REPUBLIC OF RWANDA

MINISTRY OF EDUCATION

Rwanda Education Board

# MATHEMATICS SYLLABUS FOR ADVANCED LEVEL S4-S6

Kigali, 2015

# MATHEMATICS SYLLABUS FOR ADVANCED LEVEL S4 – S6

## Kigali, 2015

## FOREWORD

The Rwanda Education Board is honoured to provide syllabuses which serve as both official documents and as a guide to competence-based teaching and learning. These syllabuses ensure consistency and coherence in the delivery of quality education across all levels of general education in Rwandan schools.

The Rwandan education philosophy aims to ensure that young people at every level of education achieve their full potential in terms of relevant knowledge, skills and appropriate attitudes in order to prepare them to be well integrated into society and make full use of employment opportunities.

In line with efforts to improve the quality of education, the government of Rwanda emphasises the importance of aligning the syllabus, teaching and learning and assessment approaches in order to ensure that the system is producing the kind of citizens the country needs. Many factors influence what children are taught, how well they learn and the competences they acquire, particularly the relevance of the syllabus, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials available. The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market has necessitated the shift to a competence-based syllabus. With the help of the teachers, whose role is central to the success of the syllabus, learners will gain appropriate skills and be able to apply what they have learned in real life situations. Hence they will make a difference not only to their own lives but also to the success of the nation.

I wish to sincerely extend my appreciation to the people who contributed to the development of this document, particularly REB and its staff who organised the whole process from its inception. Special appreciation goes to the development partners who supported the exercise throughout. Any comment or contribution would be welcome for the improvement of this syllabus.

**GASANA I.Janvier**
**Director General REB**

## ACKNOWLEDGEMENT

I wish to sincerely extend my special appreciation to the people who played a major role in the development of this syllabus. It would not have been successful without the participation of a range of education stakeholders and the financial support from different donors. For this, I would like to express my deep gratitude.My thanks firstly goes to the Rwanda Education leadership who supervised the curriculum review process and the Rwanda Education Board staff who were involved in the conception and writing of the syllabus. I wish to extend my appreciation to teachers from pre-primary to university level for their valuable efforts during the conception of the syllabus.

I owe gratitude to the different education partners such as UNICEF, UNFPA, DFID and Access to Finance Rwanda for their financial and technical support. We also value the contribution of other education partner organisations such as CNLG, AEGIS Trust, Itorero ry'Igihugu, Center for Gender Sudies, Gender Monitoring Office, National Unit and Reconciliation Commission, RBS, REMA, Handicap International, Wellspring Foundation, Right To Play, MEDISAR, EDC/L3, EDC/Akazi Kanoze, Save the Children, Faith Based Organisations, WDA, MINECOFIN and local and International consultants. Their respective initiatives, co- operation and support significantly contributed to the successful production of this syllabus by the Curriculum and Pedagogical Material Production Department (CPMD).

**Dr. Joyce Musabe**

Head of CPMD,

## LIST OF PARTICIPANTS WHO WERE INVOLVED IN THE DEVELOPMENT OF THE SYLLABUS

**Rwanda Education Board**

1. Dr. MUSABE Joyce, Head of CPMD, as a Facilitator
2. RUTAKAMIZE Joseph, Director of Science Unit in CPMD
3. KAYINAMURA Aloys, Mathematics Curriculum Specialist : Team leader
4. NYIRANDAGIJIMANA Anathalie, Specialist in charge of Pedagogic Norms

**Teachers and Lecturers**

1. BIBUTSA Damien, Mathematics teacher at Ecole des Sciences de Musanze
2. HABINEZA NSHUTI Jean Clément, Mathematics teacher at Ecole Secondaire de Nyanza
3. NDARA Lula Jerry, Mathematics teacher at EFOTEK
4. UNENCAN MUNGUMIYO Dieudonné, Mathematics teacher at Lycée de Kigali

**Other resource persons**

MUREKERAHO Joseph, National consultant

**Quality assurer/editors**

Dr Alphonse Uworwabayeho (PhD), University of Rwanda (UR), College of Education

# TABLE OF CONTENTS

FOREWORD ....... iv

ACKNOWLEDGEMENT ....... v

LIST OF PARTICIPANTS WHO WERE INVOLVED IN THE DEVELOPMENT OF THE SYLLABUS ....... vi

1. INTRODUCTION: ....... 9

1.1. Background to curriculum review ....... 9

1.2. Rationale of teaching and learning mathematics ....... 9

1.2.1. Mathematics and society ....... 9

1.2.2. Mathematics and learners ....... 10

1.2.3. Competences ....... 10

2. PEDAGOGICAL APPROACH ....... 14

2.1. The role of the learner ....... 14

2.2. Role of the teacher ....... 15

2.3. Special needs education and inclusive approach ....... 16

3. ASSESSMENT APPROACH ....... 16

3.1. Types of assessment ....... 17

3.2. Record keeping ....... 18

3.3. Item writing in summative assessment ....... 18

3.4. Reporting to parents ....... 20

4. RESOURCES ....... 20

4.1. Materials needed for implementation ....... 20

4.2. Human resource ....... 21

5. SYLLABUS UNITS ..... 22
5.1. Presentation of the structure of the syllabus units ..... 22
5.2. Mathematics program for secondary four ..... 23
5.3. Mathematics program for secondary five ..... 43
5.4. Mathematics program for secondary six ..... 61
6. REFERENCES ..... 79
7. APPENDIX: SUBJECTS AND WEEKLY TIME ALOCATION FOR A'LEVEL ..... 80

# 1. INTRODUCTION

## 1.1. Background to curriculum review

The motivation for reviewing the mathematics syllabus at advanced level was to ensure that the syllabus is responsive to the needs of the learner and to shift from objective and knowledge-based learning to competence-based learning. The review puts more emphasis on skills and competencies. The coherence within the existing content isy benchmarked with syllabi elsewhere as examples best practice.

The new Mathematics syllabus guides the interaction between the teacher and the learners in the learning process and highlights the competencies a learner should acquire during and at the end of each unit of learning.

Learners will have the opportunity to apply Mathematics in different contexts, and see its importance in daily life. Teachers help the learners appreciate the relevance and benefits for studying this subject at advanced level.

The new Mathematics syllabus is prepared for all science combinations with Mathematics as the core subject where it has to be taught in seven periods per week.

## 1.2. Rationale of teaching and learning mathematics

### 1.2.1. Mathematics and society

Mathematics plays an important role in society through abstraction and logic, counting, calculation, measurement, systematic study of shapes and motion. It is also used in natural sciences, engineering, medicine, finance and social sciences. The applied Mathematics such as statistics and probability play an important role in game theory, in the national census process, in scientific research, et cetera. In addition, some cross-cutting issues such as financial awareness are incorporated into some of the Mathematics units to improve the social and economic welfare of Rwandan society. Mathematics is key to the Rwandan educational ambition of developing a knowledge-based and technology-led economy since it provides learners with all required knowledge and skills to be used in different learning areas. Therefore, Mathematics is an important subject as it supports other subjects. This new curriculum will address gaps in skills and attitudes in the current Rwanda Education system.

### 1.2.2. Mathematics and learners

Learners need enough basic mathematical competences to be effective members of Rwandan society including the ability to estimate, analyse, interpret statistics, assess probabilities and read commonly used mathematical representations and graphs.

Therefore, mathematics equips learners with knowledge, skills and attitudes necessary to enable them to succeed in an era of rapid technological growth and socio-economic development. Mastery of basic mathematical ideas and calculations makes learners confident in problem-solving. It enables the learners to be systematic, creative and self confident in using mathematical language and techniques to reason, think critically and develop imagination, initiative and flexibility of mind. In this regard, learning of Mathematics needs to include practical problem-solving activities with opportunities for students to plan their own investigations in order to develop their mathematical competence and confidence.

As new technologies have had a dramatic impact on all aspects of life, wherever possible in Mathematics learners should gain experience of a range of ICT equipment and applications.

### 1.2.3. Competences

Competence is defined as the abilityto perform a particular task successfully, resulting from having gained an appropriate combination of knowledge, skills and attitudes.

The Mathematics syllabus gives the opportunity for learners to develop different competences, including the generic competences .

Basic competences are addressed in the stated broad subject competences and in objectives highlighted on a year basis and in each unit of learning. The generic competences, basic competences that must be emphasised and reflected in the learning process, are briefly described below and teachers will ensure that learners are exposed to tasks that help the learners acquire these skills.

**Generic Competences and Values**

**Critical and problem solving skills:** Learners use different techniques to solve mathematical problems related to real life situations. They are engaged in mathematical thinking, they construct, symbolise, apply and generalise mathematical ideas. The acquisition of such skills will help learners to think imaginatively and broadly to evaluate and find solutions to problems encountered in all situations.

**Creativity and innovation**: The acquisition of such skills will help learners to take initiatives and use imagination beyond the knowledge provided to generate new ideas and construct new concepts. Learners will improve these skills through Mathematics contests and Mathematics competitions, etc.

**Research:** This will help learners to find answers to questions basing on existing information and concepts as well as explain phenomena based on findings from information gathered.

**Communication in official languages:** Learners communicate effectively their findings through explanations, construction of arguments and drawing relevant conclusions. Mathematics teachers, irrespective of not being teachers of language, will ensure the proper use of the language of instruction by learners. This will help learners to communicate clearly and confidently and convey ideas effectively through speaking and writing and using the correct language structure and relevant vocabulary.

**Cooperation, inter-personal management and life skills:** Learners are engaged in cooperative learning groups to promote higher achievement rather than competitive and individual work.

This will help learners to cooperate with others as a team in whatever tasks are assigned and to practice positive ethical moral values and respect for the rights, feelings and views of others. Learners will perform practical activities related to environmental conservation and protection. They will also advocate for personal, family and community health, hygiene and nutrition and respond creatively to the variety of challenges encountered in life.

**Lifelong learning:** The acquisition of such skills will help learners update their knowledge and skills with minimum external support and to cope with the evolution of advances in knowledge for personal fulfillment in areas that need improvement and development.

**Broad Mathematics Competences**

During and at the end of the learning process, the learner can:

- Develop clear, logical, creative and coherent thinking.
- Master basic mathematical concepts and use them correctly in daily life problem solving.
- Express clearly, comprehensibly, correctly and precisely in verbal and/or written form all the reasons and calculations leading to the required result whenever finding a solution to any given exercise.
- Master the presented mathematical models and identify their applications in his/her environment.
- Demonstrate mathematical interest and research curiosity in theories and their applications.
- Use the acquired mathematical concepts and skills to persue further study (colleges, higher institutions and universities).
- Use acquired mathematical skills to develop work spirit, team work, self-confidence and time management without supervision.
- Use ICT tools to explore Mathematics (such as calculators, computers, mathematical software).
- Demonstrate a sense of research, curiosity and creativity in their areas of study.

**Mathematics and developing competences**

The national policy documents based on national aspirations identify some 'Basic Competences' alongside the 'Generic Competences'' that will develop higher order thinking skills and help students learn subject content and promote the application of acquired knowledge and skills.

Through observations, constructions, the use of symbols, applying and generalising mathematical ideas and presenting information during the learning process, the learner will not only develop deductive and inductive skills but also acquire cooperation, communication, critical thinking and problem solving skills. This will be realised when learners make presentations leading to inferences and conclusions at the end of the learning unit. This will be achieved through learner group work and cooperative learning which in turn will promote interpersonal relations and teamwork.

The acquired knowledge in learning Mathematics should develop a responsible citizen who adapts to scientific reasoning and attitudes and develops confidence in reasoning independently. The learner should show concern for individual attitudes and environmental protection and comply with the scientific method of reasoning. The scientific method should be applied with the necessary rigor and intellectual honesty to promote critical thinking while systematically pursuing a line of thought.

The selection of types of learning activities must focus on how the learners are able to demonstrate such competencies throughout and at the end of the learning process.

# 2. PEDAGOGICAL APPROACH

The change to a competence-based curriculum is about transforming learning, ensuring that learning is deep, enjoyable and habit-forming.

## 2.1. The role of the learner

In the competence-based syllabus, the learner is the principal actor in his/her education. He/she is not an empty bottle to fill. Taking into account the initial capacities and abilities of the learner, the syllabus lists learning activities under each unit and reflects appropriate engagement of the learner in the learning process.

The teaching and learning processes will be tailored towards creating a learner-friendly environment based oncapabilities, needs, experience and interests. Therefore, the following are some of the roles or the expectations from learners:

- Learners construct the knowledge either individually or in groups in an active way. From the learning theory, learners move in their understanding from concrete through pictorial to abstract. Therefore, the opportunity should be given to learners to manipulate concrete objects and to use models.
- Learners are encouraged to use a hand-held calculator. This stimulates mathematical ability as it isused, bothin work environment and in scientific applications. Frequent use of calculators can enhance learners' understanding and mastery of arithmetic.
- Learners work on one competence at a time in the form of concrete units with specific learning objectives broken down into knowledge, skills and attitude.
- Learners will be encouraged to do research and present their findings through group work activities.
- A learner is co-operative: learners work in heterogeneous groups to increase tolerance and understanding.
- Learners are responsible for their own participation and for ensuring the effectivness of their work.

- Help is sought from within the group and the teacher is asked for help only when the whole group agrees to ask a question.
- The learners who learn at a faster pace do not do the task alone and then the others merely sign off on it.
- Participants ensure the the effective contribution of each member to clear explanation and argumentation to improve English literacy and to develop a sense of responsibility,increasing self-confidence, public speech ability, etc.

### 2.2. Role of the teacher

In the competence-based syllabus, the teacher is a facilitator, organiser, advisor and conflict solver.
The specific duties of the teacher in a competence-based approach are the following:

- He/she is a facilitator: his/her role is to provide opportunities for learners to encounter problems that interest and challenge them and that, with appropriate effort, they can solve. This requires an elaborated preparation to plan the activities, the place they will be carried out and the required assistance.
- He/she is an organiser: his/her role is to organise the learners in the classroom or outside and engage them through participatory and interactive methods through the learning processes as individuals, in pairs or in groups. To ensure that the learning is personalised, active, participativeand co-operative the teacher must identify the needs of the learners, the nature of the learning to be done, and the means to shape learning experiences accordingly.
- He/she is an advisor: he/she provides counseling and guidance for learners in need. He/she comforts and encourages learners by valuing their contributions in the class activities.
- He/she is a conflict-solver: most of the competence-based activities are performed in groups. When the members of a group have problems such as the attribution of tasks they should find the intervention of the teacher useful and constructive as a unifying element.

- He/she is ethical: he/she teaches by example by being impartial, by being a role-model and by caring for individual needs, especially for slow learners and learners with physical impairments, through special assistance by providing remedial activities or reinforcement activities. One should notice that this list is not exhaustive.

### 2.3. Special needs education and inclusive approach

All Rwandans have the right to access education regardless of their different needs. The underpinnings of this provision would naturally hold that all citizens benefit from the same menu of educational programs. The possibility of this assumption is the focus of special needs education. The critical issue is that we have persons/ learners who are totally different in their ways of living and learning as opposed to the majority. The difference can either be emotional, physical, and sensory or intellectually learning challenged, traditionally known as mental retardation.

These learners equally have the right to benefit from the free and compulsory basic education in the nearby ordinary/mainstream schools. Therefore, the school's role is to enrol them and also set strategies to provide relevant education for them. The teacher therefore is requested to consider each learner's needs during the teaching and learning process. Assessment strategies and conditions should also be standardised to the needs of these learners. Detailed guidance for each category of learners with special education needs is provided for in the guidance for teachers.

## 3. ASSESSMENT APPROACH

Assessment evaluates the teaching and learning processes through collecting and interpreting evidence of an individual learner's learning progress and to make a judgment about a learner's achievements measured against defined standards. Assessment is an integral part of the teaching and learning process. In the new competence-based curriculum assessment must also be competence-based, whereby a learner is given a complex situation related to his/her everyday life and asked to try to overcome the situation by applying what he/she has learned. Assessment will be organised at the following levels: School-based assessment, district examinations, national assessment (LARS) and national examinations.

## 3.1. Types of assessment

### 3.1.1 Formative assessment

Formative assessment helps to check the efficiency of the process of learning. It is done within the teaching/learning process. Continuous assessment involves formal and informal methods used by schools to check whether learning is taking place. When a teacher is planning his/her lesson, he/she should establish criteria for performance and behavioral changes at the beginning of a unit. Then at the end of every unit, the teacher should ensure that all the learners have mastered the stated key unit competences based on the criteria stated, before going on to the next unit. The teacher will assess how well each learner masters both the subject and the generic competences described in the syllabus, and from this the teacher will gain a picture of the all-round progress of the learner. The teacher will use one or a combination of the following: (a) observation (b) pen and paper (c) oral questioning.

### 3.1.2 Summative assessment

When assessment is used to record a judgment of a competence or the performance of the learner, it serves a summative purpose. Summative assessment gives a picture of a learner's competence or progress at any specific moment. The main purpose of summative assessment is to evaluate whether learning objectives have been achieved. The results are used for the ranking or grading of learners, for deciding on progression, for selection into the next level of education and for certification. This assessment should have an integrative aspect whereby a student must be able to show mastery of all competences.

Summative assessment can be internal school-based assessment or external assessment in the form of national examinations. School-based summative assessment should take place once at the end of each term and once at the end of the year. School summative assessment average scores for each subject will be weighted and included in the final national examinations grade. School-based assessment average grades will contribute a certain percentage as teachers gain more experience and confidence in assessment techniques. In the third year of the implementation of the new curriculum it will contribute 10% of the final grade, but will be progressively increased. Districts will be supported to continue their initiatives to organize a common test per class for all the schools to evaluate the performance and the achievement level of learners in each individual school. External summative assessment will be done at the end of P6, S3 and S6.

### 3.2. Record keeping

This is gathering facts and evidence from assessment instruments and using them to judge the student's performance by assigning an indicator against the set criteria or standard. Assessment procedures generate data in the form of scores which will be carefully recorded and stored in a portfolio. These scores will contribute to remedial actions and alternative instructional strategies. They will also be used to provide feedback to the learner and their parents to check the learning progress and to provide advice as well as being used in the final assessment of the students.

This portfolio is a folder (or binder or even a digital collection) containing the student's work as well as the student's evaluation of the strengths and weaknesses of their work. Portfolios reflect not only the work produced (such as papers and assignments), but also provide a record of the activities undertaken over time as part of student learning. It will also serve as a verification tool for each learner that he/she attended the whole learning activity before he/she undergoes the summative assessment for the subject.

### 3.3. Item writing in summative assessment

Before writing a question paper, a plan or specification of what is to be tested or examined must be developed that shows the units or topics to be tested, the number of questions in each level of Bloom's taxonomy and the marks allocation for each question. In a competencebased curriculum, questions from higher levels of Bloom's taxonomy should be given more weight than those from the knowledge and comprehension level.

Before developing a question paper, the item writer must ensure that the test or examination questions are tailored towards competence based assessment by doing the following:

- Identify topic areas to be tested on from the subject syllabus.
- Outline subject matter content to be considered as the basis for the test.
- Identify learning outcomes to be measured by the test.
- Prepare a table of specifications.
- Ensure that the verbs used in the formulation of questions do not require memorisation or recall answers only but test for broad competences as stated in the syllabus.

**Structure and format of the examination:**

There will be one paper in Mathematics at the end of Secondary. The paper will be composed of two sections, where the first section will be composed with short answer items or items with short calculations which include the questions testing for knowledge and understanding, investigation of patterns, quick calculations and applications of Mathematics in real life situations. The second section will be composed with long answer items or answers with demonstrations, constructions, high level reasonning, analysis, interpratation and drawing conclusions, investigation of patterns and generalisation. The items for the second section will emphasise the mastering of mathematical facts, the understanding of mathematical concepts and its applications in real life situations. In this section, the assessment will find out not only what skills and facts have been mastered, but also how well learners understand the process of solving a mathematical problem and whether they can link the application of what they have learned to the context or to the real life situation. The time required for the paper is three hours (3hrs).

The following topic areas have to be assessed: Trigonometry; algebra; analysis; linear algebra; geometry; statistics and probability. Topic areas with more weight will have more emphasis in the second section where learners should have the right to choose to answer 3 items out of 5.

### 3.4. Reporting to parents

The wider range of learning in the new curriculum means that it is necessary to think again about how to share learners' progress with parents. A single mark is not sufficient to convey the different expectations of learning that are outlined in the learning objectives. The most helpful reporting is to share what students are doing well and where they need to improve.

## 4. RESOURCES

### 4.1. Materials needed for implementation

The following list shows the main materials and equipment needed in the learning and teaching process:

- Materials to encourage group work activities and presentations: Computers (desktops and laptops) and projectors; manila papers and markers
- Materials for drawing and measuring geometrical figures/shapes and graphs: Geometric instruments, ICT tools such as geogebra, Microsoft student ENCARTA
- Materials for enhancing research skills: Textbooks and internet (the list of the textbooks to consult is given in the reference at the end of the syllabus and those books can be found in printed or digital copies)
- Materials to encourage the development of mathematical models: scientific calculators, Math type, Matlab, etc.

The technology used in the teaching and learning of Mathematics should be regarded as tools to enhance the teaching and learning process and not to replace teachers.

### 4.2. Human resource

The effective implementation of this curriculum requires a joint collaboration of educators at all levels. Given the material requirements, teachers are expected to accomplish their noble role as stated above. School head teachers and directors of studies are required to make a follow-up and assess the teaching and learning of Mathematics. These combined efforts will ensure bright future careers and lives for learners as well as the contemporary development of the country.

In a special way, a teacher of Mathematics at ordinary level should have a firm understanding of mathematical concepts at the leavel he/she teaches. He/she should be qualified in Mathematics and have a firm ethical conduct. The teacher should possess the qualities of a good facilitator, organiser, problem solver, listener and adviser. He/she is required to have basic skills and competence of guidance and counseling because students may come to him/her for advice.

**Skills required for the teacher of Mathematics**

The teacher of Mathematics should have the following skills, values and qualities:

- Engage learners in variety of learning activities.
- Use multiple teaching and assessment methods.
- Adjust instruction to the level of the learners.
- Use creativity and innovation in the teaching and learning process.
- Be a good communicator and organiser.
- Be a guide/ facilitator and a counsellor.

- Manifest passion and impartial love for children in the teaching and learning process.
- Link the use of Mathematics with other subjects and real life situations.
- Have a good mastery of Mathematics content.
- Have good classroom management skills.

# 5. SYLLABUS UNITS

## 5.1. Presentation of the structure of the syllabus units

The Mathematics subject is taught and learnt in advanced level of secondary education as a core subject in S4, S5 and S6 respectively. At every grade, the syllabus is structured in topic areas, sub-topic areas where applicable and then further broken down into units to promote the uniformity, effectivness and efficiency of teaching and learning Mathematics. The units have the following elements:

1. Each unit is aligned with the number of periods
2. Each unit has a Key Unit Competency theachievement of which is pursued by all teaching and learning activities undertaken by both the teacher and the learners
3. Each Key Unit Competence is broken into three types of Learning Objectives as follows:
    a. *Type I:* Learning Objectives relating to Knowledge and Understanding (*Type I* Learning Objectives are also known as Lower Order Thinking Skills or LOTS)
    b. *Type II a*nd *Type III:* These Learning Objectives relate to the acquisition of skills, attitudes and values (*Type II* and *Type III* Learning Objectives are also known as Higher Order Thinking Skills or HOTS) – These Learning Objectives are actually considered to be the ones targeted by the present reviewed curriculum.
4. Each Unit has content that indicates the scope of coverage of what is to be taught and learnt in line with the stated Learning Objectives

5. Each Unit suggests a non-exhaustive list of Learning Activities that are expected to engage learners in an interactive learning process as much as possible (learner-centred and participatory approach)
6. Finally, each Unit is linked to other subjects, the Assessment Criteria and the Materials (or Resources) that are expected to be used in the teaching and learning process.

The Mathematics syllabus for Advanced level has got 7 Topic Areas: Trigonometry, Algebra, Analysis, Linear algebra, Geometry, Statistics and Probability and these topic areas are found in each of the three grades of the advanced level which are S4, S5 and S6. As for units, they are 16 in S4, 10 in S5 and 9 in S6.

## 5.2. Mathematics program for secondary four

### 5.2.1 Key competences at the end of secondary four

After completion of secondary 4, the Mathematics syllabus will help the learner to:

- Use the trigonometric concepts and formulas in solving problems related to trigonometry
- Think critically and analyse daily life situations efficiently using mathematical logic concepts and infer conclusions
- Model and solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities
- Represent graphically simple numerical functions
- Perform operations on linear transformation and solve problems involving geometric transformations
- Determine algebraic representations of lines, straight lines and circles in 2D
- Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation
- Use matrices and determinants of order 2 to solve systems of linear equations and to define transformations of 2D
- Use counting techniques and concepts of probability to determine the probability of possible outcomes of events occurring under equally likely assumptions.

### 5.2.2. Mathematics units for secondary four

<table>
<tr><td colspan="3">Topic Area: TRIGONOMETRY</td><td colspan="2">Sub-topic Area: Trigonometric circle and identities</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 1: Fundamentals of trigonometry.</td><td>No. of periods: 26</td></tr>
<tr><td colspan="5">Key Unit Competence: Use trigonometric circle and identities to determine trigonometric ratios and apply them to solve related problems.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Contents</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle – know special values (30$^o$, 45$^o$, 60$^o$).<br>- Convert radians to degree and vice versa.<br>- Differentiate between complementary angles, supplementary angles and co-terminal angles</td><td>-Represent graphically sine, cosine and tangent, functions and, together with the unit circle, use to relate values of any angle to the value for a positive acute angle<br>-Use trigonometry, including the sine and cosine rules, to solve problems involving triangles</td><td>- Appreciate the relationship between the trigonometric values for different angles<br>- Verify reasonableness of answers when solving problems</td><td>-Trigonometric concepts:<br>o Angle and its measurements<br>o Unit circle<br>o Trigonometric ratios<br>o Trigonometric identities<br>-Reduction to functions of positive acute angles<br>-Triangles and Applications:<br>o Bearing<br>o Air Navigation<br>o Inclined plane…</td><td>- Mental task – imagine a point on the edge of a wheel – as the wheel turns how high is the point above the centre? – sketch the graph<br>- Practical – on graph paper draw circle of 10cm radius and measure half chord length and distance from the centre to chord for angles (say multiples of 15$^o$) – plot the graphs – use calculator to determine which is sine and cosine. What is the radius of the calculator's circle? – unit circle<br>- Use of dynamic geometry and graph plotting to illustrate relationship e.g. geogebra<br>- In groups use unit circle and graphs to determine the relationship between trigonometric functions of any angle and the value for an acute positive angle – generalise.<br>- Group investigation -What angle subtends an arc length equal to the radius? – define a radian, make a table of equivalences<br>- Derive trigonometric identities, sine and cosine rules<br>- Apply trigonometry to practical problems involving triangles and angles.</td></tr>
<tr><td colspan="5">Links to other subjects: Physics (optics, wave, electricity), geography, architecture, engineering.</td></tr>
</table>

| |
|---|
| ***Assessment criteria:*** *Apply trigonometric concepts to solve problems involving triangles and angles.* |
| ***Materials:*** *Geometric instruments (ruler,T-square,compass), graph paper, digital technology including calculators.* |

| **Topic Area: ALGEBRA** | | | **Sub-topic Area: Mathematical logic and applications** | |
|---|---|---|---|---|
| **S.4 MATHEMATICS** | **Unit 2:** Propositional and predicate logic. | | | **No. of periods :** 14 |
| **Key Unit Competence:** Use mathematical logic to organise scientific knowledge and as a tool of reasoning and argumentation in daily life. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Distinguish between statement and proposition.<br>- Convert into logical formula composite propositions and vice versa.<br>- Draw the truth table of a composite proposition.<br>- Recognize the most often used tautologies<br>- (E g. De Morgan's Laws). | - Use mathematical logic to infer conclusion from given proposition.<br>- Evaluate claims, issues and arguments, and identify mistakes in reasoning and prove the validity or invalidity of arguments in ordinary discourse.<br>- Show that a given logic statement is tautology or a contradiction. | - Judge situations accurately and act with equality.<br>- Observe situations and make appropriate decisions.<br>- Appreciate and act with thoughtfulnes: grasp and demonstrate carefulness.<br>- Develop and show mutual respect.<br>- Demonstrate broad-mindedness. | - Generalities:<br>○ Introduction and fundamental definitions.<br>- Propositional logic:<br>○ Truth tables.<br>○ Logical connectives.<br>○ Tautologies and contradictions.<br>- Predicate Logic:<br>○ Propositional functions.<br>○ Quantifiers.<br>- Applications:<br>○ Set theory.<br>○ Electric circuits. | - Practical: deduce if given statement is a proposition or not.<br>- Group investigation: perform research in advance in the library about propositional logic. |
| **Links to other subjects:** *Physics (Use of De Morgan's Laws in Electricity), biology.* | | | | |

| |
|---|
| ***Assessment criteria:*** *Learner is able to use mathematical logic to organise scientific knowledge and as a tool of reasoning and argumentation in daily life* |
| ***Materials:*** *Manila papers.* |

<table>
<tr><td colspan="3">Topic Area: ALGEBRA</td><td colspan="2">Sub-topic Area: Numbers and operations</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 3: Binary operations.</td><td>No. of periods:14</td></tr>
<tr><td colspan="5">Key Unit Competence: Use mathematical logic to understand and perform operations using the properties of algebraic structures.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Define a group, a ring , an integral domain and a field.<br>- Demonstrate that a set is (or is not) a group, a ring or a field under given operations.<br>- Demonstrate that a subset of a group is (or is not) a sub group.</td><td>- Determine the properties of a given binary operation.<br>- Formulate, using adequate symbols, a property of a binary operation and its negation.<br>- Construct the Cayley table of order 2,3,4.<br>- Discover a mistake in an incorrect operation.</td><td>- Appreciate the importance and the use of properties of binary operations.<br>- Show curiosity, patience, mutual respect and tolerance in the study of binary operations .</td><td>- Definitions and properties of: a group, a ring, a field<br><br>- Algebraic structures: group, ring, field</td><td>- Discuss in groups, patiently in mutual respect and tolerance, the main facts about binary operations and algebraic structures and choose a reporter to present the work<br>- With an example of binary operation define a group, ring, field</td></tr>
<tr><td colspan="5">Links to other subjects: Physics (electricity in calculation of numerical values from a formula), entrepreneurship (economics: calculation of data from sales), computer science.</td></tr>
<tr><td colspan="5">Assessment criteria: Learners should be able to explain why grouping, interchanging, distributing is correct or not depending on the context and carry out binary operations and determine their properties.</td></tr>
<tr><td colspan="5">Materials: Digital technology including calculators.</td></tr>
</table>

| **Topic Area:ALGEBRA** | | **Sub-topic Area: Numbers and operations** | | |
|---|---|---|---|---|
| **S.4 MATHEMATICS** | **Unit 4:** Set $\mathbb{R}$ of real numbers. | | | **No. of periods:** 24 |
| **Key Unit Competence:** Think critically using mathematical logic to understand and perform operations on the set of real numbers and its subsets using the properties of algebraic structures. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Match a number and the set to which it belongs.<br>- Define a power, an exponential, a radical, a logarithm, the absolute value of a real number. | - Classify numbers into naturals, integers, rational and irrationals.<br>- Determine the restrictions on the variables in rational and irrational expressions.<br>- Illustrate each property of a power, an exponential, a radical, a logarithm, the absolute value of a real number.<br>- Use logarithm and exponentials to model simple problems about growth, decay, compound interest, magnitude of an earthquake. Transform a logarithmic expression to equivalent power or radical form and vice versa.<br>- Rewrite an expression containing "absolute value" using order relation. | - Appreciate the importance and the use of properties of operations on real numbers.<br>- Show curiosity for the study of operations on real numbers. | - **Absolute value and its properties.**<br>- **Powers and radicals.**<br>- **Decimal logarithms and properties.** | - Group investigation: perform research in advance in the library about sets of numbers (natural numbers, integers, rational numbers and irrational numbers).<br>- Mental task: what are the main facts about sets of real numbers?<br>- Apply operations on a set of real numbers to illustrate relation to arithmetic. |
| **Links to other subjects:** *Physics: converting temperature from degree Celsius to degree Fahrenheit, converting seconds to minutes and vice versa. Entrepreneurship and economics: organisation and computation of data from sales. Chemistry: the decay process. Biology: growth of bacteria. Geography: magnitude of an earthquake.* | | | | |
| ***Assessment criteria:*** *Learners should be able to use mathematical logic to understand and perform operations on the set of real numbers and its subsets using the properties of algebraic structures.* | | | | |
| ***Materials:*** *Graph papers, manila papers, digital technology including calculators.* | | | | |

<table>
<tr><td colspan="3">Topic Area:<br>ALGEBRA</td><td colspan="2">Sub-topic Area: Equations and inequalities</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 5: Linear equations and inequalities</td><td>No. of periods: 12</td></tr>
<tr><td colspan="5">Key Unit Competence: Model and solve algebraically or graphically daily life problems using linear equations or inequalities.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- List and clarify the steps in modeling a problem by linear equations and inequalities.</td><td>- Solve equations and inequalities in one unknown.<br>- Solve parametric equations and inequalities in one unknown.<br>- Solve simultaneous equations in two unknowns.<br>o Use equations in soling mathematical problems involving supply and demand, linear motions, electric circuitsalancing equations).</td><td>- Appreciate, value and care for situations involving to quadratic equations and quadratic inequalities in daily life situation<br>- Show curiosity about quadratic equations and quadratic inequalities</td><td>- Equations and inequalities in one unknown.<br>- Parametric equations and inequalities in one unknown.<br>- Simultaneous equations in two unknowns.<br>- Applications:<br>o Economics (problems about supply and demand analysis)<br>o Physics (linear motions, electric circuits).<br>o Chemistry (balancing equations).</td><td>- Group investigation: discuss in groups the importance and necessity of linear equations and inequalities and how it takes place in the trade.<br>- Practical: solve linear equations and simultaneous equations on graph paper.</td></tr>
<tr><td colspan="5">Links to other subjects: Physics: kinematics. Chemistry, economics.</td></tr>
<tr><td colspan="5">Assessment criteria: Learners should be able to model and solve algebraically or graphically daily life problems using linear equations or inequalities.</td></tr>
<tr><td colspan="5">Materials: Geometric instruments (ruler-square), digital technology including calculator.</td></tr>
</table>

<table>
<tr><td colspan="3">Topic Area: ALGEBRA</td><td colspan="2">Sub-topic Area: Linear equations and inequalities</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 6: Quadratic equations and inequalities.</td><td>No. of periods: 18</td></tr>
<tr><td colspan="5">Key unit competence: Model and solve algebraically or graphically daily life problems using quadratic equations or inequalities</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr>
<td>- Define a quadratic equation.<br>- Be able to solve problems related to quadratic equations.</td>
<td>- Apply critical thinking by solving any situation related to quadratic equations (economics problems, finance problems).<br>- Determine the sign, sum and product of a quadratic equation.<br>- Discuss the parameter of a quadratic equation and a quadratic inequality.<br>- Explain how to reduce bi-quadratic equations to a quadratic equation and other equations of a degree greater than 2.</td>
<td>- Appreciate, value and care for situations involving to quadratic equations and quadratic inequalities in daily life situations.<br>- Show curiosity about quadratic equations and quadratic inequalities.</td>
<td>- Equations and inequalities in one unknown.<br>- Parametric equations and inequalities in one unknown.<br>- Simultaneous equations in two unknowns.<br>- Applications:<br>○ Physics (projectile motions)<br>○ Masonry (arched shape).</td>
<td>- Group investigation: discuss in a group the importance and necessity of a quadratic equation and a quadratic inequality and how it takes place in finance problems, economics problems and physics -sketch on graph paper the graph of quadratic equations.</td>
</tr>
<tr><td colspan="5">Links to other subjects: Physics: kinematics. Masonry, economics, finance.</td></tr>
<tr><td colspan="5">Assessment criteria: Learners are able to model and solve algebraically or graphically daily life problems using quadratic equations or inequalities.</td></tr>
<tr><td colspan="5">Materials: Geometric instruments (ruler, T-square), digital technology including calculator.</td></tr>
</table>

<table>
<tr><td colspan="4">Topic Area: ANALYSIS</td><td>Sub-topic Area: Functions</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 7: Polynomial, rational and irrational functions.</td><td>No. of periods: 14</td></tr>
<tr><td colspan="5">Key Unit Competence: Use concepts and definitions of functions to determine the domain of rational functions and represent them graphically in simple cases and solve related problems.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Identify a function as a rule and recognise rules that are not functions.<br>- Determine the domain and range of a function.<br>- Construct compositions of functions.<br>- Find whether a function is even, odd, or neither.<br>- Demonstrate an understanding of operations on polynomials, rational and irrational functions, and find the composite of two functions.</td><td>- Perform operations on functions<br>- Apply different properties of functions to models and solve related problems in various practical contexts.<br>- Analyse, model and solve problems involving linear or quadratic functions and interpret the results.</td><td>- Increase self-confidence and determination to appreciate and explain the importance of functions.<br>- Show concern, patience, mutual respect and tolerance when solving problems about polynomial, rational and irrational functions.</td><td>- Factorisation of polynomials.<br>- Generalities on numerical functions:<br>o Definitions.<br>o Domain and range of a function.<br>o Operations<br>o Parity of a function (odd or even).<br>- Application of rational and irrational functions:<br>o Physics (free fall, projectile).<br>o Economics (cost of commodity or marginal cost).<br>o Chemistry (rate of reaction).</td><td>- Practical: study algebraically and graphically polynomial functions.<br>- Practical: discuss in groups patiently in mutual respect and tolerance, different operations on factorisations.<br>- Model or interpret the problems related to polynomial functions.</td></tr>
<tr><td colspan="5">Links to other subjects: : Physics: use a quadratic function to model the fall of a ball. Economics: use of polynomials to represent the cost of producing “x” units of a commodity, or marginal cost. Chemistry: use polynomial to express the rate of reaction in chemistry.</td></tr>
<tr><td colspan="5">Assessment criteria: Learner is able to use concepts and definitions of functions to determine the domain of rational functions and represent them graphically in simple cases and solve related problems.</td></tr>
<tr><td colspan="5">Materials: Pair of compasses, graph paper, ruler, digital technology including calculators.</td></tr>
</table>

| **Topic Area: ANALYSIS** | | **Sub-topic Area: Limits, differentiation and integration** | | |
|---|---|---|---|---|
| **S.4 MATHEMATICS** | **Unit 8:** Limits of polynomial, rational and irrational functions. | | | **No. of periods:** 14 |
| **Key Unit Competence:** Evaluate correctly limits of functions and apply them to solve related problems | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define the concept of limit for real-valued functions of one real variable.<br>- Evaluate the limit of a function and extend this concept to determine the asymptotes of the given function. | - Calculate limits of certain elementary functions.<br>- Develop introductory calculus reasoning.<br>- Solve problems involving continuity.<br>- Apply informal methods to explore the concept of a limit including one sided limits.<br>- Use the concepts of limits to determine the asymptotes to the rational and polynomial functions. | - Show concern for the importance, use and determination of limits of functions.<br>- Appreciate the use of intermediate-value theorem. | - **Concepts of limits:**<br>○ Neighborhood of a real number.<br>○ Limit of a variable.<br>○ Definition and graphical interpretation of limit of a function.<br>○ One-sided limits.<br>○ Squeeze theorem.<br>○ Limits of functions at infinity.<br>○ Operations on limits.<br>- **Indeterminate cases:**<br>$\frac{\infty}{\infty}, \frac{0}{0}, \infty-\infty, 0.\infty$<br>- **Applications:**<br>○ Continuity of a function at a point or on interval.<br>○ Asymptotes. | - Learners discuss in groups to evaluate the limit of a function at a point both algebraically and graphically, extend this understanding to determine the asymptotes.<br>- Learners represent on graph paper limits of some chosen functions and draw the possible asymptotes. |
| **Links to other subjects:** *Physics: calculation of velocity, acceleration using concepts of limits.* | | | | |
| ***Assessment criteria:*** *Learners are able to evaluate correctly limits of functions and apply them to solve related problems.* | | | | |
| ***Materials:*** *Manila papers, graph paper, ruler, markers, digital technology.* | | | | |

| Topic Area: ANALYSIS | | | Sub-topic Area: Limits, differentiation and integration | |
|---|---|---|---|---|
| **S.4 MATHEMATICS** | **Unit 9:** Differentiation of polynomials, rational and irrational functions and their applications. | | | **No. of periods:** 21 |
| **Key Unit Competence:** Use the gradient of a straight line as a measure of rate of change and apply this to tangent and normal of curves in various contexts and use the concepts of differentiation to solve and interpret related rates and optimisation problems in various contexts. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Evaluate derivatives of functions using the definition of derivative.<br>- Define and evaluate from first principles the gradient at a point.<br>- Distinguish between techniques of differentiation to use in an appropriate context. | - Use properties of derivatives to differentiate polynomial, rational and irrational functions.<br>- Use first principles to determine the gradient of the tangent line to a curve at a point.<br>- Apply the concepts of and techniques of differentiation to model, analyse and solve rates or optimisation problems in | - Appreciate the use of gradient as a measure of rate of change (economics).<br>- Appreciate the importance and use of differentiation in kinematics (velocity, acceleration).<br>- Show concern on derivatives to help in the understanding of optimisation problems. | - **Concepts of derivative of a function:**<br>○ Definition.<br>○ Differentiation from first principles.<br>○ High order derivatives.<br>- **Rules of differentiation**<br>- **Applications of differentiation :**<br>○ Geometric interpretation of derivatives:<br>Equation of the tangent to a curve;<br>Equation of normal to a curve.<br>○ Mean value theorem for derivatives: Lagrange's theorem.<br>○ Rolle's theorem<br>○ Hospital's theorem.<br>○ Variations of a function (maximum and minimum values, critical points, inflexion points, concavity, stationary points, increasing and decreasing function). | - Group investigation: determine the gradient of different functions at a point using definition of derivatives, from first principles, chain rule, and interpret the results.<br>- Practical task: represent on graph paper the gradient of a straight line and interpret it geometrically in various practical problems.<br>- In groups, learners use different techniques of differentiation to model, analyse and solve rates or optimisation problems.<br>- In groups, learners determine rate of change from various practical problems and interpret the results. |

| | | | | |
|---|---|---|---|---|
| | different situations. | | ○ Rates of change problems: Gradient as a measure of rate of change; Kinematic meaning of derivatives.<br>○ Optimisation problems. | |

**Links to other subjects:** *Physics, economics.*

***Assessment criteria:*** *Learners are able to use the gradient of a straight line as a measure of rate of change and apply this to tangent or normal of curves in various contexts and use the concepts of differentiation to solve and interpret related rates and optimisation problems in various contexts.*

***Materials:*** *Manila paper, graph paper, digital technology including calculators.*

| **Topic Area: LINEAR ALGEBRA** | | | **Sub-topic Area: Vectors** | |
|---|---|---|---|---|
| **S.4 MATHEMATICS** | **Unit 10:** Vector spaces of real numbers. | | | **No. of periods:** 16 |
| **Key Unit Competence**: Determine the magnitude and angle between two vectors and be able to plot these vectors and point out dot product of two vectors. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define the scalar product of two vectors.<br>- Give examples of scalar product.<br>- Determine the magnitude of a vector and angle between two vectors. | - Calculate the scalar product of two vectors.<br>- Analyse a vector in term of size.<br>- Calculate the angle between two vectors. | - Apply and transfer the skills of dot product, magnitude to other areas of knowledge. | - **Vector spaces** $\mathbb{R}^2$:<br>○ Definitions and operations on vectors.<br>○ Properties of vectors $\mathbb{R}^2$.<br>○ Sub-vector spaces.<br>○ Linear combination of vectors.<br>○ Basis and dimension.<br>- **Euclidian vector space** $\mathbb{R}^2$<br>○ Dot product and properties.<br>○ Modulus or magnitude of vectors<br>○ Angle between two vectors. | - In groups:<br>○ Learners discuss about the scalar product of two vectors.<br>○ Determine the magnitude of vector and measure the angle between two vectors.<br>○ Learners should be given a task to determine the magnitude and angle between two vectors and plot these vectors and point out dot product of two vectors. |
| **Links to other subjects:** *Physics: dynamics. Geography.* | | | | |
| ***Assessment criteria:*** *Learners are able to determine the magnitude of a vector and the angle between two vectors and to be able to plot these vectors and point out dot product of two vectors.* | | | | |
| ***Materials:*** *Manila papers, graph papers, geometric instruments : rulers , T-square , protractors, computers.* | | | | |

<table>
<tr><td colspan="5">Topic Area: LINEAR ALGEBRA Sub-topic Area: Linear transformations in 2D</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 11: Concepts and operations on linear transformations in 2D.</td><td>No. of periods: 14</td></tr>
<tr><td colspan="5">Key Unit Competence: Determine whether a transformation of $IR^2$ is linear or not and perform operations on linear transformations.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr>
<td>- Define and distinguish between linear transformations in 2D.<br>- Define central symmetry, orthogonal projection of a vector, identical transformation<br>- Define a rotation through: an angle about the origin.reflection in the x axis in y axis, in the line $y = x$<br>- Show that a linear transformation is isomorphism in 2D or not.</td>
<td>- Perform operations on linear transformations in 2D.<br>- Construct the composite of two linear transformations in 2D.<br>- Determine whether a linear transformation in 2D is isomorphism or not.<br>- Determine the analytic expression of the inverse of an isomorphism in 2D.</td>
<td>- Appreciate the importance and the use of operations on transformation in 2D.<br>- Show curiosity for the study of operations on transformations in 2D.</td>
<td>- Linear transformation in 2D.<br>○ Definitions and properties.<br>- Geometric transformations.<br>○ Definitions and properties.<br>- Kernel and range.<br>- Operations on transformations.</td>
<td>In groups:<br>- Learners will be given a task to discuss whether each operation in 2D is linear or not.<br>- Learners will be given a task, and be asked to:<br>○ Perform operations on linear transformations, such as:<br>○ Construct the composite of two linear transformations in 2D.<br>○ Solve f(v)= 0 and determine kernel and range of f(v).<br>○ Determine whether a linear transformation in 2D is isomorphism or not. and determine the analytic expression of the inverse of an isomorphism in 2D.</td>
</tr>
<tr><td colspan="5">Links to other subjects: Physics: quantum physics.</td></tr>
<tr><td colspan="5">Assessment criteria: Learners are able to demonstrate that a transformation of $IR^2$ is linear and perform operations on linear transformations.</td></tr>
<tr><td colspan="5">Materials: Manila papers, graph papers, geometric instruments: ruler, pair of compasses, square, digital technology including calculators.</td></tr>
</table>

<table>
<tr><td colspan="4">Topic Area: LINEAR ALGEBRA</td><td colspan="2">Sub-topic Area: Linear transformation in 2D</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 12: Matrices of and determinants of order 2.</td><td colspan="2">No. of periods: 12</td></tr>
<tr><td colspan="6">Key Unit Competence: Use matrices and determinants of order 2 to solve systems of linear equations and to define transformations of 2D.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr>
<td>- Define the order of a matrix.<br>- Define a linear transformation in 2D by a matrix.<br>- Define operations on matrices of order 2.<br>- Show that a square matrix of order 2 is invertible or not.</td>
<td>- Reorganise data into matrices.<br>- Determine the matrix of a linear transformation in 2D.<br>- Perform operations on matrices of order 2.<br>- Construct the matrix of the composite of two linear transformations in 2D.<br>- Construct the matrix of the inverse of an isomorphism of $IR^{2}$.</td>
<td>- Appreciate the importance and the use of matrices in organising data.<br>- Show curiosity for the study of matrices of order 2.</td>
<td>- Matrix of a linear transformation:<br>○ Definition and operations.<br>- Matrices of geometric transformations<br>- Operations on matrices:<br>○ Equality of matrices.<br>○ Addition.<br>○ Multiplication by a scalar.<br>○ Multiplication of matrices.<br>○ Transpose of a matrix.<br>○ Inverse of a square matrix.<br>- Determinant of a matrix of order 2<br>○ Definition.</td>
<td>- In groups:<br>○ Learners should be given a task to reorganise given data into matrices and be asked to perform different operations on matrices by calculating their determinant.<br>○ Learners in groups discuss how a matrix of order 2 is invertible.<br>- Learners should perform research about the importance and use of matrices for example in physics, economics, entrepreneurship or sports, and report the findings.</td>
</tr>
</table>

| | | | | |
|---|---|---|---|---|
| | - Determine the inverse of a matrix of order 2. | | ○ Applications of determinants. | |

**Links to other subjects:** *Physics: matrices of Lorenz transformations. Entrepreneurship and economics: oOrganisation of data from sales.*

***Assessment criteria:*** *Learners are able to use matrices and determinants of order 2 to solve systems of linear equations and to define transformations.*

***Materials:*** *Manila papers, markers.*

| Topic Area: GEOMETRY | | | Sub-topic Area: Plane geometry | |
|---|---|---|---|---|
| **S.4 MATHEMATICS** | **Unit 13:** Points, straight lines and circles in 2D. | | | **No. of periods:** 21 |
| **Key Unit Competence:** Determine algebraic representations of lines, straight lines and circles in 2D. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define the coordinate of a point in 2D.<br>- Define a straight line knowing its:<br>○ 2 points.<br>○ Direction vector.<br>○ Gradient. | - Represent a point and / or a vector in 2D - Calculate the distance between two points in 2D and the mid - point of a segment in 2D.<br>- Determine equations of a straight line (vector equation, parametric equation, Cartesian equation).<br>- Apply knowledge to find the centre, radius, and diameter to find out the equation of a circle.<br>- Perform operations to determine the intersection of a circle and a line. | - Appreciate that a point is a fixed position in a plane.<br>- Show concern patiently and mutual respect in representations and calculations.<br>- Be accurate in representations and calculations.<br>- Manifest a team spirit and think critically in problem solving related to the position of straight lines in 2D. | - **Points in 2D:**<br>○ Cartesian coordinates of a point.<br>○ Distance between two points.<br>○ Mid-points of a line segment.<br>- **Lines in 2D:**<br>○ Equations of line:Vector equation; Parametric equations; Cartesian equation<br>- **Problems on points and straight lines in 2D:**<br>○ Positions.<br>○ Angles.<br>○ Distance.<br>- **Definition of a circle.**<br>- **Cartesian equation of a circle.**<br>- **Problems involving position of a circle and a point or position of circle and lines in 2D.** | - In groups learners discusd the distance between two vectors.<br>- Learners represent on graphs some points, straight lines and circles in 2D.<br>- Learners will be given a task in groups or individually, to determine the magnitude of vectors and measure the angle between two vectors.<br>- Learners represent on graph paper some chosen points, lines and/or circles and determine their parametric or Cartesian equations. |

| |
|---|
| **Links to other subjects:** *Physics, chemistry, geography, topography.* |
| ***Assessment criteria:*** *Learners are able to determine algebraic representations of lines, straight lines and circles in the 2D.* |
| ***Materials:*** *Manila paper, graph paper, geometric instruments ruler, T-square), digital technology including calculators.* |

<table>
<tr><td colspan="3">Topic Area: STATISTICS AND PROBABILITY</td><td colspan="2">Sub-topic Area: Descriptive statistics</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 14: Measures of dispersion.</td><td>No. of periods: 7</td></tr>
<tr><td colspan="5">Key Unit Competence: Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Define the variance, standard deviation and the coefficient of variation.<br>- Analyse and interpret critically data and infer conclusions.</td><td>- Determine the measures of dispersion of a given statistical series.<br>- Apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.<br>- Express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.</td><td>- Appreciate the importance of measures of dispersion in the interpretation of data.<br>- Show concern on how to use the standard deviation as measure of variability of data.</td><td>- Variance.<br>- Standard deviation (including combined set of data).<br>- Coefficient of variation.<br>- Application:<br>o Problems to include measure of dispersion and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.<br>o Problems to include measure of dispersion and express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.</td><td>- In groups, learners will be given a task and be asked to:<br>o Discuss about the measures of dispersion, interpret them and represent their findings.<br>o Represent data on graph paper, interpret them and infer conclusions.<br>o Perform research on given problems arising from various situations in daily life, investigate them to include the standard deviation, and represent their findings.</td></tr>
<tr><td colspan="5">Links to other subjects: Physics, biology, chemistry, geography, finance, economics.</td></tr>
<tr><td colspan="5">Assessment criteria: Learners are able to extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.</td></tr>
<tr><td colspan="5">Materials: Manila paper, graph paper, ruler, digital technology including calculators.</td></tr>
</table>

| Topic Area: STATISTICS AND PROBABILITY | | | Sub-topic Area: Combinatorial analysis and probability | |
|---|---|---|---|---|
| **S.4 MATHEMATICS** | **Unit 15:** Combinatorics. | | | **No. of periods:** 18 |
| **Key Unit Competence:** Use combinations and permutations to determine the number of ways a random experiment occurs. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define the combinatorial analysis.<br>- Recognise whether repetition is allowed or not, and if order matters or not in performing a given experiment.<br>- Construct Pascal's triangle.<br>- Distinguish between permutations and combinations. | - Determine the number of permutations and combinations of "n" items, "r" taken at a time.<br>- Use counting techniques to solve related problems.<br>- Use properties of combinations. | - Appreciate the importance of counting techniques.<br>- Show concern on how to use the counting techniques. | - **Counting techniques:**<br>○ Venn diagrams.<br>○ Tree diagrams.<br>○ Contingency table.<br>○ Multiplication principles.<br>- **Arrangement and permutations:**<br>○ Arrangements with or without repetition.<br>○ Permutations with or without repetition.<br>- **Combinations:**<br>○ Definitions and properties.<br>○ Pascal's triangles.<br>○ Binomial expansion. | Learners will be given:<br>- A mental task, and be asked to imagine that you are a photographer sitting a group in a row for pictures. You need to determine how many different ways you can seat the group. Learners find out.<br>- Questions in groups, and be asked about counting techniques to use for example " In how many different ways could a committee of 5 people be chosen from a class of 30 students? ".<br>- A task of using the letters from their proper words and be asked to create their own words,e.g: use letters of "MISSISSIPPI", without prior instructions, to create news words then give feedback. |
| **Links to other subjects:** *English, physics, biology, chemistry, geography, finance, economics, medical sciences.* | | | | |
| ***Assessment criteria:*** *Learners are able to calculate accurately combinations or permutations of "n" items , "r" taken at a time.* | | | | |
| ***Materials:*** *Manila papers, graph paper, ruler, digital components including calculators.* | | | | |

<table>
<tr><td colspan="3">Topic Area: STATISTICS AND PROBABILITY</td><td colspan="2">Sub-topic Area: Combinatorial analysis and probability</td></tr>
<tr><td>S.4 MATHEMATICS</td><td colspan="3">Unit 16: Elementary probability.</td><td>No. of periods: 7</td></tr>
<tr><td colspan="5">Key Unit Competence: Use counting techniques and concepts of probability to determine the probability of possible outcomes of events occurring under equally likely assumptions.</td></tr>
<tr><th colspan="3">Learning Objectives</th><th rowspan="2">Content</th><th rowspan="2">Learning Activities</th></tr>
<tr><th>Knowledge and understanding</th><th>Skills</th><th>Attitudes and values</th></tr>
<tr><td>- Define probability and explain probability as a measure of chance.<br>- Distinguish between mutually exclusive and non-exclusive events and compute their probabilities.</td><td>- Use and apply properties of probability to calculate the number of possible outcomes of occurring events under equally likely assumptions.<br>- Determine and explain expectations from an experiment with possible outcomes.</td><td>- Appreciate the use of probability as a measure of chance.<br>- Show concern for patience, mutual respect, tolerance and curiosity in the determination of the number of possible outcomes of a random experiment.</td><td>- Concepts of probability:<br>○ Random experiment.<br>○ Sample space.<br>○ Event.<br>○ Definition of probability of an event under equally likely assumptions.<br>- Properties and formulas.</td><td>- Learners discuss gambling problems and report their results to the group.<br>- Learners are given a task of sitting 3 men and 4 women at random in a row. In groups, they discuss the probability that all the men are sitted together then they give feedback.</td></tr>
<tr><td colspan="5">Links to other subjects: Biology: genetics. Economics, physics: quantum physics. Chemistry.</td></tr>
<tr><td colspan="5">Assessment criteria: Learners are able to use counting techniques and concepts of probability to determine the probability of possible outcomes of occurring events under equally likely assumptions.</td></tr>
<tr><td colspan="5">Materials: Manila paper, graph paper, ruler, digital technology including calculators.</td></tr>
</table>

## 5.3. Mathematics program for secondary five

### 5.3.1 Key competencies at the end of secondary five

After completion of secondary 5, the mathematics syllabus will help the learner to:

- Extend the use of the trigonometric concepts and transformation formulas to solve problems involving trigonometric equations, inequalities and/or trigonometric identities.
- Use arithmetic, geometric and harmonic sequences, including convergence, to understand and solve problems arising in various contexts.
- Solve equations involving logarithms or exponentials and apply them to model and solve related problems.
- Study and represent graphically a numerical function.
- Apply theorems of limits and formulas to solve problems involving differentiation including optimisation.
- Study linear dependence of vectors of $IR^3$ and perform operations on linear transformations of $IR^3$ using vectors.
- Extend the use of matrices and determinants to order 3 to solve problems in various contexts.
- Use algebraic representations of lines, spheres and planes in 3D space and solve related problems.
- Extend the understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines.
- Solve problems using Bayes theorem and data to make decisions about likelihood and risk.

### 5.3.2 Mathematics units for secondary five

<table>
<tr><td colspan="3">Topic Area: TRIGONOMETRY</td><td colspan="2">Sub-topic Area: Trigonometric circle and identities</td></tr>
<tr><td>S.5 MATHEMATICS</td><td colspan="3">Unit 1: Trigonometric functions and equations.</td><td>No. of periods: 35</td></tr>
<tr><td colspan="5">Key Unit Competence: Solve trigonometric equations, inequalities and related problems using trigonometric functions and equations.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Contents</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr>
<td>- Show how to use transformation formulas to simplify the trigonometric expressions.<br>- Extend the concepts of trigonometric ratios and their properties to trigonometric equations.<br>- Analyse and discuss the solution of trigonometric inequalities.</td>
<td>- Apply the transformation formulas to simple trigonometric expressions.<br>- Use trigonometric functions and equations to model and solve problems involving trigonometry concepts.<br>- Apply trigonometric functions and equations to perform so far operations in complex numbers, calculation in integration and solving problems including harmonic motion in physics.</td>
<td>- Appreciate the use and importance of trigonometric functions and equations to understand problems arising in complex numbers, in integration in harmonic motion.<br>- Appreciate the relationship between trigonometry and other subjects.<br>- Show concern for patience, mutual respect, tolerance and curiosity in the solving and discussion about problems involving</td>
<td>- Transformation formulas:<br>o Addition and subtraction formulas.<br>o Double-angle and half-angle formulas.<br>o Sum, difference and product formulas.<br>- Trigonometric equations.<br>- Trigonometric inequalities.<br>- Applications:<br>o Euler’s form in complex numbers.<br>o Integration.<br>o Harmonic motion in physics.</td>
<td>- In groups, learners discuss how to simplify trigonometric expressions using transformation formulas – solve problems involving trigonometric equations and inequalities.</td>
</tr>
</table>

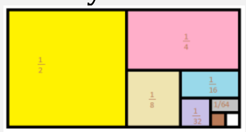
| | | | | |
|---|---|---|---|---|
| | - Solve any other problems related to trigonometry.<br>- Analyse and explain the solutions. | trigonometric functions and equations. | | |

**Links to other subjects:** *Physics: harmonic motion. Complex numbers: Euler's formula. Analysis: integration of trigonometric functions.*

***Assessment criteria:*** *Learners are able to apply trigonometry functions, transformation formulas and equations to solve problems related to trigonometry.*

***Materials:*** *Geometric instruments (ruler,T-square, compass), graph paper, calculators.*

| Topic Area: ALGEBRA | | | | Sub-topic Area: Number patterns |
|---|---|---|---|---|
| **S.5 MATHEMATICS** | **Unit 2:** Sequences. | | | **No. of periods:** 25 |
| **Key Unit Competence:** Understand, manipulate and use arithmetic, geometric and harmonic sequences, including convergence. | | | | |
| **Learning Objectives** | | | **Contents** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define a sequence and determine if a given sequence increases or decreases, converges or not.<br>- Define and understand arithmetic progressions and their properties.<br>- Determine the value of "n", given the sum of the first "n" terms of arithmetic progressions.<br>- Show how to apply formulas to determine the "n$^{th}$" term and the sum of the first "n" terms of arithmetic progressions.<br>- Extend the concepts of arithmetic progression to harmonic sequences.<br>- Define and explain geometric progressions | - Use basic concepts and formulas of sequences to find the value "n", given the sum of the first "n" terms of arithmetic progressions - the "n$^{th}$" term and the sum of the first "n"terms of arithmetic progressions.<br>- Use basic concepts and formulas of sequences to find the value "n", given the sum of the first "n" terms of arithmetic progressions - the "n$^{th}$" term and the sum of the first "n"terms of geometric progressions.<br>- Apply the concepts of sequences to solve problems involving arithmetic, harmonic or geometric sequences. | - Appreciate the relationship between the sequences and other subjects to understand occurring situations (in economics: value of annuity, future value of money; in physics: harmonic motion) values for different angles.<br>- Show concern for patience, mutual respect, tolerance and curiosity to discuss about sequences and their applications. | - **Generalities on sequences.**<br>- **Arithmetic and harmonic sequences.**<br>- **Geometric sequences.**<br>- **Applications:**<br>○ Problems including population growth.<br>○ Problems including compound and simple interests.<br>○ Half-life and decay problems in radioactivity.<br>○ Bacteria growth problems in biology. | - Group led approach: Learners should be given a task of folding a piece of paper to make them understand the meaning of geometric sequences, and think what should be the last term to the infinity:<br>$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \frac{1}{2^n}$<br>- Group investigation: If the bank rates increase or decrease unexpectedly, learners discuss or investigate how in the next n- years:<br>○ they come out ahead, the deal stays fair. |



| | | | | |
|---|---|---|---|---|
| and their properties.<br>- Determine the value of "n", given the sum of the first "n" terms of geometric progressions<br>- Show how to apply formulas to determine specific terms,the "$n^{th}$" term and the sum of the first "n"terms of geometric progressions. | | | | |

| |
|---|
| **Links to other subjects:** *Demography: populaton growth problems. Economics: compound and simple interests. Chemistry: half-life and decay problems in radioactivity. Biology: bacteria growth problems.* |
| ***Assessment criteria:*** *Learners are able to applyconcepts of sequences to solve problems involving arithmetic, harmonic or geometric sequences.* |
| ***Materials:*** *Geometric instruments (ruler, T-square, compass), graph papers, digital technology including calculators, manila paper.* |

<table>
<tr><td colspan="4">Topic Area: ALGEBRA</td><td colspan="2">Sub-topic Area: Equations and inequalities</td></tr>
<tr><td>S.5 MATHEMATICS</td><td colspan="4">Unit 3: Logarithmic and exponential equations.</td><td>No. of periods: 14</td></tr>
<tr><td colspan="6">Key Unit Competence: Solve equations involving logarithms or exponentials and apply them to model and solve related problems.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Contents</td><td colspan="2" rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Define logarithm or exponential equations using properties of logarithms in any base.<br>- State and demonstrate properties of logarithms and exponentials.<br>- Carry out operations using the change of base of logarithms.</td><td>- Use the properties of logarithms to solve logarithmic and exponential equations.<br>- Convert the logarithm to exponential form.<br>- Apply logarithms or exponential to solve rates problems, mortgage problems, population growth problems.</td><td>- Appreciate the use of logarithmic equations to model and solve problem involving logarithms such radioactive-decay problems, carbon dating problems, problems about alcohol and risk of car accident, etc.<br>- Show concern on patience, mutual respect and tolerance in solving problems involving logarithmic or exponential equations.</td><td>- Logarithmic equations, including natural logarithms.<br>- Exponential equations.<br>- Application:<br>○ Interest rate problems.<br>○ Mortgage problems.<br>○ Population growth problems.<br>○ Radioactive decay problems.<br>○ Earthquake problems.<br>○ Carbon dating problems.<br>○ Problems about alcohol and risk of car accident.</td><td colspan="2">In group or individually, learners:<br>- Once they have the shape of a logarithmic graph, learners can shift it vertically or horizontally, stretch it, shrink it, reflect it, check answers with it, and the most important is to interpret the graph.<br>- Given for example a growth or decay situation, learners after investigating the situation, write an exponential function and evaluate it for a given input.</td></tr>
<tr><td colspan="6">Links to other subjects: Demography: population growth problems. Economics: interest rates problems, annuity value of money etc.</td></tr>
<tr><td colspan="6">Assessment criteria: Learners are able to apply concepts of logarithmic and exponential equations to solve problems involving logarithms or exponentials.</td></tr>
<tr><td colspan="6">Materials: Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators, manila paper.</td></tr>
</table>

<table>
<tr><td colspan="2">Topic Area: ALGEBRA</td><td colspan="3">Sub-topic Area: Equations and inequalities</td></tr>
<tr><td>S.5 MATHEMATICS</td><td colspan="3">Unit 4: Solving equations by numerical method.</td><td>No. of periods: 21</td></tr>
<tr><td colspan="5">Key Unit Competence: To be able to use numerical methods e.g Newton-Raphson method to approximate solution to equations.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Contents</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Illustrate numerical techniques for approximating solutions to equations and be aware of their limitations.</td><td>- Use numerical methods to approximate solutions of equations.<br>- Select a numerical method appropriate to a given problem.<br>- Derive error estimates for approximate solutions to equations.</td><td>- Appreciate that equations can only be solved approximately using numerical methods.</td><td>- Numerical methods:<br>○ Linear interpolation and extrapolation.<br>○ Location of roots: by graphical and analytical methods.<br>○ Iterative methods: Newton Raphson Method (general formula and tolerance limit).<br>○ Bisection methods.<br>○ Fixed point iteration.</td><td>- Work in groups to identify approximate solutions to equations that are not easily solved analytically e.g. $x^x = 1000$, start by plotting a graph and getting closer to a solution via interval bisection.<br>- In groups research the Newton Raphson method and apply to the solution of various equations – identifying when the method fails.</td></tr>
<tr><td colspan="5">Links to other subjects: Physics: kinematics, civil engineering, economics, finance.</td></tr>
<tr><td colspan="5">Assessment criteria: Learners are able to use numerical methods e.g Newton-Raphson method to approximate solution to equations.</td></tr>
<tr><td colspan="5">Materials: Graph paper, manila papers, markers, geometric instruments and some software to graph and compute numerical values.</td></tr>
</table>

| Topic Area: ANALYSIS | | Sub-topic Area: Limits, differentiation and integration | | |
|---|---|---|---|---|
| **S.5 MATHEMATICS** | **Unit 5:** Trigonometric and inverse trigonometric functions. | | | **No. of periods:** 37 |
| **Key Unit Competence:** Apply theorems of limits and formulas of derivatives to solve problems of refraction of light in a prism, simple harmonic motion problems, and optimisation including trigonometric or inverse trigonometric functions. | | | | |
| **Learning Objectives** | | | **Contents** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Extend the concepts of function, domain, range, period, inverse function, limits to trigonometric functions.<br>- Extend the concepts of limits and/or differentiation to modeland solve problems involving trigonometric or inverse trigonometric functions. | - Apply concepts and definition of limits, to calculate the limits of trigonometric functions and remove their indeterminate forms – calculate also their high derivatives.<br>- Derive techniques of differentiation to model and solve problems related to trigonometry.<br>- Apply technique of differentiation to solve problems involving trigonometric or inverse trigonometric functions such as refraction of light in prism, simple harmonic motion problems, | - Appreciate that questions of refraction of light in prism, simple harmonic motion problems, optimisation, involving trigonometri c functions can be solved using concepts of limits and/or techniques of derivatives. | - **Trigonometric functions:**<br>○ Generalities:<br>- Definitions<br>- Domain and range of a function<br>- Parity of a function (odd or even)<br>- Periodic functions<br>○ Limits, including indeterminate cases $\frac{0}{0}$ , $0.\infty$<br>○ Differentiation of trigonometric functions – extend this to high derivatives.<br>- **Inverse trigonometric functions.** | - Learners in groups plot the graphs of trigonometric functions e.g $y = \sin x$ or $y = a\sin bx$ and investigate it, learners discuss its period, then find its domain of definition and range.<br>○ Generalise this activity to other trigonometric functions.<br>○ Calculate high derivatives of these trigonometric functions.<br>- Derive techniques of differentiation to differentiate trigonometric or inverse trigonometric functions and apply them to solve related practical problems such as:<br>○ Refraction of light in prism.<br>○ Simple harmonic motion problems etc. |

| | | | | |
|---|---|---|---|---|
| | optimisation. | | - **Applications:**<br>○ Refraction of light in prism.<br>○ Simple harmonic motion problems. | |

**Links to other subjects:** *Physics: refraction of light in prism, simple harmonic motion.*

***Assessment criteria:*** *Learners are able to apply theorems of limits and formulas of derivatives to solve problems of refraction of light in prism, simple harmonic motion problems and optimisation, including trigonometric or inverse trigonometric functions.*

***Materials:*** *Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators.*

| **Topic Area: LINEAR ALGEBRA** | | | | **Sub-topic Area: Vectors in 3D** |
|---|---|---|---|---|
| **S.5 MATHEMATICS** | **Unit 6:** Vector space of real numbers | | | **No. of periods:** 21 |
| **Key Unit Competence:** Study linear dependence of vectors of $\mathbb{R}^3$, solve problems related to angles using the scalar product in $\mathbb{R}^3$ and use the vector product to solve mensuration problems in $\mathbb{R}^3$. | | | | |
| **Learning Objectives** | | | **Contents** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define a basis and the dimension of a vector space and give examples of bases of $\mathbb{R}^3$.<br>- Define addition of vectors of $\mathbb{R}^3$, and multiplication of a vector of $R^3$ by a scalar.<br>- Define the dot product and the cross product of two vectors in a three-dimensional vector space and list their properties. | - Perform operations on vectors in 3 dimensions.<br>- Express a vector of $\mathbb{R}^3$ as linear combination of given vectors.<br>- Determine a basis and the dimension of a subspace of $\mathbb{R}^3$.<br>- Calculate the resultant of two vector quantities in three-dimensional space.<br>- Determine the dot product and the | - Appreciate the usefulness of vectors of $\mathbb{R}^3$ in the description of quantities such as force, velocity - Express a vector of $\mathbb{R}^3$ as linear combination of vectors.<br>- Appreciate the importance of the dot product as a measure of parallelism and the cross | - **Vector spaces $\mathbb{R}^3$:**<br>○ Definitions and operations on vectors.<br>○ Properties of vectors $\mathbb{R}^3$.<br>○ Sub-vector spaces.<br>○ Linear combination of vectors.<br>○ Basis and dimension.<br>- **Euclidian Vector space $\mathbb{R}^3$:**<br>○ Dot product and properties.<br>○ Modulus or magnitude of vectors.<br>○ Angle between two vectors. | Learners perform specific tasks in group, patiently, in mutual respect and tolerance such as:<br>- Draw a three-dimensional coordinate system and plot some chosen points and represent the corresponding vectors.<br>- Choose some learners to simulate points and vectors in three-dimensional space and ask the audience to describe vectors and related operations.<br>- Study vectors in a three-dimensional coordinate system to describe quantities such as force, velocity and acceleration. |

| | | | | |
|---|---|---|---|---|
| - Define the magnitude of a three-dimensional vector and list its properties<br>- Distinguish between the dot product and the cross product. | vector product of two vectors in a three-dimensional space and use them to solve practical related problems.<br>- Explain geometrically the dot product and the cross product. | product as a measure of perpendicularit y. | ○ Vector product and properties. | |

**Links to other subjects:** *Physics: force, velocity,acceleration.*

***Assessment criteria:*** *Learners are able to apply linear dependence of vectors of $\mathbb{R}^3$ to solve problems related to angles using the scalar product in $\mathbb{R}^3$ and use the vector product to solve also mensuration problems in $\mathbb{R}^3$.*

***Materials:*** *Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators.*

| **Topic Area: LINEAR ALGEBRA** | | | **Sub-topic Area: Linear transformation in 3D** | |
|---|---|---|---|---|
| **S.5 MATHEMATICS** | **Unit 7:** Matrices and determinants of order 3. | | | **No. of periods:** 35 |
| **Key Unit Competence:** Apply matrix and determinant of order 3 to solve related problems. Demonstrate that a transformation of $IR^3$ is linear and perform operations on linear transformations of $IR^3$ using vectors. | | | | |
| **Learning Objectives** | | | **Contents** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define operations on matrices of order 3.<br>- Illustrate the properties of determinants of matrices of order 3.<br>- Show that a square matrix of order 3 is invertible or not.<br>- Define a linear transformation in 3D by a matrix.<br>- Define and perform operations on linear transformations of $\mathbb{R}^3$.<br>- Express analytically the inverse of an isomorphism of $\mathbb{R}^3$.<br>- Discuss with respect to a parameter the solutions of a system | - Perform operations on matrices of order 3.<br>- Calculate the determinants of matrices of order 3.<br>- Explain using determinant whether a matrix of order 3 is invertible or not.<br>- Determine the inverse of a matrix of order 3.<br>- Reorganise data into matrices.<br>- Determine the matrix of a linear transformation in 3D.<br>- Apply matrices to solve related problems (e.g in physics).<br>- Apply the concepts of linear transformation to determine and perform various operations on linear transformations of $\mathbb{R}^3$.<br>- Use the properties of linear transformation of $\mathbb{R}^3$ to construct the analytic expression of the inverse of an isomorphism | - Appreciate the importance of matrices of order 3 and their determinants in organising data and solving related problems.<br>- Appreciate the importance of operations on linear transformations of $\mathbb{R}^3$ and their properties. | - **Matrix of a linear transformation:**<br>○ Definition and operations.<br>- **Operations on matrices:**<br>○ Equality of matrices.<br>○ Addition.<br>○ Multiplication by a scalar.<br>○ Multiplication of matrices.<br>○ Transpose of a matrix.<br>○ Inverse of a square matrix.<br>- **Determinant of a matrix of order 3:**<br>○ Definition.<br>- **Applications of determinants.** | - Learners discuss in groups patiently, in mutual respect and tolerance, how to organise data into matrices of order 3 and apply these concepts to solve related problems.<br>- Learners discuss in groups and perform operations on linear transformations of $\mathbb{R}^3$.<br>- Learners discuss in groups, with respect to a parameter, the solutions of a system of three linear equations in three unknowns. |

| | | | | |
|---|---|---|---|---|
| of three linear equations in three unknowns. | of $\mathbb{R}^3$<br>- Use Cramer's rule to solve a system of three linear equations in three unknowns.<br>- Apply properties of determinants to solve problems related to matrices of order 3. | | | |
| **Links to other subjects:** *Physics: expressing force, velocity, acceleration. Engineering.* | | | | |
| ***Assessment criteria:*** *Learners are able to apply matrix and determinant of order 3 to solve related problems. Demonstrate that a transformation of* $IR^3$ *is linear and perform operations on linear transformations of* $IR^3$ *using vectors.* | | | | |
| ***Materials:*** *Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators.* | | | | |

| Topic Area: GEOMETRY | | | Sub-topic Area: Space geometry | |
|---|---|---|---|---|
| **S.5 MATHEMATICS** | **Unit 8:** Points, straight lines and sphere in 3D. | | | **No. of periods:** 34 |
| **Key Unit Competence:** Use algebraic representations of points, lines, spheres and planes in 3D space and solve related problems. | | | | |
| **Learning Objectives** | | | **Contents** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define by its coordinates the position of a point in 3D.<br>- Define a line using points and direction vector.<br>- Define the position vectors of plane.<br>- Define the positions of a line and a sphere in 3D. | - Represent a point and/or a vector in 3D - Calculate the distance between two points in 3D and the mid - point of a segment in 3D.<br>- Determine equations of a straight line (vector equation, parametric equation, Cartesian equation).<br>- Explain the position vector of a plane - Determine vector equation, parametric equations of a plane and/or the Cartesian equation of a plane - Invent the Cartesian equation of a plane using parametric equations. | - Appreciate the importance of the measure of angle between two line segments.<br>- Be accurate in calculation to determine Cartesian and/or parametric equations of a line – be aware of the usefulness of the equations of a plane<br>- Think critically in problem solving related to the equations of lines and planes.<br>- Appreciate the importance of the | - Points in 3D:<br>○ Cartesian coordinates of a point, distance between two points, mid-points of a line segment.<br>- Lines in 3D:<br>○ Equations of line:<br>○ Vector, parametric equations, Cartesian equation.<br>- Planes in 3D:<br>○ Determination of a plane in 3D.<br>○ Equations of line:<br>○ Vector, parametric, Cartesian equation.<br>- Problems on points and straight lines in 3D:<br>○ Positions, angles, | - Learners represent on graph papers some points in 3D<br>- Learners investigate lines, distance and line segments. and measure angles, distance between them and present their findings.Learners represent on graph papers some chosen planes and determine its parametric or Cartesian equations and present their findings.In groups, learners discuss about the position of sphere:<br>○ Point – Sphere.<br>○ Line – Sphere.<br>○ Plane – Sphere. |

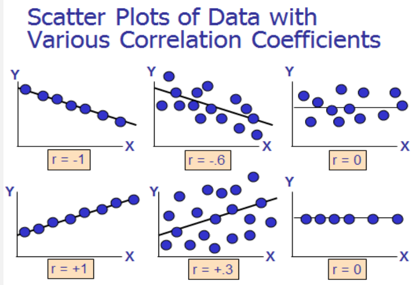
| | | | | |
|---|---|---|---|---|
| | - Apply knowledge to find the centre, radius, and diameter to find out the equation of a sphere.<br>- Perform operations to determine the intersection of a sphere and a plane; intersection of a sphere and a line. | distance in space to determine the equation of a sphere. | distance.<br>- Sphere:<br>○ Definition.<br>○ Cartesian equation of a sphere.<br>○ Positions of sphere in 3D:<br>- Point – Sphere.<br>- Line – Sphere.<br>- Plane – Sphere. | |

**Links to other subjects:** *Geometry: distance and positions. Physics: distances. Chemistr: molecular structures. Engineering: architecture.*

***Assessment criteria:*** *Learners are able to solve problems of positions, distances, angles about points, lines and planes, and sphere in 3D.*

***Materials:*** *Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators.*

<table>
<tr><td colspan="4">Topic Area: STATISTICS AND PROBABILITY</td><td colspan="2">Sub-topic Area: Descriptive statistics</td></tr>
<tr><td>S.5 MATHEMATICS</td><td colspan="3">Unit 9: Bivariate statistics.</td><td colspan="2">No. of periods: 10</td></tr>
<tr><td colspan="6">Key Unit Competence: Extend understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Contents</td><td rowspan="2" colspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Define the covariance, coefficient of correlation and regression lines.<br>- Analyse, interpret data critically then infer conclusion.</td><td>- Determine the coefficient of correlation, covariance and regression lines of bivariate data of dispersion of a given statistical series.<br>- Apply and explain the coefficient of correlation and standard deviation as the more convenient measure of the variability in the</td><td>- Appreciate the importance of regression lines and coefficient of correlation to analyse, interpret data to infer conclusion - Predict event e.g after analysing the population growth of a given country, we can make a decision about the future</td><td>- Covariance.<br>- Correlation coefficient.<br>- Regression lines.<br>- Applications:<br>○ Data analysis, interpretation and prediction problems in various areas (biology, business, engineering, geography, demography).</td><td colspan="2">- Learners discuss in groups, the correlation between class results and rank in school for example. They investigate them, they analyse the relationship between them, and check how the coefficient of correlation reflects the amount of variability that is shared between them and what they have in common. They finally infer conclusion.<br>- Learners plot visually data on scatter diagram or scatter plot to represent a correlation between two variable. Analyse the graph, infer conclusion using coefficient of correlation to make predictions about the variables studied.<br>- E.g</td></tr>
</table>

| | | | | |
|---|---|---|---|---|
| | interpretation of data. | generation. | |  |

| |
|---|
| **Links to other subjects:** *Geography: spatial statistics research, air pollution in different year. Biology: bio-statistic. Chemistry, demography: population growth.* |
| ***Assessment criteria:*** *Learners are able to extend understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines.* |
| ***Materials:*** *Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators.* |

<table>
<tr><td colspan="4">Topic Area: STATISTICS AND PROBABILITY</td><td colspan="2">Sub-topic Area: Probability</td></tr>
<tr><td>S.5 MATHEMATICS</td><td colspan="3">Unit 10: Conditional probability and Bayes theorem.</td><td colspan="2">No. of periods: 20</td></tr>
<tr><td colspan="6">Key Unit Competence: Solve problems using Bayes theorem and use data to make decisions about likelihood and risk.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Contents</td><td rowspan="2" colspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr><td>- Extend the concept of probability to explain it as a measure of chance.<br>- Compute the probability of an event B occurring when event A has already taken place.<br>- Interpret data to make decision about likelihood and risk.</td><td>- Apply theorem of probability to calculate the number possible outcomes of occurring independent events under equally likely assumptions.<br>- Determine and explain results from an experiment with possible outcomes<br>- Apply Bayes theorem to calculate the number of possible outcomes of occurring independent events under equally likely assumptions.</td><td>- Appreciate the use of probability theorem as measure of chance.<br>- Show concern on patience, mutual respect, tolerance and curiosity about the possible outcomes of event B occurring when event A has already taken place.<br>- Appreciate the use of Bayes theorem to determine the probability of event B occurring when event A has already taken place.</td><td>- Conditional probability:<br>○ Probability of event B occurring when event A has already taken place.<br>○ Basic formulae and properties of conditional probability.<br>○ Independent events.<br>○ Probability tree diagram.<br>- Bayes theorem and its applications.</td><td colspan="2">- Learners discuss in groups patiently in mutual respect and tolerance the number of possible outcomes of event B occurring when even A has already taken place.<br>- In a given task, learners use Bayes theorem to determine the probability of event B occurring when event A has already taken place.</td></tr>
<tr><td colspan="6">Links to other subjects: Geography, biology, chemistry, demography.</td></tr>
<tr><td colspan="6">Assessment criteria: Learners are able to solve problems using Bayes theorem and use data to make decisions about likelihood and risk.</td></tr>
<tr><td colspan="6">Materials: Manila paper, markers, digital technology including calculators.</td></tr>
</table>

## 5.4. Mathematics program for secondary six

### 5.4.1 Key competences at the end of secondary six

After completion of secondary 6, the mathematics syllabus will help the learner to:

- Extend understanding of sets of numbers to complex numbers.
- Solve polynomial equations in the set of complex numbers and solve related problems in physics.
- Extend the use of concepts and definitions of functions to determine the domain of logarithmic and exponential functions.
- Use integration as the inverse of differentiation and as the limit of a sum and apply them to finding area and volumes to solve various practical problems.
- Use differential equations to solve related problems that arise in a variety of practical contexts.
- Relate the sum and the intersection of subspaces of a vector space by the dimension formula.
- Determine the kernel and the image of a linear transformation and use the results to infer the properties of a linear transformation.
- Transform a matrix to its equivalent form using elementary row operations.
- Determine algebraic representations of conics in the plane and use them to represent and interpret physical phenomena.
- Use probability density functions of a random variable to model events, including binomial, Poisson and Normal distributions.

### 5.4.2 Mathematics units for secondary six

| **Topic Area: ALGEBRA** | | **Sub-topic Area: Numbers and operations** | | |
|---|---|---|---|---|
| **S.6 MATHEMATICS** | **Unit 1:** Complex numbers. | | **No. of periods:** 36 | |
| **Key Unit competence:** Perform operations on complex numbers in different forms and use complex numbers to solve related problems in physics (voltage and current in alternating current), computer science (fractals), trigonometry (Euler's formula to transform trigonometric expressions). | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Identify the real part and the imaginary part of a complex number<br>- Convert a complex number from one form to another<br>- Represent a complex number on Argand diagram<br>- State De Moivre's formula and Euler's formula | - Apply the properties of complex numbers to perform operations on complex numbers in algebraic form, in polar form or in exponential form<br>- Find the modulus and the square roots of a complex number<br>- Solve in the set of complex numbers a linear or quadratic equation<br>- Use the properties of complex numbers to factorize a polynomial and to solve a polynomial equation in the set of complex numbers<br>- Apply De Moivre's formula and Euler's formula to transform trigonometric | - Appreciate the importance of complex numbers in solving problems about alternating current<br>- in Physics ,fractals in Computer science, etc<br>- Show orderliness in the performance of tasks about complex numbers | - **Concepts of complex numbers:**<br>o Definition and structures<br>o Algebraic structures<br>- **Algebraic form of Complex numbers.**<br>o Definition and properties of "i"<br>o Operations: Addition, subtraction, multiplication, powers, conjugate and division.<br>o Modulus of a complex number.<br>o Square roots in the set $\mathbb{C}$ of complex numbers.<br>o Equations in the set $\mathbb{C}$ of complex numbers.<br>o Polynomials in the set $\mathbb{C}$ of complex numbers.<br>- **Geometric representation of complex numbers.**<br>- **Polar form of complex numbers.**<br>o Definition. | - Mental work: use definition of the multiplication of complex numbers to determine the complex number whose square is -1 and draw conclusion about the properties of "i".<br>- Constructing the points representing the nth roots of a complex number on Argand diagram and drawing conclusion about the polygon obtained by joining the points representing the nth roots.<br>- Learners derive properties of operations on complex numbers in trigonometric form and apply complex numbers to transform trigonometric formulas.<br>- Use the internet to determine |

| | | | | |
|---|---|---|---|---|
| | expressions<br>- Solve linear trigonometric equations using complex numbers | | o Modulus and argument of a complex number.<br>o Operations.<br>o De Moivre's formula.<br>o $N^{th}$ roots of a complex number.<br>o Construction of regular polygons.<br>- **Exponential forms of complex numbers:**<br>o Definition and operations.<br>o Euler's formula of complex numbers.<br>- **Application of complex numbers.**<br>o Product to sum formulas in trigonometry.<br>o Solution of linear trigonometric equations.<br>o Alternating current problems in physics. | the generation of fractals by complex numbers and print the different shapes to present in class. |

**Links to other subjects:** *Physics: alternating current. Computer science: fractal**s.***

***Assessment criteria:*** *Learners are able to perform operations on complex numbers in different forms and use complex numbers to solve related problems in physics (voltage and current in alternating current), computer science(fractals), trigonometry (Euler's formula to transform trigonometric expressions).*

***Materials:*** *IT equipment.*

| Topic Area: ANALYSIS | | | Sub-topic Area: Limits, differentiation and integration | |
|---|---|---|---|---|
| **S.6 MATHEMATICS** | **Unit 2:** Logarithmic and exponential functions. | | | **No. of periods:** 28 |
| **Key Unit Competence:** Extend the concepts of functions to investigate fully logarithmic and exponential functions, finding the domain of definition, the limits, asyrmptotes, variations, graphs, and model problems about interest rates, population growth or decay, magnitude of earthquake, etc. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - State the restrictions on the base and the variable in a logarithmic function.<br>- Extend the concept of functions to investigate fully logarithmic and exponential functions.<br>- Perform operations on logarithmic | - Transform a logarithm from a base to another.<br>- Find the domain and the range of a logarithmic or an exponential function.<br>- Calculate limits of logarithmic and exponential functions.<br>- Determine possible | - Show concern on the importance of logarithmic and exponential functions in solving problems such as carbon dating in chemistry.<br>- Develop patience, dedication and commitment in solving problems about logarithmic and exponential | - **Logarithmic functions:**<br>○ Domain of definition.<br>○ Limits of logarithmic functions and their applications to continuity and asymptotes.<br>○ Differentiation and its applications.<br>○ Curve sketching.<br>- **Exponential functions:**<br>○ Domain of definition.<br>○ Limits of logarithmic functions and their applications to continuity and asymptotes.<br>○ Differentiation and its applications.<br>○ Curve sketching. | - Learners use scientific calculators to evaluate logarithms and exponentials of real numbers; they conclude about the domain (the allowed input values) and the range (the set of possible outputs).<br>- Learners may use software, such as Geogebra, to graph logarithmic and exponential functions and report to class their findings about the general trend of the graphs.<br>- Practical: plot on graph paper, from the table of values, the points resulting from logarithmic and exponential functions and obtain the curves.<br>- Derive formulas about differentiation of logarithmic and exponential functions.<br>- Discuss in groups the applications of logarithms and exponentials in real life |

| | | | | |
|---|---|---|---|---|
| and exponential functions in any base.<br>- Recall the differentiation formulas for logarithmic and exponential functions to perform different operations or solve problems. | asymptotes of a logarithmic or an exponential function.<br>- Determine the derivative of a logarithmic or an exponential function.<br>- Solve related problems involving logarithms.<br>- Sketch the graph of a logarithmic or an exponential function. | functions.<br>- Show mutual respect, tolerance, in the discussion in group about logarithmic and exponential functions. | - **Applications of logarithmic and exponential functions:**<br>o Interest rate problems.<br>o Mortgage problems.<br>o Population growth problems.<br>o Radioactive decay problems.<br>o Earthquake problems.<br>o Carbon dating problems.<br>- **Problems about alcohol and risk of car accidents.** | and report the results. |

**Links to other subjects:** *English: as medium of communication. Physics: Newton's law of cooling. Geography: magnitude of an Earthquake. Economics: compounded interest. Biology: population growth. Chemistry: carbon dating.*

***Assessment criteria:*** *Learner is able to investigate fully logarithmic and exponential functions,finding the domain of definition, the limits, asyrmptotes, variations, graphs and model problems about interest rates, population growth or decay, magnitude of earthquake etc.*

***Materials:*** *Graph paper, ruler, digital technologies including calculators.*

| Topic Area: ANALYSIS | | Sub-topic Area: Limits, differentiation and integration | | |
|---|---|---|---|---|
| **S.6 MATHEMATICS** | **Unit 3:** Taylor and Maclaurin's expansions. | | | **No. of periods:** 14 |
| **Key Unit Competence:** Use Taylor and Maclaurin's expansion to solve problems about approximations, limits and integration. Extend the Maclaurin's expansion to Taylor series. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - State the Taylor's formula for a function and the conditions for the expansion to be valid.<br>- State the Maclaurin's formula for a function and the conditions for the expansion to be valid.<br>- List the first few terms of Maclaurin's expansion of $(1+x)^n$, cosx, sinx, $e^x$ and .ln(1+x), in each case, state the conditions for the expansion to be valid. | - Derive the Maclaurin's series for $(1+x)^n$, cosx, sinx, $e^x$ and .ln(1+x).<br>- Use Maclaurin's series to approximate an irrational number or the roots of a transcendental equation.<br>- Apply Maclaurin's series to the determination of asymptotic behavior of a curve.<br>- Calculate a definite integral using Maclaurin's expansion.<br>- Use properties of Maclaurin's expansion to perform operations on Maclaurin's series (addition, multiplication by a scalar, multiplication and division of functions). | - Show concern on the importance of Taylor and Maclaurin's and Taylor's series in solving problems about approximation, limits, etc.<br>- Appreciate the contribution of each member in the discussion about Taylor and Maclaurin's series, show mutual respect | - **Generalities on series:**<br>○ Definition.<br>○ Convergence of series.<br>- **Power series:**<br>○ Definition and properties.<br>- **Taylor series.**<br>- **Maclaurin series.**<br>- **Applications:**<br>○ Approximation of an irrational number by a rational number.<br>○ Approximation of the roots of equations.<br>○ Calculation of limits.<br>○ Evaluating | In groups:<br>- Use graphical approach to demonstrate the behaviour of different series, to discover Taylor polynomials and to investigate the relationship between the functions they approximate.<br>- Derive different Taylor and Maclaurin's formulas and use them to approximate the value of an irrational number, solutions of transcendental equations, asymptotic behaviour of a curve, approximation of definite integral, etc. and report the result to class. |

| | | | | |
|---|---|---|---|---|
| | | and tolerance in the discussion in groups.. | definite integrals, etc. | |

**Links to other subjects:** *English: terminologies such as series, convergence. Physics: path of a moving particle.*

***Assessment criteria:*** *Learner is able to use Taylor and Maclaurin's expansion to solve problems about approximations, limits and integration.*

***Materials:*** *Graph paper, scientific calculators.*

<table>
<tr><td colspan="3">Topic Area: ANALYSIS</td><td colspan="2">Sub-topic Area: Limits, differentiation and integration</td></tr>
<tr><td>S.6 MATHEMATICS</td><td colspan="3">Unit 4: Integration.</td><td>No. of periods: 42</td></tr>
<tr><td colspan="5">Key Unit Competence: Use integration as the inverse of differentiation and as the limit of a sum then apply it to find the area of plane surfaces, volumes of solids of revolution, lengths of curved lines.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr>
<td>- Define the differential of a function<br>- Interpret geometrically the differential of a function<br>- List the differentiation formulas<br>- Clarify the relationship between derivative and antiderivative of a function<br>- Illustrate the use of basic integration formulas<br>- Extend the concepts of indefinite</td>
<td>- Use differentials to approximate a function and to calculate the percentage error in an estimation<br>- Calculate integrals. Using appropriate techniques<br>- Use properties of integrals to simplify the calculation of integrals<br>- Calculate a limit of a sum to infinity as a definite integral<br>- Apply definite integrals to calculate the area, volume, arc length</td>
<td>- Show concern on the importance of integral calculus in solving problems from daily life.<br><br>- Appreciate various techniques of integration and show patience, commitment and tolerance in the evaluation of</td>
<td>- Differentials:<br>○ Definitions and operations on increments.<br>○ Properties of differentials.<br>○ Applications:<br>- Approximation<br>- Calculation of error.<br>- Indefinite integrals:<br>○ Antiderivatives.<br>○ Definition and properties.<br>○ Techniques of integration:<br>- Basic integration formulas.<br>- Integration by change of variables.<br>- Integration by parts.<br>- Integration of rational functions by partial fractions.<br>- Integration of trigonometric functions.<br>- Definite integrals</td>
<td>- Graphical approach: considering consecutive subintervals, learners calculate the areas of corresponding rectangles, then introduce the concept of integral as sum to infinity, when the width tends to zero.<br>- Learners determine the area of square, rectangle, trapezium, triangle and circle – determine also the volume of sphere, cylinder and cone – calculate the circumference of circle.</td>
</tr>
</table>

| | | | | |
|---|---|---|---|---|
| integrals to definite integrals. | - Analyze the convergence of an improper integral<br>- Use integrals to solve problems in Physics (work), Economics (marginal and total cost), etc. | integrals | ○ Definition.<br>○ Properties.<br>○ Techniques of integration.<br>○ Applications of definite integrals:<br>- Calculation of area of a plane surface.<br>- Calculation of volume of a solid of revolution.<br>- Calculation of length of curved lines.<br>- Applications in physics (work, energy.<br>- **Improper integrals:**<br>○ Definition.<br>○ Calculation of improper integrals. | |

**Links to other subjects:** *English, physics: work, energy.*

***Assessment criteria:*** *Learners are able to use integration as the inverse of differentiation and as the limit of a sum then apply it to find the area of plane surfaces, volumes of solid of revolution, lengths of curved lines.*

***Materials****: Manila paper, calculators, graph papers, ruler, markers.*

| **Topic Area: ANALYSIS** | | | **Sub-topic Area: Limits, differentiation and integration** | |
|---|---|---|---|---|
| **S.6 MATHEMATICS** | **Unit 5:** Differential equations. | | **No. of periods:** 21 | |
| **Key Unit Competence:** Use ordinary differential equations of first and second order to model and solve related problems in physics, economics, chemistry and biology. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Extend the concepts of differentiation and integration to ordinary differential equations.<br>- State the order and the degree of an ordinary differential equation.<br>- Express the auxiliary quadratic equation of a homogeneous linear differential equation of second order with constant coefficients.<br>- Predict the form of the particular solution of an | - Determine whether an ordinary differential equation of first order is with separable variables, homogeneous or linear.<br>- Use appropriate method to solve an ordinary differential equation of first order.<br>- Solve an ordinary linear differential equation of first order by "variation of constant" and by "integrating factor".<br>- Solve an ordinary linear differential | - Appreciate the use of differential equations in solving problems occurring from daily life.<br>- Show patience, commitment and dedication when solving a differential equation or modelling a problem using differential equations.<br>- When discussing in groups the solution of a differential equation, make sense of other learners' thinking, | - **Definition and classification.**<br>- **1st Order differential equation:**<br>○ Differential equation with separable variables.<br>○ Simple homogeneous differential equations.<br>○ Linear differential equations.<br>○ Applications.<br>- **2nd Order differential equation:**<br>○ Linear equations with constant coefficients:<br>- The right hand side is equal to zero.<br>- The right hand side is a polynomial function.<br>- The right hand side is | - Mental task: imagine the motion of a child on a swing. Express the displacement as function of time. Differentiate the function to find the velocity and acceleration, and then express the relation between the function and its derivatives. Report your results.<br>- Use graph plotting to illustrate the general solution of a differential equation.<br>- Discuss in groups the solutions of a differential equation with respect to a parameter and present the result to the class, show ability to communicate your thinking and reasoning.<br>- Use internet to find the applications of differential equations in sciences and report your findings to the class.<br>- Derive the general solutions of differential equations. |

| | | | | |
|---|---|---|---|---|
| ordinary linear differential equation of second order. | equation of second order.<br>- Use differential equations to model and solve problems in physics (simple harmonic motion), economics (point elasticity), etc. | show tolerance and mutual respect. | a trigonometric function.<br>- The right hand side is an exponential function. | - Solve ordinary differential equations of first and second orders. |

**Links to other subjects:** *Physics: simple harmonic motion. English: integrating factor). Chemistry: radioactive decay. Economics: point elasticity and demand function etc.*

***Assessment criteria:*** *Learner is able to use ordinary differential equations of first and second order to model and solve related problems in physics, economics, chemistry, biology, etc.*

***Materials:*** *Geometric instruments, graph paper, calculators, ICT equipment.*

<table>
<tr><td colspan="5">Topic Area: LINEAR ALGEBRA Sub-topic Area: Vectors</td></tr>
<tr><td>S.6 MATHEMATICS</td><td colspan="3">Unit 6: Intersection and sum of subspaces.</td><td>No. of periods: 14</td></tr>
<tr><td colspan="5">Key Unit Competence: Relate the sum and the intersection of subspaces of a vector space by the dimension formula.</td></tr>
<tr><td colspan="3">Learning Objectives</td><td rowspan="2">Content</td><td rowspan="2">Learning Activities</td></tr>
<tr><td>Knowledge and understanding</td><td>Skills</td><td>Attitudes and values</td></tr>
<tr>
<td>- Define the intersection and the sum of subspaces of a vector space.<br>- State the dimension formula.<br>- List the conditions for a vector space to be qualified as direct sum of its subspaces.</td>
<td>- Perform the intersection and the addition of subspaces of a vector space.<br>- Determine a basis of the intersection and a basis of the sum of the subspaces of a vector space.<br>- Apply the concepts of bases to calculate the dimension of the intersection and the dimension of the sum of given subspaces of a vector space.<br>- Apply the concept of the dimension formula to determine whether a vector space is direct sum or</td>
<td>- Appreciate the relationship between the dimensions of the intersection, the sum and the subspaces of a vector space.<br>- Show patience, commitment and dedication when verifying through the dimension formula whether a vector space is direct sum of its subspaces or not.</td>
<td>- Intersection of subspaces:<br>o Definition.<br>o Dimension of the intersection of subspaces.<br>- Sum of subspaces:<br>o Definition.<br>o Dimension of the sum of subspaces.<br>- Dimension formula.<br>- Direct sum of subspaces.</td>
<td>- Draw a Venn diagram for the vector space and its subspaces, and shade in color the intersection of the subspaces.<br>- Study in group the properties of the intersection and the sum of subspaces and present to the class the result with the ability to communicate the thinking and reasoning.<br>- Demonstrate that the intersection and the sum of two subspaces of a vector space is a subspace of the vector space.<br>- Construct two subspaces of a given vector space and show that their union is not a subspace of the given vector space.</td>
</tr>
</table>

| | | | | |
|---|---|---|---|---|
| | not of its given subspaces. | | | |

**Links to other subjects:** *Geometry: vectors. Physics: vectors. Engineering: vectors.*

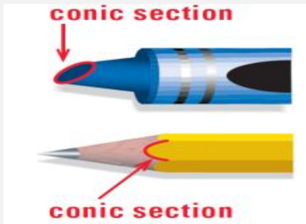
***Assessment criteria:*** *Learners are able to relate the sum and the intersection of subspaces of a vector space by the dimension formula.*

***Materials:*** *Manila paper, markers.*

| Topic Area: LINEAR ALGEBRA | | | Sub-topic Area: Linear transformations | |
|---|---|---|---|---|
| **S.6 MATHEMATICS** | **Unit 7:** Transformation of matrices. | | | **No. of periods:** 29 |
| **Key Unit Competence:** Transform matrices to an echelon form or to a diagonal matrix and use the results to solve simultaneous linear equations or to calculate the nth power of a matrix. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define the kernel, the image, the nullity and the rank of a linear transformation.<br>- State the dimension formula for linear transformations.<br>- Carry out the elementary row operations on matrices.<br>- Define Eigen values and eigenvectors of a square matrix.<br>- Discuss the diagonalisation of a matrix of order 2. | - Use definitions and properties to determine the kernel and the image of linear transformation.<br>- Apply the concept of dimension to calculate the nullity and the rank of linear transformation.<br>- Use elementary row operations to obtain a matrix in echelon form and to solve simultaneous linear equations.<br>- Determine a basis in which the matrix of a linear transformation of $R^2$ is diagonal and use the transition matrix to verify the result. | - Appreciate the use of the transformation of matrices in the simplification of the expression of a linear transformation.<br>- Verify the correctness of the diagonal matrix to develop self-esteem. | - **Kernel and range:**<br>○ Definitions.<br>○ Nullity and rank of a linear mapping.<br>○ Dimension formula for linear mapping.<br>- **Elementary row operations:**<br>○ Elementary row operations<br>○ Row reducing matrices to echelon form and applications:<br>- inverse of a matrix.<br>- Gauss elimination.<br>- **Diagonalisation of matrices of order 2:**<br>○ Characteristic polynomial.<br>○ Eigen values, Eigen spaces and Eigen vectors of a matrix of order 2.<br>○ Diagonalisation of | In groups:<br>- Learners discuss in pairs the transformation of matrices to diagonal matrices and they report and represent the results.<br>- Derive the formulas and determine the diagonal matrices.<br>- Calculate the $n^{th}$ power of a square matrix after transforming the matrix to diagonal form.<br>- Perform research using internet, library, etc. about transformation of matrices and their applications in real life and report, present results to class. |

| | | | | |
|---|---|---|---|---|
| | | | matrices of order 2.<br>○ Power of a matrix. | |

| |
|---|
| **Links to other subjects:** *English: terminologies. Geometry: matrices of plane transformations. Economics: organisation of data.* |
| ***Assessment criteria:*** *Learners are able to transform matrices to an echelon form or to diagonal matrix and use the results to solve simultaneous linear equations or to calculate the nth power of a matrix.* |
| ***Materials:*** *Manila paper, calculators.* |

| **Topic Area: GEOMETRY** | | | **Sub-topic Area: Plane geometry** | |
|---|---|---|---|---|
| **S.6 MATHEMATICS** | **Unit 8:** Conics. | | | **No. of periods:** 35 |
| **Key Unit Competence:** Determine the characteristics and the graph of a conic given by its Cartesian, parametric or polar equation. Find the Cartesian, parametric and polar equations of a conic from its characteristics. | | | | |
| **Learning Objectives** | | | **Content** | **Learning Activities** |
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define geometrically a conic as the intersection of a plane and a cone and classify conics from the position of the intersecting plane.<br>- Express, in Cartesian form, the standard equation of a parabola, an ellipse and a hyperbola.<br>- Classify conics based on the value of the eccentricity. | - Find the characteristics of a conic given its equation (centre, axes, vertices, tangent, normal, shapes, asymptotes, foci, directrices, eccentricity, etc.).<br>- Determine the Cartesian, parametric and polar equations of a conic from its characteristics.<br>- Derive the standard equation of a conic, in standard form.<br>- Use diagonalisation of matrices to simplify the equation of an | - Show patience and determination when analysing the general equation of a conic.<br>- Appreciate the importance of conics in the naturally occurring shapes, such as rainbow, the modelling of paths of particles such as projectiles, satellites, and manmade shapes such aerial, | - **Parabola** :<br>○ Definition.<br>○ Cartesian and parametric equations.<br>○ Graphical representation of a parabola.<br>○ Characteristics of a parabola (Vertex, axis of symmetry, focus, directrix, tangent and normal).<br>○ General equation.<br>- **Ellipse:**<br>○ Definition.<br>○ Cartesian and parametric equations.<br>○ Graphical representation of an ellipse.<br>○ Characteristics of an ellipse (Centre of symmetry, vertices, axes of symmetry, | - Visual approach: Learners analyze conics from sections of planes and cones:<br><br><br>- Matching conics to their standard forms.<br>- Analyzing and graphing conics from their equations.<br>- Deriving formulas related to conics (e.g. equation of the tangent).<br>- Analyse the general |

| | | | | |
|---|---|---|---|---|
| | ellipse or a hyperbola. | bridge, etc. | Eccentricity, foci, directrices, tangent and normal).<br>o General equation.<br>- **Hyperbola:**<br>o Definition.<br>o Cartesian and parametric equation.<br>o Graphical representation of a hyperbola.<br>o Characteristics of a hyperbola (Centre of symmetry, vertices, axes of symmetry, asymptotes, Eccentricity, foci, directrices, tangent and normal).<br>o General equation.<br>- **Polar coordinates:**<br>o Definition.<br>o Conversion of polar coordinates to Cartesian coordinates and vice versa.<br>o Straight line, circle and conics in polar coordinates. | equation of an ellipse and a hyperbola to obtain the graph. |
| **Links to other subjects:** *Architecture: shape of bridge. Physics: path of particle. Chemistry, astronomy: motion of satellites.* | | | | |
| ***Assessment criteria:*** *Learners are able to determine the characteristics and the graph of a conic given by its Cartesian, parametric or polar equation and can find the Cartesian, parametric and polar equations of a conic from its characteristics.* | | | | |
| ***Materials:*** *Geometric instruments (ruler, compass, T-square), IT equipment, conic sections from cones and planes.* | | | | |

| **Topic Area: STATISTICS AND PROBABILITY** | | **Sub-topic Area: Probability** |
|---|---|---|
| **S.6 MATHEMATICS** | **Unit 9:** Random variables. | **No. of periods:** 33 |

**Key Unit Competence:** Calculate and interpret the parameters of a random variable (discrete or continuous) including binomial and the Poisson distributions.

| **Learning Objectives** | | | **Content** | **Learning Activities** |
|---|---|---|---|---|
| **Knowledge and understanding** | **Skills** | **Attitudes and values** | | |
| - Define a random variable.<br>- Identify whether a given random variable is discrete or continuous.<br>- Define the parameters of a discrete random variable.<br>- Learn in which situation the Binomial distribution applies and state its parameters. | - Use the concepts of statistics to compare frequency distribution to probability distribution.<br>- Calculate and interpret the parameters of a random variable (discrete or continuous).<br>- Construct the probability distribution of a discrete random variable.<br>- Determine whether a function can serve as probability density function or not.<br>- Apply binomial and the Poisson distributions to solve related problems. | - Appreciate the use of the random variable in the interpretation of statistical data. | - **Discrete and finite random variable:**<br>○ Probability distribution.<br>○ Expected value, variance and standard deviation of a discrete random variable.<br>○ Cumulative distribution function.<br>○ Binomial and<br>○ The Poisson distribution.<br>- **Continuous random variables**<br>○ Probability density function.<br>○ Expected value, variance and standard deviation of a discrete random variable. | -Mental task: imagine a policeman recording the numbers of cars crossing a certain junction in different periods of the day, the nurse recording the weights of infants at birth in a hospital. Classify the random variables as continuous or discrete.<br>-Construction of probability distribution of discrete random variable.<br>-Calculation of the expected value, the variance and the standard deviation.<br>-Graphing the probability distribution. |

**Links to other subjects:** *Economics, entrepreneurship, biology, geography: analysis of data.*

***Assessment criteria:*** *Learners are able to calculate and interpret the parameters of a random variable (discrete or continuous) including binomial and the Poisson distributions.*

***Materials:*** *Calculator, graph paper, manila paper.*

# 6. REFERENCES


1. Alpers, B. (2013). *A Framework for Mathematics Curricula in Engineering Education*: *A Report of the Mathematics Working Group.* European Society for Engineering Education (SEFI): Brussels
2. Balakrishnan, V.K . (1995). *Schaum's Outline, Combinatorics.*
3. Bird, J. ( 2003). *Engineering Mathematics* ( $4^{th}$ Edition).
4. Bronson, R. (2003). Schaum's Outline, Differential equation.
5. CrashwshaW,J.,& Chambers,J.(2001).*Advanced Level Statistics with worked examples* (Fourth Edition). Nelson Thornes Ltd:UK
6. Curriculum Planning and Development Division (2006). *Secondary Mathematics Syllabuses*. Ministry of Education: Singapore
7. Frank Ayres, F., & Moyer,E.R.(1999). *Schaum' s Outline Series Trigonometry* ($3^{rd}$ Edition).
8. National Curriculum Development Centre (2008).Mathematics teaching syllabus: Uganda Certificate of Education Senior1-4. Ministry of Education and Sports: Uganda.
9. National Curriculum Development Centre (2013). *Uganda Advanced Certificate of Education: Teaching Syllabi for Physics and Mathematics* (Volume 2). Ministry of Education and Sports: Uganda.
10. National Curriculum Development Centre (2013). *Uganda Advanced Certificate of Education: Subsidiary Mathematics teaching syllabus*. Ministry of Education and Sports: Uganda.
11. Rich, B. & Thomas, C. (2009). *Schaum's Outline, Geometry : Fourth Edition.*
12. Rwanda Education Board (2010).*Advanced Level Mathematics for Science Combination*. Ministry of Education: Rwanda
13. Rwanda Education Board (2014). *Mathematics Curriculum For Physics - Chemistry - Biology (PCB) Combination (Elective) Advanced Level.* Ministry of Education: Rwanda
14. Sanchez , A.V, & Ruiz, M.P. (2008). *Competence-Based Learning : A proposal for the assessment of Generic Competences(Eds)*.
15. Shampiyona, A. (2005). *Mathématiques 6.* Imprimerie de Kigali: Kigali
16. Spiegel,M. R.; Schiller, J. & Srinivasan, A. (2009). *Schaum's Outline, Probability and Statistics: Third Edition.*
17. Stewart, J. ( 2008 ). *Calculus* ( $6^{th}$ Edition)
18. Stewart, J. (2008). *Calculus Early Transcendental*, sixth edition. McMaster University: US.
19. Wrede,R., Murray,D., & Spiegel, R. (2010). *Schaum's Outline, Advanced calculus, $3^{rd}$ Edition*.

## 7. APPENDIX: SUBJECTS AND WEEKLY TIME ALOCATION FOR A'LEVEL

| **Subjects in Secondary 4-6** | | **Number of periods per week (1 period = 40 min.)** | | |
|---|---|---|---|---|
| **Core subjects** | | **S4** | **S5** | **S6** |
| 1. Mathematics | | 7 | 7 | 7 |
| 2. Physics | | 7 | 7 | 7 |
| 3. Computer Science | | 7 | 7 | 7 |
| 4. Chemistry | | 7 | 7 | 7 |
| 5. Biology | | 7 | 7 | 7 |
| 6. Geography | | 7 | 7 | 7 |
| 7. History | | 7 | 7 | 7 |
| 8. Economics | | 7 | 7 | 7 |
| 9. Literature in English | | 7 | 7 | 7 |
| 10. Kinyarwanda major | | 7 | 7 | 7 |
| 11. Kiswahili major | | 7 | 7 | 7 |
| 12. French major | | 7 | 7 | 7 |
| 13. Religion major | | 7 | 7 | 7 |
| 14. Entrepreneurship | | 6 | 6 | 6 |
| 15. General Studies and Communication Skills | | 3 | 3 | 3 |
| 16. Subsidiary Mathematics | | 3 | 3 | 3 |
| Electives Subjects | 17. English minor | 4 | 4 | 4 |
| | 18. French minor | 4 | 4 | 4 |
| | 19. Kinyarwanda minor | 4 | 4 | 4 |
| | 20. Kiswahili minor | 4 | 4 | 4 |
| Co-curricular Activities | Religious activities | 2 | 2 | 2 |
| | Sports/ Clubs | 2 | 2 | 2 |
| | Computer/library | 2 | 2 | 2 |