Mathematics

For

Rwandan Primary

Schools

Pupil's book

5

Kigali, 2022

FOREWORD

Dear Pupil,

Rwanda Basic Education Board is honoured to present to you this Mathematics book for Primary Five which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of Mathematics subject. The Rwandan educational philosophy is to ensure that you achieve full potential at every level of education which will prepare you to be well integrated in society and exploit employment opportunities.

The government of Rwanda emphasizes the importance of supporting teaching and learning materials with the syllabus to facilitate your learning process. Many factors influence what you learn, how well you learn and the competences you acquire. Those factors include the instructional materials available among others. Special attention was paid to the activities that facilitate the learning process in which you can develop your ideas and make new discoveries during concrete activities carried out individually or with peers.

In competence-based curriculum, learning is considered as a process of active building and developing knowledge and meanings by the learner where concepts are mainly introduced by an activity, a situation or a scenario that helps the learner to construct knowledge, develop skills and acquire positive attitudes and values. For effective use of this textbook, your role is to:

- Work on given activities which lead to the development of skills
- Share relevant information with other learners through presentations, discussions, group work and other active learning techniques such as role play, case studies, investigation and research in the library, on internet or outside
- Participate and take responsibility for your own learning
- Draw conclusions based on the findings from the learning activities.

I wish to sincerely extend my appreciation to the people who contributed towards the development of this textbook, particularly REB staff who organized the whole process from its beginning. Special gratitude goes to teachers, illustrators and designers who carefully worked to successful completion of this textbook. Any comment or contribution would be welcome for the improvement of this textbook for the next edition.

Dr. MBARUSHIMANA Nelson
Director General, REB

ACKNOWLEDGEMENT

I wish to sincerely extend my special appreciation to people who played a major role in development and editing of this Mathematics book for Primary Five. It would not have been successful without the participation of different partners that I would like to express my deep gratitude.

My thanks go to the Rwanda Basic Education Board leadership and staff who were involved and supervised the whole activity of in-house textbook Elaboration. I also wish to extend my appreciation to primary school teachers, lecturers and different education experts for their valuable support.

Joan MURUNGI
Head of CTLR Department

Content

UNIT 1

Reading, writing, comparing and calculating whole numbers up to 1 000 000

1.1 Reading and writing numbers in words up to 1 000 000

(a) Reading and writing in words

Activity 1.1

- Study these numbers:
  100 001   280 465   405 230   729 111   999 999
- Write each of them on little slips of papers. Pick a slip of paper.
- Now, arrange yourselves according to your numbers.
- Read your number aloud then write your number in words.

Tip:

When writing a number in words, group the digits of the number. Look at the example below.

Example 1.1

Write 976 946 in words.

Solution

We can group 976 946 as:

976 000 – nine hundred seventy six thousand

900 – nine hundred

40 – forty

6 – six

Therefore 976 946 is nine hundred seventy six thousand nine hundred and forty six.

Practice Activity 1.1

1. Read and write each of the following numbers in words.
   (a) 671 379 (b) 286 748 (c) 910 842 (d) 263 450
2. Discuss how to write the followings numbers in words.
   (a) 716 809 (b) 604 382 (c) 862 059 (d) 345 671
3. A factory packed 447 313 text books in cartons for sale. Write this in words. Explain the steps you used to arrive at your answer.
4. A Green Belt Movement planted 527 174 tree seedlings. Write this in words. Explain steps to your answer.

(b) Reading and writing numbers in figures

Activity 1.2

(a) Write these numbers in figures.
   (i) Six hundred thirty three thousand four hundred and five.
   (ii) One hundred thousand and eleven.
   (iii) Nine hundred seven thousand one hundred and seven.

(b) Now match these numbers in figures to the correct number in words.

Number in figures	Number in words
909 090	One million
1 000 000	One hundred thousand
100 000	Three hundred eighty six thousand two hundred and eleven
386 211	Nine hundred nine thousand and ninety

Make a class presentation to explain your answer.

Example 1.2

Write eight hundred twenty five thousand two hundred and thirty four in figures:

Solution

Thousands		825 000
Hundreds		200
Tens		30
Ones	+	4
		825 234

Practice Activity 1.2

1. Write each of the following numbers in figures.
   (a) Seven hundred and six thousand five hundred and eighteen.
   (b) One hundred and three thousand six hundred and four.
   (c) Nine hundred thousand nine hundred and nine.
   (d) Five hundred thousand and five.
2. Discuss how to write the following in figures.
   (a) Six hundred and fifty thousand.
   (b) Eight hundred and eight thousand eight hundred and eight.
   (c) Two hundred thirty four thousand one hundred and eleven.
   (d) Four hundred seventy one thousand two hundred and thirty five.
3. I had three hundred ninety eight thousand seven hundred and sixty six Rwandan Francs. Write the amount I had in figures..
4. During an election, five hundred forty seven thousand seven hundred and fifty voted for the winning member of parliament. How many votes in figures did she get? Show your working steps.

1.2 Place value and comparing numbers

(a) Place value of numbers up to 7 digits

Activity 1.3

- Study these numbers.

  (a) 100 000 (b) 473 625 (c) 999 999

  Write the numbers on paper cutouts.

Name the place value of each digit.

Write the place value of each digit.

Say the place value of each digit. Present your findings.

- What is the next number after 999 999? What is the place value of digit 1 in your answer? Discuss your answer.

Example 1.3

Write the place value of each digit in the number 235 176.

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
2	3	5	1	7	6

Solution

2 – Hundred Thousands

3 – Ten Thousands

5 – Thousands

1 – Hundreds

7 – Tens

6 – Ones

Practice Activity 1.3

1. Identify the place value of each digit in the number.

   (a) 560 438 (b) 189 274 dam^2 (c) 908 346

2. Identify the digit in the place value of hundred thousands. Justify your answer.

   (a) 964 815 (b) 321 456 (c) 811 943 kg

3. Write the place value of the coloured digit. Discuss and present your answer.

   (a) 19**8** 065 (b) **7**46 138 *l* (c) 6**4**0 404 (d) 245 **6**89

(b) Comparing numbers using <, > or =

Activity 1.4

- Write the following numbers on small paper cutouts.

  $\boxed{6}$, $\boxed{2}$, $\boxed{4}$, $\boxed{8}$, $\boxed{9}$, $\boxed{1}$

  Arrange the numbers to form:
  - the largest number.
  - the smallest number.

  (i) Use > to compare the numbers you have formed.
  (ii) Use < to compare the numbers you have formed.
- Repeat the activity above with other digits. Explain your answers.
- Where do you compare quantities? Discuss how you do it.

Example 1.4

By using <, = or > compare: 356 481 —— 353 406

Solution

Step 1: Write the two numbers on place value chart.

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
3	5	6	4	8	1
3	5	3	4	0	6

Step 2: Compare the digits from the left towards the right. The hundred thousands digits are the same. So are the ten thousands digits. Thousands digits are 6 and 3, but 6 > 3. Therefore, 356 481 > 353 406

Tip:

- To compare whole numbers: Check digits of numbers at the same place values. Start at the left and compare the digits in the greatest place value position. The greater number has a greater digit at the greatest place value.
- We use: < for "less than", > for "greater than" and = for "equal to".

Practice Activity 1.4

1. By using <, = or > compare the following pair of numbers.
   (a) 440 040 ☐ 440 040 (b) 657 000 ☐ 675 000
   (c) 649 362 ☐ 639 462 (d) 831 647 ☐ 861 347
2. Use <, = or > to compare the following. Discuss your answers.
   (a). 531 926 ☐ 513 926 (b) 100 000 ☐ 1 000 000
   (c) 210 034 ☐ 201 034 (d) 245 689 ☐ 245 689
3. The number of adults in a certain district was 136 895. The number of children was 136 989 for the same district.
   (i) Were there more adults or children?
   (ii) Were there fewer adults or children? Discuss your answers.
4. Anne is a coffee farmer. Anne made 550 000 Frw from sales of her coffee over four months. Musabe is a business person. Musabe made 630 000 Frw from sales in his shop over four months. Who made more money? Explain your answer.

1.3 Addition of 3 or more whole numbers of 7 digits with/without carrying

Activity 1.5

Use an abacus or objects of different colours to add the following:

(a) 100 204 + 551 480 + 226 102.

(b) 128 539 + 300 856 + 15 210. Explain your answer.

What did you observe while carrying out the addition?

Example 1.5

Work out:

(a) 272 142 + 203 512 + 402 123 (b) 472 598 + 284 706 + 163 075

Solution

Align according to place values.

(a)

$$\begin{array}{r} 272\ 142 \\ 203\ 512 \\ +\ 402\ 123 \\ \hline 877\ 777 \\ \hline \end{array}$$

Steps

Add ones	2 + 2 + 3 = 7 write 7 ones
Add tens	4 + 1 + 2 = 7 write 7 tens
Add hundreds	1 + 5 + 1 = 7 write 7 hundreds
Add thousands	2 + 3+ 2 = 7 write 7 thousands
Add tens thousands	7 + 0 + 0 = 7 write 7 ten thousands
Add hundred thousands	2 + 2 + 4 = 8 write 8 hundred thousands

(b)

$$\begin{array}{r} {}^{2\,1\,1\ \ 1\,1} \\ 472\ 598 \\ 284\ 706 \\ +\ 163\ 075 \\ \hline 920\ 379 \\ \hline \end{array}$$

Steps: Add

Ones	8 + 6 +5 = 19 ones, write 9 in ones carry 1 ten to tens
Tens	9 + 0 + 7 + 1 = 17 tens, write 7 tens write 7 in tens carry 1 to hundreds.
Hundreds	5 + 7 + 0 + 1 = 13 hundreds, write 3 in hundreds carry 1 to thousands.
Thousands	2 + 4 + 3 + 1 = 10 thousands, write 0 in thousands carry 1 to ten thousands.
Ten thousands	7 + 8 + 6 + 1 = 22 ten thousands, write 2 in ten thousands carry 2 to hundred thousands.
Hundred thousands	4 + 2 + 1 + 2 = 9 hundred thousands.

Practice Activity 1.5

1. Work out:
   (a) 430 526 + 323 250 + 122 102 =
   (b) 252 143 + 235 322 + 400 312 =
   (c) 283 054 + 3 002 + 415 621 =
   (d) 311 052 + 203 932 + 132 003 =
2. Work out the following. Present your answers.
   (a) 39 845 + 105 523 + 351 214 =
   (b) 193 584 + 258 907 + 391 358 =
   (c) 552 797 + 25 895 + 188 253 =
   (d) 340 020 + 215 322 + 104 052 + 340 606 =

3. A poultry farmer sold 252 797 chickens in one year. The next year he sold 391 358 chicken. The third year he sold 193 583 chickens. How many chickens did he sell in 3 years?
4. A business man invested 442 300 Frw in the first year of business. The second year he invested 442 100 Frw. The third year he invested 115 600 Frw. How much money did he invest in total? Explain your answer.
5. At a peace campaign rally, there were 8 430 women. Men were 5 660 and children were 7 200. How many people attended the rally? Discuss the steps involved in calculating your answer.

1.4 Subtraction of 2 whole numbers of 7 digits with/without borrowing

Activity 1.6

Use an abacus to subtract:

(a) 398 450 – 352 150 (b) 852 757 – 193 583 (c) 710 534 – 40 203

What do you notice in subtracting a, b and c? Explain the borrowing from one place value to the next to carry out subtraction in b and c.

(d) A company packed 763 389 packets of milk on Monday. 539 897 packets were sold at the same day. How many packets of milk remained to be sold on Tuesday?

Example 1.6

Subtract: (a) 258 950 – 50 300 (b) 874 450 – 583 750

Solution

(a)

$$\begin{array}{r} 258\ 950 \\ -\quad 50\ 300 \\ \hline 208\ 650 \\ \hline \end{array}$$

Steps: Subtract
Ones 0 – 0 = 0 write 0
Tens 5 – 0 = 5 write 5
Hundreds 9 – 3 = 6 write 6
Thousands 8 – 0 = 8 write 8
Ten thousands 5 – 5 = 0 write 0
Hundred thousands 2 – 0 = 2 write 2.

(b)

$$\begin{array}{r} \overset{7}{\cancel{8}}\,{}^{1}7\ \overset{3}{\cancel{4}}\,{}^{1}4\,5\,0 \\ -\ 583\ 750 \\ \hline 290\ 700 \\ \hline \end{array}$$

Steps: Subtract Ones 0 – 0 = 0 write 0
Tens 5 – 5 = 0 write 0
Hundreds 4 – 7 not possible. Borrow 10 from 4 thousand to get 14 – 7 = 7 write 7. Thousands 3 – 3 = 0 write 0
Ten thousands 7 – 8, not possible borrow 10 from 8 to get 17 – 8 = 9 write 9
Hundred thousands 7 – 5 = 2 write 2.

Practice Activity 1.6

Subtract

1. (a) 808 210 – 205 210 = (b) 394 930 – 192 620 =
   (c) 888 980 – 56 360 = (d) 393 588 – 372 475 =
2. Subtract the following numbers and explain your answer.
   (a) 855 157 – 398 480 = (b) 875 864 – 557 993 =
   (c) 480 734 – 469 372 = (d) 736 425 – 463 758 =
3. A farmer harvested 404 040 kilograms of maize. He sold 345 678 kilograms. How many kilograms of maize did he remain with? Discuss the steps involved in calculating your answer.
4. A school uses 840 020 litres of water in a term. The school was closed a week earlier. They had used 710 229 litres of water. How many litres of water was not used? Discuss the steps involved in calculating your answer.

1.5 Quick multiplication of a 3 digit number by 5, 9, 11, 19, 25, 49 and 99

Activity 1.7

- Compose 3 digit numbers and quick multiply by 5, 9, 11, 19, 25, 49 and 99. Solve them and make a presentation to the class.
- Solve this problem. We are 5 in our group. Each of us has 520 Frw. How much money do we have?
- Now compose problems related to real life. Solve and present them to class.

Example 1.7

Quick multiply the following. Discuss your steps.

(a) 726 × 99 (b) 436 × 25 (c) 352 × 19 (d) 1 325 × 11

Solution

(a) 726 × 99

Step 1

Multiply the number by 100

726 × 100 = 72 600

Step 2

Subtract the original 3 digit number from your result.

72 600 – 726 = 71 874.

Thus, 726 × 99 = 71 874

Task:

Now devise a way to quick multiply 726 × 9. Explain your steps.

(b) 436 × 25

Step 1

Multiply the number by 100

436 × 100 = 43 600

Step 2

Divide the result by 4.

$$\begin{array}{r|l} & 10900 \\ 4 & 43600 \\ & -41 \\ & \overline{036} \\ & -36 \\ & \overline{000} \end{array}$$

Thus, 436 × 25 = 10 900

Task:

Now compose steps to follow to quick multiply 1 343 × 5. Discuss your steps.

(c) 352 × 19

Step 1

Multiply 352 by 2.

352 × 2 = 704

Step 2

Multiply the result by 10.

704 × 10 = 7 040

Step 3

Subtract the original number from your result.

7 040 – 352 = 6 688.

Thus, 352 × 19 = 6 688

Task:

Now devise a way to quick multiply 2 423 × 49. Explain your steps.

(d) 1 325 × 11

Steps: Add two consecutive digits of 1 325. Then put them between 1 and 5. Start from the right side, proceed to the left.

1. Note the right most digit of 1 325 is 5
2. 5 + 2 = 7, Note 7 to the left of 5 → (75)
3. 2 + 3 = 5, Note 5 to the left of 7 → (575)
4. 3 + 1 = 4, Note 4 to the left of 5 → (4575)
5. Note 1 first digit to the left → (14575)

Thus, 1 325 × 11 = 14 575

Practice Activity 1.7

1. Quick multiply the following:

   (a) 883 × 5 (b) 827 × 9 (c) 618 × 11 (d) 704 × 19

2. Use quick multiplication to solve the following. Discuss the steps to your answers.

   (a) 567 × 25 (b) 430 × 49 (c) 525 × 99

   (d) 629 × 5 (e) 449 × 9

3. A library has 113 shelves with 99 books each. How many books are in the library?

4. 25 schools have 215 pupils each in a certain province. How many pupils are in those 25 schools? Explain your answer.
5. A school bought 19 boxes of pencils. Each box had 144 pencils. How many pencils are there? Explain and present your answer.
6. A school farm collected 99 eggs daily. Each egg was sold at 110 Frw. Calculate the money the school got from sale of eggs daily.
7. There are 49 pupils in the P5 class. During a mathematics lesson, each pupil brought 125 counters. How many counters were brought altogether?
8. 25 pupils in a school are given milk. Milk is in 500 millilitre packets. The pupils take a packet of milk every day. How many packets are required in 7 weeks? Why is it important to drink milk?

1.6 Multiplication of whole numbers by a 3 digit number

Activity 1.8

- Solve this problem.

  A shopkeeper had 112 bottles of juice. He sold each of them at 350 Frw. How much money did he get?
- Now compose problems related to real life. Solve and present them to the class.

Example 1.8

Multiply 365×241

Solution

$$\begin{array}{rl} 365 & \\ \times\ 241 & \\ \hline 365 & (365 \times 1) \\ 1460 & (365 \times 40) \\ +\ 730 & (365 \times 200) \\ \hline 87965 & \\ \hline \end{array}$$

Steps

- Multiply by ones. 365 × 1 = 365
- Multiply by tens. 365 × 4 = 1 460
- Multiply by hundreds. 365 × 2 = 730

365 × 241 = 87 965.

Practice Activity 1.8

Work out:

1. (a) 833 × 410 (b) 581 × 611 (c) 648 × 212
   (d) 439 × 326 (e) 788 × 423 (f) 373 × 465

2 Solve the following problems and discuss your answers
   (a) 349 × 247 (b) 943 × 333 (c) 317 × 149 (d) 623 × 261

3. There are 258 hotels in a certain country. In one holiday season, each hotel received 415 visitors. How many visitors were there during that season? Explain your answer.

4. 135 rings are needed to decorate the ceiling of each room in a hotel. The hotel has 221 similar rooms. Explain how many rings are needed?

5. A district received 375 cartons of exercise books. Each carton holds 180 exercise books. How many exercise books were received? Explain your answer.

6. A wholesaler received 247 cartons of juice. Each carton costs 950 Frw. Explain how much money he paid?

7. The government gave 790 textbooks to each of the 183 primary schools in a district. Explain how many textbooks were given altogether?

1.7 Division without a remainder of a 3 digit number by a 2 digit number

Activity 1.9

(a) A teacher had 120 exercise books. The books are to be equally shared by 24 pupils. How many books did each pupil get?

(b) During a birthday party a packet of sweets with 120 sweets was shared equally by 15 pupils. Discuss how many sweets each pupil got.

- Now explain situations where sharing occurs in daily life.
- Name some items or things that are often shared.

Example 1.9

Work out: 875 ÷ 25

Solution

$$\begin{array}{r|l} & 35 \\ 25 & 875 \\ - & 75 \\ & 125 \\ - & 125 \\ & 0 \end{array}$$

Steps

Let us try to divide from 8.

8 ÷ 25 is not possible.

We try 87 ÷ 25. This gives: 87 ÷ 25 = 3 rem 12.

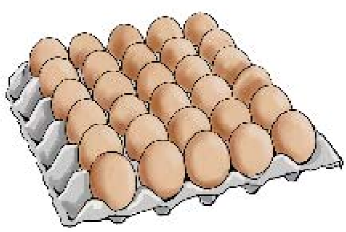
Record 3 above 8. Drop 5 to be with 12. This gives 125.

125 ÷ 25 = 5 rem 0.

Thus, 875 ÷ 25 = 35

Practice Activity 1.9

1. Divide the following:
   (a) 792 ÷ 18 (b) 768 ÷ 32 (c) 391 ÷ 23
   (d) 858 ÷ 22 (e) 405 ÷ 15 (f) 390 ÷ 30
2. Work out the following and discuss your answer.
   (a) 688 ÷ 43 (b) 714 ÷ 17 (c) 759 ÷ 23 (d) 861 ÷ 21
3. Margaret has 180 eggs. She packs them in trays of 30 eggs each. How many trays did she fill? Explain your answer.



4. A farmer had 468 seedlings planted in 18 rows. Each row had an equal number of seedlings. Explain how many seedlings were in each row?

5. The teacher shared 516 books equally among 43 pupils. How many books did each pupil get? Explain your answer.

Revision Activity 1

1. Write the numbers below in words.
   (a) 382 640   (b) 942 108
2. Write the numbers below in figures.
   (a) Nine hundred seventy seven thousand six hundred thirty one.
   (b) Four hundred eighty two thousand seven hundred sixty five.
3. What is the place value of the coloured digits?
   (a) 981 555   (b) 436 914
4. Use <, = or > to compare
   (a) 677 931 ☐ 977 631
   (b) 848 756 ☐ 848 657
5. To conserve environment, members of community planted trees during rainy season. In the first region, they planted 187 255 trees. In the second region, they planted 320 316 trees. In the third region they planted 439 230 trees.
   (a) Find the total number of trees planted in the three regions.
   (b) Calculate the difference of the highest and lowest number of trees they planted. Justify your answer.
   (c) Discuss the importance of conserving our environment.
6. In a certain village, 840 kg of maize seeds were donated by a certain farmer. Thirty five families shared them equally and planted in their farms. How many kilograms of maize seeds did each family get?
7. A distributor delivered 99 cartons of books to each of 265 schools. This was during book delivery program. Each carton had 25 books.
   (a) Using quick multiplication to find the number of cartons of books delivered to the schools.
   (b) Suppose one carton was to be given to each school. How many books would have to be delivered to 228 schools? Why should we have books?

Word list

Place values	Numerals	Addition	Subtraction
Multiplication	Division	Digits	Abacus
Compare	Calculate		

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 2 Addition and subtraction of integers

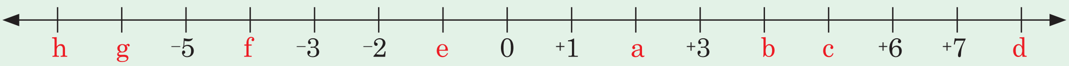
2.1 Location of positive and negative numbers on a number line

Let us do the activity below.

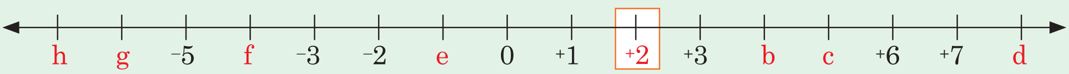
Activity 2.1

Do the following.

- On manila paper, draw the number line shown below.

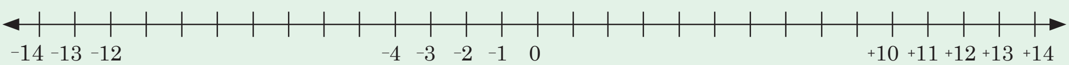


- Make paper cutouts. Write different numbers from $^{-}7$ to $^{+}8$ on paper cutouts.
- Fix the paper cutouts on the number line. These numbers should replace the written letters. For example, fix $\boxed{2}$ on a, as shown below.



Activity 2.2

- Draw a number line on the chalkboard, as shown below.



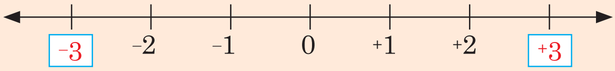
- Make number cards to show different numbers. This is from $^{-}14$ to $^{+}14$. Use manila paper cutouts or used carton cutouts.
- In turns, pick a number card and place it on the correct point on the board.

Example 2.1

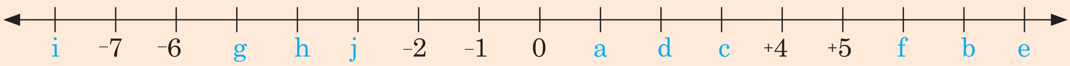
(a) Draw a number line. Show $\boxed{+3}$ and $\boxed{-3}$.

Solution

Numbers with opposite signs are located on opposite sides of 0.



(b) From the number line below, write the integers represented by letters.



Solution

a = +1	b = +7	c = +3	d = +2	e = +8
f = +6	g = -5	h = -4	i = -8	j = -3

Tip:

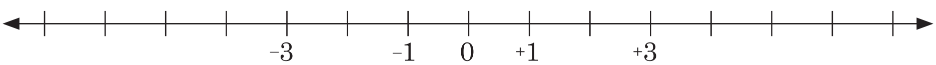
When a number is positive it is located on the right side of 0. A negative number is located on left side of 0.

Practice Activity 2.1

1. Make these number cards.

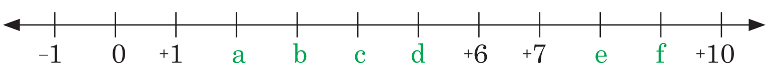
   $\boxed{-5}$, $\boxed{+5}$, $\boxed{+2}$, $\boxed{-2}$, $\boxed{+7}$, $\boxed{-7}$

   Locate them on the number line below.

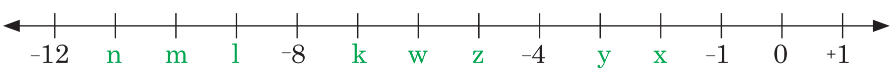


2. Look at the number lines below. Write the integers represented by the letters.

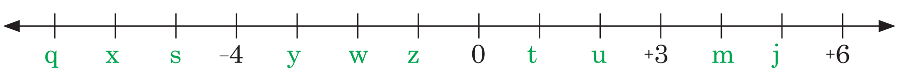
(a)

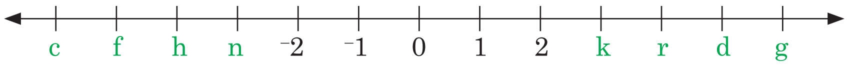


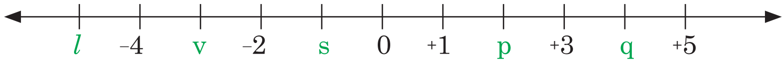
(b)



(c)



(d) 

(e) 

2.2 Comparison and ordering of integers

(a) Comparing integers using a number line

Activity 2.3

Draw a number line. Use it to compare the following. Tell the integer that is greater. Tell the integer that is smaller. Explain your answer.

(i) $^{-}3$ and $^{+}2$ (ii) $^{-}4$ and $^{+}4$

(iii) $^{-}5$ and $^{-}2$ (iv) $^{+}1$ and $^{+}6$

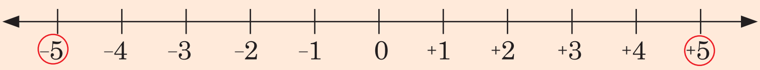
Tip:

Integers on the right side of 0 are greater than those on the left. Positive numbers are greater than negative numbers.

Example 2.2

Use a number line to compare $^{-}5$ and $^{+}5$.

Solution

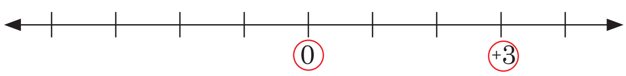


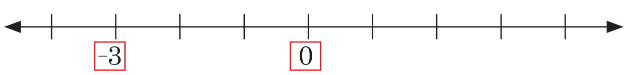
$^{+}5$ is greater than $^{-}5$

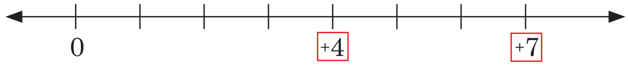
A number to the right is greater than a number to the left on a number line.

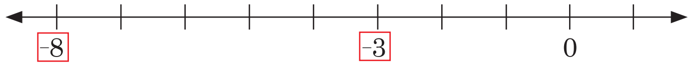
Practice Activity 2.2

1. Study the number lines below. Tell which integer is greater in each number line.

(a) 

(b) 

(c) 

(d) 

2. Use a number line to compare the following. Which one is greater?

(a) $^{+}2$ and $^{-}2$ (b) $^{+}4$ and $^{-}4$ (c) $^{-}1$ and $^{+}5$

(d) $^{+}1$ and $^{+}5$ (e) 0 and $^{-}6$ (f) 0 and $^{+}9$

(b) Ordering integers and comparing integers using <, > or =

Activity 2.4

- Draw a number line. Have written paper cutouts for $^{-}10$, $^{-}9$, $^{-}8$, $^{-}7$, $^{-}5$, $^{-}4$, $^{-}3$, 0, $^{+}3$, $^{+}4$, $^{+}5$, $^{+}6$, $^{+}7$, $^{+}8$, $^{+}9$, $^{+}10$. Fix them on the number line.
- Use the integers position to arrange $^{-}10$, $^{-}7$, 0, $^{+}7$, $^{+}2$ from;

(i) Smallest to largest

(ii) Largest to smallest

(iii) Use < or > or = to compare

(a) $^{-}10 \square ^{-}7$ (b) $^{+}2 \square ^{-}7$

Tip: We use > for 'greater than', < for 'less than' and = for 'equal to'.

Ordering numbers can mean to arrange numbers from the smallest to the largest. It can also means to arrange numbers from the largest to the smallest. Arranging/ordering numbers from the smallest to the largest is called ascending order. For example $^{+}4$, $^{+}5$, $^{+}6$, $^{+}7$.

Ordering/arranging numbers from the largest to the smallest is called descending order. For example; $^{+}4$, $^{+}3$, $^{+}2$, $^{+}1$, 0.

Look at the example below.

Example 2.3

1. Use > or < or = to compare the integers given below.

(a) $^{+}3 \square ^{+}1$ (b) $^{-}6 \square ^{+}2$ (c) $^{+}5 \square 5$

2. (a) Arrange in ascending order: $^{+}3$, $^{-}4$, 0, $^{+}6$

(b) Arrange in descending order: $^{+}6$, $^{+}1$, $^{-}1$, $^{-}3$, $^{+}2$

Solution

1. (a) $^+3 > 1$. On a number line 3 is farther to the right side than 1 from 0.

   (b) $^-6 < {}^+2$. On a number line $^-6$ is to the left side while $^+2$ is to the right side of 0.

   (c) $^+5 = 5$. It is the same point on number line.

2. (a) In ascending order, start from the smallest to the largest:

   $^-4, 0, {}^+3, {}^+6$

   (b) In descending order, start from the largest to the smallest:

   $^+6, {}^+2, {}^+1, {}^-1, {}^-3$

Practice Activity 2.3

1. Use >, < or = to compare the integers given below.

   (a) $^-10$ ☐ $^+3$ (b) $^-15$ ☐ 0 (c) $^+3$ ☐ $^+3$

   (d) $^-6$ ☐ $^+4$ (e) $^+6$ ☐ 0 (f) $^+4$ ☐ $^-2$

2. Use >, < or = to compare the integers below. Discuss your answers.

   (a) $^-5$ ☐ $^+1$ (b) $^+7$ ☐ $^+9$ (c) 0 ☐ $^+8$

   (d) $^+10$ ☐ $^-6$ (e) $^-11$ ☐ $^+6$ (f) $^+11$ ☐ $^+11$

3. Arrange the integers in ascending order

   (a) $^-3, {}^+4, {}^-5, {}^-1$ (b) $^+20, {}^-15, {}^+4, {}^-11$ (c) $^-22, {}^-11, {}^-20, {}^+11$

4. Arrange the integers in descending order.

   (a) $^+1, {}^-2, {}^-8, {}^+9$ (b) $^+10, {}^-10, {}^+24, {}^-5$ (c) $^-3, {}^-5, {}^-1, {}^-8$

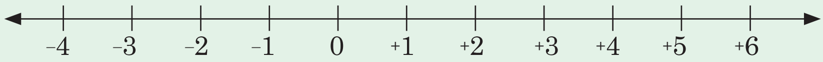
2.3 Addition of integers

(a) Addition of integers using a number line

Activity 2.5

- You need the following materials: white powder or dry loose soil, tape measure, manila paper.
- Make a number line from $^-4$ to $^+6$ on the field.

Steps:



Now use it to work out (–3) + (+4). Follow these steps.

(i) Stand at –3. Move 4 steps to the right. Where do you stop? That is the answer to (–3) + (+4).

(ii) Repeat this for (+4) + (–3). What do you get?

- On a paper, draw the number line you made. Discuss the steps you followed to find your answer.

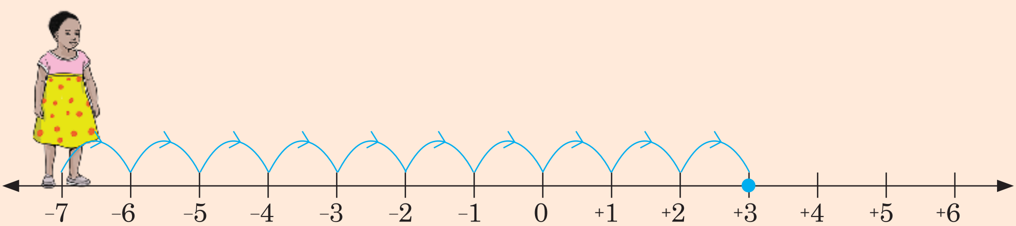
Example 2.4

On a number line, work out (–7) + (+10)

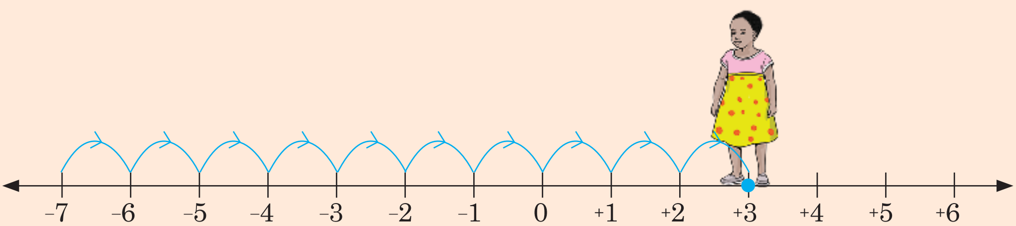
Solution

Stand at –7. Move 10 steps to the right.

Where do you stop?



(Start at –7) You stop at +3



(Stops at +3)

Therefore, (–7) + (+10) = +3

Practice Activity 2.4

Using a number line, work out:

1. (-1) + (+3)
2. (-9) + (+4)
3. (-10) + (+5)
4. (-2) + (-3)
5. (+3) + (-4)
6. (+10) + (-7)

Use a number line to add the following integers. Explain your answer.

7. (-6) + (-2)
8. (+8) + (-2)
9. (-15) + (-12)
10. (-13) + (-1)

(b) Addition of integers without using a number line

Activity 2.6

1. Work out the following without using a number line.
   (a) (-3) + (+4) (b) (+4) + (-3) (c) (-3) + (-4) (d) (+3) + (+4)
2. Try your addition without using a number line.
   (a) (-3) + (+4) (b) (+3) + (-4) (c) (-3) + (-4) (d) (+3) + (+4)
3. From your working in number 1 and 2, which method was easier? Discuss your steps in each case.

Tip:

(i) When adding numbers with the same sign, the answer takes that sign.
For example,
$(^-3) + (^-4) = {}^-(3 + 4) = {}^-7$
$(^+3) + (^+4) = {}^+(3 + 4) = {}^+7$

(ii) When adding numbers with different signs, the answer takes the sign of the larger number.
For example, $(^-3) + (^+4) = {}^+(4 - 3) = {}^+1$
$(^+3) + (^-4) = {}^-(4 - 3) = {}^-1$

Example 2.5

Work out:

$(^-8) + (^-6)$

Solution

$(^-8) + (^-6) = {}^-(8 + 6) = {}^-14$

Practice Activity 2.5

Work out the following.

1. (-8) + (+5)
2. (-6) + (+2)
3. (-20) + (+16)
4. (-2) + (+10)
5. (-3) + (-3)
6. (-12) + (+6)

Work out the following. Explain your steps.

7. (-11) + (+10)
8. (+4) + (+4)
9. (-12) + (+1)
10. (-9) + (+4)

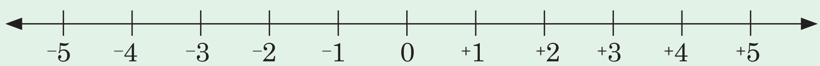
2.4 Subtraction of integers

(a) Subtraction of integers using a number line

Activity 2.7

You need the following materials: white powder or dry loose soil, tape measure, manila paper.

Make a number line from -5 to +5 on the field.



Use your number line to subtract: (a) (-1) – (+3) (b) (-1) – (-3)

Follow these steps:

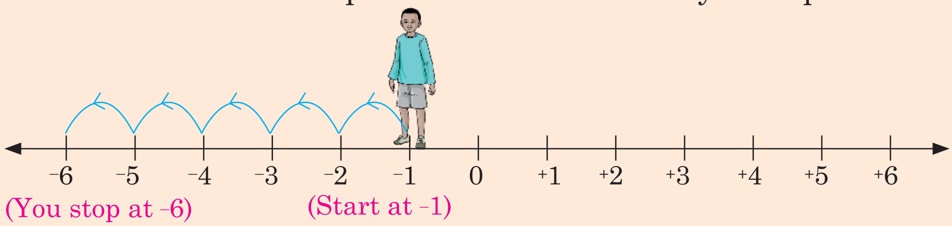
- For (-1) – (+3); start at -1 move 3 steps backwards (to the left), where do you stop?
- For (-1) – (-3); start at -1, move 3 steps backward of backward. Backward of backward results in forward movement (to the right). Where do you stop? Explain your answer.

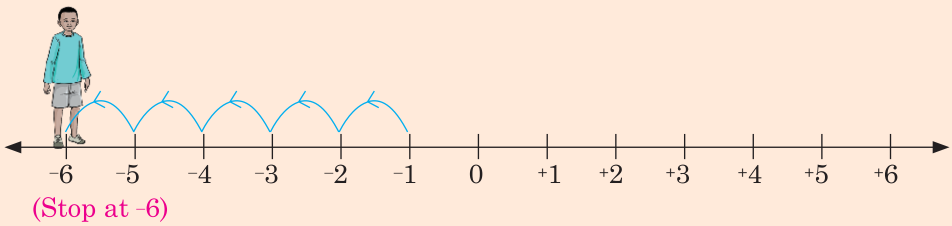
Example 2.6

Using a number line, work out (-1) – (+5).

Solution

Stand at -1. Move 5 steps to the left. Where do you stop?





Therefore (-1) – (+5) = -6

Practice Activity 2.6

Work out the following using a number line.

1. (+8) – (+3)
2. (-6) – (+2)
3. (-8) – (+3)
4. (+7) – (+9)
5. (+7) – (+3)
6. (+9) – (+9)

Use number line to subtract the following. Explain the steps followed.

7. (-4) – (+1)
8. (+2) – (+8)
9. (+4) – (+4)
10. (+13) – (+10)

(b) Subtraction of integers without using a number line

Activity 2.8

Work out the following without using a number line.

1. (-1) – (+3)
2. (-1) – (-3)

Follow these steps:

1. (-1) – (+3) = -(3 + 1)
   Explain your steps.
2. (-1) – (-3) = (-1) + (+3) = 3 – 1

Tip:

Since backward of backward results in forward movement, then

$(^-1) - (^-3) = (^-1) + (^+3) = (^+3) - (^+1)$

Example 2.7

Work out:

(a) $6 - 3$ (b) $(^-6) - (^+3)$ (c) $(^+6) - (^-3)$ (d) $(^-6) - (^-3)$

Solution

We work out as shown below:

(a) $6 - 3 = 3$

(b) $(^-6) - (^+3) = {^-}(6 + 3) = {^-}9$

(c) $(^+6) - (^-3) = 6 + 3 = 9$

(d) Recall that $- {^-}3$ is replaced by $^+3$ (Backward of backward is forward)

Thus $(^-6) - (^-3) = {^-}6 + 3 = {^-}3$

Practice Activity 2.7

Work out the following.

1. $(^+12) - (^+10)$
2. $(^-15) - (^+5)$
3. $(^-7) - (^-3)$
4. $(^-8) - (^+2)$
5. $(^+16) - (^-3)$
6. $(^-11) - (^+6)$

Work out the following. Discuss your answers.

7. $(^+7) - (^+13)$
8. $(^-10) - (^-8)$
9. $(^+9) - (^-4)$
10. $(^-6) - (^-13)$
11. $(^-4) - (^-12)$
12. $(^-2) - (^-3)$

2.5 Additive inverses of numbers

Activity 2.9

Add the following integers.

1. $(^-4) + (^+4) =$
2. $(^-5) + (^+5) =$
3. $(^-6) + (^+6) =$
4. $(^-7) + (^+7) =$
5. $(^-8) + (^+8) =$
6. $(^-9) + (^+9) =$

What do you notice when you add?

State five other positive and negative integers. State their additive inverse. Write them on flash cards. Present your findings.

Tip: For every integer, there is another integer such that the sum of the two integers is zero. The pair of integers whose sum is zero are additive inverses.

Example 2.8

(a) Work out the following.

(i) $(^-2) + (^+2) =$ (ii) $(^-4) + (^+4) =$

(b) Find the additive inverse of: (i) $^-9$ (ii) $^+6$

Solution

(a) (i) $(^-2) + (^+2) = 0$

(ii) $(^-4) + (^+4) = 0$

(b) Additive inverse of (^+a) is (^-a), so that $(^+a) + (^-a) = 0$. So we have:

(i) additive inverse of $^-9$ is $^+9$.

(ii) additive inverse of $^+6$ is $^-6$.

Practice Activity 2.8

Write the additive inverse of the following.

1. $^-3$
2. $^-10$
3. $^-11$
4. $^-14$
5. $^-15$
6. $^+4$

Find additive inverses of the following. Explain your answer.

7. $^+6$
8. $^+8$
9. $^+10$
10. $^+12$
11. $^-7$
12. $^-8$
13. $^+9$
14. $^+8$
15. $^+15$

2.6 Solving problems involving addition and subtraction of integers

Activity 2.10

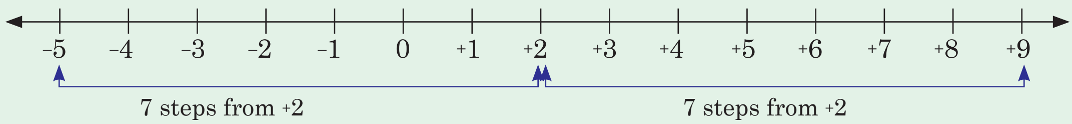
Read the instructions and solve the puzzle. Use the distance between integers to find the position.

I am 7 steps from $^+2$. I am greater than 2.

Draw a number line to show integers.

7 steps from $^{+}2$ is either $^{-}5$ or $^{+}9$.

I am greater than $^{+}2$. Give the answer now.



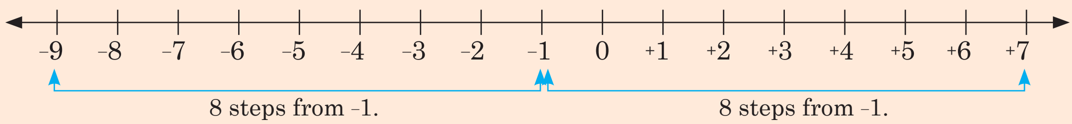
Justify your answer.

Example 2.9

I am 8 steps from $^{-}1$. I am greater than $^{+}6$. Where am I?

Solution

Draw a number line.



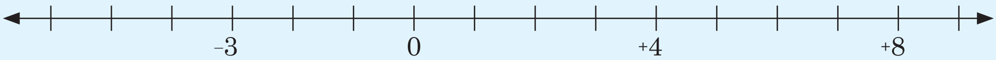
I am either at $^{-}9$ or $^{+}7$. But I am greater than $^{+}6$. We know $^{-}9 < {}^{+}6$ and $^{+}7 > {}^{+}6$. Therefore, I am $^{+}7$.

Practice Activity 2.9

1. I am a positive number. Exactly 11 steps from 0. What am I?
2. Mary is 17 steps from $^{+}7$. She is next to $^{-}9$. Where is she?
3. The temperature of town A is $^{+}15$ °C at noon. In the evening its temperature is $^{+}6$ °C. What is the difference in the temperature?
4. I am a negative number. Exactly 9 steps from $^{-}1$. Where am I?
5. A positive number is 8 steps away from $^{-}3$. It is greater than 4. What is the number?
6. A number is 15 steps from $^{+}10$. The number is less than $^{-}4$. What is the number?

Revision Activity 2

1. Write the missing numbers on the number line.



2. Look at the pairs of integers. Which one is greater? Describe how you got the answer.

   (a) $^-4$ and $^+4$ (b) $^-3$ and $^-9$

   (c) $^+6$ and $^+10$ (d) $^+7$ and $^-5$

3. Use <, > or = to compare the following. Discuss and present your answers.

   (a) $^+5 \ \square \ ^+11$ (b) $^-8 \ \square \ ^-2$

   (c) $^+11 \ \square \ ^-11$ (d) $^-9 \ \square \ ^+4$

4. Use a number line to work out the following.

   (a) $(^+6) + (^+2) =$ (b) $(^-7) - (^+3) =$

   (c) $(^+9) - (^+4) =$ (d) $(^-10) + (^+3) =$

5. Work out the following. Discuss steps to your answers.

   (a) $(^+10) - (^+5) =$ (b) $(^+6) + (^+3) =$

   (c) $(^-10) + (^+4) =$ (d) $(^-6) - (^+3) =$

6. What is the additive inverse of the following integers?

   (a) $^-2$ (b) $^-6$

   (c) $^+7$ (d) $^+5$

7. Jackline is 10 steps away from $^+3$. She is standing on a negative number less than $^-6$. Where is she standing? Justify your answer.

Word list

Integer	Positive integer	Negative integer
Locate	Steps	Compare
Order	Arrange	Distance
Additive inverses		

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 3 Prime factorisation and divisibility tests

3.1 Prime factorisation of numbers and its uniqueness

Activity 3.1

Prime factorise the following numbers. Explain the steps to your answer.

(a) 60 (b) 180

Tip:

A prime number is a number that has only two different factors. That is 1 and itself.

Some examples of prime numbers are 2, 3, 5, 7, 11 and 13. We can write a number using its prime factors.

Look at the following:

Prime factorise 40.

Prime factors	Number
2	40
2	20
2	10
5	5
	1

$40 = 2 \times 2 \times 2 \times 5$

Example 3.1

Prime factorise 30

Solution

Prime factors	Number
2	30
3	15
5	5
	1

$30 = 2 \times 3 \times 5$

Practice Activity 3.1

Write each of the following as product of its prime factors.

1. 40	2. 120	3. 170	4. 80	5. 200

Prime factorise the following numbers. Explain your answer.

6. 320	7. 540	8. 670	9. 560	10. 132
11. 366	12. 144	13. 266	14. 470	15. 920

3.2 Using indices as shorthand for repeated factors

Activity 3.2

Factorise the following numbers. Use indices (or powers) to show repeated prime factors.

(a) Prime factorise 120.

(b) Prime factorise 280. Explain what you have noticed.

Tip:

We can express numbers as products of prime factors. We can use powers or indices on repeated prime factors. For example, prime factorise.

(a) 68 (b) 16.

Express them using indices.

(a)

Prime factors	Number
2	68
2	34
17	17
	1

$68 = 2 \times 2 \times 17$

$= 2^2 \times 17$ (This is because 2×2 is such that 2 is repeated two times)

(b)

Prime factors	Number
2	16
2	8
2	4
2	2
	1

$16 = 2 \times 2 \times 2 \times 2$

$= 2^4$ (This is because 2 is repeated 4 times)

We read this as 2 to the power of 4 or 2 raised to 4.

Example 3.2

Prime factorise 60. Show the prime factors using indices.

Solution

Prime factors	Number
2	60
2	30
3	15
5	5
	1

$60 = 2 \times 2 \times 3 \times 5$

$60 = 2^2 \times 3 \times 5$

2^2 is 2 to the power 2 or 2×2 (2 two times)

Therefore $60 = 2^2 \times 3 \times 5$ has been written using factors in powers/indices.

Now prime factorise 72. Show the prime factors using indices. Explain the steps you followed to arrive at your answer. Present your findings.

Practice Activity 3.2

Prime factorise the following numbers. Show their prime factors using indices (or powers).

1. 27
2. 75
3. 36
4. 76

Prime factorise the following. Express prime factors in indices form and explain.

5. 98
6. 48
7. 25
8. 64
9. 45
10. 106
11. 54
12. 74

3.3 Calculation of the Lowest Common Multiple (LCM)

What is the multiple of a number? When you have two numbers, like 5 and 6, you can list their multiples. There will be a common multiple. The smallest of the common multiples is the Least Common Multiple.

Now, do the following activity.

Activity 3.3

Find the Lowest Common Multiple of;

(a) 3, 9 and 12
(b) 3 , 6 and 9
(c) 3, 4 and 8
(d) 4, 5, and 8

Find some examples where you can apply the LCM to daily life. Discuss your findings.

Example 3.3

(a) What is the Lowest Common Multiple of 6, 12 and 18?

Solution

2	6	12	18
2	3	6	9
3	3	3	9
3	1	1	3
	1	1	1

Steps

1. Divide 6, 12 and 18 by prime numbers. Start with the smallest till all have been reduced to 1.
2. Multiply the prime numbers $2 \times 2 \times 3 \times 3 = 36$.
3. The product is the LCM.

Therefore Lowest Common Multiple of 6, 12 and 18 is 36.

(b) What is the Lowest Common Multiple of 4, 6 and 8?

Solution

In indices form:

$4 = 2 \times 2 = 2^2$

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2 = 2^3$

The LCM of 4, 6, 8 is $2^3 \times 3 = 8 \times 3$

$= 24.$

2^3 has the greatest indices for 2.

Practice Activity 3.3

1. Find the LCM of the following numbers.
   (a) 2, 5 and 10 (b) 5, 6 and 9 (c) 2, 6 and 8
2. Find the LCM of the numbers below. Present your answers.
   (a) 6, 15 and 20 (b) 4, 5 and 10 (c) 3, 4 and 5
3. Find the LCM of the numbers below. Explain the steps to your answer.
   (a) 4, 5 and 12 (b) 4, 6 and 9 (c) 6, 15 and 10
   (d) 12, 18 (e) 10, 15, 9

3.4 Calculation of Greatest Common Factors (GCF)

What is the GCF of 18, 12 and 24? Start dividing with the smallest prime factor that divides all the numbers. Continue dividing until there is no other prime factor that can divide all the numbers.

Prime factors	Numbers		
2	18	12	24
3	9	6	12
	3	2	12

Hint:
There is no common divisor for 3, 2, 12. So we stop division.

Therefore the GCF of 12, 18 and 24 is:

$= 2 \times 3$

$= 6$

Activity 3.4

Find the Greatest Common Factor (GCF) of the following numbers.

(a) 36 and 39 (b) 42 and 48

(c) 9, 18 and 27 (d) 15, 30 and 35

Explain the steps to your answer.

Discuss daily life examples where you use the GCF.

Example 3.4

Find the Greatest Common Factor (GCF) of 28, 42 and 56.

Solution

Method 1

Start dividing by the smallest prime number that divides all the numbers.

Prime factors	Numbers		
2	28	42	56
7	14	21	28
	2	3	4

Therefore GCF is $2 \times 7 = 14$.

Method 2

Express 28, 42 and 56 in indices forms:

$28 = 2 \times 2 \times 7 = 2^2 \times 7$

$42 = 2 \times 3 \times 7$

$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$

Observation on Common factors in indices:

2, 2^2, 2^3 and 7 are common. 3 is not common.

So, GCF is $2 \times 7 = 14$.

We use common factors with lowest indices.

Practice Activity 3.4

Find the Greatest Common Factor (GCF) of the numbers below.

1. 14, 20 and 36
2. 24, 36 and 40
3. 72, 84 and 108
4. 84, 140 and 224
5. 42, 70 and 112
6. 220 and 360

Calculate the GCF of the following. Discuss your steps.

7. 54 and 90
8. 45, 60 and 750
9. 250, 450 and 750
10. 180, 360 and 630

3.5 Divisibility test for 2

Activity 3.5

Divide the following numbers by 2.

(a) 3 241 (b) 573 428 (c) 361 800 (d) 520 042

- Which numbers are divisible by 2? Check their last digits. What do you notice?
- Which numbers are not divisible by 2? Check their last digits. What do you notice?
- What can you say about the last digit of the numbers divisible by 2? Present your findings.

Tip:

A number is divisible by 2 if the last digit is an even number or zero.

Example 3.5

1. Is 49 140 divisible by 2?

Solution

The last digit in 49 140 is 0.

Therefore the number 49 140 is divisible by 2.

2. Test if the following are divisible by 2.

   (a) 90 712 (b) 90 721

Solution

(a) The last digit 2 in 90 712 is an even number.
Therefore the number 90 712 is divisible by 2.

(b) The last digit 1 is an odd even number.
Therefore, 90 721 is not divisible by 2.

Practice Activity 3.5

Which of the following numbers are divisible by 2?

1. 4 480	2. 6 429	3. 5 258
4. 21 224	5. 49 242	6. 15 504
7. 470 881	8. 636 027	9. 36 085

Test and write numbers divisible by 2. Discuss how you found your answers.

10. 52 100	11. 148 516	12. 462 946
13. 90 712	14. 54 213	15. 41 768
16. 87 742	17. 49 112	18. 214 332

3.6 Divisibility test for 3

Activity 3.6

- Divide the following numbers by 3.

(a) 39 (b) 214 (c) 171 (d) 8 811

Find the sum of the digits of each number above. Divide the sum for each number by 3. What do you discover?

Present your findings.

Tip:

A number is divisible by 3 if the sum of its digits is a multiple of 3.

Example 3.6

(a) Test if 1 824 is divisible by 3?

(b) Test if 23 416 is divisible by 3.

Solution

(a) • Add the digits for the number 1 824.
$1 + 8 + 2 + 4 = 15$. Now, $15 \div 3 = 5$. So 15 is divisible by 3.
Therefore, 1 824 is divisible by 3.

(b) • Add the digits for 23 416. We have: $2 + 3 + 4 + 1 + 6 = 16$. Now $16 \div 3 = 5$ with remainder of 1.
So, 16 is not divisible by 3.
Therefore, 23 416 is not divisible by 3.

Practice Activity 3.6

Test and give the numbers that are divisible by 3. Explain the steps to your answers.

1. 1 836	2. 5 613	3. 9 786
4. 6 123	5. 56 004	6. 23 112
7. 62 172	8. 456 312	9. 214 701
10. 306 171	11. 178 123	12. 363 114
13. 100 456	14. 690 390	15. 120 300

3.7 Divisibility test for 4

Activity 3.7

- Divide the following numbers by 4.
  (a) 2 472 (b) 2 814 (c) 17 936
  Which of them are divisible by 4?
- Test whether the last 2 digits of each number is divisible by 4 or not. What do you notice?
  Present your findings.

Tip: A number is divisible by 4 if the last 2 digits form a number divisible by 4, or the 2 last digits are 00.

Example 3.7

(a) Is 456 312 divisible by 4?
(b) Is 106 526 divisible by 4?

Solution

(a) The last 2 digits of 456 312 forms 12. Now, $12 \div 4 = 3$.
So, 12 is divisible by 4.
Therefore, 456 312 is divisible by 4.

(b) The last 2 digits of 106 526 forms 26. Now, $26 \div 4 = 6$ with remainder of 2. So, 26 is not divisible by 4.
Therefore, 106 526 is not divisible by 4.

Practice Activity 3.7

Test which of these numbers are divisible by 4.

1. 839 016
2. 7 936
3. 49 424
4. 873 008
5. 990 004
6. 182 510
7. 52 850
8. 91 044
9. 41 928

Test for numbers divisible by 4. Discuss your steps.

10. 3 148
11. 98 541
12. 83 710
13. 426 940
14. 201 084
15. 390 712

3.8 Divisibility test for 5

Activity 3.8

Divide the following numbers by 5.

(a) 99 000 (b) 27 435 (c) 47 861 (d) 78 390

Which numbers are divisible by 5? Check their last digit.

Which numbers are not divisible by 5? Check their last digit.

What do you notice about the last digit of numbers divisible by 5?

Discuss your findings.

Tip:

A number is divisible by 5 if its last digit is 0 or 5.

Example 3.8

Which of the following numbers is divisible by 5?

(a) 56 480 (b) 225 445 (c) 741 024

Solution

(a) 56 480 has the last digit 0. Therefore, 56 480 is divisible by 5.

(b) 225 445 has the last digit 5. Therefore, 225 445 is divisible by 5.

(c) 741 024 has the last digit 4. Therefore, 741 024 is not divisible by 5.

Practice Activity 3.8

Test to find the numbers are divisible by 5.

1. 487 200
2. 578 425
3. 140 265
4. 859 420
5. 718 426
6. 419 347

Test for numbers divisible by 5. Explain your steps.

7. 736 920
8. 878 945
9. 572 315
10. 640 635
11. 670 670
12. 654 285
13. 563 759
14. 410 458
15. 369 000

3.9 Divisibility test for 6

Activity 3.9

Look at the numbers below.

(a) 336 (b) 690 (c) 4 878 (d) 194 (e) 736

Divide the numbers by 2.
Divide the numbers by 3 again.
Divide the same numbers by 6.
What do you notice about the numbers?
Discuss your findings.

Tip:

A number is divisible by 6 if it is also divisible by 2 and 3.

Example 3.9

Which of the numbers below is divisible by 6? Explain your steps.

(a) 2 700 (b) 458 716

Solution

(a)
- The last digit for 2 700 is 0. So 2 700 is divisible by 2.
- 2 + 7 + 0 + 0 = 9. The sum of the digits of 2 700 is 9. So 9 is divisible by 3. Therefore, 2 700 is divisible by 3.
- Finally, 2 700 is divisible by 6.

(b)
- The last digit of 458 716 is 6. Now, 6 is an even number. Thus, 458 716 divisible by 2.
- 4 + 5 + 8 + 7 + 1 + 6 = 31. The sum of the digits of 458 716 is 31. Now, 31 ÷ 3 = 10 reminder 1, or 31 is not divisible by 3. Thus, 458 716 is not divisible by 3.
- Finally, 458 716 is not divisible by 6.

Practice Activity 3.9

Test and give numbers that are divisible by 6.

1. 70 032	2. 54 451	3. 46 008
4. 82 092	5. 14 256	6. 85 728

Test to find numbers divisible by 6. Discuss your steps.

7. 458 710	8. 51 200	9. 216
10. 144	11. 928	12. 93 621
13. 3 759	14. 48 780	15. 56 800

3.10 Divisibility test for 8

Activity 3.10

- Divide the numbers below by 8.

(a) 5 328 (b) 17 428 (c) 93 640

- Now form a number from the last three digits of each number. Divide your number by 8. What do you notice? Explain your observations.

Tip: A number is divisible by 8 if the last three digits form a number divisible by 8, or are 000 .

Example 3.10

Investigate for the numbers that are divisible by 8.

(a) 404 320 (b) 200 072 (c) 323 638

Solution

Check if the number formed by the last 3 digits is divisible by 8.

(a) From 404 320, the last digits form 320. Now 320 ÷ 8 = 40. Since 320 is divisible by 8, thus 404 320 is divisible by 8.

(b) From 202 072, the last 3 digits form 072. Now 072 ÷ 8 = 9. Since 072 is divisible by 8, thus 202 072 is divisible by 8.

(c) From 323 638, the last 3 digits form 638. Now 638 ÷ 8 = 79 with remainder of 6, is not divisible by 8. Thus, 323 638 is not divisible by 8.

Practice Activity 3.10

Test and give the numbers that are divisible by 8.

1. 842 056
2. 300 400
3. 642 323
4. 374 816
5. 322 642
6. 138 648
7. 183 257
8. 768 265
9. 543 120

Test and write the numbers that are divisible by 8. Explain your steps.

10. 679 168
11. 217 800
12. 436 756
13. 374 912
14. 276 480
15. 248 263

3.11 Divisibility Test for 9

Activity 3.11

- Divide these numbers by 9.
  (a) 8 109 (b) 2 916 (c) 20 007 (d) 108 450
- Add the digits of the numbers given above.
  Divide the sum of the digits by 9. Are they all divisible by 9?
- What do you notice about numbers divisible by 9? Present your findings.

Tip:

A number is divisible by 9 if the sum of its digits forms a number divisible by 9.

Example 3.11

Which of the following numbers is divisible by 9?

(a) 64 737 (b) 607 131 (c) 128 000

Solution

Step 1: Add the digits of the numbers.

(a) 64 737 : 6 + 4 + 7 + 3 + 7 = 27

(b) 607 131 : 6 + 0 + 7 + 1 + 3 + 1 = 18

(c) 128 000 : 1+ 2+ 8+ 0 + 0 + 0 = 11

Step 2: Divide the sum by 9. State which numbers are divisible by 9.

(a) 27 ÷ 9 = 3. Therefore 64 737 is divisible by 9.

(b) 18 ÷ 9 = 2. Therefore 607 131 is divisible by 9.

(c) 11 ÷ 9 = 1 with remainder of 2. Therefore 128 000 is not divisible by 9.

Practice Activity 3.11

1. Test and write the numbers that are divisible by 9.
   (a) 98 541 (b) 49 041 (c) 903 132
   (d) 383 121 (e) 394 020 (f) 42 568
   (g) 34 679 (h) 721 800 (i) 530 280
2. Test and write the numbers divisible by 9. Discuss your answer.
   (a) 713 610 (b) 819 234 (c) 999 045
   (d) 515 230 (e) 304 133

3.12 Divisibility test for 10

Activity 3.12

Divide the following numbers by 10.

(a) 8 730 (b) 6 940 (c) 5 285 (d) 94 000 (e) 20 184

Which numbers are divisible by 10?

Which numbers are not divisible by 10?

Check the numbers that are not divisible by 10 again? What are their last digits? Discuss your observations.

Tip:

A number is divisible by 10 if it ends with 0.

Example 3.12

Which of the following numbers are divisible by 10?

(a) 49 140 (b) 199 000 (c) 447 861 (d) 872 930

Solution

The numbers with a last digit of 0 are:

(a) 49 140 (b) 199 000 and (d) 872 930

Therefore 49 140, 199 000, 872 930 are divisible by 10.

(c) 447 861 is not divisible by 10. It ends with 1.

Practice Activity 3.12

1. Which of the following numbers are divisible by 10?
   (a) 1 000 000 (b) 405 330 (c) 555 355
   (d) 725 660 (e) 554 740
2. Test which numbers are divisible by 10. Discuss and present your findings.
   (a) 874 930 (b) 582 140 (c) 529 900
   (d) 81 420 (e) 793 004
3. List five numbers that are divisible by 10.
4. Workers unloaded a lorry with 50 000 books. The books are to be shared by 10 schools. How many books did each school get?

3.13 Divisibility test for 11

Look at 2 463. The digits 4, 3 are alternate. Similarly, 2 and 6 are alternate digits. Let us do the activity below.

Activity 3.13

- Get the sums of the alternate digits in each of the following. Then find their differences.

  (a) 3 190 (b) 3 465 (c) 2 376 (d) 18 931
- Divide each of the numbers by 11. Check the difference of alternate digits for those numbers divisible by 11. Present your findings.

Tip: If the difference of the sums of alternate digits is 0, 11 or a multiple of 11, then the number is divisible by 11.

Example 3.13

1. Is 23 760 divisible by 11?

Solution

- Add alternate digits: $(2 + 7 + 0) = 9$ and $(3 + 6) = 9$
- Find their difference $9 - 9 = 0$

  Difference is 0. Therefore, 23 760 is divisible by 11.

2. Is 934 010 divisible by 11?

Solution

- Add alternate digits: $(9 + 4 + 1) = 14$

  $(3 + 0 + 0) = 3$

  Find their difference

  $14 - 3 = 11$

  Difference is 11. Therefore, 934 010 is divisible by 11.

3. Is 575 814 divisible by 11?

Solution

- Add alternate digits: $(5 + 5 + 1) = 11$

  $(7 + 8 + 4) = 19$
- Find their difference

  $19 - 11 = 8$. The difference 8 is not divisible by 8.

  Therefore, 575 814 is not divisible by 11.

Practice Activity 3.13

1. Test which of the numbers below is divisible by 11?
   (a) 469 246 (b) 329 856
   (c) 986 832 (d) 912 857
2. Write the numbers that are divisible by 11. Discuss your test.
   (a) 102 762 (b) 105 820
   (c) 862 211 (d) 422 939
3. Test for numbers divisible by 11 from below. Explain your steps.
   (a) 352 274 (b) 329 835
   (c) 422 940 (d) 9 625

3.14 Divisibility test for 12

Activity 3.14

- Divide the following numbers by 12.
  (a) 1 524 (b) 1 320 (c) 3 936 (d) 2 544 (e) 5 076
  Divide each number by 3. Divide each of the numbers by 4.
  Are all the numbers divisible by 12, 3 and 4?
- Which numbers are divisible by both 3 and 4? Which numbers are divisible by 12? Discuss your findings.

Tip:

A number is divisible by 12 if it is divisible by both 3 and 4.

Example 3.14

Test and state the number that is divisible by 12.
182 844 or 644 346

Solution

Hint: Carry out divisibility tests for both 3 and 4.

Step 1: Divisibility test for 3. Add the digits and divide by 3.

- 182 844: $1 + 8 + 2 + 8 + 4 + 4 = 27$. Now $27 \div 3 = 9$.
  Thus, 182 844 is divisible by 3.

- 644 346 $\longrightarrow$ 6 + 4 + 4 + 3 + 4 + 6 = 27. Now 27 ÷ 3 = 9. Thus, 644 346 is divisible by 3.

Step 2: Divisibility test for 4. Divide the number formed by the last 2 digits of each number by 4.

- From 182 844; we have 44 ÷ 4 = 11. So 182 844 is divisible by 4.
- From 644 346; 46 ÷ 4 = 11 with remainder of 2. Therefore, 46 is not divisible by 4.

  So 644 346 is not divisible by 4.

Step 3: Conclusion: A number divisible by 12.

- From Steps 1 and 2, 182 844 is divisible by both 3 and 4. Therefore, 182 844 is divisible by 12.
- From Steps 1 and 2, 644 346 is divisible by 3 and not by 4. Thus, 644 346 is not divisible by 12.

Practice Activity 3.14

Find the numbers that are divisible by 12.

1. 3 360
2. 2 724
3. 9 684
4. 8 676
5. 89 184
6. 58 968
7. 39 300
8. 26 716
9. 541 656

Test and write the numbers divisible by 12. Discuss your steps.

10. 933 216
11. 753 072
12. 665 580
13. 582 100
14. 403 560

Revision Activity 3

1. Prime factorise the numbers below using indices.
   (a) 240 (b) 300 (c) 1 000
2. Find the Lowest Common Multiple of the following.
   (a) 6, 9 and 12 (b) 4, 8 and 10
   (c) 8, 10 and 1 (d) 10, 12 and 15
3. Find the Greatest Common Factor of the following. Explain your answer.
   (a) 48, 40 and 72 (b) 100, 120 and 150

4. Identify the numbers divisible by 2 below.
   (a) 649 426 (b) 241 233 (c) 792 400
5. Which of the following numbers are divisible by 3?
   (a) 300 012 (b) 400 560 (c) 450 106
6. Name the numbers that are divisible by 4. Present your answers.
   (a) 480 120 (b) 820 440 (c) 541 610
7. Which numbers are divisible by 5?
   (a) 400 255 (b) 426 451 (c) 728 400
8. Identify the numbers that are divisible by 6. Explain your answers.
   (a) 403 560 (b) 67 260 (c) 2 724
9. Name the numbers divisible by 8. Discuss your steps.
   (a) 868 562 (b) 480 240 (c) 976 861
10. Which of the numbers below is divisible by 9?
   (a) 810 720 (b) 820 503 (c) 413 333
11. Which of the numbers is divisible by 10?
   (a) 716 300 (b) 633 420 (c) 660 855
12. Name the numbers divisible by 11. Explain your answers.
   (a) 467 181 (b) 891 484 (c) 541 656
13. Which of the following numbers are divisible by 12? Discuss your steps.
   (a) 891 480 (b) 556 680 (c) 497 185

Word list

Prime factorisation	Divisible
Prime numbers	Indices (powers)
Lowest Common Multiple	Greatest Common Factor
Divisibility tests	Natural numbers

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 4

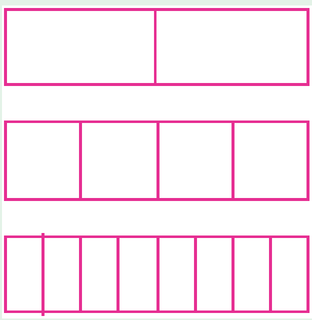
Equivalent fractions and operations

4.1 Concept of equivalent fractions using models

Activity 4.1

- Cut three strips of paper as shown below.
  They are the same sizes.

- Fold paper A into two equal parts.
- Fold paper B into four equal parts.
- Fold paper C into eight equal parts.
- Open them and draw lines along the folds.



A — Shade $\frac{1}{2}$ in A

B — Shade $\frac{2}{4}$ in B

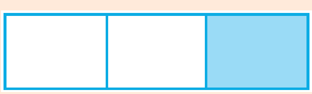
C — Shade $\frac{4}{8}$ in C

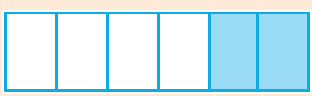
- Compare the sizes of shaded parts in A, B and C. What do you notice? Present your findings.

Example 4.1

Shade $\frac{1}{3}$ and $\frac{2}{6}$ on paper cutouts of the same size. Compare $\frac{1}{3}$ and $\frac{2}{6}$.

Solution


$\frac{1}{3}$


$\frac{2}{6}$

From the diagram $\frac{1}{3} = \frac{2}{6}$

Thus $\frac{2}{6}$ is equivalent to $\frac{1}{3}$. We write as $\frac{1}{3} = \frac{2}{6}$.

Practice Activity 4.1

Shade the equivalent fractions below.

1. $\frac{1}{5}$ $\frac{\quad}{10}$

2. $\frac{1}{4}$ $\frac{\quad}{8}$

3. $\frac{1}{6}$ $\frac{\quad}{12}$

4. $\frac{1}{8}$ $\frac{\quad}{16}$

Shade the equivalent fractions. Explain your steps.

5. $\frac{1}{7}$ $\frac{\quad}{14}$

6. $\frac{1}{2}$ $\frac{\quad}{6}$

7. $\frac{1}{9}$ $\frac{\quad}{18}$

8. $\frac{2}{8}$ $\frac{\quad}{16}$

Activity 4.2

Cut three strips of papers. They should be the same size.

A Fold paper A into three equal parts.

B Fold paper B into six equal parts.

C Fold paper C into twelve equal parts.

Open then draw lines on the folds.

A Shade $\frac{2}{3}$

B Shade $\frac{4}{6}$

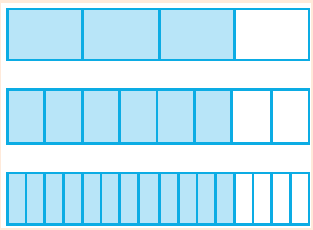
C Shade $\frac{8}{12}$

Compare the sizes of the shaded parts. What do you notice? Discuss your finding.

Example 4.2

Shade $\frac{3}{4}$, $\frac{6}{8}$ and $\frac{12}{16}$ on same size of paper cutouts. Compare the size of shaded parts. What do you notice?

Solution



$\frac{3}{4}$

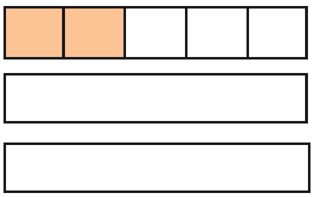
$\frac{6}{8}$

$\frac{12}{16}$

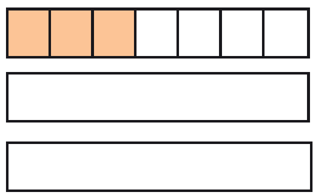
- The shaded parts are equal sizes.
- Thus, $\frac{3}{4}$, $\frac{6}{8}$ and $\frac{12}{16}$ are equivalent fractions. $\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$

Practice Activity 4.2

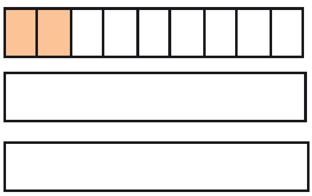
Shade two more equivalent fractions in each case.

1. 

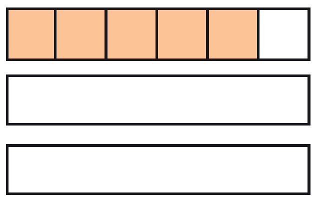
$\frac{2}{5}$, $\frac{?}{10}$, $\frac{?}{15}$

2. 

$\frac{3}{7}$, $\frac{?}{14}$, $\frac{?}{21}$

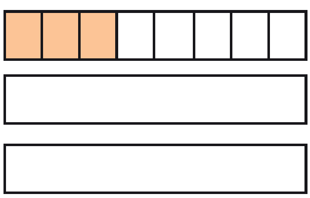
3. 

$\frac{2}{9}$, $\frac{?}{18}$, $\frac{?}{27}$

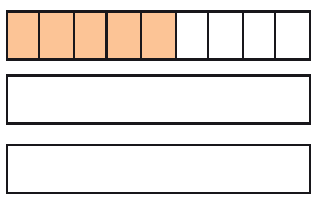
4. 

$\frac{5}{6}$, $\frac{?}{12}$, $\frac{?}{18}$

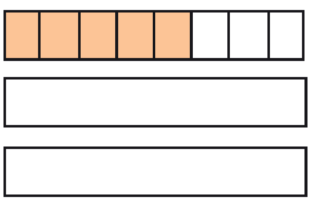
Shade two more equivalent fractions in each case. Discuss the steps to your answers.

5. 

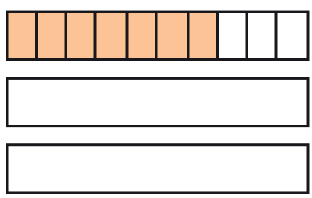
$\frac{3}{8}$, $\frac{?}{16}$, $\frac{?}{24}$

6. 

$\frac{5}{9}$, $\frac{?}{18}$, $\frac{?}{27}$

7. 

$\frac{5}{8}$, $\frac{?}{16}$, $\frac{?}{24}$

8. 

$\frac{7}{10}$, $\frac{?}{20}$, $\frac{?}{30}$

Activity 4.3

- Draw and shade $\frac{2}{3}$ in A. Shade the equivalent fraction in B. Make paper cutouts and compare their sizes.

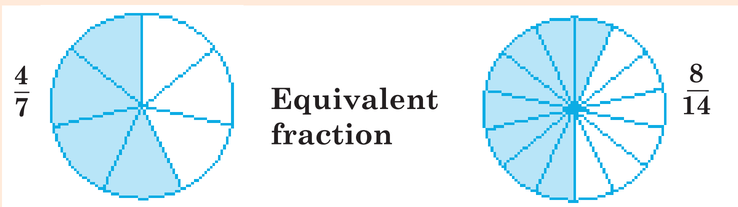
- Use circular models like those above to find equivalent fractions.
  (i) $\frac{3}{4}$ (ii) $\frac{4}{5}$ Present your findings.

Example 4.3

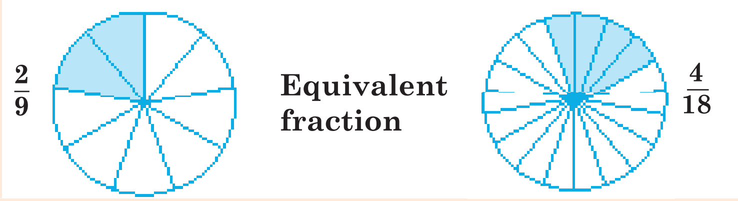
Using models show equivalent fractions of: (a) $\frac{4}{7}$ (b) $\frac{2}{9}$.

Solution

(a)



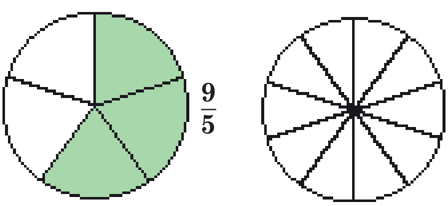
(b)



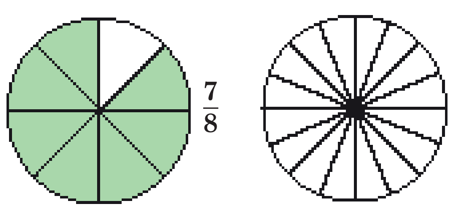
Practice Activity 4.3

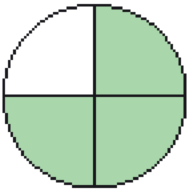
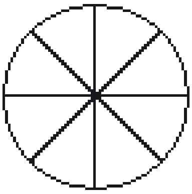
Shade the equivalent fraction of each of the fractions given below.

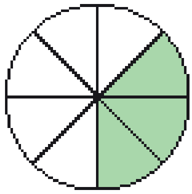
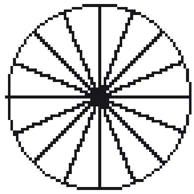
1.



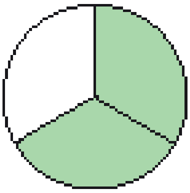
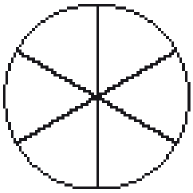
2.

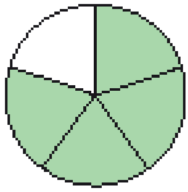
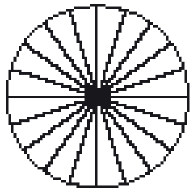


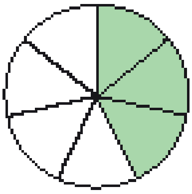
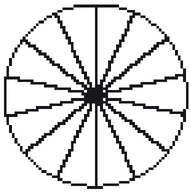
3.  $\frac{3}{4}$ 

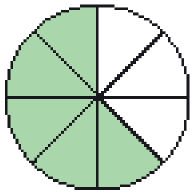
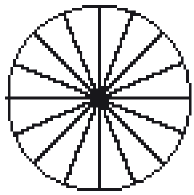
4.  $\frac{3}{8}$ 

Shade the equivalent fraction in each case. Explain your answer.

5.  $\frac{2}{3}$ 

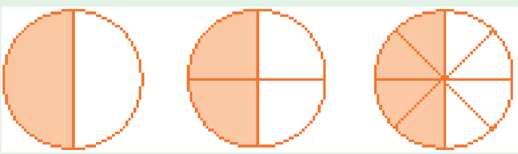
6.  $\frac{4}{5}$ 

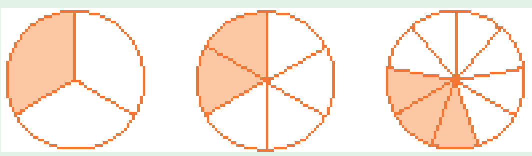
7.  $\frac{3}{7}$ 

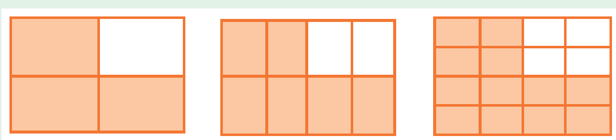
8.  $\frac{5}{8}$ 

Activity 4.4

Write the shaded fractions. In each case, are they equivalent? Explain your answer.

(a) 

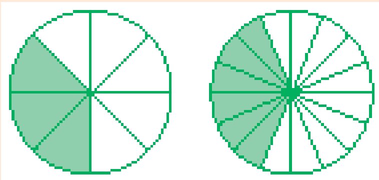
(b) 

(c) 

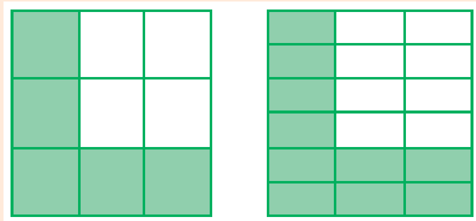
Example 4.4

Write the shaded fraction and its equivalent fraction.

(a)



(b)



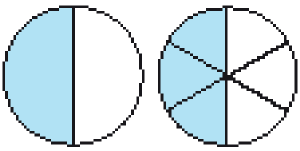
Solution

(a) $\frac{3}{8} = \frac{6}{16}$ (b) $\frac{5}{9} = \frac{10}{18}$

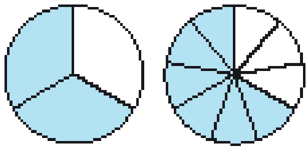
Practice Activity 4.4

A. Write the equivalent fractions for the models below.

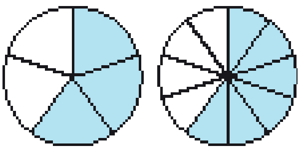
1.



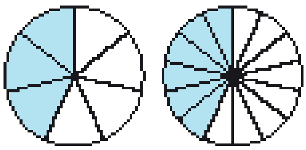
2.



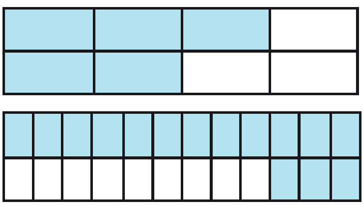
3.



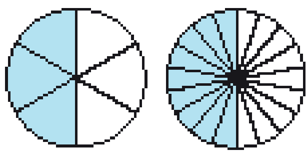
4.



5.

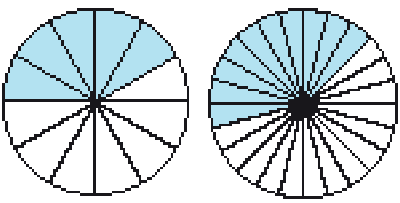


6.

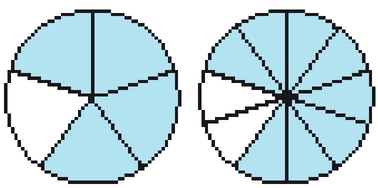


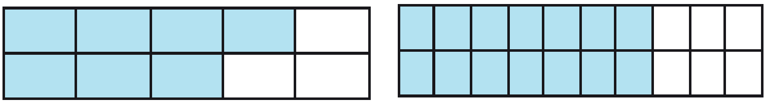
B. Write the equivalent fractions for the models. Discuss your answer.

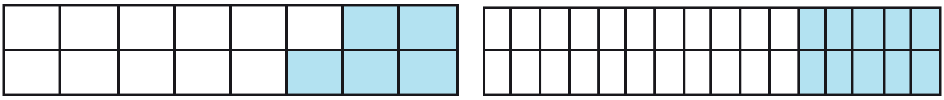
1.



2.

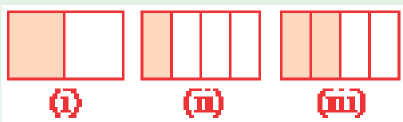


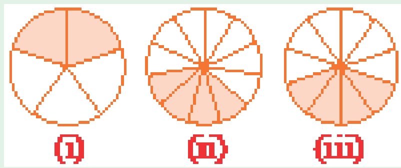
3. 

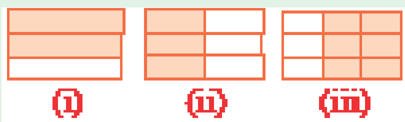
4. 

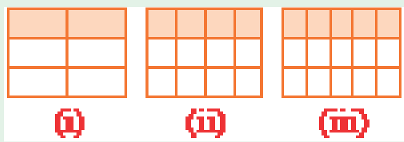
Activity 4.5

Identify the shaded parts showing equivalent fractions. Write the equivalent fractions. Discuss and present your findings.

(a) 

(b) 

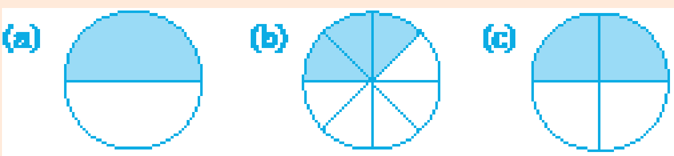
(c) 

(d) 

Example 4.5

(i) Which of the shaded parts show equivalent fractions?

(ii) Write the equivalent fractions.



Solution

(i) The shaded parts in (a) and (c) are equal.

(ii) The fraction in (a) is $\frac{1}{2}$. The fraction in (c) is $\frac{2}{4}$.

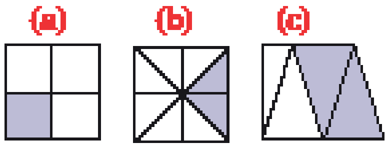
The equivalent fractions shown are $\frac{1}{2} = \frac{2}{4}$.

Practice Activity 4.5

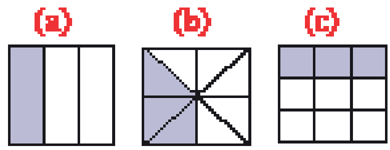
(i) Which shaded parts show equivalent fractions in each case?

(ii) Write the equivalent fractions in each case. Discuss your answer.

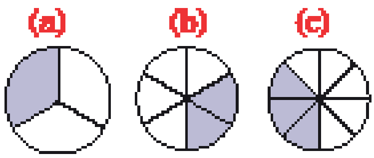
1.



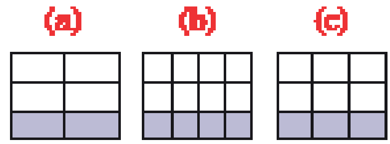
2.



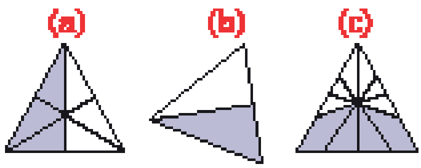
3.



4.



5.



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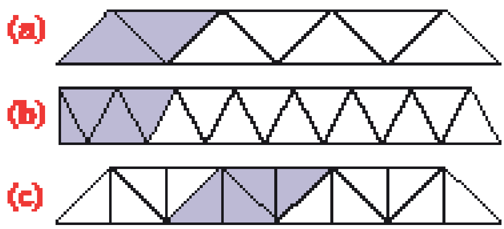
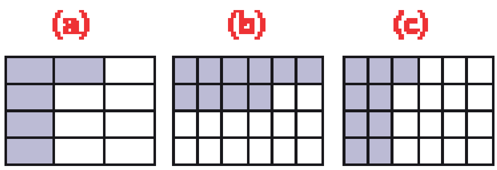


7.



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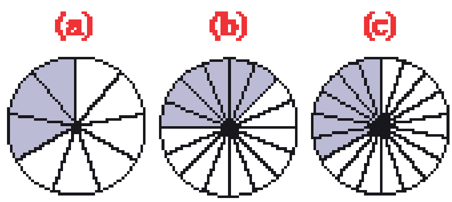
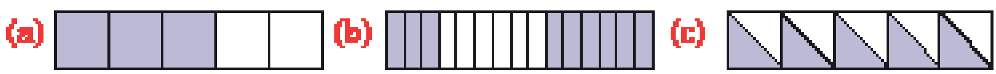
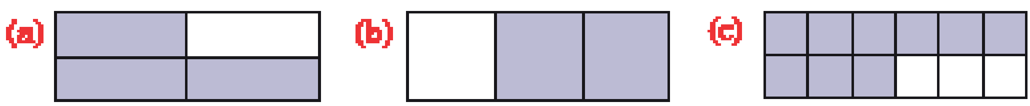


9.



10.



4.2 Calculation of equivalent fractions

Activity 4.6

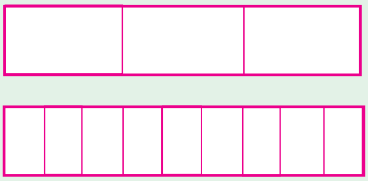
- Multiply $\frac{2}{3}$ by $\frac{3}{3}$ as $\frac{2}{3} \times \frac{3}{3} = \frac{2 \times 3}{3 \times 3} =$

  What fraction do you get?

  Use the same size paper cutouts to shade.

  $\frac{2}{3}$

  $\frac{6}{9}$

  Compare $\frac{2}{3}$ and $\frac{6}{9}$. Are they equivalent or not?

- Repeat the same steps to find equivalent fractions of:

  (a) $\frac{4}{5}$ (b) $\frac{5}{6}$ (c) $\frac{3}{8}$ Explain your answer.

- Where do we use equivalent fractions in daily life?

Tip:

- To find the equivalent fraction, multiply both denominator and numerator by a whole number.
- A whole number to use include 2, 3, 4, 5, … If you multiply by 1, you get the same fraction.

Example 4.6

Write 3 equivalent fractions of $\frac{3}{7}$.

Solution

Multiply by $\frac{2}{2}$: $\frac{3}{7} \times \frac{2}{2} = \frac{6}{14}$ Multiply by $\frac{3}{3}$: $\frac{3}{7} \times \frac{3}{3} = \frac{9}{21}$

Multiply by $\frac{4}{4}$: $\frac{3}{7} \times \frac{4}{4} = \frac{12}{28}$

Therefore $\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{12}{28}$

Three equivalent fractions of $\frac{3}{7}$ are $\frac{6}{14}$, $\frac{9}{21}$, $\frac{12}{28}$ among others.

Practice Activity 4.6

Find the equivalent fractions of the given fraction. Then fill in the missing blanks and explain your answer.

1. $\frac{1}{4} = \frac{\square}{52}$
2. $\frac{2}{3} = \frac{\square}{24}$
3. $\frac{3}{10} = \frac{\square}{50}$
4. $\frac{5}{6} = \frac{\square}{54}$
5. $\frac{\square}{7} = \frac{16}{28}$
6. $\frac{\square}{9} = \frac{27}{81}$
7. $\frac{\square}{50} = \frac{34}{100}$
8. $\frac{1}{11} = \frac{3}{\square}$
9. $\frac{3}{9} = \frac{\square}{27}$
10. $\frac{7}{10} = \frac{21}{\square}$
11. $\frac{4}{9} = \frac{16}{\square}$
12. $\frac{2}{3} = \frac{\square}{12}$
13. $\frac{2}{\square} = \frac{4}{6}$
14. $\frac{7}{8} = \frac{\square}{32}$
15. $\frac{3}{4} = \frac{\square}{12}$

Activity 4.7

Find two equivalent fractions for each fraction below. Justify your answer.

(i) $\frac{4}{5}$ (ii) $\frac{4}{7}$ (iii) $\frac{4}{9}$ (iv) $\frac{27}{81}$

Example 4.7

Find two equivalent fractions for: (a) $\frac{4}{11}$ (b) $\frac{5}{9}$

Solution

(a) Multiply $\frac{4}{11}$ by $\frac{2}{2}$ to find the first one. Multiply $\frac{4}{11}$ by $\frac{3}{3}$ to find the next fraction.

- $\frac{4}{11} \times \frac{2}{2} = \frac{8}{22}$
- $\frac{4}{11} \times \frac{3}{3} = \frac{12}{33}$

Two equivalent fractions of $\frac{4}{11}$ are $\frac{8}{22}$ and $\frac{12}{33}$.

(b) Multiply $\frac{5}{9}$ by $\frac{2}{2}$ to find the first one. Multiply $\frac{5}{9}$ by $\frac{3}{3}$ to find next one.

- $\frac{5}{9} \times \frac{2}{2} = \frac{10}{18}$
- $\frac{5}{9} \times \frac{3}{3} = \frac{15}{27}$

Two equivalent fractions of $\frac{5}{9}$ are $\frac{10}{18}$ and $\frac{15}{27}$.

Practice Activity 4.7

A. Find two equivalent fractions for the following fractions.

1. $\frac{5}{8}$
2. $\frac{3}{7}$
3. $\frac{2}{3}$
4. $\frac{6}{11}$
5. $\frac{3}{10}$

B. Find two equivalent fractions for the following fractions. Discuss and present your findings.

1. $\frac{7}{9}$
2. $\frac{8}{12}$
3. $\frac{3}{5}$
4. $\frac{6}{7}$
5. $\frac{9}{13}$
6. $\frac{6}{9}$
7. $\frac{4}{6}$

Activity 4.8

Find three equivalent fractions for each of the following:

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{5}{6}$ (d) $\frac{5}{9}$

Discuss the steps you have followed to calculate them.

Example 4.8

Find three equivalent fractions for (a) $\frac{3}{10}$ (b) $\frac{4}{7}$

Solution

- Multiply both numerator and denominator by 2. Repeat steps with 3 then with 4.
  This is the same multiply each fraction by $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}$ among others.

(a) • $\frac{3}{10} \times \frac{2}{2} = \frac{6}{20}$ • $\frac{3}{10} \times \frac{3}{3} = \frac{9}{30}$ • $\frac{3}{10} \times \frac{4}{4} = \frac{12}{40}$

Some equivalent fractions of $\frac{3}{10}$ are $\frac{6}{20}, \frac{9}{30}, \frac{12}{40}$. Find other examples.

(b) • $\frac{4}{7} \times \frac{2}{2} = \frac{8}{14}$ • $\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$ • $\frac{4}{7} \times \frac{4}{4} = \frac{16}{28}$

Some equivalent fractions of $\frac{4}{7}$ are $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}$. Find more examples.

Practice Activity 4.8

A. Find three equivalent fractions for each of the following.

1. $\frac{1}{5}$ 2. $\frac{8}{15}$ 3. $\frac{5}{9}$ 4. $\frac{5}{12}$ 5. $\frac{5}{6}$

B. Write three equivalent fractions for each of the following. Explain your answer.

1. $\frac{3}{8}$ 2. $\frac{4}{5}$ 3. $\frac{3}{4}$

C. Find three equivalent fractions for the following. Discuss and present your findings.

1. $\frac{7}{16}$ 2. $\frac{1}{2}$ 3. $\frac{5}{8}$ 4. $\frac{7}{8}$

Activity 4.9

- Find the equivalent fractions for
  (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$
- From your answers, write equivalent fractions with a denominator of 12 for $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

  What is the easiest way to find your answer? Justify your answer.

Example 4.9

Change $\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{4}$ to equivalent fractions with the denominator 20.

Solution

We find equivalent fractions with a denominator of 20 as follows:

- For $\frac{1}{2}$, we have $20 \div 2 = 10$. So the equivalent fraction for $\frac{1}{2} \times \frac{10}{10} = \frac{10}{20}$
- For $\frac{1}{5}$, we have $20 \div 5 = 4$. So the equivalent fraction for $\frac{1}{5} \times \frac{4}{4} = \frac{4}{20}$
- For $\frac{1}{10}$, we have $20 \div 10 = 2$. So the equivalent fraction for $\frac{1}{10} \times \frac{2}{2} = \frac{2}{20}$
- For $\frac{1}{4}$, we have $20 \div 4 = 5$. So the equivalent fraction for $\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$

Practice Activity 4.9

1. Find the equivalent fraction with the denominator 30 for the following.

   (a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{15}$

2. Find the equivalent fraction with the denominator 48 for the fractions below. Then explain your answer.

   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{96}$ (e) $\frac{1}{8}$

3. Change the fractions below so that their denominators are 60. Discuss and present your findings.

   (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{20}{600}$ (e) $\frac{10}{120}$ (f) $\frac{7}{15}$

4.3 Addition of fractions with different denominators using equivalent fractions

Activity 4.10

- Find the equivalent fraction with the denominator 6 for $\frac{2}{3}$.
- Find the equivalent fraction with the denominator 8 for $\frac{1}{4}$.
- Now, use your results to add the following. Discuss your findings.

(a) $\frac{2}{3} + \frac{1}{6} = \frac{\square}{6} + \frac{1}{6} = \frac{\square + 1}{6} =$ (b) $\frac{1}{4} + \frac{3}{8} = \frac{\square}{8} + \frac{3}{8} = \frac{\square + 3}{8} =$

Example 4.10

Add: $\frac{1}{3} + \frac{4}{9}$

Solution

Denominators 3 and 9 are different. We change $\frac{1}{3}$ into an equivalent fraction with 9 as its new denominator.

- $\frac{1}{3} = \frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$

Thus, $\frac{1}{3} + \frac{4}{9} = \frac{3}{9} + \frac{4}{9} = \frac{3+4}{9} = \frac{7}{9}$.

Practice Activity 4.10

Fill in the missing numbers.

1. $\frac{1}{10} + \frac{3}{5} = \frac{1}{10} + \frac{\square}{10} = \frac{1 + \square}{10} =$

2. $\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{\square}{8} = \frac{3 + \square}{8} =$

3. $\frac{2}{3} + \frac{1}{9} = \frac{\square}{9} + \frac{1}{9} = \frac{\square + 1}{9} =$

4. $\frac{1}{5} + \frac{3}{10} = \frac{\square}{10} + \frac{3}{10} = \frac{\square + 3}{10} =$

Fill in the missing numbers. Discuss the steps you have followed.

5. $\frac{3}{7} + \frac{1}{14} = \frac{\square}{14} + \frac{1}{14} = \frac{\square + 1}{14} =$

6. $\frac{1}{2} + \frac{2}{8} = \frac{\square}{8} + \frac{2}{8} = \frac{\square + 2}{8} =$

7. $\frac{3}{4} + \frac{1}{8} = \frac{\square}{8} + \frac{1}{8} = \frac{\square + 1}{8} =$

8. $\frac{1}{3} + \frac{1}{6} = \frac{\square}{6} + \frac{1}{6} = \frac{\square + 1}{6} =$

9. $\frac{2}{3} + \frac{3}{9} = \frac{\square}{9} + \frac{3}{9} = \frac{\square + 3}{9} =$

10. $\frac{3}{10} + \frac{1}{5} = \frac{3}{10} + \frac{\square}{10} = \frac{3 + \square}{10} =$

Activity 4.11

- Find the equivalent fraction with the denominator 6 for $\frac{1}{2}$. Now use an equivalent fraction to add $\frac{1}{2} + \frac{1}{6}$. Justify your answer.
- Find the equivalent fraction with the denominator 10 for $\frac{1}{2}$. Then add $\frac{1}{2} + \frac{1}{10}$. Explain the steps you followed.

Example 4.11

Work out $\frac{1}{3} + \frac{1}{6}$.

Solution

Denominators 3 and 6 are different.

We get an equivalent fraction of $\frac{1}{3}$ with a denominator of 6. Then we add the two fractions.

Equivalent fraction of $\frac{1}{3} = \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$.

We add as follows: $\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{2 + 1}{6} = \frac{3}{6}$.

Practice Activity 4.11

Use equivalent fractions to add the following.

1. $\frac{1}{2} + \frac{1}{4} =$
2. $\frac{1}{4} + \frac{1}{8} =$
3. $\frac{1}{4} + \frac{1}{12} =$
4. $\frac{1}{2} + \frac{1}{8} =$
5. $\frac{1}{5} + \frac{1}{10} =$
6. $\frac{1}{3} + \frac{1}{9} =$

Work out the following. Discuss and present your findings.

7. $\frac{1}{2} + \frac{1}{10} =$
8. $\frac{1}{2} + \frac{1}{12} =$
9. $\frac{1}{7} + \frac{1}{14} =$
10. $\frac{1}{6} + \frac{1}{3} =$
11. $\frac{1}{12} + \frac{1}{6} =$
12. $\frac{1}{6} + \frac{1}{18} =$

Activity 4.12

1. Use equivalent fractions to add the following:

   (a) $\frac{2}{5} + \frac{3}{10}$ (b) $\frac{3}{7} + \frac{3}{14}$

Steps:

- Find the equivalent fraction of $\frac{2}{5}$ with the denominator 10. Then add it to $\frac{3}{10}$.
- Find the equivalent fraction of $\frac{3}{7}$ with the denominator 14. Then add it to $\frac{3}{14}$.

2. Now compose a problem like the ones above. Solve and present your findings to the class.

Example 4.12

Work out $\frac{2}{3} + \frac{3}{12}$

Solution

- Find the equivalent fraction of $\frac{2}{3}$ with the denominator 12.
- To get 12, multiply 3 by 4. So $\frac{2}{3} = \frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$.
- Now $\frac{2}{3} + \frac{3}{12} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.

Practice Activity 4.12

Use equivalent fractions to work out:

1. $\frac{2}{5} + \frac{3}{10} =$
2. $\frac{1}{12} + \frac{3}{4} =$
3. $\frac{1}{2} + \frac{3}{8} =$
4. $\frac{2}{3} + \frac{1}{9} =$
5. $\frac{1}{4} + \frac{3}{8} =$
6. $\frac{3}{7} + \frac{1}{14} =$

Work out the following. Present your findings.

7. $\frac{1}{2} + \frac{5}{12} =$
8. $\frac{2}{3} + \frac{1}{6} =$
9. $\frac{3}{5} + \frac{1}{10} =$
10. $\frac{2}{9} + \frac{2}{3} =$
11. $\frac{2}{7} + \frac{3}{14} =$
12. $\frac{5}{12} + \frac{3}{6} =$

Activity 4.13

A. • Find equivalent fractions with the denominator 6 for:

(i) $\frac{1}{3}$ (ii) $\frac{1}{2}$

What is the easiest way to do that?

• Now add $\frac{1}{2} + \frac{1}{3}$.

B. • Using equivalent fractions, add the following fractions. What answers did you get?

(a) $\frac{1}{2} + \frac{1}{5}$ (b) $\frac{1}{3} + \frac{1}{4}$

• Now write two fractions with different denominators. Add your fractions and present your findings.

Tip: When finding equivalent fractions to add the fractions:

(i) Identify the different denominators.

(ii) Check if denominators are multiples of each other. Then use the largest denominator as a common denominator.

(iii) Where denominators are not multiples of each other, find the lowest common multiple for them. For example, 2 and 3 have a common multiple of 6. So common denominator is 6.

(iv) Add fractions with a common denominator.

Example 4.13

Use the LCM to add

$\frac{1}{3} + \frac{2}{5}$. Explain the steps you followed.

Solution

- 3 and 5 are different denominators. Their common denominator is $3 \times 5 = 15$.

  Equivalent fractions are:

  $\frac{1}{3} = \frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$

  $\frac{2}{5} = \frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$

Thus, $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{5+6}{15} = \frac{11}{15}$

Practice Activity 4.13

Work out the following.

1. $\frac{1}{2} + \frac{3}{5} =$
2. $\frac{6}{7} + \frac{2}{3} =$
3. $\frac{1}{3} + \frac{6}{7} =$
4. $\frac{1}{2} + \frac{1}{3} =$
5. $\frac{1}{2} + \frac{1}{5} =$
6. $\frac{3}{4} + \frac{2}{5} =$

Find the sum of the fractions below. Discuss your answer.

7. $\frac{4}{5} + \frac{2}{6} =$
8. $\frac{1}{3} + \frac{1}{4} =$
9. $\frac{2}{3} + \frac{3}{4} =$
10. $\frac{3}{5} + \frac{2}{7} =$
11. $\frac{1}{7} + \frac{1}{6} =$
12. $\frac{3}{5} + \frac{1}{4} =$

4.4 Addition of fractions with different denominators using LCM

Revision

Lowest Common Multiple is written as LCM.

Find the LCM of the following numbers.

1. 3, 9
2. 4, 6
3. 10, 12
4. 3, 9, 6
5. 7, 9

Activity 4.14

Add the following fractions using the LCM. Discuss your findings.

(i) $\frac{1}{4} + \frac{1}{8}$ (ii) $\frac{1}{10} + \frac{1}{5}$ (iii) $\frac{1}{6} + \frac{2}{3}$

Example 4.14

Use the LCM to add

$\frac{1}{3} + \frac{1}{15}$. Explain your steps.

Solution

Find LCM of 3 and 15

3	3	15
5	1	5
	1	1

LCM is $3 \times 5 = 15$

We can now use the LCM to add.

Method 1

$$\frac{1}{3} + \frac{1}{15} = \frac{(15 \div 3) \times 1 + (15 \div 15) \times 1}{15} = \frac{5+1}{15} = \frac{6}{15}$$

Method 2

- The common denominator is the LCM.
- We rewrite $\frac{1}{3}$ as $\frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$

  So $\frac{1}{3} + \frac{1}{15} = \frac{5}{15} + \frac{1}{15} = \frac{5+1}{15} = \frac{6}{15}$

Practice Activity 4.14

Using the LCM, work out:

1. $\frac{1}{8} + \frac{1}{2} =$
2. $\frac{1}{6} + \frac{1}{12} =$
3. $\frac{1}{4} + \frac{1}{8} =$
4. $\frac{1}{4} + \frac{1}{12} =$
5. $\frac{1}{7} + \frac{1}{14} =$
6. $\frac{1}{2} + \frac{1}{4} =$

Use the LCM to work out the following questions. Discuss your answer.

7. $\frac{1}{7} + \frac{1}{21} =$
8. $\frac{1}{5} + \frac{1}{10} =$
9. $\frac{1}{12} + \frac{1}{24} =$
10. $\frac{1}{12} + \frac{1}{4} =$
11. $\frac{1}{15} + \frac{1}{3} =$
12. $\frac{1}{6} + \frac{1}{18} =$

Activity 4.15

Add the fractions below using the LCM.

(i) $\frac{1}{9} + \frac{2}{3}$ (ii) $\frac{3}{7} + \frac{3}{6}$ (iii) $\frac{3}{8} + \frac{2}{5}$

What steps did you follow?

Example 4.15

Use the LCM to add $\frac{2}{5} + \frac{2}{6}$.

Solution

Find the LCM of 5 and 6.

$$\begin{array}{r|l} 2 & 6 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 5 & 5 \\ 1 & \end{array}$$

LCM is $2 \times 3 \times 5 = 30$

We use the LCM to rename the fractions.

- $\frac{2}{5}$ as $\frac{2}{5} \times \frac{6}{6} = \frac{12}{30}$
- $\frac{2}{6}$ as $\frac{2}{6} \times \frac{5}{5} = \frac{10}{30}$

Thus, $\frac{2}{5} + \frac{2}{6} = \frac{12}{30} + \frac{10}{30} = \frac{12 + 10}{30} = \frac{22}{30}$

We can do it in short form as:

$$\frac{2}{5} + \frac{2}{6} = \frac{(30 \div 5) \times 2 + (30 \div 6) \times 2}{30}$$

$$= \frac{12 + 10}{30} = \frac{22}{30}$$

Practice Activity 4.15

Use the LCM to work out the following.

1. $\frac{1}{6} + \frac{2}{7} =$
2. $\frac{1}{3} + \frac{5}{12} =$
3. $\frac{2}{3} + \frac{3}{10} =$
4. $\frac{3}{5} + \frac{2}{7} =$
5. $\frac{1}{6} + \frac{2}{3} =$
6. $\frac{2}{5} + \frac{1}{2} =$

Use the LCM to work out the following. Explain your steps.

7. $\frac{4}{9} + \frac{3}{7} =$
8. $\frac{4}{12} + \frac{1}{9} =$
9. $\frac{2}{6} + \frac{2}{9} =$
10. $\frac{2}{8} + \frac{3}{6} =$
11. $\frac{3}{7} + \frac{1}{4} =$
12. $\frac{2}{7} + \frac{1}{6} =$

Activity 4.16

Add the following fractions using the LCM.

(a) $\frac{3}{6} + \frac{5}{10}$ (b) $\frac{5}{6} + \frac{3}{18}$ (c) $\frac{4}{8} + \frac{6}{12}$

Explain your steps.

Example 4.16

Use the LCM to add $\frac{3}{9} + \frac{2}{3}$

Solution

Find the LCM of 3 and 9.

3	3	9
3	1	3
	1	1

LCM is $3 \times 3 = 9$

Add: $\frac{3}{9} + \frac{2}{3} = \frac{(9 \div 9) \times 3 + (9 \div 3) \times 2}{9} = \frac{3+6}{9} = \frac{9}{9} = 1$

Practice Activity 4.16

Use the LCM to work out the following.

1. $\frac{8}{12} + \frac{1}{4} =$
2. $\frac{4}{6} + \frac{8}{24} =$
3. $\frac{4}{8} + \frac{3}{6} =$
4. $\frac{5}{7} + \frac{6}{21} =$
5. $\frac{12}{36} + \frac{5}{9} =$
6. $\frac{18}{30} + \frac{2}{5} =$

Use the LCM to work out the following. Justify your answer.

7. $\frac{9}{15} + \frac{4}{10} =$
8. $\frac{16}{28} + \frac{3}{7} =$
9. $\frac{12}{18} + \frac{2}{6} =$
10. $\frac{7}{8} + \frac{4}{32} =$
11. $\frac{10}{12} + \frac{1}{6} =$
12. $\frac{9}{24} + \frac{5}{8} =$

4.5 Addition of more fractions with different denominators

Look at the following.

1. $\frac{1}{3}, \frac{1}{4}, \frac{3}{4}, \frac{4}{5}$: In these fractions the numerator is smaller than the denominator. These are called proper fractions.
2. $\frac{6}{4}, \frac{11}{5}, \frac{14}{6}, \frac{4}{3}$: In these fractions the numerator is bigger than the denominator. Such fractions are known as improper fractions.
3. $\frac{6}{4}$ can be written as $\frac{4}{4} + \frac{2}{4} = 1 + \frac{2}{4}$, written as $1\frac{2}{4}$ or $1\frac{1}{2}$.

   $1\frac{1}{2}$ is called a mixed number. It has a whole number and a fraction.

Activity 4.17

- Convert $\frac{11}{5}$, $\frac{7}{6}$, $\frac{9}{8}$, $\frac{7}{2}$ into mixed numbers.
- Add the following using the LCM.

  (a) $\frac{7}{2} + \frac{7}{6}$ (b) $\frac{11}{5} + \frac{9}{8}$

  What type of fraction is the answer you get? Explain.

Example 4.17

Add and write the answer as a mixed number: $\frac{5}{8} + \frac{3}{4}$

Solution

The LCM of 8 and 4 is 8.

Add: $\frac{5}{8} + \frac{3}{4} = \frac{5+6}{8} = \frac{11}{8}$

$\frac{11}{8} = 11 \div 8 =$

$$\begin{array}{r} 1 \\ 8\,\overline{)\,11} \\ -\ 8 \\ \hline 3 \end{array}$$

1 — whole number

Therefore, $\frac{11}{8} = 1\frac{3}{8}$

Practice Activity 4.17

Work out the following.

1. $\frac{4}{5} + \frac{4}{7}$
2. $\frac{7}{8} + \frac{3}{4}$
3. $\frac{4}{9} + \frac{9}{10}$

Work out the following questions. Give your answers as mixed numbers. Explain your steps.

4. $\frac{5}{6} + \frac{3}{7}$
5. $\frac{4}{5} + \frac{2}{3}$
6. $\frac{2}{3} + \frac{1}{2}$
7. $\frac{5}{4} + \frac{4}{3}$
8. $\frac{8}{7} + \frac{9}{8}$
9. $\frac{3}{2} + \frac{4}{3}$

4.6 Addition of mixed numbers with different denominators

Look at these

(a) Write $3\frac{3}{11}$ as an improper fraction.

$$3\frac{3}{11} = 3 \text{ wholes and } \frac{3}{11} = \frac{(3 \times 11)}{11} + \frac{3}{11}$$

$$= \frac{33 + 3}{11} = \frac{36}{11}$$

(b) Write $2\frac{1}{6}$ as an improper fraction.

$$2\frac{1}{6} = 2 \text{ whole and } \frac{1}{6} = \frac{(2 \times 6)}{6} + \frac{1}{6} = \frac{12 + 1}{6} = \frac{13}{6}$$

Activity 4.18

- Write $2\frac{1}{5}$, $3\frac{1}{6}$, $3\frac{3}{8}$, $4\frac{1}{9}$ as improper fractions.
- Add (a) $2\frac{1}{5} + 3\frac{1}{4}$ (b) $3\frac{3}{8} + 4\frac{1}{9}$ What answer did you get? Explain your findings.

Example 4.18

Add: (a) $2\frac{1}{6} + 3\frac{1}{6}$ (b) $5\frac{3}{4} + 2\frac{2}{3}$

Explain your steps.

Solution

(a) $2\frac{1}{6} + 3\frac{1}{6}$

Add whole numbers and add fractions separately.

$$2\frac{1}{6} + 3\frac{1}{6} = (2 + 3) + \left(\frac{1}{6} + \frac{1}{6}\right) = 5\frac{2}{6} \text{ or } 5\frac{1}{3}$$

(b) $5\frac{3}{4} + 2\frac{2}{3}$

- Add whole numbers
- Add fractions using LCM

$$5\frac{3}{4} + 2\frac{2}{3} = (5 + 2) + \left(\frac{3}{4} + \frac{2}{3}\right) = 7\frac{9 + 8}{12}$$

$$= 7\frac{17}{12}\left(\frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}\right)$$

$$= 7 + 1\frac{5}{12} = 8\frac{5}{12}$$

Practice Activity 4.18

Using the LCM to work out the following.

1. $3\frac{3}{8} + 1\frac{1}{4} =$
2. $4\frac{1}{5} + 3\frac{1}{2} =$
3. $2\frac{2}{9} + 1\frac{3}{4} =$

Using the LCM, add the following. Discuss your answer.

4. $1\frac{3}{5} + 2\frac{1}{2} =$
5. $2\frac{1}{6} + 5\frac{1}{3} =$
6. $1\frac{3}{8} + 3\frac{3}{4} =$
7. $3\frac{5}{7} + 4\frac{1}{3} =$
8. $3\frac{9}{10} + 4\frac{4}{5} =$
9. $4\frac{3}{7} + 2\frac{3}{4} =$

4.7 Word problems for addition of fractions

Activity 4.19

Mum wanted to prepare a good meal for lunch. She then bought $\frac{1}{8}$ kilogram of beef and $\frac{1}{2}$ kilogram of liver. Find the weight of both beef and liver. Discuss the steps to your answer. Present your findings.

Example 4.19

Karen wanted to fence her farm. She had $\frac{1}{8}$ kg of nails in her store. She bought $\frac{1}{2}$ kg more nails. Find the total mass of nails Karen had.

Solution

She had $\frac{1}{8}$ kg.

She bought $\frac{1}{2}$ kg.

Total is $\frac{1}{8}$ kg + $\frac{1}{2}$ kg. We write $\frac{1}{2}$ as $\frac{1}{2} = \frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$. Then we add.

$$\frac{1}{8}\text{ kg} + \frac{1}{2}\text{ kg} = \left(\frac{1}{8} + \frac{4}{8}\right)\text{ kg} = \frac{1+4}{8} = \frac{5}{8}\text{ kg.}$$

Karen used $\frac{5}{8}$ kg to fence her farm.

Example 4.20

During a prayer meeting in the church, $\frac{1}{3}$ of the people were women. $\frac{3}{5}$ of the people were children. The rest were men. Calculate the fraction of people for women and children.

Solution

Fraction of women is $\frac{1}{3}$, fraction of children is $\frac{3}{5}$.

Total is $\frac{1}{3} + \frac{3}{5}$.

LCM of 3 and 5 is got as:

$$\begin{array}{c|c} 3 & 3 \\ 1 & 1 \end{array} \qquad \begin{array}{c|c} 5 & 5 \\ 1 & \end{array}$$

LCM is $3 \times 5 = 15$.

We then use LCM to add as below.

$$\frac{1}{3} + \frac{3}{5} = \frac{1 \times 5 + 3 \times 3}{15} = \frac{5 + 9}{15} = \frac{14}{15}$$

Practice Activity 4.19

1. Carene was celebrating her birthday. Her mother bought her a cake. Carene shared the cake with her mother. Carene ate $\frac{4}{9}$ of the cake. Her mother took $\frac{1}{3}$ of the same cake. The remaining part of the cake was eaten by her father. What is the total fraction of the cake eaten by Carene and her mother? Explain why we should share what we have.
2. In the morning, a cook wanted to make some tea. He mixed $\frac{1}{4}$ litre of milk and $\frac{1}{8}$ litre of water. He then boiled them. Find the amount of tea in litres he made. Discuss your steps.
3. During a sports day, a pupil wanted to carry a bottle of water. The bottle had $\frac{1}{3}$ litre of clean water. He added $\frac{1}{2}$ litre more clean water into the bottle. Calculate the amount of clean water he had in the bottle. Justify your answer. Tell the importance of drinking clean water.
4. A farmer had inherited $\frac{4}{7}$ ha of land from his parents in 2014.

   In 2016, the farmer bought $\frac{7}{10}$ ha of land to expand his investment activities. Determine the size of land he had altogether in 2016. Explain some importance of farming.

5. In a community work to clean the streets, adults and children participated. The fraction of men was $\frac{1}{3}$ of all people. $\frac{1}{4}$ of all people were women and the rest were children. Find the fraction for both men and women of all the people. Discuss your steps. What other community work do we do?

4.8 Subtraction of fractions with different denominators using equivalent fractions

Activity 4.20

- Find the equivalent fraction with the denominator 6 for $\frac{2}{3}$.
- Find the equivalent fraction with the denominator 8 for $\frac{1}{2}$.
- Find the equivalent fraction with the denominator 12 for $\frac{5}{6}$.

(a) $\frac{2}{3}-\frac{1}{6}=\frac{\square}{6}-\frac{1}{6}=\frac{\square-1}{6}=$

(b) $\frac{7}{8}-\frac{1}{2}=\frac{7}{8}-\frac{\square}{8}=\frac{7-\square}{8}=$

(c) $\frac{5}{6}-\frac{5}{12}=\frac{\square}{12}-\frac{5}{12}=\frac{\square-5}{12}=$

(d) $\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{\square}{4}=\frac{3-\square}{4}=$

What answer did you get? Discuss your steps to answers.

Example 4.21

$\frac{7}{8}-\frac{3}{4}$

Solution

Calculate the equivalent fraction for $\frac{3}{4}$ with the denominator of 8.

$\frac{3}{4}=\frac{6}{8}$

Subtract $\frac{6}{8}$ from $\frac{7}{8}$.

Therefore $\frac{7}{8}-\frac{6}{8}=\frac{1}{8}$.

Practice Activity 4.20

A. Use equivalent fractions to work out the following.

1. $\frac{1}{4}-\frac{1}{8}=\frac{\square}{8}-\frac{1}{8}=\frac{\square-1}{8}=$

2. $\frac{5}{8}-\frac{1}{4}=\frac{5}{8}-\frac{\square}{8}=\frac{5-\square}{8}=$

3. $\frac{5}{6}-\frac{1}{3}=\frac{5}{6}-\frac{\square}{6}=\frac{5-\square}{6}=$

4. $\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{\square}{4}=\frac{3-\square}{4}=$

B. Work out the following using equivalent fractions. Discuss your answer.

1. $\frac{7}{8} - \frac{1}{2} = \frac{7}{8} - \frac{\square}{8} = \frac{7 - \square}{8}$

2. $\frac{4}{5} - \frac{1}{10} = \frac{\square}{10} - \frac{1}{10} = \frac{\square - 1}{10} =$

3. $\frac{5}{9} - \frac{1}{3} = \frac{5}{9} - \frac{\square}{9} =$

4. $\frac{1}{3} - \frac{1}{6} = \frac{\square}{6} - \frac{1}{6} =$

C. Use equivalent fractions to work out the following questions. In each case present your findings.

1. $\frac{7}{10} - \frac{1}{5} = \frac{7}{10} - \frac{\square}{10} =$

2. $\frac{2}{3} - \frac{2}{9} = \frac{\square}{9} - \frac{2}{9} =$

3. $\frac{3}{10} - \frac{1}{5} = \frac{3}{10} - \frac{\square}{10} =$

4. $\frac{4}{7} - \frac{2}{14} = \frac{\square}{14} - \frac{2}{14} =$

Activity 4.21

Discuss how to solve the following. Use equivalent fractions to solve them.

(i) $\frac{1}{6} - \frac{1}{12}$ (ii) $\frac{1}{4} - \frac{1}{8}$ (iii) $\frac{1}{5} - \frac{1}{10}$

Follow these steps.

(i) Find the equivalent fraction of $\frac{1}{6}$ with the denominator 12. Subtract $\frac{1}{6} - \frac{1}{12}$.

(ii) Find the equivalent fraction of $\frac{1}{4}$ with the denominator 8. Subtract $\frac{1}{4} - \frac{1}{8}$.

(iii) Find the equivalent fraction of $\frac{1}{5}$ with the denominator 10. Subtract $\frac{1}{5} - \frac{1}{10}$.

Example 4.22

Work out $\frac{1}{3} - \frac{1}{9}$.

Solution

Find the equivalent fraction of $\frac{1}{3}$ with the denominator 9.

$\frac{1}{3} = \frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$

Now, $\frac{1}{3} - \frac{1}{9} = \frac{3}{9} - \frac{1}{9} = \frac{3 - 1}{9} = \frac{2}{9}$.

Practice Activity 4.21

A. Solve using equivalent fractions.

1. $\frac{1}{2} - \frac{1}{10} =$ 2. $\frac{1}{2} - \frac{1}{8} =$ 3. $\frac{1}{3} - \frac{1}{12} =$

B. Use equivalent fractions to solve. Discuss your answer.

1. $\frac{1}{3} - \frac{1}{6} =$ 2. $\frac{1}{7} - \frac{1}{14} =$ 3. $\frac{1}{9} - \frac{1}{18} =$

4. $\frac{1}{2} - \frac{1}{12} =$ 5. $\frac{1}{6} - \frac{1}{18} =$

C. Work out using fractions. Present your answers to the class.

1. $\frac{1}{10} - \frac{1}{20} =$ 2. $\frac{1}{4} - \frac{1}{12} =$ 3. $\frac{1}{4} - \frac{1}{16} =$ 4. $\frac{1}{5} - \frac{1}{15} =$

Activity 4.22

Use equivalent fractions to solve the following.

(i) $\frac{3}{4} - \frac{1}{3}$ (ii) $\frac{5}{6} - \frac{2}{5}$ (iii) $\frac{3}{5} - \frac{1}{2}$

- Use these steps to solve(i): Common multiple of 3 and 4 is $3 \times 4 = 12$. Rewrite $\frac{3}{4}$ and $\frac{1}{3}$ with the denominator 12. Subtract them.
- Follow similar steps for (ii) and (iii). Explain your answer.

Example 4.23

Work out $\frac{7}{8} - \frac{2}{3}$.

Solution

- The common denominator for 8 and 3 is $8 \times 3 = 24$.
  We find equivalent fractions for $\frac{7}{8}$ and $\frac{2}{3}$ with the denominator 24.
- $\frac{7}{8} = \frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$
- $\frac{2}{3} = \frac{2}{3} \times \frac{8}{8} = \frac{16}{24}$
- Now, $\frac{7}{8} - \frac{2}{3} = \frac{21}{24} - \frac{16}{24} = \frac{21-16}{24} = \frac{5}{24}$

Practice Activity 4.22

Using equivalent fractions, work out the following.

1. $\frac{9}{10} - \frac{3}{6} =$
2. $\frac{5}{6} - \frac{7}{9} =$
3. $\frac{7}{8} - \frac{1}{3} =$
4. $\frac{8}{9} - \frac{1}{5} =$
5. $\frac{2}{3} - \frac{2}{4} =$
6. $\frac{6}{7} - \frac{2}{5} =$

Work out the following using equivalent fractions. Discuss your answer in each case.

7. $\frac{7}{8} - \frac{2}{3} =$
8. $\frac{2}{3} - \frac{4}{7} =$
9. $\frac{2}{3} - \frac{3}{5} =$
10. $\frac{7}{9} - \frac{1}{4} =$
11. $\frac{5}{7} - \frac{2}{5} =$
12. $\frac{3}{10} - \frac{2}{9} =$

4.9 Subtraction of fractions with different denominators using the LCM

Activity 4.23

Subtract these fractions using Lowest Common Multiple (LCM).

(a) $\frac{1}{2} - \frac{1}{4}$ (b) $\frac{1}{2} - \frac{1}{3}$ (c) $\frac{1}{4} - \frac{1}{6}$ (d) $\frac{1}{4} - \frac{1}{8}$

Discuss the steps you followed.

Example 4.24

Use the LCM to subtract $\frac{1}{6} - \frac{1}{9}$.

Solution

Find the LCM of 6 and 9.

2	6	9
3	3	9
3	1	3
	1	1

LCM is $2 \times 3 \times 3 = 18$

Method 1

We now get the common denominator as the LCM = 18. So $\frac{1}{6} - \frac{1}{9}$ are rewritten as:

$\frac{1}{6} = \frac{1}{6} \times \frac{3}{3} = \frac{3}{18}$ and $\frac{1}{9} = \frac{1}{9} \times \frac{2}{2} = \frac{2}{18}$

Thus, $\frac{1}{6} - \frac{1}{9} = \frac{3}{18} - \frac{2}{18} = \frac{3-2}{18} = \frac{1}{18}$

Method 2

We can use the LCM to subtract as follows:

$$\frac{1}{6} - \frac{1}{9} = \frac{(18 \div 6) \times 1 - (18 \div 9) \times 1}{18} = \frac{3-2}{18} = \frac{1}{18}$$

Practice Activity 4.23

A. Use the LCM to work out:

1. $\frac{1}{3} - \frac{1}{7} =$
2. $\frac{1}{3} - \frac{1}{9} =$
3. $\frac{1}{5} - \frac{1}{10} =$
4. $\frac{1}{2} - \frac{1}{9} =$
5. $\frac{1}{4} - \frac{1}{12} =$
6. $\frac{1}{5} - \frac{1}{6} =$

B. Use LCM to work out the following. Then explain the steps to your answer.

1. $\frac{1}{6} - \frac{1}{7} =$
2. $\frac{1}{8} - \frac{1}{9} =$
3. $\frac{1}{9} - \frac{1}{10} =$
4. $\frac{1}{10} - \frac{1}{12} =$
5. $\frac{1}{7} - \frac{1}{9} =$
6. $\frac{1}{10} - \frac{1}{12} =$

Activity 4.24

Subtract the following using their LCM.

(a) $\frac{5}{6} - \frac{3}{8}$ (b) $\frac{5}{6} - \frac{2}{9}$ (c) $\frac{6}{8} - \frac{2}{3}$

Discuss how you arrived at your answer.

Example 4.25

Work out using the LCM: $\frac{2}{3} - \frac{1}{5}$.

Solution

- Find LCM of 3 and 5

3	3	5
5	1	5
	1	1

LCM is $3 \times 5 = 15$

Method 1

- The common denominator is 15.

$\frac{2}{3} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$

$\frac{1}{5} = \frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$

$\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{10-3}{15} = \frac{7}{15}$

Method 2

$\frac{2}{3} - \frac{1}{5} = \frac{(15 \div 3) \times 2 - (15 \div 5) \times 1}{15}$

$= \frac{10-3}{15}$

$= \frac{7}{15}$

Practice Activity 4.24

Work out using the LCM.

1. $\frac{6}{7} - \frac{3}{4} =$
2. $\frac{8}{9} - \frac{8}{12} =$
3. $\frac{2}{3} - \frac{2}{7} =$
4. $\frac{5}{6} - \frac{5}{8} =$
5. $\frac{5}{6} - \frac{1}{2} =$
6. $\frac{8}{9} - \frac{2}{3} =$

Using the LCM work out the following. Discuss your steps.

7. $\frac{1}{2} - \frac{1}{12} =$
8. $\frac{6}{8} - \frac{1}{3} =$
9. $\frac{2}{6} - \frac{2}{9} =$
10. $\frac{7}{9} - \frac{7}{12} =$
11. $\frac{2}{3} - \frac{2}{10} =$
12. $\frac{4}{5} - \frac{3}{4} =$

4.10 Subtraction of whole numbers and fractions

Activity 4.25

- Write $16\frac{1}{2}$, $20\frac{9}{10}$, $6\frac{5}{8}$, $9\frac{1}{9}$ as improper fractions.
- Now, work out: (a) $23 - 20\frac{9}{10}$ (b) $15 - 9\frac{1}{9}$

What answers did you get? Discuss your steps.

Example 4.26

Work out using the LCM.

(a) $2 - \frac{1}{3}$ (b) $3\frac{3}{4} - 1\frac{1}{4}$

Solution

(a) $2 - \frac{1}{3} = 1 + \left(\frac{3}{3} - \frac{1}{3}\right)$ Borrow 1 from 2 and change it to three thirds.

$= 1 + \frac{2}{3} = 1\frac{2}{3}$

(b) First subtract whole numbers. Then subtract the fractions

$3\frac{3}{4} - 1\frac{1}{4} = (3 - 1) + \left(\frac{3}{4} - \frac{1}{4}\right) = 2\frac{2}{4} = 2\frac{1}{2}$

Practice Activity 4.25

Use the LCM to work out.

1. $8\frac{5}{6} - 3\frac{2}{6}$
2. $6\frac{3}{5} - 3\frac{1}{5}$
3. $4\frac{7}{8} - 1\frac{3}{8}$
4. $5\frac{5}{9} - 2\frac{2}{9}$

Use the LCM to subtract. Explain your steps.

5. $4\frac{1}{4} - 1\frac{1}{4}$
6. $4\frac{7}{10} - 1\frac{2}{5}$
7. $5 - \frac{7}{10}$
8. $6 - \frac{1}{2}$
9. $7 - \frac{4}{7}$
10. $9 - 3\frac{3}{4}$

4.11 Subtraction of mixed numbers with different denominators

Activity 4.26

Work out the following using the LCM.

$1\frac{1}{4} - \frac{1}{4}$, $3 - \frac{1}{2}$, $4\frac{1}{4} - \frac{1}{8}$, $5 - \frac{3}{4}$. Discuss the steps to your answer.

Example 4.27

(a) Work out $4\frac{1}{3} - 1\frac{2}{3}$

Solution

$4\frac{1}{3} - 1\frac{2}{3} = (4 - 1) + \left(\frac{1}{3} - \frac{2}{3}\right)$ Subtract whole numbers first. Then the fractions.

$= 3 + \left(\frac{1}{3} - \frac{2}{3}\right)$ Borrow 1 from 3 and change it to thirds then subtract $\left(1 = \frac{3}{3}\right)$

$$= 2 + \left(\frac{3}{3} + \frac{1}{3} - \frac{2}{3}\right)$$

$$= 2 + \left(\frac{4}{3} - \frac{2}{3}\right) = 2\frac{2}{3}$$

(b) Work out $5\frac{1}{4} - 1\frac{5}{6}$

Solution

$5\frac{1}{4} - 1\frac{5}{6} = (5 - 1) + \left(\frac{1}{4} - \frac{5}{6}\right)$ Subtract whole numbers first then the fractions.

$= 4 + \frac{3 - 10}{12}$ Borrow 1 from 4 and change it to twelfths $(1 = \frac{12}{12})$

$= 3 + \frac{12 + 3 - 10}{12} = 3 + \frac{15 - 10}{12}$

$= 3\frac{5}{12}$

Practice Activity 4.26

A. Work out using the LCM.

1. $4\frac{1}{2} - 1\frac{4}{5}$ 2. $6\frac{1}{2} - 2\frac{5}{6}$ 3. $2\frac{1}{4} - \frac{2}{3}$

B. Work out using the LCM and discuss your answers.

1. $5\frac{1}{4} - \frac{3}{4}$ 2. $4\frac{1}{5} - 1\frac{7}{8}$ 3. $2\frac{1}{2} - \frac{3}{4}$

5. $6\frac{2}{3} - 3\frac{5}{6}$ 5. $7\frac{3}{4} - 3\frac{2}{3}$ 6. $2\frac{1}{4} - 1\frac{1}{4}$

7. $4\frac{7}{9} - 3\frac{2}{3}$ 8. $3\frac{7}{11} - 1\frac{1}{2}$ 9. $4\frac{1}{6} - 1\frac{6}{9}$

4.12 Word problems for subtraction of fractions

Activity 4.27

Use the LCM to solve the following.

(i) A farmer has $\frac{7}{8}$ ha of land. A $\frac{1}{2}$ ha from the land is planted with crops. The rest is for the homestead. How much land is used for the homestead?

(ii) A storage tank weighed $\frac{3}{4}$ tonnes when full of water. After five days, the family had used water from the tank in washing clothes and cleaning utensils. The weight of the water in the tank became $\frac{2}{5}$ tonnes. Calculate the weight of water used by the family in five days. Explain your steps.

Example 4.28

A mother had $\frac{5}{9}$ litres of milk for her baby. During the day, the baby drank some of the milk. The milk that remained was $\frac{1}{3}$ litres. How much milk did the baby drank during the day?

Solution

Amount of milk initially is $\frac{5}{9}$ litres.

Remaining milk is $\frac{1}{3}$ litres.

Let us rename $\frac{1}{3}$ as:

$\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$

Milk drank $= \left(\frac{5}{9} - \frac{1}{3}\right)$ litres

$= \left(\frac{5}{9} - \frac{3}{9}\right)$ litres $= \left(\frac{5-3}{9}\right)$ litres

$= \frac{2}{9}$ litre

Example 4.29

During a lunch in the school, a pupil was served with $\frac{1}{3}$ litres of milk for good health. She drank some of the milk immediately and reserved the remainder for 4:00 p.m usage. The amount of milk that remained for 4:00 p.m was $\frac{1}{4}$ litres. How much milk did she drank during the lunch?

Solution

Initial amount of milk is $\frac{1}{3}$ litres. Remaining amount for 4:00 p.m is $\frac{1}{4}$ litres.

Milk drank is $\left(\frac{1}{3} - \frac{1}{4}\right)$ litres

Find the LCM of 3 and 4.

2	3	4
2	3	2
3	3	1
	1	1

LCM = 2 × 2 × 3 = 12

Now, $\left(\frac{1}{3} - \frac{1}{4}\right)$ litre $= \frac{(12 \div 3) \times 1 - (12 - 4) \times 1}{12}$ litres $= \frac{4-3}{12}$ litres

$= \frac{1}{12}$ litres

Practice Activity 4.27

1. During an activity on measuring length, Jane and Michael had different sticks. Jane had a stick that is $\frac{7}{8}$ m long. Michael had a stick that is $\frac{5}{6}$ m long. By how many metres is Jane's stick longer than Michael's stick?

2. A teacher had $\frac{3}{4}$ metres of a thread. The teacher used $\frac{1}{2}$ metre from the thread to mend a cloth. What length of thread remained? Explain your answer.

3. During a rainy season, a certain school harvested rainwater. The school tank became full and its weight was $\frac{9}{12}$ tonnes. For one week, pupils used tank water to clean their classes. There was no rain during the week. The weight of water in the tank finally was $\frac{1}{2}$ tonnes. Calculate the weight of water used in cleaning activity. Discuss your steps. Why should we keep our classes clean?

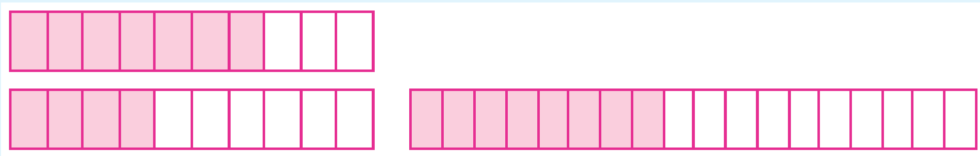
4. Peggy took a cake whose mass was $\frac{1}{2}$ kg to school. During the tea-break, she shared her cake with her friend. The mass of the cake her friend ate was $\frac{2}{8}$ kg. Peggy ate the rest of the cake. Find the mass of the cake eaten by Peggy during the break. Who ate larger part of the cake? Discuss your steps. Why should we make friends with others?

5. In a class activity, a pupil found the fraction of boys and girls in his class. He observed that boys form $\frac{3}{7}$ of the class.
   (a) What fraction of the class were girls? Justify your answer.
   (b) Find the difference of the fraction of girls and boys.
   (c) What roles do boys and girls play in our community?

6. A worker wanted to paint the furniture in a hotel. He bought $\frac{3}{4}$ litre of white paints. He painted his table using his paints. After completing his work, the amount of paint that remained was $\frac{3}{8}$ litre. Calculate the amount of paint he used to paint the table. Explain your steps. Tell the importance of painting.

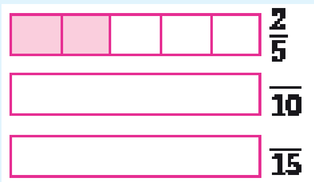
Revision Activity 4

1. Name the shaded fractions. Give one other equivalent fraction for each.

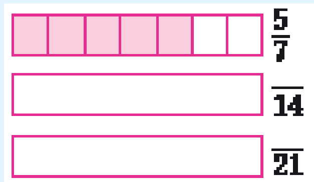


2. Shade 2 more equivalent fractions.

(a)



(b)



3. Write 2 equivalent fractions for the following fractions.

(a) $\frac{3}{8}$ (b) $\frac{3}{11}$

4. Change the fractions below to have the denominator 24.

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{3}{8}$

5. Add using equivalent fractions. Explain your answer.

(a) $\frac{2}{3} + \frac{4}{9}$ (b) $\frac{3}{12} + \frac{5}{6}$ (c) $\frac{3}{10} + \frac{3}{5}$

6. Subtract using equivalent fractions. Discuss.

(a) $\frac{5}{8} - \frac{1}{2}$ (b) $\frac{3}{4} - \frac{1}{12}$ (c) $\frac{2}{3} - \frac{1}{9}$

7. Add using the Lowest Common Multiple. Explain your answer.

(a) $\frac{3}{7} + \frac{5}{6}$ (b) $\frac{2}{9} + \frac{2}{12}$ (c) $1\frac{1}{6} + 3\frac{1}{4}$

8. Subtract using the Lowest Common Multiple. Discuss your answer.

(a) $\frac{1}{5} - \frac{1}{7}$ (b) $\frac{7}{8} - \frac{7}{9}$ (c) $4 - \frac{2}{5}$

(d) $3\frac{1}{2} - 1\frac{1}{2}$ (e) $5\frac{3}{4} - 2\frac{2}{3}$ (f) $1\frac{1}{4} - \frac{2}{3}$

9. Two pupils measured the distance from their school to their homes. The distance from Maryarita's home to the school was $\frac{5}{8}$ km. The distance from Cylde's home to the school was $\frac{1}{4}$ km. Every morning, each pupil walks to the school.
   (a) Who walks longer distance every morning? And by how many kilometres?
   (b) Explain importance of doing physical exercise.

10. During a public meeting, men, women and children attended. $\frac{1}{3}$ of the people in the meeting were men. Children were $\frac{1}{4}$ of the people. The rest were women.
   (a) What fraction were both men and children?
   (b) What fraction were women? Justify your answer.
   (c) What fraction were both men and women?
   (d) Find the fraction for both women and children. Discuss your steps.
   (e) Tell the roles played by men and women in our society.

Word list

Equivalent fractions	Models
Denominators	Least Common Multiple
Concept	Determining

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 5

Multiplication and division of decimals

5.1 Decimal fractions

A tenth

Activity 5.1

Materials needed: knife and an orange.

Steps:

(a) Cut an orange into ten equal parts.

A whole orange

Ten pieces of orange

A tenth$=\frac{1}{10}$

(b) Name each part you have cut.

(c) Write $\frac{4}{10}$ in words.

(d) Explain your observations to the class. Discuss.

A hundredth

Activity 5.2

Materials: a pair of scissors, manila paper, a ruler, a pencil

Steps:

(a) Draw a square of 10 cm. Do it on manila paper.

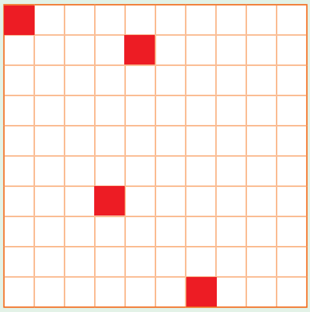
(b) Draw smaller squares each measuring 1 cm inside the bigger square.

(c) Count the number of small squares.

(d) Shade four of the small squares.

(e) What fraction have you shaded?

(f) Write this fraction in words.



$\frac{4}{100}$ = Four Hundrendths

(g) Present and explain your work.

A thousandth

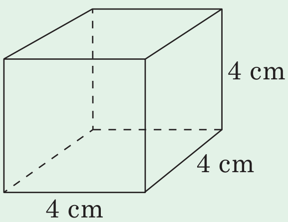
Activity 5.3

Materials: a pair of scissors, manila paper, a ruler, a pencil

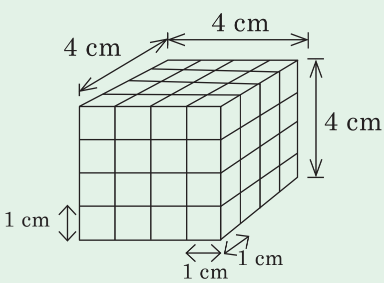
Steps

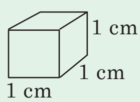
Do the following;

– Draw a cube measuring 4 cm.



– In it, draw 1 cm cubes.





– Count the number of 1 cm cubes you have drawn. Let us make a 4 cm cube using wet clay. We can cut a 1 cm cube using a sharp knife.

– Repeat the process above for a cube with 10 cm sides. What decimal fraction is $\frac{705}{1000}$? Write in words. Discuss your findings.

Tip:

$\frac{1}{10}$ = one tenth = 0.1. The decimal fraction is read as zero point one.

$\frac{1}{100}$ = one hundredth = 0.01. The decimal fraction is read as zero point zero one.

$\frac{1}{1000}$ = one thousandth = 0.001. The decimal fraction is read as zero point zero zero one.

Example 5.1

(i) Read and write the decimals in words.

(a) 0.02 (b) 0.3 (c) 0.005

(d) 0.85 (e) 0.850

(ii) Write the following in figures.

(a) Three hundredths (b) Seven tenths

(c) Nine thousandths (d) Four point nine

Solution

(i) (a) 0.02 is read as zero point zero two. It is written as two hundredths.

(b) 0.3 is read as zero point three. It is written as three tenths.

(c) 0.005 is read as zero point zero zero five. It is written as five thousandths.

(d) 0.85 is read as zero point eight five. It is written as eighty five hundredths.

(e) 0.850 is read as zero point eight five zero. It is written as eight hundred fifty thousandths.

(ii) (a) 0.03 (b) 0.7 (c) 0.009 (d) 4.9

Practice Activity 5.1

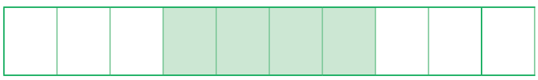
Work out the following:

1. Fill in the table below.

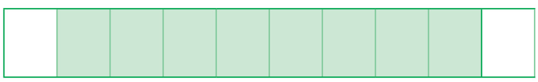
	Written in figures	Read as	Written in words
(a)	0.1	zero point one	one tenth
(b)	0.8	___	___
(c)	0.01	___	one hundredth
(d)	___	zero point zero eight	___
(e)	0.706	___	___
(f)	___	___	four and twenty three thousandths

2. From each diagram below, write the fraction and decimal fraction.

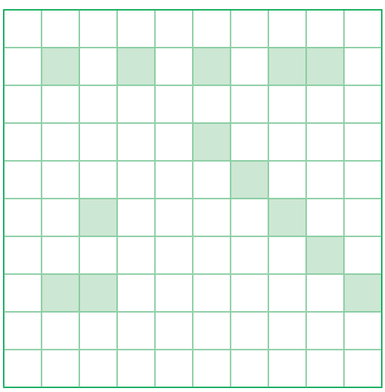
(a)  $\frac{2}{10} = 0.2$

(b)  ___ = 0.4

(c)  = ___

(d)  = ___

3. Study the figure below. Explain your observation.



(a) Write the decimal fraction for the shaded part.

(b) Write the decimal fraction for the part that is not shaded.

4. Read and write the following decimal fractions in words. Present your answer.

(a) 0.256 (b) 2.513 (c) 436.2
(d) 196.261 (e) 0.75 (f) 0.4

5. Read and write the following decimal fractions in figures. Discuss your answer.

(a) Zero point two three five.
(b) Zero point three seven eight.
(c) Six hundredths.
(d) Eight hundred seven thousandths.
(e) Four thousand and two hundredths.
(f) Six and two tenths.

5.2 Place value of decimals

Let us do the activity below.

Activity 5.4

Fill in the following table.

	Place value for decimals					
	Decimal	Ones	Decimal point	Tenths	Hundredths	Thousandths
(a)	0.23	0	.	2	3	
(b)	–	0	.	0	0	4
(c)	0.456					
(d)	9.25					
(e)	–	0	.	0	0	5

Discuss your results.

Tip: Tenths, hundredths and thousandths are examples of place values for decimals. For example, the place values of the digits in 3.647 are:

Decimal point
3.647 ← Thousandths
Hundredths
Tenths
Ones

Practice Activity 5.2

1. Fill in the following table.

		Place value							
	Decimal number in figures	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths
(a)	0.25				0	.	2	5	
(b)	–				0	.	4	5	8
(c)	5.236								
(d)	84.5								
(e)	–		6	2	5	.	0	1	
(f)	467.2								
(g)	4 321.036								
(h)	57.3								
(i)	0.934								
(j)	5 000.62								
(k)	100.423								

2. What is the place value of the digit 4 in the following numbers?
   (a) 356.4 (b) 236.254 (c) 196.456
   (d) 0.004 (e) 0.245
3. Discuss and write the name of the place value of the digit 2 in the following numbers.
   (a) 56.235 (b) 43.325 (c) 0.002
   (d) 9.362 (e) 156.267
4. Present the place value of the digit 3 in the following numbers.
   (a) 925.53 (b) 0.023 (c) 123.564
   (d) 135.267 (e) 85.364

5.3 Comparing and Ordering decimal numbers

We use these symbols to compare decimals.

$<$ means less than. For example, $0.009 < 0.01$.

$>$ means greater than. For example, $0.02 > 0.01$.

$=$ means equals to. For example, $0.1 = 0.10 = 0.100$.

Now do the activity below.

Activity 5.5

Compare these decimals. Use $>$, $<$ or $=$.

(a) $0.3 \square 0.4$

(b) $0.07 \square 0.09$

(c) $0.001 \square 0.009$

(d) $0.01 \square 0.010$

(e) $0.2 \square 0.02 \square 0.002$

Explain your answers.

Tip:

- Tenths are greater than hundredths and thousandths.
- Thousandths are less than hundredths and tenths.

Example 5.2

Which is greater?

Compare the following. Use $>$, $<$, $=$.

(a) $0.2 \square 0.4$

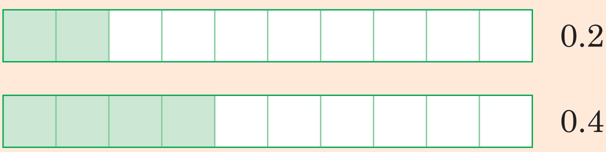
(b) $0.05 \square 0.08$

(c) $0.009 \square 0.004$

(d) $0.009 \square 0.04 \square 0.1$

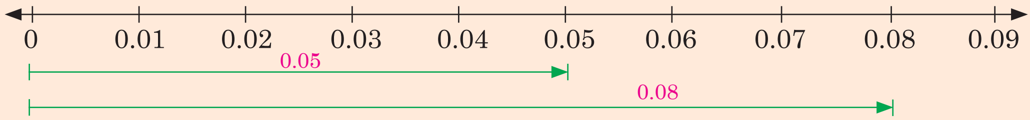
Solution

(a) Draw similar strips below. Shade 0.2 and 0.4.



From the diagram, 0.4 is greater than 0.2. We can say 0.2 is less than 0.4. Thus, $0.2 < 0.4$.

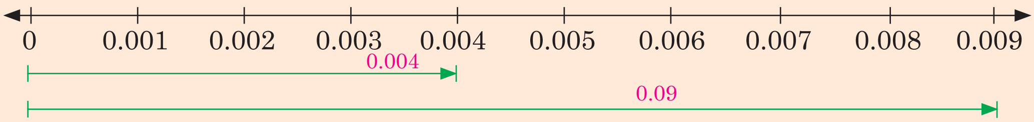
(b) Draw a number line and represent the numbers on it.



0.05 is less than 0.08.

Thus, 0.05 < 0.08

(c) Draw a number line and represent the numbers 0.009 and 0.004 on it.



0.009 is greater than 0.004.

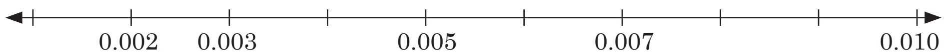
We write 0.009 > 0.004

(d) If we draw number lines, tenths is greater than hundredths. Hundredths is greater than thousandths. Thus, 0.009 < 0.04 < 0.1. Draw this on a paper and discuss your answer.

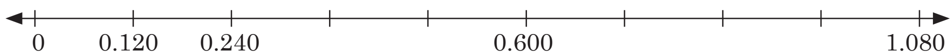
Practice Activity 5.3

1. Copy and complete the number lines below.

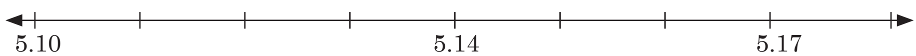
(a)



(b)



(c)



2. Use >, < and = to fill the blanks correctly.

(a) 0.005 ☐ 0.007 (b) 0.003 ☐ 0.008

(c) 3.40 ☐ 3.040 (d) 0.77 ☐ 0.770

(e) 0.825 ☐ 0.826 (f) 0.23 ☐ 0.023

3. Use >, < and = to compare the following. Discuss your answer.

(a) 0.006 ☐ 0.007 (b) 4.105 ☐ 3.05

(c) 0.9 ☐ 0.8 (d) 0.77 ☐ 0.770

4. Arrange the following from the smallest to the largest. You can use >, < or =. Present your answers.

(a) 0.01, 0.05, 0.02, 0.04 (b) 0.006, 0.003, 0.005, 0.007

(c) 0.452, 0.252, 0.436 (d) 0.5, 0.4, 0.6, 0.8

5. A farmer collected 10 eggs and harvested 10 apples. The mass of each egg was 23 g while each apple was 25 g.
   (a) Find the mass of the eggs in kilograms.
   (b) Find the mass of the apples in kilograms.
   (c) Compare the total mass of eggs and apples using >.
   Which items had smaller mass? Explain.
6. A farmer recorded the amount of milk from her farm as follows:

Days of the week	Monday	Tuesday	Wednesday	Thursday
Amount of milk	0.25 hl	0.23 hl	0.34 hl	0.30 hl

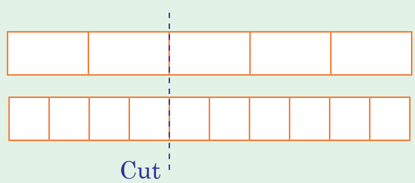
   (a) In which day had the farmer recorded the highest amount of milk?
   (b) Arrange the recorded amount of milk from the largest to the smallest. Justify your answer.

5.4 Conversion of fractions to decimals

We can change a fraction into a decimal. For example $\frac{3}{10} = 0.3$.
Do the activity below.

Activity 5.6

- Get two strips of manila paper that are the same size.
- Fold one paper into five equal parts. Cut out two of the five parts.
- Then fold the second paper strip into ten equal parts. Cut out four of the ten parts.



- Compare the parts you have cut. Write them as fractions.
- What did you discover?
- Now, write the cut parts as decimals.
- Explain your observations.

Example 5.3

(a) Write $\frac{2}{5}$ as a decimal.

Solution

Method 1: Divide 2 by 5.

$$\begin{array}{r|l} & 0.4 \\ \hline 5 & 20 \\ & -\underline{20} \\ & 00 \end{array}$$

Method 2

$\frac{2}{5}$ Multiply the numerator and denominator by a common number. Make the denominator 10.

$\frac{2}{5} \times \frac{2}{2} = \frac{4}{10}$. This is four tenths.

Thus $\frac{4}{10} = 0.4$

(b) Write $\frac{15}{25}$ as a decimal

Solution

Method 1: Divide 15 by 25.

$$\begin{array}{r|l} & 0.6 \\ \hline 25 & 150 \\ & -\underline{150} \\ & 000 \end{array}$$

Method 2

$\frac{15}{25}$ Multiply the numerator and denominator by a common number. Make the denominator 100.

$\frac{15}{25} \times \frac{4}{4} = \frac{60}{100}$. This is sixty hundredths.

Thus $\frac{60}{100} = 0.60$

Example 5.4

Write $\frac{23}{40}$ as a decimal.

Solution

Method 1

$\frac{23}{40}$ Check: What can you multiply by 40 to get 100 or 1 000? $25 \times 40 = 1\,000$. Thus multiply 23×25 and 25×40.

$\frac{23}{40} \times \frac{25}{25} = \frac{575}{1000}$. We have 575 thousandths.

Thus $\frac{23}{40} = \frac{575}{1000} = 0.575$

Method 2

$$
\begin{array}{r|l}
 & \ \ 0.575 \\
\hline
40 & 230 \\
 & -200 \\
\hline
 & \ \ \ 300 \\
 & -\ 280 \\
\hline
 & \ \ \ \ 200 \\
 & -\ \ 200 \\
\hline
 & \ \ \ \ \ 00
\end{array}
$$

$\frac{23}{40} = 0.575$

Practice Activity 5.4

1. Write the following as decimals.

(a) $\frac{5}{10}$ (b) $\frac{2}{5}$ (c) $\frac{6}{25}$

(d) $\frac{5}{20}$ (e) $\frac{21}{25}$ (f) $\frac{8}{1000}$

2. Write the following as decimals.

(a) $\frac{2}{50}$ (b) $\frac{5}{20}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

(e) $\frac{4}{5}$ (f) $\frac{25}{10}$ (g) $\frac{3}{8}$

3. Match the fractions to the decimals. Present.

	Fractions	Decimals
(a)	$\frac{8}{100}$	0.1
(b)	$\frac{8}{1000}$	0.9
(c)	$\frac{1}{10}$	0.008
(d)	$\frac{9}{10}$	0.08
(e)	$\frac{165}{100}$	0.165
(f)	$\frac{165}{1000}$	1.65

4. John gave an orange to four pupils to share equally. Find the decimal fraction of the orange each got. Use a diagram to present decimal fraction of an orange got by each pupil.

5.5 Conversion of decimals to fractions

We can change a decimal into a fraction. For example, $0.5 = \frac{5}{10} = \frac{1}{2}$. Do the activity below.

Activity 5.7

Change the following into fractions.

(a) 0.8 (b) 0.7 (c) 0.45 (d) 0.658

What steps do you follow?

Tip:

To convert a decimal into a fraction, know the decimal places. For example 0.4 is 4 tenths, 0.40 is 40 hundredths etc. Thus, $0.4 = \frac{4}{10}$; $0.40 = \frac{40}{100}$. We then simplify the fraction. Look at the following example.

Example 5.5

Convert into fractions.

(a) 0.475 (b) 0.89

Solution

(a) $0.475 = \frac{475}{1000}$

$$\frac{\overset{\overset{19}{\cancel{95}}}{\cancel{475}}}{\underset{\underset{40}{\cancel{200}}}{\cancel{1000}}} = \frac{19}{40}$$

Remember:

(i) 0.475 is 475 thousandths. So $0.475 = \frac{475}{1000}$.

(ii) Simplify by dividing with a common number.

(b)

0.89 is 89 hundredths.

So $0.89 = \frac{89}{100}$.

Check if it can be simplified. **No.**

Therefore, $0.89 = \frac{89}{100}$

Practice Activity 5.5

1. Change into fractions.
   (a) 0.75 (b) 0.455 (c) 0.625 (d) 0.075

2. Change the following decimals into fractions.
   (a) 0.41 (b) 0.009 (c) 1.8 (d) 0.62
   (e) 0.136 (f) 0.005 (g) 1.45 (h) 0.28

3. Write the following as fractions and explain how to simplify.
   (a) 0.75 (b) 0.52 (c) 0.5 (d) 0.006
   (e) 0.25 (f) 2.4 (g) 20.4 (h) 17.125

4. Which one is greater? Justify your answer.
   (a) $\frac{3}{5}$ or 0.007 (b) $\frac{1}{5}$ or 0.75 (c) $\frac{2}{5}$ or 0.25

5. Discuss and arrange the following from the smallest to the largest.
   (a) 0.56, $\frac{3}{10}$, 0.09 (b) $\frac{3}{10}$, 0.84, 0.25 (c) 0.44, $\frac{1}{4}$, 0.5

6. Match the decimals to the fractions.

	Decimals	Fractions
(a)	0.180	$\frac{900}{1000}$
(b)	0.018	$\frac{18}{10}$
(c)	1.8	$\frac{180}{1000}$
(d)	0.900	$\frac{9}{10}$
(e)	0.9	$\frac{9}{100}$
(f)	0.09	$\frac{18}{1000}$

5.6 Multiplication of decimal fractions

We can multiply a decimal number by a whole number. We can also multiply a decimal number by a decimal number. Let us study the following activity.

Activity 5.8

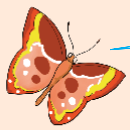
- Cut 2 oranges into halves. Each half is 0.5 of an orange. Put three halves together. What decimal number is three halves?
- Now, multiply the following:
  (i) 0.5×3 (ii) 0.5×6 (iii) 0.5×0.5
  What do you notice?
- Present your findings.

Example 5.6

Multiply the following:

(a) 0.8×4 (b) 0.2×0.7 (c) 0.4×0.16

Solution



Express decimal number in terms of place values e.g. 0.8 is $\frac{8}{10}$ or 8 tenths. Multiply whole numbers and denominators separately.

(a) 0.8×4 is $\frac{8}{10} \times \frac{4}{1} = \frac{8 \times 4}{10 \times 1}$

$= \frac{32}{10} = 3.2$

(b) 0.2×0.7 is $\frac{2}{10} \times \frac{7}{10} = \frac{2 \times 7}{10 \times 10}$

$= \frac{14}{100} = 0.14$

(c) 0.4×0.16 is $\frac{4}{10} \times \frac{16}{100} = \frac{4 \times 16}{10 \times 100}$

$= \frac{64}{1000} = 0.064$

Practice Activity 5.6

1. Work out the following.

(a) 0.06×7 (b) 2.2×7 (c) 3.502×2

(d) 7.04×4 (e) 15.23×8 (f) 0.105×9

(g) 2.66×11 (h) 6.35×11 (i) 6.9×33

2. Multiply each of the following decimal fractions.

(a) 0.2×0.6 (b) 0.14×0.2 (c) 1.5×0.02

(d) 0.17×0.3 (e) 0.2×0.04 (f) 1.5×1.2

(g) 1.3×3.3 (h) 1.3×1.5 (i) 0.93×0.7

3. Multiply the following.

(a) 2.25×10 (b) 0.039×10 (c) 0.245×10

(d) 8.91×10 (e) 35.4×10 (f) 116.7×10

4. Multiply the following. Justify your answers.

(a) 0.089×100 (b) 2.533×100 (c) 33.52×100

(d) 1.485×100 (e) 4.008×100 (f) 22.7×100

5. Multiply the following. Discuss and present your steps.

(a) $0.006 \times 1\,000$ (b) $4.005 \times 1\,000$ (c) $21.06 \times 1\,000$

(d) $13.507 \times 1\,000$ (e) $0.015 \times 1\,000$ (f) $0.267 \times 1\,000$

6. A motorcycle consumes 1 litre of petrol to cover 5.25 km. Calculate the distance it would cover with 1.5 litres of petrol.

7. I cut an orange into ten equal pieces. How many pieces of tenths would I cut from 9 oranges? Explain your answer.

8. 20 pupils were each given 0.5 loaf of bread. How many loaves of bread were given in total? Discuss your answer.

9. A small bottle holds 0.3 litres of milk. Discuss how much milk is held by 12 such small bottles?

5.7 Division of decimal fractions

Let us do the activity below.

Activity 5.9

- Cut an orange into two equal parts.
- Share half of the orange equally among 4 pupils. What fraction of orange do each of the four get? Now, discuss the following.

  (i) $0.5 \div 4$ (ii) $0.5 \div 5$

  (iii) $0.5 \div 0.5$ (iv) $0.005 \div 0.04$
- Present your findings.

Example 5.7

Work out 5 ÷ 0.2

Solution

Write it as a fraction

$5 \div 0.2 = \frac{5}{0.2}$

(How can we make the denominator a whole number? Multiply both the numerator and the denominator by 10)

$$\frac{5}{0.2} = \frac{5 \times 10}{0.2 \times 10} = \frac{50}{2}$$

$$= 25$$

Example 5.8

Work out 3.609 ÷ 0.03

Solution

Write it as a fraction

$3.609 \div 0.03 = \frac{3.609}{0.03}$

(Since the denominator is in hundredths, multiply it by $\frac{100}{100}$)

$$\frac{3.609}{0.03} = \frac{3.609 \times 100}{0.03 \times 100}$$

$$= \frac{\overset{120.3}{\cancel{360.9}}}{\cancel{3}\,_1} = 120.3$$

Tip:

Identify number of decimals in the denominator. If it is in tenths, multiply by $\frac{10}{10}$. If it is in hundredths, multiply by $\frac{100}{100}$. If the denominator is in thousandths, multiply by $\frac{1000}{1000}$.

Practice Activity 5.7

1. Work out the following.

   (a) 0.2 ÷ 5 (b) 0.44 ÷ 1.1 (c) 6.4 ÷ 1.6

   (d) 4 ÷ 0.02 (e) 1.792 ÷ 0.07 (f) 2.4 ÷ 0.08

2. Work out the following. Discuss the steps you followed.

   (a) 12.22 ÷ 26 (b) 8.648 ÷ 0.23 (c) 0.13 ÷ 0.05

3. A roll of cloth 540 m long was cut into equal pieces, each 3.6 m. Each piece was enough to make a dress. Calculate the number of dresses made from the roll.

4. The perimeter of a rectangular piece of land is 525 m. Poles are put at a fixed spacing of 0.25 m. How many poles are required to fence the entire piece of land? Justify your answer.

5. A quarter of an orange is shared equally by 5 pupils. What size of orange does each pupil get? Explain your steps to answer.

5.8 Mixed operations for multiplication and division

Activity 5.10

Work out the following

(a) $0.6 \times 0.2 \div 0.04$ (b) $\frac{0.02 \times 0.6}{0.04}$

Example 5.9

Work out the following.

(a) $0.06 \times 0.2 \div 0.04$ (b) $\frac{0.1 \times 0.36}{0.09}$

Solution

(a) $$0.06 \times 0.2 \div 0.04 = \frac{0.06 \times 0.2}{0.04}$$

$$= \frac{0.06 \times 0.2 \times 100}{0.04 \times 100}$$

$$= \frac{6 \times 0.2}{4} = \frac{3 \times 0.2}{2}$$

$$= \frac{3 \times 0.1}{1} = 0.3$$

(b) $$\frac{0.1 \times 0.36}{0.09} = \frac{0.1 \times 0.36 \times 100}{0.09 \times 100}$$

$$= \frac{0.1 \times 36}{9} = 0.1 \times 4$$

$$= 0.4$$

Practice Activity 5.8

1. Work out the following.

(a) $0.4 \times 0.2 \div 0.8$ (b) $0.5 \times 0.2 \div 0.4$ (c) $0.04 \times 0.2 \div 0.4$

(d) $5 \times 1.6 \div 0.08$ (e) $29.14 \times 9.2 \div 0.2$

2. Work out and explain the steps you followed.

(a) $\frac{1.2 \times 1.2}{1.2}$ (b) $\frac{4 \times 0.4}{0.16}$ (c) $\frac{0.3 \times 0.03}{0.009}$

Revision Activity 5

1. Multiply 0.23 by 0.23.
2. Divide $3 \div 0.3$
3. What is the place value of 3 in 264.235?
4. Change $\frac{1}{2}$ into a decimal fraction. Discuss the steps followed.
5. Write 0.236 as a fraction.
6. Arrange in order starting from the smallest to the largest.
   0.02, 0.85, 0.26. Present your answers.
7. Use >, <, or = to fill in the blanks. Discuss your answers.
   (a) 0.081 ☐ 0.095 (b) 0.25 ☐ 0.205 (c) 0.65 ☐ 0.650
8. Read and write these in figures.
   (a) Six hundred sixty seven thousandths.
   (b) Seventy two hundredths.
   (c) One and one tenth.
9. Write $\frac{2}{10}$ as a decimal.
10. Work out $1.44 \div 1.2$.
11. Share two oranges equally among twenty people. What decimal fraction would each person get? Discuss your steps.
12. Work out $0.2 \times 0.5 \div 0.01$. Explain the steps you followed.
13. Solve $\frac{5.2 \times 0.2}{0.05}$
14. Identify the place value of the digit 3 in 0.253. Explain your steps.
15. Write 52.067 in words.

Word list

Tenths	Hundredths	Thousands	Place value	
Figures	Words	Decimals	Fractions	
Convert	Matching	Multiply	Divide	>, <, =

Task

Do the following.

(i) Read each word aloud to your friend.
(ii) Write the meaning of each of the words above. Discuss with your friend.
(iii) Write sentences using each of the words above. Read with your friend.

UNIT 6 Application of direct proportions

6.1 Concept of direct proportion

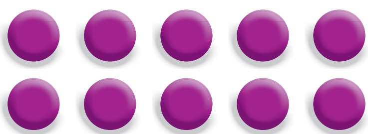
Let us do the activity below. We will then explain the concept of direct proportion.

Activity 6.1

Have your five books or counters. Record your counters as shown.

Have two of you put their counters together. Record the number of your counters.

Carry on for up to four of you and fill in the blanks accordingly.

Number of pupils	Counters
	 5
	 10
	
	

What have you observed? Explain your observations.

Tip:

It is clear that pupils increase from 1 to 2. In the same way, counters increase from 5 to 10.

We note: $\frac{2}{1} = \frac{10}{5}$, increase in the same way.

We can say 2 pupils have 10 counters. When we decrease the pupils from 2 to 1, the counters reduce from 10 to 5.

We note $\frac{1}{2} = \frac{5}{10}$, decrease in the same way.

Activity 6.2

Materials: water, $\frac{1}{2}$ litre bottles, 1 litre bottles, similar cups.

- Pour water into the $\frac{1}{2}$ litre bottle and the 1 litre bottle.

Bottles

Cup

Water in bucket

- Pour water from the $\frac{1}{2}$ litre bottle into the cups. How many cups are filled?
- Pour water from the 1 litre bottle into the cups. How many cups are filled?

  Now fill in the table below.

Litres of water	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$
Cups filled							

- Divide: 1 litre $\div \frac{1}{2}$ litre and their respective number of cups. What do you notice? Explain your findings.

Example 6.1

(a) On a scale drawing, 1 cm represents 10 km of road. What length of road is represented by 3 cm?

Solution

1 cm represents 10 km

3 cm represent __?_

It is clear that 3 cm will represent a longer length of road.

Thus, 3 cm represents $3 \times \frac{10}{1}$ km = 30 km

(b) I take 30 minutes to walk to school. How much time do I need to:

(i) walk to school and back home?

(ii) walk to school and back home for 5 days?

Solution

The distance to school from home is fixed. I take 30 minutes one way.

(i) To walk to school and back home is two ways.
I take 30 minutes × 2 = 60 minutes = 1 hour

(ii) In 1 day, walking to and from school, I take 60 minutes.
In 5 days, I take 60 minutes × 5 = 300 minutes
or 1 h × 5 = 5 hours.

Tip: Rule for direct proportion

(i) When one quantity increases, the second quantity increases in a similar way.

(ii) When one quantity decreases, the second quantity decreases in a similar way.

Practice Activity 6.1

Work out the following

1. Fill in the table below.

Number of pupils	1	2	3	4	5	___
Quantity of bread eaten	$\frac{1}{2}$	1	___	___	___	20

2. Study the table below. Fill in the missing numbers. Justify your answers.

Number of pupils	1	___	4	20	___	35
Exercise books they have	4	8	___	___	100	___

3. I have six water tanks. Each tank holds 1 500 litres of water. How many litres of water can my tanks hold? Explain your answer.
4. It takes 2 minutes to walk round the school field once. How long does it take to walk round the school 7 times? Discuss your answer.
5. We are twenty pupils. Each of us is 10 years old. What is the total of our ages?

6.2 Ratios and direct proportion

In direct proportions, we compare quantities of different items. For example, 1 boy has 2 books. A boy and books are different items. In this case, 1:2 is the ratio of boy to books.

Let us study the activity below.

Activity 6.3

- Give three books to each volunteer.
- Now, have 2 volunteers put their books together. How many books do they have?
- Have four volunteers put their books together. How many books do they have?
- Divide: $\frac{\text{number of books}}{\text{number of volunteers}}$. This is the book to volunteer ratio.
- Count the number of boys and girls in your group. What ratio is it?
- Explain your findings.
- Discuss situations where ratios are used in daily life.

Example 6.2

Three children had nine sweets.

(a) What is the ratio of child to sweets?

(b) How many sweets are needed for 4 children?

Solution

(a) Ratio $= \dfrac{\text{number of children}}{\text{number of sweets}}$

$= \dfrac{\text{3 children}}{\text{9 sweets}}$

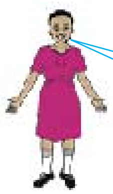
= 1 child to 3 sweets or 1 : 3

(b) 1 child gets 3 sweets.

4 children get 3 × 4 sweets

= 12 sweets

Note: From Example 6.2(a);



3 children to 9 sweets is a ratio. It is simplified be 1 : 3 by dividing by 3.

Practice Activity 6.2

Work out the following.

1. Express each of the ratios in its simplest form.

   (a) 8:24 (b) 21:42 (c) 9:27 (d) 16:12

   (e) 18:8 (f) 24:16 (g) 8:50

2. A minibus carries 28 passengers. A bus carries 64 passengers. What is the ratio of passengers carried by minibus to bus? Present your answer.
3. The weight of Pierre's mathematics book is 420 g. The weight of his dictionary is 560 g. What is the ratio of his mathematics book to his dictionary?
4. Dusabimana has 360 oranges and 120 mangoes. Find the ratio of her mangoes to her oranges. Discuss your answer.
5. In a game park, there are 120 giraffes and 360 antelopes. Find the ratio of giraffe to antelope in its simplest form.
6. The ratio of mass of maize to rice was 3:5. The total mass of maize and rice was 96 kg. Find the mass of rice and maize. Explain your steps to answer.

7. A class has 56 pupils. There are 14 boys in the class. Find the ratio of boys to girls in the class.
8. The mass of a pupil's book is 300 g. The mass of a teacher's book is 900 g. Find the ratio of the masses of teacher's book to pupil's book.

   Present your results.
9. Observe and find the ratio of items in your home or school. For example, the ratio of:
   (a) Number of teachers to pupils in your school.
   (b) Boys to girls in your class.
   (c) Number of cups to plates at home.

   Discuss your results.

6.3 Problems involving direct proportion

Activity 6.4

Solve the problems below:

- A family uses 90 litres of water every day from their tank.
  (a) How many days will it take the family to use 2 700 litres of water from the tank?
  (b) How many litres of water from the tank are used in 20 days? Explain your steps.
  (c) Discuss examples where you use direct proportions.

Example 6.3

In a peace rally, 3 speakers talk to people in 7 districts.

How many speakers are needed for 42 districts? Discuss the importance of peace in our country.

Solution

3 speakers talk to 7 districts.

We can write this as $\frac{3}{7}$ speakers per district.

Then 42 districts require $42 \times \frac{3}{7}$ speakers

$$= (6 \times 3) \text{ speakers}$$

$$= 18 \text{ speakers}$$

18 speakers can talk about peace in 42 districts.

Practice Activity 6.3

1. The weight of eight copies of P5 mathematics book is 480 g. What is the weight of one copy?
2. A car travels 12 km on one litre of petrol. How many kilometres will the car travel on three litres?
3. In transport business, three minibuses carry 54 passengers. How many passengers will 8 such minibuses carry? Discuss the importance of transport business.
4. A car uses 3 litres of petrol to travel 72 kilometres. How much petrol does it use in a journey of 648 kilometres? Explain your answer.
5. A house cleaner uses 8 litres of water everyday to clean a house. How many days will 64 litres of water last for his work?
6. A mother feeds her baby with 4 glasses of milk every day. This is in order to keep the baby healthy. How many days will the baby take to drink 132 glasses of milk? Discuss your steps to answer.
7. The weight of 15 boys is 300 kg. The boys have the same weight. Calculate the weight of one boy? Present your answer.
8. One aircraft carries 100 passengers. How many passengers are carried by 6 such aircrafts? Explain your answer.
9. Three tractors can dig 10 acres of land in a day during farming season. How many tractors are needed to dig 30 acres in a day during the season? Justify your answer. Explain importance of farming.

Activity 6.5

(a) In a certain school, there are 700 pupils. The ratio of boys to girls is 3:4. Find the number of boys and girls in the school.

(b) The ratio of boys to girls was 3:5 in a group. 24 girls left the group and 24 boys joined the group. The ratio of boys to girls became 5:3. How many boys and girls were in the original group? Explain the steps used.

Example 6.4

1. In a certain town, the ratio of adults to children is 4:5. The number of adults is 400.
   (a) Calculate the number of children in the town.
   (b) Every Monday to Friday, 100 adults go for their jobs away from the town. Similarly, 200 children go to their schools away from the town. Find the total number of adults and children in the town on Tuesday at 10:00 a.m.
2. The ratio of girls to boys was 5:3 originally. Later 24 boys joined the group and 24 girls left the group. The ratio of girls to boys became 3:5.
   (a) How many boys and girls were in the original group?
   (b) How many boys and girls were in the final group?

Solution

1. (a) Ratio of adult to children = 4:5 = 400 : children

   Clearly number of children = 5 × 100 = 500

   or $\frac{4}{9} \times$ total population = 400

   Total population = $\frac{9}{4} \times 400 = 900$, here adults are 400.

   So children are 900 – 400 = 500.

   (b) We know, there are 400 adults and 500 children.

   On Tuesday, we have (400 – 100) adults = 300 adults and (500 – 200) children = 300 children.

   Total number of adults and children are 300 + 300 = 600.

2. (a) Let the number of boys be b and girls be g in original group.

   Thus, g:b = 5:3. We can simplify as $(\frac{g}{b} : 1) = (\frac{5}{3} : 1)$. So $\frac{g}{b} = \frac{5}{3}$

   or $g = \frac{5}{3}b$

In the final group, boys are b + 24 and girls are g – 24

$(g - 24) : (b + 24) = 3:5$ or $\left(\frac{g-24}{b+24}\right) : 1 = \frac{3}{5} : \frac{5}{5} = \frac{3}{5} : 1$

So, $\frac{g-24}{b+24} = \frac{3}{5}$

(but $g = \frac{5}{3}b$)

$\frac{\frac{5}{3}b - 24}{b + 24} = \frac{3}{5}$. We now cross-multiply:

$(\frac{5}{3}b - 24) \times 5 = 3 \times (b + 24)$. Then solve for b.

$\frac{25}{3}b - 120 = 3b + 72$

$\frac{25}{3}b - 3b = 120 + 72$

$\frac{25b - 9b}{3} = 192$

$\frac{16b}{3} = 192$

$16b = 3 \times 192$

$b = \frac{3 \times \overset{\overset{12}{\cancel{96}}}{\cancel{192}}}{\underset{\cancel{8}_1}{\cancel{16}}}$

$= 3 \times 12$

$b = 36$

but $g = \frac{5}{3}b = \frac{5}{\cancel{3}_1} \times \overset{12}{\cancel{36}} = 5 \times 12$

$= 60$

The original group had 36 boys and 60 girls.

(b) The final group had

(60 – 24) girls = 36 girls and (36 + 24) boys = 60 boys

Practice Activity 6.4

1. A certain farmer has goats and chickens in her farm. The ratio of goats to chickens is 3:5 in her farm. The total number of chickens and goats is 320.

   (a) How many chickens are there in her farm?

(b) Calculate the number of goats in her farm. Explain your steps.

(c) The farmer sold 20 goats and 80 chickens so as to get money for school fees. Find the ratio of goats to chickens after selling her animals. Why is it important to educate children?

2. In a church wedding, the ratio of children to adults was 3:4. The total number of adults and children was 175. Later, 18 children and 5 adults left the church. The ratio of children to adults became 3:5.

   (a) How many children and adults were there initially?

   (b) Find the number of children in the church after 18 of them left.

   (c) Find the number of adults in the church after 5 of them left.

   (d) Suppose, the ratio of men to women was 2:3 initially in the church.

      (i) Discuss how many women were present in the church?

      (ii) Discuss how many men were present in the church?

3. In a shop, the ratio of number of shirts to trousers was 5:6. The shopkeeper bought 10 more trousers and 10 more shirts. The new ratio of shirts to trousers became 7:8.

   (a) Calculate the original number of shirts.

   (b) Calculate the original number of trousers.

   (c) Calculate the new number of trousers.

   (d) Calculate the new number of shirts. Explain importance of selling.

4. During a sports day, the ratio of boys to girls was 5:6 in the morning. At midday, 170 more boys and 180 more girls came. The ratio of boys to girls became 7:8.

   (a) Find the number of girls in the morning.

   (b) Find the number of boys in the morning.

   (c) Find the number of boys at midday.

   (d) Find the number of girls at midday.

   Discuss your answers. Why are sports important?

Revision Activity 6

1. Study the following table. It shows the number of teachers to pupils in a certain country. Fill in the missing numbers. State the ratio used and explain your observation.

Number of teachers	1	2	3	4	5	—	7
Number of pupils	30	—	—	—	150	180	—

2. In a school, each pupil is given 5 exercise books. How many exercise books will 30 pupils get from the school?
3. Abel's farm has 20 mango trees and 100 coffee trees. Find the ratio of mango to coffee trees on his farm. Why do we plant trees?
4. At breakfast 2 loaves of bread are served to 8 children. How many loaves are needed for 64 children? Tell the importance of breakfast.
5. For good health, a pupil should drink 5 glasses of water every day. Calculate how many glasses of water a pupil should drink in 10 days for good health.
6. It takes 40 minutes to walk to the market. How much time do I need to walk to the market and back home every day for one week?
7. In a game park, there are 120 lions and 240 antelopes. Find the ratio of lions to antelopes in its simplest form?
8. A wagon travels 30 km in $1\frac{1}{2}$ hours. How many kilometres will it travel in a $\frac{1}{2}$ hour? Explain the steps followed.
9. Dusabimana travelled from 6:00 a.m to 10:00 a.m on Monday to the field. He was covering 60 km every hour. What was the total distance of his journey? Present your answer.
10. In class activity, it takes a pupil 40 minutes to write a composition. How many compositions should the pupil write in 160 minutes? Discuss your steps to answer. Tell the importance of writing composition and keeping time.

Word list

Direct proportion	Ratios	Original group	Final group
Increases	Decreases	Similar way	

Task

Do the following.

(i) Read each word aloud to your friend.
(ii) Write the meaning of each of the words above. Discuss with your friend.
(iii) Write sentences using each of the words above. Read with your friend.
(iv) Give daily life examples where you apply direct proportion.

UNIT 7 Solving problems involving measurements of length, capacity and mass

7.1 Revision problems on length, capacity and mass

Revision work 7

1. Complete the conversions.

(a) Length

km	hm	dam	m	dm	cm	mm
1	0	0	0	0	0	0
				1	0	0

From table;
- 1 dm = 100 mm
- 1 km = ___ mm
- 1 hm = ___ m
- 1 dam = ___ m
- 1 m = ___ cm
- 1 m = ___ mm

(b) Capacity

hl	dal	l	dl	cl	ml
1	0	0	0	0	0

From table;
- 1 *l* = 1 000 ml
- 1 hl = ___ *l*
- 1 dal = ___ *l*
- 1 *l* = ___ dl
- 1 *l* = ___ cl

(c) Mass

kg	hg	dag	g	dg	cg	mg
1	0	0	0	0	0	0
					1	0

From table;
- 1 g = 10 dg
- 1 kg = ___ hg
- 1 dag = ___ g
- 1 kg = ___ mg
- 1 g = ___ mg

(d)

t	q	–	kg
	1	0	0
		1	0

From table;
1 t = ___ kg

2. Convert the following into centimetres. Explain your steps.
   (a) 30 mm (b) 60 mm (c) 0.7 km
3. Convert the following measurements into millimetres.
   (a) 40 cm (b) 2.4 cm (c) 0.85 m (d) 0.5 km
4. Convert the following measurements into metres.
   (a) 260 cm (b) 4 000 cm (c) 6 km (d) 60 cm
5. Convert the following measurements into kilometres.
   (a) 600 cm (b) 360 000 mm (c) 800 m (d) 14 000 cm
6. Convert the following measurements into decimetres. Explain the steps followed.
   (a) 600 cm (b) 4 000 mm (c) 120 dam
   (d) 6 dam (e) 2 km
7. Convert the following measurements into decametres and present your findings.
   (a) 1 000 000 mm (b) 10 000 cm
   (c) 200 m (d) 20 km
8. Convert the following into hectometres. Discuss your steps.
   (a) 3 000 dam (b) 12 000 cm (c) 1 000 m (d) 10 dm
9. Change the following into litres. Present your answers.
   (a) 30 dl (b) 105 dl (c) 1 050 ml (d) 2 500 ml
10. Write the following weights in tonnes.
    (a) 3 450 kg (b) 2 050 kg (c) 170 000 kg
11. (a) Subtract 2 m 6 dm 4 cm from 9 m. Give answer in dm.
    (b) A person's stride is 90 cm. How many strides can she take in a distance of 27 dam to her school? Explain your steps to answer.
12. Work out the following. Discuss your answers.
    (a) 4.5 kg + 13.6 dag = ____ kg
    (b) 4 hl – 20 dal = ___ litres
    (c) 2 dam 3 m × 5 = ___ hm

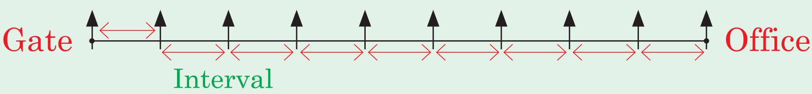
7.2 Number of intervals between objects on an open line

Activity 7.1

(a) Measure the length of the major paths that are in the school compound. For example:

- (i) The path from the school gate to the staff room.
- (ii) The path from the staff room to the P5 classroom.
- (iii) The path from the P5 classroom to the assembly grounds.

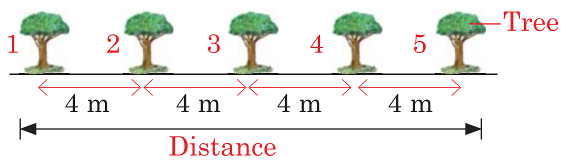
(b) Make a 0.9 m stick. Use it to mark fixed distances from one point to another. Fixed distances are called intervals. Make markings by standing along the lines at intervals of 0.9 m. Look at the figure below.



- (i) How many of you stood along the line?
- (ii) How many intervals are there? Present your finding.
- (iii) What other ways could you stand along the line?

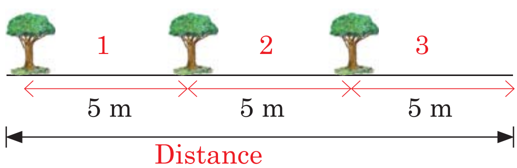
Tip: Look at an open line below:

1.



- 4 m is the interval.
- Distance = 4 m × 4 = 16 m
- Number of intervals = 4 or (5 – 1)
- Number of trees = 5 or (4 + 1)
- Number of trees = $\frac{\text{Distance}}{\text{Interval}} + 1$ e.g. $\frac{16 \text{ m}}{4 \text{ m}} + 1 = 4 + 1 = 5$ trees.

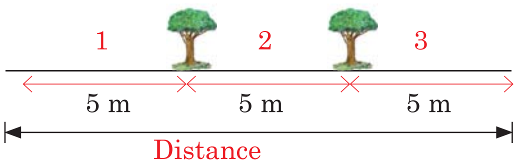
2. Interval space left at an end.



- 5 m is the interval.

- Distance = 5 m × 3 = 15 m
- Number of intervals = 3
- Number of trees = 3
- In this case, number of trees = $\frac{\text{Distance}}{\text{Interval}}$ e.g. $\frac{15\text{ m}}{5\text{ m}}$ = 3 trees.

3.



- 5 m is the interval
- Distance = 5 m × 3 = 15 m
- Number of intervals = 3
- Number of trees = 2
- In this case, number of trees = $\frac{\text{Distance}}{\text{Interval}} - 1$ e.g $\frac{15\text{ m}}{5\text{ m}} - 1 = 2$ trees.

Example 7.1

A man's stride is 10 dm long. He walks a distance of 10 dam. How many strides does he take to cover the distance?

Solution

Change 10 dam to dm.

km	hm	dam	m	dm	cm	mm
	1	0	0	0		

From the conversion table, 10 dam = 1,000 dm

Now, Interval = Stride's length = 10 dm. Distance = 1,000 dm or 10 dam.

So we calculate the number of strides:

Number of strides = $\frac{\text{Distance}}{\text{Interval}} + 1$

$$\frac{100\cancel{0}}{1\cancel{0}} + 1 = 100 + 1 = 101 \text{ strides}$$

Practice Activity 7.1

1. A road is 2 km long. Trees were planted 2 m apart along one side of the road. An interval of 2 m was left at one end without a tree due to an existing shop. How many trees were planted along the road?
2. A path is 5 dam long. Trees are to be planted at intervals of 5 dm on

both sides. How many trees are needed?

3. Electric poles are fixed along one side of 16 km section of road. This was to light the road. The poles are placed 10 m apart from each other. How many poles are fixed? Discuss why it is important to light the road.

4. A farmer planted crops in straight lines. In each line, an interval gap was left without crop for easy movement at both ends. There are 10 lines. Each line is 20 m. The interval for plants in each line is 0.5 m.

   (a) How many plants are in each line? Justify your answer.

   (b) How many plants are in 10 lines? Discuss your answer.

5. A farmer planted 20 trees along a terrace of his land. The trees were planted at intervals of 2 m. What is the length of the terrace planted with trees? Present your findings. Why is it important to plant trees along terraces?

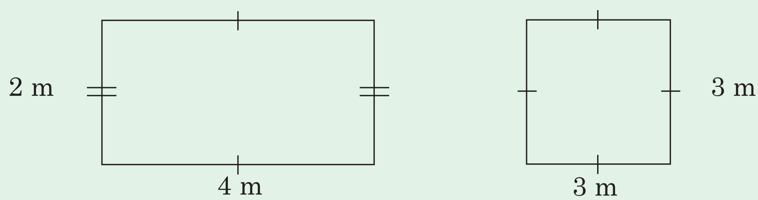
6. 21 vegetables were planted along a straight line in a garden. The vegetables were planted at fixed intervals. The line was 30 m long. Work out the length of the interval, then explain your answer. Why should we have a kitchen garden?

7. A section of a road is 3 km long. Flowers were planted 200 cm apart alongside the road. There were two rows of flowers on each side of the road. How many flowers were planted along the road? Discuss your answers.

7.3 Finding the number of intervals on a closed line

Activity 7.3

- Measure a 1 metre long stick.
- Make a square and a rectangle on the ground using measured stick. Let their perimeters be 12 m.



- Starting at one corner, fix small stones at equal intervals of 1 m.

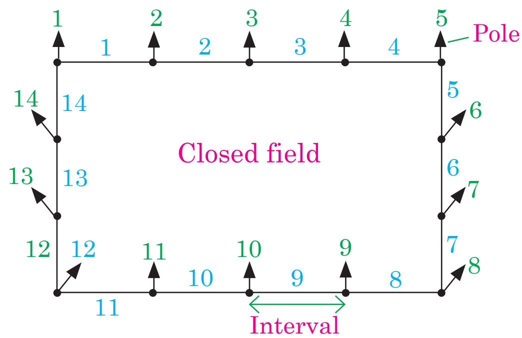
(i) How many small stones have you used?

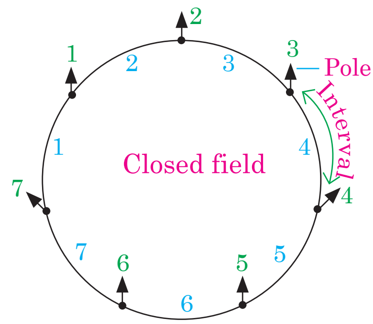
(ii) How many intervals are there?

– Tell the daily life situation relating to the activity.

– Discuss your findings.

Tip: Look at these figures:





On every closed line or field;

- Number of intervals = number of poles.
- Number of intervals × interval length = distance of closed field line.

  Look at the example below.

Example 7.2

The length of a rectangular piece of land is 48 m by 12 m. Poles were fixed at intervals of 2 m to fence it.

(a) What is the distance round the land?

(b) How many poles were used to fence the land?

Solution

(a) Distance round the land = perimeter

$= 48 \text{ m} + 12 \text{ m} + 48 \text{ m} + 12 \text{ m}$

$= 120 \text{ m}$

(b) Number of poles $= \dfrac{\text{Distance round or perimeter}}{\text{Interval length}}$

$= \dfrac{120 \text{ m}}{2 \text{ m}} = 60 \text{ poles}$

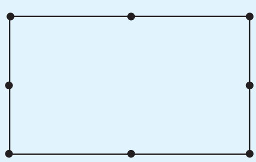
Practice Activity 7.2

1. In a town, a square plot has sides of 50 m. Poles were fixed to fence it at intervals of 2 m. How many poles were used? Where can you fence and why?
2. A circular fish pond has a circumference of 154 m. Poles are to be fixed at intervals of 3.5 m. How many poles are required to fence around the entire pond? Tell the importance of rearing fish.
3. A rectangular tank was built to store water. The tank had a length of 125 m and a width of 100 m. It was fenced using 150 poles fixed at equal intervals. Calculate the length of each interval.
4. The fence distance round an animal park is 4.2 km. 210 trees are planted along the park fence at equal intervals. Calculate the length of each interval in metres. Explain your answer. Why do we fence round the animal park?
5. A farmer fenced her cassava farm using 50 poles. The poles were spaced at equal intervals of 2.5 m. Calculate the distance round her farm in decametres. Discuss your answer. Explain the importance of cassava.
6. In a certain town, trees were planted round it. The trees were equally spaced at intervals of 6 m. The distance round the town was 402 m. Find the number of trees round the town. What have you observed?

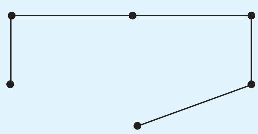
Revision Activity 7

1. Name the types of lines below. Why?

(a)



(b)



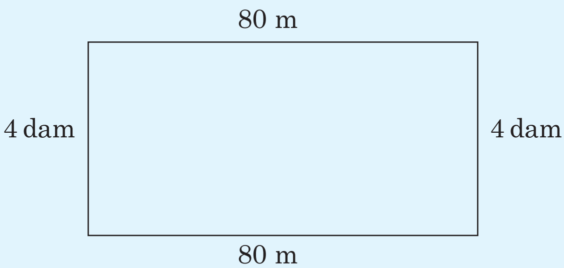
2. Convert:

(i) 25 dm = ___ hm (ii) 300 mm = ___ dam

(iii) 3.45 t = ___ kg (iv) 30 dl = ___ l

3. In an athletic competition, an athlete ran a distance of 80 dam. The length of her stride was 80 cm. How many strides did she take?

4. While walking along the road, I counted 91 trees in a straight line. The trees were equally spaced at intervals of 8 m. Find the distance of the road planted with trees. Explain your answer.
5. 41 poles were put up to fence one side of a field. The length of the side was 12 dam. Find the interval between the poles in metres.
6. Look at the piece of land below.



   It was to be fenced using posts at intervals of 3 m. How many posts were to be used? Present your work. What materials do you need for fencing?
7. In an open field, 7 trees were planted to prevent soil erosion. The interval between the trees was 5 m. Calculate the distance of field that was planted. Why should we prevent soil erosion?
8. 75 trees were planted around a field. They were equally spaced at intervals of 4 m. Calculate the distance of the field that was planted. Justify and present your answer.

Word list

Length	Capacity	Mass	Interval
Open line	Closed line	Distance	

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 8 Solving problems involving time intervals

8.1 Converting units of time

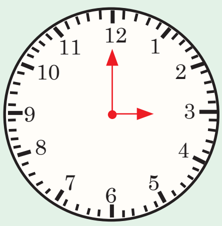
- In P4 we learnt units of time. We have various units of measuring time. They include seconds (s), minutes (min) and hours (h).
- Hours, minutes and seconds are related. Fill in the table.

1 hour	— minutes
— minute	60 seconds

(a) Converting hours into minutes

Activity 8.1

1. From the clock face below, identify the hour and minute hands.



How many minutes equal an hour? Explain.

2. Convert the following hours into minutes. Then present your findings.
   (a) 3 hours (b) 4 hours

Tip: 1 hour is equal to 60 minutes. 1 h = 60 min

Example 8.1

A tourist took 6 hours in her visit to the animal park. How many minutes did the tourist take in the park?

Solution

1 h = 60 minutes.

6 h = 6 × 60 min = 360 min

Practice Activity 8.1

Convert the hours given below into minutes.

1. 2 hours
2. $6\frac{1}{2}$ hours
3. 14 hours
4. 5 h
5. 12 h
6. 22 h
7. In a cross-country, an athlete run for 2 hours. How many minutes did the athlete run? Discuss the importance of participating in athletics.
8. A man drove a car for $5\frac{1}{4}$ hours from town A to town B. How much time in minutes did it take him? Justify your answer.
9. A traditional music festival lasted for $3\frac{1}{2}$ hours. How many minutes did the festival last? Why is it important to participate in traditional music festival?
10. A teacher took her pupils to visit Akagera National Park for 6 hours. How long did their visit last in minutes? Explain your answer.

(b) Converting minutes into hours

Activity 8.2

Convert the following minutes into hours. Discuss the steps followed.

(a) 360 minutes (b) 480 minutes

Tip: 60 minutes equals to one hour. $\boxed{60 \text{ min} = 1 \text{ h}}$

Example 8.2

(a) A National drama festival lasted for 180 minutes. How many hours is this?

(b) In a school, an annual general meeting lasted for 200 minutes. Calculate the number of hours and minutes it took. Explain your working steps.

Solution

(a) 60 min = 1 h.

180 min = ___ h.

180 min = (180 ÷ 60)h = 3 h.

$$\begin{array}{r|l} & \text{3 h} \\ \hline 60 & \text{180 min} \\ - & \underline{\text{180 min}} \\ & \text{00} \end{array}$$

(b) We know, 60 min = 1 h

200 min = ___ h ___ min

So 200 min is (200 ÷ 60) h

$$(200 \div 60) = \begin{array}{r|l} & \text{3 h} \\ 60 & \text{200 min} \\ - & \underline{\text{180 min}} \\ & \text{20 min} \end{array}$$

By dividing 200 by 60, we get 3 h and 20 min remained.

Thus, 200 min = 3 h 20 min.

Practice Activity 8.2

1. Convert the following into hours.
   (a) 120 min (b) 360 min (c) 840 min
   (d) 420 min (e) 240 min (f) 720 min
2. Convert the following into hours and minutes. Explain your answers.
   (a) 72 min (b) 130 min (c) 90 min
   (d) 61 min (e) 190 min (f) 320 min
3. A football match took 100 min. How many hours and minutes did the match take? Discuss your steps.
4. In a certain village, community work of cleaning the road took 425 minutes. How many hours and minutes did the work take? Why should we participate in community work? Explain.
5. Some workers were managing a river. They took 725 min to finish their work. How many hours and minutes did their work take? Explain your steps. Why should we manage our rivers?

8.2 Converting hours into seconds

Activity 8.3

1. Get a real clockface.
2. Identify the seconds hand (the longest hand)
3. Carefully observe the time it takes the second hand to make one minute.
4. Finally observe the number of seconds that it takes to make one hour.

Convert the following into seconds.

(a) 2 hours (b) 4 hours

What steps have you followed? Explain.

Tip: From table,

$1 \text{ h} = 60 \text{ min}$

$1 \text{ min} = 60 \text{ s}$

Therefore $1 \text{ h} = (60 \times 60)\text{s} = 3\,600 \text{ s}$.

Conversion fact: $\boxed{1 \text{ h} = 3\,600 \text{ s}}$

Example 8.3

(a) Convert 3 h into seconds.

(b) A public meeting took 6 h 50 s. How many seconds was the public meeting?

Solution

Conversion fact:
1 h = 3 600 s

(a) $3 \text{ h} = (3 \times 3\,600) \text{ s} = 10\,800 \text{ s}$

(b) $6 \text{ h} = 6 \times 3\,600 \text{ s} = 21\,600 \text{ s}$

The public meeting took 6 h 50 s.

Thus, $6 \text{ h } 50 \text{ s} = 21\,600 \text{ s} + 50 \text{ s}$

$= 21\,650 \text{ s}$

Practice Activity 8.3

1. Convert the following into seconds.
   (a) $1\frac{1}{2}$ hours (b) 5 hours (c) $10\frac{1}{4}$ hours
2. Convert the following into seconds. Discuss your steps.
   (a) 3 hours (b) 12 hours (c) $2\frac{3}{4}$ hours
3. A bus took 10 h 30 s to travel from Town A to Town B. Find the time in seconds that the bus travelled between the two towns.
4, A farmer spent 6 h 57 s to plant maize in his farm. How much time in seconds did the farmer take? Explain your steps to answer.
5. A tractor took 2 hours to dig a piece of land. Calculate how much time the tractor took in seconds?
6. A church prayer session lasted for $3\frac{1}{2}$ hours in the morning. How many seconds did the prayer session last? Why do people pray?

8.3 Changing days into hours

Activity 8.4

1. Discuss the number of hours that are there from Tuesday midnight up to Wednesday midnight.
2. I went to camp on Monday 8:00 a.m. I came back on Friday at 8:00 a.m.
   (a) How many days was I in the camp?
   (b) How many hours was I in the camp? Discuss your answer.

Tip: There are 24 hours in one day. $\boxed{1 \text{ day} = 24 \text{ h}}$

Example 8.4

(a) How many hours are in 5 days?

(b) Workers took $10\frac{1}{2}$ days to make terraces for controlling soil erosion. How many hours did the workers took?

Solution

(a) 1 day = 24 hours

So, 5 days $= 5 \times 24$ h

$= 120$ h

Therefore, 5 days have 120 hours.

(b) 1 day = 24 hours

So, $\frac{1}{2}$ day $= \frac{1}{2} \times 24$ hours

$= 12$ hours

10 days $= 10 \times 24$ h

$= 240$ h

Workers took $10\frac{1}{2}$ days.

Therefore, $10\frac{1}{2}$ days $= 12$ h $+ 240$ h

$= 252$ hours.

Practice Activity 8.4

1. Find the number of hours in the following:

   (a) 10 days (b) 15 days (c) 6 days (d) 4 days

   (e) 11 days (f) 13 days (g) $11\frac{1}{2}$ days (h) 28 days

2. Joyce became the best in her classroom performance. Her parents then permitted her to visit and stayed at her uncle's place for 14 days. How many hours did Joyce spend at her uncle's place? Why should you work hard?

3. A National music festival took place over $5\frac{1}{2}$ days. How many hours is this? Justify your answer.

4. Due to sickness, John was admitted to the hospital for $20\frac{1}{2}$ days. How many hours did John stay in the hospital? Discuss importance of hospitals.

5. An activity of making a road took one week in a certain place. How many hours did it take? Discuss why we should make roads.

8.4 Changing hours into days

Activity 8.5

(a) • How many hours are there from midnight to midday?
- How many hours are there from midday to midnight?
- How many hours are in 1 day? Discuss your findings.

(b) Now discuss and convert the following into days.

   (i) 24 h (ii) 48 h (iii) 120 h

(c) Present your findings.

Tip: A day has 24 hours. A day starts at midnight up to the next midnight.

$\boxed{24 \text{ h} = 1 \text{ day}}$

Example 8.5

(a) How many days are in 144 hours?

(b) A marriage counsellor took 182 hours to counsel two couples and solve marriage problem. How many days and hours were these?

Solution

(a) 24 hours = 1 day

168 h = ____ days

168 h = (168 ÷ 24) days

$$\begin{array}{r|l} & 6 \text{ days} \\ \hline 24 & 144 \text{ h} \\ & -\underline{144} \text{ h} \\ & 00 \end{array}$$

144 hours = 6 days

(b) 24 hours = 1 day

182 h = __ days __ hours

$$\begin{array}{r|l} & 7 \text{ days} \\ \hline 24 & 182 \text{ h} \\ & -\underline{168} \text{ h} \\ & 14 \text{ h} \end{array}$$

182 hours = 7 days 14 hours

Practice Activity 8.5

1. How many days are in
   (a) 120 hours? (b) 216 hours?
   (c) 720 hours? (d) 432 hours?
2. How many days and hours are in
   (a) 571 hours? (b) 612 hours? (c) 520 hours?
   (d) 192 hours? (e) 242 hours?
3. A youth camp took place over 312 hours. How many days did the camp take? Explain your answer.
4. A tour took 249 hours to visit East African countries by bus. How many days did the tour take? Explain your answer. Why should you tour other countries?
5. A boarding school went for mid-term holidays for 144 hours. How many days was the holiday? Discuss your answer.
6. A semi-desert place did not receive rain for 6 760 hours. How many days was the place without rain? Explain your steps. Why should we conserve our environment?

8.5 Finding time intervals

Activity 8.6

(a) What time do you go to school in the morning? What time do you go back home in the evening from school? Find the length of time between the two events.

(b) Find the time between the sunrise and the sunset. Justify your answer.

(c) How long is the time for your Mathematics lesson? Present your answer.

Tip:

- Duration – Length of time in which a particular event takes place.
  It is calculated as; Duration = Ending time – Starting time
- Starting and Ending time can be given different names. For example;
  (a) For a journey starting time is departure time. Ending time is arrival time.
  (b) For a match of football, starting time is kick-off time. Ending time is stoppage time.

Example 8.6

A doctor was on duty in hospital from 8:00 a.m. to 12:00 noon. How long was she on for duty?

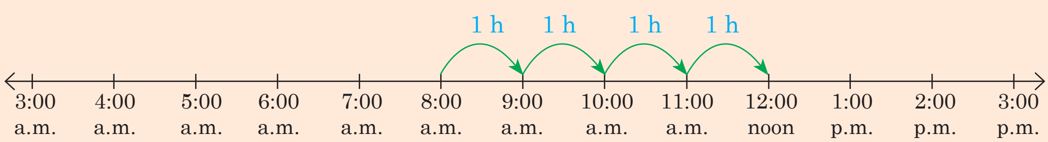
Solution

Method 1

We subtract

Duration = 12:00 – 8:00 = 4 hours

Method 2



The time is 4 hours.

Tip: Duration can be addition of lengths of time. For example;

Duration from 8:00 a.m. to 2:00 p.m. is

$(12 - 8)$ h + 2 h = $(4 + 2)$ h = 6 h.

Confirm this from the time number line above.

Practice Activity 8.6

1. How many hours are there from
   (a) 9:00 a.m. to 3:00 p.m.?
   (b) 4:00 a.m. to 1:00 p.m.?
   (c) 6:00 a.m. to 4:00 p.m.?
   (d) 7:00 a.m. to 6:00 p.m.?
2. A baby slept at 8:15 a.m. and woke up at 10:15 a.m. How long did the baby sleep? Explain your answer.
3. A train left from one station at 10:45 a.m. It arrived at the next station at 12:45 p.m.. Calculate the time it took from one station to the next.
4. A football match started at 2:00 p.m. and ended at 3:30 p.m. How long did the match take? Discuss your answer.
5. A class had lessons from 9:45 a.m. to 11:45 a.m. How long did the lessons take? Justify your answer.
6. A religious meeting started at 1:00 p.m. and ended at 6:30 p.m. How long was the meeting? Why should we be religious? Present your answer.
7. A mathematics lesson started at 10:00 a.m. and ended at 12:30 p.m. Calculate the duration of the lesson. Discuss why we should study Mathematics.

8.6 Addition involving time

Activity 8.7

- Study the questions below. Then answer the questions correctly.
  (a) A car travelled from town A to town B in 2 hours. It then travelled to town C in 3 hours. Find the total duration of travel.
  (b) A meeting took 5 h 45 min in the morning and 1 h 25 min in the afternoon. Find the duration for the meeting.

- Now add the following durations.

  (i) 2 h + 3 h =

  (ii) 5 h 45 min + 1 h 25 min = ____ h ___ min

  (iii) 3 h 30 min + 6 h 30 min = ____ h ___ min

  Explain your answers.

Example 8.7

(a) Add 6 h + 9 h

(b) A train travelled from town P to Q over 7 h 55 min. Then it travelled to town R over 2 h 15 min. Find the total time taken to travel from P to Q to R.

(c) A mathematics lesson started at 8:40 a.m.. It lasted for 1 h 20 min. At what time did the lesson end?

Solution

(a) 6 h + 9 h = 15 hours

(b) Time taken = 7 h 55 min + 2 h 15 min

= (7 h + 2 h) + (55 min + 15 min)

= 9 h + 70 min. (Now 70 min = 1 h 10 min)

= 9 h + 1 h 10 min = (9 h + 1 h) + 10 min

= 10 h 10 min

(c) Starting time = 8.40 a.m., duration = 1 h 20 min

Ending time = starting time + duration = 8:40 a.m. + 1 h 20 min

= 10:00 a.m.

$$\begin{array}{rl} & \overset{1}{8}{:}40 \text{ am} \\ + & 1{:}20 \\ \hline & 10{:}00 \text{ am} \\ \hline \end{array}$$

Note: 60 min = 1 h, so you carry over 1 h.

Practice Activity 8.7

Work out the following:

1. 2 h + 1 h
2. 6 h + 7 h
3. 1 h 30 min + 4 h 30 min = ___ h ___ min
4. 12 h 37 min + 3 h 48 min = ___ h ___ min

Work out the following. Discuss your steps.

5. 10 min 5 s + 30 min 5 s = ___ min ___ s
6. 14 h 18 min + 12 h 32 min = ___ day ___ h ___ min
7. In a car race, twenty cars drove from the first town to a second town. The winner took 1 h 20 min to reach the second town. He then drove to a third town in 1 h 45 min. Calculate the total time the winner drove.
8. We stayed in assembly for 20 minutes. We then took 30 minutes to clean the school compound. Calculate the duration of the two events.
9. A meeting started at 11:45 a.m.. It went on for 1 hour 45 minutes. What time did the meeting end? Explain your steps.
10. We went for lunch at 12:45 p.m. We took a 1 h 15 min lunch break. When did the lunch break end? Discuss your steps.

8.7 Subtraction involving time

Activity 8.8

Look at the following questions.

Work them out:

(a) 18 h 35 min – 11 h 45 min = _____ h ____ min

(b) 4 h – 1 h 30 min = _____ h ____ min

(c) A cross-country race ended at 12:35 p.m.. The duration of the race was 2 h 10 min. At what time did the race begin?

Present your answers.

Example 8.8

Work out:

(a) 9 h 15 min – 5 h 45 min = ____ h ____ min

(b) A meeting started at 8:00 a.m. It ended at 11 a.m. There was a health break from 9:00 a.m. to 9:30 a.m.

(i) How long was meeting?

(ii) Find the duration of the break.

(c) An aircraft left Kigali for Kenya on Monday. It then came back to Kigali and arrived at 12:40 a.m. on Tuesday. The journey had taken total time of 4 h 45 min. At what time did the aircraft leave Kigali for Kenya?

Solution

(a) 9 h 15 min – 5 h 45 min =

- Start with 15 min – 45 min. It is not possible. Borrow 1 h to have 60 min + 15 min = 75 min. So (75 – 45) min = 30 min.
- Remember you borrowed 1 h from 9 h. We have; 8 h – 5 h = 3 h.

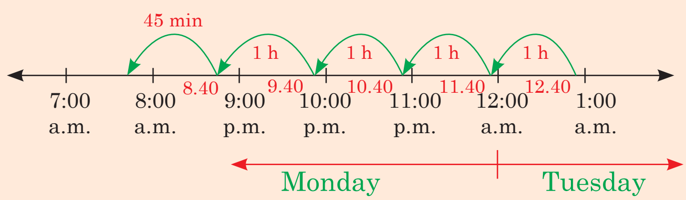
Thus, 9 h 15 min – 5 h 45 min = 3 h 30 min.

(b) (i) Duration = Ending time – starting time

= 11:00 a.m. – 8:00 a.m. = 3 h

(ii) 9:30 a.m. – 9:00 a.m. = 30 min

(c)



We subtract 4 h from 12:40 a.m..

We stop at 8:40 p.m. previous day (Monday).

We now have 8:40 p.m. – 45 min = 7:55 p.m.

The aircraft left Kigali at 7:55 p.m. on Monday.

Practice Activity 8.8

Work out the following.

1. 4 h – 2 h =
2. 8 h – 5 h =
3. 32 h 20 min – 20 h 10 min = ____ h ____ min

Work out the following. Explain your steps.

4. 16 h 30 min – 12 h 45 min = ____ h ____ min
5. 6 days 12 h – 3 days 9 h = ____ days ____ h
6. A volleyball match ended at 11:15 a.m. It was 3 h 20 min long. At what time did the match start?
7. A football match stopped at 7:15 p.m. in Amahoro Stadium. It was 2 h 26 min long. Discuss what time the match started?
8. A *taxi* took a total of 3 h from the time it left the airport to Kigali city. The *taxi* had stopped at a restaurant for 1 h 40 min for lunch.
   (a) Calculate the travel time of the *taxi*.
   (b) Solve: 3 h – 1 h 40 min = ___ h ___ min
9. A train arrived in a certain town at 4:30 p.m. It had taken 10 h 20 min of travel from the previous station. What time had it departed from the previous station? Discuss.
10. Work out 18 h 15 min – 4 h 38 min = ___ h ___ min. Present your answer.

Revision Activity 8

1. How many hours are there from
   (a) 9:00 a.m. to 3:00 p.m.?
   (b) 4:00 a.m. to 1:00 p.m.?
   (c) 10:00 a.m. to 4:00 p.m.?
   (d) 3:00 a.m. to 3:00 p.m.?
   (e) 6:00 a.m. to 4:00 p.m.?
2. Calculate how many hours until noon for the following times.
   (a) 4:00 a.m.
   (b) 8:00 a.m.
   (c) 6:00 a.m.
   (d) 11:00 p.m.
   (e) 7:00 a.m.
3. I woke up at 3:00 a.m. and two hours later the cock crowed. What time did the cock crow? Discuss.

4. Change into days.
   (a) 168 hours (b) 480 hours (c) 720 hours (d) 285 hours
5. Change into hours. Explain the steps followed.
   (a) 5 days (b) 12 days (c) 180 min (d) 1 440 min
   (e) 3 600 s (f) 7 200 s (g) 360 min (h) 25 200 s
6. Change into hours and minutes. Explain the steps followed.
   (a) 206 minutes (b) 156 minutes (c) 236 minutes
7. Change into minutes.
   (a) 15 hours (b) 6 hours (c) 10 hours (d) 15 hours
8. Change into seconds.
   (a) 2 min (b) 56 min (c) 6 h (d) 3 h
9. A farmer worked for four hours in the morning on Saturday. In the afternoon she worked for three hours. Find the total time the farmer worked on Saturday. Discuss and present your answer.
10. A competence mathematics activity started at 8:40 a.m. It ended at 9:55 a.m. Find the duration of the activity.

Word list

Units of time	Seconds	Minutes	Hours
Days	Duration	Time interval	
Length of time	Starting time	Ending time	

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 9 Money and its financial applications

9.1 Simple budgeting

(a) Uses and role of money in our lives

Activity 9.1

- Discuss the uses and roles of money in our lives.
- Now create a roleplay about the uses and roles of money in our lives.
- Present your roleplay to the class.

Example 9.1

Sibomana is a Primary 5 pupil. He was given 500 Frw by his uncle. State two ways he can use the money.

Solution

- Sibomana can use the money to buy a geometric set.
- He can use the money to buy exercise books, pens and pencils.

Practice Activity 9.1

1. State three ways a mother can use with 8 000 Frw.
2. State two ways a primary five pupil can use with 5 000 Frw.
3. State two roles of money in a family.
4. Explain two roles of money at a religious centre.
5. Explain ways in which a school uses money.

(b) Sources of money

Activity 9.2

1. Discuss different ways
   (i) An individual can get money.
   (ii) A family can get money.
   (iii) A school can get money.
2. Explain how money can help
   (i) individuals (ii) the family (iii) the school

Note:

- The different ways we can get money are known as sources of money.
- We should only get money through legal ways. Legal ways of earning money include working on farms, working at a job, doing business, etc.
- We should not get money through illegal sources. Illegal sources include stealing, robbing, taking bribes, corruption, etc.

Rule: Never get money through illegal sources.

Example 9.2

1. A primary five pupil stated the following ways of getting money.
   (i) Begging
   (ii) Working on a farm for pay.
   (iii) Stealing
   Which one is a legal source of money?
2. Explain why begging and stealing are not good ways of getting money.

Solution

1. Working on a farm for pay.
2. Begging encourages laziness. Stealing is a crime punishable according to the law.

Practice Activity 9.2

1. Dusabimana stated the following as ways of getting of money.
   - Doing business
   - Fishing
   - Begging
   - Washing a car for pay

   Which one is a bad source?
2. State two ways a family can get money.
3. Explain how a school can get money through school activities.
4. Explain ways through which a community can get money.
5. Gilbert named the following as sources of money:
   - Stealing bananas
   - Picking tea leaves for sale
   - Taking a bribe

   (a) Which one is a good source of money? Explain why.

   (b) Which of the above are bad ways to get money? Discuss your answers.

(c) Budgeting and setting priorities

Needs are things that we cannot do without. These include food, shelter and clothes.

Wants are things we would like to have, but we can do without them. This may include toys, video games, ice creams and radio.

Money should be spent on the most important things first. After our needs are taken care of, we can choose to spend on wants. This is called setting priorities.

Activity 9.3

- Study the needs and wants below.

  Food, clothes, car, television set, ice cream, shelter, school fees, shoes, education, water.

  (i) Which are needs?

  (ii) Which are wants?

  (iii) List the needs and wants in order of priority.

  (iv) Explain why you classified the items as wants or needs.

Example 9.3

A family got 50 000 Frw from sales at their shop.

The family had the following projects they wanted to do:

(i) Painting house at the cost of 12 000 Frw.

(ii) Buying family food for 20 000 Frw.

(iii) Buying clothes for 12 000 Frw.

(iv) Paying school fees worth 16 000 Frw.

(a) Order the family needs according to priority.

(b) (i) How much money does the family require to meet their budget?

(ii) Do they have the required amount?

(iii) What can the family do?

(c) State the item that can be done later and explain why.

Solution

(a) (i) Buying family food

(ii) Buying clothes

(iii) Paying school fees

(iv) Painting house

(b) (i) (12 000 + 20 000 + 12 000 + 16 000) Frw = 60 000 Frw.

(ii) They don't have enough money to do everything.

(iii) Budget money according to priority.

(c) Painting house at the cost of 12 000 Frw. From budget 2 000 Frw is available. The family can wait and do it once they have more money.

Practice Activity 9.3

1. List three most important needs.
2. Nzigiyimana plans to do the following.

(i)	Buy food for his family	(ii)	Buy a television set
(iii)	Pay for holiday trip	(iv)	Buy a car
(v)	Buy clothes	(vi)	Construct a house

(a) List Nzigiyimana's needs in order of priority.

(b) List Nzigiyimana's wants in order of priority.

3. Suppose your family has 20 000 Frw. List three things your family can spend the money in order of priority.
4. Suppose you have 3 000 Frw. List the things you can do with the money. Start from the most important to the least important. Explain why the items have been classified as most important or least important.
5. A school plans to do the following projects. They have 200 000 Frw to spend.
   - (i) Dig a well of water at 35 000 Frw.
   - (ii) Paint two classrooms at 40 000 Frw.
   - (iii) Construct a toilet at 70 000 Frw,
   - (iv) Buy 20 school desks at 60 000 Frw.

   (a) Order the above projects according to priority.

   (b) How much more money does the school need to do all projects?

   (c) State which project can be done later. Then explain why it can be done later.

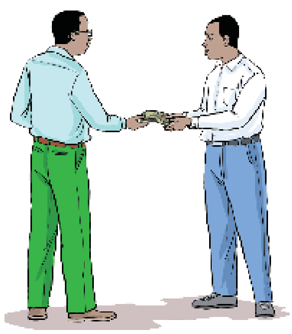
9.2 Ways of transferring money

Activity 9.4

Study the following ways of transferring money.

(a)

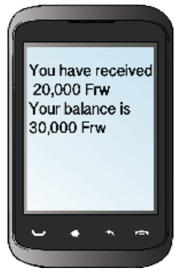
(b)



(c)

(d)

(e)



(i) Name each of the ways used to transfer money in diagrams above.

(ii) Discuss how money can be transferred using the methods above.

(iii) From the methods above, explain the most convenient way of transferring

(a) a large sum of money.

(b) small sums of money.

Practice Activity 9.4

1. State 2 methods Ndayisaba can use to send money to his cousin.
2. State a method Paul can use to send money for school fees to his daughter at school.
3. Dusabimana has 200 000 Frw. State the most convenient method she can use to transfer these money to her mother. Justify your answer.
4. Mugiraneza urgently wants to send 10 000 Frw to his worker. State one method he can use. Explain your choice.
5. Amina wants to send 5 000 Frw to her father. State one method she can use. Explain why you chose that method.

9.3 Saving and borrowing money

Activity 9.5

Mugiraneza and Niyirera each had 1 000 Frw from their parents. Mugiraneza bought a ruler worth 300 Frw. He also bought biscuits worth 400 Frw for his friends. Niyirera bought a ruler worth

300 Frw and saved the rest. During the week, their teacher asked them to buy a geometrical set worth 500 Frw.

(i) How much did each pupil spend?

(ii) How much did Mugiraneza save?

(iii) How much did Niyirera save?

(iv) How much does Mugiraneza need to borrow from Niyirera? Why does he need to borrow money?

(v) Who spent the money wisely. Explain your answer.

Hint:

Borrowed money is not free. It has to be returned to the lender.

Example 9.4

Musabe earns a salary of 150 000 Frw in a month. He spends his money as follows:

Rent: 30 000 Frw

School fees: 35 000 Frw

Food: 25 000 Frw

Transport: 15 000 Frw

He saves the remaining money.

(i) How much does he spend in total each month?

(ii) How much does he save each month?

(iii) Why do you think it is important for Musabe to save?

Solution

(i) Money spent = (30 000 + 35 000 + 25 000 + 15 000) Frw
= 105 000 Frw

(ii) Savings = Money earned – money spent
= (150 000 – 105 000) Frw
= 45 000 Frw

(iii) For future use or to use in case of an emergency.

Practice Activity 9.5

1. Look at the flash cards below. They contain different ways of saving and borrowing money.

A	B	C
Only spend on what you can afford.	Bank loans	Walking to school to save on transport

D	E	F
Avoid impulse buying	Peer to peer lending	Working in farms to assist parents

(a) Which flash cards show how to save money?

(i) ____

(ii) ____

(iii) ____

(b) Which flash cards show how one can borrow money?

(i) ____

(ii) ____

(iii) ____

2. Discuss the importance of saving money.
3. Explain the importance of borrowing money.
4. Imanarera earns 100 000 Frw in a month. She spends 20 000 Frw on rent, 25 000 Frw school fees, 15 000 Frw on transport and 18 000 Frw on food.

(i) How much did she spend altogether?

(ii) How much did she save?

(iii) Explain why it is important for Imanairere to save money.

9.4 Different currencies and converting currencies

Activity 9.5

- Study these currencies. They are from different countries. Name these currencies.

- Present your answers to the class.

Tip:

Different currencies have different values in relation to our currency (Frw).

Let us now look at the following activity.

Activity 9.6

Study the table below. It was displayed in a Forex shop in Kigali on 14/02/2016 at 10.00 a.m.

Currency	Frw
1 USD	750.20
1 EUR	850.70
KSh 1	7.56
1 UGSh	0.23

(i) Kamanutsi had 300 USD. How many Frw are these?

(ii) Mukahirwa had 20 000 Frw. How many EUR are these?

(iii) Nzikobankunda had 600 UGX. How many Frw are these? Explain your steps.

(iv) Which one was the strongest currency as compared to the Frw? Justify your answer.

Tip:

You can convert Frw to any other currency. Always be sure to use current exchange rates.

Example 9.5

Convert 10 000 Frw into

(i) US dollars (USD)

(ii) Euros (EUR)

(iii) Kenya shillings (KSh)

(iv) Uganda shillings (UGSh)

Exchange rates as on 10/1/2016 at 3 p.m

Currency	Frw
1 USD	740
1 EUR	836
KSh 1	7.25
4.60 UGSh	1

Solution

(i) 740 Frw = 1 USD

Thus, 10 000 Frw = (10 000 ÷ 740) USD = 13.51 USD

Practically, this would be given as 13 USD and you would lose 0.51 USD. But you can give an extra 370 Frw and get 14 USD.

(ii) 836 Frw = 1 EUR

Thus, 10 000 Frw = (10 000 ÷ 836) EUR = 11.96 EUR.

(iii) 7.25 Frw = KSh 1

Thus, 10 000 Frw = KSh (10 000 ÷ 7.25) = KSh 1 379.31

Practically, this is converted as KSh 1 379.30 or more easily as KSh 1 379.

(iv) 1 Frw = 4.60 UGSh

Thus, 10 000 Frw = 10 000 × 4.60 UGSh = 46 000 UGSh

Practice Activity 9.6

Use the exchange rates given in Example 9.5 in this activity

1. Convert 5 000 Frw into
   (i) Kenyan shillings
   (ii) Ugandan shillings
   (iii) Euros
   (iv) US dollars

Convert the following into Rwandan Francs. Discuss why one would convert his/her currencies into Frw.

2. 100 USD
3. 50 EUR
4. 20 000 UGSh
5. KSh 2 000

Revision Activity 9

1. State two uses of money for a primary 5 pupil.
2. State two uses of money for a family.
3. State two uses of money for a school.
4. State two sources of money.
5. Write two ways a family can get money.
6. State two ways a school can get money.
7. Below are some sources of money. Which one is not a good way of getting money? Explain your answer.
   – Working for pay on a farm.
   – Salary from employment.
   – Slashing overgrown grass for a wage.
   – Stealing from a friend.
8. The following are needs and wants. State and explain the most important need.
   - Food
   - Clothes
   - Television
   - House
9. Daliya has the following needs and wants. List them from the most important to the least important. Then explain your reasoning.
   - Holiday camp
   - School fees
   - Food
   - Clothes

10. State three ways of transferring money from one destination to another.
11. State and describe two ways of saving money.
12. State and explain two ways of borrowing money.
13. Explain why you should budget before spending.
14. Study the currency exchange table below. It was observed on 20/1/2016 at 11 a.m in a Forex shop.

Currency	Rate (Frw)
1 USD	740
1 EUR	830
KSh 1	7.25
4.60 UGSh	1

(a) Convert 15 000 Frw into
   (i) EUR (ii) USD (iii) KSh (iv) UGSh
(b) Convert KSh 500 into Frw. Discuss your answer.
(c) Convert 20 000 UGSh into Frw. Discuss your answer.

Word list

Simple budgeting	Priorities	Wants	Needs
Sources of money	Uses/roles of money	Setting priorities	
Transferring money	Different currencies	Converting currencies	

Task

Do the following.

(i) Read each word aloud to your friend.
(ii) Write the meaning of each of the words above. Discuss with your friend.
(iii) Write sentences using each of the words above. Read with your friend.

UNIT 10

Sequences that include whole numbers, fractions and decimals

10.1 Ordering whole numbers according to their size in increasing order

Activity 10.1

1. Rwandan athletes participated in the national athletics trials. They participated in the following races:
   10 000 m, 800 m, 100 m, 200 m, 400 m and 5 000 m.
   Arrange the races in order of increasing distance. Justify your answer.
2. Discuss situations where you arrange quantities in increasing order.

Tip:

We can arrange numbers in increasing order. This is done by ordering/ arranging them from the smallest to the largest.

Example 10.1

Describe how the following numbers can be arranged in increasing order.

46 295, 45 690, 68 925

Solution

By checking the digits in the highest place value to the digits in the lowest place value.

The numbers 46 295, 45 690, 68 925 arranged in increasing order are 45 690, 46 295, 68 925.

Practice Activity 10.1

Arrange the numbers below in increasing order.

1. 5 000 m, 4 000 m, 9 000 m

2. 637 045, 705 365, 673 045, 637 450
3. 491 279, 137 004, 397 080, 491 792
4. 26 734, 62 374, 62 347, 63 437
5. 431 209, 413 209, 431 290, 413 029
6. 584 039, 548 039, 854 390, 458 309

Describe how to arrange the following in increasing order.

7. 783 165, 738 165, 783 615, 731 865
8. 627 558, 627 585, 672 558, 672 855
9. 97 862, 83 052, 78 962, 97 628
10. 413 500, 431 500, 134 500, 351 400

10.2 Ordering whole numbers according to their size in decreasing order

Activity 10.2

1. The table below shows the number of soccer fans who attended the CECAFA tournament. The data is for the first four matches.

Matches	Number of fans (attendance)
1st Match	9 284
2nd Match	10 150
3rd Match	10 020
4th Match	12 345

- Which match had
  - (i) the lowest attendance?
  - (ii) highest attendance?
- Now arrange the match attendance in decreasing order. Justify your answer.

2. Tell examples where you can arrange quantities in decreasing order.

Tip:

We can arrange numbers in decreasing order. This is done by arranging the numbers from the largest to the smallest.

Example 10.2

Explain how to arrange the following in decreasing order.

43 250, 42 420, 43 502, 40 352

Solution

By checking the numbers from the digit with the highest value to the lowest value.

The numbers arranged in decreasing order are

43 502, 43 250, 42 420, 40 352

Practice Activity 10.2

1. Arrange the following numbers in decreasing order.
   (a) 213 456, 213 564, 213 546, 213 645
   (b) 23 451, 23 514, 23 145, 23 415
   (c) 860 720, 806 720, 860 270, 860 027
2. Arrange the following in decreasing order. Discuss your steps.
   (a) 602 097, 632 097, 602 039, 600 397
   (b) 708 540, 785 040, 780 504
   (c) 234 567, 243 567, 235 467
3. Four farmers sold their farm produce. The money they got are recorded in the table below.

Farmer	1st	2nd	3rd	4th
Money (FRw)	150 150	155 100	115 500	150 015

   (a) Order the money of the four farmers in decreasing order.
   (b) Which farmer got the highest amount of money. Explain your answer.

10.3 Simple sequences that include fractions

Activity 10.3

- Discuss the sequence given below and the pattern used.

  $10, 11\frac{1}{2}, 13, 14\frac{1}{2}, 16, 17\frac{1}{2},$ ___, ___

- Write more numbers on flash cards with a pattern that follows $1\frac{1}{2}$ in increasing order.

- Formulate more tasks on sequences with a pattern that follows $1\frac{1}{2}$ in increasing order.
- Explain the pattern used.

Example 10.3

Study the sequence below.

$20, 21\frac{1}{2}, 23, 24\frac{1}{2}, 26, 27\frac{1}{2},$ ____

Explain the steps involved to find the next number.

Solution

Find the differences in between the numbers.

$20, 21\frac{1}{2}, 23, 24\frac{1}{2}, 26, 27\frac{1}{2},$ ____

Difference $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$

(To get the next number, you add $1\frac{1}{2}$ to the previous number).

The next number will be: $27\frac{1}{2} + 1\frac{1}{2}$

$= 29$

Practice Activity 10.3

Find the next numbers in the sequences below.

1. $15, 16\frac{1}{2}, 18, 19\frac{1}{2}, 21,$ ___, ___
2. $36, 34\frac{1}{2}, 33, 31\frac{1}{2}, 30,$ ___,
3. $25, 26\frac{1}{2}, 28, 29\frac{1}{2},$ ___, ___
4. $60, 61\frac{1}{2}, 63, 64\frac{1}{2}, 66,$ ___

Explain the steps involved in calculating the next numbers.

5. $50, 52\frac{1}{2}, 55, 57\frac{1}{2}, 60,$ ___, ___
6. $25\frac{1}{4}, 25, 24\frac{3}{4}, 24\frac{1}{2}, 24\frac{1}{4},$ ___

7. 40, $41\frac{1}{2}$, 43, $44\frac{1}{2}$, 46, $47\frac{1}{2}$, ___

8. 70, $71\frac{1}{2}$, 73, $74\frac{1}{2}$, 76, $77\frac{1}{2}$, ___

9. 100, $101\frac{1}{2}$, 103, $104\frac{1}{2}$, 106, $107\frac{1}{2}$, ___

10. 80, $82\frac{1}{2}$, 85, $87\frac{1}{2}$, 90, ___

10.4 Simple sequences that include decimals

Activity 10.4

- Discuss the sequence given below and discover the pattern used.
  10, 10.5, 11, 11.5, 12, 12.5, ___, ___
- Form your own sequences involving decimals. Make presentation to the class.

Example 10.4

Find the next numbers in the sequence below.

5, 5.5, 6, 6.5, 7, 7.5, ___, ___

Solution

Find the difference in between the numbers.

	5,		5.5,		6,		6.5,		7,		7.5,		___,	___
Difference		0.5		0.5		0.5		0.5		0.5				

The numbers are increasing by 0.5. Find the next number in the sequence by adding 0.5 to 7.5. Then find the following number.

$7.5 + 0.5 = 8$

$8 + 0.5 = 8.5$

The next numbers in the sequence are 8 and 8.5. The sequence is 5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5

Practice Activity 10.4

Find the next numbers in the sequences below.

1. 1, 1.5, 2, 2.5, 3, 3.5, 4, ___, ___
2. 14.5, 13, 11.5, 10, 8.5, 7, ___, ___
3. 2.5, 5, 7.5, 10, ___
4. 21, 21.5, 22, 22.5, 23, ___

Find the next number in the sequences below. Explain your steps.

5. 70, 73.5, 77, 80.5, 84, 87.5, ___, ___
6. 19, 18.5, 18, 17.5, 17, 16.5, 16 ___
7. 30, 30.5, 31, 31.5, 32, 32.5, ___
8. 90, 90.5, 91, 91.5, 92, 92.5, ___, ___
9. 80, 80.5, 81, 81.5, 82, 82.5, ___, ___
10. 50, 50.5, 51, 51.5, 52, 52.5, ___

10.5 Sequence with constant differences

Activity 10.5

- Discuss the sequence given. Discover the pattern used and find the next number.

  25, 28, 31, 34, 37, ___
- Now, form your own sequences with constant differences. Then make a presentation to the class.

Tip: Sequences with constant differences are called arithmetic progressions.

Example 10.5

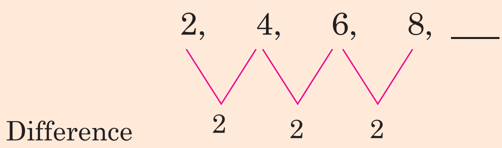
Express the steps involved in getting the next number in the sequence below.

2, 4, 6, 8, _____

Solution

Method 1

Steps:
- Find the difference between two consecutive numbers.
- Observe the pattern of the differences.



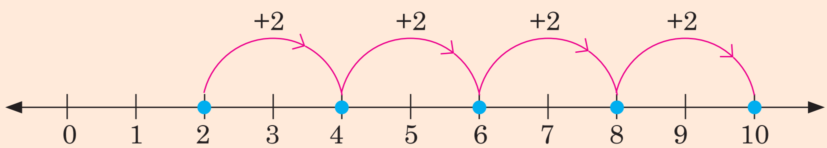
Observation: The difference is 2. Therefore we add 2 to 8. So, 8 + 2 = 10

The sequence is 2, 4, 6, 8, 10.

Method 2

We find the missing number using a number line.

On a number line, we have;



Moving in +2 steps. The pattern is 2, 4, 6, 8, 10.

Practice Activity 10.5

Find the missing numbers in the sequences below.

1. 35, 37, 39, 41, 43, ___, ___
2. 7, 11, 15, 19, 23, ___, ___
3. 20, 25, 30, 35, 40, ___, ___
4. 70, 74, 78, 82, 86, ___, ___
5. 25, 28, 31, 34, 37, ___, ___

Find the mising numbers in the sequences below. Explain your steps.

6. 40, 43, 46, 49, ___, ___
7. 60, 64, 68, 72, ___
8. 2, 5, 8, 11, 14, ___
9. 52, 57, 62, 67, ___, ___
10. 22, 28, 34, 40, 46, ___

10.6 Sequences with constant ratios

Activity 10.6

Look at the sequences given below.

(a) 1, 2, 4, 8, 16, ___, ___, ___ (b) 3, 9, 27, 81, ___

(i) Find the missing numbers.

(ii) Describe the pattern you have discovered.

(iii) Form your own sequences with the pattern you discovered. Then make a presentation to the class.

Tip: We can have a sequence with constant ratios. These are called geometric progressions.

Example 10.6

Discuss the steps involved to extend the sequences below.

5, 15, 45, ___

Solution

Find the constant ratio by dividing each number by the previous one.

$\frac{15}{5} = \frac{45}{15} = 3$

The constant ratio is 3. Multiply $45 \times 3 = 135$

Therefore the sequence is 5, 15, 45, 135.

Practice Activity 10.6

Find the next numbers in the following sequences.

1. 1, 2, 4, 8, ___, ___
2. 16, 32, 64, ___
3. 3, 9, 27, 81, ___
4. 4, 16, 64, ___

Find the missing numbers in the sequences below. Explain your steps.

5. 1, 5, 25, 125, ___
6. 10, 100, 1 000, ___
7. 176, 88, 44, 22, ___, ___
8. 2, 8, 32, ___, ___
9. 3, 27, 243, ___
10. 6, 12, 24, ___, ___

10.7 Sequences with regularly changing differences

Activity 10.7

- Discuss the patterns in the sequences below. Find the next numbers in the sequences.

  (i) 1, 3, 6, 10, 15, ___ (ii) 4, 5, 8, 14, 24, ___

  What do you notice?
- Study the sequence: 2, 3, 5, 7, 11, ___, ___. Explain the rule used in finding the sequence and make a presentation.

Example 10.7

Describe the steps involved to find the next numbers in the sequence below.

2, 3, 6, 12, 22, ___, ___

Solution

Steps:
- Find the difference between two consecutive numbers.
- Observe the pattern of the differences.

2, 3, 6, 12, 22

Difference $\underbrace{1 \quad 3}_{2} \underbrace{\quad 6}_{3} \underbrace{\quad 10}_{4}$

Observation: The difference is increasing. Numbers added to get difference has number added greater by 1 than previous. That is we add 2, then 3, then 4. So add 5, then 6 as shown.

2, 3, 6, 12, 22 37, 58

Changing difference 1 3 6 10 15 21

Arithmetic difference $\underbrace{2 \quad 3}_{+1} \underbrace{\quad 4}_{+1} \underbrace{\quad 5}_{+1} \underbrace{\quad 6}_{+1}$

Hint:
2 + 1 = 3
3 + 3 = 6
6 + 6 = 12
12 + 10 = 22
22 + 15 = 37
37 + 21 = 58

Thus, the sequence is: 2, 3, 6, 12, 22, 37, 58.

Practice Activity 10.7

Find the next numbers in the sequences below.

1. 1, 4, 10, 19, ___, ___
2. 12, 13, 16, 22, 32, ___, ___
3. 50, 52, 55, 59, ___, ___
4. 8, 11, 15, 20, 26, ___, ___

Discuss the patterns in the sequences below. Then find the missing numbers.

5. 20, 23, 27, 32, 38, ___
6. 70, 75, 81, 88, 96, ___, ___
7. 31, 32, 35, 41, 51, ___
8. 44, 45, 48, 54, 64, ___
9. 62, 63, 66, 72, ___
10. 100, 101, 104, 110, 120, ___

10.8 Sequences where the difference is geometric

Activity 10.8

- Look at the sequence: 11, 23, 47, 95, ___.
- Find the difference between consecutive numbers. What pattern is the difference? Explain your steps.
- Now, find the next number.

  11, 23, 47, 95, ___
- Form your own sequences like the sequence above. Make posters for your sequences and present to the class.

Example 10.8

Find the next number in the sequence below. Explain your steps.

10, 21, 43, 87, ___

Solution

Steps:

- Find the difference between consecutive numbers.
- Observe the pattern of the differences.
- Find the next number using the pattern.

10, 21, 43, 87, $(88 + 87) = 175$

Difference 11 22 44 $44 \times 2 = 88$

The difference follows a geometric pattern. The next difference is $44 \times 2 = 88$.

So add $88 + 87 = 175$.

Thus, the sequence is:

10, 21, 43, 87, 175.

Practice Activity 10.8

Find the next numbers in the sequences below.

1. 1, 3, 7, 15, ___
2. 2, 5, 11, 23, ___, ___
3. 3, 7, 15, 31, ___, ___
4. 6, 13, 27, 55, ___
5. 12, 25, 51, 103, ___

Find the next number in the sequences below. Discuss your steps and present.

6. 5, 11, 23, 47, ___
7. 8, 17, 35, 71, ___
8. 20, 41, 83, 167, ___
9. 7, 15, 31, 63, ___
10. 4, 9, 19, 39, ___

Revision Activity 10

1. Arrange the following from the smallest to the largest.
   (a) 2 300, 3 200, 2 003, 3 002
   (b) 5 732, 7 532, 7 352, 5 372

2. Arrange the following in decreasing order.

(a) 9 481, 9 841, 9 148, 9 099

(b) 23 452, 23 425, 23 245, 25 254

(c) 11 000, 10 100, 10 010, 10 001

3. Find the next number in the sequence below. Explain your steps.

(a) 5, $6\frac{1}{2}$, 8, $9\frac{1}{2}$, ___

(b) 30, $31\frac{1}{2}$, 33, $34\frac{1}{2}$, 36, ___

(c) 40, $41\frac{1}{2}$, 43, $44\frac{1}{2}$ 46, ___

(d) 7, 7.5, 8, 8.5, 9, ___

(e) 14, 17, 20, 23, ___

(f) 52, 60, 68, 76, ___

(g) 30, 31, 33, 37, 45, ___

(h) 12, 13, 16, 22, 32, ___

(i) 11, 23, 47, 95, ___

(j) 21, 43, 87, 175, ___

(k) 7, 15, 31, 63, ___

4. Use a number line to find the next number in the following sequences. Explain your pattern.

(a) 3, 6, 9, 12, ___

(b) 1, 5, 9, 13, ___

5. Use geometric patterns to determine the next number in the sequences below. Discuss your patterns.

(a) 1, 4, 9, 16, ___

(b) 3, 7, 11, 15, ___

Word list

Sequence	Fractions	Decimals
Increasing	Decreasing	Constant ratios
Geometric difference		

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

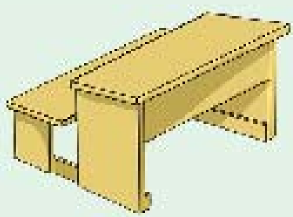
UNIT 11 Drawing and construction of angles

11.1 Parallel lines, intersecting lines and transversals

Activity 11.1

- Observe the tables, desk, chairs and walls of the class.





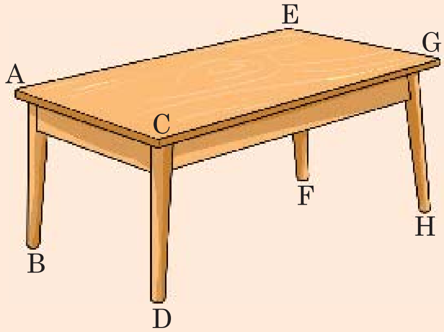

- You can observe other objects in your class like boxes or cartons.
- Identify different lines such as;
  (i) straight lines.
  (ii) lines that meet.
  (iii) lines that do not meet.
  (iv) Present your findings.

Tip:

- A line joins two points on a flat surface.
- When the lines do not meet, they are said to be parallel.
- When two lines meet, we say they have intersected.
- When straight lines intersect they form angles.

Example 11.1

Below is a picture of a table. Study and answer the following questions.



(i) Identify the lines that do not meet.
(ii) Identify the lines that meet.
(iii) Identify lines that cut parallel lines.

Solution

(i)
- Lines AB, CD, EF and GH do not meet.
- Lines AC and EG do not meet.
- Lines AE and CG do not meet.

(ii)
- Lines AB, AE and AC meet at A.
- Lines AC, GC and DC meet at C.
- Lines AE, GE and FE meet at E.
- Lines CG, HG and EG meet at G.

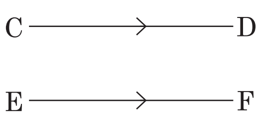
(iii) Lines that cut parallel lines are:
- CG cutting DC and HG.
- EG cutting FE and HG.
- AE cutting AB and FE.
- AC cutting AB and DC.

Tip:

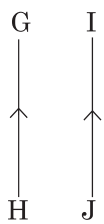
(i) A straight line

(ii) Parallel lines

(a)

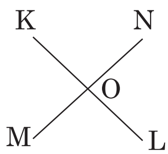


(b)



Parallel lines do not meet. CD is parallel to EF. We write CD//EF.

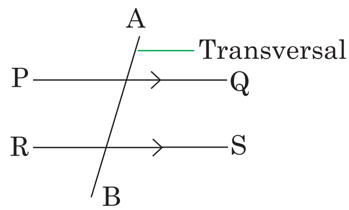
(iii) Intersecting lines



Intersecting lines meet at a point. Point O is intersection point.

(iv) The transversal

- The transversal cuts parallel lines.
- PQ//RS
- AB is transversal to PQ and RS.



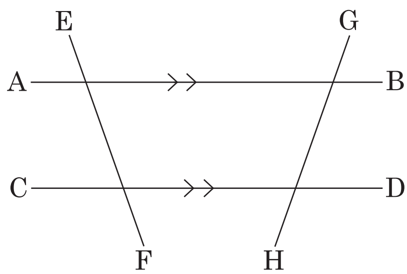
Task: Now, draw your own:

(i) parallel lines (ii) intersecting lines (iii) transversal

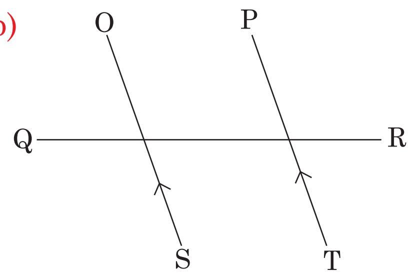
Practice Activity 11.1

1. Name lines that are parallel to each other from the diagrams below.

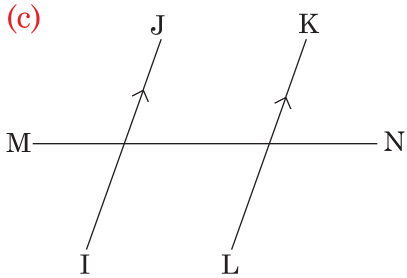
(a)



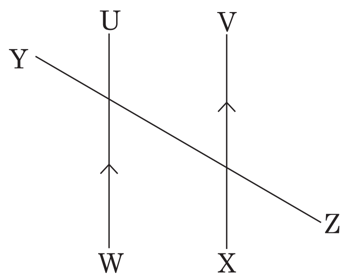
(b)



(c)

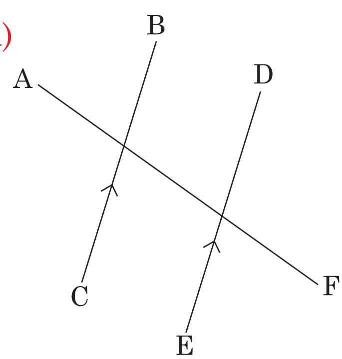


(d)

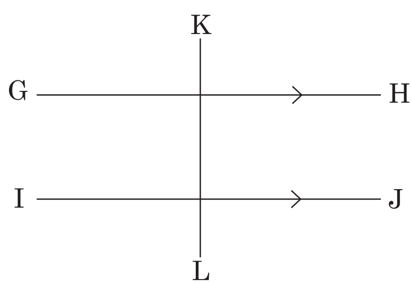


2. Identify the transversals in the diagrams below.

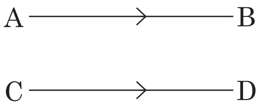
(a)



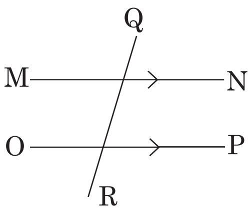
(b)

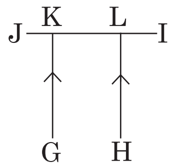


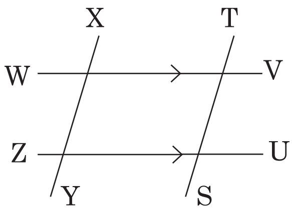
3. Observe the diagrams below. Explain and justify straight lines, parallel lines and intersecting line.

(a) 

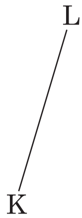
(b) 

(c) 

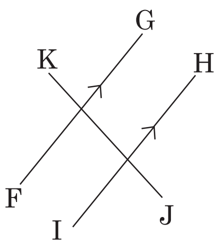
(d) 

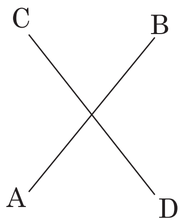
(e) 

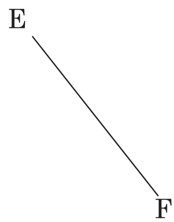
(f) 

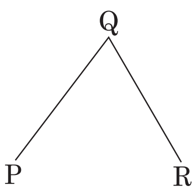
(g) 

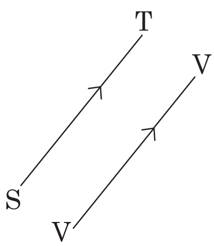
4. Explain how you would identify intersecting lines from the lines drawn below.

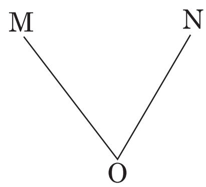
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

11.2 Perpendicular lines

Activity 11.2

- Observe the walls of the classroom. Measure the angle where the walls are meeting. What do you notice? Present your observation.
- Observe the windows of the classroom. Look at the corners where the frames join each other. Measure the angle. What is the angle?

Example 11.2

Observe the rectangle drawn below.

Measure each of the angles at the corners of rectangle using a protractor. Present what you notice about each angle to the class.

Solution

Each of the angles is 90°.

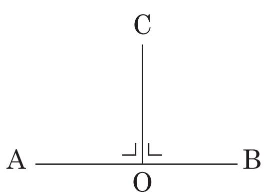
Tip:

Lines AB meet line BD forming a right angle. A right angle is 90°.

(a) From the rectangle in Example 11.2, we note:

- Lines that intersect at 90° are called perpendicular lines.
- Line AB is perpendicular to line BD.
- Line BD is perpendicular to line DC.
- Line BA is perpendicular to line AC. Give more examples.

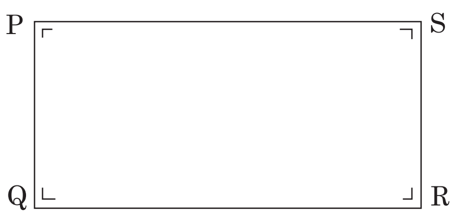
(b) Lines CO and AOB are perpendicular in the figure below. The symbols in the diagram shows the angles are 90°.



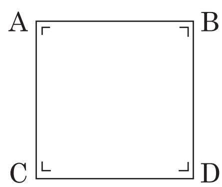
Practice Activity 11.2

1. Identify the perpendicular lines in the following diagrams.

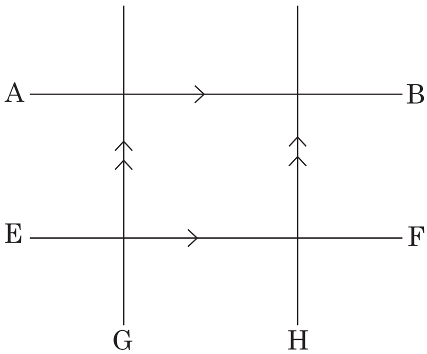
(a)



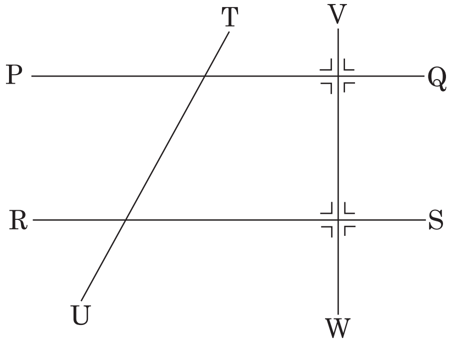
(b)



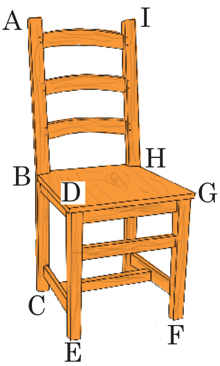
(c)



(d)



2. Observe the frame of the chair below.

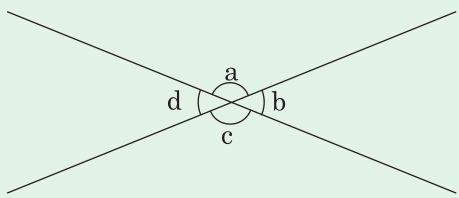


Which lines are perpendicular to each other? Explain.

11.3 Properties related to angles formed by intersecting lines

Activity 11.3

Study the figure below. Draw any two intersecting lines. Note angles a, b, c and d.

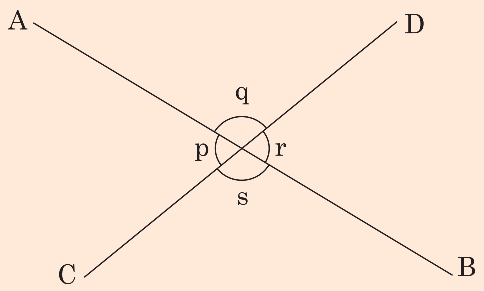


(i) Use a protractor to measure the angles a, b, c, d.
What do you notice about: a and c, and d and b?

(ii) Add angle; a + b, c + d, a + b + c + d and give the sum. Present your results.

Example 11.3

Two straight lines AB and CD intersect as shown below.



(i) Use a protractor to measure the angles p, q, r and s.

(ii) Find the sum of p + q, r + s.

(iii) What do you notice about angle q and s?

(iv) What do you notice about angle p and r?

(v) Find the sum of angle p, q, r and s.

Solution

(i) Angle p = 70°

Angle q = 110°

Angle r = 70°

Angle s = 110°

(ii)
- Angle p + angle q:

  $p + q = 70° + 110°$

  $= 180°$
- Angle r + angle s:

  $r + s = 70° + 110°$

  $= 180°$

(iii) Angles q and s are equal.

(iv) Angles p and r are equal.

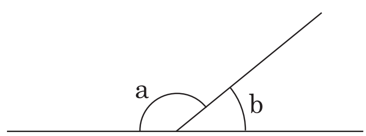
(v) Sum of angles p, q, r, s

$p + q + r + s = 70° + 110° + 70° + 110°$

$= 360°$

Tip:

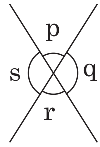
(i) Angles on straight line add up to 180°. They are called supplementary angles.



$a + b = 180°$

(ii) • Angles at a point add up to 360°.

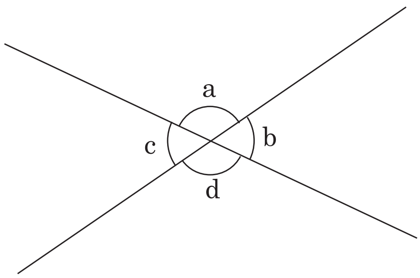
• Angles p and r are equal.
Angles s and q are equal.



Angles p and r or s and q are called opposite angles.

Practice Activity 11.3

1. Use a protractor to measure the angles shown by the letters below.



(a) Angle a =

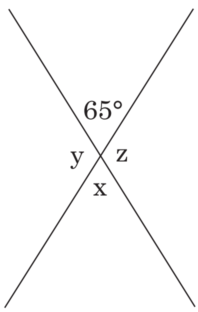
(b) Angle b =

(c) Angle c =

(d) Angle d =

(e) What do you notice about angle a and angle d?

2. In the diagram below the value of one of the angles is given.



(a) Explain the steps involved to find the size of the following:

(i) Angle x =

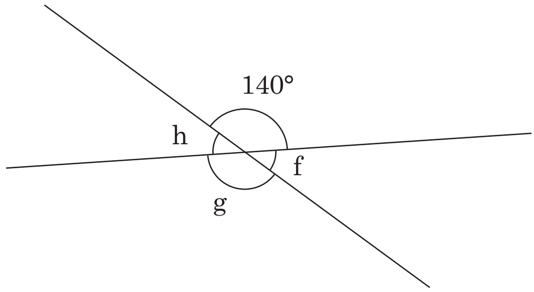
(ii) Angle y =

(iii) Angle z =

(b) Angle y is opposite to angle ___.

(c) Angle x + angle z = ___

3. Find the size of angles marked f, g and h. Explain your steps.



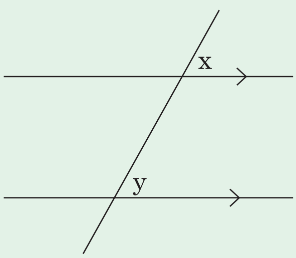
11.4 Angle properties of parallel lines

Corresponding angles

Activity 11.4

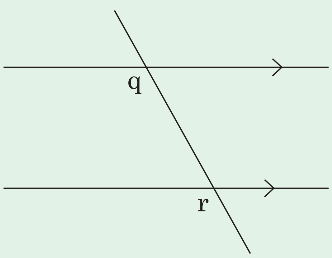
Measure and discuss the angles marked with letters given in the diagrams below.

(i)



What do you notice about the size of angle x and y?

(ii)

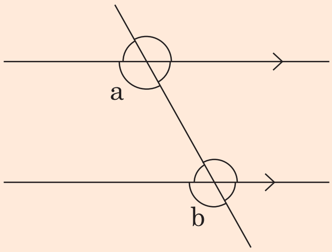


Explain what you can notice about the size of angle q and r.

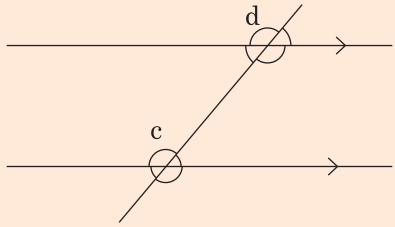
Example 11.4

Use a protractor to find the angles marked with letters and explain their relationship.

(a)



(b)



Solution

(a) a = 120°
b = 120°
Angles a nd b are equal.

(b) c = 130°
d = 130°
Angles c and d are equal.

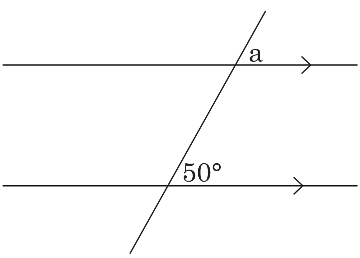
Tip:

- When a transversal intersects two parallel lines, two pairs of equal angles are formed as shown above. They are called corresponding angles.
- We can say angle **a** corresponds to angle **b.** Angle **c** corresponds to angle **d**.
- Corresponding angles are equal.

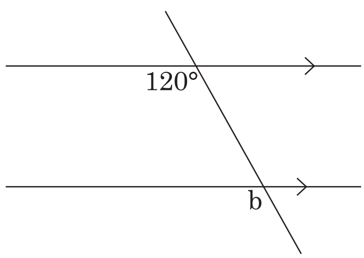
Practice Activity 11.4

Find the value of the angles marked with letters.

1.

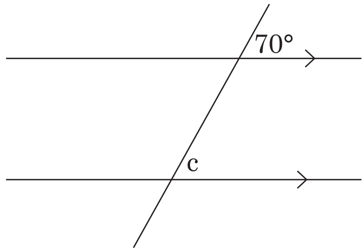


2.

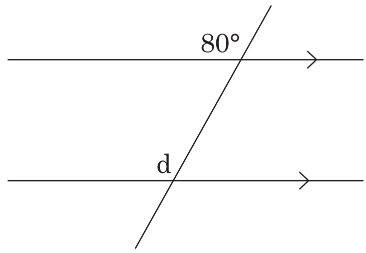


Find the angles marked with letters. Justify your answer.

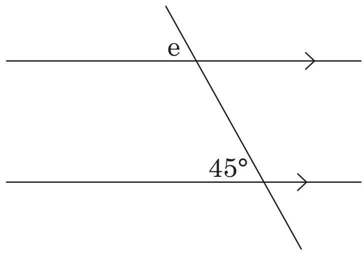
3.



4.



5.

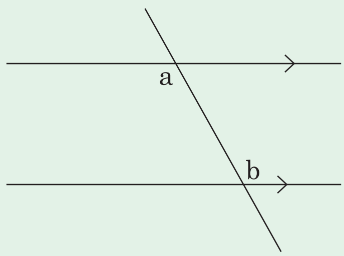


11.5 Alternate angles

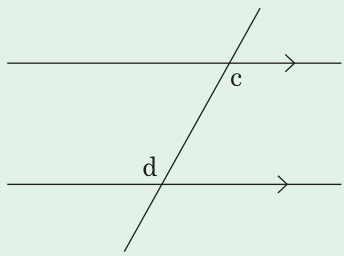
Activity 11.5

Use a protractor to measure and discuss the angles marked with letters on the diagram given below.

(a)



(b)

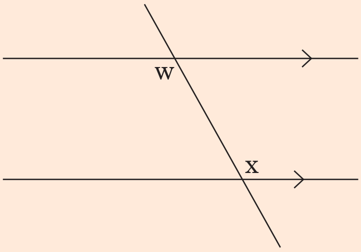


- Measure angles a, b, c and d.
- What do you notice about angle a and angle b?
- Explain the relationship between angles c and angle d.

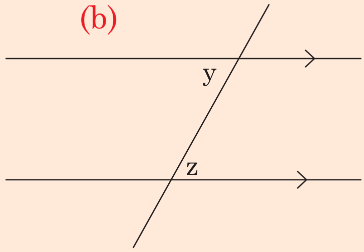
Example 11.5

Use a protractor to measure the angles marked with letters. Discuss the relationship between angles w and x? What is common between y and z?

(a)



(b)



Solution

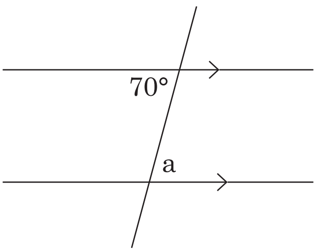
(a) w = 120°
x = 120°
Angles **w** and **x** are equal.

(b) y = 45°
z = 45°
Angles y and z are equal.

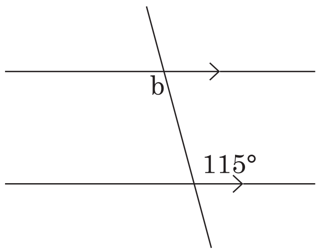
Practice Activity 11.5

Find the value of the angles marked with letters and explain the relationship.

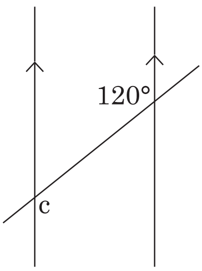
1.



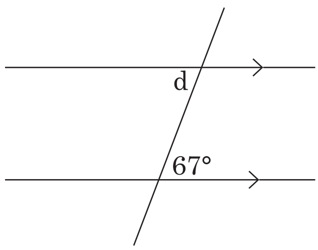
2.



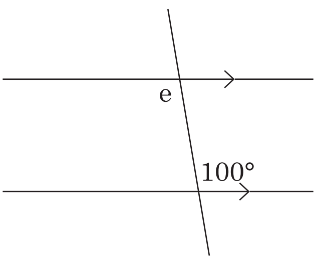
3.



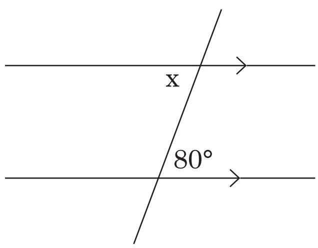
4.

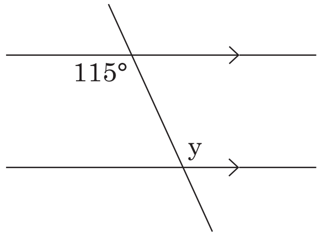


5.



6.

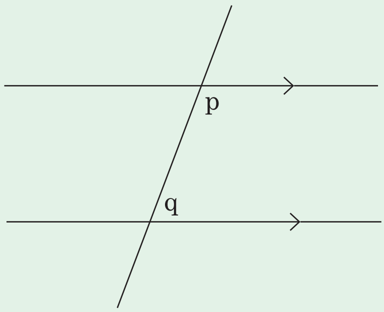


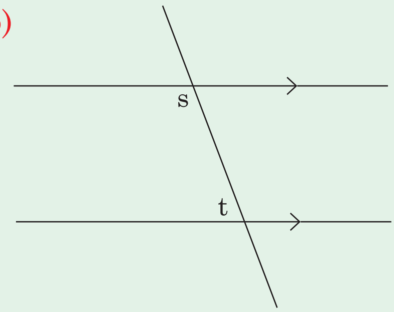
7. 

11.6 Co-interior angles

Activity 11.6

Discuss and measure the angles marked with letters.

(a) 

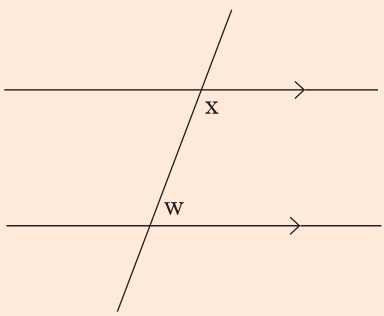
(b) 

- Add angles p + q, s + t.
- What do you notice with the sum of angle p and angle q?
- What do you notice with the sum of angle s and t? Explain.

Tip: Co-interior angles add up to 180°.

Example 11.6

(a) Measure angles x and w from the figure below.



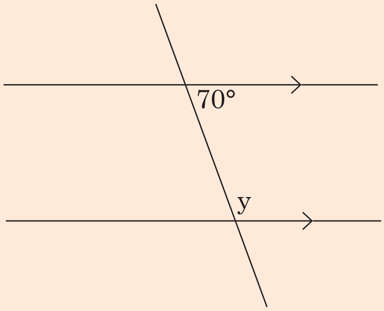
Solution

By measuring using a protractor,

angle w = 70°

angle x = 110°

(b) Find the size of angle y in the diagram below.



Solution

Co-interior angles add up to 180°.

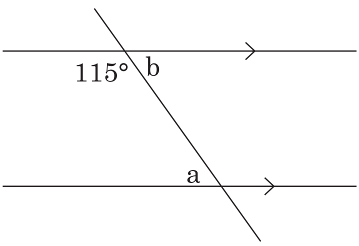
Therefore y + 70° = 180°

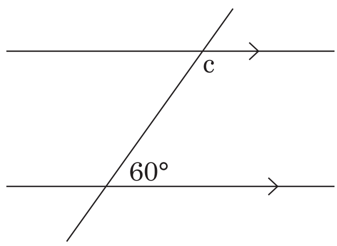
y = 180° – 70°

y = 110°

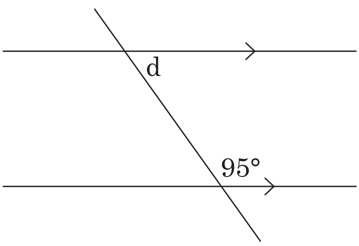
Practice Activity 11.6

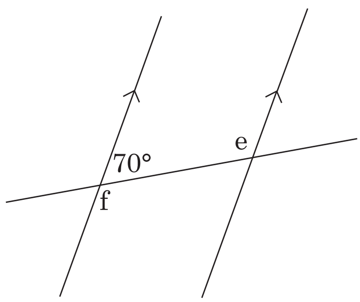
Find the value of the angles marked with letters.

1. 

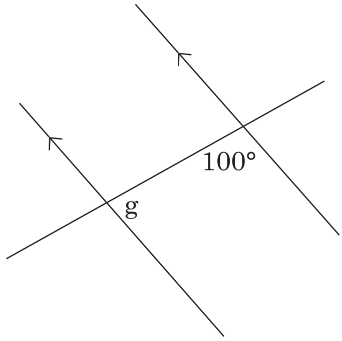
2. 

Find the size of angles marked with letters. Justify your answers.

3. 

4. 

5.



11.7 Drawing angles with a protractor

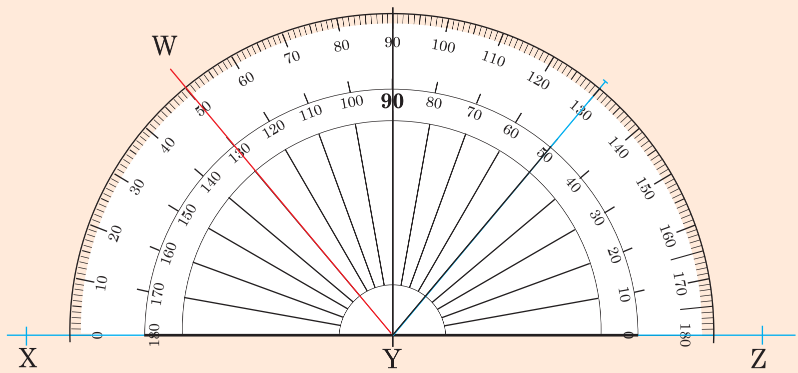
Activity 11.7

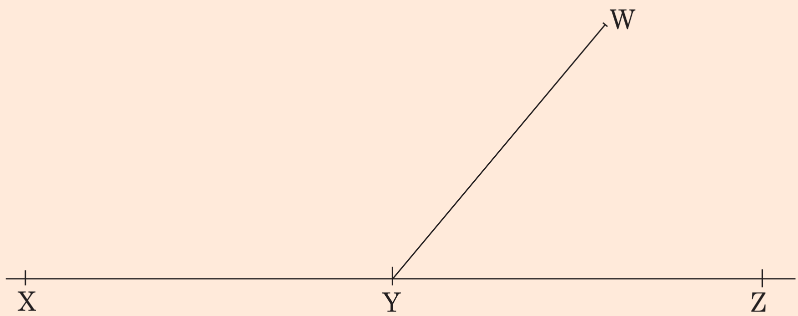
- Draw a straight line, AB = 10 cm. Mark its centre O. Have centre O meet with the centre of your protractor. Mark a point where the angle is 50° from left side.
- Repeat the steps above but do it from the right side of protractor. Discuss your steps.

Example 11.7

How to draw an angle;

- Draw a straight line and mark a point Y on it.
- Place the protractor on the line so that the centre point lies on Y and the line passes through the zero marks of the lines and outer scales. We can draw an angle between 0° and 180° by using either of the scales.
- To draw an angle of 130° by using the outer scale. Mark a point W on the paper at the 130° mark.
- Mark two more points X and Z on the line as shown in the figure.
- Draw a line from Y through W.





- Angle XYW = 130°
- YW also passes through the 50° mark of the inner scale. Therefore angle ZWY = 50°.

Practice Activity 11.7

Use a protractor and a ruler to draw the following angles.

1. 120°   2. 60°   3. 70°   4. 110°   5. 80°

Use a protractor and a ruler to draw the following angles. Justify whether you have used the inner or outer scale.

6. ABC = 100°   7. DEF = 85°   8. PQR = 95°
9. GEF = 40°   10. XYZ = 140°

11.8 Bisection of angles (Using folding)

Activity 11.8

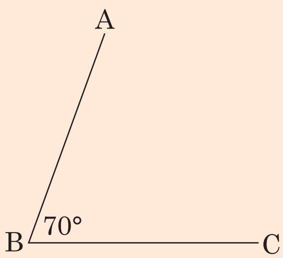
- Draw angles 120°, 90°, 80° on a paper.
- Cut out the angles. Fold each of the paper angles into two equal halves.

- Cut out the angles along the line created from folding.
- Measure the angles of each of the halves.
- Present your findings to the class.

Tip: To bisect an angle, we divide its size into 2 equal parts. When you bisect 90°, you get 45°.

Example 11.8

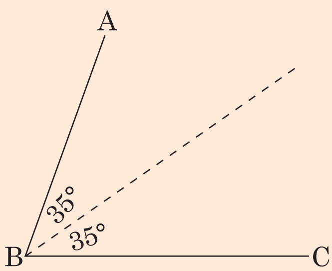
Bisect the angle given below by folding.



Solution

Step I: Draw angle ABC on a paper. Make a paper cutout for the angle. Fold the angle ABC into two equal parts.

Step II: Unfold the angle. Draw a line along the folded part as shown by the dotted line.



Step III: Measure the size of the angle on each of the two pieces. What do you notice?

Practice Activity 11.8

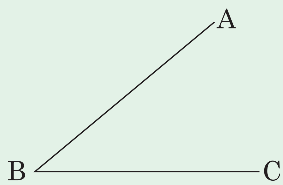
Draw the following angles. Then explain how to bisect them using folding.

1. 60°	2. 100°	3. 80°
4. 120°	5. 90°	6. 130°
7. 140°	8. 50°	9. 180°

11.9 Bisecting angles using a pair of compasses and a ruler

Activity 11.9

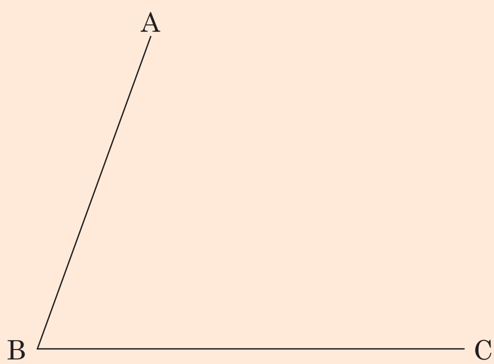
- Look at the angle below.



- Make a paper cutout of the angle above. Fold it in half. What do you get?
- Use a pair of compasses to bisect the angle ABC. What steps do you follow? Discuss your steps.

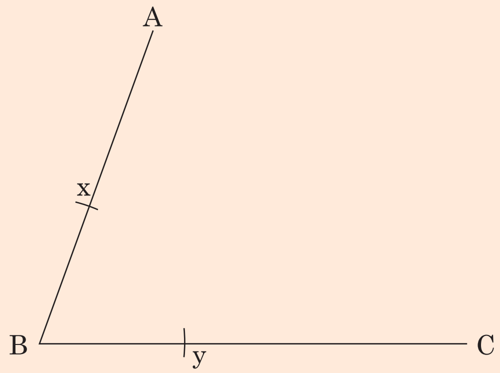
Example 11.9

Describe the steps involved in bisecting the angle below.

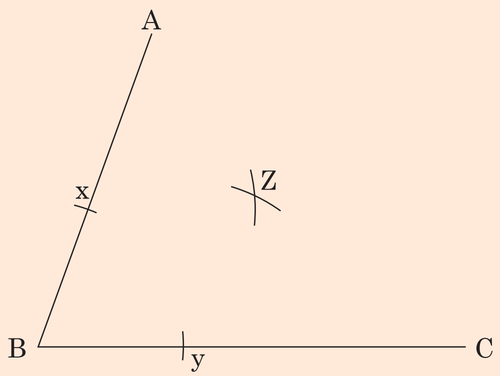


Solution

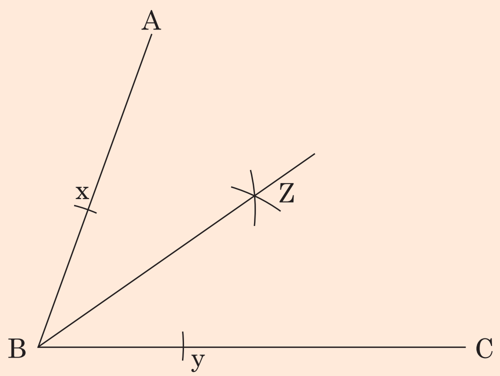
Step I: At point B, make an arc of any radius to cut line AB and BC at point x and y respectively.



Step II: With points x and y as the centres, make arcs to intersect at point Z.



Step III: Using a ruler, draw a line from point B to Z.



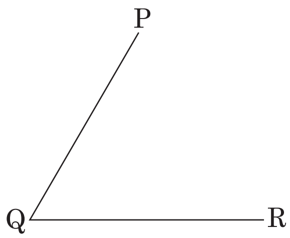
Line BZ divides angle ABC into two equal parts.

Angle ABZ is equal to angle CBZ.

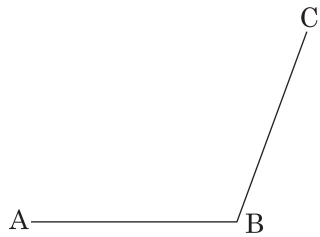
Practice Activity 11.9

Using a pair of compasses and a ruler, bisect the following angles.

1.

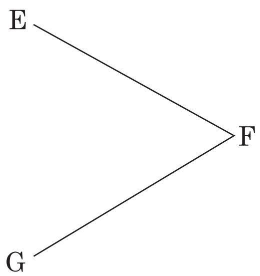


2.

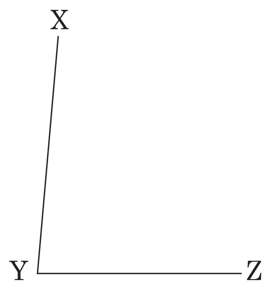


Bisect the following angles using a ruler and a pair of compasses. Explain your steps.

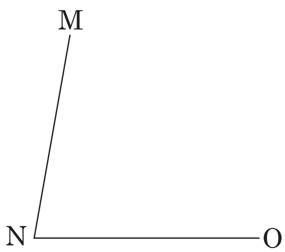
3.



4.



5.



11.10 Constructing 90°, 45° and 22.5° angles

Activity 11.10

(a)
- Make a paper cutout for 90°.
- Fold it to make an angle of 45°.
- Fold the paper cutout for 45° to form 22.5 °

(b)
- Now construct an angle of 90°. Use a pair of compasses and a ruler.
- Bisect 90° to have 45°.
- Bisect 45° to have 22.5°.

Tip:

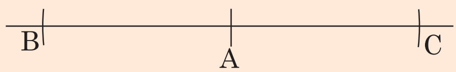
- Starting with 90° you bisect to have 45°.
- When you bisect 45° you get 22.5°

Example 11.10

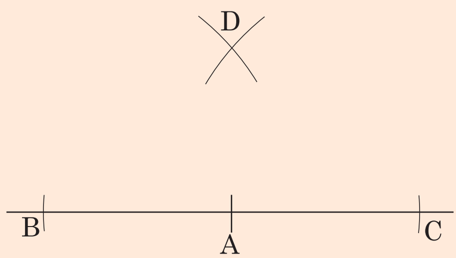
Explain the steps involved in constructing 90° using a ruler and a pair of compasses only.

Solution

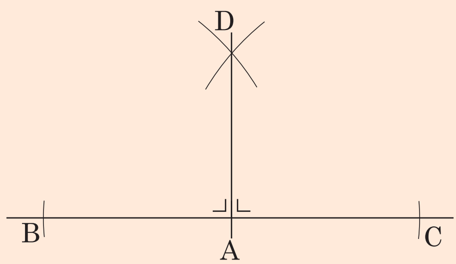
Step I: Mark an arc A on a straight line. Using A as the centre, make two other arcs on both sides of point A. Label them as B and C.



Step II: Increase the radius and use points B and C as centres to draw arcs intersecting at point D above the line.



Step III: Join point A to D.



Angle DAC = 90°

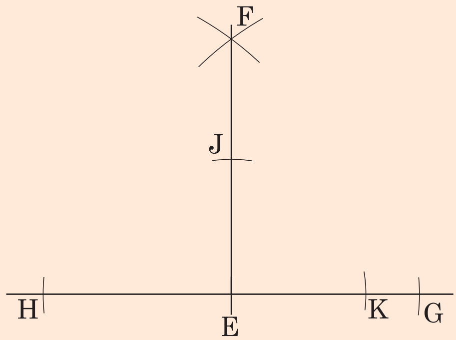
Angle BAD = 90°

Example 11.11

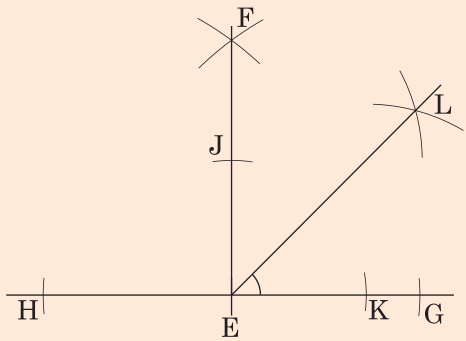
Describe the steps involved in constructing 45° using a pair of compasses and a ruler

Solution

Step I: Construct 90°. With point E as the centre, mark arcs to cut line EF and EG at point J and K respectively.



Step II: With J and K as the centres, mark arcs to intersect at point L.



Angle LEG = 45°. Angle FEL = 45°.

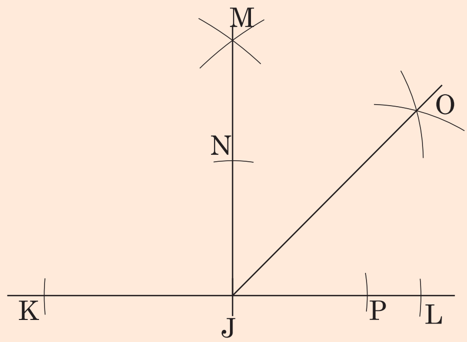
Example 11.12

Explain the steps involved in constructing 22.5° using a ruler and a pair of compasses.

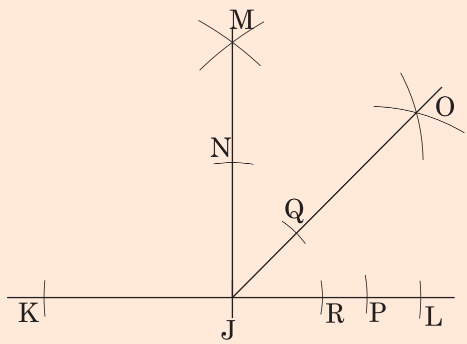
Solution

Hint: Construct 90° then bisect the angle to have 45°. Bisect 45° to have 22.5°.

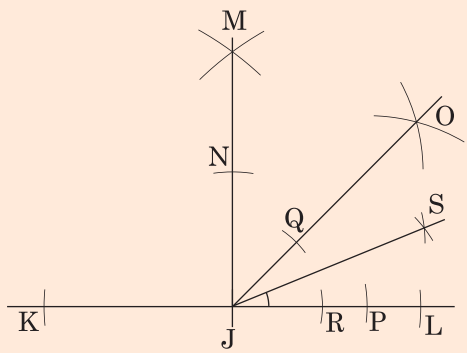
Step I: Construct 90°. Bisect angle 90° to have MJO = OJL = 45°.



Step II: With point J as the centre, mark arcs. The arcs cut line OJ at Q and line JL at R.



Step III: With Q and R as the centres, mark arcs to intersect at point S.



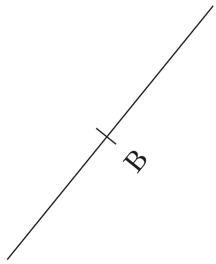
Angle SJL is half of 45°. Angle SJL = 22.5°.

Practice Activity 11.10

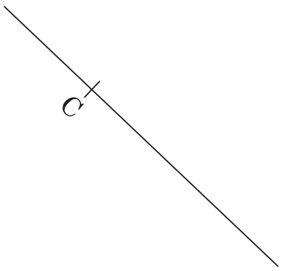
1. Construct 90° angle at the points marked with letters on the lines below.

(a)

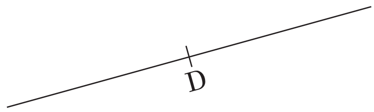
(b)



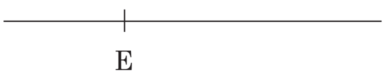
(c)



(d)



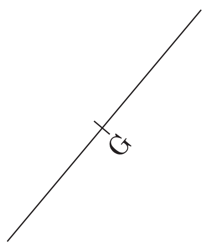
(e)



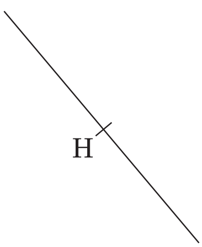
2. Explain the steps involved in constructing a 45° angle at the points marked with letters.

(a)

(b)

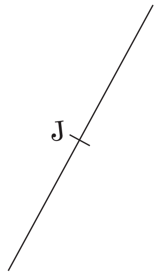


(c)



3. Explain the steps involved in constructing a 22.5° angle at the points marked with letters.

(a) 

(b) 

11.11 Constructing 60°, 30° and 15° angles

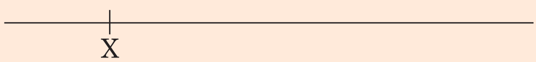
Activity 11.11

(a) Construct 60° angle using a pair of compasses and a ruler.
- State the steps involved in constructing a 60° angle.

(b) Construct a 30° angle using a pair of compasses.
- Discuss the steps involved in constructing a 30° angle.

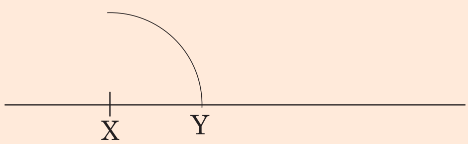
Example 11.13

(a) Construct a 60° angle at point X using a ruler and a pair of compasses.

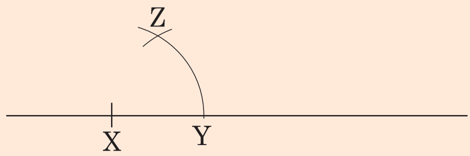


Solution

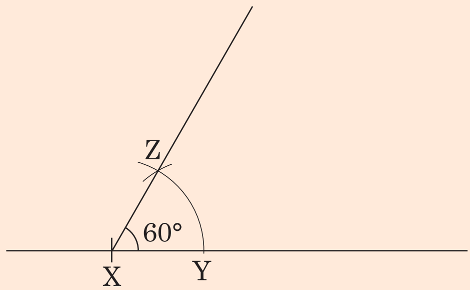
Step I: Use point X as the centre to mark an arc of any radius to cut the line at Y.



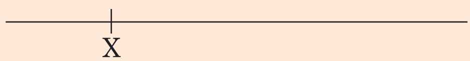
Step II: Keep the same radius and use Y as the centre. Draw another arc to intersect the first arc at Z.



Step III: Draw a line through XZ. AngleYXZ = 60°.



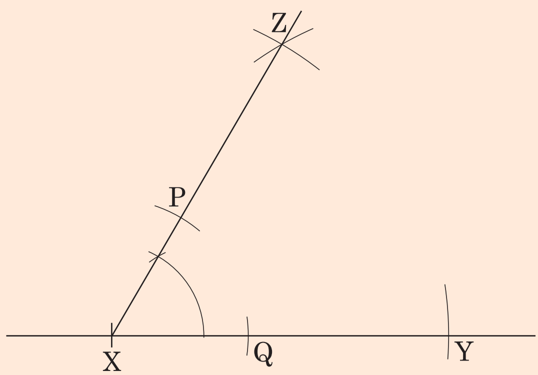
(b) Use a pair of compasses and a ruler to construct a 30° angle at X. Discuss your steps.



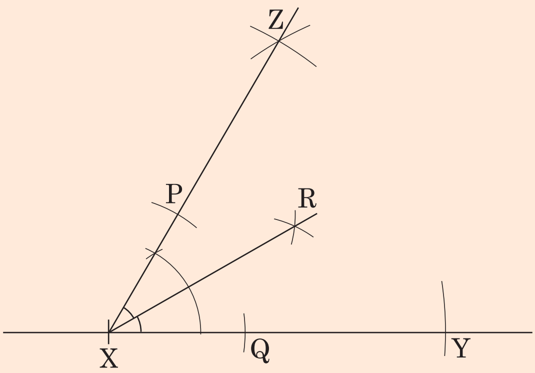
Solution

Hint: Construct a 60° angle then bisect it to get 30°.

Step I: Construct 60° at X. With X as the centre, draw arcs to cut line XZ and XY at point P and Q. See the diagram below.



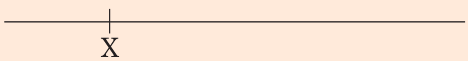
Step II: With points P and Q as the centre, draw arcs to intersect at point R. Join X to R. Measure angle YXR.



Angle YXR = RXZ = 30°.

Example 11.14

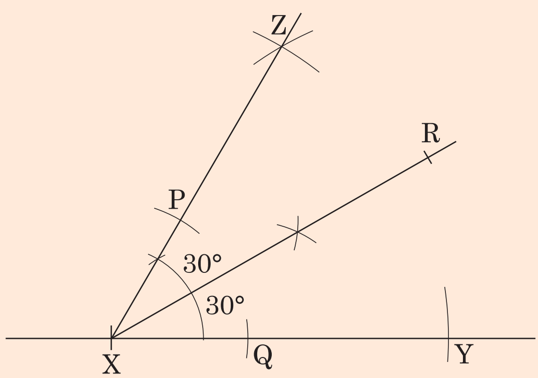
Construct a 15° angle at point X using a ruler and a pair of compasses. Discuss your steps.



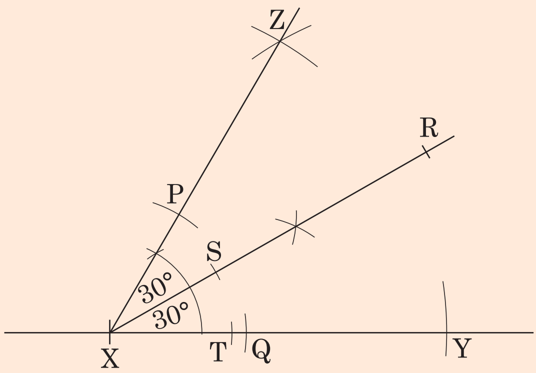
Solution

Hint: Construct a 60° angle. Bisect 60° to get 30°. Bisect 30° to get 15°.

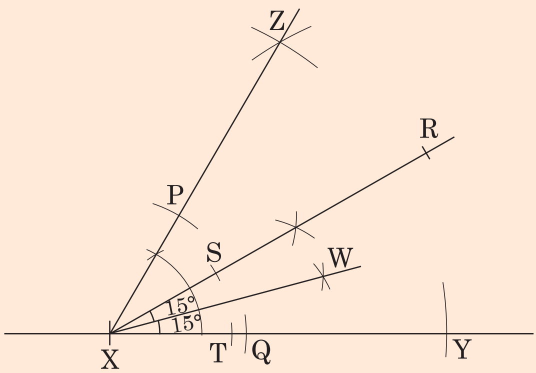
Step I: Construct a 60° angle at X. Bisect 60° to get 30°.



Step II: With point X as the centre, draw arcs to cut XR at S and XY at T.



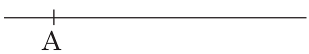
Step III: With S and T as the centres, draw arcs to intersect at point W. Then join XW. Measure angles WXR and YXW.

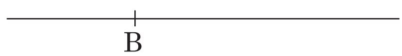


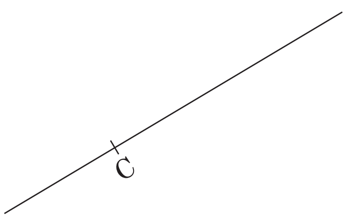
Angle RXW = WXY = 15°.

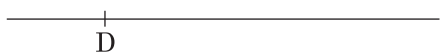
Practice Activity 11.11

1. Construct a 60° angle at the points marked with letters.

(a) 

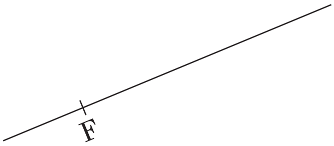
(b) 

(c) 

(d) 

(e) 

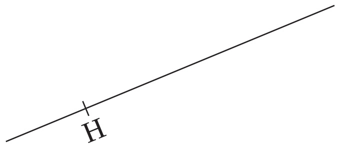
(f)



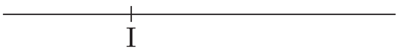
2. Construct a 30° angle using a ruler and a pair of compasses at the points marked with letters. Discuss your steps.

(a)

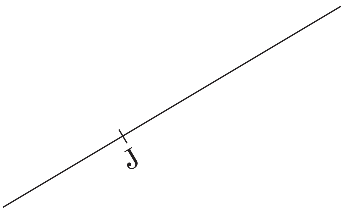
(b)



(c)

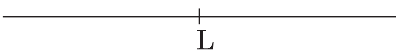


(d)

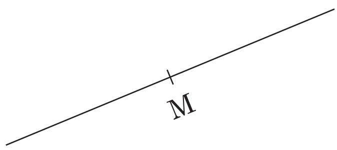


3. Construct a 15° angle using a ruler and a pair of compasses at the points marked with letters. Explain your steps.

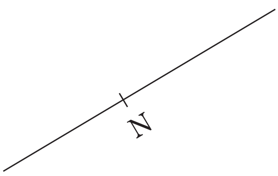
(a)



(b)



(c)



11.12 Constructing angles 120° and 150° angles

Activity 11.12

- Construct 120° and 150° angles using a ruler and a pair of compasses. Discuss your steps.

Example 11.15

Construct the following angles using a ruler and a pair of compasses. Describe your steps.

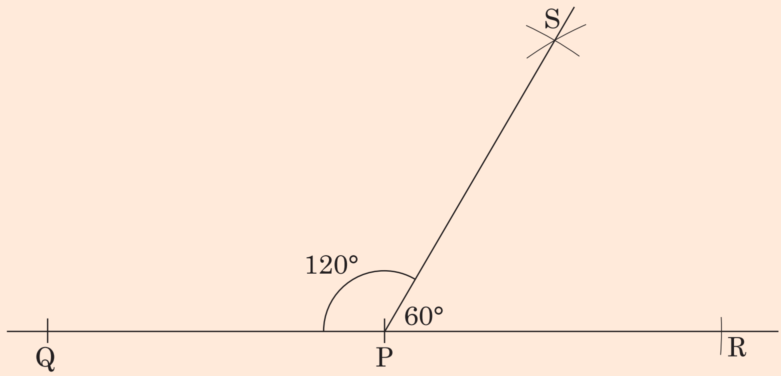
(i) 120° (ii) 150°

Solution

(i) Constructing a 120° angle at point P.

Hint: 120° = 180° – 60°. We construct a 60° angle on a straight line. The supplementary of 60° is 120°.

Step I: Construct a 60° angle on one side of point P on line QPR.

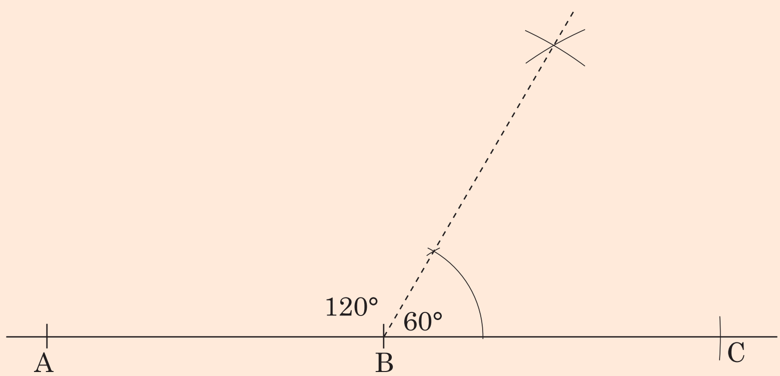


Angles SPQ = 120°.

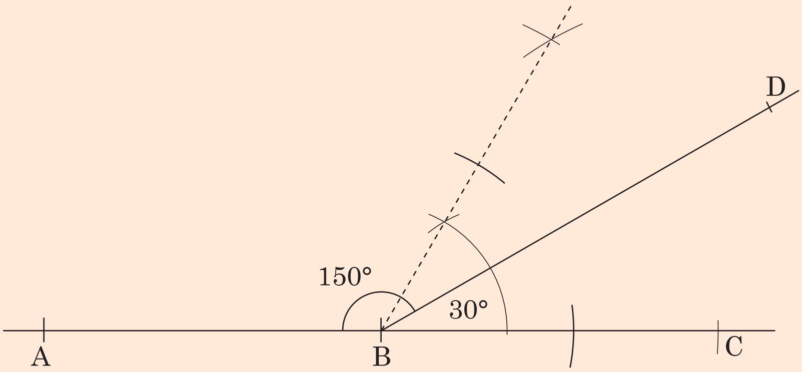
(ii) Constructing a 150° angle

Hint: 150° = 180° – 30°. We construct a 30° angle on a straight line. The supplementary of 30° is 150°.

Step I: Construct a 60° angle at point B on line ABC.



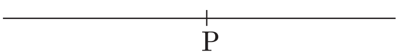
Step II: Bisect a 60° angle to get 30°.



Since ABC = 180° and CBD = 30°, then angle DBA = 150°. Measure it to confirm.

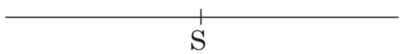
Practice Activity 11.12

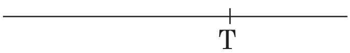
1. Construct a 120° angle at the points marked with letters. Use a pair of compasses and a ruler.

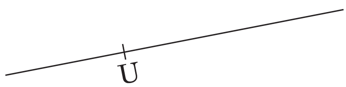
(a) 

(b) 

(c) 

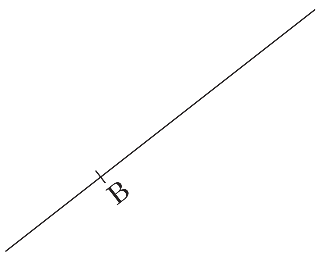
(d) 

(e) 

(f) 

2. Discuss the steps involved in constructing a 150° angle at the point marked with letters. Use a pair of compasses and a ruler.

(a) 

(b) 

(c) 

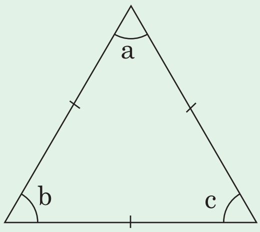
(d) 

11.13 Angle sum of a triangle

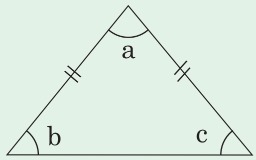
Activity 11.13

- Study the following triangles. We have labelled its angles with letters a, b, c.
- In each case, use a protractor to measure angles a, b and c.

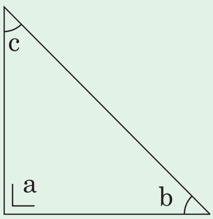
(i)



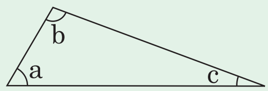
(ii)



(iii)

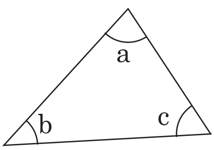


(iv)



- In each case, find the sum of a + b + c. What do you notice?
- Present your findings in class.

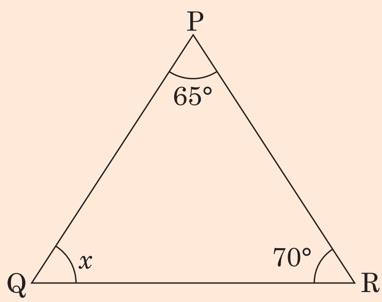
Tip: For any triangle, the sum of its interior angles is 180°.



- a, b, c are the interior angles of the triangle.
- a + b + c = 180°

Example 11.16

In triangle PQR below, two of the angles are given. Find the value of angle x.



Solution

The interior angles of a triangle add up to 180°.

Therefore $x + 65° + 70° = 180°$

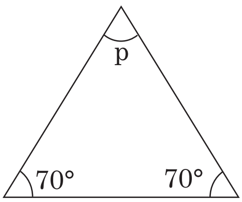
$x + 135° = 180°$

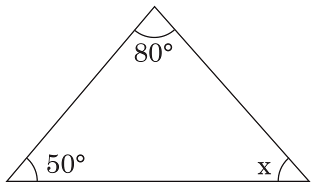
$x = 180° - 135°$

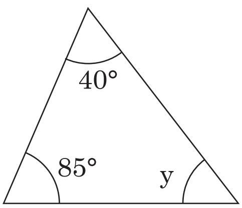
$x = 45°$

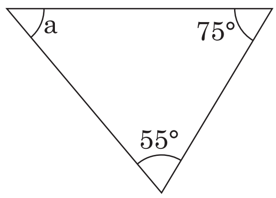
Practice Activity 11.13

Study each triangle below. Then find the value of the angles marked with letters.

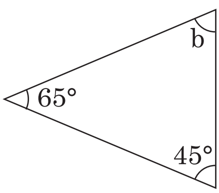
1.


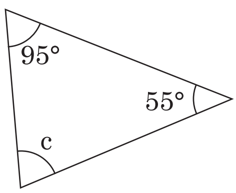
2.


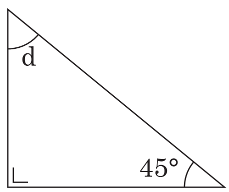
3.


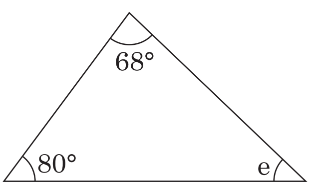
4.


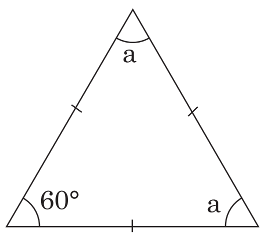
Explain the steps involved in getting the angles marked with letters in each of the following.

5.


6.


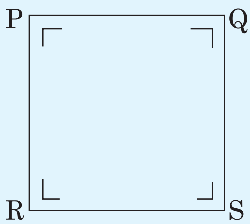
7.


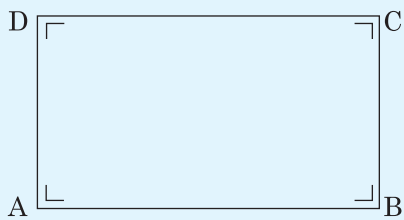
8.


9.


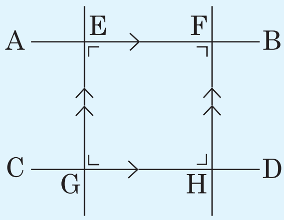
Revision Activity 11

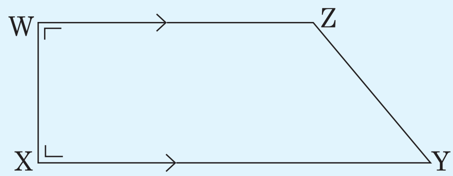
1. State two items in your classroom that have parallel sides.
2. In the diagram below, identify lines that are parallel.

(a) 

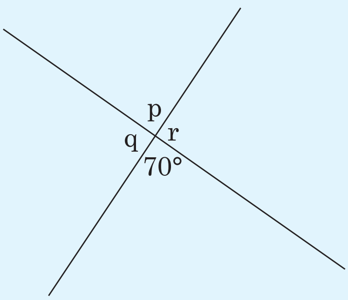
(b) 

3. In the figures below, identify the lines that are perpendicular.

(a) 

(b) 

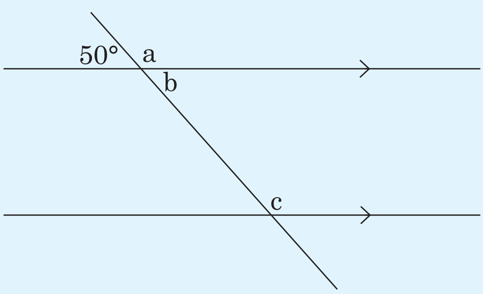
4. The diagram below represents intersecting lines.



Find the value of the angles marked with letters

(i) p = ______ (ii) q = ______ (iii) r = ______

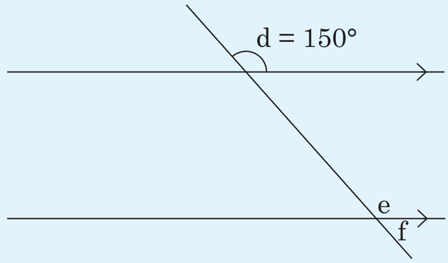
5. Study the diagram drawn below.



Find the size of the angles marked

(i) a (ii) b (iii) c

6. In the diagram below, angle d = 150°.



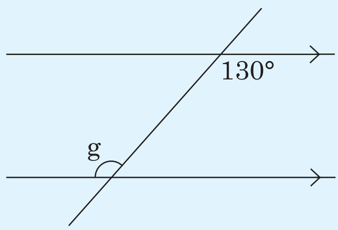
Find the size of the angles marked

(i) e (ii) f

(iii) Explain the relationship between angle *d* and angle *e*.

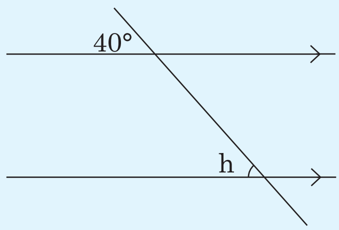
7. Study the transversals and parallel lines below. In each case, find the size of the marked angle. Explain your steps.

(a)



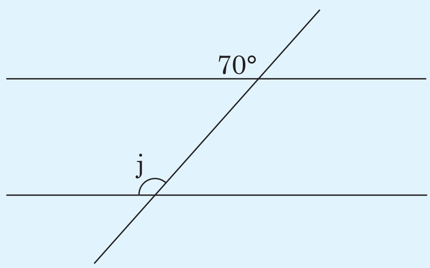
angle g = _____

(b)



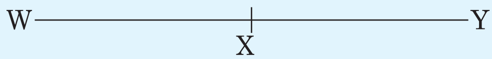
angle h = _____

(c)



angle j = _____

8. Draw a 80° angle using a protractor.
9. Explain the steps involved in constructing a 60° angle using a ruler and a pair of compasses.
10. Explain the steps involved in constructing a 150° angle at point X.



Word list

Parallel lines	Intersecting lines	A transversal
Perpendicular line	Vertically opposite angles	Supplementary angles
Corresponding angles	Alternate angles	Co-interior angles
Construct	Draw	A pair of compasses
Angle properties	Protractor	Bisect

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 12

Interpreting and constructing scale drawings

12.1 Concept of scale drawing

Activity 12.1

Draw diagrams to represent the following in your books.

(a) Your classroom.

(b) The distance between your classroom and the office.

(c) The chalkboard.

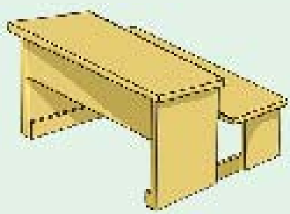
Measure and record actual distances of real objects you have drawn.

Measure the lengths of your drawings.

Compare the actual measurements with their drawing measurements. Which distances are bigger? Make a presentation to the class.

Activity 12.2

- Measure the length of the top of the desk drawn below. Record its length in cm.



- Now measure the length of your actual desk in cm.
- Compare the drawing length with the actual length.

Tip: Actual distances may not be possible to fit in a drawing on a paper. We draw them to the size of the paper using a shorter distances. When we do that, we have drawn objects to a scale. For example, a distance of 12 km road can be drawn as 12 cm on paper. This way, we have drawn to scale.

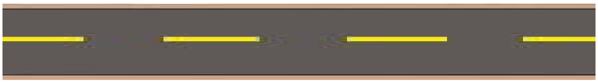
Practice Activity 12.1

Study the following:

(a) What are their actual lengths?

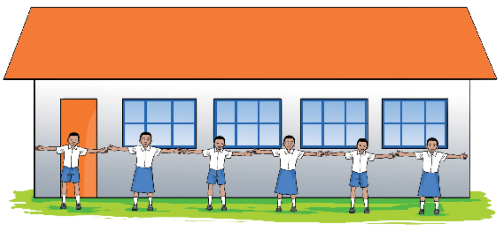
(b) Measure their drawing lengths.

(c) Explain why a diagram of a large object can fit on paper.

1.  Section of road

2.  Pencil

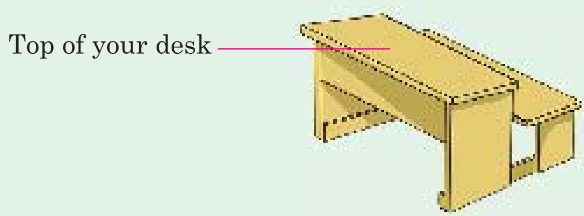
3.  Stick

4.  Classroom

12.2 Finding scale

Activity 12.3

- Cover the top of your desk with sheets of paper. How many sheets of paper have you used?



- Measure the actual width and length of your desk. Record them.

- Try to draw the top of your desk to cover 1 page paper. What are the width and length of your drawing? What scale have you used?
- Make presentations to the class.

Tip: To find a scale;

(i) Measure drawing length

(ii) Measure the actual length of the object in the same unit as the drawing length.

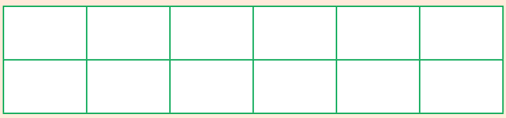
(iii) Scale = $\frac{\text{Drawing length}}{\text{Actual length}}$ = Drawing length : Actual length.

Example 12.1

(a) 12 sheets of paper fit on top of a desk. The top of a desk is drawn on one piece of paper. Find the scale.

Solution

- 12 sheets covering top of desk.



- To draw one sheet, I divide actual lengths by 12.

  The scale is 1:12

(b) The actual length of a path is 20 m. It is drawn using a line 5 cm long. What scale has been used?

Solution

Working in same units;

$$\text{Scale} = \frac{\text{Drawing length}}{\text{Actual length}} = \frac{5\text{ cm}}{20\text{ m}} = \frac{\overset{1}{\cancel{5}}\,\text{cm}}{\underset{4}{\cancel{20}} \times 100\text{ cm}} = \frac{1}{400} = 1{:}400$$

Note: A scale has no units. Why?

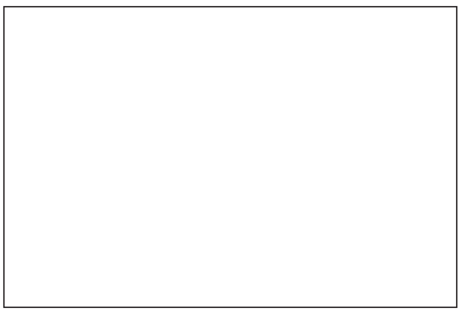
Practice Activity 12.2

1. The actual lengths for various items were measured. Their drawing lengths were recorded as follows.

	Item (s)	Actual length	Drawing length	Scale used
(a)	Length of a desk	2 m	2 cm	2:200 = 1:100
(b)	Width of a table	1.5 m	5 cm	___
(c)	Length of a ruler	30 cm	10 cm	___
(d)	Diameter of a cup	7 cm	3.5 cm	___
(e)	Length of a road	40 km	10 cm	___
(f)	Perimeter of a classroom	6 dam	30 cm	___

Calculate the scale used and fill in the table above accordingly.

2. The actual distance for a section of road is 25 km. It is drawn on a map using a 5 cm line. Explain how to find the scale of the map.
3. A flag post is drawn to scale. Its drawing height is 5 cm. Suppose the actual height is 10 m. Find the scale used to draw the flag post.
4. The actual length of the Nyabarongo River is 300 km. On a map, it is represented by a 30 cm long line. Discuss how to find the scale used of the map.
5. The actual perimeter of a rectangular plot is 100 m. The plot was drawn to scale as shown below.



   (a) Explain how to find the length and width of the plot in the drawing.
   (b) Explain how to find the perimeter of the scale drawing.
   (c) Explain the scale used in drawing the plot.

12.3 Constructing scale drawings

Activity 12.4

- Measure the actual length of your classroom. Use a scale to draw a line to represent the length of your classroom.

- Measure the width and length of your classroom. Use a scale to draw the shape of the floor.
- What is the actual distance from
  (i) your classroom to assembly?
  (ii) your classroom to the office?

Draw a simple map showing distances for:

Classroom point to office point and assembly point. Use a suitable scale.

Tip: To make a scale drawing

(i) Know the actual distances

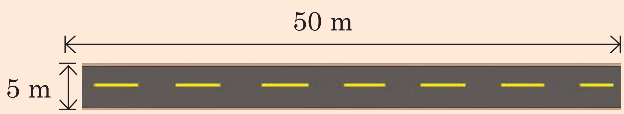
(ii) Choose a good scale

(iii) Find drawing distances

(iv) Draw the diagram

Example 12.2

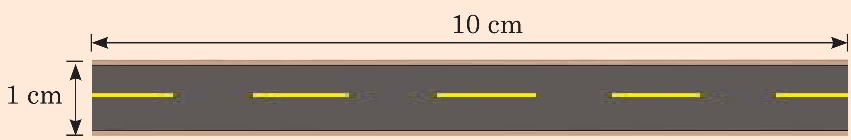
Look at the sketch of a section of a road.



Explain how you would use a scale of 1:500 to show the road on paper.

Solution

1 cm represents 500 cm or 5 m. From 50 m, the drawing length is $\frac{50}{5}$ = 10 cm. From 5 m, we have a drawing width of $\frac{5}{5}$ = 1 cm.



Example 12.3

A dining hall measures 40 m long and 35 m wide. Using a scale 1:1 000, explain how to make a scale drawing of the hall.

Solution

The scale 1:1 000, means 1 cm represents 1 000 cm. This also means

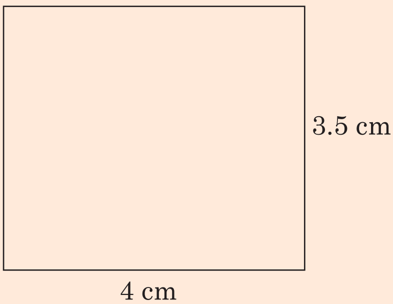
1 cm represents $\frac{10\cancel{0}\cancel{0}}{1\cancel{0}\cancel{0}}$ m (changing cm into metres)

1 cm represents 10 m

Drawing length of dining hall $\frac{4\cancel{0}}{1\cancel{0}}$ cm = 4 cm

Drawing width of dining hall $\frac{35}{10}$ cm = 3.5 cm

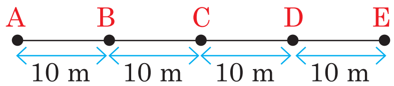
The dining hall will be represented by a rectangle. The rectangle is 4 cm long and 3.5 cm wide. The rectangle is drawn below.



Practice Activity 12.3

1. Using a scale of 1:100, make a scale drawing of the following.
   (a) A rectangle measuring 7 m by 3 m.
   (b) An equilateral triangle with 5 m sides.
   (c) A square with 4 m sides.
2. Using a scale of 1:500 000, draw lines to represent the following distances.
   (a) 15 km (b) 26.5 km (c) 45 km (d) 10 km
3. Using a scale 1:1 000, describe how to draw lines to represent the following length. Present to the class.
   (a) 7 200 cm (b) 8 000 cm (c) 6 800 cm
4. (a) Measure the actual distances of the following:
   (i) Length and height of chalkboard.
   (ii) Length and width of playground.
   (b) Discuss how to draw them to scale.

5. Look at the sketch below. The actual distances are stated.



Draw the diagram to a scale 1:400.

(a) What is the drawing length from A to B?

(b) What is the drawing distance AE?

(c) Explain your answers in (a) and (b).

12.4 Finding actual distance

Activity 12.5

- Measure the length of the line below. It represents a ruler.

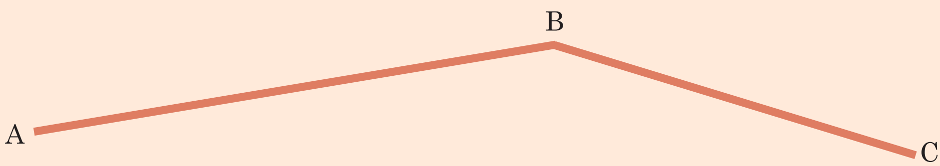
The scale used is 1:10

Find the actual length of the ruler.

- Consider 15 cm ruler, 30 cm ruler and a metre-rule. Which of them has its length represented by AB? Discuss your answer.

Example 12.4

In a map, a section of road is represented by the line below.



The scale used is 1:10 000

(a) Measure line AB, BC. What distance is AC through B?

(b) Interpret the scale.

(c) Find the actual distance AB and AC. Find the actual distance of the section of the road.

Solution

(a) Drawing lengths: AB = 7 cm, BC = 5 cm.

AC through B = (7 + 5) cm = 12 cm.

(b) The scale 1:10 000 means; 1 cm drawing length represents 10 000 cm or 100 m actual distance on the road.

(c) Actual distance;

AB = 7 × 100 m = 700 m

BC = 5 × 100 m = 500 m

Distance of the road = 700 m + 500 m = 1 200 m or 1.2 km

Tip:

To find actual distance

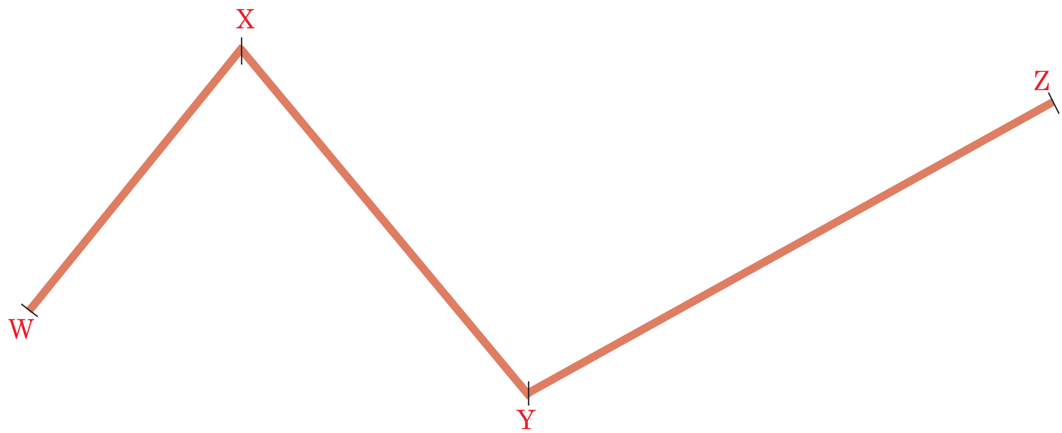
(i) Measure drawing length.

(ii) Interpret the scale.

(iii) Use formula, Actual distance = drawing length × value of scale (represented by 1 cm).

Practice Activity 12.4

1. The drawing length for a section of a river is 10 cm. The scale used was 1:2 500. Find the actual length of the section of the river (in m).
2. Below is a section of road joining towns W, X, Y, Z. Measure the drawing lengths and fill in the table accordingly. The scale used was 1: 100 000. Calculate the actual distances.

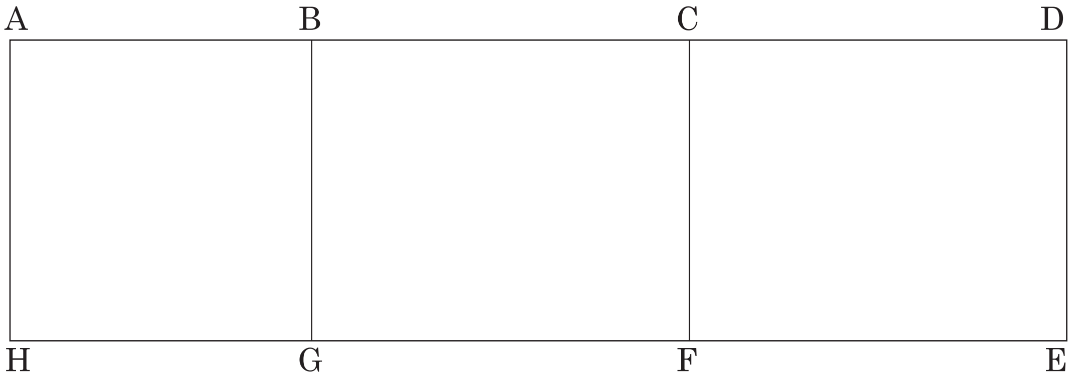


	Distances between	Drawing length	Actual distance in km
(a)	WX		
(b)	YZ		
(c)	XY		
(d)	WZ		

3. Given the scale 1:200 000, explain how to find the actual lengths of the following.

(a) 5 cm (b) 2.5 cm (c) 3.2 cm (d) 8 cm

4. Below is a scale drawing of a floor of classrooms.



Scale used is 1:40 000

Explain how to find the actual distances in metres for the following distances.

(i) AB (ii) BC (iii) CD (iv) DE

(v) AH (vi) FE (vii) AD (viii) GE

12.5 Finding the drawing length

Activity 12.6

- Measure the actual distance from your classroom to the assembly grounds. Use a metre rule or tape measure.
- Measure the actual lengths of your classroom. Record your results in the table below.
- Use a scale 1:1 000 to find drawing lengths for each object.

		Actual distance	Drawing length
(a)	Classroom to assembly		
(b)	Length of classroom		
(c)	Width of classroom		

Present your findings.

Example 12.5

The sketch below shows actual distances between towns P, Q, R and S. It is to be represented in a scale drawing. The scale to use is 1:200 000.

(a) Interpret the scale.

(b) Explain how to get the drawing measurements between the towns.

(i) PQ (ii) QR (iii) RS

(c) Make a scale drawing of the distances between towns.

Solution

(a) The scale 1:200 000 means 1 cm represents 200 000 cm or 2 km. Thus, 1 cm on the drawing represents 2 km of actual distance.

(b) (i) 1 cm represents 2 km. So PQ = 12 km.

12 km is represented by $(\frac{12}{2})$ cm = 6 cm.

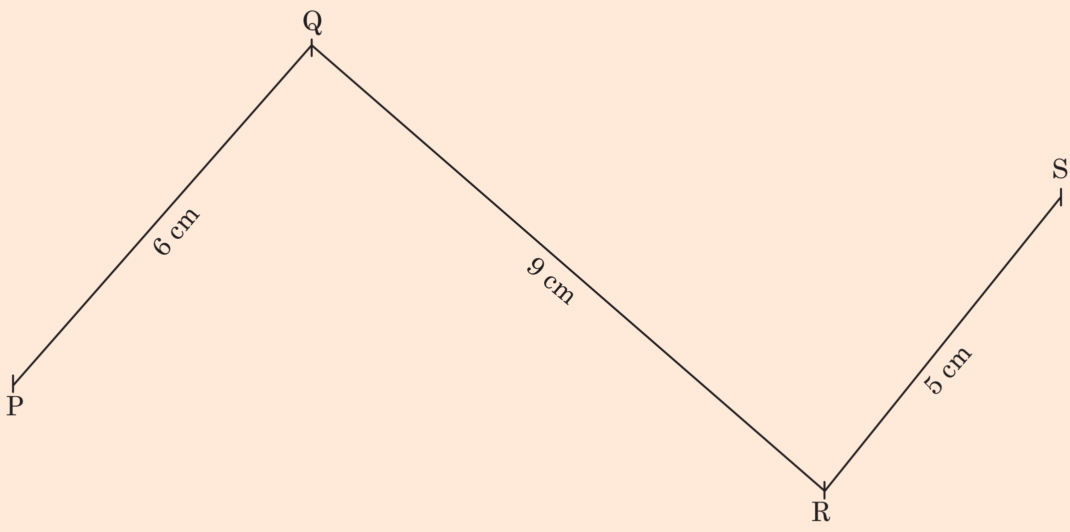
(ii) Actual distance QR = 18 km.

Drawing length for QR = $(\frac{18}{2})$ cm = 9 cm.

(iii) Actual distance RS = 10 km.

Drawing length for RS is $(\frac{10}{2})$ cm = 5 cm

(c)



Practice Activity 12.5

1. Use the scale 1:10 000. Find the drawing lengths for these:
   (a) A section of river that is 350 m
   (b) A section of road that is 820 m
   (c) The length of school path that is 225 m
2. Use the scale 1: 100 000. Find the drawing length for each of the following:
   (a) A road joining towns PQ = 60 km
   (b) A railway line joining towns XY = 225 km
   (c) A length of river joining two provinces = 200 km.
3. The distance between two towns is 120 km. Use a scale of 1:300 000. Find the drawing length for the two towns.
4. Using the scale of 1:2 000, make the following scale drawings. Then explain your work.
   (a) Rectangular field measuring 80 m by 60 m.
   (b) A path which is 240 m long.
5. Using the scale of 1:30 000, make scale drawings of the following. Then discuss your work.
   (a) A square field with sides of 1 200 m.
   (b) A section of a road which is 2 700 m.

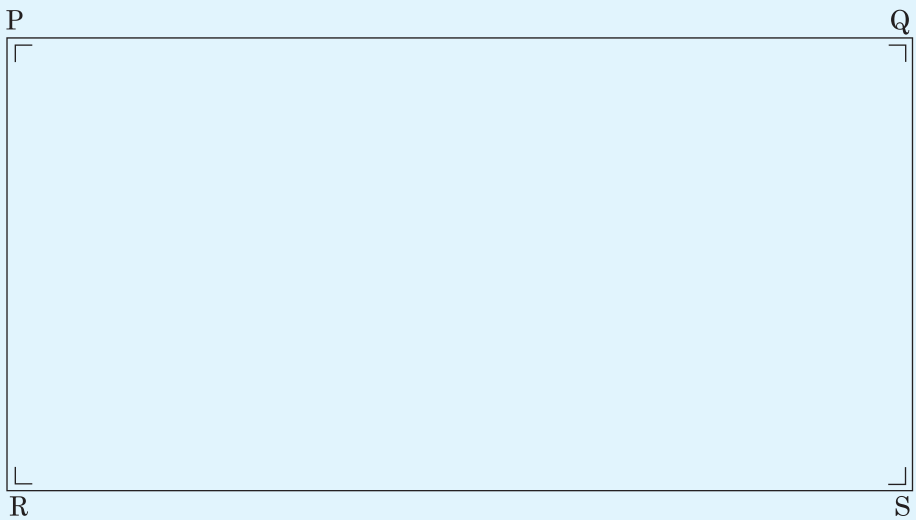
6. A road between two towns is 56 km long. It is represented in a map with a scale of 1:1 000 000. What is the drawing length of the road in the map? Justify your answer.

Revision Activity 12

1. A section of river is 1 720 m. It is drawn on a map. The drawing length is 17.2 cm. Find the scale used in the map.
2. A building measures 24 m by 10 m. It is to be drawn to scale on a paper measuring 30 cm by 20 cm. Discuss the appropriate scale to be used.
3. Choose appropriate scales to be used to draw the following lengths?

   (a) 820 cm (b) 60 m (c) 40 km
4. What scale was used to make the following drawings? Measure the drawing lengths. The actual measurements are given.

   (a)

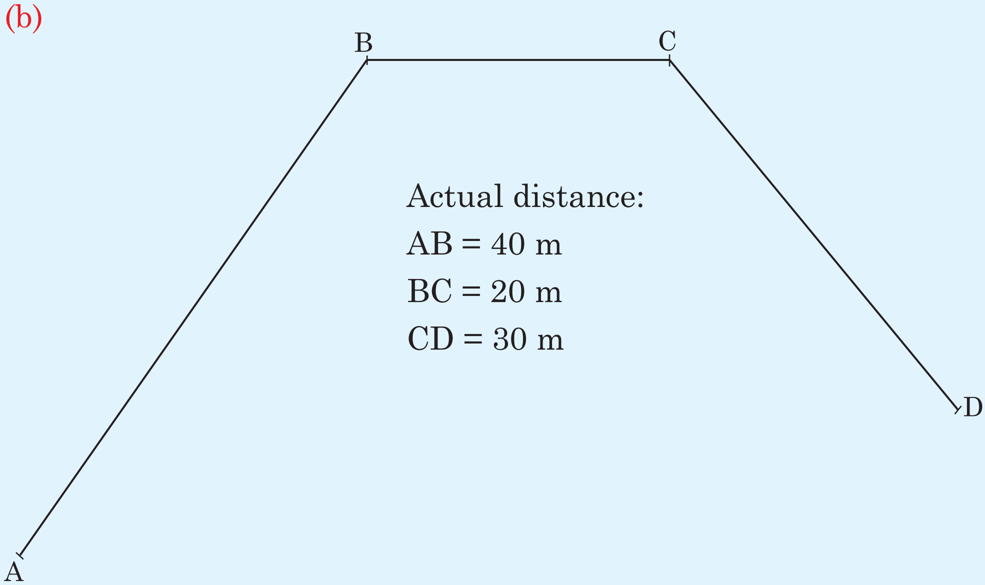


Actual distances:

PQ = 60 m

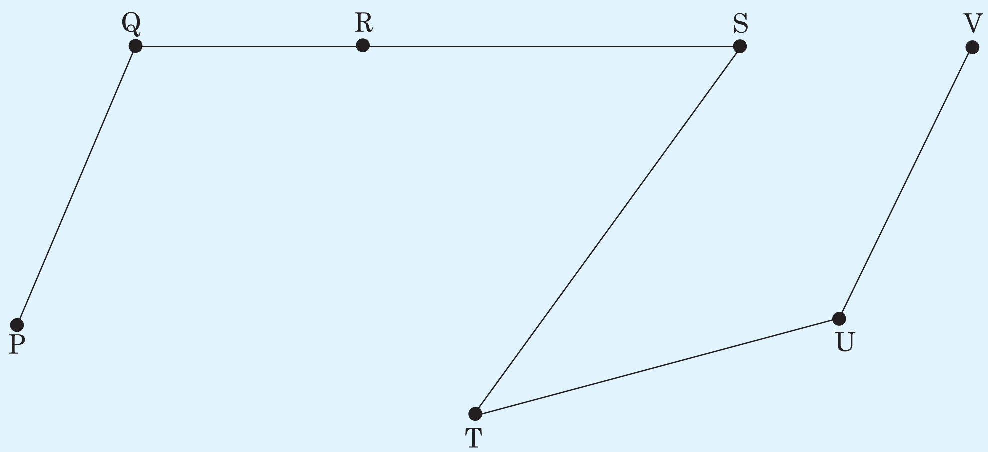
QS = 30 m

(b)



5. In a scale drawing, a scale of 1:20 000 was used. Find the drawing length for a section of road that is 840 m.

6. The diagram below is drawn to scale. It shows the roads joining various towns.



The actual distances are as follows:

PQ	QR	RS	ST	TU	UV
28 km	21 km	35 km	42 km	35 km	28 km

(a) Explain how to find the drawing lengths between;

(i) PQ (ii) QR (iii) RS

(iv) ST (v) TU (vi) UV

(b) Find the actual distance between Q and S through R.

(c) Explain how to find the scale used for the map.

7. The figure below has been drawn to scale at 1:500.

(a) Measure its length. (b) Measure its width.

(c) Interpret the scale. (d) Find the actual length.

(e) Find the actual width. (f) Discuss your answer.

8. Using the scale 1:1 000, make scale drawings of the following and discuss your answers.

(a) Rectangle measuring 40 m by 20 m.

(b) Square field whose sides are 50 m.

(c) A rectangular field measuring 80 m by 60 m.

9. In a map, a scale of 1:300 000 is used. Discuss the steps to follow to calculate the drawing length for these distances:

(a) 21 km (b) 27 km (c) 36 km (d) 15 km

Word list

Scale	Actual length	Drawing length
Scale drawing	Actual distance	

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

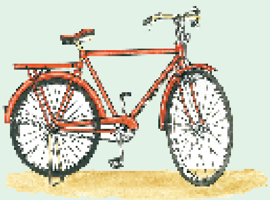
(iii) Write sentences using each of the words above. Read with your friend.

UNIT 13 Calculating the circumference of a circle and the volume of cuboids and cubes

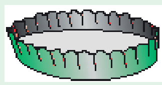
13.1 The circumference of a circle

Activity 13.1

Study the diagrams below.





(i) Trace the circular path on each of them.

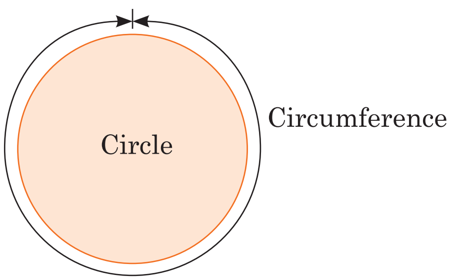
(ii) Which objects with circular paths have you brought?

(iii) Wrap a string around the circular path of the object. Untie the string and measure its length using a ruler.

(iv) Discuss your findings.

Tip:

The distance around a circular object is called the circumference.



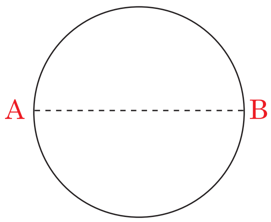
What are the circumferences of the objects you measured in Activity 13.1?

Practice Activity 13.1

1. Identify circular objects in the school compound.
2. Measure their circumferences.
3. Why do you think these objects are circular? Discuss.

13.2 Finding pi (π)

Look at the circle below.



The distance AB is diameter of the circle.

The diameter of a circle is a straight line which passes through its centre.

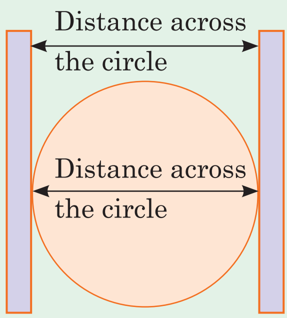
Activity 13.2

Have the following materials:

Rulers, tape measure, string, circular objects, manila papers

Steps

(i) Measure the diameter of the circular objects you have, as shown below. The distance must pass through the centre of the circle.



Note: Half of the diameter is called the radius.

(ii) Prepare a chart on a manila paper as shown in the table below.

(iii) Now measure the circumference of the objects you have. Record the circumference and diameter of each object on a chart.

(iv) Divide: Circumference ÷ diameter = C ÷ D of each object.

Object	Circumference (C)	Diameter (D)	C ÷ D

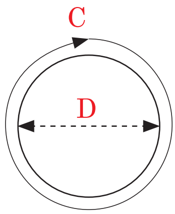
What do you notice about the results of C ÷ D? Explain.

Tip: The result of: circumference ÷ diameter is called pi. The symbol for pi is π.

Pi (π) = circumference (C) ÷ diameter (D)

$$= \frac{\text{Circumferemce (C)}}{\text{Diameter (D)}}$$

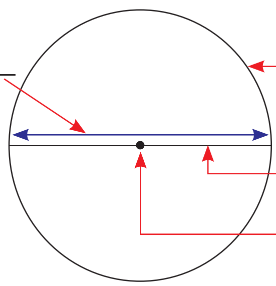
$$\pi = \frac{C}{D}$$



Note: Pi (π) is $\frac{22}{7}$ or $3\frac{1}{7}$ or 3.14

13.3 Calculating the circumference of a circle

Let us remember the parts of a circle as shown below. Discuss and name the parts labelled a – d.



Activity 13.3

Refer to your chart. Multiply π by diameter ($\pi \times D$) of any object. What do you get? Compare your answer with the circumference (C) of the object.

What do you notice? Discuss your finding.

Tip: $\pi \times D = C$ or $C = \pi D$

Example 13.1

(a) Find the circumference of a circle whose diameter is 14 cm.

Take $\pi = \frac{22}{7}$.

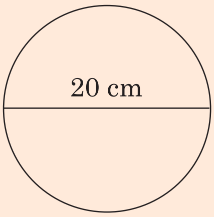
Solution

$c = \pi d$

$c = \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}}$ cm

$c = 44$ cm

(b) Find the circumference of the circle below. Take $\pi = 3.14$.



Solution

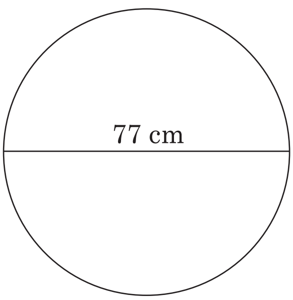
$C = \pi d$

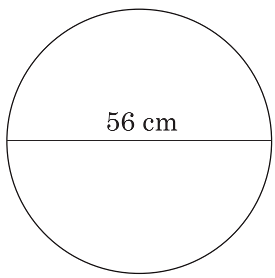
$C = 3.14 \times 20$ cm

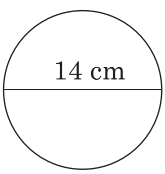
$c = 62.8$ cm

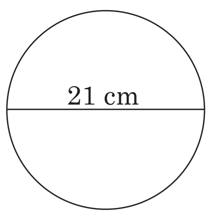
Practice Activity 13.2

A. Find the circumferences of the circles. Take $\pi = \frac{22}{7}$

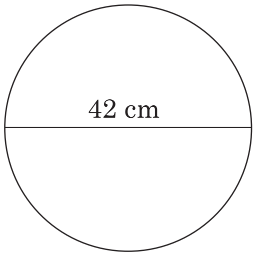
1. 

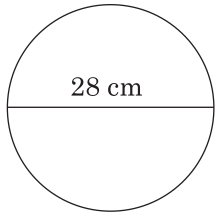
2. 

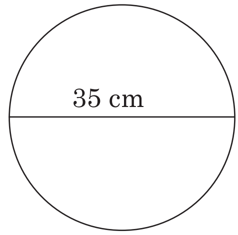
3. 

4. 

B. Find the circumference of the circles below. Take $\pi = 3.14$

1. 

2. 

3. 

C. Discuss and find the answers to the following questions. Present your findings.

1. The diameter of a circular ring is 21 cm. What is its circumference?
2. A bicycle wheel has a diameter of 98 cm. What is the circumference of the wheel?
3. The diameter of the circle at the centre of a football field is 9.8 m.
   (a) What is the circumference of the circle?
   (b) A player ran round the circle three times. What distance did he cover?

Example 13.2

(a) What is the circumference of a circle whose radius is $10\frac{1}{2}$ cm?

Take $\pi = \frac{22}{7}$

Solution

You are given the radius as $10\frac{1}{2}$ cm.

Therefore we use the formula $c = 2\pi r$ to find the circumference.

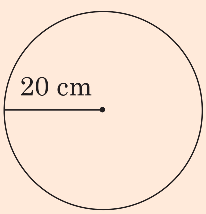
$$c = 2\pi r$$

$$c = \frac{22}{7} \times 2 \times 10\frac{1}{2} \text{ cm}$$

$$c = \frac{22}{\cancel{7}_1} \times \overset{1}{\cancel{2}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{2}_1} \text{ cm}$$

$$c = 66 \text{ cm}$$

(b) Find the circumference of the circle below. Take $\pi = 3.14$.



Solution

$C \;= 2\pi r$

$C \;= 2 \times 3.14 \times 20$ cm

$\;\;\;\;= 125.6$ cm

(c) A circular flower garden has a radius of 7 m. It was fenced all around the circumference. What is the length of the fence?

Solution

Length of the fence = circumference of garden

Length of fence = πD

Remember diameter = $2 \times$ radius

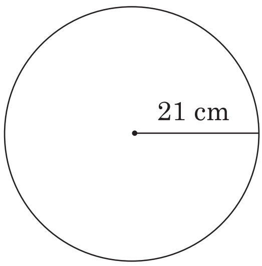
Diameter = $2 \times 7 = 14$ m

Length of fence = $\frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} = 44$ m

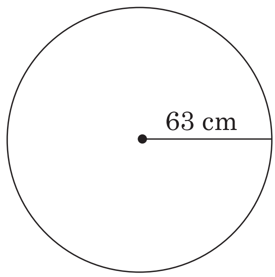
Practice Activity 13.3

1. What is the circumference of each of the circles below? Take $\pi = \frac{22}{7}$

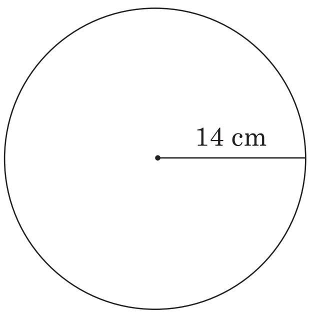
(a)



(b)



(c)



2. The radius of a circle is 15 cm. What is its circumference?
3. The radius of the lid of a bucket is 45 cm. Calculate the circumference of the lid. Discuss your steps to answer.
4. The radius of a circular water tank is $3\frac{1}{2}$ m. A rope is tied around the tank. What is the length of the rope? Explain how you arrived at your answer.

Activity 13.4

Referring to your chart in Activity 13.2.

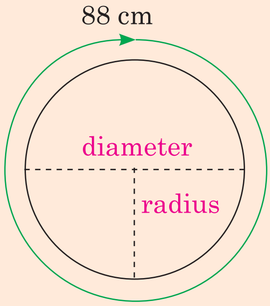
What is $c \div \pi = \frac{c}{\pi}$?

Compare your answer to the diameter (D) of each object. Present your results.

Tip: The formula for calculating the diameter of a circle is Circumference ÷ pi, $d = \frac{C}{\pi}$

Example 13.3

1. The circumference of a circle is 88 cm.



(a) Find its diameter. Take $\pi = \frac{22}{7}$

(b) Find its radius.

Solution

Remember $d = \frac{c}{\pi}$, therefore;

(a) $d = \frac{c}{\pi}$

$d = 88 \div \frac{22}{7}$

$= \overset{4}{\cancel{88}} \times \frac{7}{\underset{1}{\cancel{22}}}$

$d = 28$ cm

(b) $2r = d$

$\frac{2r}{2} = \frac{28}{2}$ cm

$r = 14$ cm

To get the radius, divide the diameter by 2.

2. The circumference of circular log of wood is 77 cm. What is its diameter?

Take $\pi = \frac{22}{7}$

Solution

$d = \frac{c}{\pi}$ or $c \div \pi$

$= 77 \div \frac{22}{7}$

$= \overset{7}{\cancel{77}} \times \frac{7}{\underset{2}{\cancel{22}}}$

$= \frac{49}{2} = 24\frac{1}{2}$ cm

Practice Activity 13.4

Copy and complete the table below. Take $\pi = \frac{22}{7}$

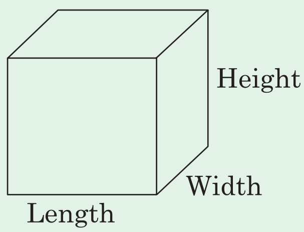
	Circumference	Diameter	Radius
1.	154 m		
2.	77 cm		
3.	44 cm		

4. The circumference of a circle is 110 m. Calculate its diameter.
5. The circumference of a roundabout is 30.8 m. Calculate its radius. Take $\pi = \frac{22}{7}$. Discuss your steps.
6. The circumference of a circle is 439.6 cm. What is its diameter? Take $\pi = 3.14$. Discuss your steps.
7. The circumference of a circular building is 628 m. What is the radius of the building? Take $\pi = 3.14$. Explain your answer.
8. A wire which is 88 cm long was cut to form a ring. What is the diameter of the wheel? Explain how you arrived at the answer.

13.4 Cubes and cuboids

Activity 13.5

- Discuss and identify the length, width and height of different boxes.



- Prepare a chart like one shown below.
- Measure the length, width and height of different boxes.
- Record your results in your chart.

Cubes and cuboids chart

Box	Length	Width	Height

- In which boxes are

(a) the length, width and height equal? Explain.

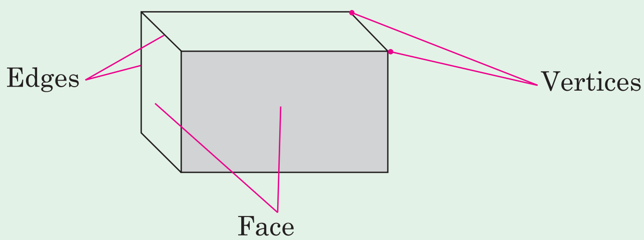
(b) the length, width and height different? Justify your results.

Tip: The boxes whose three sides are equal are called **cubes**.
The boxes whose three sides are different are called **cuboids**.

13.5 Properties of cubes and cuboids

Activity 13.6

Study the cuboid below.



Make a chart like the one below on manila paper.

Properties of cubes and cuboids			
Cube	**Faces**	**Edges**	**Vertices**
Cuboids	**Faces**	**Edges**	**Vertices**

Take the cubes and the cuboids in turns. Count their vertices, faces and edges. Record them in your chart. Discuss your results.

Activity 13.7

Make a chart as shown below.
Study cubes and cuboids then fill in your chart appropriately.

Properties	**Cube**	**Cuboid**
Opposite faces equal		
Opposite edges equal		
Have 8 vertices		
Have 6 faces		

Have 12 edges		
All faces are equal		

(a) Which of the boxes are cubes? Justify your answers.
(b) Which of the boxes are cuboids? Justify your answers.
(c) Discuss the similarities between cubes and cuboids.
(d) List the difference between cubes and cuboids.

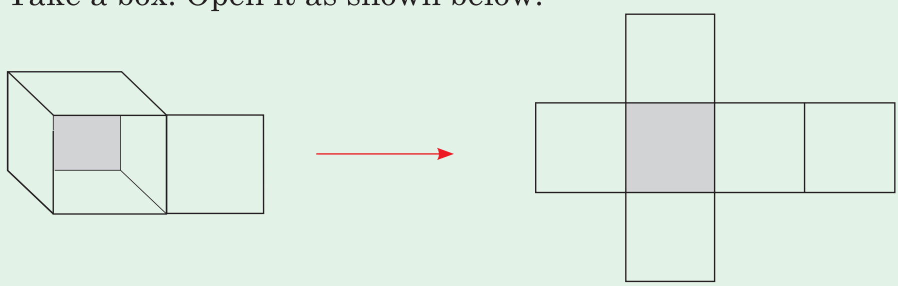
Practice Activity 13.5

1. How many vertices does a cube have?
2. How many edges are there in a cuboid?
3. How many faces are there in a cube?
4. How many faces are there in cuboid?
5. What is the product of edges and vertices in a cube?
6. Observe the shapes of objects in the classroom.
   (a) Which ones are cubes? Why are they cubes?
   (b) Which ones are cuboids? Explain why they are cuboids.

13.6 Nets of cubes and cuboids

Activity 13.8

- Take a box. Open it as shown below.



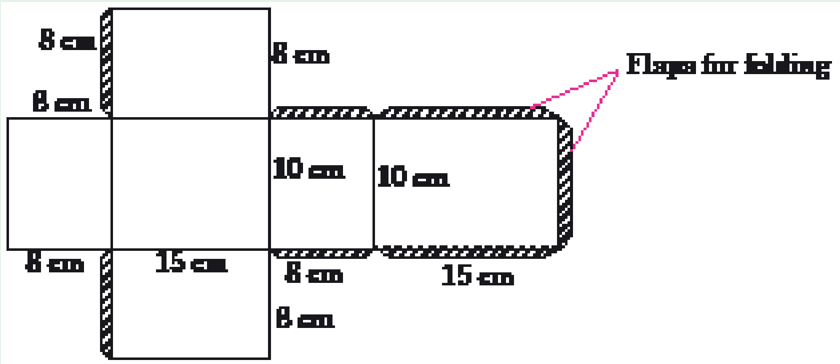
- The completely opened shape of the box is called a net. To make a cube or a cuboid, we must first prepare a '**net**'.
- Open other cubes and cuboids to form different nets.
- How many faces are in each net?
- Fold a net to form a cube or cuboid.
- Fold nets to form a cube or cuboid. Present your model.

Making nets

Activity 13.9

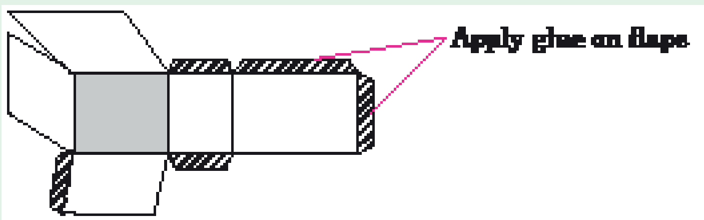
Have the following materials: manila paper, pair of scissors, ruler, glue.

On manila paper, draw the net of a cuboid. Its measurements should be length 10 cm, width is 8 cm and height is 6 cm.

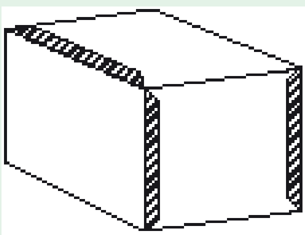


Cut out the net from the manila paper. Make sure the net has flaps which are about 1 cm wide.

Fold the net to make a cuboid. Make sure the edges are neatly folded along the lines.



Apply glue on the inner parts of the flaps. Describe your steps.



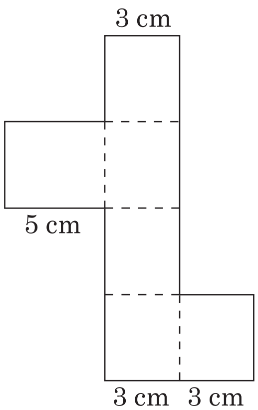
Practice Activity 13.6

1. Make the following cubes and cuboids.
   (a) Length 10 cm, width 10 cm and height 10 cm.
   (b) Length 20 cm, width 15 cm and height 10 cm.

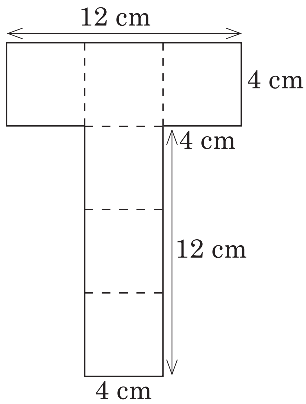
You need a pencil, a ruler, manila papers, a pair of scissors and glue.

2. Draw the following on manila paper. Cut the nets out. Fold them along the dotted lines.
   (a) Which of the nets will make a cube?
   (b) If the net does not make a cube, explain reasons why you think it does not.

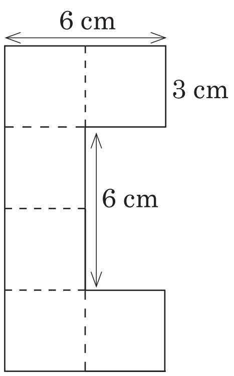
(i)



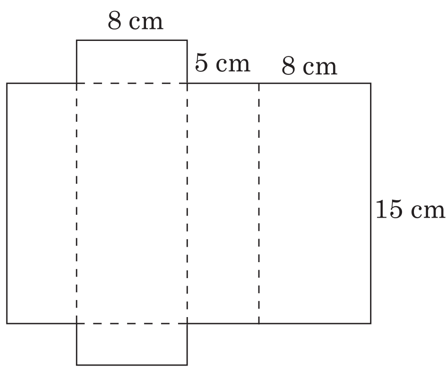
(ii)



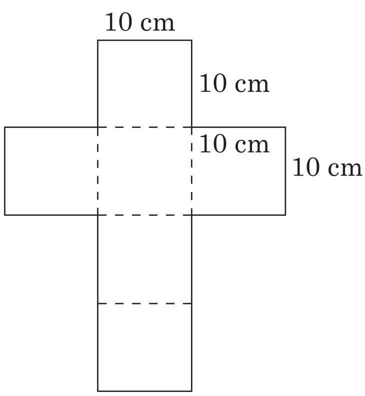
(iii)



(iv)



(v)

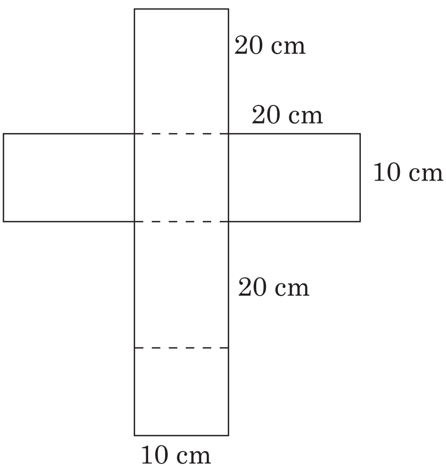


3. Draw the shapes below using the given measurements. Cut the shapes out. Fold them along the dotted lines.

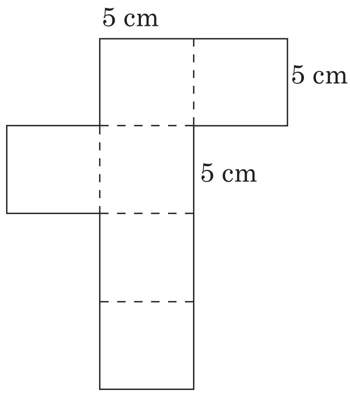
(a) Which of the nets make cuboids?

(b) Why do the other nets NOT make cuboids? Explain your answers.

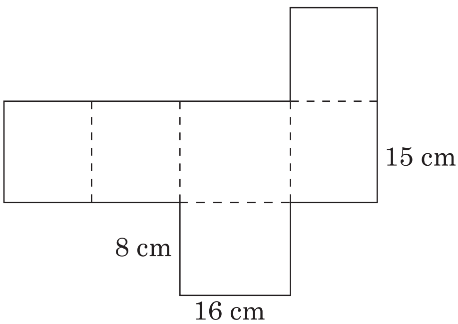
(i)



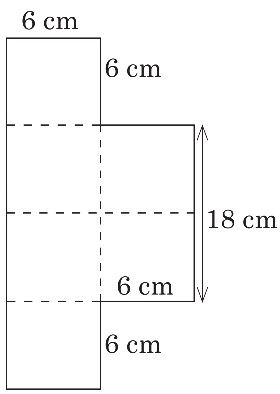
(ii)



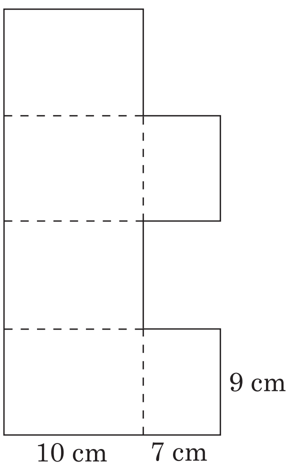
(iii)



(iv)



(v)



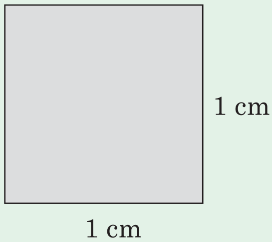
(c) How would you re-arrange some of the nets to make cuboids?

13.7 Calculating the volume of cubes and cuboids

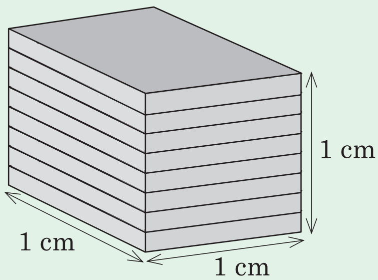
Volume of cubes

Activity 13.10

Cut out square pieces of manila paper with sides of 1 cm each.



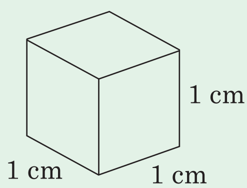
- Observe the space occupied by one square card. The area of the square card is 1 cm^2.
- Stack the square card up to a height of 1 cm.



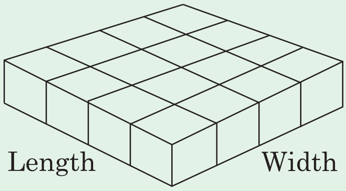
- Take the cubes and cuboids that you made from the previous activities. Which ones occupy bigger space? Which ones occupy less space?
- Compare the space occupied by your exercise book to that occupied by the text book.
- Discuss the space occupied by various objects in the classroom.

Activity 13.11

- Make several cubes like one shown below. This is a unit cube.



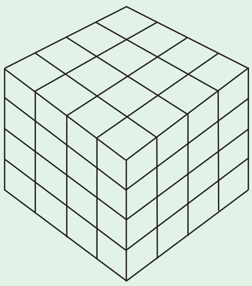
- Make a layer like the one below using unit cubes.



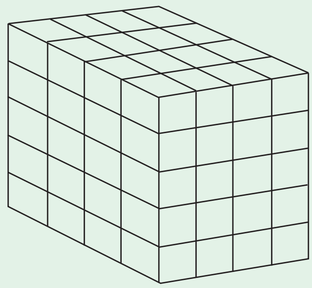
(i) How many cubes are there along the length?
(ii) How many cubes are there along the width?
(iii) How many cubes are there along the height?
(iv) Count the number of cubes in the layer.
(v) Calculate the number of cubes in the layer.

- By adding similar layers on top, make the following:

(a)



(b)



(i) How many cubes are along the length?
(ii) How many cubes are along the width?
(iii) How many layers are in the stack? Explain.
(iv) How many cubes form each stack? This is the volume of the stack. Discuss your results.
(v) Now let us calculate the volume as follows:
Volume = cubes along length × cubes along width × cubes along height.

Tip:

From Activity 13.11, above, in stack (a);
Its length = 4 cm
Its width = 4 cm
Its height = 4 cm
Its volume = length × width × height
= 4 cm × 4 cm × 4 cm = 64 cm^3

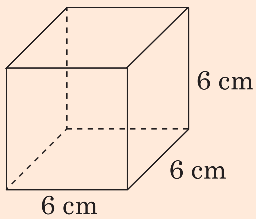
Now, calculate the volume of cuboid (b) using the formula.

Volume = length × width × height

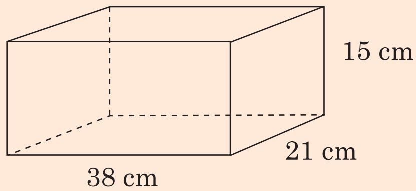
Example 13.4

Find the volume of the diagrams below.

(a)



(b)



(c) A rectangular box is 65 cm long, 40 cm wide and 28 cm high. Calculate the volume of the box.

Solution

(a) Volume = length × width × height

$= 6 \text{ cm} \times 6 \text{ cm} \times 6 \text{ cm} = 216 \text{ cm}$

(b) Volume = length × width × height

$= 38 \text{ cm} \times 21 \text{ cm} \times 15 \text{ cm} = 1\ 1970 \text{ cm}^3$

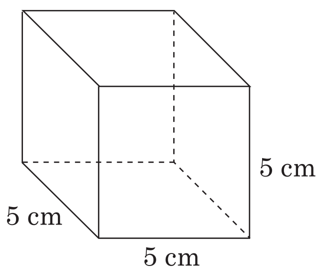
(c) Volume = length × width × height

$= 65 \text{ cm} \times 40 \text{ cm} \times 20 \text{ cm} = 72\ 800 \text{ cm}^3$

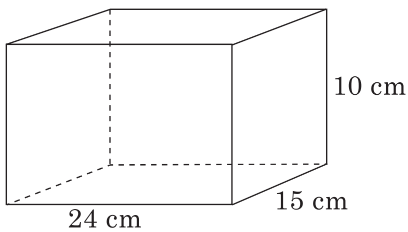
Practice Activity 13.7

Calculate the volume of each of the following.

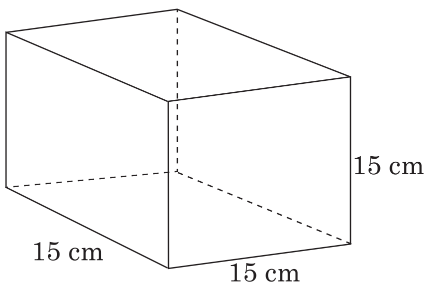
1.



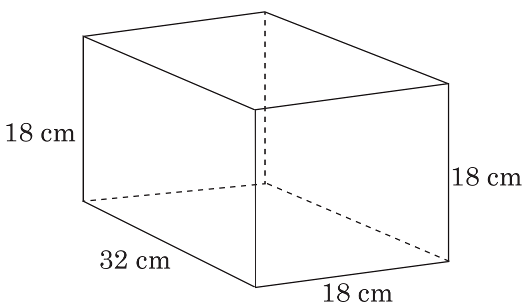
2.



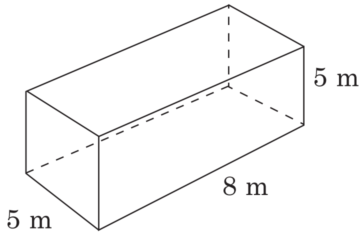
3.



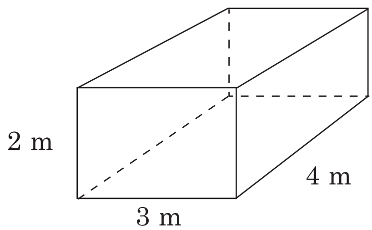
4.



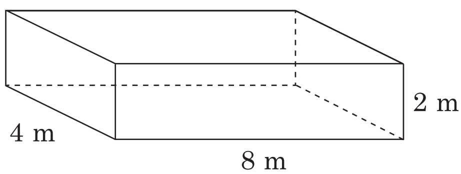
5. A rectangular tank measures 4.3 m long, 2.4 m wide and 1.5 m high. Calculate the volume of the tank. Justify your answer.
6. A large carton measures 64 cm long, 32 cm wide and 30 cm high. What is the volume of the carton? Where do we use a carton? Discuss.
7. The figure below represents a water tank. If it is filled with water, what is the volume of the water in it in cubic metres? Explain how you arrived at your answer.



8. A cube has 18 cm sides. What is its volume when it is a quarter full?
9. The figure below represents a water tank. It was half filled with water. How much water was in it? Discuss your steps.



10. The figure below represents a swimming pool. How much water in m^3 are in it when it is half full? Explain how you found your answer. Tell importances of a swimming pool.



Example 13.5

A box is 30 cm long, 20 cm wide and 15 cm high. What is the volume of the box?

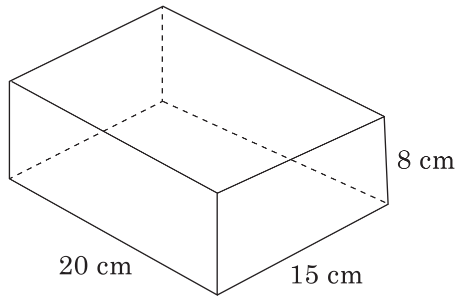
Volume $= \text{length} \times \text{width} \times \text{height} = l \times w \times h$

$= (30 \times 20 \times 15)\ cm^3$

$= 9\ 000\ cm^3$

Practice Activity 13.8

1. What is the volume of a cube whose sides are 20 cm. Explain how you arrive at the answer.
2. A rectangular water tank (cuboid) measures 4 m long, 3 m wide and 2 m high. What is its volume?
3. A building brick is 20 cm long, 15 cm wide and 8 cm high. What is its volume? Discuss your steps.



4. A box is 35 cm long, 22 cm wide and 18 cm high. Calculate $\frac{2}{3}$ of its volume. Present your answer.
5. An underground tank is 8 m long, 6 m wide and 10 m high. How much water in m^3 is required to fill it? Explain how you arrive at your answer.

13.8 Finding one dimension of a cuboid

Activity 13.12

Playing a game – The missing dimension

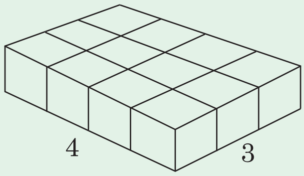
Required items: – 48 cubes each with 5 cm sides.
– Chart like the one below.

Play this game.

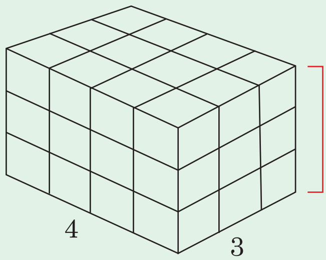
	Number of cubes (volume)	Number of cubes along length	Number of cubes along width	Number of cubes along height
(a)	36	4	3	
(b)	48		4	2
(c)	24	3		2
(d)	42	7	2	

Arrange unit cubes along the given sides.

Example: From (a) arrange as below.



Arrange such layers to have 36 unit cubes. How many layers are there? That is the number of cubes along the height.



3 layers

Repeat similar steps for (b) to (d). Fill in the missing blanks. Play in turns. Find out the shortest method to get the missing dimension. Present your method to the class.

Example 13.6

(a) The volume of a cuboid is 420 cm^3. It has a length of 10 cm and width 7 cm. What is its height?

Solution

Volume = l × w × h

420 cm^3 = 10 cm × 7 cm × h

420 cm^3 = 70 cm^2 × h

420 cm^3 ÷ 70 cm^2 = h

$h = \frac{42\cancel{0}}{7\cancel{0}}$ cm

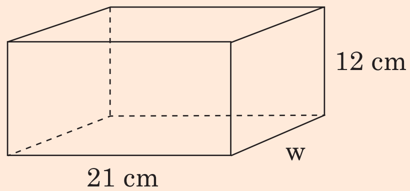
$$\begin{array}{r|l} & 6 \\ \hline 70 & 420 \\ & -\,420 \\ \hline & 0 \end{array}$$

h = 6 cm

The length is 6 cm.

Note: height = $\frac{\text{volume}}{\text{length} \times \text{width}}$

(b) Study the cuboid below. Its volume is 4 536 cm^3.



Find its width.

Solution

V = length × width × height

4 536 cm^3 = 21 cm × w × 12 cm

4 536 cm^3 = 21 cm × 12 cm × w

4 536 cm^3 = 252 cm^2 × w

4 536 cm^3 ÷ 252 cm^2 = w

$\frac{4536}{252}$ cm = w

18 cm = w

width = 18 cm

Note: width = $\frac{\text{volume}}{\text{length} \times \text{height}}$

(c) A rectangular tank has a volume of 7 m^3. It is 2 m wide and 1.4 m high. Find the length of the tank.

Solution

Volume = length × width × height

7 m^3 = length × 2 m × 1.4 m

= length × 2.8 m^2

Length = 7 m^3 ÷ 2.8 m^2

= $(\frac{7}{2.8})$ m = 2.5 m

Note: Length = $\frac{\text{volume}}{\text{width} \times \text{height}}$

Form an equation to find the missing number.

L × w × h = V

where L = ?

w = 2 m

h = 1.4 m

V = 7 m^3

Therefore,

L × 2 × 1.4 = 7

2.8 L = 7

$\frac{2.8\text{ L}}{2.8} = (\frac{7}{2.8})$ m

L = 2.5 m

Practice Activity 13.9

Copy and complete the table below.

	Length	Width	Height	Volume
1.	4 m	5 m	12 m	
2.	12 m	6 m		288 m^3
3.	36 cm		50 cm	25 200 cm^3
4.		50 cm	50 cm	125 000 cm^3

5. A carton has a volume of 142 560 cm^3. Its width is 36 cm and its height is 72 cm. Form an equation and calculate its length.
6. The volume of a rectangular water tank is 414 720 cm^3. Its length is 144 cm while its width is 36 cm. Calculate its height. Explain how you arrive at your answer.
7. A log of wood is in the shape of a cuboid. It is 35 cm long and 30 cm high. Its volume is 25 200 cm^3. How wide is the log?
8. A container is 40 cm wide and 18 cm high. Its volume is 21 600 cm^3. What is its length?
9. A box has a volume of 160 m^3. It is 8 m long and 5 m wide. What is its height?
10. A water tank has a length of 3.8 m, 2.5 m wide. Its volume is 38 m^3. What is its height?

13.9 Find the height of a cuboid given its volume and base area

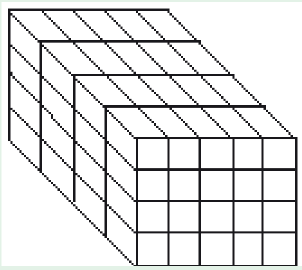
Activity 13.13

Collect several unit cubes. Use them to make different cuboids. In turns, player 1 makes a cuboid and states how many cubes are in it. Player 2 states how many unit cubes are in one layer. Player 3 states how many layers make the height.

Each correct answer given is awarded 3 marks. A wrong answer is not awarded any mark.

The cuboid is dismantled. Player 2 makes his or her own cuboid.

Player 3 gets the cubes in one layer. Player 1 gets the layers along the height.



Record your results in the table below.

Number of cubes for the stack (volume)	**Number of cubes in one layer (base area)**	**Number of cubes along height**

Compare: Volume ÷ base and height.

What do you notice? Discuss your findings.

Discuss and match the following.

Activity	**Quantity**
Cubes on shorter side at the base	Volume
Cubes along height	Base area
Cubes in stack	Height
Cubes in layer	Width

Justify your answer.

Example 13.7

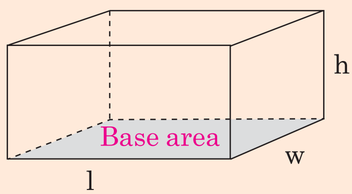
The volume of the cuboid is 72 000 cm^3. Its base area is 2 400 cm^2. What is its height?

Solution

Volume = l × w × h

Volume = base area × height

Therefore, height = $\frac{\text{volume}}{\text{base area}}$



Height $= \frac{\overset{30}{\cancel{72000}}}{\underset{1}{\cancel{2400}}}$

= 30 cm

Note: Height = $\frac{\text{volume}}{\text{base area}}$

Practice Activity 13.10

Copy and complete the table below.

	Volume	Base area	Height
1.	3 150 cm^3	315 cm^2	
2.	576 cm^3	144 cm^2	
3.	250 m^3	50 m^2	

4. The volume of a cuboid is 18 000 cm^3. Its base is a square of side 30 cm. What is
   (a) its base area? Explain how you got answer.
   (b) its height?
5. The base of a tank is a rectangle whose length is 90 cm and width 50 cm. Its volume is 10 000 cm^3. Calculate its height.
6. The volume of a rectangular water tank is 8 000 cm^3. It has a base area of 160 cm^2. What is its height? Explain your answer.
7. The floor of a classroom measures 8 m long and 7 m wide. The volume of the classroom is 168 m^3. What is the height of the classroom. Compare this with the height of your classroom. Discuss your results.
8. To make a brick, a mason used 11 220 cm^3 of mortar. He made a brick whose length was 34 cm and width 22 cm. Calculate the height of the brick.
9. The volume of an underground tank is 84 m^3. Its base area is 28 m^2. How deep is the tank?

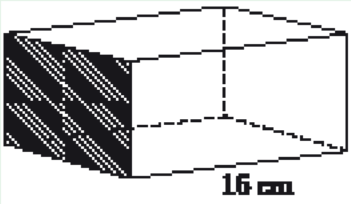
13.10 Finding the area of a face of a cuboid

Activity 13.14

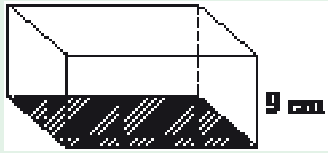
Study the following cuboids. Discuss and find the area of their shaded faces. The volume of each cuboid is 1 728 cm^3. The shaded face becomes the base and the given length is the height.

Remember base area × height = volume.

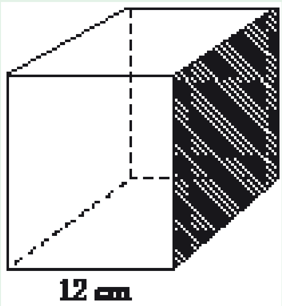
(a)



(b)



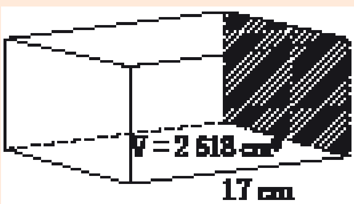
(c)



Present your findings and how you got your answer to the class.

Example 13.8

Calculate the area of the shaded face of the cuboid below. Its volume is 2 618 cm^3.



Solution

Volume of the cuboid = 2 618 cm^3, length = 17 cm.

$$\text{Area of shaded face} = \frac{\text{volume}}{\text{length}}$$

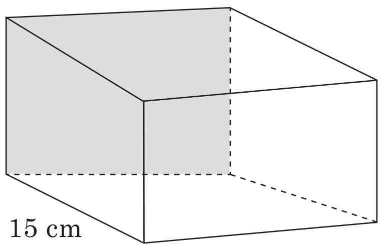
$$= \frac{2\,618 \text{ cm}^3}{17 \text{ cm}}$$

$$= 154 \text{ cm}^2$$

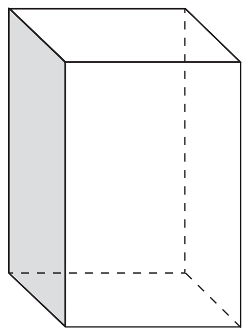
Practice Activity 13.11

1. Find the area of the shaded faces of the given cuboids. The volume (V) and one dimension have been given in each case.

(a) $V = 3\ 120\ \text{cm}^3$

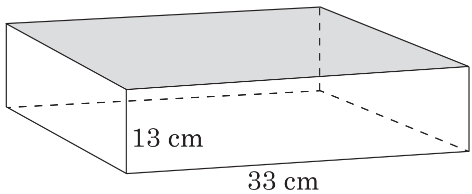


(b) $V = 1\ 428\ \text{cm}^3$

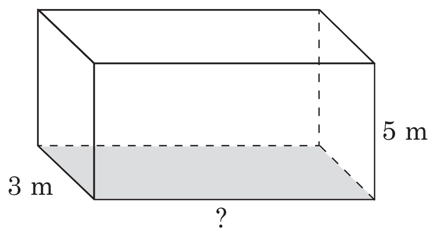


2. Study the figures below:

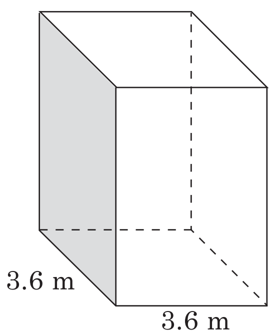
(a) $V = 8\ 151\ \text{cm}^3$



(b) $V = 90\ \text{m}^3$



(c) $V = 41.472\ \text{m}^3$



(i) Calculate the area of the shaded part.

(ii) Discuss and find the missing dimension of each figure.

(iii) Explain how you calculate the missing dimension.

3. A carpenter made a rectangular wooden box with a volume of $1.44\ \text{m}^3$. Its length was 1.5 m and width 0.8 m.

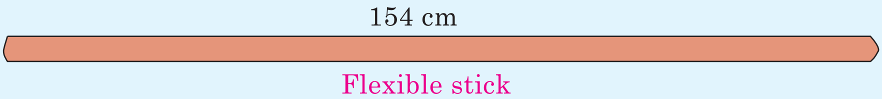
(a) What is its area? Discuss your steps.

(b) Find its height.

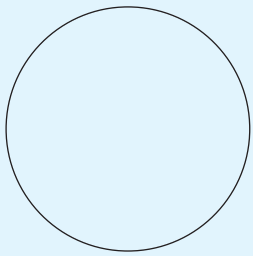
(c) How did you get its height? Explain.

Revision Activity 13

1. Collect a flexible stick that is 154 cm long.



Fold it into a circle to form a wheel. Use tape to attach the ends.

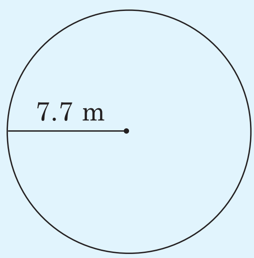


(a) What is the circumference of the wheel made from the flexible stick?

(b) What is the diameter of the wheel. (Use a ruler to measure)

2. What is the circumference of a wheel whose diameter is 28 cm? Take $\pi = \frac{22}{7}$.

3. What is the circumference of the circle below. Take $\pi = \frac{22}{7}$.



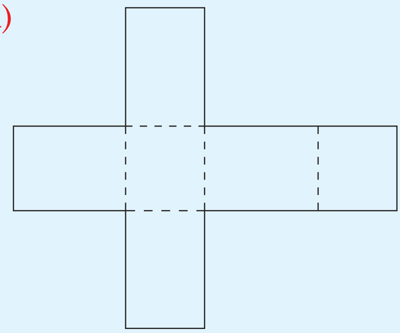
4. How many faces are there in an open cube?

5. What is the product of the edges and the vertices of a cuboid?

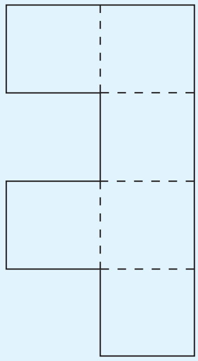
6. The following nets are folded to form cubes or cuboids. Indicate the ones that will form:

(i) cubes (ii) cuboids (iii) neither cuboids nor cubes

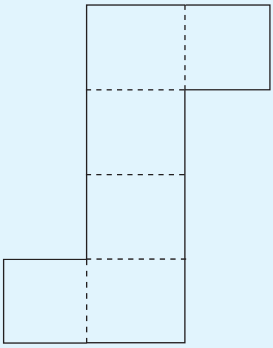
(a)



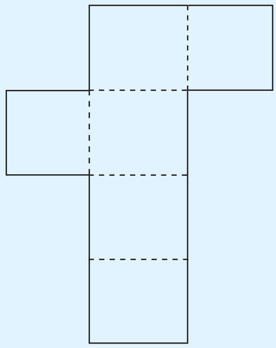
(b)



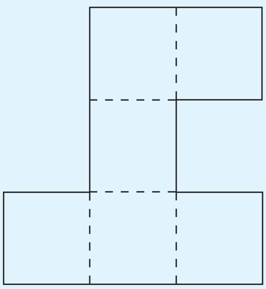
(c)



(d)



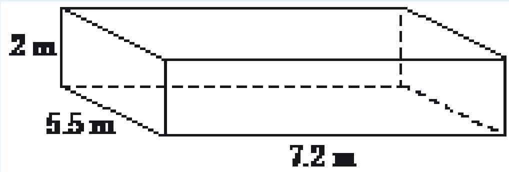
(e)



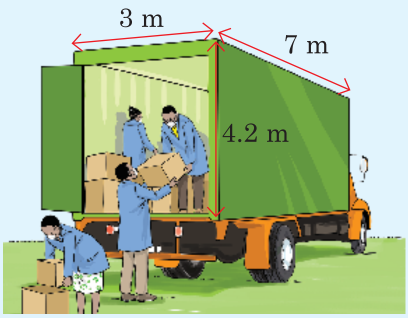
7. To make a roundabout, the contractor needed a circumference of 88 m. What diameter does he use to get the circumference? Explain your answer.

8. Small bottles packed in small packets measuring 10 cm long, 10 cm wide and 20 cm high were packed in a carton measuring 80 cm long, 60 cm wide and 40 cm high. How many bottles fit in the box? Explain your answer.

9. What is the volume of the cuboid below.

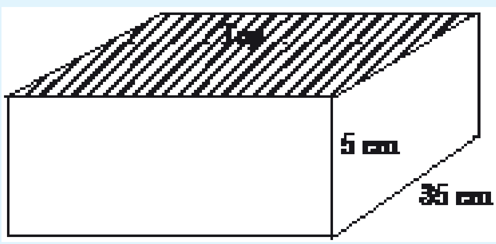


10. What is the volume of a cube whose sides are 18 cm? Explain how you arrive at your answer.
11. Study the container for the truck below. It is for carrying loads like cartons of books.



The container measures 7 m long, 3 m wide and 4.2 m high. Find the volume of the container. Justify your answer.

12. The volume of a cube is 64 cm^3. What is the measurement of one of its sides? Present the process of arriving at the answer.
13. The volume of a box is 405 m^3. It is 15 m long and 9 m wide. What is its height? Discuss your steps.
14. The diagram below represents a box made up of cardboards.



Its volume is 4 530 cm^3. What is the area of its top? Explain how you get the answer.

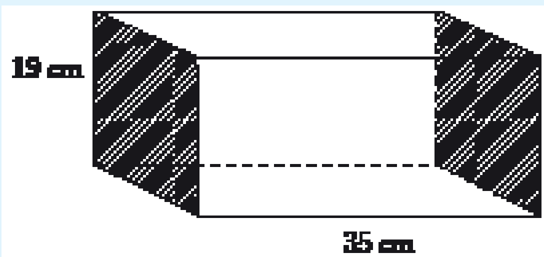
15. Find the length of a cuboid whose volume is 87 360 cm^3. It is 48 cm wide and 35 cm high.

16. The diagram below represents a water tank. Its volume is 32 m^3. What is its depth?

Present the process of getting the answer.

17. A rectangular container had two faces made from metal sheets. The shaded part represents the metal sheets. Its volume is 19 285 cm^3. Calculate the area of the metal sheet used to make the shaded parts.

Explain your answer.



Word list

Diameter	Radius	Circumference
Cube	Cuboid	Net
Circle	Base area	Height
Width	Length	

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 14 Statistics

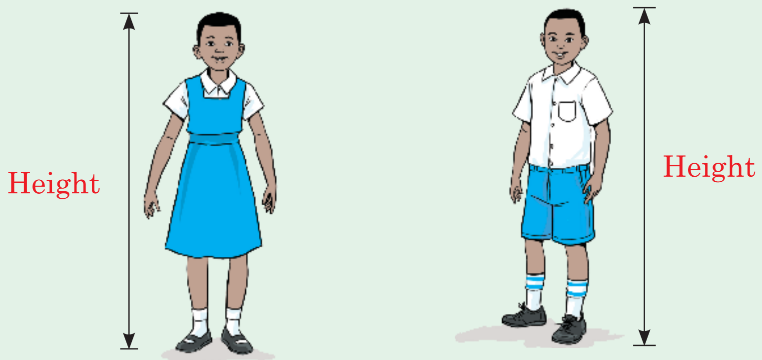
14.1 Continuous and Discrete Quantitative data

Activity 14.1

1. **Question:** What type of data are your heights?

   Materials: Tape measure or ruler.

   Steps:



Girl Boy

(i) In groups, measure your heights. Record your results in the table below.

(ii) Observe the values you record.

Name						
Height						

(a) Name the type of data you collect. Discuss your answer.

(b) Does your data have various values that include decimals?

2. Now, form a question to collect data for
   - distance to school compound.
   - time taken to get to school.

   Follow the same process you used in (1).

Activity 14.2

1. **Question:** How many brothers and sisters do you have?

   Steps:

   (i) In groups, make a chart below. State and record your number of sisters and brothers.

Number of pupils	Number of sisters	Number of brothers

   (ii) Observe the values you record. Do they take various values? Do your values include decimals? Explain your answers.

2. Now, follow the same steps and form questions to collect data on:
   - Shoe sizes worn by adults.
   - Shoe sizes worn by children.

Task

Discuss the differences in the type of data from Activity 14.1 and 14.2. Show a summary of the type of data you collected.

Tip:

- Data with numerical values is called quantitative data.
- The values for numerical data can be whole numbers only. Such data is discrete.
- The values for numerical data can take any number including decimals. Such data is continuous.

Example 14.1

From Activity 14.1 and 14.2, state the type of quantitative data you collected.

Solution

(a) Discrete quantitative data include: number of brothers or sisters you have and shoe size worn by different people.

(b) Continuous quantitative data include: distance from home to school, time taken to get to school and heights of pupils in class.

(c) Discuss and name other discrete and continuous quantitative data.

Practice Activity 14.1

1. The following data was collected by a group of pupils.

Pupil	Jean Paul	Anitha	Bernard	Fike
Height	1.40 m	1.38 m	1.43 m	1.42 m

   (i) What type of data was collected?
   (ii) Is the data discrete or continuous? Explain your answer.

2. A class collected numerical data on the following.
   (a) Shoe sizes worn by pupils in school.
   (b) Time taken to run twice round the field.
   (c) Distance from home to school for group members.
   (d) Number of parents for different pupils in a class.

   Draw a table and group the data accordingly. Discuss your answers.

	Discrete	**Continuous**
(i)		
(ii)		

3. Shyaka and Filonne collected the following data.

   (a)

Pupil	Anne	Nadege	Innocent	Martin
Number of friends	2	4	6	2

   (b)

Colour of car passing by school	Red	White	Blue	Grey
Number of cars	10	20	7	14

   In each case, identify the type of quantitative data?
   Explain your answer.

14.2 Representing data using bar charts

Activity 14.3

Represent the data you collect in Activity 14.1 and 14.2 using a bar chart.

Explain your bar chart.

Tell cases where you can represent data using bar graphs.

Example 14.2

The table below contains data about the number of cars in a car park.

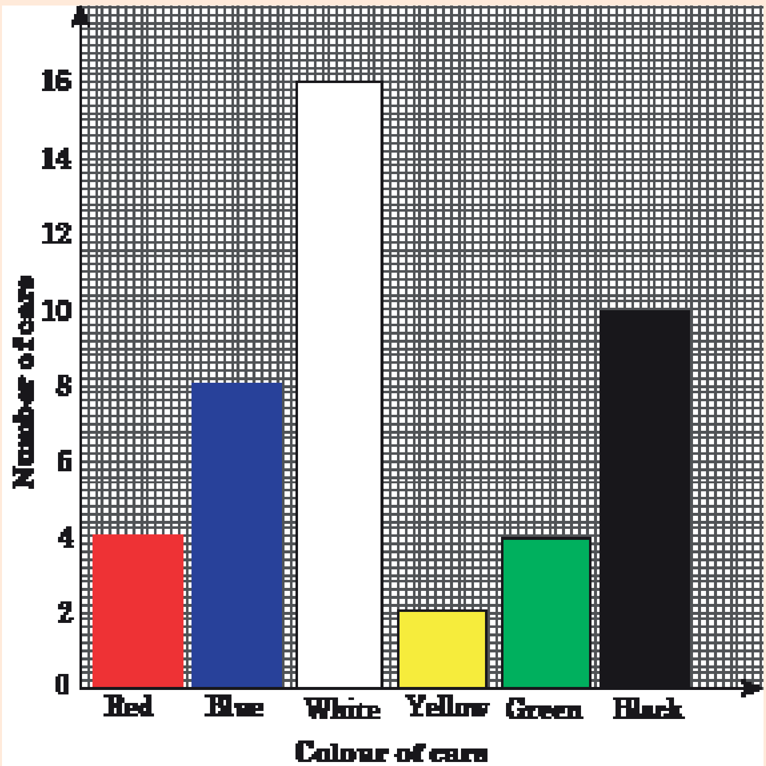
Red	Blue	White	Yellow	Green	Black
4	8	16	2	4	10

Represent the data in a bar chart.

Solution

Steps:

- Draw horizontal and vertical axes. Label them as shown.
- Choose a good scale to allow you to plot the data easily.
- Mark the length of bars as per the number of cars for each colour. Draw them.



Practice Activity 14.2

1. The table below contains data for heights of family members.

Family member	Father	Mother	1st child	2nd child	3rd child
Height	1.9 m	1.7 m	1.5 m	1.4 m	1.2 m

Represent the data using a bar chart.

2. The table below contains data for shoe sizes for different people.

Person	Elise	Janvier	Nadege	Phionah
Shoe size	6	8	4	7

Represent the data using a bar chart.

3. The table below contains data about the number of books each pupil has.

Name	Olivier	Thierry	Elie	Christine	Carene
Number of books	10	12	8	15	13

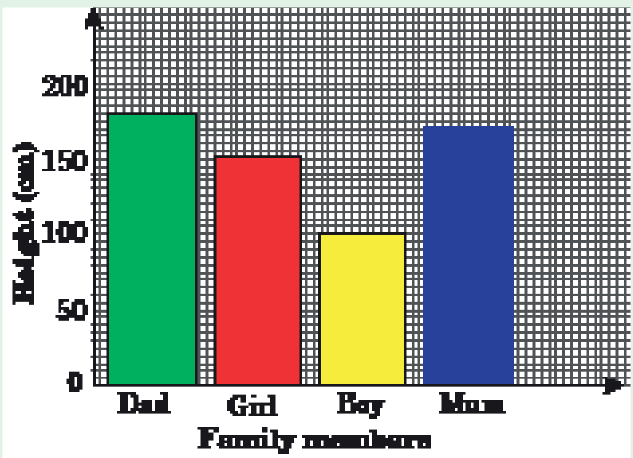
Represent the data using bar chart.

4. What type of data are those in questions 2 and 3?

14.3 Interpreting bar charts

Activity 14.4

Study the bar chart below.



(a) What is the bar chart about? Explain.

(b) Read the height of:

(i) Dad (ii) Mum (iii) Boy (iv) Girl

Tip: In interpreting bar charts

(i) Read the lengths of bars and the information they represent. Check the vertical axis.

(ii) State the information represented by each bar. Check the horizontal axis.

Example 14.3

From Activity 14.4, answer the following.

(a) What information is shown by the bar chart?

(b) Who is the tallest?

(c) Who is 150 cm tall?

(d) How is bar chart important to you?

Solution

(a) On the vertical axis, we have height (in cm).

On the horizontal axis, we have family members.

The information shown is the heights of family members.

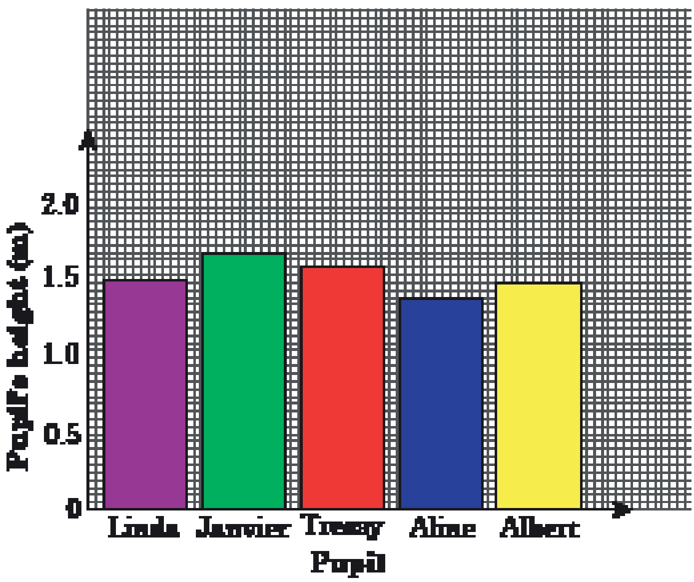
(b) Dad is the tallest. His height is 180 cm.

(c) Starting from vertical axis, at 150 cm put a ruler. Draw a dotted line to see which bar is at 150 cm.

The bar representing the Girl is at 150 cm. The girl is 150 cm tall.

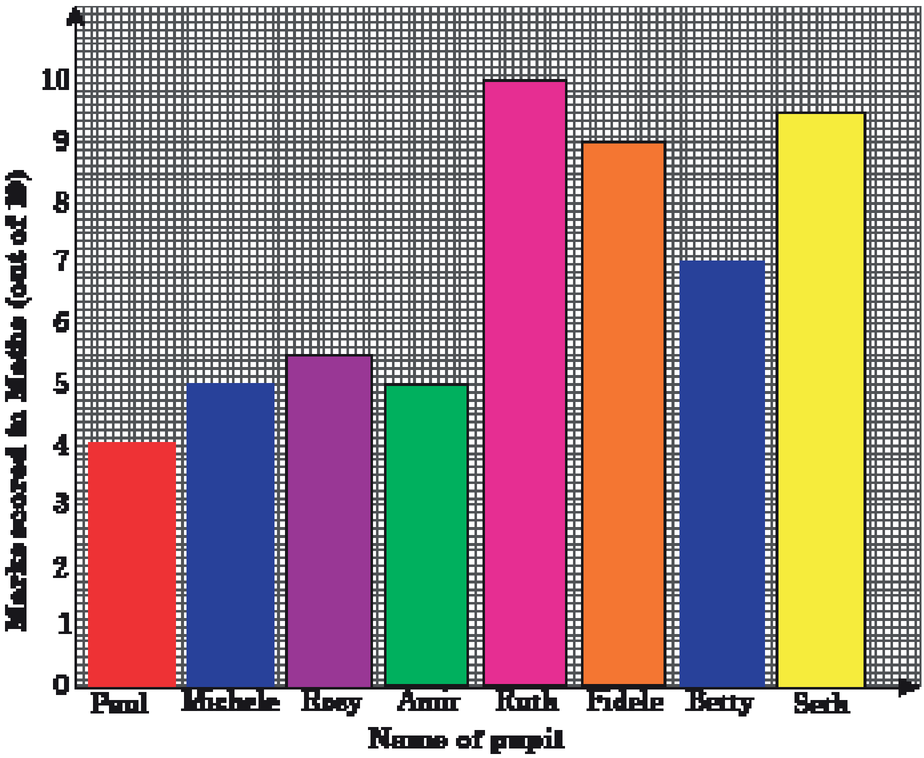
Practice Activity 14.3

1. Look at the bar graph below.



(a) How many pupils measured their heights?
(b) What information is shown on the graph?
(c) Who is the shortest?
(d) How many metres tall is the tallest pupil?
(e) Which pupils were the same height?

2. Study the bar chart below.



(a) What is the bar chart about?
(b) How many pupils did the mathematics activity?
(c) What is the highest score?
(d) Who scored the lowest mark?
(e) Which pupils scored the same mark?
(f) Who scored the highest mark?
(g) How many more marks did Ruth score than Rosy?
(h) What score did Seth get?

14.4 Representing data using line graphs

Activity 14.5

Materials: Metre rule or tape measure.

- Measure the length of your shadow at the following times. Record your findings in the following table.

Time	8:00 a.m	10:00 a.m	12:00 noon	2:00 p.m	4:00 p.m
Length of your shadow					

- Represent your data using a line graph. Discuss your steps.

  Where can you use line graphs to represent data in daily life?

Tip: To draw a line graph;

(i) Choose a suitable scale for all values.

(ii) Draw axes and label them.

(iii) Plot points on the graph.

(iv) Join the points using straight line.

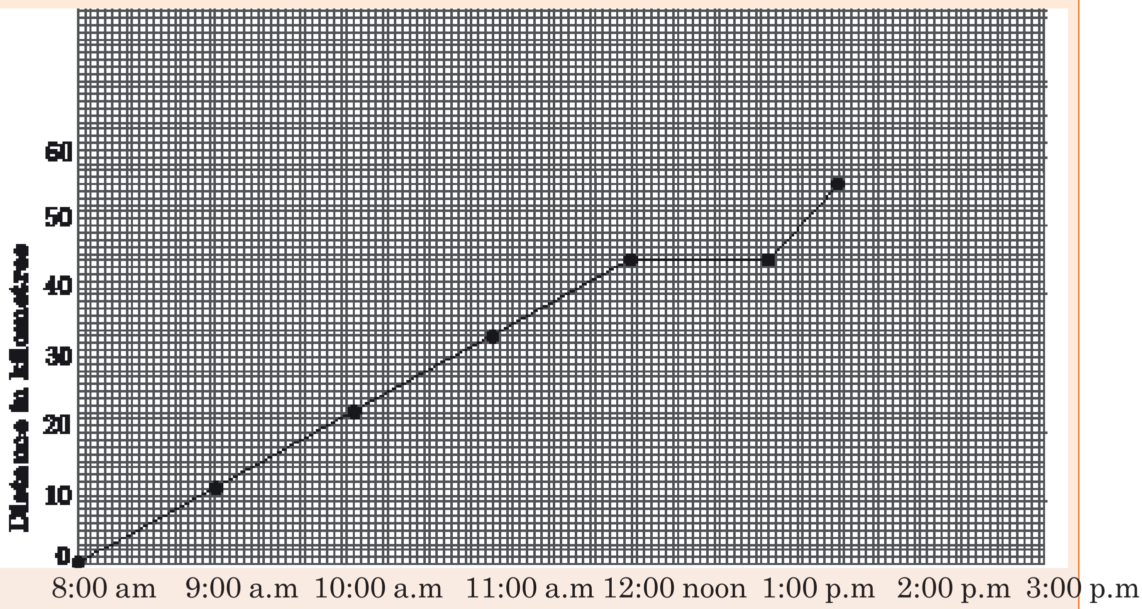
Example 14.4

Below is data for the distance covered by a cyclist at different times.

Time	8:00 a.m	9:00 a.m	10:00 a.m	11:00 a.m	12:00 p.m	1:00 p.m	2:00 p.m
Distance (km)	0	11 km	22 km	33 km	44 km	44 km	55 km

Represent the data using a line graph.

Solution



Practice Activity 14.4

1. Study the data in the table below. It shows the distance travelled by a motorist.

Time	10:00 a.m	11:00 a.m	12:00 noon	1:00 p.m	2:00 p.m	3:00 p.m	4:00 p.m
Distance covered (km)	0	50	100	150	150	250	350

Represent the data using a line graph.

2. The data below is the amount of water used by a family after every two hours.

Time	7:00 a.m	9:00 a.m	11:00 a.m	1:00 p.m	3:00 p.m	5:00 p.m
Water used (litres)	0	30	60	60	90	120

(a) Represent the data using a line graph.

(b) Why was the amount of water the same from 11:00 a.m to 1:00 p.m? Explain your answer.

3. A pupil did a number of competence exams. She got the following marks at different times.

Time	Week 1	Week 2	Week 3	Week 4	Week 5
Marks scored (%)	40	58.5	80	96.5	100

Represent the data using a line graph.

Was the pupil improving in performance or not? Explain your answer.

4. A farmer planted a crop. She measured its height after every two months. She recorded the data below.

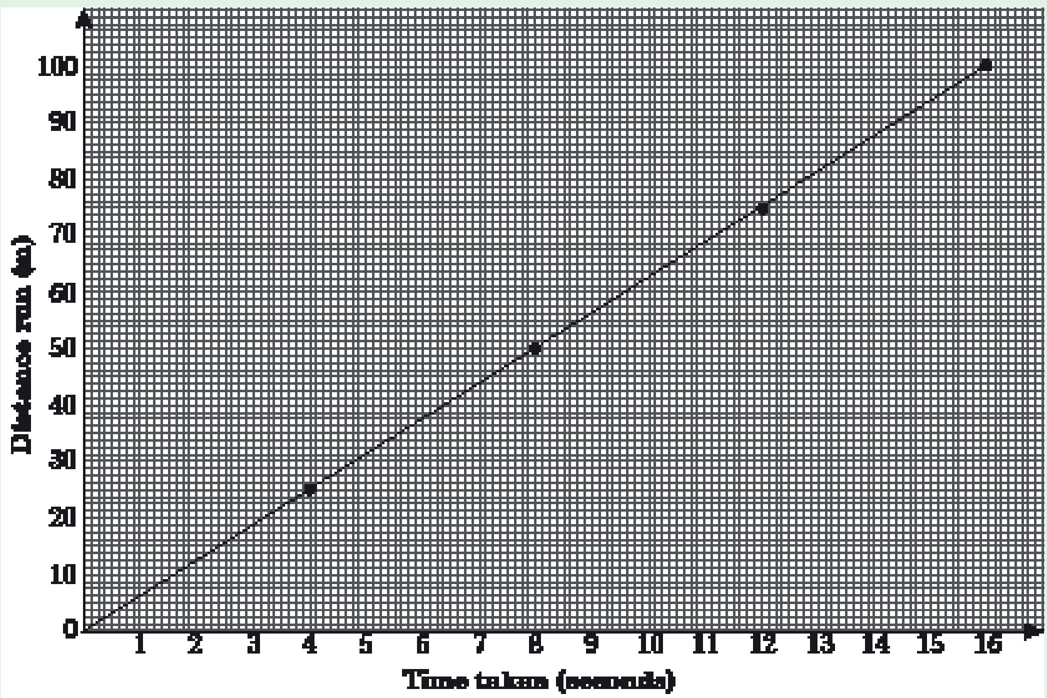
Time	0	2 months	4 months	6 months	8 months	10 months
Height (cm)	0	50	100	150	180	200

Represent the data using a line graph. Discuss your steps to draw an accurate line.

14.5 Interpreting line graphs

Activity 14.6

Look at the line graph below. It represents data for distance covered by an athlete.



(i) What is the line graph about?

(ii) How many seconds does the athlete take to cover 100 m? Explain the steps you used to find the answer.

Example 14.5

Study the graph in Activity 14.6. Answer these questions:

(a) What distance does the athlete cover in 4 seconds?

(b) How long does it take the athlete to cover 75 m?

(c) Find the distance the athlete covered in 8 seconds.

(d) How is line graph important to you?

Solution

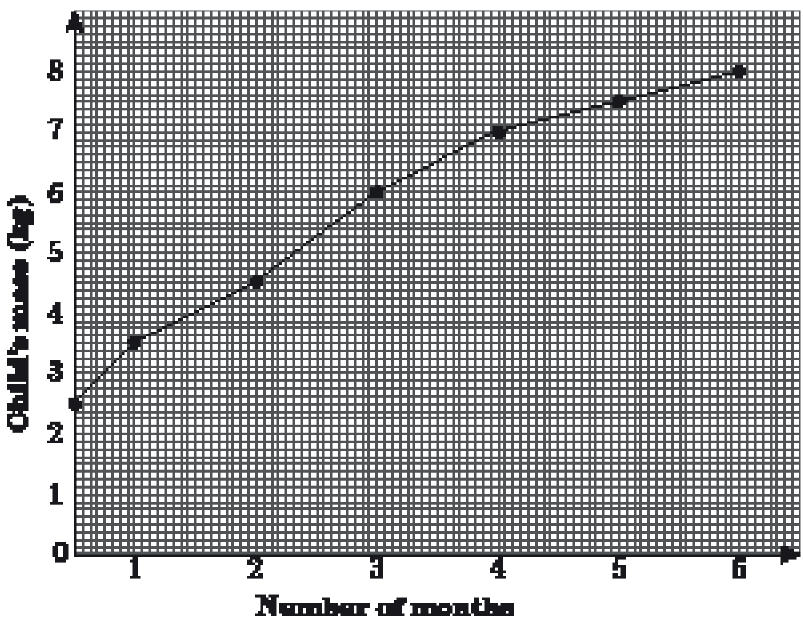
(a) Put the ruler vertically at 4 seconds. It cuts the line at 25 m.

(b) Put the ruler horizontally at 75 m. It cuts the line at 12 seconds.

(c) In 8 seconds, the athlete covered 50 m.

Practice Activity 14.5

1. Look at the graph below.



(a) What is the graph about?

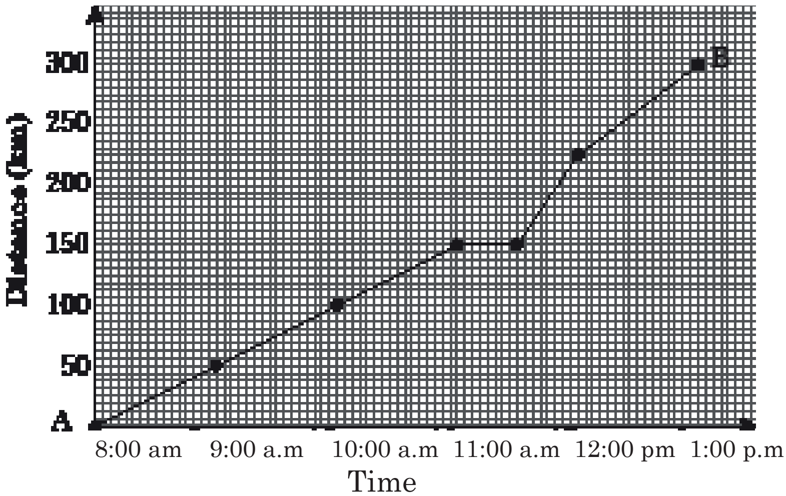
(b) What is the mass of the child at birth?

(c) Read the mass of the child at 3rd month.

(d) At what month is the mass of the child 7 kg?

(e) What is the change in the child's mass from the 3rd to the 4th month?

2. A motorist started a journey from town A to B. The data is represented in the graph below.



(i) How far is town A from B?

(ii) What time did the motorist start the journey?

(iii) What time did the motorist start to rest?

(iv) What distance did the motorist cover from 11:00 a.m to 11:30 a.m?

(v) Find the time taken to reach town B after resting.

Revision Activity 14

1. In a school the following data was recorded.

Class	P1	P2	P3	P4	P5	P6
Number of pupils	50	47	48	45	45	40

(a) Name the type of quantitative data above. Explain.

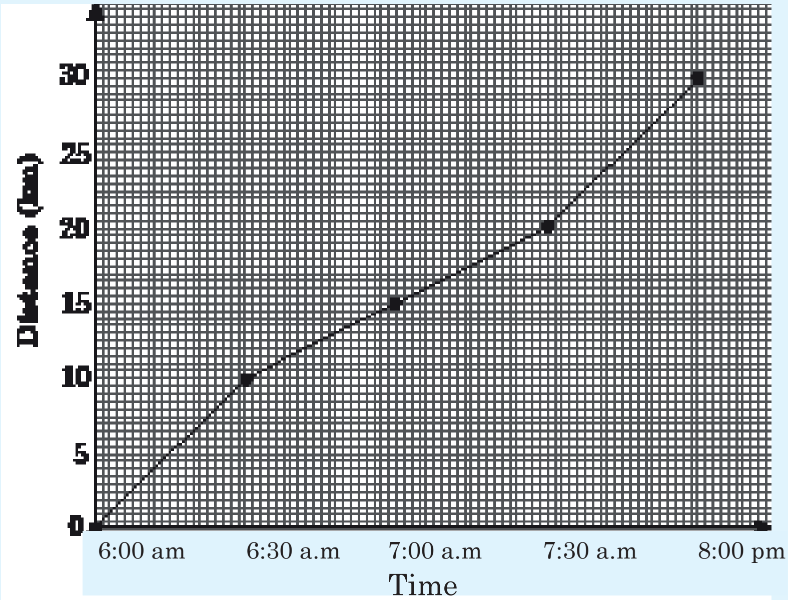
(b) Represent the data using a bar chart. What do you notice?

(c) From your bar graph;

(i) Which class had the highest number of pupils?

(ii) Which two classes had the same number of pupils?

2. In a cross-country race, a top athlete was involved. The time and distance covered are represented below.



(a) What was the distance ran by the athlete?

(b) At what time did the athlete complete the race?

(c) At what time did the race begin? Explain your answer.

(d) How much distance had the athlete completed at 7:00 a.m?

(e) At what time had the athlete run 10 km? Discuss the steps to your answer. What are the importance of interpreting line graphs?

3. The amount of rainfall was recorded for a certain town.

Month	January	February	March	April	May	June
Rainfall (mm)	10	2.5	15	30	45	20

(a) What type of quantitative data was collected?

(b) Represent the data using a line graph.

(c) From your graph, which month had the highest rainfall? Explain.

Word list

Quantitative data
Continuous quantitative data
Discrete quantitative data
Record
Collect data
Represent data
Interpret data
Bar graph
Line graph

Task

Do the following.

(i) Read each word aloud to your friend.

(ii) Write the meaning of each of the words above. Discuss with your friend.

(iii) Write sentences using each of the words above. Read with your friend.

UNIT 15 Probability

15.1 Vocabulary of chance

Activity 15.1

Look at the vocabulary of chance. Read each of them.

Impossible	Certain	Sure
Unlikely	Likely	
Equally likely	Even chance	Equal chance

What is the meaning of each of them? Use each of them to compare events.

Let us now do the following activities. Each activity has different events. Occurrence of an event involves chance. You will learn the vocabulary of chance for different events.

Activity 15.2

(a) Inside your class, what is there?
   (i) Is it certain that there is a pupil?
   (ii) Is it impossible to find a pupil in class?
   (iii) Can you find a book in your class? Which vocabulary of chance can you use? Certain or impossible?

(b) Go outside your classroom. What is there in your school?
   Are you likely or unlikely to find the following? Explain.
   - a tree
   - a lion
   - a bird
   - grass
   - a car
   - a tea plant
   - a cow
   - a motor bike
   - other things in your area

(c) Toss a coin. What side is likely to face up? Head or tail.

Repeat several times, count the heads and tails.

(i) Does head and tail have equal chance to face up?

(ii) Is it possible to have both head and tail face up at once?

(iii) Is it likely to have either head or tail face up in a toss?

Which vocabulary of chance can you use here in (ii) and (iii)?

- Why do referees toss a coin before starting a football match? Discuss your answer.

Tip:

In tossing a coin, either head or tail face up.

(i) It is sure or certain to see either head or tail in a toss.

(ii) It is not possible to have both head and tail face up at once in a toss. It is impossible for a coin to face up and give outcome of both head and tail at once.

(iii) It is equally likely to see either tail or head in a toss. Both head and tail have equal chance.

(iv) When tossing a coin several times, it is unlikely to observe heads only or tails only. It is likely you will observe heads and tails.

Practice Activity 15.1

Use the correct vocabulary of chance in each of the following sentences. Discuss your answers.

1. In a museum, you are ___ to see preserved animals.
2. In animal park, you are ___ to see a wild animal in cage.
3. Inside your class, it is ___ to see a living lion lying next to you.
4. It is ___ to see a bird flying by your school.
5. It is ___ that you have tails only in several tosses of the coin.

15.2 Conducting experiments and chances

You have tossed a coin in Activity 15.2. This is an example of simple experiment involving chance. Every experiment is such that the results are predictable. Results are called outcomes. The outcomes for tossing a coin are heads or tails. Let us now do more experiments of chance.

Activity 15.3

- Toss a coin 20 times. Record the results in the table below. For example, if in 1st throw, head faces up, then tick (✓) head in table.

Throw	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Head	✓																				
Tail																					

- Count the total number of heads and write this in the total. Count the total number of tails and write this in the total.

Compare your results with the other groups.

1. What are the chances of getting a tail in a throw? Explain.
2. What are the chances of getting a head in a throw? Explain.
3. When you toss a coin, which side is likely to face up? Why?
4. Does the outcome depend on what was observed previously? Why?

Fill in the table below by ticking the appropriate box. Discuss your answer.

	Vocabulary of chance applicable				
Possible outcome in tossing a coin	**Certain**	**Impossible**	**Likely**	**Unlikely**	**Equally likely**
Getting a head					
Getting a tail					
Getting tail and head at once					
Getting no tail and no head					

Tip:

- When tossing a coin once,

  The chances of getting a head or tail $= \frac{\text{Outcome observed}}{\text{Total possible outcomes}}$.

  When you toss a coin, it is either head or tail that faces up.
- In tossing a coin several times,

  The chances of getting a head $= \frac{\text{Number of heads observed}}{\text{Total possible outcomes}}$

Activity 15.4

Toss a dice 48 times and record the outcomes.

For example, if after 48 tosses, 1 faces up four times, record as below.

Faces of dice	1	2	3	4	5	6
Tally for outcome	////					
Total	4					

- Find the totals for each number. Using the result from your table, make a bar graph.
- Compare your results as a class. Add the results from all the class members for each number. Make a table, and draw a bar graph from the table.
- Discuss the following:

(a) Find the chances of rolling:

(i) 1 (ii) 3 (iii) 6

(b) Do some scores have better chances than others?

(i) What is the chance of getting an even number?

(ii) What is the chance of getting an odd number?

(iii) Is getting an even number more likely than getting an odd number?

(c) Compare the chance of rolling a 2 in dice and observing a head when tossing a coin twice. Which is greater? Or are they the same?

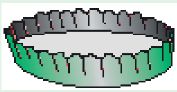
Tip:

Chance of getting a number when throwing dice once $= \dfrac{\text{Score observed}}{\text{Total possible scores}} = \dfrac{1}{6}$.

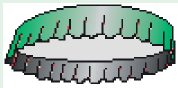
Chance of getting a score when throwing a dice many times $= \dfrac{\text{Results for a score}}{\text{Total results recorded}}$

Activity 15.5

Take a bottle top and throw it twenty times. Record the results.



Face up



Face down

Throw	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Face up																				
Face down																				

Compare your results with the rest of the class.

Discuss the following:

- Does the bottle top behave the same way as the coin?
- Why does it behave that way? Explain.

Practice Activity 15.2

1. In an activity, a pupil tossed a coin once.
   (i) What was the possible outcome?
   (ii) What was the chance of a head facing up?
2. In an experiment, a group toss a coin 20 times. They recorded the results below. For head facing up, they recorded H. For tail facing up, they recorded T.

Throw	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Head facing up	H	H		H				H		H	H		H				H	H	H	
Tail facing up			T		T	T	T		T			T		T	T	T				T

   (i) Find the total number of heads (H) that faced up.
   (ii) Find the total number of tails (T) that faced up.
   (iii) What was the chance of getting the head (H)?
   (iv) Find the chance of getting the tail (T).
   (v) Did the result of the previous toss affect the next result?

3. In an experiment, two pupils toss a dice 48 times. They recorded their results as below.

Face of dice	1	2	3	4	5	6	**Total**
Total times it faced up	9	8	8	7	8	8	48

(i) Represent the data on a bar graph.

(ii) Which face had the highest chance of showing up? What was its chance?

(iii) Which face showed up the least? What was the chance of getting it?

(iv) What was the chance of getting the face 5? Discuss your answer.

4. A bottle top was tossed 10 times. The results for facing up or facing down are below.

Times	1	2	3	4	5	6	7	8	9	10
Facing down	✓	✓								
Facing up			✓	✓	✓	✓	✓	✓	✓	✓

(i) What was the total number of results observed?

(ii) Find the total number for bottle top facing down.

(iii) Find the total number of bottle top facing up.

(iv) Does tossing a bottle top give a fair chance to either face? Why? Explain your answer.

Revision Activity 15

1. A pupil went to bathe by a river. Use vocabulary of chance to fill in the following.

(i) It is ___ he will wet his body. (certain, unlikely)

(ii) It is ___ he will bathe without getting wet. (certain, impossible)

(iii) It is ___ he will use soap. (unlikely, likely)

(iv) It is ___ that he will wet and then dry his body. (equally likely, unlikely)

(v) It is ___ that he will bathe without soap. (likely, unlikely)

2. Eric and Olive conducted an experiment. They tossed a coin ten times. They recorded their results below.

Throw	1	2	3	4	5	6	7	8	9	10
Head faced up	✓	✓		✓			✓	✓		✓
Tail faced up			✓		✓	✓			✓	

✓ shows the result that was observed at each throw.

(i) How many times did heads faced up?
(ii) How many times did tails faced up?
(iii) Find the total times heads and tails faced up. Explain your steps.
(iv) What was the chance of getting a head?
(v) What was the chance of getting a tail?
(vi) Was it possible to have both head and tail at once in a toss? What was the chance of having both a head and a tail at once? Discuss your answer.
(vii) If a bottle top was used, would the results be the same? Explain your answer.
(viii) Tell the importance of learning probability.

Word list

Vocabulary of chance	Likely	Unlikely	Equally likely
Even chance	Sure	Impossible	Toss a coin
Toss a dice	Toss a bottle top		Experiment

Task

Do the following.

(i) Read each word aloud to your friend.
(ii) Write the meaning of each of the words above. Discuss with your friend.
(iii) Write sentences using each of the words above. Read with your friend.