners communicate and convey information and ideas through speaking when they are presenting their work.

5 minutes	**Step 2:** Request a reporter from each group to present the work on the chalkboard.	Reporter represents the work. Learners interact through questions and comments. **Answers:** 1. No value 2. 0 3. 1	Self confidence: learners will gain self confidence competence when they are presenting their work
2 minutes	**Step 3:** Capture the main points from the presentation of the learners and summarise them as follows: Given that $f(x)$ is polynomial, then the domain of definition is the set of real numbers. That is $Domf = \mathbb{R}$ Given that $f(x) = \frac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are polynomials, then the domain of definition is the set of real numbers excluding all values where the denominator is zero. That is $Domf = \{x \in \mathbb{R}: h(x) \neq 0\}$	Take notes in their note books.	The fact of being convinced without fighting peace, education and values are developed too.
Conclusion **3 minutes** **7 minutes**	Request learners to give the main points of the learmed lesson Request learners to do exercises 3 in their respective groups. Goes round to check the progress of the discussion, and intervenes where necessary.	Summarize the learned lesoon Work in groups	

5 minutes	Request some learners to answer to the questions in exercises 3 on chalkboard. Ensures that the learners understood the learned lesson and decide to repeat the lesson or to continue with new lesson next time.	Do exercises on chalkboard.	
3 minutes	Give to the learners an individual evaluation (quiz). **Lead into next lesson** Give to the learners the the homework which will be an activity for the next lesson	Do the given quiz individually.	

Example 2

Academic year: …….. Term: THREE
School: …… Subject: MATHEMATICS
Teacher's name: …….... Class + Combination: S4.....

Term	Date	Subject	Class	Unit N°	Lesson N°	Duration	Class size
3	.....	Mathematics	.....	8	.....	40 minutes	.....

Type of Special Educational Needs and number of learners

Two slow learners: Creation of a fun environment and relate new concepts with previous one.

Four talented learners: To encourage them to do their own research and learned lesson or lesson to be learned next time and explain to each other.

Topic area:	Linear algebra
Sub-topic area:	Linear transformation in 2 dimensions.
Unit title	Matrices and determinants of order 2.
Key Unit Competence:	Use matrices and determinants of order 2 to solve other related problems such as organization of data in a shopping, in Cryptography, in Physics (problems about quantum or circuits), …
Title of the lesson	Inverse of matrix
Instructional objective	**Given scientific calculator and matrix of order two, the learner should be able to find the inverse of matrix of order two accurately.**
Plan for this Class	Location: Classroom Learners are organized into groups of 6 and they have to do activity 2 in their groups.
Learning Materials	Calculator
References	Learner's Book

Description of teaching and learning activity

In groups of 6 learners, each group will do the activity 2 and will report the result to the class. At the end of the lesson, they are given an assignment to be discussed as an iactivity of the next lesson "Solving simultaneous equations".

Timing for each step	Teacher's activities	Learners' activities	Competences and cross cutting issues to be addressed
Introduction **5 minutes**	Ask questions on previous lesson. **Question:** Find the following determinants 1. $\begin{vmatrix} 11 & 0 \\ 6 & 3 \end{vmatrix}$ 2. $\begin{vmatrix} 6 & -2 \\ 1 & 3 \end{vmatrix}$ 3. $\begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix}$ 4. $\begin{vmatrix} 3 & 5 \\ 6 & 5 \end{vmatrix}$	Respond to questions on the chalkboard. **Answers:** 1. 27 2. 20 3. 0 4. -15	Students are developing communication skills when they are explaining and sharing ideas.
Body of the lesson **10 minutes**	**Step 1:** Form groups of at least 6 learners each Request the learners to do activity 2 in their groups. Goes round to check the progress of the discussion, and intervenes where necessary.	In their groups, learners will do activity 2 in Exercise books.	Cooperation and interpersonal management developed through working in groups

10 minutes	**Step 2:** Rerter from each group to present the work on the chalkboard.	Reporter represents the work. Learners interact through questions and comments. **Answers:** • $\begin{pmatrix} 3 & -2 \\ -6 & 10 \end{pmatrix}$ • $\begin{vmatrix} 10 & 2 \\ 6 & 3 \end{vmatrix} =$ $= 30 - 12 = 18$ • $\frac{1}{18}\begin{pmatrix} 3 & -2 \\ -6 & 10 \end{pmatrix}$	
5 minutes	**Step 2:** Request a reporter from each group to present the work on the chalkboard.	Reporter represents the work. Learners interact through questions and comments. Answers: $\frac{1}{18}\begin{pmatrix} 3 & -2 \\ -6 & 10 \end{pmatrix}$ The product is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ which is the identity matrix	Communication: learners communicate and convey information and ideas through speaking when they are presenting their work. Self confidence: learners will gain self confidence competence when they are presenting their work
2 minutes	**Step 3:** Capture the main points from the presentation of the learners and summarise them as follows: Consider the following matrix $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$	Take notes in their note books.	

	The inverse of **A** is $A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$		
Conclusion **3 minutes**	If the determinant is zero, the matrix has no inverse. Request learners to give the main points of the learned lesson	summarize the learned lesson Work in groups	
7 minutes **5 minutes** **3 minutes**	Request learners to do exercises 2 in their respective groups Goes round to check the progress of the discussion, and intervenes where necessary. Request some learners to answer to the questions in exercises 2 on chalkboard. Ensures that the learners understood the learned lesson and decide to repeat the lesson or to continue with new lesson next time. Give to the learners an individual evaluation (quiz). **Lead into next lesson** Give to the learners the the homework which will be an activity for the next lesson	Do exercises on chalkboard. Do the given quiz individually.	

Effective tips for brighter learners

The teacher must encourage learners to;

1. explain to each other as best as they can on the lesson learned.
2. have an exercise book, Homework book and note book.
3. have geometric instruments and scientific calculator.
4. worked exercises and Homework and check their answers to gain practice with every lesson.
5. do their own research on learned lessons or lessons to be learned next time.

Effective tips for slow learners

Mathematics may be challenging for a slow learner, but not impossible. Slow learners also want to learn mathematics, but due to lack of learning ability they are not able to learn faster. The following are some techniques which can help slow learners:

1. Slow learners need more time to understand any problem or to find out the answer. Give extra time to slow learners. This will increase their confidence. Do not pressurise learners to perform on time beyond their ability. This will only decrease confidence.
2. Slow learners need extra attention. With a small learner's group, you can effectively respond to each learner.
3. Environment is more potent than willpower. Create a fun environment for learners. Use new learning techniques, especially for slow learners. Teacher can provide mathematics games and activities to learners.
4. Build a helpful environment for learners. Encourage learners to ask questions and let them feel free to ask for any help.
5. Most of the slow learners face difficulty to understand the

new concepts. Try to relate the new concepts with previous concepts. This will help them to catch the new concepts relatively fast.

6. One of the best ways to teach maths to all learners is to explain its concepts using real life examples.
7. Whenever possible, provide opportunities to show them their work. Let the learners teach you about mathematics. This will help learners to reduce mathematics fear.
8. Because slow learners need more time to understand the concepts, frequent reviewing can help them out. Reviewing mathematics concepts time to time will allow them to master the maths concepts.
9. Slow learners tend to have lack of confidence, if you pressurise them for time management or anything, this will only reduce their confidence.
10. Slow learners tend to have low confidence. Low confidence impedes anyone's learning ability. If you reward them time to time, this will help them to raise their confidence.

Extension knowledge and ideas for teachers

The following are the most important principles in mathematics teaching.

Principle 1: Let it make sense

Let us strive to teach for understanding of mathematical concepts and procedures, the "why" something works, and not only the "how".

The "how" something works is often called procedural understanding; the learner knows how to work or solve a linear equation. It is often possible to learn the "how" mechanically without understanding why something works. Procedures learned this way are often forgotten very easily.

The relationship between the "how" and the "why" - or between

procedures and concepts - is complex. One doesn't always come totally before the other, and it also varies from learner to learner.

Try alternating the instruction; teach how to solve a linear equation, and let the learner practice. Then explain why it works.

As a teacher, don't totally leave a topic until the learner knows both "how" and understands the "why".

The teacher can often test a learner's understanding of a topic by asking him/her "Tell me an example of where linear equation is used in daily life."

Principle 2: Remember the goals

Teacher must:

- cover the curriculum by the end of school year.
- make sure the learners have a lot formative and summative assessments.

Generally, teacher must:

- enable the learners to understand information around us.
- prepare learners for further studies in mathematics.
- let learners see some beauty of mathematics and learn them to like it.

Principle 3: Know tools

First of all, of course comes a black or white board or paper — something to write on, then we have pencils, compass, protractor, ruler, eraser.... and the book the teacher is using.

Then we have computer software, interactive activities, animated lessons and such. There are workbooks, fun books, work texts, books, and online tutorials.

The teacher has to start somewhere, probably with the basics, and then add to his/ her "toolbox" little by little as you have opportunity. It's important to learn how to use any tool that the teacher might acquire.

Basic tools:

- The board and/or paper to write on. Essential. Easy to use.
- The learners' book and teacher's guide.
- The extras:
- Computer and projector.
- Internet connection.

If a computer lab is available at the school, the teacher can show the learners how ICT is used in mathematics. For example;

- writing mathematical expression using Microsoft Office Word or other software tools like Math-Type.
- sketching a function in Cartesian plane using Microsoft Office Excel.
- determine the mean, standard deviation, variance,... of a set of data using Microsoft Office Excel formulas.
- finding the limit and derivative of a function using MATLAB software...

Principle 4: Living and Loving Mathematics

Mathematics teachers have to ensure that they;

- use maths often in daily life.
- like mathematics.
- love mathematics.
- are happy to teach mathematics.

Some ideas for the teacher:

- let it make sense. This alone can usually make quite a difference and learners will stay interested.
- read through some fun maths books. Get to know some interesting maths topics besides just schoolbook arithmetic. There are lots of story books (maths readers) that teach maths concepts.
- consider including some maths history if you have the time.
- when you use maths in your daily life, explain how you're doing it, and include the children if possible. Figure it out together.

Additional activities

Research activities

Activity 1

Make research in advance in the library about Sets of numbers (natural numbers, integers, rational numbers and irrational numbers).

Activity 2

Discuss the importance and necessity of linear equations and inequalities and how it takes place in the trade.

Activity 3

Discuss the importance and necessity of a quadratic equation and a quadratic inequality and how it takes place in Finance problems, Economics problems, Physics.

Activity 4

Model or interpret the problems related to polynomial functions.

Activity 5

Make research about the importance and use of matrices for example in Physics, Economics, Entrepreneurship, Sports, ..., and report the findings.

Activity 6

Use the letters from your proper words and create your own words, e.g: use letters of “MISSISSIPI”, without prior instructions, to create news words.

Other activities

Activity 7

Place the sixteen squares in an envelope. Divide learners into groups (groups of three work well). Give each group an envelope of squares, and instruct them to match equivalent expressions to create one large square, lining up equivalent values.

Learners will review and learn common equivalents and will be better prepared to deal comfortably with trigonometric functions in more advanced studies. If a group seems to be having difficulty, suggest identifying one of the corner squares to get started.

Activity 8

Imagine that you have boarded an airplane. The rows are numbered from 1 to 30, and there are six seats per row, three on each side of the isle. Seats in each row are labeled A through F. Using that information,

a) How many seats are in the airplane?
b) What are your chances of sitting in row number 7?
c) What are your chances of sitting in a window seat?
d) What are your chances of sitting in an "A" seat?
e) What are your chances of sitting in an even-numbered row?

Learners will be better prepared to deal comfortably with combinatorial.

Activity 9

Learners are given a task of sitting 3 men and 4 women at random in a row. In groups, they discuss about the probability that all the men are seated together then they give feedback.

Learners will be better prepared to deal comfortably with probability.

Activity 10

Imagine you are a professional mathematician and you are working on developing a mathematical model to describe the dynamics of a City. You first have to identify all the variables (quantities that vary) and constants (quantities that stay the same) playing a role in that City.

Make a list of all the 'players' or 'elements' in this setting. Some examples are: the car park, the hawkers or the number of shops on the first floor.

Listed below are some more examples of 'players' or 'elements' in this context. Between this list and your own examples, decide which are variables (with quantities that vary) and which are constants (with quantities that stay the same). Will any of these be both? If so, what would this depend on?

Consider the number of:

- police women and men who work at the police department that is in charge of security at the complex.
- car parks.
- people employed by the municipal corporation that is in charge of civic maintenance of the complex.
- parking lot attendants.
- hawkers.
- escalators.
- shop owners whose shop is on the first floor.
- restaurant owners on the ground floor.
- electricity supply companies.
- visitors who desire to purchase a laptop.

Identify variables and constants in the real-life situation of the City.

Learners will be better prepared to deal comfortably with linear equations.

Activity 11

The following equations each show the relationship between household income per week (x) and household expenditure on a particular good per week (y):

1. $y = 100 + 0.3x$
2. $y = 200 - 0.15x$
3. $y = 25$
4. $y = -200 + 0.05x$

Answer the following questions relating to equations (1)-(4).

a) What are these equations called by Economists?

b) To what sort of good do you think that each equation relates? Explain your answers.

c) In each case, calculate weekly expenditure on the good for a household earning 1000 per week.

d) In each case, where applicable, find the income level at which expenditure equals zero.

Learners will be better prepared to deal comfortably with linear equations.

Activity 12

Draw a square on board or paper, and draw one diagonal into it. Make the sides of the square to be, say, 5 units. Then make the picture to be a right triangle by wiping out the two sides of square. How can you find the length of the longest side of the triangle?

Learners will be better prepared to deal comfortably with quadratic equations.

General Methodology

Follow the following three steps when teaching any lesson.

Introduction

Reviews previous lesson through asking the learners some questions. If there is no previous lesson ask them pre-knowledge questions on the day lesson.

Body of the lesson

Give an activity to learners that will be done in groups or individually. Invite one or more groups for presentation of their work to other groups. If the activity is individual ask one or more learners to present his/her work to others. After activities capture the main points from the presentation of the learners and summarize them.

Conclusion

Ask learners what did they learn in day lesson. Request learners to do exercises in their respective groups. Request learners to correct exercises on chalkboard and give them individual evaluation. Remember to give homework to the learners. Give them two home works: one for the lesson of the day and another which will be activity for the next lesson.

Unit 1

Fundamentals of Trigonometry

Aim

Use trigonometric circle and identities to determine trigonometric ratios and apply them to solve related problems.

Guidance on introductory activity 1

Student-teachers work on the introductory activity to understand the use of trigonometry.

Let them read and do the introductory activity 1 in the Student-teacher's book.

Make sure that all student-teachers are activating and performing well.

Through class discussions, let student-teachers think of different ways of application of trigonometry.

Through different examples, help student-teachers to understand the importance of trigonometry by showing their application in real life for example in construction, satellite systems and astronomy, naval and aviation industries, land surveying and in cartography (creation of maps) and so on.

Answer of introductory activity 1

Pythagoras theorem is not enough for finding the height of the given cathedral. By using sine rule, the required height can be determined as

$$\frac{\sin 60^0}{h} = \frac{\sin 30^0}{280} \Leftrightarrow h\sin 30^0 = 280\sin 60^0 \Leftrightarrow \frac{h}{2} = \frac{280\sqrt{3}}{2}$$

$$\Leftrightarrow h = 280\sqrt{3} \simeq 484.97m$$

Objectives

By the end of this unit, the learners will:

- define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle – know special values.
- convert radians to degree and vice versa.
- use trigonometric identities.
- apply trigonometric formulae in real world problems.

Vocabulary

Trigonometry, radian, grade, sine, cosine, secant, cosecant, tangent and cotangent.

Contents

1 Trigonometric concepts

Recommended teaching time: 9 periods

This section introduces the conversion of angles in their units of measurements (degrees, radians and grades). It looks at trigonometric ratios, trigonometric number of an angle and trigonometric identities.

$\frac{D}{180} = \frac{R}{\pi} = \frac{G}{200}$, where D stands for degree, R for radians, G for grades and $\pi = 3.14...$

In a right triangle

$$\sin\alpha = \frac{opposite\ side}{hypotenuse}, \cos\alpha = \frac{adjacent\ side}{hypotenuse}, \tan\alpha = \frac{opposite}{adjacent}$$

$$\csc\alpha = \frac{hypotenuse}{opposite} = \frac{1}{\sin\alpha}, \sec\alpha = \frac{hypothenuse}{adjacent} = \frac{1}{\cos\alpha}$$

$$and\ \cot\alpha = \frac{adjacent}{opposite} = \frac{1}{\tan\alpha}$$

2 Triangle and applications

Recommended teaching time: 6 periods

This section introduces the sine and cosine rule and their applications.

The cosine law says that

$$\begin{cases} a^2 = b^2 + c^2 - 2bc\cos A \\ b^2 = a^2 + c^2 - 2ac\cos B \\ c^2 = a^2 + b^2 - 2ab\cos C \end{cases}$$

The sine law is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Materials, Methodology, Answers to activities and exercises

1 Trigonometric Concepts

Activity 1

Materials

Manila paper if not sheets of paper or notebook, Markers or pens, Instruments of geometry

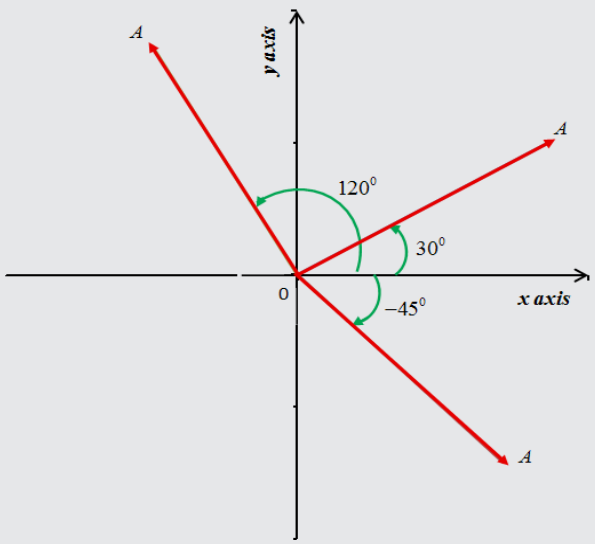
Methodology

To form a group of atleast four learners and facilitate them during this activity.

To check if each group is rotating the given vector correctly on the manila paper (or sheet of paper).

Invite one group for representation of its work to other group.

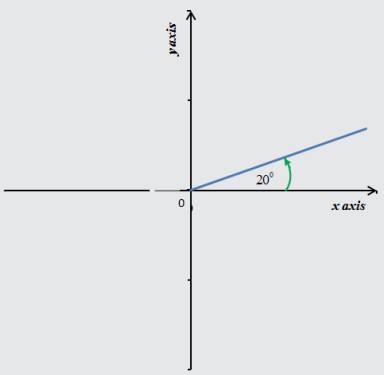
Answers



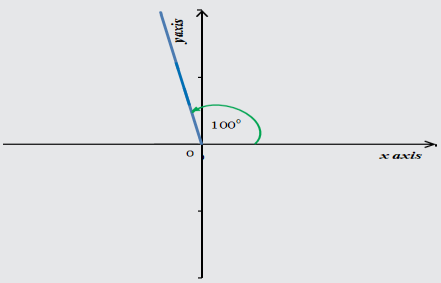
Answers to Exercise 1

1.

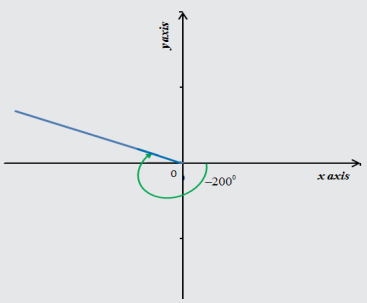
a)



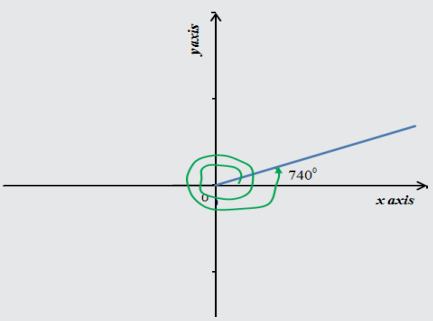
b)



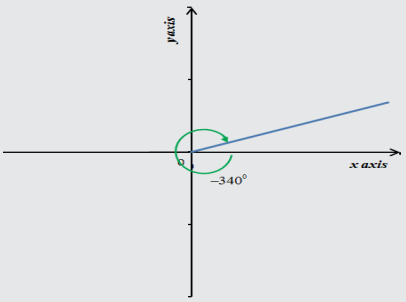
c)



d)



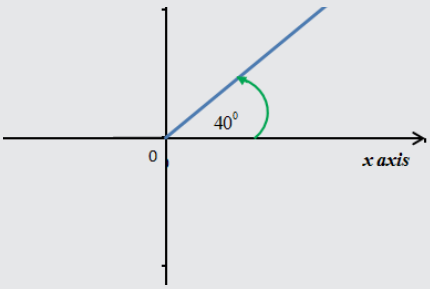
e)



740^0 and -340^0 are co terminal to 20^0

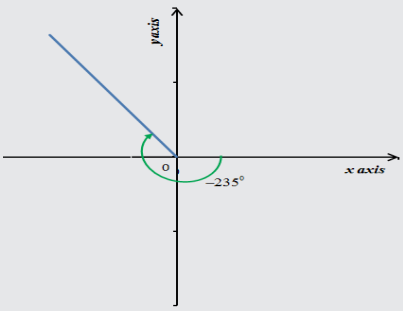
2.

a)



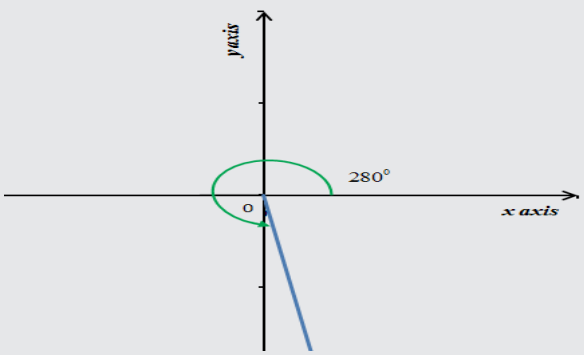
40^0 is in first quadrant

b)



-235^0 is in second quadrant

c)



280^0 is in fourth quadrant

Activity 2

Materials

Notebook, pens, scientific calculator

Methodology

This is a pairing activity. Facilitate every learner during this activity.

Get different answers from learners and emphasize on conversion.

Answers

1. $\frac{80}{3}$ 2. $\frac{2480}{9}$ 3. π 4. $\frac{3\pi}{2}$

Answers to Exercise 2

1. $\frac{11}{10}\pi$ radians , 198^0 2. $\frac{62\pi}{9}$ radians , $\frac{12400}{9}$ grades
3. 72^0 4. $5^0\ 36'\ 00''$

Activity 3

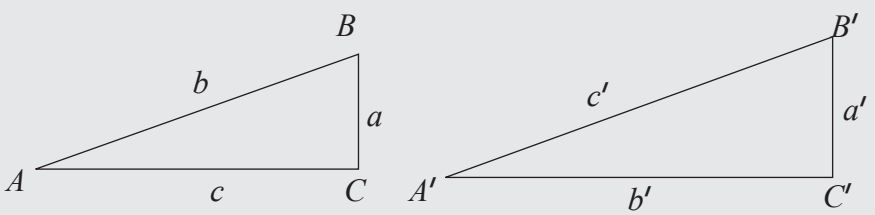
Materials

Note book, Pens, Instruments of geometry

Methodology

Group work and guide learners during this activity of drawing a right triangle and show different ratios in that triangle.

Answers



The longest side is called the hypotenuse.

Angle	$\frac{opposite\ side}{hypotenuse}$	$\frac{adjacent\ side}{hypotenuse}$	$\frac{opposite\ side}{adjacent\ side}$
A	$\frac{a}{c}$	$\frac{b}{c}$	$\frac{a}{b}$
B	$\frac{b}{c}$	$\frac{a}{c}$	$\frac{b}{a}$
C	$\frac{c}{c}=1$	$\frac{0}{c}=0$	$\frac{c}{0}$ **does not exist**

Angle	$\frac{opposite\ side}{hypotenuse}$	$\frac{adjacent\ side}{hypotenuse}$	$\frac{opposite\ side}{adjacent\ side}$
A’	$\frac{a'}{c'}$	$\frac{b'}{c'}$	$\frac{a'}{b'}$
B’	$\frac{b'}{c'}$	$\frac{a'}{c'}$	$\frac{b'}{a'}$
C’	$\frac{c'}{c'}=1$	$\frac{0}{c'}=0$	$\frac{c'}{0}$ **does not exist**

From Thales’ theorem: $\frac{a}{c}=\frac{a'}{c'}, \frac{b}{c}=\frac{b'}{c'}, \frac{c}{c}=1=\frac{c'}{c'}$

For both triangle, the ratio

- Opposite side to the considered angle and hypotenuse is the same
- Adjacent side and hypotenuse is the same
- Opposite side to the considered angle and adjacent side is the same.

Answers to Exercise 3

$\csc B=\frac{15}{9}=\frac{5}{3}$ $\quad \csc A=\frac{15}{12}=\frac{5}{4}$

$\sec B=\frac{15}{12}=\frac{5}{4}$ $\quad \sec A=\frac{15}{9}=\frac{5}{3}$ $\quad \csc C=1$

$\cot B=\frac{12}{9}=\frac{4}{3}$ $\quad \cot A=\frac{9}{12}=\frac{3}{4}$ $\quad \sec C$ *does not exist*

$\cot C=0$

Activity 4

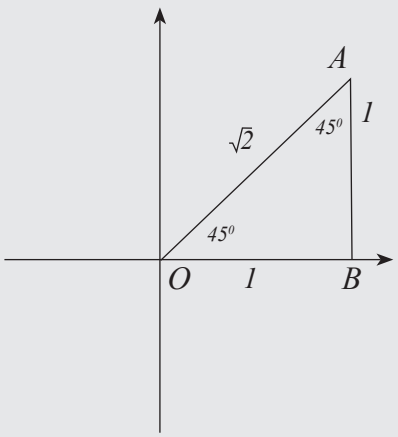
Materials

Note book, Pens and calculator

Methodology

Group work and Questioning for facilitating learners during this activity and developing brainstorming.

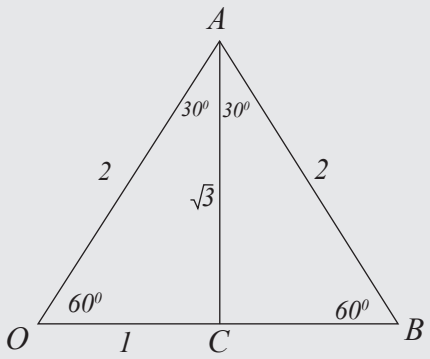
Answers



From the diagram

$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = \frac{1}{1} = 1$



From ΔOAC,

$\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}$, $\csc 60^0 = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$,

$\sec 60^\circ = 2$,

$\cot 60^0 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$,

From ΔOAC,

$\sin 30^0 = \frac{1}{2}, \cos 30^0 = \frac{\sqrt{3}}{2}, \tan 30^0 = \frac{\sqrt{3}}{3}, \csc 30^0 = 2, \sec 30^0 = \frac{2\sqrt{3}}{3},$

$\cot 30^0 = \sqrt{3}$

Answers to Exercise 4

1. 1	2. 1	3. $\frac{\sqrt{3}}{3}$
4. 0	5. does not exist	6. 0
7. does not exist	8. does not exist	9. 0
10. does not exist	11. 0	

Activity 5

Materials

Note book, Pens, Instruments of geometry

Methodology

Group work , facilitating learners **(HINT: to construct trigonometric circle)** for this activity then questioning.

Answers

	Quadrant			
Value	*I*	*II*	*III*	*IV*
sin	+	+	-	-
cos	+	-	-	+
tan	+	-	+	-

Answers to Exercise 5

a) Second quadrant
b) Second quadrant
c) First quadrant
d) First quadrant
e) Third quadrant
f) Second quadrant

Activity 6

Materials

Note book, Pens and calculator

Methodology

Group work, brainstorming for this activity then questioning.

Answers

In this triangle,

$$\sin\theta = \frac{y}{r},\ \cos\theta = \frac{x}{r},\ \sin\alpha = \frac{x}{r},\ \cos\alpha = \frac{y}{r}$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = \left(\sin\theta\right)^2 + \left(\cos\theta\right)^2 = \sin^2\theta + \cos^2\theta$$

and then $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\alpha + \cos^2\alpha = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

Answers to Exercise 6

1. 1
2. 0
3. $\dfrac{1+\tan a}{3-\tan a}$

2 Triangle and Applications

Activity 7

Materials

Manila paper or Note book, markers or Pens

Methodology

Facilitate learners in Group work, then questioning .

Answers

1. $\cos A = \dfrac{AX}{b}$
2. $b^2 = h^2 + (AX)^2 \Rightarrow h^2 = b^2 - (AX)^2$
3. $a^2 = h^2 + (XB)^2 \Rightarrow h^2 = a^2 - (XB)^2$
4. $h^2 = b^2 - (AX)^2$ and $h^2 = a^2 - (XB)^2$ gives $b^2 - (AX)^2 = a^2 - (XB)^2$

   But $XB = c - AX$, then

   $b^2 - (AX)^2 = a^2 - (c - AX)^2$

   $\Leftrightarrow b^2 - (AX)^2 = a^2 - \left(c^2 - 2cAX + (AX)^2\right)$

   $\Leftrightarrow b^2 - (AX)^2 = a^2 - c^2 + 2cAX - (AX)^2$

   $\Leftrightarrow b^2 + c^2 - 2cAX = a^2$

   But $\cos A = \dfrac{AX}{b} \Rightarrow AX = b\cos A$. Then

   $\Leftrightarrow b^2 + c^2 - 2cb\cos A = a^2$

$\Leftrightarrow a^2 = b^2 + c^2 - 2bc\cos A$

Answers to Exercise 7

1. $9.43\,cm$ 2. $c = 21.7\,cm$ 3. $A = 12.7^0,\ B = 22.3^0, c = 14.4\,cm$

Activity 8

Materials

Manila paper or Note book, markers or Pens

Methodology

Work in Group, then control participation of learners in answering this activity.

Answers

1. $\sin B = \frac{h}{a},\ \sin A = \frac{h}{b}$.

   $h = a\sin B$ and $b\sin A = h$, then $a\sin B = b\sin A$ or $\frac{a}{\sin A} = \frac{b}{\sin B}$

2. $\sin A = \frac{k}{c},\ \sin C = \frac{k}{a}$.

   $k = c\sin A$ and $k = a\sin C$, then $c\sin A = a\sin C$ or $\frac{c}{\sin C} = \frac{a}{\sin A}$

3. Now, $\frac{a}{\sin A} = \frac{b}{\sin B}$ and $\frac{c}{\sin C} = \frac{a}{\sin A}$. This gives $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Answers to Exercise 8

1. $8.45\,cm$
2. $A = 56.8^0,\ C = 81.2^0$ or $A = 123.2^0,\ C = 14.8^0$
3. $6.18\,cm$

Answers to revision exercise

1. a) $\dfrac{\sin a(1+\cos a)}{(1-\cos a)(1+\cos a)} = \dfrac{\sin a(1+\cos a)}{\sin^2 a}$

$= \dfrac{1+\cos a}{\sin a}$

b) $\dfrac{1}{\cos^2 a} + \dfrac{1}{\sin^2 a} = \dfrac{\sin^2 a + \cos^2 a}{\cos^2 a \sin^2 a}$

$= \dfrac{1}{\cos^2 a}\dfrac{1}{\sin^2 a}$

$= \sec^2 a \csc^2 a$

c)

$$\left(\sec^2 a + \tan^2 a\right)\left(\sec^2 a - \tan^2 a\right) = \left(\sec^2 a + \tan^2 a\right)\left(\frac{1}{\cos^2 a} - \frac{\sin^2 a}{\cos^2 a}\right)$$

$$= \left(\sec^2 a + \tan^2 a\right)\left(\frac{1-\sin^2 a}{\cos^2 a}\right)$$

$$= \left(\sec^2 a + \tan^2 a\right)\left(\frac{\cos^2 a}{\cos^2 a}\right)$$

$$= \sec^2 a + \tan^2 a$$

d) $\sqrt{\dfrac{(1-\cos a)(1-\cos a)}{(1+\cos a)(1-\cos a)}} = \sqrt{\dfrac{(1-\cos a)^2}{\sin^2 a}} = \dfrac{1-\cos a}{\sin a}$

2. $\tan\theta = 3.18$

3. $\cos\theta = -0.8$; $\tan\theta = -0.75$

4. $\cos 14^0 = \sqrt{1-\sin^2 14^0}$ *or* $\cos 14^0 = \sin\left(90^0 - 14^0\right) = \sin 76^0$

5. $x \approx 37.7$ 6. $p = 60^0$ 7. $h \approx 116.6$ 8. $4.12m$ 9. 7^0

10. a) $88.6\,km$ b) $179.3\,km$ 11. $954km, 133^0$ 12. $76.5m/s$

13. 81.2189^0 14. $117^0\,39'\,26''$ 15. $2^0\,56'$ 16. 75.3^0

Unit 2

Set of Real Numbers

Aim

Think critically using mathematical logic to understand and perform operations on the set of real numbers and its subsets using the properties of algebraic structures.

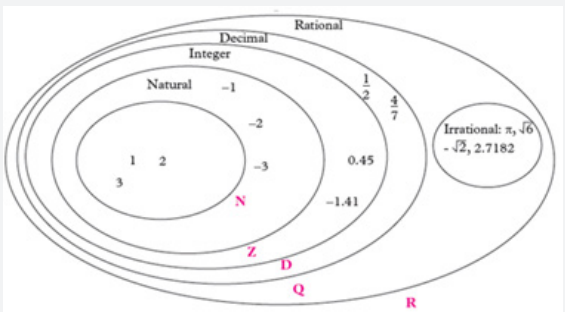
Guidance on introductory activity 2

- Form small groups of learners and guide them to work on the introductory activity.
- Provide learning materials accordingly to the given activities and give clear guidance and instructions to perform the activities.
- Give time to learners to read and analyse the given activity and let them discuss about different possible solutions of the problem.
- Walk around in different groups to provide advice and facilitations where necessary and remind them to justify and support their answer / findings.
- Lead learners to recognize that the given activity should get different answers depending on the considered set of number.
- Basing on learners' experience, prior knowledge and abilities shown in answering the questions for the introductory activity, use different questions to prompt them to give their predictions and ensure that you arouse their curiosity on what is going to be leant in the unit 4

- After presenting their finding, the teacher harmonizes and guides class discussions and interventions.

Answer of introductory activity 2

1. Lead learners to know that in the question1, set of numbers they already know from senior one (S1) in secondary schools, are: $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \ldots$.
2. The numbers we use in counting plus zero are called Natural numbers; integers are numbers which have either negative or positive sign and includes zero. The set of integers is represented by $\mathbb{Z}$; the set of **rational numbers** $\mathbb{Q}$ and the set of irrational numbers I form the set of real numbers. The set of real numbers is denoted by $\mathbb{R}$
3. Some examples of numbers in each set:



4. The relationship between set of numbers is as follows: Natural numbers are part of integers, integers are part of rational numbers, rational numbers and irrational numbers are pats of real numbers. Therefore, $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Objectives

By the end of this unit, the learners will:

- define a power, an exponential, a radical, a logarithm, the absolute value of a real number
- illustrate each property of a power, an exponential ,a radical, a logarithm, the absolute value of a real number.
- use logarithm and exponentials to model simple problems about growth, decay, compound interest, magnitude of an earthquake...

Vocabulary

Logarithm

Contents

1 Absolute value and its properties

Recommended teaching time: 3 periods

This section introduces the absolute value of a real number and properties of absolute value.

2 Powers and radicals

Recommended teaching time: 3 periods

This section introduces powers and radicals in set of real numbers and their properties.

3 Decimal logarithm

Recommended teaching time: 6 periods

This section looks at decimal logarithm, properties and applications.

Materials, Methodology, Answers to activities and exercises

1 Absolute Value and its Properties

Activity 1

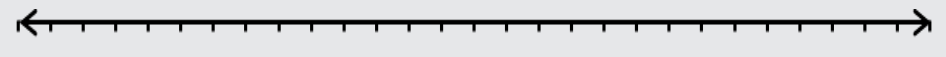
Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

Facilitate learners in Group work, then questioning .

Answers



1. 8 units 2. 8 units 3. $\frac{1}{2}$ units 4. 13 units

Answers to Exercise 1

1. -6 or 6 2. -1 3. -3 or 9
4. $\frac{-5}{2}$ or $\frac{3}{2}$ 5. 1 or 5

Activity 2

Materials

Notebook, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning .

Answers

1. 3 and 3 2. 15 and 15 3. 3 and 13

Answers to Exercise 2

1. 5 2. 20 3. 28 4. -24 5. -2

2 Powers and Radicals

Activity 3

Materials

Notebook, pens and calculator.

Methodology

Facilitate learners in pairs, then questioning.

Answers

1.

Week	Dollars
One	2=2
Two	$2\times 2 = 2^2 = 4$
Three	$2\times 2\times 2 = 2^3 = 8$
Four	$2\times 2\times 2\times 2 = 2^4 = 16$
Five	$2\times 2\times 2\times 2\times 2 = 2^5 = 32$

2. $2^7 = 128$, $2^{10} = 1024$
3. No, his parent will not agree with his suggestion. Allowance is very increasing at certain time.

Answers to Exercise 3

1. x^5 2. $5x^2y^6$ 3. $2y$ 4. 0 5. $\frac{1}{4}$

Activity 4

Materials

Note book, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. 9 2. 6 3. -3 4. 2

Answers to Exercise 4

1. ab^2c 2. abc 3. $\frac{2}{3}$ 4. x 5. $\frac{xy^2}{2}$

Activity 5

Materials

Note book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $4\sqrt{2}$ 2. $-\sqrt{3}$ 3. $\sqrt{6}$ 4. $\sqrt{3}$

Answers to Exercise 5

1. $3\sqrt{5}$ 2. $2\sqrt{3}$ 3. $3\sqrt{7}$ 4. 12 5. $13\sqrt{5}$ 6. $2\sqrt{3}$

Activity 6

Materials

Note book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $\frac{\sqrt{2}}{2}$ 2. $\frac{2\sqrt{5}-\sqrt{15}}{10}$

3. $\frac{-2+2\sqrt{6}}{5}$ 4. $\frac{-3-\sqrt{6}+\sqrt{10}+\sqrt{15}}{2}$

Answers to Exercise 6

1. $\frac{5\sqrt{7}}{7}$ 2. $1-\sqrt{2}$ 3. $\frac{\sqrt{12}-\sqrt{18}-\sqrt{20}+\sqrt{50}}{4}$

4. $\frac{6\sqrt{6}-8\sqrt{2}}{23}$ 5. $\frac{a\sqrt{d}-\sqrt{bd}}{d}$ 6. $\frac{-3\sqrt{3}+6\sqrt{6}-2\sqrt{2}+8}{7}$

3 Decimal Logarithm

Activity 7

Materials

Note book, pens and scientific calculator.

Methodology

Facilitate learners in pairs, then questioning.

Answers

1. 0 2. 1 3. 2

4. 3 5. 4 6. 5

Answers to Exercise 7

1. a) $a > b$ b) $a < 6$
2. a) 2.17 c) 0.30
3. a) -2 b) -1.62 c) -1.176

Answers to revision exercise

1. a) yz b) $2a^3b^6$ c) $2\sqrt{2}$
2. a) $\frac{3\sqrt{35}+\sqrt{14}}{14}$ b) $2\sqrt{2}+2\sqrt{5}+\sqrt{6}+\sqrt{15}$
   c) $\frac{-2-\sqrt{6}+\sqrt{10}+\sqrt{15}}{3}$
3. a) 1.08 b) -0.35 c) 0.56
4. a) → iii b) → ii c) → i
5. 96 6. 64

Linear, Quadratic Equations and Inequalities

Aim

Model and solve algebraically or graphically daily life problems using linear and quadratic equations or inequalities.

Guidance on introductory activity 3

- Invite learners to work in groups where they read and analyse the problem in introductory activity 5
- During instruction, tel them that they can use a library or computer lab to search on the definition of linear equation and its application in real life.
- Ask learners to complete the table found in introductory activity by using the information obtained from research.
- Invite all groups to present their findings to the whole class.
- Basing on their experience, results from their own research, prior knowledge and abilities shown in answering the questions for this activity, use different questions to facilitate learners give their predictions and ensure that you arouse their curiosity on what is going to be leant in this unit.

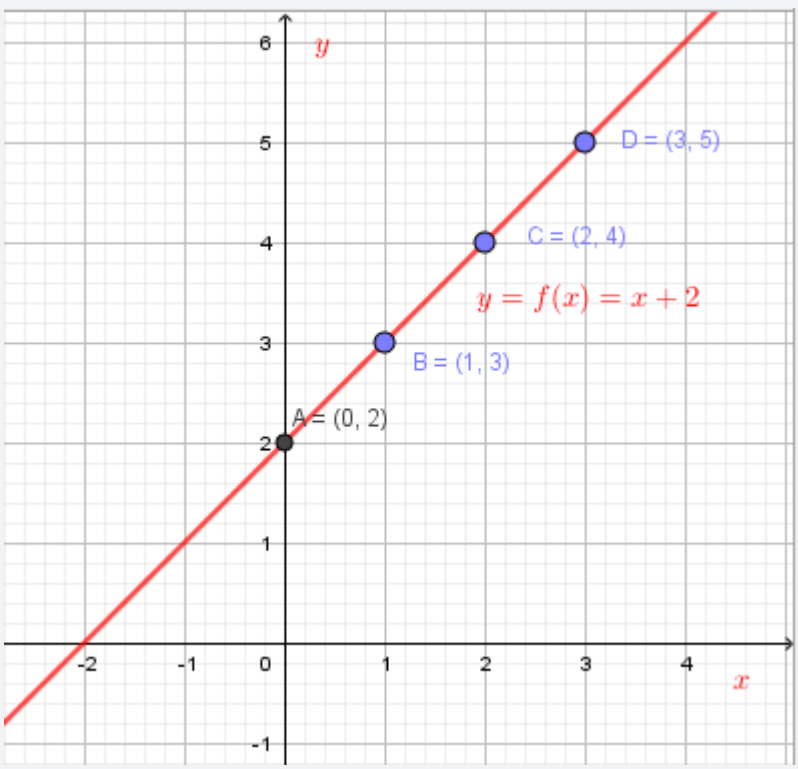
Answer of introductory activity 3

1. Refer to the student's book and verify answers for students.
2. If x is the number of pens for a learner, the teacher decides to give him/her two more pens. A learner with one pen will have $(1+2)\,\text{pens} = 3\,\text{pens}$

a) $y = f(x) = x + 2$

X	-2	-1	0	1	2	3	4
$y = f(x) = x + 2$	0	1	2	3	4	5	6
(x,y)	(-2;0)	(-1;1)	(0,2)	(1;3)	(2;4)	(3;5)	(4;6)

b) The graph obtained is the following:



c) The graph obtained is a line.

d) d) $y = x + 2$ this is a linear equation because its graph is a line. Identically, $x + 2 \geq 0$ is a linear inequality.

3. Students will give different examples. Verify whether the solution involves the linear equation.

4. a) $y = -16t^2 + 1600$, for $y = 1000$, we have $1000 = -16t^2 + 1600$

Solve this equation to find the time requested. $t = \frac{\sqrt{600}}{4} \approx 6.1$

The jumper is in free fall for about 6 seconds.

b) Table of value:

t	0	1	2	3	4	5	6
y	1600	1616	1664	1744	1856	2000	2176

Objectives

By the end of this unit, the learners will:

- solve equation of the first degree and second degree.
- solve inequality of the first degree and second degree.
- solve a system of linear equations.
- use equations and inequalities to solve word problems.
- apply equations and inequalities in real life problems.

Vocabulary

Parameter

Contents

1 Equations and inequalities in one unknown

Recommended teaching time: 3 periods

This section looks at the method used to solve equation and inequality of the first degree.

2 Simultaneous equations

Recommended teaching time: 7 periods

This section looks at the method used to solve simultaneous equations and second degree in one unknown.

3 Applications

Recommended teaching time: 3 periods

This section looks at application of equations in daily life.

Materials, Methodology, Answers to activities and exercises

1 Equations and Inequalities in One Unknown

Activity 1

Materials

Note book, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. 4	2. 2	3. -3
4. -34	5. 3	6. 14

Answers to Exercise 1

1. 4	2. 0	3. 5	4. 15	5. -4
6. 5	7. $\frac{1}{2}$	8. $-\frac{7}{3}$	9. -1	10. 10

Activity 2

Materials

Note book and pens.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $AB=0 \Leftrightarrow A=0 \ or \ B=0$ Either $x+1=0$ or $x-1=0$
2. $AB=0 \Leftrightarrow A=0 \ or \ B=0$ Either $2x-3=0$ or $x=0$
3. Cross product: $2(2x-3)=x$, with $x \neq 0$

Answers to Exercise 2

1. $S=\{-2,5\}$ 2. $S=\left\{-\frac{9}{2}\right\}$ 3. $S=\left\{-8,-\frac{1}{2}\right\}$
4. $S=\left\{\frac{47}{11}\right\}$

Activity 3

Materials

Note book and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. All numbers less than 5
2. All positive numbers
3. All numbers between -4 and 12
4. All numbers less than or equal to 100

Answers to Exercise 3

1. $]-\infty,9[$ 2. $]-\infty,10[$ 3. $]-\infty,5]$
4. $\left]\frac{26}{3},+\infty\right[$ 5. $[5,+\infty[$ 6. $]-\infty,4[$
7. $]-\infty,+\infty[$ or IR 8. $\{\ \}$ 9. $]-\infty,-11]$ 10. $]-3,+\infty[$

Activity 4

Materials

Note book and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

In each case, first construct the sign table. The solution will be given by interval showing negative values for $<$

Answers to Exercise 4

1. $]-\infty,-3[\cup]3,+\infty[$
2. $\left[\frac{3}{4},1\right]$
3. $]-\infty,-5[\cup]-1,2[$
4. $\left]-\frac{4}{3},-1\right[$
5. $]-\infty,-2[\cup[3,+\infty[$

Activity 5

Materials

Note book, pens and instruments of geometry

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $S=\{x\in IR:|x|>4\}$
2. $S=\{x\in IR:|x|<6\}$

Answers to Exercise 5

1. $]-\infty,-2[\cup]3,+\infty[$
2. $\left]-\infty,-\frac{9}{2}\right]\cup\left[\frac{3}{2},+\infty\right[$
3. $]0,3[$

Activity 6

Materials

Note book, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. Let x be the age of the son, then the age of father has $x+30$ years old. Set $x+30-5=4(x-5)$ and then solve.

   Here some learners can take x as the age of the father.

2. Let x be the money Betty had originally. Set

$$x-\frac{x}{5}-\frac{1}{2}\left(x-\frac{x}{5}\right)-7000=13000$$

Answers to Exercise 6

1. The two numbers are 8 and 17.
2. The two numbers are 84 and 36.
3. Length of the rectangle is 24 m and breadth of the rectangle is 12 m.
4. Present age of Ron is 6 years and present age of Aaron is 1 year.
5. Sam scored less than 3 goals, which means that Sam could have scored 0, 1 or 2 goals. Alex scored 3 more goals than Sam did, so Alex could have scored 3, 4, or 5 goals.
6. His average speed running is greater than 13 km/h and his average speed cycling is greater than 26 km/h

2 Simultaneous equations in two unknown and equations of the second degree

Activity 7

Materials

Note book, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $\begin{cases} x+y=12 & \times -1 \\ 2x+y=4 & \times 1 \end{cases} \Leftrightarrow \begin{cases} -x-y=-12 \\ 2x+y=4 \end{cases}$

$x=-8$

2. $\begin{cases} 3x-y=20 & \times 1 \\ -x+2y=4 & \times 3 \end{cases} \Leftrightarrow \begin{cases} 3x-y=20 \\ -3x+6y=12 \end{cases}$

$5y=32$

3. $\begin{cases} x-2y=10 & \times 1 \\ 2x+y=14 & \times 2 \end{cases} \Leftrightarrow \begin{cases} x-2y=10 \\ 4x+2y=28 \end{cases}$

$5x=38$

There are many different possible numbers

Answers to Exercise 7

1. Infinity solutions
2. $x=y=0$
3. $x=3, y=\frac{2}{3}$
4. $x=-\frac{1}{8}, y=\frac{15}{8}$
5. $x=y=-1$
6. $x=\frac{7}{3}, y=\frac{1}{3}$

Activity 8

Materials

Note book, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $x = 5 + y$ and $5 + y + 2y = 6 \Leftrightarrow 5 + 3y = 6$
2. $x = 10 - 2y$ and $-3(10 - 2y) + 2y = 12 \Leftrightarrow -30 + 8y = 12$
3. $x = -10 - y$ and $4(-10 - y) + y = 0 \Leftrightarrow -40 - 3y = 0$

   Some learners can first find the value of y from any equation

Answers to Exercise 8

1. Infinity solutions
2. $x = y = 0$
3. $x = \frac{3}{2},\ y = \frac{9}{2}$
4. $x = \frac{13}{4},\ y = \frac{15}{4}$
5. $x = \frac{3}{5}, y = \frac{2}{5}$
6. $x = 1, y = 0$

Activity 9

Materials

Note book, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. 0
2. 17
3. 25

Answers to Exercise 9

1. $x = 2, y = 0$
2. $x = y = 1$
3. $x = \frac{19}{9}, y = \frac{4}{9}$
4. $x = -\frac{3}{8}, y = \frac{5}{8}$
5. $x = 1, y = -\frac{2}{3}$
6. $x = \frac{70}{13}, y = \frac{16}{13}$

Activity 10

Materials

Note book, pens, instruments of geometry and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. For $3x+y=10$

   Let $x=3$, then $y=1$.

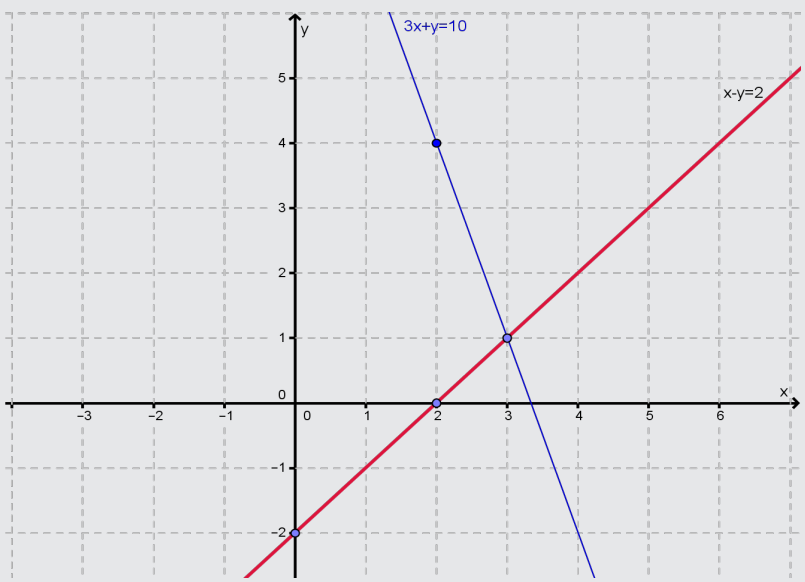
   Let $y=4$, then $x=2$. We have $(3,1)$ and $(2,4)$

   For $x-y=2$

   Let $x=0$, then $y=-2$.

   Let $y=0$, then $x=2$. We have $(0,-2)$ and $(2,0)$

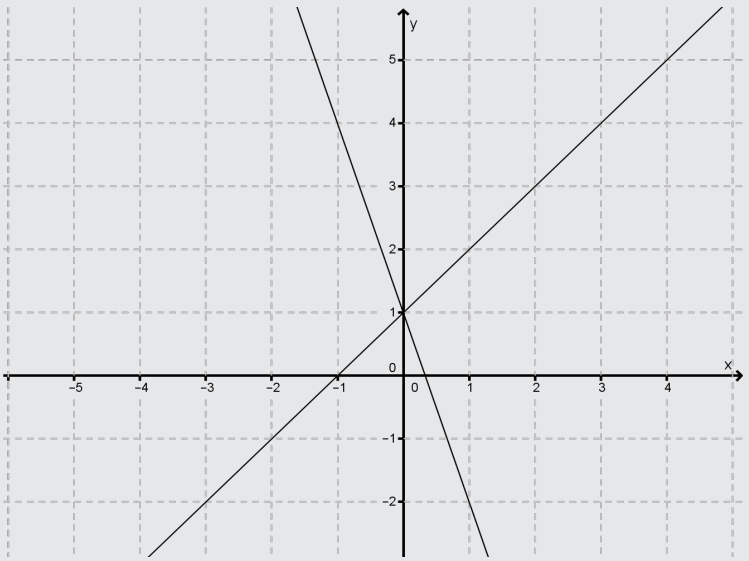
2. Graph



3. The point of intersection is $(3,1)$

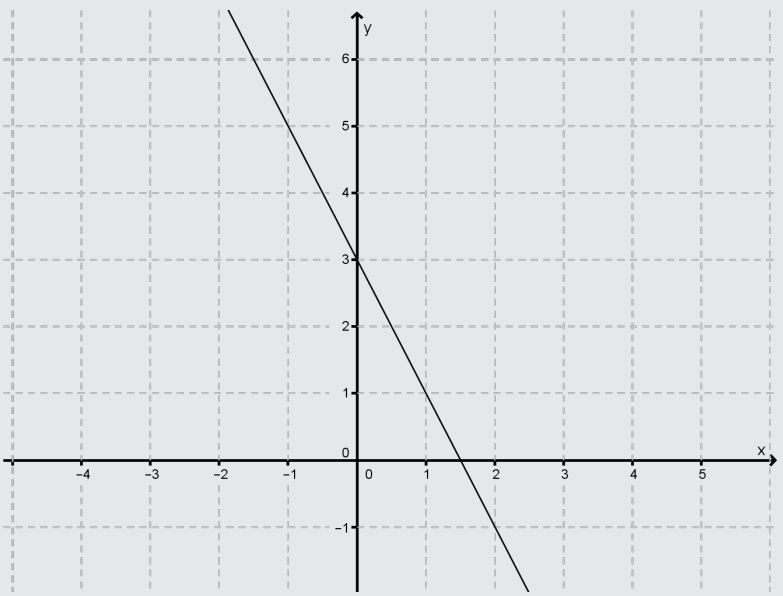
Answers to Exercise 10

1. Graph



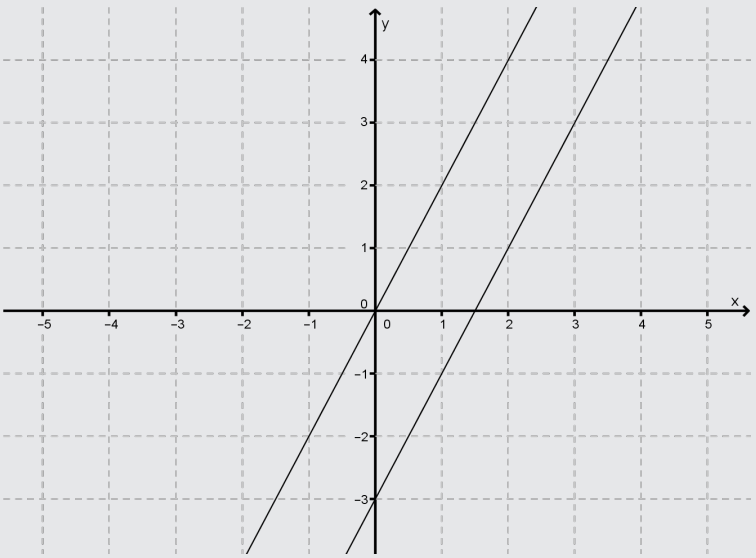
$x = 0,\ y = 1$

2. Graph



The two lines coincide. There is infinite number of solution

3. Graph



Parallel and distinct lines. No solution.

Activity 11

Materials

Note book, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. Let x = cost per cat, y = cost per dog

$$\begin{cases} 164x + 24y = 4240 \\ x = 2y \end{cases}$$

Then solve the system

2. Let x = cost of water slide, y = cost of Ferris wheel

$$\begin{cases} 3x + 3y = 17.70 \\ 2x + 3y = 15.55 \end{cases}$$

Then solve the system

Answers to Exercise 11

1. There are 5 multiple choice questions.

   There are 15 T/F questions.
2. The small pitcher holds 2 cups of water.

   The large pitcher holds 4 cups of water.
3. Number of students in van is 8 and number of students in bus is 22 students
4. 34
5. The speed of the boat in still water is 12 miles/hour and the speed of the current is 9 miles/hour.

3 Quadratic Equations

Activity 12

Materials

Exercise books, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

$y = -16x^2 + 1600$

$1000 = -16x^2 + 1600$

$-16x^2 = -600$

$x^2 = 37.5 \Rightarrow x = \pm 6.1$

The jumper is in free fall for about 6.1s

Answers to Exercise 12

1. {-4, 2} 2. {-1, 3} 3. {-2, -1} 4. $\left\{-\frac{\sqrt{77}}{6}, \frac{\sqrt{77}}{6}\right\}$

Activity 13

Materials

Exercise books, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning

Answers

a) $y = ax^2 + bx + c, a \neq 0$

$y = ax^2 + bx + c, a \neq 0$

$\Leftrightarrow y = \left(ax^2 + bx\right) + c$

$\Leftrightarrow y = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right] + c$

$\Leftrightarrow y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$

$\Leftrightarrow y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

Hence $y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ if $a \neq 0$ as required

b) From a) $2x^2 - 7x - 4 = 0$

$\Leftrightarrow 2\left(x^2 + \frac{7}{2}\right) - 4 = 0 \quad \Leftrightarrow 2\left(x + \frac{7}{4}\right)^2 + \left(-4 - \frac{7^2}{8}\right) = 0$

$\Leftrightarrow 2\left(x + \frac{7}{4}\right)^2 = \left(-4 - \frac{7^2}{8}\right) = 0$

Answers to Exercise 13

1. -8; 5 2. 4; 9 3. -1½; 2 4. -3; 1⅓

Activity 14

Materials

Exercise books, Pens, Scientific calculator.

Methodology

In pairs, teacher facilitates learners in developing their critical thinking

Answers

1. 2 and 2
2. 2 and 3
3. -3 and -4
4. 1 and $\frac{1}{2}$
5. -7 and 5

Answers to Exercise 14

1. $S=\{1,11\}$
2. $S=\{-7,5\}$
3. $S=\varnothing$
4. $S=\left\{\frac{1}{3},2\right\}$
5. $S=\{-11,11\}$

Activity 15

Materials

Exercise books, Pens, Scientific calculator.

Methodology

In group work, teacher facilitates learners in developing their critical thinking

Answers

1. x^2+3x-4
2. $3x^2-21x+30$
3. x^2+3x+2
4. $6x^2-66x+144$
5. $x^2-4x-12$

In each case the original form is the factor form

Answers to Exercise 15

1. $(x-8)(x-2)$
2. No factor form
3. $(2x-1)(3x-1)$
4. $(x-1)(x+5)$
5. $(x+2)(4x-1)$

Activity 16

Materials

Exercise books, Pens

Methodology

In groups of two, teacher facilitates learners in developing their critical thinking

Answers

1. $u^2 - 2u + 2 = 0$
2. $6u^2 + 5u + 1 = 0$
3. $u^2 - 13u + 36 = 0$

Answers to Exercise 16

1. $S = \{-3, -2, 2, 3\}$
2. $S = \left\{1, \sqrt[3]{6}\right\}$
3. $S = \{-3, -1, 1, 3\}$
4. $S = \{-6, -5, 5, 6\}$

Activity 17

Materials

Exercise books, Pens, Scientific calculator.

Methodology

In group work, teacher facilitates learners in developing their critical thinking

Answers

$$\left(\sqrt{4+\sqrt{12}}\right)^2 = \left(\sqrt{x}+\sqrt{y}\right)^2$$

$$\Leftrightarrow 4+\sqrt{12} = x + 2\sqrt{xy} + y$$

$$\Leftrightarrow 4+\sqrt{12} = x + y + \sqrt{4xy}$$

$$\begin{cases} x+y=4 \\ 4xy=12 \end{cases} \Rightarrow \begin{cases} x+y=4 \\ xy=3 \end{cases}$$

We need two numbers such that their sum is 4 and their product is 3

$\Rightarrow x=3,\ y=1$ or $x=1,\ y=3$

Answers to Exercise 17

1. $\sqrt{5}-1$ 2. $1+\sqrt{2}$ 3. $\sqrt{2}+\sqrt{3}$

Activity 18

Materials

Exercise books, Pens, Scientific calculator.

Methodology

In group work, teacher facilitates learners in developing their critical thinking

Answers

1. $\sqrt{x+8}=x+2 \Leftrightarrow x+8=(x+2)^2$
2. $x+8=x^2+4x+4$

   $x^2+3x-4=0$

   Either $x=1$ or $x=-4$
3. We test these two values to the given equation. We see that -4 is a false solution and it must be deleted. The only solution is 1.

Answers to Exercise 18

1. $S=\{162\}$ 2. $S=\varnothing$ 3. $S=\varnothing$

Activity 19

Materials

Exercise books, Pens, Scientific calculator.

Methodology

In group work, teacher facilitates learners in developing their critical thinking

Answers

1. $ax^2+bx+c+\frac{b}{x}+\frac{a}{x^2}=0$ 2. $a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$

3. $ay^2+by+c-2a=0$

Answers to Exercise 19

1. $s=\left]-\infty,5-3\sqrt{5}\right[\cup\left]5+3\sqrt{5},+\infty\right[$

2. $s=\left[\frac{1}{3},\frac{1}{2}\right]$ 3. $s=\mathbb{R}$ 4. s=∅ 5. $s=[8,9]$

4 Applications

Activity 20

Materials

Note book, pens.

Methodology

Facilitate learners for their research

Answers

1. Linear equations can be used in daily life in many different ways like:
   - In economics, supply and demand analysis
   - Linear motion
   - Balancing equation.
2. Quadratic equations are used in daily life like:
   - Calculating areas
   - Figuring out a profit
   - In athletics
   - Finding speeds and so on
   - There are many different answers.

Answers to revision exercise

1. a) -19 b) 0 or -9 c) 9 d) 1
   e) 90 f) 1 g) 4 h) 3

2. a) $]-\infty,-1[$ b) $]1,\infty[$ c) $]-2,0[$

   d. $\left]-\infty,-\frac{5}{3}\right]$ e) $]-\infty,-1[\cup[1,\infty[$ f) $]-\infty,-4[\cup]-1,\infty[$

   g) $]-\infty,-9[\cup]-1,2[$

3. a) 7, 10 b) $\frac{-45\pm\sqrt{2569}}{8}$ c) $5\pm2\sqrt{6}$

   d) $]-\infty,2]\cup[5,\infty[$ e) $\left[\frac{1}{3},\frac{1}{2}\right]$ f) $\mathbb{R}$

   g) $\varnothing$ h) $\varnothing$ i) $\mathbb{R}$

   j) $]-\infty,-1[\cup[2,3]$ k) $-\infty,\ 2\left[\cup\right]3,+\infty[$

4. a) 30 b) $\frac{26}{3}$ c) -2 d. 1

   e) 3 f) 0 or 2 g) $-\frac{13}{8}$

5. a) $\pm\frac{\sqrt{2}}{2}$ b) 25 c) $\varnothing$ d) $-\frac{1}{8}$ *or* 125

6. a) $\sqrt{2}-1$ b) $\sqrt{4}+\sqrt{3}$ c) $2\sqrt{2}-\sqrt{7}$

   d) $2\left(\sqrt{2}+1\right)$ e) $\sqrt{6}-\sqrt{3}$

7. 18, 59

8. \$4, \$7

9. The number is 40 . The two parts are 15 and 25.

10. Robert: 10 years, his father: 40 years.

11. 25 and 30.

12. 39° and 51°

13. Each chair: $125, each table: $165.

14. The velocity is between 5 m/s and 15 m/s between 1 second and 3 seconds after it is thrown.

15. The width must be between 1 m and 7 m (inclusive) and the length is 8m width

16. 12 metres by 16 metres

17. -33 and -34

18. 1.5 metres

19. 9.75 metres on a side

Unit 4

Polynomial, Rational and Irrational functions

Aim

Use concepts and definitions of functions to determine the domain of rational functions and represent them graphically in simple cases and solve related problems…

Guidance on introductory activity 4

Help the learners to take into their mind the general idea of the whole unit: definition, classification (polynomial, rational and irrational functions), operations (composite, inverse, etc), qualities (such as being odd, even, etc.), some specific sets (such as domain, range, etc.).

You can proceed as follows:

- Give clear instructions for learners to form small groups and to work on the introductory activity;
- As they are discussing, circulate around to note the relevancy of the discussion and to provide guidance where necessary;
- Facilitate working, especially the straggling learners
- Ensure that the learners have understood what the unit will be about and they are eager to learn; you can observe this through a clear and concise presentation of a group chosen randomly and the degree of attention other students are paying to the presentation;
- Sustain the curiosity of the learners by a proper management of your class;

- A prior knowledge of software such as geogebra and malmath would be very useful for you and for your students;

Answer of introductory activity 4

1. **i.** The use (what something is made for);

   ii. A quantity whose value depends on the value of another quantity

   iii. A quantity whose value depends on the value of another quantity

2. **i.** – Independent variable: x

   – Dependent variable: y

   ii. – Independent variable: r

   – Dependent variable: A

   iii. – Independent variable: A

   – Dependent variable: S

3. a) Irrational function b) Rational function

   c)Irrational function d)Polynomial function

   e) Rational function

4. i. Domain: $Domf =]-\infty;1[\cup]1;+\infty[$

   Range: $\text{Im}\, f =]-\infty;0[\cup]0;+\infty[$: The function $y = \dfrac{4x-4}{(x-1)^2}$ is equivalent to $y = \dfrac{4}{x+1}$;the only value that y cannot assume is 0;

   ii. Domain: $Domf =]-\infty;+\infty[$: as any expression, r can assume any value;

   Range: $\text{Im}\, f = [0;+\infty[$: the expression $A = \pi r^2 \geq 0$, for any value of r

   iii. As A can always take positive values or can be zero, then $\text{Dom}\, f = [0;+\infty[$ and the range is $\text{Im}\, f = [0;+\infty[$

Objectives

By the end of this unit, the learners will:

- demonstrate an understanding of operations on polynomials, rational and irrational functions, and find the composite of two functions.
- identify a function as a rule and recognize rules that are not functions.
- determine the domain and range of a function.
- construct composition of functions.
- find whether a function is even, odd, or neither.

Vocabulary

Domain of definition, odd function, even function.

Contents

1 Generalities on numerical functions

Recommended teaching time: 7 periods

This section looks at generalities on numerical function: domain, range, parity.

2 Applications

Recommended teaching time: 2 periods

This section looks at some applications of functions in daily life.

Materials, Methodology, Answers to activities and exercises

1 Generalities on numerical functions

Activity 1

Materials

Exercise books and pens.

Methodology

Facilitate learners in pairs, then questioning.

Answers

$1 \to 3$ $2 \to 4$ $3 \to 2$
$4 \to 5$ $5 \to 5$

Answers to Exercise 1

1. The first two relations
2. $Domain = \{a, b, d, e\}$, $Codomain = \{1, 2, 3, 4, 5, 6, 7\}$, $Range = \{1, 2, 3, 4\}$
3. a) 8 b) 0 c) $2d + 4$ d) $a = -4$
4. We have been made one to many by colonialists, NDI UMUNYARWANDA is making us many to one

Activity 2

Materials

Exercise books, pens

Methodology

Facilitate learners in pairs, then questioning.

Answers

1. Polynomial function 2. Rational function
3. Irrational function

Activity 3

Materials

Note book, pens

Methodology

Facilitate learners in pairs, then questioning.

Answers

1. No value 2. 0 3. 1

Answers to Exercise 2

1. $Domf = \mathbb{R}$ 2. $Domf = \mathbb{R}$ 3. $Domf = \mathbb{R} \setminus \{5\}$
4. $Domf = \mathbb{R} \setminus \{3,5\}$ 5. $Domf = \mathbb{R}$

Activity 4

Materials

Exercise books, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $\left]-\infty,-\frac{1}{2}\right[$ 2. None 3. $[-1,2[$

Answers to Exercise 3

1. $[2,+\infty[$ 2. $]-\infty,-6] \cup [1,+\infty[$
3. $\mathbb{R} \setminus \{-4\}$ 4. $]-\infty,-5[\cup]5,+\infty[$ 5. $]-4,+\infty[$

Activity 5

Materials

Exercise books, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $\dfrac{2\left(x^2-1\right)}{2x-3}$
2. $\dfrac{-2\left(x^2-x-2\right)}{2x-3}$
3. $\dfrac{x^2+2x+1}{2x-3}$
4. $\dfrac{1}{2x-3}$

Answers to Exercise 4

1. $2x^3+8x-5$
2. $6x^5-13x^4+28x^3-30x^2+25x-12$
3. $\left[-\dfrac{3}{2},+\infty\right[$
4. $]-4,+\infty[$
5. $2x^3+4x^2-9x-6$

Activity 6

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answer

1. $f\left(-x\right)=x^2-2x+3,-f\left(x\right)=-x^2-2x-3,f\left(-x\right)\neq-f\left(x\right),\ f\left(-x\right)\neq f\left(x\right)$
2. $f\left(-x\right)=\sqrt[3]{-x^3-x}=-\sqrt[3]{x^3+x},\ -f\left(x\right)=-\sqrt[3]{x^3+x},\ f\left(-x\right)=-f\left(x\right)$
3. $f\left(-x\right)=\dfrac{x^2-3}{x^2+1},\ -f\left(x\right)=\dfrac{-x^2+3}{-x^2+1},\ f\left(-x\right)=f\left(x\right)$

Answers to Exercise 5

1. Neither odd nor even
2. Neither odd nor even
3. Odd
4. Even
5. Neither odd nor even

Activity 7

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answer

1. $3x^2-1$
2. $3\left(3x^2+4x+1\right)$

Answers to Exercise 6

1. $\left(f\circ g\right)(x)=-3,\ \left(g\circ f\right)(x)=2$
2. $\left(f\circ g\right)(x)=72x^2+6x-3,\ \ \left(g\circ f\right)(x)=12x^2+6x-18$
3. $\left(f\circ g\right)(x)=72x^2+6x-3,\ \ \left(g\circ f\right)(x)=12x^2+6x-18$

Activity 8

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answer

1. $x=y-1$
2. $x=\dfrac{y+2}{3}$
3. $x=\dfrac{y+3}{2y+1}$

Answers to Exercise 7

1. $\frac{x-2}{5}$ 2. $\frac{-x-2}{7}$ 3. $\frac{2x+1}{x+2}$

2 Applications

Activity 9

Materials

Exercise book and pens.

Methodology

Facilitate learners for their research.

Answers

Polynomials and functions are used in different ways in daily life:

- Polynomials are used to graph curves.
- Business people also use polynomials to model markets.
- Functions are important in calculating medicine, building structures

Answers to revision exercise

1. a and d
2. a) (iii) b) (i) c) (iii) d) (ii)
3. a) 14 b) 50 c) 2 d) 11 e) $3t^2+2$
4. a) 21 b) $-\frac{5}{3}$ c) $\frac{\sqrt[3]{5}+2}{\sqrt[3]{5}}$ d) $\frac{\pi+1}{\pi-1}$ e) $\frac{a}{a-2}$
5. a) does not exist b) -8 c) $\frac{1}{3}$ d) $\frac{10}{31}$ e) does'nt exist
6. -3
7. 1 or $-\frac{3}{2}$

8. $a=1,\ b=-1,\ g(-4)=17$

9. a) $\{y: 0 \le y \le 7\}$ one to one b) $\{y: 0 \le y \le 9\}$ many to one
   c) $\{y: 0 < y \le 1\}$ one to one d) $\{y \in \mathbb{R}: y \neq 0\}$ one to one

10. (a) $]-\infty, 3[\cup]3, +\infty[$ b) $\left]-\infty, -\frac{7}{5}\right[\cup \left]-\frac{7}{5}, +\infty\right[$
    c) $]-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty[$ d) $]-\infty, -2] \cup [1, +\infty[$ e) $]-\infty, +\infty[$
    f) $]-\infty, +\infty[$ g) $]-\infty, -3[$ h) $[5, +\infty[$
    i) $]-\infty, +\infty[$ j) $]-\infty, -3] \cup [2, +\infty[$

11. a) $]-\infty, +\infty[$ b) $]-\infty, -5[\cup]-5, 3[\cup]3, +\infty[$
    c) $\left]-\infty, \frac{1}{2}\right]$ d) $[-3, 4]$ e) $\left[\frac{1}{3}, +\infty\right[$
    f) $[-2, -1] \cup]1, +\infty]$ g) $]-2, 1] \cup]4, +\infty[$

12. -6

13. -3

14. a) $x^3 + 2x^2 - x - 2$ b) $x^3 + 4x^2 - 3x - 2$ c) $x^5 + 2x^4 - 5x^3 + 2x$

15. a) odd b) neither c) odd
    d) odd e) even f) odd
    g) even

16. a) $(f \circ g)(x) = 216,\ (g \circ f)(x) = 6$

    b) $(f \circ g)(x) = 1024x^{10} + 3584x^8 + 4992x^6 + 34,$

    $(g \circ f)(x) = 4x^{10} - 8x^9 + 4x^8 - 24x^5 + 24x^4 + 39$

    c) $(f \circ g)(x) = x^{12} + 4x^9 + 4x^6 - 5,$

    $(g \circ f)(x) = x^{12} - 6x^{10} + 40x^6 - 96x^2 - 63$

17. a) $\frac{-x-2}{9}$ b) $\frac{5x-2}{x+1}$ c) $\frac{2x-9}{3x-1}$

Unit 5

Limits of Polynomial, Rational and Irrational functions

Aim

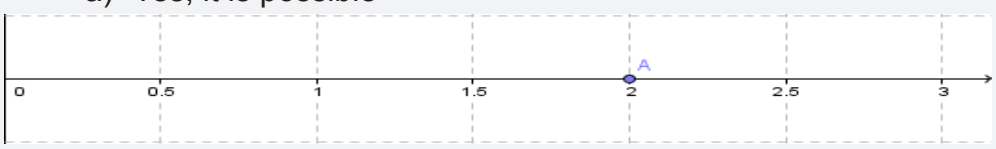
Evaluate correctly limits of functions and apply them to solve related problems.

Guidance on introductory activity 5

- Invite learners to work in group and do the activity 8 found in their Mathematics books;
- Move around in the class for facilitating where necessary and give more clarification on eventual challenges they may face during their work;
- As they are discussing, concentrate on slow learners for further explanation and provide assistance to groups in need
- Invite one member from each group to present their work;
- A teacher, harmonize the findings from presentation of learners and guide them to explore the content and examples given in the student's book where they will be able to differentiate the neighbourhood of a real number and the value of a function at a given point.
- After the lesson, guide learners to do the application activity 8 and evaluate whether lesson objectives were achieved.

Answer of introductory activity 1

a) Yes, it is possible



b) Open intervals: $]1.9;2.1[$ and $]1.99;2.01[$

c) When x approaches 2 , $f(x)$ approaches 4

$2.00001 \approx 2$ and $3.999985000 \approx 4$

$1.99999 \approx 2$ and $4.000015000 \approx 4$

Objectives

By the end of this unit, the learners will:

- calculate limits of certain elementary functions.
- apply informal methods to explore the concept of a limit including one sided limits.
- solve problems involving continuity.
- use the concepts of limits to determine the asymptotes to the rational and polynomial functions.

Vocabulary

Limit, continuity, asymptote.

Contents

1 Concepts of limits

Recommended teaching time: 3 periods

This section looks at the method used to evaluate the limit of a given function algebraically and graphically.

2 Indeterminate cases

Recommended teaching time: 3 periods

This section looks at generalities and the methods used to remove the indeterminate cases such as $\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, \infty-\infty$.

3 Applications

Recommended teaching time: 3 periods

This section looks at some applications of limits: continuity of a function and asymptotes.

Materials, Methodology, Answers to activities and exercises

1 Concepts of limits

Activity 1

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

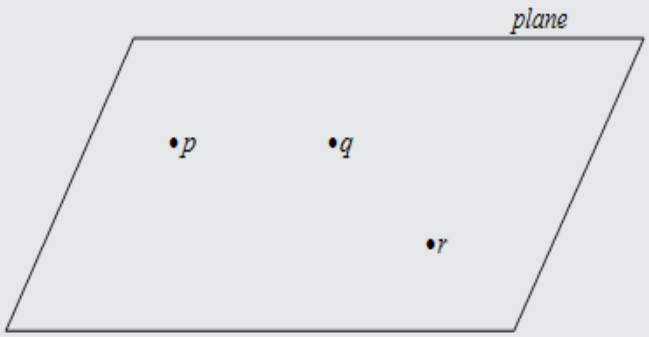
Answers

Lesotho is completely surrounded by South Africa. Swaziland is surrounded by South Africa and Mozambique.

Answers to Exercise 1

1. San Marino, a state surrounded by Italy.
   Vatican City, a state forming part of Rome, thereby surrounded by Italy.
2. $(-6,0)$, $(-7,-1)$, $(-6,-4)$. There are many possible answers (all open intervals containing -5)
3. No. Because no small disk on the circle around any point of the circle

4. The following plane is a neighborhood of points *p*, *q* and *r*.



Activity 2

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $\frac{3}{4}$ 2. 2 3. 98

Answers to Exercise 2

1. 2 2. -8 3. 1 4. -8 5. 12

Activity 3

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. 1.8 2. 1.9 3. 1.99 4. 2.15
5. 2.03 6. 2.003

Answers to Exercise 3

1. 7 2. 1 3. 0 4. Does not exist

Activity 4

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. a) -65.6 b) -99 c) -199
   d) 201 e) 101 e) 67.6
2. a) $+\infty$ b) $-\infty$ c) indeterminate
   d) $-\infty$ e) $-\infty$ f) $+\infty$
   g) indeterminate

Answers to Exercise 4

1. $\frac{1}{2}$ 2. 0 3. ∞
4. $\lim\limits_{x\to -4^-} \frac{x+1}{x+4} = +\infty$, $\lim\limits_{x\to -4^+} \frac{x+1}{x+4} = -\infty$
5. $\lim\limits_{x\to 3^-} \frac{x^2+2x+1}{x-3} = -\infty$, $\lim\limits_{x\to 3^+} \frac{x^2+2x+1}{x-3} = +\infty$

Activity 5

Materials

Exercise books, pens

Methodology

Facilitate learners in pairs, then questioning.

Answers

1. 1 2. 2

The limit at 1 does not exist since one sided limits are not equal.

Answers to Exercise 5

1. 2
2. Does not exist
3. $\lim_{x\to -1^-} h(x) = -\infty,\ \lim_{x\to -1^+} h(x) = +\infty$,

   $\lim_{x\to 1^-} h(x) = -\infty,\ \lim_{x\to 1^+} h(x) = +\infty$,

   $\lim_{x\to -\infty} h(x) = 1,\ \lim_{x\to +\infty} h(x) = 1$

Activity 6

Materials

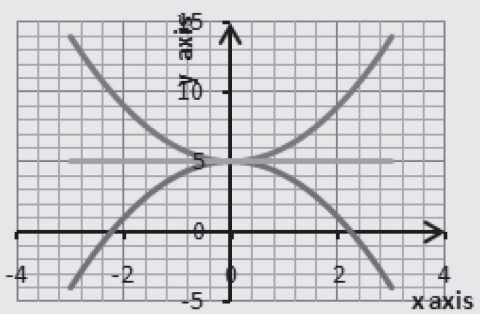
Exercise books, pens, instruments of geometry and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1.



The curve of $h(x)=5$ lies between other two curves and the three curves meet at the same point $(0,5)$

$\lim_{x\to 0} f(x)=\lim_{x\to 0} g(x)=\lim_{x\to 0} h(x)=5$

2. a) -3, -3 b) 0, -1, -1

c) -2, 5, $-\frac{2}{5}$ d) 1, 6, 6 e) 289, 289

A constant can be moved through a limit sign,

Limit of sum is the sum of limits,

Limit of quotient is the quotient of limits, provided that the denominator is not zero,

Limit of a power is the power of limit.

Answers to Exercise 6

1. 0
2. a) 3 b) -5832
3. a) $\infty-\infty$ is indeterminate form not zero

b) $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)=\lim_{x\to 0^+}\frac{x-1}{x^2}=\frac{0-1}{\left(0^+\right)^2}=\frac{-1}{0^+}=-\infty$

x	$-\infty$	0	1	$+\infty$
$x-1$	-	0	+	
x^2	+	0	+	
$\frac{x-1}{x^2}$	-	|| -	0	+

Hence, $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x^2}\right)=-\infty$

2 Indeterminate cases

Activity 7

Materials

Exercise books, pens.

Methodology

Facilitate learners in pairs, then questioning.

Answers

a) $x-1$ b) $x-2$

Answers to Exercise 7

1. $+\infty$ 2. $-\infty$ 3. 20

Activity 8

Materials

Exercise books, pens

Methodology

Facilitate learners in pairs, then questioning.

Answers

a) $\sqrt{x^2-2}-3$ b) $\sqrt{x-2}+1$

Answers to Exercise 8

1. $\lim\limits_{x\to 4^-}\dfrac{\sqrt{x^2-6}-10}{x-4}=+\infty,\ \lim\limits_{x\to 4^+}\dfrac{\sqrt{x^2-6}-10}{x-4}=-\infty$

2. -1

3 Applications

Activity 9

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

a) 4 b) 4

$f(2)$ and $\lim_{x\to 2} f(x)$ exist and are equal.

Answers to Exercise 9

1. The function is not continuous at $x=-3$ and $x=5$
2. $k=6$
3. $a=-1, b=1$

Activity 10

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. 9, 4 2. 3

Answers to Exercise 10

1. Eliminable discontinuity
2. Jump discontinuity
3. Discontinuity of second kind

4. Jump discontinuity
5. No point of discontinuity

Activity 11

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $a=-1,\ b=1$ 2. $a=-3,\ b=-2$ 3. $a=-3,\ b=-2$

Answers may vary.

Answers to Exercise 11

Consider the given interval and apply theorem

Activity 12

Materials

Exercise books, pens.

Methodology

Facilitate learners in pairs, then questioning.

Answers

As x increases or decreases the curve comes closer and closer to the line B. As x approaches 3 from the right or from the left, the curve comes closer and closer to the line A.

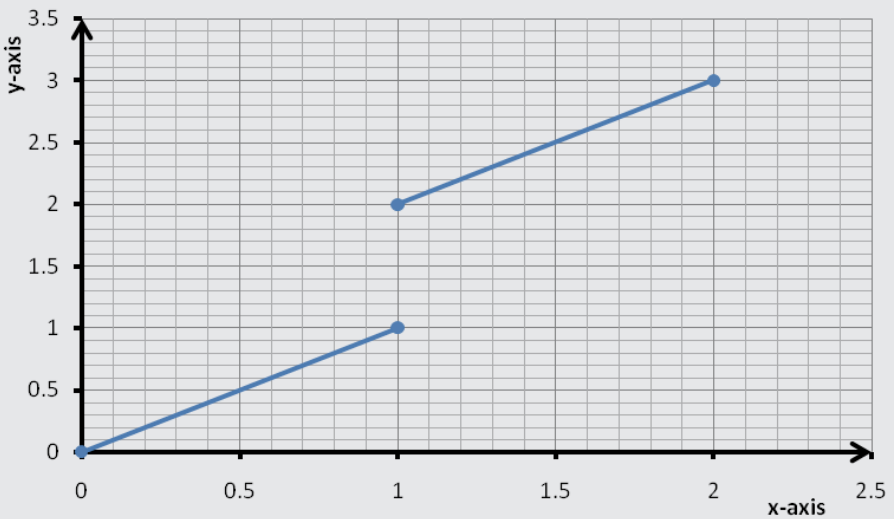
Answers to Exercise 12

1. Horizontal asymptote: $y=1$, vertical asymptotes: $x=-1,\ x=0$
2. Horizontal asymptote: $y=0$

3. Vertical asymptote: $x=\frac{9}{4}$, oblique asymptote: $y=\frac{1}{4}x+\frac{21}{16}$
4. No asymptote
5. Horizontal asymptote: $y=3$, vertical asymptotes: $x=-2,\ x=2$

Answers to revision exercise

1. $-\frac{1}{2}$ 2. 2 3. 2 4. $\frac{1}{2}$
5. 8 6. 1 7. 4 8. 2
9. Does not exist 10. -6
11. $\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=f(2)$. The given function is continuous at $x=2$
12. 0
13. $x=1$



14. $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$. The given function is continuous at $x=0$

    $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=f(1)$. The given function is continuous at $x=0$

15. Eliminable discontinuity at $x=4$

16. Discontinuity of the second kind at $x = -2$
17. Discontinuity of the first kind at $x = 3$
18. Vertical asymptote $x = a$
19. No asymptotes
20. Vertical asymptote $x = 1$, oblique asymptotes $y = -x - \frac{1}{2}$ and $y = x + \frac{1}{2}$
21. Vertical asymptote $x = 0$
22. Vertical asymptote $x = 2$, oblique asymptote $y = 2x + 2$
23. Vertical asymptote $x = -1$, oblique asymptote $y = x + 1$
24. Vertical asymptote $x = 2$, oblique asymptote $y = x + \frac{1}{2}$
25. Vertical asymptote $x = \frac{5}{3}$, horizontal asymptote $y = \frac{2}{3}$
26. No asymptotes
27. Vertical asymptote $x = 0$, oblique asymptote $y = ax$
28. Vertical asymptotes $x = 1$ and $x = -1$, horizontal asymptote $y = 2$
29. Vertical asymptote $x = a$
30. Oblique asymptote $y = \frac{x}{2} - \frac{1}{2}$
31. Vertical asymptote $x = \frac{1}{2}$
32. Horizontal asymptotes: $y = -1$ and $y = 1$
33. Horizontal asymptotes: $y = 0$ and $y = -1$, vertical asymptote $x = 0$

Unit
6

Differentiation of Polynomial, Rational and Irrational functions

Aim

Use the gradient of a straight line as a measure of rate of change and apply this to line tangent and normal to curves in various contexts and use the concepts of differentiation to solve and interpret related rates and optimization problems in various contexts.

Guidance on introductory activity 6

- Give clear instructions for learners to form small groups and to work on the introductory activity;
- The learners have encountered gradients before. Ask them what they recall about the concept of gradients
- As they are discussing, circulate around to note the relevancy of the discussion and to provide guidance where necessary;
- Facilitate working, especially the straggling learners
- Ensure that the learners have understood what the unit will be about and they are eager to learn; you can observe this through a clear and concise presentation of a group chosen randomly and the degree of attention other students are paying to the presentation;
- Sustain the curiosity of the learners by a proper management of your class;
- A prior knowledge of software such as geogebra and malmath would be very useful for you and for your learners.

Answer of introductory activity 6

1.

a) *we* have $x_0 = 1$ *and* $h = \Delta x = 1$

The slope is given by $m_p = \dfrac{\Delta y}{\Delta x} = \dfrac{f(x_0+h)-f(x_0)}{(x_0+h)-x_0} = \dfrac{4-2}{2-1} = 2$

b) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\left[(x+h)^2+1\right]-(x^2+1)}{h}$

$= \dfrac{x^2+2hx+1-x^2-1}{h}$

$= \dfrac{2hx}{h} = 2x$

$f'(x) = 2x$

for $x_0 = 1 \Rightarrow f'(x_0) = f'(1) = 2$

The slope $m_p = f'(x_0) = 2$

2. **Possible answers:**

The derivative of a function $f(x)$ with respect to x is denoted by $f'(x)$ or $\dfrac{d}{dx}f(x)$ and defined as $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ provided that the limit exists.

Or the **derivative of a function** $y = f(x)$ of a variable x is a measure of the rate at which the value y of the **function** changes with respect to the change of the variable x. It is called the **derivative** of $f(x)$ with respect to x.

Objectives

By the end of this unit, the learners will:

- use properties of derivatives to differentiate polynomial, rational and irrational functions.
- use first principles to determine the gradient of the tangent line to a curve at a point.
- apply the concepts of and techniques of differentiation to model, analyze and solve rates or optimization problems in different situations.

Materials

Instrument of geometry, scientific calculator.

Vocabulary

Differentiation, derivative.

Contents

1 Concepts of derivative of a function

Recommended teaching time: 3 periods

This section introduces the concepts of derivative of a function at point.

2 Rules of differentiation

Recommended teaching time: 3 periods

This section looks at rules of differentiation:

- Multiplication with a scalar
- Sum or difference
- Product
- Quotient
- Inverse of a function
- Power
- Composite function,...

It also introduces the successive derivatives and chain rule.

3 Applications

Recommended teaching time: 3 periods

This section looks at some applications of derivatives: tangent and normal lines, rate of change, critical points, extrema, extreme value theorem, Fermat theorem, variation and concavity of a function, Rolle 's Theorem and L'Hôpital's Rule.

Materials, Methodology, Answers to activities and exercises

1 Concepts of derivative of a function

Activity 1

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $\frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

2. Q will coincide with P

3. $m_{\tan} = \lim_{x \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

4. $m_{\tan} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$

Answers to Exercise 1

1. 1 2. 12 3. $8x - 1$ 4. $8x + 3$ 5. 0

Activity 2

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. 4, 0 2. 3, 1

Answers to Exercise 2

1. Not differentiable at 1
2. Not differentiable at 2
3. Not differentiable at 0
4. Not differentiable at 4
5. 0

2 Rules of differentiation

Activity 3

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{c-c}{h}$$
$$=0$$

2. $$f'(x_0)=\lim_{x\to x_0}\frac{x^n-x_0^n}{x-x_0}=\lim_{x\to x_0}\frac{\left(x^{n-1}+x^{n-2}x_0+\ldots+x_0^{n-1}\right)(x-x_0)}{x-x_0}$$
$$=\lim_{x\to x_0}\left(x^{n-1}+x^{n-2}x_0+\ldots+x_0^{n-1}\right)\lim_{x\to x_0}\frac{(x-x_0)}{x-x_0}$$
$$=\left(x_0^{n-1}+x_0^{n-2}x_0+\ldots+x_0^{n-1}\right)x'$$
$$=\left(x_0^{n-1}+x_0^{n-1}+\ldots+x_0^{n-1}\right)x'$$
$$=nx_0^{n-1}x' \quad =nx_0^{n-1} \qquad (1) \text{ as we have n terms}$$

Answers to Exercise 3

1. 0
2. $3x^2+6x+3$
3. $\dfrac{4x+1}{2\sqrt{2x^2+x-2}}$

Activity 4

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1.

$$(cf)'(x_0)=\lim_{x\to x_0}\frac{(cf)(x)-(cf)(x_0)}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{cf(x)-cf(x_0)}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{c\left[f(x)-f(x_0)\right]}{x-x_0}$$

$$=c\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$$

$$=cf'(x_0)$$

$$(cf)'=cf'$$

2.

$$(f\cdot g)'(x_0)=\lim_{x\to x_0}\frac{(f\cdot g)(x)-(f\cdot g)(x_0)}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{f(x)g(x)-f(x_0)g(x_0)}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{f(x)g(x)-f(x_0)g(x)+f(x_0)g(x)-f(x_0)g(x_0)}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{\big(f(x)-f(x_0)\big)g(x)+f(x_0)\big(g(x)-g(x_0)\big)}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{\big(f(x)-f(x_0)\big)g(x)}{x-x_0}+\lim_{x\to x_0}\frac{f(x_0)\big(g(x)-g(x_0)\big)}{x-x_0}$$

$$=\lim_{x\to x_0}\frac{\big(f(x)-f(x_0)\big)}{x-x_0}\lim_{x\to x_0}g(x)+\lim_{x\to x_0}f(x_0)\lim_{x\to x_0}\frac{\big(g(x)-g(x_0)\big)}{x-x_0}$$

$$=f'(x_0)g(x_0)+f(x_0)g'(x_0)$$

$$=\big(f'g+fg'\big)(x_0)$$

$$(f\cdot g)'=f'g+fg'$$

Answers to Exercise 4

1. $3x^2-4x+6$ 2. $8x-17$ 3. $15x^2-20x$ 4. 6

Activity 5

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answers

$$(f \pm g)'(x_0) = \lim_{x \to x_0} \frac{(f \pm g)(x) - (f \pm g)(x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{f(x) \pm g(x) - f(x_0) \mp g(x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \pm \lim_{x \to x_0} \frac{g(x) - g(x_0)}{x - x_0}$$

$$= f'(x_0) \pm g'(x_0)$$

$$(f \pm g)' = f' \pm g'$$

Answers to Exercise 5

1. $-8x + 7$ 2. $750x^5 - 105x^4$ 3. $96x^3 - 6x^2$

Activity 6

Materials

Exercise books, pens.

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $$\left(\frac{1}{f}\right)'(x_0) = \lim_{x \to x_0} \frac{\left(\frac{1}{f}\right)(x) - \left(\frac{1}{f}\right)(x_0)}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{\frac{1}{f(x)} - \frac{1}{f(x_0)}}{x - x_0}$$

$$= \lim_{x \to x_0} \frac{f(x_0) - f(x)}{f(x) f(x_0)(x - x_0)}$$

$$= \lim_{x \to x_0} \frac{f(x_0) - f(x)}{(x - x_0)} \lim_{x \to x_0} \frac{1}{f(x) f(x_0)}$$

$$= -\lim_{x \to x_0} \frac{f(x) - f(x_0)}{(x - x_0)} \lim_{x \to x_0} \frac{1}{f(x) f(x_0)}$$

$$= -f'(x_0) \frac{1}{f(x_0) f(x_0)}$$

$$= \frac{-f'(x_0)}{\left[f(x_0)\right]^2}$$

2. $\left(\frac{1}{f}\right)' = \frac{-f'}{f^2}$

$$\left(\frac{f}{g}\right)' = \left(f \frac{1}{g}\right)'$$

$$= f' \frac{1}{g} + f\left(\frac{1}{g}\right)'$$

$$= \frac{f'}{g} + f\left(-\frac{g'}{g^2}\right)$$

$$= \frac{f'}{g} - \frac{fg'}{g^2}$$

$$= \frac{f'g - fg'}{g}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Answers to Exercise 6

1. $\frac{18x^5 + 12x^3 + 6}{2x^2 + 4x + 1} - \frac{\left(3x^6 + 3x^4 + 6x - 9\right)(4x + 4)}{\left(2x^2 + 4x + 1\right)^2}$

2. $\frac{-3x^2 - 4x}{\left(x^3 + 2x^2 + 6\right)^2}$

Activity 7

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. x^2+5x-3 2. $2x+5$ 3. $2x+3$
4. $2x+5$ 5. 1 6. $2x+5$

Results in 2. and 6. are the same.

Answers to Exercise 7

1. 2 2. x 3. $4x^3+12x$
4. 0 5. 0

Activity 8

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $6x^5+5x^4+9x^2-4x+1$ 2. $30x^4+20x^3+18x-4$
3. $120x^3+60x^2+18$ 4. $360x^2+120x$ 5. $720x+120$

Answers to Exercise 8

1. $3360x^3$ 2. 0
3. $\frac{-6}{(x-2)^2}$ 4. 0

3 Applications of differentiation

Activity 9

Materials

Exercise books, pens, calculator and instrument of geometry

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. Solve $-x^2+3x=3x$. You obtain $x=0$ and also $y=0$
   Some learners can draw the two function in Cartesian plane to see the intersection.
2. 3
3. They are the same.

Answers to revision exercise

1. a) $4x^3$ b) $12x^2$ c) $16x$ d) $2x-4$
2. a) $2x+y=-6$ b) $y=-\frac{4}{3}$
3. a) 44 or 4 b) $\frac{84}{3}$
4. a) 5, 0 b) $6x-6$, 6
   c) $6x^2-10x+4$, $12x-10$ d) $3x^2+\frac{2}{x^2}$, $6x-\frac{4}{x^3}$
5. a) $(19.6-9.8t)ms^{-1}$, $-9.8ms^{-2}$ b) 2s c) 19.6m
   d) 0.586s, 3.41s
6. $a=-3$, $b=-12$
7. a) 3, -9 b) $\frac{32}{27},0$ c) $\frac{32}{27},-\frac{49}{27}$

8. $8\dfrac{ft^2}{ft}$ 9. $16\pi\dfrac{m^3}{m}$

10. a) Increases on $\left(-\infty,-\dfrac{2}{\sqrt{3}}\right)$ and $\left(\dfrac{2}{\sqrt{3}},+\infty\right)$, decreases on $\left(-\dfrac{2}{\sqrt{3}},\dfrac{2}{\sqrt{3}}\right)$

b) Increases on $(-2,0)$ and $(2,+\infty)$, decreases on $(-\infty,-2)$ and $(0,2)$

c) Increases on $(-\infty,3)$ and $(5,+\infty)$, decreases on $(3,5)$

11. $\dfrac{1-y}{2+x}$ 12. $\dfrac{2x+y}{3y^2-x}$ 13. $\dfrac{2-2xy^3}{3x^2y^2+1}$

14. $-\dfrac{3x^2+2xy}{x^2+4y}$ 15. $\begin{cases} y'=-14(3-2x)^6 \\ y''=168(3-2x)^5 \\ y'''=-1680(3-2x)^4 \end{cases}$

16. $\begin{cases} y'=-12(x-1)^{-3} \\ y''=36(x-1)^{-4} \\ y'''=-144(x-1)^{-5} \end{cases}$ 17. $\begin{cases} y'=\dfrac{1}{3}x^{-\frac{2}{3}}+\dfrac{1}{3}x^{-\frac{4}{3}} \\ y''=-\dfrac{2}{9}x^{-\frac{5}{3}}-\dfrac{4}{9}x^{-\frac{7}{3}} \\ y'''=\dfrac{10}{27}x^{-\frac{8}{3}}+\dfrac{28}{27}x^{-\frac{10}{3}} \end{cases}$

18. $\begin{cases} y'=\dfrac{5}{2}x^{\frac{3}{2}}+\dfrac{3}{2}x^{-\frac{1}{2}} \\ y''=\dfrac{15}{4}x^{\frac{1}{2}}-\dfrac{3}{4}x^{-\frac{3}{2}} \\ y'''=\dfrac{15}{8}x^{-\frac{1}{2}}+\dfrac{9}{8}x^{-\frac{5}{2}} \end{cases}$

Unit 7

Vector Space of Real numbers

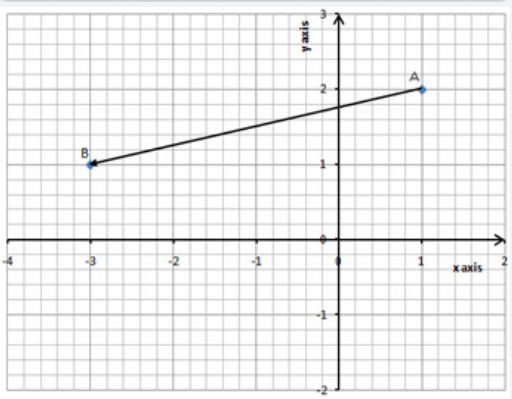
Aim

Determine the magnitude and angle between two vectors and to be able to plot these vectors and point out dot product of two vectors.

Guidance on introductory activity 7

- Form groups of students and invite learners to work on the introductory activity found in student's book
- Move around each group in order to give the support where it is necessary
- Guide the learners to present their findings and help them to harmonize their findings basing on their experience, prior knowledge and abilities shown in answering the questions for this activity.
- Open a discussion with the students on how the vectors are drawn in the plane. This will lead to the introduction of the vector space of real number.

Answer of introductory activity 7



Objectives

By the end of this unit, the learners will:

- find the norm of a vector.
- calculate the scalar product of two vectors.
- calculate the angle between two vectors.
- apply and transfer the skills of vectors to other area of knowledge.

Vocabulary

Linear combination, dependent, independent, dimension

Contents

1 Euclidean vector space IR^2

Recommended teaching time: 6 periods

This section looks at the scalar product of two vectors, magnitude or modulus of a vector and angle between two vectors.

Materials, Methodology, Answers to activities and exercises

1 Euclidian space IR^2

Activity 1

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in pair, then questioning.

Answers

a) 11 b) 0

Answers to Exercise 1

1. a) 5 b) $\sqrt{10}$

2. a) $\|\vec{u}\| = \sqrt{41}$ $\|\vec{v}\| = \sqrt{10}$ b) $\vec{u}.\vec{v} = -7$

Activity 2

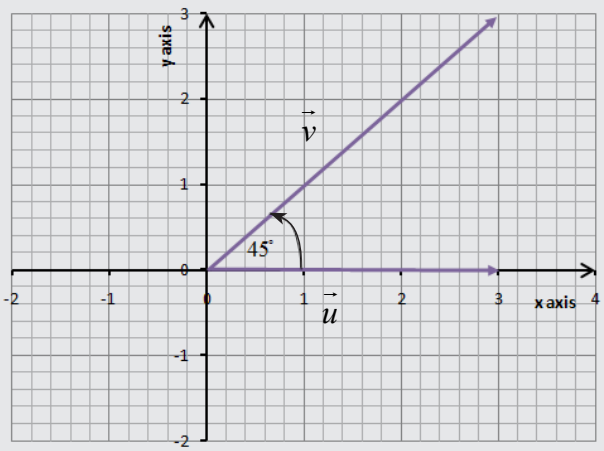
Materials

Exercise books, pens and instrument of geometry

Methodology

Facilitate learners in Group work, then questioning.

Answers



The vector drawn in a. is the adjacent side with length 3 and the vector drawn in b. is the hypotenuse. The other side is the opposite side with length 3.

Then $\frac{adjacent\ side}{hypothenuse} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$

Also $\frac{opposite\ side}{hypothenuse} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$

Answers to Exercise 2

1. a) 53.84^0 b) 40.43^0

2. a) 3 b) $-\frac{4}{3}$ c) $\frac{-26\sqrt{3}+48}{3}$

Answers to revision exercise

1. a) $\sqrt{153}$ b) $\sqrt{97}$ c) $2\sqrt{34}$

2. a) 29 b) -20 c) 4

3. $\frac{13\sqrt{290}}{290}$, 40.2^0, $0.70\,rad$

4. a) 16.3 degrees b) 36.9 degrees

5. a) 6 b) $-\frac{3}{2}$ c) -9 or 1

Unit

8

Matrices and Determinants of order two

Aim

Use matrices and determinants of order 2 to solve systems of linear. equations and to define transformations of 2 dimensions.

Guidance on introductory activity 8

- Form groups of students and invite learners to work on the introductory activity found in student's book
- Move around each group in order to give the support where it is necessary
- Guide the learners to present their findings and help them to harmonize their findings basing on their experience, prior knowledge and abilities shown in answering the questions for this activity.
- Open a discussion with the students on how the numbers were presented before solving the problems. This will lead to the introduction of the matrix concept.

Answer of introductory activity 8

a)

Cocks	Rabbits	Prices
5	4	35,000
3	6	30,000

These numbers can also be presented as follows: $\begin{pmatrix} 5 & 4 \\ 3 & 6 \end{pmatrix}$ and $\begin{pmatrix} 35000 \\ 30000 \end{pmatrix}$ respectively.

c) Let x be the cost of one cock and y be the cost of one rabbit, then , $\begin{cases} 5x+4y=35,000 \\ 3x+6y=30,000 \end{cases}$

$$\begin{cases} 5x+4y=35,000 \times(3) \\ 3x+6y=30,000 \times(-5) \end{cases} \Rightarrow \begin{cases} 15x+12y=105,000 \\ -15x-30y=-150,000 \end{cases}$$

$$\Rightarrow -18y=-45,000 \Rightarrow y=2,500$$

If we replace y in the first equation we obtain

$$5x+4(2500)=35,000 \Rightarrow 5x=25,000 \Rightarrow x=5,000$$

Thus the cost of 1 cock is 5,000Frw and the cost of one rabbit is 2,500Frw.

Objectives

By the end of this unit, the learners will:

- define matrices.
- perform operations on matrices of order 2.
- determine the inverse of a matrix of order 2.

Materials

Manila papers, markers, calculator

Vocabulary

Transpose

Contents

1 Square matrices of order two

Recommended teaching time: 3 periods

This section introduces the concepts of matrices of order two: operations, inverse.

2 Determinants and inverse of matrices

Recommended teaching time: 6 periods

This section introduces determinant, inverse and application.

Materials, Methodology, Answers to activities and exercises

1 Square matrices of order two

Activity 1

Materials

Exercise books, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

$$\begin{array}{l} cell\ \ phones \\ computers \end{array}\left|\begin{pmatrix} 20 & 45 \\ 31 & 23 \end{pmatrix}\right.$$

Answers to Exercise 1

There are many answers. Some of them

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ -20 & 13 \end{pmatrix}, \begin{pmatrix} 1 & -3 \\ 8 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 6 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} -11 & 3 \\ 37 & 18 \end{pmatrix}$$

Activity 2

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. $\begin{pmatrix} 34 & 34 \\ 15 & 22 \end{pmatrix}$ 2. $\begin{pmatrix} 19 & -2 \\ 9 & 16 \end{pmatrix}$ 3. $\begin{pmatrix} 13 & 6 \\ 4 & 10 \end{pmatrix}$, $\begin{pmatrix} 7 & 3 \\ 10 & 4 \end{pmatrix}$

Answers to Exercise 2

1. a) $\begin{pmatrix} 5 & 0 \\ 3 & 2 \end{pmatrix}$ b) $\begin{pmatrix} -9 & -12 \\ 1 & 2 \end{pmatrix}$ c) $\begin{pmatrix} -12 & -6 \\ 8 & 4 \end{pmatrix}$

d) $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ e) $\begin{pmatrix} 294 & 147 \\ 98 & 49 \end{pmatrix}$

2. $x = 7, y = -1, z = \frac{-3}{2}$

a) $A = \begin{pmatrix} 8 & -3 \\ 1 & 1 \end{pmatrix}$ b) $A^t = \begin{pmatrix} 8 & 1 \\ -3 & 1 \end{pmatrix}$

2 Determinants of matrices of order two

Activity 3

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. 1 2. 0 3. 18 4. 114

Answers to Exercise 3

1. 2 2. 0 3. 0 4. 2

Activity 4

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

$$\frac{1}{18}\begin{pmatrix} 3 & -2 \\ -6 & 10 \end{pmatrix}$$

Answers to Exercise 4

1. $A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$ 2. No inverse 3. No inverse

4. $\left(A^t\right)^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$ 5. No inverse

Activity 5

Materials

Exercise books, pens

Methodology

Facilitate learners in pair, then questioning.

Answers

1. $\begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$ 2. $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 19 \\ 10 \end{pmatrix}$

Answers to Exercise 5

1. $x=3,\ y=4$
2. Infinity solution
3. $x=1,\ y=1$
4. $x=2, y=0$

Answers to revision exercise

1. a) $x=-4,\ y=11$ b) $x=2,\ y=-1$ c) $x=5,\ y=19$
2. 4, -11
3. $-\frac{1}{2}$ or 3
4. a) $x=3,\ y=-2$ b) $x=1,\ y=-\frac{2}{3}$ c) $x=\frac{3}{2},\ y=\frac{1}{2}$

Unit 9

Measures of Dispersion

Aim

Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.

Guidance on introductory activity 9

- In small groups or pairs, let learners read and do the introductory activity in the students' book.
- Facilitate learners to think on different ways of getting solutions.
- Through question-answer, facilitate learners to understand how statistics is important /used in everyday life.
- After a given time invite learners to present their findings and harmonize them.
- From presentations, the teacher decides to engage the class into discussions that help to the introduction of the unit.

Answer of introductory activity 9

1. The table below shows the types and the number of sold fruits in one week.

Type of fruit	A (Banana)	B (Orange)	C (Pineapple)	D (Avocado)	E (Mango)	F (apple)
Number of fruits sold	1100	962	1080	1200	884	900

a) The highest number of fruits sold is 1200 (Avocadoes)

b) The least number of fruits sold is 884 (mangoes)

c) The total number of fruits sold during the week is 6126 fruits

d) The average number of fruits sold per day is $\frac{6126}{6} = 1021$

2.

a) The mean mark of the class is

$$\frac{3+5+6+3+8+7+8+4+8+6}{10} = \frac{58}{10} = 5.8 \ .$$

b) The mark that was obtained by many students is 8

c) Comparing the mean mark of the class and the mark for every student-teacher, one can find that 4 students have the marks (3 , 4 and 5) below the mean, 2 students scored the mark near the mean while 4 students have scored higher marks than the mean. Mathematics tutor should prepare remedial activities for students whom their marks are below and near the mean

Objectives

By the end of this unit, the learners will:

- determine the measures of dispersion of a given statistical series.
- apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.
- express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.

Vocabulary

Variance, Standard deviation, Coefficient of variation

Contents

1 Variance

Recommended teaching time: 2 periods

Variance measures how far a set of numbers is spread out. A variance of zero indicates that all the values are identical. Variance is always non-negative: a small variance indicates that the data points tend to be very close to the mean and hence to each other, while a high variance indicates that the data points are very spread out around the mean and from each other.

2 Standard deviation

Recommended teaching time: 1 period

The standard deviation has the same dimension as the data, and hence is comparable to deviations from the mean. We define the **standard deviation** to be the square root of the variance.

3 Coefficient of variation

Recommended teaching time: 2 periods

The coefficient of variation measures variability in relation to the mean (or average) and is used to compare the relative dispersion in one type of data with the relative dispersion in another type of data.

4 Applications

Recommended teaching time: 1 period

A large standard deviation indicates that the data points can spread far from the mean and a small standard deviation indicates that they are clustered closely around the mean.

Standard deviation is often used to compare real-world data against a model to test the model.

Materials, Methodology, Answers to activities and exercises

1 Variance

Activity 1

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

x	f	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$	$f\left(x-\bar{x}\right)^2$
12	4	-4.875	23.76563	95.0625
13	2	-3.875	15.01563	30.03125
15	1	-1.875	3.515625	3.515625
19	4	2.125	4.515625	18.0625
21	5	4.125	17.01563	85.07813
	16			231.75

Answers to Exercise 1

1. $\frac{81}{25}$ 2. 6 3. $\frac{308}{81}$ 4. $\frac{11}{25}$ 5. $\frac{38}{9}$

2 Standard deviation

Activity 2

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

x	f	x^2	fx	fx^2
3	2	9	6	18
4	3	16	12	48
5	5	25	25	125
7	1	49	7	49
9	6	81	54	486
	17		104	726

Answers to Exercise 2

1. $\frac{3\sqrt{2}}{2}$
2. $\frac{2\sqrt{1224499}}{7}$
3. $\frac{4\sqrt{10}}{7}$
4. $\sqrt{2}$
5. $\frac{2\sqrt{34}}{7}$

3 Coefficient of variation

Activity 3

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

x	f	x^2	fx	fx^2
10	10	100	100	1000
14	2	196	28	392
16	14	256	224	3584
18	8	324	144	2592
20	6	400	120	2400
	40		616	9968

Answers to Exercise 3

1. 54.51%
2. 20.79%
3. 55.12%
4. 22.22%
5. 43.92%

Answers to revision exercise

1. 53.6
2. $Mean = 32, S.D = 12.36, Median = 32.61$
3. a. 8, 9, 6 b) 194, 194, 195
4. 24, 1.19
5. a) 775, 120250 b) 157cm, 5.89cm
6. 21
7. 8
8. 7
9. $2-3\bar{x}$, $9\sigma^2$
10. 22.9^0, 3.27^0
11. 12; 4, 3
12. 5.10
13. 5
14. 3.74
15. 14.4, 1.68
16. 11, 14
17. $\bar{x} = 6.4, \sigma = 2.42$

    a) $\bar{x} = 6.4 + 3 = 9.4, \sigma = 2.42$

b) $\bar{x} = 3 \times 6.4 = 19.2, \sigma = 3 \times 2.42 = 7.25$

c) $\bar{x} = 0.9 \times 6.4 = 5.76, \sigma = 0.9 \times 2.42 = 2.17$

d) $\bar{x} = 1.05 \times 6.4 = 6.72, \sigma = 1.05 \times 2.42 = 2.54$

18. 19 19. 6, 4

Unit 10

Elementary Probability

Aim

Use combinations and permutations to determine the number of ways a random experiment occurs.

Guidance for the introductory activity 10

- In small groups or pairs, let learners read and do the introductory activity in the students' book.
- Form small groups of learners and give them instructions on how to work on the given introductory activity;
- Walk around to each group and ask probing questions leading them to determine the total number of roads from A to C via B;
- Ask each group to share their answers with neighbouring group and ask them to support each other for improvement;
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a facilitator, harmonize their answers highlighting that there is a technique of finding the total number of outcomes for a given random experiment;
- Use different probing questions and guide them to explore examples given in the student's book and lead them to determine total number of outcomes for a given random experiment using: Venn diagram, tree diagram or a table.

- Guide them to discover that if a sequence of n events in which the first one has n_1 possibilities, the second with n_2 possibilities the third with n_3 possibilities, and so forth until n_k, the total number of possibilities of the sequence will be

  $= n_1.n_2.n_3...\mathrm{n_k}$

- From presentations, the teacher decides to engage the class into discussions that help to the introduction of the unit.

Answer of introductory activity 10

1. Many answers may be found by students
2. To find all possible roads, students can use allows to join points or a try and fail method.

   $\Omega = \left\{ AB_1C_1, AB_1C_2, AB_1C_3, AB_2C_1, AB_2C_2, AB_2C_3 \right\}$ so they are 6.

3. a) There are 25 black cards in an ordinary deck of 52 cards.

   b) $P(\mathrm{A}) = \dfrac{n}{number\ of\ allcards} = \dfrac{26}{52} = 0.5$

   c) $P(A) = \dfrac{Number\ of\ outcome\mathrm{s\ inE}}{Total\ \mathrm{nu}\ mber\ of\ outcome\mathrm{s\ in}\ the\ sample\ space} = \dfrac{n(\mathrm{E})}{n(\Omega)}$

Objectives

By the end of this unit, the learners will:

- determine the sample space of an experiment.
- explain different concepts relating to events.
- find probability of different events.
- determine probability of an event with an equiprobable sample space.

Materials

Manila papers, markers, deck of 52 playing cards, calculator

Vocabulary

Permutation, arrangement, factorial, combination, probability, sample space and event.

Contents

1 Permutations and arrangements

Recommended teaching time: 6 periods

This section introduces the concepts of permutations and arrangements. It looks at addition and multiplication principles.

2 Combinations

Recommended teaching time: 3 periods

This section looks at combinations of r unlike objects selected from n different objects. It also looks at binomial theorem and Pascal's triangle.

3 Concepts of probability

Recommended teaching time: 3 periods

Probability is the chance that something will happen-how likely it is that some event will happen. A random experiment is an experiment that, atleast theoretically, may be repeated as often as we want and whose outcome cannot be predicted, the roll of a die. Each time an experiment is repeated, an elementary outcome is obtained. The set of all elementary outcomes of a random experiment is called the sample space, which is denoted by Ω. Sample space may be discrete or continuous. An event is a set of elementary outcomes. That is, it is a subset of the sample space.

4 Properties and formulas

Recommended teaching time: 3 periods

The probability of an event $A \subset \Omega$, is a real number obtained by applying to A the function P defined by

$$P(A) = \frac{number\ of\ \ favourable\ outcomes}{number\ of\ \ possible\ outcomes} = \frac{\#A}{\#\Omega}$$

Addition probability law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Materials, Methodology, Answers to activities and exercises

1 Permutations and arrangements

Activity 1

Materials

Exercise book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

a) $AB_1C_1, AB_1C_2, AB_1C_3, AB_2C_1, AB_2C_2, AB_2C_3$
$AB_3C_1, AB_3C_2, AB_3C_3, AB_3C_1, AB_3C_2, AB_3C_3$

12 roads

b) $H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6$: 12 outcomes

Answers to Exercise 1

1. 20 2. 2000000

Activity 2

Materials

Exercise book and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

- All possible different words from three letters A, B and C (not necessarily sensible):

  ABC, ACB, BAC, BCA, CAB,CBA. Number of possible arrangements for three letters A, B and C is 6

- All possible different arrangements:

  ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CBAD, CBDA, CABD, CADB, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA. Number of possible arrangements for four letters A, B, C and D is 24

Answers to Exercise 2

1. a) 19! b) 17!3!
2. a) 60 b) 1

Activity 3

Materials

Exercise book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

- All possible different arrangements:
  BOOM, BOMO, BMOO, OBOM, OMOB, OMBO, OBMO, OOMB, OOBM, MOOB, MOBO, MBOO. Number of possible arrangements is 12

- All possible different arrangements:
  CLASS, CLSAS, CLSSA, CSLSA, CSSLA, CSSAL, CALSS, CASLS, CASSL, CSLAS, CSASL, CSALS, LCASS, LCSSA, LCSAS, LACSS, LASCS, LASSC, LSCAS, LSACS, LSASC, LSSAC, LSSCA, LSCSA, ACLSS, ACSLS, ACSSL, ALCSS, ALSCS, ALSSC, ASSLC, ASSCL, ASCLS, ASLCS, ASLSC, ASCSL, SCLAS, SCALS, SCSAL, SCSLA, SCLSA, SCASL, SLCAS, SLCSA, SLACS, SLASC, SLSAC, SLSCA, SACLS, SACSL, SALCS, SALSC, SASLC, SASCL, SSACL, SSALC, SSLAC, SSLCA, SSCLA, SSCAL.

  Number of possible arrangements is 60

Answers to Exercise 3

1. 5040 2. 26! 3. 1260

Activity 4

Materials

Exercise book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

- There are 24 different ways.

Answers to Exercise 4

1. 24 2. 10!

Activity 5

Materials

Exercise book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

- You must pick one or the other but not both

Answers to Exercise 5

1. (3+15+60+180+360+360) or 978
2. 4
3. 336

Activity 6

Materials

Exercise book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

- Selection: NOW, permutations:
  $\mathrm{NOW, NWO, ONW, OWN, WNO, WON}$
- Selection: KOW, permutations:
  $\mathrm{KOW, KWO, OKW, OWK, WKO, WOK}$
- Selection: KNW, permutations:
  $\mathrm{KNW, KWN, NKW, NWK, WKN, WNK}$
- Selection: KNO, permutations: $\mathrm{KNO, KON, NKO, NOK, OKN, ONK}$
  In total, there are 24 possible permutations.

Answers to Exercise 6

1. ${}^{7}P_{4} = 840$
2. 20
3. ${}^{7}P_{5}$
4. ${}^{26}P_{10}$

2 Combinations

Activity 7

Materials

Exercise book, pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

- There are 18 groups

Answers to Exercise 7

1. ${}^{10}C_4 \times {}^{12}C_2$ 2. ${}^{9}C_4 \times {}^{10}C_5$

Activity 8

Materials

Exercise book and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Power	Coefficient of powers of a and b					Binomial expression
0	1					$(a+b)^0$
1	1	1				$(a+b)^1$
2	1	2	1			$(a+b)^2$
3	1	3	3	1		$(a+b)^3$
4	1	4	6	4	1	$(a+b)^4$

Answers to Exercise 8

1. 240 2. 0 3. 9039811410
4. $x^7+28x^6+336x^5+2240x^4+8960x^3+21504x^2+28672x+16384$
5. $8x^3-36x^2+54x-27$

3 Concepts of probability

Activity 9

Materials

Exercise books and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

1. a) 52, b) 4, c) 1 2. 1

Answers to Exercise 9

1. c 2. d 3. d

4. a) Exhaustive: $X \cup Y \cup Z = \Omega$

   b) Exhaustive: $X \cup Y \cup Z = \Omega$

   c) Exhaustive: $X \cup Y \cup Z = \Omega$

4 Properties and formulas

Activity 10

Materials

Exercise books and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

a) 11 b) 4, $\frac{4}{11}$ c) 7, $\frac{7}{11}$

d) (i) Empty set

(ii) $\{O, A, I, I, P, R, B, B, L, T, Y\}$

(iii) $\{P, R, B, B, L, T, Y\}$

(iv) $\{O, A, I, I\}$

Answers to Exercise 10

1. a) $\frac{2}{11}$ b) $\frac{2}{11}$

2. a) $\frac{13}{19}$ b) $\frac{3}{19}$ c) $\frac{3}{19}$

Activity 11

Materials

Exercise books, pens and calculator

Methodology

Facilitate learners in Group work, then questioning.

Answers

a) $\frac{2}{3}$ b) $\frac{2}{3}$

The two results are the same.

Answers to Exercise 11

1. $\frac{5}{6}$ 2. $\frac{1}{2}$

Activity 12

Materials

Exercise books and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 0 d) 1 e) 1

Answers to Exercise 12

1. $\frac{8}{15}$ 2. $\frac{11}{30}$

Activity 13

Materials

Exercise books and pens

Methodology

Facilitate learners in Group work, then questioning.

Answers

a) 1 b) 1

The two results are the same

Answers to Exercise 13

a) $\frac{3}{8}$ b) $\frac{5}{8}$ c) $\frac{1}{32}$

Answers to revision exercise

1. a) 144 b) 72
2. 216, 72, 108
3. 18
4. 10!
5. 8!
6. 24
7. a) 210 b) $\frac{1}{110}$ c) 600 d) 81
8. a) $n-1$ b) $(n+2)(n+1)$ c) $n(n^2+3n+1)$
9. 5040
10. 168
11. a. 240 b) 600
12. a. 81 b) 256
13. 36
14. 2522520
15. 151 a) 73 b) 78
    c) 13 d) 138
16. a) 462 b) 56 c) 20
17. 1260
18. a) 5 b) 85 c) 365
19. a) 126 b) 280

20. 480, 172800, 462, 425

21. a) $27+27x+9x^2+x^3$

b) $125+150x+60x^2+8x^3$

c) $16+32x+24x^2+8x^3+x^4$

d) $16-32x+24x^2-8x^3+x^4$

e) $32y^5+80y^4x+80y^3x^2+40y^2x^3+10yx^4+x^5$

f) $32x^5-240x^4y+720x^3y^2-1080x^2y^3+810xy^4-243y^5$

g) $x^4-4x^2+6-\frac{4}{x^2}+\frac{1}{x^4}$ h) $x^5-10x^3+40x-\frac{80}{x}+\frac{80}{x^3}-\frac{32}{x^5}$

22. $1+12x+66x^2+220x^3$ 23. $64x^5+160x^{-1}+20x^{-7}$

24. 0, 1 (trivial) and 6 25. 2

26. 30.43168 27. $a^{10}-30a^9x+405a^8x^2-3240a^7x^3$

28. $1+10x+\frac{95}{2}x^2+\frac{285}{2}x^3+\frac{4845}{16}x^4$

29. $16,\ \frac{1}{8}$

30. a) ${}^{10}C_5\times3^5$ b) ${}^{12}C_8\times4^{10}$ c) ${}^{6}C_4\times3^2\times2^4$ d) $2^4\left(2\times{}^{10}C_5+{}^{10}C_6\right)$

31. a) 1.0937 b) 0.9860837 c) 0.9044 d) 973.9

32. a) 0.97980 b) 10.1980 c) 2.0199 d) 1.01943

e) 2.05828

33. $1-\frac{3}{2}x+\frac{15}{8}x^2-\frac{51}{16}x^3, |x|<\frac{1}{2}$ 34. 2, -9, 29, -82

35. a) ${}^{8}C_3\times5^5\times3^3$ b) $-{}^{7}C_3\times7^4\times2^3$

36. a) 560 b) -590625 c) -720 d) -448

e) 1966080 f) $-\frac{7}{144}$

37. $\pm\frac{2}{3}$ 38. ${}^{12}C_8\times4^4\times x^8$ 39. -20

40. $\frac{3}{2},\frac{1}{2},\frac{1970}{1393}$ 41. All of above 42. Landing on red

43. Choose a letter at random from the word SCHOOL

44. Choosing a yellow jelly bean

45. Landing on a number less than 8

46. $\frac{9}{20}$ 47. $\frac{10}{21}$ 48. $\frac{1}{3}$ 49. $\frac{21}{46}$

50. $\frac{1}{221}$ 51. $\frac{3}{13}$

52. a) 0.5 b) 0.14 c) 0.07 d) 0.43

53. a) $\frac{1}{6}$ b) $\frac{5}{126}$