MATHEMATICS BOOK FOR TTCs

TUTORS' GUIDE

YEAR

1

OPTION:

EARLY CHILDHOOD AND LOWER PRIMARY EDUCATION (ECLPE)

FOREWORD

Dear Tutor,

Rwanda Basic Education Board is honoured to present the tutor's guide for Mathematics in the option of ECLPE which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of the mathematics content. The Rwandan educational philosophy is to ensure that student-teachers achieve full potential at every level of education which will prepare them to be well integrated in society and exploit employment opportunities.

Specifically, TTC curriculum was reviewed to train quality teachers who will confidently and efficiently implement the Competence Based Curriculum in pre-primary and primary education. The rationale of the changes is to ensure that TTC leavers are qualified for job opportunities and further studies in Higher Education in different programs under education career advancement.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate their learning process. Many factors influence what they learn, how well they learn and the competences they acquire. Those factors include the relevance of the specific content, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials.

The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market has necessitated the shift to a competence-based curriculum. After a successful shift from knowledge to a competence based curriculum in general education, TTC textbooks also were elaborated to align them to the new curriculum.

The book provides active teaching and learning techniques that engage student teachers to develop competences. In view of this, your role is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize group discussions for student-teachers considering the importance of social constructivism suggesting that learning occurs more effectively when the student-teachers work collaboratively with more knowledgeable and experienced people.
- Engage student-teachers through active learning methods such

as inquiry methods, group discussions, research, investigative activities and group and individual work activities.

- Provide supervised opportunities for student-teachers to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation.
- Support and facilitate the learning process by valuing student-teachers' contributions in the class activities.
- Guide student-teachers towards the harmonization of their findings.
- Encourage individual, peer and group evaluation of the work done in the classroom and use appropriate competence-based assessment approaches and methods.

To facilitate you in your teaching activities, the content of this book is self-explanatory so that you can easily use it. It is divided in 3 parts:

The part I explains the structure of this book and gives you the methodological guidance;

The part II gives the sample lesson;

The part III details the teaching guidance for each concept given in the student book.

Even though this Teacher's guide contains the guidance on solutions for all activities given in the student-teacher's book, you are requested to work through each question before judging student-teacher's findings.

I wish to sincerely express my appreciation to the people who contributed towards the development of this book, particularly, REB staff, TTC Tutors, Teachers from general education for their technical support. A word of gratitude goes also to the Head Teachers and TTCs principals who availed their staff for various activities.

Dr. MBARUSHIMANA Nelson

Director General, REB

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Joan MURUNGI

Head of CTLR Department

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PART I: GENERAL INTRODUCTION

1.1. The structure of the guide

The tutor's guide of Mathematics is composed of three parts:

The Part I concerns general introduction that discusses methodological guidance on how best to teach and learn Mathematics, developing competences in teaching and learning, addressing cross-cutting issues in teaching and learning and Guidance on assessment.

Part II presents a sample lesson plan. This lesson plan serves to guide the teacher on how to prepare a lesson in Mathematics.

The Part III is about the structure of a unit and the structure of a lesson. This includes information related to the different components of the unit and these components are the same for all units. This part provides information and guidelines on how to facilitate student teachers while working on learning activities. More other, all application activities from the textbook have answers in this part.

1.2. Methodological guidance

1.2.1.Developing competences

Since 2015 Rwanda shifted from a knowledge based to a competency-based curriculum for pre-primary, primary, secondary education and recently the TTC curriculum. This called for changing the way of learning by shifting from tutor centred to a learner centred approach. Teachers are not only responsible for knowledge transfer but also for fostering student-teachers' learning achievement and creating safe and supportive learning environment. It implies also that student-teachers have to demonstrate what they are able to transfer the acquired knowledge, skills, values and attitude to new situations.

The competence-based curriculum employs an approach of teaching and learning based on discrete skills rather than dwelling on only knowledge or the cognitive domain of learning. It focuses on what student-teacher can do rather than what student-teacher knows. Student-teachers develop competences through subject unit with specific learning objectives broken down into knowledge, skills and attitudes through learning activities.

In addition to the competences related to Mathematics, student-teachers also develop generic competences which should promote the development of the higher order thinking skills and professional skills in Mathematics teaching. Generic competences are developed throughout all units of Mathematics as follows:

Generic competences	Ways of developing generic competences
Critical thinking	All activities that require student-teachers to calculate, convert, interpret, analyse, compare and contrast, etc. have a common factor of developing critical thinking into student-teachers
Creativity and innovation	All activities that require student-teachers to plot a graph of a given algebraic data, to organize and interpret statistical data collected and to apply skills in solving problems of economics have a common character of developing creativity into student-teachers
Research and problem solving	All activities that require student-teachers to make a research and apply their knowledge to solve problems from the real-life situation have a character of developing research and problem solving into student-teachers.
Communication	During Mathematics class, all activities that require student-teachers to discuss either in groups or in the whole class, present findings, debate …have a common character of developing communication skills into student-teachers.
Co-operation, interpersonal relations and life skills	All activities that require student-teachers to work in pairs or in groups have character of developing cooperation and life skills among student-teachers.

Lifelong learning	All activities that are connected with research have a common character of developing into student-teachers a curiosity of applying the knowledge learnt in a range of situations. The purpose of such kind of activities is for enabling student-teachers to become life-long student-teachers who can adapt to the fast-changing world and the uncertain future by taking initiative to update knowledge and skills with minimum external support.
Professional skills	Specific instructional activities and procedures that a tutor may use in the class room to facilitate, directly or indirectly, student-teachers to be engaged in learning activities. These include a range of teaching skills: the skill of questioning, reinforcement, probing, explaining, stimulus variation, introducing a lesson; illustrating with examples, using blackboard, silence and non verbal cues, using audio – visual aids, recognizing attending behaviour and the skill of achieving closure.

The generic competences help student-teachers deepen their understanding of Mathematics and apply their knowledge in a range of situations. As student-teachers develop generic competences they also acquire the set of skills that employers look for in their employees, and so the generic competences prepare student-teachers for the world of work.

1.2.2. Addressing cross cutting issues

Among the changes brought by the competence-based curriculum is the integration of cross cutting issues as an integral part of the teaching learning process-as they relate to and must be considered within all subjects to be appropriately addressed. The eight cross cutting issues identified in the national curriculum framework are: *Comprehensive Sexuality Education, Environment and Sustainability, Financial Education, Genocide studies, Gender, Inclusive Education, Peace and Values Education, and Standardization Culture.*

Some cross-cutting issues may seem specific to particular learning areas/subjects but the tutor need to address all of them whenever an opportunity arises. In addition, student-teachers should always be given an opportunity during the learning process to address these cross-cutting issues both within and out of the classroom.

Below are examples of how crosscutting issues can be addressed:

Cross-Cutting Issue	Ways of addressing cross-cutting issues
Comprehensive Sexuality Education: The primary goal of introducing Comprehensive Sexuality Education program in schools is to equip children, adolescents, and young people with knowledge, skills and values in an age appropriate and culturally gender sensitive manner so as to enable them to make responsible choices about their sexual and social relationships, explain and clarify feelings, values and attitudes, and promote and sustain risk reducing behaviour.	Using different charts and their interpretation, Mathematics tutor should lead student-teachers to discuss the following situations: "Alcohol abuse and unwanted pregnancies" and advise student teachers on how they can instil student-teachers to fight those abuses. Some examples can be given in powers and properties , logarithms and properties , and statistics
Environment and Sustainability: Integration of Environment, Climate Change and Sustainability in the curriculum focuses on and advocates for the need to balance economic growth, society well-being and ecological systems. Student-teachers need basic knowledge from the natural sciences, social sciences, and humanities to understand to interpret principles of sustainability.	Using Real life models or student-teachers' experience, Mathematics Tutor should lead student teachers to illustrate the situation of "population growth" and discuss its effects on the environment and sustainability. Some examples in proportional change, logarithms, and polynomial functions
Financial Education: The integration of Financial Education into the curriculum is aimed at a comprehensive Financial Education program as a precondition for achieving financial inclusion targets and improving the financial capability of Rwandans so that they can make appropriate financial decisions that best fit the circumstances of one's life.	Through different examples and calculations on interest rate problems, total revenue and total cost, Mathematics Tutor can lead student teachers to discuss how to make appropriate financial decisions. Some examples in ratios and proportions, statistics, equations, polynomial functions

Gender: At school, gender will be understood as family complementarities, gender roles and responsibilities, the need for gender equality and equity, gender stereotypes, gender sensitivity, etc.	Mathematics Tutor should address gender as cross-cutting issue through assigning leading roles in the management of groups to both girls and boys and providing equal opportunity in the lesson participation and avoid any gender stereotype in the whole teaching and learning process.
Inclusive Education: Inclusion is based on the right of all student-teachers to a quality and equitable education that meets their basic learning needs and understands the diversity of backgrounds and abilities as a learning opportunity.	Firstly, Mathematics Tutors need to identify/recognize student-teachers with special needs. Then by using adapted teaching and learning resources while conducting a lesson and setting appropriate tasks to the level of student-teachers, they can cater for student-teachers with special education needs. They must create opportunity where student-teachers can discuss how to cater for student-teachers with special educational needs.
Peace and Values Education: Peace and Values Education (PVE) is defined as education that promotes social cohesion, positive values, including pluralism and personal responsibility, empathy, critical thinking and action in order to build a more peaceful society.	Through a given lesson, a tutor should: – Set a learning objective which is addressing positive attitudes and values, – Encourage student-teachers to develop the culture of tolerance during discussion and to be able to instil it in colleagues and cohabitants; – Encourage student-teachers to respect ideas for others.
Standardization Culture: Standardization Culture in Rwanda will be promoted through formal education and plays a vital role in terms of health improvement, economic growth, industrialization, trade and general welfare of the people through the effective implementation of Standardization, Quality Assurance, Metrology and Testing.	With different word problems related to the effective implementation of Standardization, Quality Assurance, Metrology and Testing, student-teachers can be motivated to be aware of health improvement, economic growth, industrialization, trade and general welfare of the people.

1.2.3. Guidance on how to help student-teachers with special education needs in classroom

In the classroom, student-teachers learn in different way depending to their learning pace, needs or any other special problem they might have. However, the tutor has the responsibility to know how to adopt his/her methodologies and approaches in order to meet the learning need of each student in the classroom. Also teachers need to understand that student with special needs, need to be taught differently or need some accommodations to enhance the learning environment. This will be done depending to the subject and the nature of the lesson.

In order to create a well-rounded learning atmosphere, teachers need to:

- Remember that student-teachers learn in different ways so they have to offer a variety of activities (e.g. role-play, music and singing, word games and quizzes, and outdoor activities);
- Maintain an organized classroom and limits distraction. This will help student-teachers with special needs to stay on track during lesson and follow instruction easily;
- Vary the pace of teaching to meet the needs of each child. Some student-teachers process information and learn more slowly than others;
- Break down instructions into smaller, manageable tasks. Student-teachers with special needs often have difficulty understanding long-winded or several instructions at once. It is better to use simple, concrete sentences in order to facilitate them understand what you are asking.
- Use clear consistent language to explain the meaning (and demonstrate or show pictures) if you introduce new words or concepts;
- Make full use of facial expressions, gestures and body language;
- Pair a learner who has a disability with a friend. Let them do things together and learn from each other. Make sure the friend is not over protective and does not do everything for the one with disability. Both student-teachers will benefit from this strategy;
- Use multi-sensory strategies. As all student-teachers learn in different ways, it is important to make every lesson as multi-sensory as possible. Student-teachers with learning disabilities might have difficulty in one area, while they might excel in another. For example, use both visual and auditory cues.

Below are general strategies related to each main category of disabilities and how to deal with every situation that may arise in the classroom. However, the list is not exhaustive because each child is unique with different needs and that should be handled differently.

- **Strategy to help student-teachers with developmental impairment:**
  - Use simple words and sentences when giving instructions;
  - Use real objects that student-teachers can feel and handle. Rather than just working abstractly with pen and paper;
  - Break a task down into small steps or learning objectives. The learner should start with an activity that s/he can do already before moving on to something that is more difficult;
  - Gradually give the learner less help;
  - Let the learner with disability work in the same group with those without disability.
- **Strategy to help student-teachers with visual impairment:**
  - Help student-teachers to use their other senses (hearing, touch, smell and taste) and carry out activities that will promote their learning and development;
  - Use simple, clear and consistent language;
  - Use tactile objects to help explain a concept;
  - If the learner has some sight, ask him/her what he/she can see;
  - Make sure the learner has a group of friends who are helpful and who allow him/her to be as independent as possible;
  - Plan activities so that student-teachers work in pairs or groups whenever possible;
- **Strategy to help student-teachers with hearing disabilities or communication difficulties**
  - Always get the learner‘s attention before you begin to speak;
  - Encourage the learner to look at your face;
  - Use gestures, body language and facial expressions;
  - Use pictures and objects as much as possible.
  - Keep background noise to a minimum.

- **Strategies to help student-teachers with physical disabilities or mobility difficulties:**
  - Adapt activities so that student-teachers who use wheelchairs or other mobility aids, can participate.
  - Ask parents/caregivers to assist with adapting furniture e.g. the height of a table may need to be changed to make it easier for a learner to reach it or fit their legs or wheelchair under;
  - Encourage peer support when needed;
  - Get advice from parents or a health professional about assistive devices if the learner has one.

Adaptation of assessment strategies:

At the end of each unit, the tutor is advised to provide additional activities to help student-teachers achieve the key unit competence. These assessment activities are for remedial, consolidation and extension designed to cater for the needs of all categories of student-teachers; slow, average and gifted student-teachers respectively. Therefore, the tutor is expected to do assessment that fits individual student.

Remedial activities	After evaluation, slow student-teachers are provided with lower order thinking activities related to the concepts learnt to facilitate them in their learning. These activities can also be given to assist deepening knowledge acquired through the learning activities for slow student-teachers.
Consolidation activities	After introduction of any concept, a range number of activities can be provided to all student-teachers to enhance/ reinforce learning.
Extended activities	After evaluation, gifted and talented student-teachers can be provided with high order thinking activities related to the concepts learnt to make them think deeply and critically. These activities can be assigned to gifted and talented student-teachers to keep them working while other student-teachers are getting up to required level of knowledge through the learning activity.

1.2.4. Guidance on assessment

Assessment is an integral part of teaching and learning process. The main purpose of assessment is for improvement of learning outcomes. Assessment for learning/ Continuous/ formative assessment intends to improve student-teachers' learning and tutor's teaching whereas assessment of learning/summative assessment intends to improve the entire school's performance and education system in general.

Continuous/ formative assessment

It is an on-going process that arises during the teaching and learning process. It includes lesson evaluation and end of sub unit assessment. This formative assessment should play a big role in teaching and learning process. The tutor should encourage individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods.

Formative assessment is used to:

- Determine the extent to which learning objectives are being achieved and competences are being acquired and to identify which student-teachers need remedial interventions, reinforcement as well as extended activities. The application activities are developed in the learner book and they are designed to be given as remedial, reinforcement, end lesson assessment, homework or assignment
- Motivate student-teachers to learn and succeed by encouraging student-teachers to read, or learn more, revise, etc.
- Check effectiveness of teaching methods in terms of variety, appropriateness, relevance, or need for new approaches and strategies. Mathematics tutors need to consider various aspects of the instructional process including appropriate language levels, meaningful examples, suitable methods and teaching aids/ materials, etc.
- Help student-teachers to take control of their own learning.

In teaching Mathematics, formative or continuous assessment should compare performance against instructional objectives. Formative assessment should measure the student-teacher's ability with respect to a criterion or standard. For this reason, it is used to determine what student-teachers can do, rather than how much they know.

Summative assessment

The assessment can serve as summative and informative depending to its purpose. The end unit assessment will be considered summative when it is done at end of unit and want to start a new one.

It will be formative assessment, when it is done in order to give information on the progress of student-teachers and from there decide what adjustments need to be done.

The assessment done at the end of the term, end of year, is considered as summative assessment so that the tutor, school and parents are informed of the achievement of educational objective and think of improvement strategies. There is also end of level/ cycle assessment in form of national examinations.

When carrying out assessment?

Assessment should be clearly visible in lesson, unit, term and yearly plans.

- Before learning (diagnostic): At the beginning of a new unit or a section of work; assessment can be organized to find out what student-teachers already know / can do, and to check whether the student-teachers are at the same level.
- During learning (formative/continuous): When student-teachers appear to be having difficulty with some of the work, by using on-going assessment (continuous). The assessment aims at giving student-teachers support and feedback.
- After learning (summative): At the end of a section of work or a learning unit, the Mathematics Tutor has to assess after the learning. This is also known as Assessment of Learning to establish and record overall progress of student-teachers towards full achievement. Summative assessment in Rwandan schools mainly takes the form of written tests at the end of a learning unit or end of the month, and examinations at the end of a term, school year or cycle.

Instruments used in assessment.

- **Observation:**

  This is where the Mathematics tutor gathers information by watching student-teachers interacting, conversing, working, playing, etc. A tutor can use observations to collect data on behaviours that are difficult to assess by other methods such as attitudes, values, and generic competences and intellectual skills. It is very important because it is used before the lesson begins and throughout the lesson since the tutor has to continue observing each and every activity.

- **Questioning**
  a) Oral questioning: a process which requires a student-teacher to respond verbally to questions
  b) Class activities/ exercise: tasks that are given during the learning/ teaching process
  c) Short and informal questions usually asked during a lesson
  d) Homework and assignments: tasks assigned to student-teachers by their tutors to be completed outside of class.

Homework assignments, portfolio, project work, interview, debate, science fair, Mathematics projects and Mathematics competitions are also the different forms/instruments of assessment.

1.2.5. Teaching methods and techniques that promote active learning

The different learning styles for student-teachers can be catered for, if the teacher uses active learning whereby student-teachers are really engaged in the learning process.

The main teaching methods used in mathematics are the following:

- **Dogmatic method** (the teacher tells the student-teachers what to do, What to observe, How to attempt, How to conclude)
- **Inductive-deductive method:** Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples.
- **Analytic-synthetic method:** Analytic method proceeds from unknown to known, 'Analysis' means 'breaking up' of the problem in hand so that it ultimately gets connected with something obvious or already known. Synthetic method is the opposite of the analytic method. Here one proceeds from known to unknown.
- **Skill Laboratory method:** Skill Laboratory method is based on the maxim "learning by doing." It is a procedure for stimulating the activities of the student-teachers and to encourage them to make discoveries through practical activities.

- **Problem solving method, Project method and Seminar Method.**

The following are some active techniques to be used in Mathematics:

- Group work
- Research
- Probing questions
- Practical activities (drawing, plotting, interpreting graphs)
- Modelling
- Brainstorming
- Quiz Technique
- Discussion Technique
- Scenario building Technique

What is Active learning?

Active learning is a pedagogical approach that engages student-teachers in doing things and thinking about the things they are doing. Student-teachers play the key role in the active learning process. They are not empty vessels to fill but people with ideas, capacity and skills to build on for effective learning. Thus, in active learning, student-teachers are encouraged to bring their own experience and knowledge into the learning process.

The role of the tutor in active learning	The role of student-teachers in active learning
– The tutor engages student-teachers through active learning methods such as inquiry methods, group discussions, research, investigative activities, group and individual work activities. – He/she encourages individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods. – He provides supervised opportunities for student-teachers to develop different competences by giving tasks which enhance critical	• A learner engaged in active learning: • Communicates and shares relevant information with peers through presentations, discussions, group work and other learner-centred activities (role play, case studies, project work, research and investigation); • Actively participates and takes responsibility for his/her own learning. • Develops knowledge and skills in active ways;

thinking, problem solving, research, creativity and innovation, communication and cooperation. – Teacher supports and facilitates the learning process by valuing student-teachers' contributions in the class activities.	– Carries out research/ investigation by consulting print/online documents and resourceful people, and presents their findings; – Ensures the effective contribution of each group member in assigned tasks through clear explanation and arguments, critical thinking, responsibility and confidence in public speaking – Draws conclusions based on the findings from the learning activities.

Main steps for a lesson in active learning approach

All the principles and characteristics of the active learning process highlighted above are reflected in steps of a lesson as displayed below. Generally, the lesson is divided into three main parts whereby each one is divided into smaller steps to make sure that student-teachers are involved in the learning process. Below are those main part and their small steps:

1) Introduction

Introduction is a part where the tutor makes connection between the current and previous lesson through appropriate technique. The teacher opens short discussions to encourage student-teachers to think about the previous learning experience and connect it with the current instructional objective. The tutor reviews the prior knowledge, skills and attitudes which have a link with the new concepts to create good foundation and logical sequencings.

2) Development of the new lesson

The development of a lesson that introduces a new concept will go through the following small steps: discovery activities, presentation of student-teachers' findings, exploitation, synthesis/summary and exercises/ application activities.

- **Discovery activity**

Step 1

- The tutor discusses convincingly with student-teachers to take responsibility of their learning
- He/she distributes the task/activity and gives instructions related to the tasks (working in groups, pairs, or individual to instigate collaborative learning, to discover knowledge to be learned)

Step 2

- The **tutor** lets student-teachers work collaboratively on the task;
- During this period the tutor refrains to intervene directly on the knowledge;
- He/she then monitors how the student-teachers are progressing towards the knowledge to be learned and boosts those who are still behind (but without communicating to them the knowledge).

- **Presentation of student-teachers' findings/productions**
  - In this episode, the tutor invites representatives of groups to present their productions/findings.
  - After three/four or an acceptable number of presentations, the teacher decides to engage the class into exploitation of student-teachers' productions.
- **Exploitation of student-teacher's findings/ productions**
  - The teacher asks student-teachers to evaluate the productions: which ones are correct, incomplete or false
  - Then the teacher judges the logic of the student-teachers' products, corrects those which are false, completes those which are incomplete, and confirms those which are correct.
- **Institutionalization or harmonization (summary/ conclusion/ and examples)**
  - The teacher summarizes the learned knowledge and gives examples which illustrate the learned content.
- **Application activities**
  - Exercises of applying processes and products/objects related to learned unit/sub-unit
  - Exercises in real life contexts

- Tutor guides student-teachers to make the connection of what they learnt to real life situations. At this level, the role of **tutor** is to monitor the fixation of process and product/object being learned.

3) Assessment

In this step the **tutor** asks some questions to assess achievement of instructional objective. During assessment activity, student-teachers work individually on the task/activity. The **tutor** avoids intervening directly. In fact, results from this assessment inform the tutor on next steps for the whole class and individuals. In some cases, the tutor can end with a homework/ assignment. Doing this will allow student-teachers to relay their understanding on the concepts covered that day. T**utor** leads them not to wait until the last minute for doing the homework as this often results in an incomplete homework set and/or an incomplete understanding of the concept.

SAMPLE LESSON

School:...

Teacher's Name:...

Term	Date	Subject	Class	Unit Nº	Lesson Nº	Duration	Class size
------	----/------/-----	MATHEMATICS	Year one	7	1 of 5	40minutes	40 student-teachers
Type of Special Educational Needs to be catered for in this lesson and number of student-teachers in each category				2 student-teachers with hearing impairment will be seat near the tutor and the use of gestures will be improved in the lesson.			

Unit title	Problems on powers, indices, radical s and common logarithms
Key Unit Competence:	Solve problems related to powers, indices, radical and common logarithms.
Title of the lesson	Definition of powers/ indices and radicals.
Instructional Objective	Through the given activities, student-teachers should be able to solve problems involving powers accurately using properties of powers which are written on flash cards.
Plan for this Class (location: in / outside)	Inside the class room.
Learning Materials (for all student-teachers)	Flash Cards, papers, Pens, Exercise Books, other supporting teaching aids such as Chalks and Chalkboard, etc...
References	Year one Student-teacher's book and Tutor's guide of Mathematics.

<table>
<tr><th>Steps and Timing</th><th colspan="2">Description of teaching and learning activities</th><th>Competences and Cross-Cutting Issues to be addressed</th></tr>
<tr><td></td><td colspan="2">Student-Teachers are organized into groups to discuss the activity 7.1.1and the examples, the reporter from one group, presents the findings and the Student-Teachers interact. The tutor facilitates Student-Teachers to capture the key concepts of the lesson through harmonization.

Finally, the Student-Teachers are assigned to individual tasks and the correction is done on the chalk board.</td><td></td></tr>
<tr><td></td><td>tutor activities</td><td>Student-teachers activities</td><td></td></tr>
<tr><td>Introduction

10 mins</td><td>Powers and related problems

Tutor distributes flash cards to student-teachers in their small group discussions and invite them to brainstorm on the activity 7.1.1;

Tutor moves around to help those who are struggling and guides them in finding definitions and properties of powers.
Tutor invites student-teachers to present their findings.

Tutor harmonizes the answers from presentation.</td><td>Student-teachers receive flashcards, discuss and brainstorm on the activity 7.1.1.

They guess the definition of power and explore properties of powers.

Group representatives present findings from groups and other student-teachers participate actively in the presentation by comments or by asking questions.</td><td>Cooperation is improved through group work: team working spirit is developed through working together in small group discussions.

Communication skills are developed through small group discussions.</td></tr>
</table>

Development of the lesson: 20mins	Tutor gives instructions, invites student-teachers to brainstorm in their small groups examples given in their books (question 1 and question 2) found in the student-teacher's book.	In their respective groups, Student-teachers discuss and brainstorm on examples given in their books (question 1 and question 2).	**Critical thinking**, **problem solving skills** and **Finance Education** are developed through analyzing and solving real life Mathematical problem. **Cooperation** and **communication** are developed during presentations and group discussions.
	Tutor moves around to each group, ask probing questions in order to help struggling student-teachers. Tutor invites student-teachers to present their findings.	Student-teachers present their findings.	**Inclusive education** is addressed by providing the remediation activities and tasks to struggling student-teachers.
Conclusion 10 min	**Summary:** Tutor guides all student-teachers to highlight the main properties of powers, their usage and to summarize the lesson of the day.	Student-teachers summarize the lesson guided by the tutor.	Communication skill is developed through small discussion on the findings and the main points of the lesson.
	Assessment -Tutor asks student-teachers to individually work out the application activity	Student-teachers work independently on the application activity 7.1.1	**Critical thinking** and **problem solving skills** are developed through analyzing and solving real life Mathematical problem.
	Tutor gives the homework to student teachers.	Write homework and ask more clarification on it.	**Critical thinking** and **problem solving skills** are developed through analyzing and solving real life Mathematical problems.
Tutor self-evaluation	To be completed after receiving the feed-back from the Student-teachers.		

UNIT: 1

SET OF NUMBERS

1.1 Key unit competence

Classify numbers into naturals, integers, rational and irrationals

1.2 Prerequisite

Student-teachers will perform well in this unit if they have a good background on

- Use correctly simple language structure, vocabulary and suitable symbols of mathematics learnt in Ordinary Level;
- Carry out numerical calculations correctly;
- Interpret simple diagrams and recognize ways in which representations can be misleading.

1.3 Cross-cutting issues to be addressed

- **Inclusive education**: promote the participation of all student-teachers while teaching
- **Peace and value Education**: During group activities, the tutor will encourage student teachers to help each other and to respect opinions of colleagues.
- **Gender:** Give equal opportunities to all student-teachers (girls and boys) to participate actively in all learning activities from the beginning to the end of the lesson.

1.4 Guidance on introductory activity 1

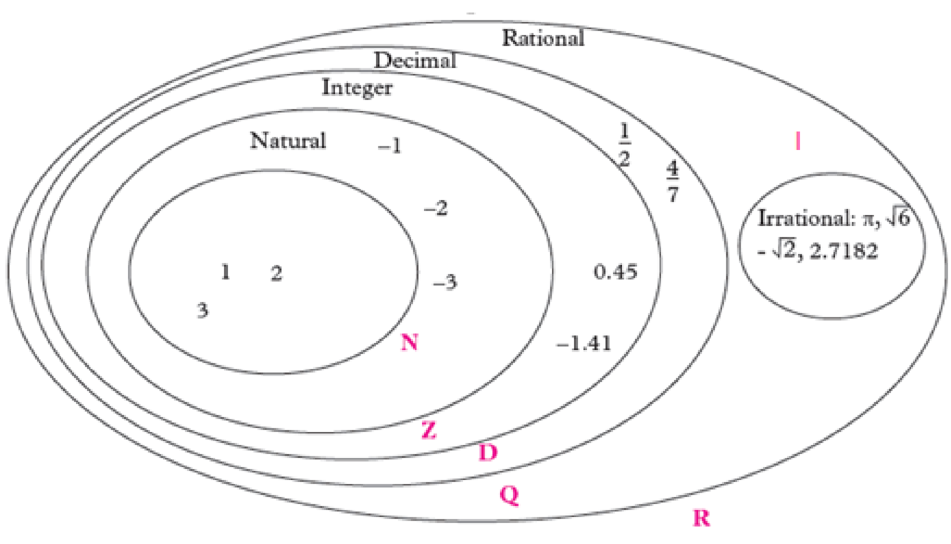
- Form small groups of student-teachers and guide them to work on the introductory activity 1;
- Through class discussions, let student-teachers think of different possible solutions and justify their validity;
- Walk around to all groups and provide pieces of advice where necessary;
- After a given time, invite student teachers to present their findings and harmonize them.

- Lead them to know that in the given activity, you can get different answers depending on the set considered. Try to arouse student-teachers' curiosity on the content for this unit.

Answer for introductory activity 1

Answers may vary.

1) Lead student-teachers to remember that from senior 1, the sets they already know are: $\mathbb{N}, \mathbb{Z}, \mathrm{D}, \mathbb{Q}, \mathbb{R}, \ldots$.
2) Numbers we use in counting are called Natural numbers except zero; integers are whole numbers which are either **negative or positive and includes zero**. The set of integers is represented by $\mathbb{Z}$; the set of limited decimal is D, the set of **rational numbers** represented by $\mathbb{Q}$ and the set of **irrational numbers** I. The set of **real numbers** is denoted by $\mathbb{R}$ and it includes the set of rational numbers and the set of irrational numbers.
3) In fact, their relationship is that $\mathbb{N} \subset \mathbb{Z} \subset D \subset \mathbb{Q} \subset \mathbb{R}$.
4) They can be summarized in the following figure with examples of numbers in each set:



1.5. List of lessons and sub-heading

No	Lesson title	Learning objectives	Periods
0	Introductory activity	Arouse the curiosity of student teacher on the content for unit 1.	1
1	Natural numbers: definition, subsets, operations and properties	- Identify natural numbers; - Work systematically to determine properties and subsets of natural numbers; - Carry out mathematical operations on natural numbers.	1
2	Definition of the set of Integers, subsets, operations and properties.	- Identify elements of the set of integers; - Determine subsets of integers; - Carry out mathematical operations on integers and explore their properties.	1
3	Rational and Irrational numbers: Definition, subsets, operations and properties	- Identify rational and irrational numbers; - Determine subsets of rational and irrational numbers; - Perform and explore operations on rational and irrational numbers and their properties; - Appreciate that rational numbers can be represented exactly as a fraction or a decimal.	1
4	Definition for the set of real numbers, subsets, operations and properties.	- Determine the hierarchy of sets on numbers and explain their relationship; - Locate different numbers on a number line; - Explain that irrational numbers cannot be expressed exactly as fractions.	1
5	End assessment		1
Total number of periods			6

Notice:

The tutor of mathematics in year one ECLPE will use group activities, assignments, home works, research and group presentations in order to use a small time to cover the content of unit 1 because it is related to the content previously seen in ordinary level.

Lesson 1: Definition, sub-sets, operations and properties of Natural numbers

a) Learning objectives:

- Identify natural numbers
- Work systematically to determine properties and subsets of natural numbers
- Carry out mathematical operations on natural numbers

b) Teaching resources:

Ruler, T-square, Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good back ground on the use of mathematics language, vocabulary and suitable symbols learnt in Ordinary Level**,** good performance in numerical calculations and interpretation of simple diagrams.

d) Learning activities

- Organize the student-teachers into small groups;
- Ask them to do the activity 1.1.1; activity 1.1.2 and activity 1.1.3 from student-teacher's book;
- After a given time, ask randomly some groups to present their findings to the whole class;
- During the harmonization, help student-teachers discover that they are using natural numbers which are used to count objects;
- Use different probing questions and guide student-teachers to explore examples and the content given in the student-teacher's book to enhance how to carry out operations in the set $\mathbb{N}$ and guide them to highlight the corresponding properties;
- After this step, guide student-teachers to do the application activity (you can chose the application activity 1.1.1, 1.1.2 or 1.1.3), assess

their competences and evaluate whether lesson objectives were achieved.

Answers for Activity 1.1.1

All of these numbers belong to the set $\mathbb{N}$ of natural numbers.

For other elements of the set of natural numbers, answers may vary but any value might be an element of $\mathbb{N} = \{0,1,2,3,\ldots\}$.

Answers for Activity 1.1.2

a) **Even numbers** are numbers which are divisible by 2 or numbers which are multiples of 2.

Odd numbers are numbers which leave a remainder of 1 when divided by 2.

Prime number is a number that has only two divisors; 1 and itself.

b) i) Odd numbers between 0 and 20 are 1,3,5,7,9,11,13,15,17 and 19.

ii) Even numbers between 0 and 20 are 2, 4, 6, 8, 10, 12, 14, 16 and 18.

iii) Prime numbers between 0 and 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

iv) 2 is the only even number that is a prime number.

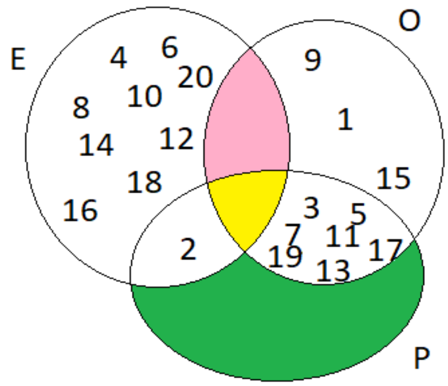
v) Odd numbers between 0 and 20 which are prime numbers are 3, 5, 7, 11, 13, 17 and 19.

c) Let E: set of even numbers between 0 and 20

O: set of odd numbers between 0 and 20

P: set of prime numbers between 0 and 20

Then, their representation on Venn diagram is as follows:



Answers for Activity 1.1.3

1. $\forall a,b,c \in \mathbb{N}$

a) $a+b=d \in \mathbb{N}$, Example: $2+3=5 \in \mathbb{N}$.

b) $a+b=b+a=d \in \mathbb{N}$. Example: $2+3=3+2=5 \in \mathbb{N}$

c) $(a+b)+c=a+(b+c)=d$ $\forall d \in \mathbb{N}$ Example:

$(2+3)+1=2+(3+1)=6 \in \mathbb{N}$

2. $\forall a,b,c,d \in \mathbb{N}$, $(a-b) \neq (b-a)$ and $(a-b)-c \neq a-(b-c)$

i) $(3-2)=1$ and $(2-3)=-1 \notin \mathbb{N}$

ii) $(7-4)-2=1$ and $7-(4-2)=5$

$(7-4)-2 \neq 7-(4-2)=5$

$1 \neq 5$

3. Given any three natural numbers a, b and c,

$(a\times b)=(b\times a)$ and $(a\times b)\times c=a\times(b\times c)$

i) $(3\times 2)=6$ and $(2\times 3)=6 \in \mathbb{N}$

ii) $(3\times 2)\times 4=24$ and $3\times(2\times 4)=24 \in \mathbb{N}$

iii) $2\times(3+4)=2\times 7=14$ and $2\times(3+4)=(2\times 3)+(2\times 4)=6+8=14$

iv) $2\times(7-4)=2\times 3=6$ and $2\times(7-4)=(2\times 7)-(2\times 4)=14-8=6$

5. $\forall a,b,d \in \mathbb{N}$,$(a:b)\neq(b:a)$, Example: $10:5=2$ and $5:10=0.5 \notin \mathbb{N}$.

e) Application Activities

Answers for application activity 1.1.1

- Ten first elements of natural numbers starting from zero are: $0,1,2,3,4,5,6,7,8$ and 9.
- Natural numbers are used in different situations of the real life to count objects of the milieu; For example to indicate time (days, hour, minute, second), to count living and non-living things, in buying, in making different payments (school fees, renting house, travelling, paying bills,…).

Answers for application activity 1.1.2

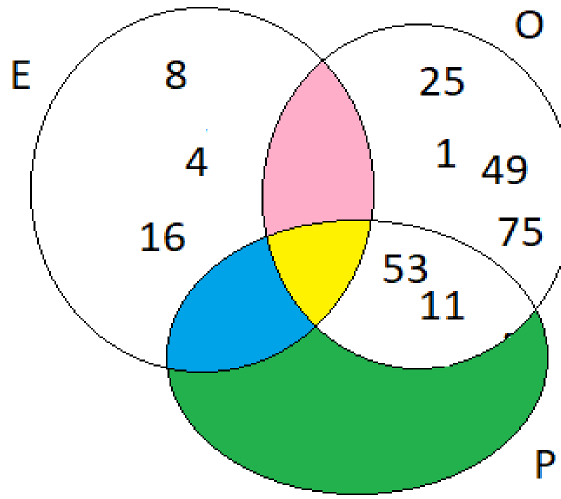
From E = {1, 4, 8, 11, 16, 25, 49, 53, 75},

a) Even numbers are 4, 8 and 16.

b) Odd numbers are 1, 11,25,49,53 and 75.

c) Prime numbers are 11 and 53.

Then, their representation on Venn diagram is as follows



Answers for application activity 1.1.3

Answers may vary with the groups of student-teachers; but for any natural number, it is easily verifiable that

1) $a+b \in \mathbb{N}$, as $\mathbb{N}$ is **closed** under addition;

$a+b=b+a$, since addition is **commutative** in $\mathbb{N}$.

$a+(b+c)=(a+b)+c$ since addition is **associative** in $\mathbb{N}$.

2) $a \times b \in \mathbb{N}$, since $\mathbb{N}$ is **closed** under multiplication

$a \times b = b \times a$, since multiplication is **commutative** in $\mathbb{N}$.

$a \times (b \times c) = (a \times b) \times c$ since multiplication is **associative** in $\mathbb{N}$.

3) $a \times (b+c) = ab + ac$ since multiplication is distributive over addition.

Lesson 2: Definition of the set of Integers, subsets, operations and properties

a) Learning objectives:

- Identify elements of the set of integers;
- Determine subsets of integers;
- Carry out mathematical operations on integers and explore their properties.

b) Teaching resources:

Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good background on concept of natural number learnt in ordinary level.

d) Learning activities

- Organize student-teachers into small groups;
- Invite them to do the following activities: Activity 1.2.1. activity 1.2.2 and activity 1.2.3 found in the student-teacher's book;
- Through group discussion invite student-teachers to do all questions of the given activities and visit each group to check if every student is engaged;
- Invite group representatives to present their findings, then help all student-teachers make content summary;
- Use different probing questions and guide student-teachers to explore examples and the content given in the student's book to enhance the elements of the set of integers, the sub sets of the set $\mathbb{Z}$, how to carry out operations in the set $\mathbb{Z}$ and guide them to highlight the corresponding properties;
- After this step, guide student-teachers to do the application activity (you can chose the application activity 1.2.1, 1.2.2 or 1.2.3), assess their competences and evaluate whether lesson objectives were achieved.

Answers for activity 1.2.1

1. The tutor guides student teachers to use dictionary and other resources like internet to search the definition of integer (negative numbers and positive numbers and compare with natural numbers).
2. a) -50m; b) +42^{0}C; c) -2m; d) +3m.

Answers for activity 1.2.2

From integers between 12 and +20, form small sets which contain;

a) Odd numbers are 13,15,17 and 19.

b) Even numbers are 14,16 and 18

c) Factor of 6 is 18

d) Multiples of 3 are 15 and 18.

Answers for activity 1.2.3

1. Work out the following on a number line:

a) (+3) + (+2) = +5 b) – (5) + - (3) = -8 c) (+4) + (-3) = +1

d) When adding a negative number to a positive number, you move on left side of the number line starting from positive number.

e) You move on left side of the number line from minuend

2. Work out the following and show your solutions on a number line.

a) (-4)-(+3) = -7 b) (+5) – (+3) = +2 c) (–6) – (-6) =0

d) You move on left side of the number line from minuend

e) You move on left side of the number line from minuend

3. Work out the following:

a) $(+5)\times(-6)=-30$ b) $(+5)\times(+6)=+30$ c) $(-5)\times(+6)=-30$

b) $(-5)\times(-6)=+30$ e)No

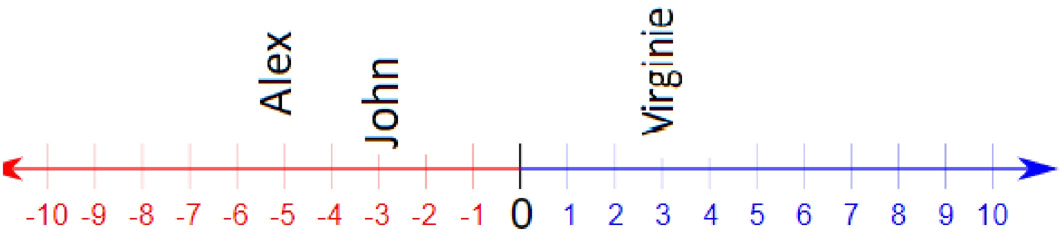
4. Work out the following and show your solutions on a number line.

a) $(-4)\div(+4)=-1$ b) $(+4)\div(+4)=+1$ c) $(-4)\div(-4)=+1$

b) $(+4)\div(-4)=-1$ e) No.

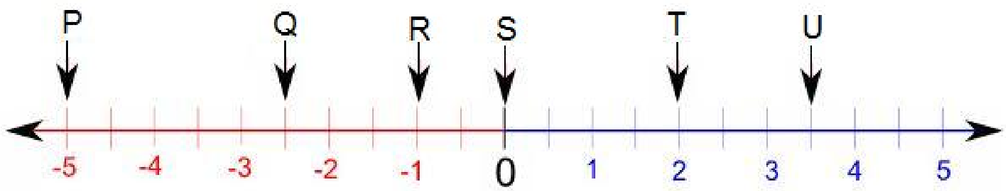
e) Application activities

Answers for application activity 1.2.1



Answers for application activity 1.2.2

1) $\{\cdots,-5,-4,-3,-2,-1,0\}$ is the set of non-positive integers.
2) From the following figure, the points P, Q, R, S, T and U are located.



Among them,

- Negative integers are P and R.
- Positive integer is T
- S is neither positive nor negative.
- Q and U are not integers.

Answers for application activity 1.2.3

If the temperature drops 3 degrees from $-23^0 C$ outside, then, temperature is $-20^0 C$.

Lesson3: Rational and Irrational numbers: Definition, subsets, operations and properties

a) Learning objectives:

- Identify rational and irrational numbers;
- Determine subsets of rational and irrational numbers;
- Perform and explore operations on rational and irrational numbers and their properties;
- Appreciate that rational numbers can be represented exactly as a fraction or a decimal.

b) Teaching resources:

T-square, ruler, student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

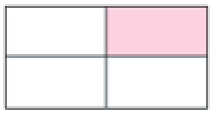
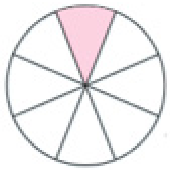
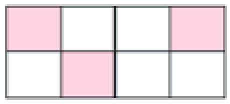
Student-teachers will perform well in this lesson if they have a good background on concept of fractions, decimals and numerical calculation learnt from Primary to Ordinary level.

d) Learning activities

- Organize student-teachers into small groups;
- Invite them to do the following activities: Activity 1.3.1. activity 1.3.2 and activity 1.3.3 found in the student-teacher's book;
- Through group discussion invite student-teachers to do all questions of the given activities and visit each group to check if every student is engaged;
- Invite group representatives to present their findings, then help all student-teachers make content summary;
- Invite student –teachers who sit on the same desk or pairs to do the activity 1.3.4 and ask them to compare the elements of the set $\mathbb{Q}$ and new elements found in this activity 1.3.4;
- Use different probing questions to guide student-teachers to explore examples and the content given in the student's book to enhance the elements of the set of integers, the sub sets of the set $\mathbb{Q}$, how to carry out operations in the set $\mathbb{Q}$ and guide them to highlight the corresponding properties;

- Guide all student- teachers to differentiate the elements of the set $\mathbb{Q}$ and irrational numbers which are grouped in the particular set I enhancing that irrational numbers cannot be written in the form of a fraction $\frac{p}{q}$;
- After this step, guide student- teachers to do the application activity (you can chose one of the application activity 1.3.1, 1.3.2 or 1.3.3 and the application activity 1.3.4), assess their competences and evaluate whether lesson objectives were achieved.

Answers for Activity 1.3.1

Fraction of			
shaded parts	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$
unshaded parts	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{5}{8}$

Fractions are elements of set of rational numbers.

Given integers a and b, with b a non-zero integer, $\frac{a}{b}$ is always a fraction.

Examples may vary because every group can give its own examples of fractions.

Answers of Activity 1.3.2

1. All integers are rational numbers.

   This statement is true. Note that $\mathbb{Z} \subset \mathbb{Q}$
2. No rational numbers are whole numbers.

   This statement is **false**. Note that $\mathbb{N} \subset \mathbb{Q}$ or $\mathbb{Q} \supset \mathbb{N}$.
3. All rational numbers are integers.

   This statement is **false**. Note that $\mathbb{Q} \not\subset \mathbb{Z}$ but $\mathbb{Z} \subset \mathbb{Q}$.
4. All whole numbers are rational numbers.

   This statement is true. Note that $\mathbb{N} \subset \mathbb{Q}$ or $\mathbb{Q} \supset \mathbb{N}$.

Answers for Activity 1.3.3

1) $\forall a,b,c,d \in \mathbb{Q}$

a) $a+b=d \in \mathbb{Q}$,

Example: $\frac{2}{3}+\frac{3}{5}=\frac{19}{15} \in \mathbb{Q}$ (to calculate the common denominator, use LCM).

b) $a+b=b+a=d \in \mathbb{Q}$.

Example:

$$\left(\frac{2}{3}+\frac{3}{5}\right)=\left(\frac{3}{5}+\frac{2}{3}\right)=\frac{19}{15} \in \mathbb{Q} \quad \text{(to calculate the common denominator, use LCM)}$$

c) $(a+b)+c=a+(b+c)=d \qquad \forall d \in \mathbb{Q}$

Example: $\left(\frac{2}{3}+\frac{3}{7}\right)+\frac{1}{2}=\frac{67}{42}$ and $\frac{2}{3}+\left(\frac{3}{7}+\frac{1}{2}\right)=\frac{67}{42} \quad \in \mathbb{Q}$

2) $\forall a,b,c,d \in \mathbb{Q}, \quad (a-b)-c \neq a-(b-c)$

$\left(\frac{3}{4}-\frac{2}{3}\right)=\frac{1}{12}$ and $\left(\frac{2}{3}-\frac{3}{4}\right)=\frac{-1}{12}$ $,thus\,(a-b)\neq(b-a)$

$\left[\left(\frac{7}{2}-\frac{4}{5}\right)-\frac{2}{3}\right]=\frac{61}{30}$ and $\left[\frac{7}{2}-\left(\frac{4}{5}-\frac{2}{3}\right)\right]=\frac{101}{30}$, thus $(a-b)-c \neq a-(b-c)$

3) Given any three rational number a, b and c,

$(a\times b)=(b\times a)$ and $(a\times b)\times c=a\times(b\times c)$

$\left(\frac{3}{4}\times\frac{1}{5}\right)=\frac{3}{20}$ and $\left(\frac{1}{5}\times\frac{3}{4}\right)=\frac{3}{20} \in \mathbb{Q}$,

Then $(a\times b)=(b\times a)$, the multiplication of rational numbers is commutative.

i) $\left(\frac{7}{4}\times\frac{3}{5}\right)\times\frac{1}{6}=\frac{7}{40}$ and $\frac{7}{4}\times\left(\frac{3}{5}\times\frac{1}{6}\right)=\frac{7}{40} \quad \in \mathbb{Q}$

Then, $(a\times b)\times c=a\times(b\times c)$, the multiplication of rational numbers is associative;

ii) $2\times(7-4)=2\times3=6$ and $2\times(7-4)=(2\times7)-(2\times4)=14-8=6$, the multiplication is distributive over addition and subtraction in the set $\mathbb{Q}$.

4) $\forall a,b,d \in \mathbb{Q}$,

$$\frac{10}{3}:\frac{5}{4}=\frac{10}{3}\times\frac{4}{5}=\frac{8}{3} \text{ and } \frac{5}{4}:\frac{10}{3}=\frac{5}{4}\times\frac{3}{10}=\frac{3}{8}\in\mathbb{Q} \text{ so } \left(\frac{10}{3}:\frac{5}{4}\right)\neq\left(\frac{5}{4}:\frac{10}{3}\right).$$

Answers for Activity 1.3.4

1) Recurring $0.66666\ldots$ as a fraction gives $0.66666\ldots=\frac{6}{10-1}=\frac{6}{9}=\frac{2}{3}$.

Notice: $q=\dfrac{recuring\ digit}{n^{th}\ position\ of\ recuring\ decimal-1}$

For our case, recurring digit is 6 and it is tenth.

2) By using calculator, carry out the following;

a) $\sqrt{2}=1.414213562\ldots$ b) $\sqrt[3]{5}=1.7099759467\ldots$

It is not possible to express these numbers as fractions because there are not terminate nor recurring decimals.

$\sqrt{2}=1.414213562\ldots$ and $\sqrt[3]{5}=1.7099759467\ldots$ are elements of the set I of irrational numbers.

Note:

The sets $\mathbb{Q}$ and I are different, they have no intersection, $\mathbb{Q}\cap I=\phi$.

e) Application Activities

Answers for application activity 1.3.1

There are three types of fractions.

- Proper Fractions: the numerator is less than the denominator.

  Examples: $\frac{1}{2},\frac{3}{4},\frac{5}{6},\frac{7}{8},\ldots$

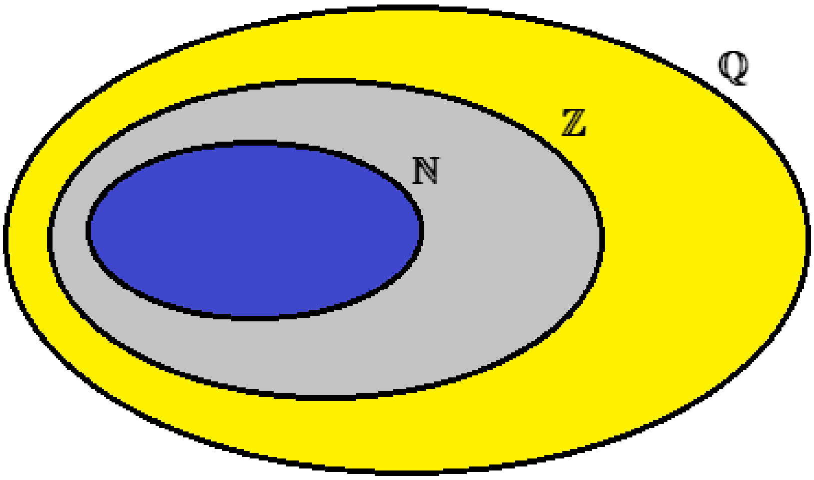
- Improper Fractions: the numerator is greater than (or equal to) the denominator.

  Examples: $\frac{3}{2},\frac{5}{4},\frac{11}{6},\frac{17}{8},\frac{19}{5},\ldots$

- Mixed Fractions: a whole number and proper fraction together.

  Examples: $1\frac{1}{2},1\frac{1}{4},1\frac{5}{6},2\frac{1}{8},3\frac{4}{5},\ldots$

Answers for application activity 1.3.2

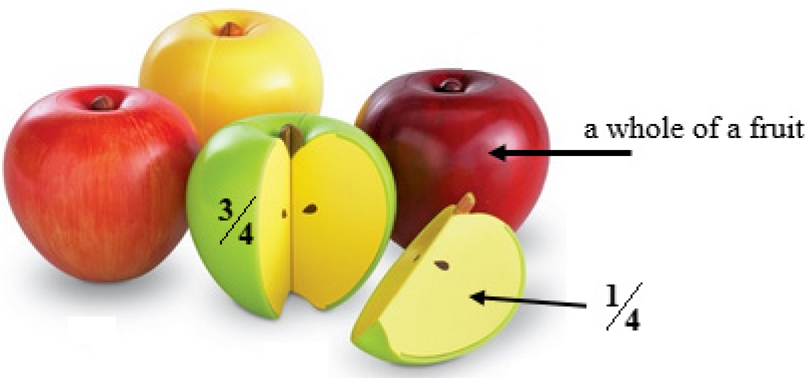


Answers for application activity 1.3.3

Answers may vary; but for any rational numbers, you take it is applicable in everywhere in our daily life. As tutor, verify whether their answers are correct. For example, we use fractions every day without even knowing it. Here are few examples you might be familiar with.

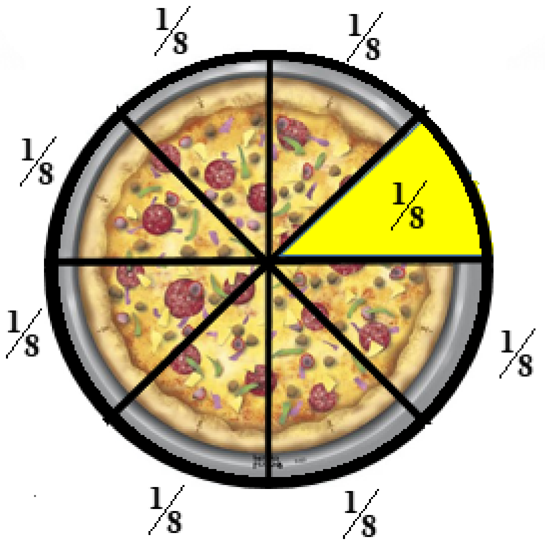
1) Fruits

Fruit is a great example. Every time we cut an apple, an orange, or any kind of fruit, we are taking a piece of the whole. We can represent those pieces as fractions. **The picture below shows pictures of apples that can be used as manipulatives**. The one that says 1/4 is already shown. For the other apples, the denominator tells us how many equal pieces those apples will be cut into.



2) Fractions in Everyday Life

A pizza is a great example of fractions! Each piece represents a part of a whole. In the picture, below, the pizza is divided into 8 pieces. If I have one piece, the fraction of pizza I am eating is one-eighth. (1/8)



Answers for application activity 1.3.4

1) The sum of rational number and irrational number is irrational.

   The given statement is always true.

   For example $\frac{1}{2}+\sqrt{2}=0.5+1.414213562\ldots=2.914213562\ldots$ is an irrational number.

2) The product of rational number and irrational number is irrational.

   The given statement is sometimes true.

   For example, $2\pi, \frac{3\pi}{4}, \frac{5}{7}\sqrt{11}, \ldots$ are irrational numbers.

   But, since 0 is a rational number and any irrational number times 0 is 0, for this case, the product is not an irrational number!

3) The sum of two irrational numbers is irrational.

   The given statement is always true.

4) The product of two irrational numbers is irrational.

   The given statement is sometimes true.

For example, $\sqrt{3}\times\sqrt{5}=\sqrt{15}$ is an irrational number but $\sqrt{3}\times\sqrt{3}=3$ is a rational number!

5) Between two rational numbers, there is an irrational number.

   The given statement is always true.

6) If you divide an irrational number by another, the result is always an irrational number.

   The given statement is sometimes true.

   For example, $\frac{\sqrt{3}}{\sqrt{6}}=\frac{1}{\sqrt{2}}$ is irrational number but $\frac{\sqrt{2}}{\sqrt{2}}=1$ is rational number.

Lesson 4: Definition of the set of real numbers, subsets, operations and properties

a) Learning objectives:

- Determine the hierarchy of sets on numbers and explain their relationship;
- Locate different numbers on a number line;
- Explain that irrational numbers cannot be expressed exactly as fractions.

b) Teaching resources:

T-square, ruler, student teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better in this lesson if they have a good understanding of set of rational numbers and the set of irrational numbers learnt in the previous lesson.

d) Learning activities

- Organize student-teachers into small groups;
- Invite them to do the following activities: Activity 1.4.1. activity 1.4.2 and activity 1.4.3 found in the student-teacher's book;
- Through group discussion invite student-teachers to do all questions of the given activities and visit each group to check if every student is engaged;

- Invite a group member from each group to present their findings, then help all student-teachers harmonize their answers and make the content summary highlighting that $\mathbb{R}=\mathbb{Q}\cup I$;
- Use different probing questions to guide student-teachers to explore examples and the content given in the student's book to enhance the elements of the set of integers, the sub sets of the set $\mathbb{R}$, how to carry out operations in the set $\mathbb{R}$ and guide them to highlight the corresponding properties;
- Guide all student-teachers to highlight the closure of the set $\mathbb{R}$ for all 4 operations (addition, subtraction, multiplication and division) and the presentation for subsets of $\mathbb{R}$ in the form of intervals;
- After this step, guide student-teachers to do the application activity 1.4.1, 1.4.2 or 1.4.3 and assess their competences and evaluate whether lesson objectives were achieved.

Answers for Activity 1.4.1

1) The set of counting numbers is a subset of natural numbers.

   This statement is **True** since $\mathbb{N}^+=\{1,2,3,\ldots\}\subset\mathbb{N}=\{0,1,2,3,\ldots\}$.
2) The intersection of the set of integers and counting numbers is the set of natural numbers.

   This statement is **False** since $\mathbb{Z}\cap\mathbb{N}^+=\mathbb{N}^+\neq\mathbb{N}=\{0,1,2,3,\ldots\}$.
3) The intersection of set of integers and natural numbers is the set of counting numbers.

   This statement is **False** since $\mathbb{Z}\cap\mathbb{N}=\mathbb{N}\neq\mathbb{N}^+=\{1,2,3,\ldots\}$.
4) The union of the set of natural numbers and counting numbers is the set of natural numbers.

   This statement is **True** since $\mathbb{N}\cap\mathbb{N}^+=\mathbb{N}=\{0,1,2,3,\ldots\}$.
5) The intersection of set of rational numbers and irrational numbers is the set of irrational numbers.

   This statement is **False** since $\mathbb{Q}\cap I=\{\ \}\neq I$.
6) The union of set of rational numbers and irrational numbers is a set of irrational numbers.

   This statement is **False** since $\mathbb{Q}\cup I=\mathbb{R}\neq I$.

Answers for Activity 1.4.2

Considering that subsets to be given by student-teachers are intervals or groups of numbers, you can verify whether they are correct or false.

Examples of subsets for the set of real numbers

Name	Description	Examples
Natural numbers	Numbers used for counting	$\{1,2,3,4,5,...\}$
Whole numbers	The natural numbers with 0	$\{0,1,2,3,4,5,...\}$
Integers	The whole numbers and the negative of the natural numbers	$\{...,-3,-2,-1,0,1,2,3,...\}$
Set of exact decimal numbers	All decimal numbers with a fixed number of decimals.	$\{-20.5,1.74,7.1112,3.4,4,5.89,...\}$
Rational numbers	Fractions	$...,-17, \quad 3, \quad 0.4, \quad 5, \quad 0.666 \quad ...$
Irrational numbers	Non-terminating, non-repeating decimals	$\{\sqrt{2}, \quad \sqrt{11}, \quad \pi, \quad 1.0010023, \quad ...\}$

Answers for activity 1.4.3

1) For any real numbers a,b and c, you verify easily that

a) $a+b$ is always a real number since $\mathbb{R}$ is **closed** under addition.

b) $a+b$ and $b+a$ always give the same answer since addition is **commutative** in $\mathbb{R}$.

c) $a+(b+c)$ and $(a+b)+c$ always give the same answer since addition is **associative** in $\mathbb{R}$.

2) For any three real numbers a,b and c, investigate the following operations : $a-b$, $a-b$ and $b-a$, $a-(b-c)$ and $(a-b)-c$

After substitution, you notice that

- $a-b$ is always a real number since $\mathbb{R}$ is **closed** under subtraction.
- $a-b \neq b-a$, which means that subtraction is not **commutative** in $\mathbb{R}$.

- $a-(b-c) \neq (a-b)-c$ which means that subtraction is not **associative** in $\mathbb{R}$.

The answer is always an element of real numbers as $\mathbb{R}$ is **closed** under subtraction.

3) For any three real numbers a, b and c, after substitution, you notice that

- $a \times b$ is always a real number since $\mathbb{R}$ is **closed** under multiplication.
- $a \times b = b \times a$, which means that multiplication is **commutative** in $\mathbb{R}$.
- $a \times (b \times c) = (a \times b) \times c$ which means that multiplication is **associative** in $\mathbb{R}$.

4) For any two real numbers a, b and c, investigate the following operations : $a \div b$, $a \div b$ and $b \div a$, $a \div (b \div c)$ and $(a \div b) \div c$

e) Application activities

Answers for application activity 1.4.1

Some examples of applications of real numbers in our daily life

- We use real numbers to express measurable quantities, e.g distance, time duration, weight, speed, in business, …
- Most mechanical devices for measuring contain part of the real number line.
- Some examples :(Marked) ruler, Protractor, Kitchen/bathroom scales, Micrometer (twice), Pressure gauge, Barometer, thermometer …
- You can read your altitude on the screen of a GPS unit - it will tell you how high or even how far below sea level you are.

Answers for application activity 1.4.2

Notation	Set description	Geometrical representation
(a,b) or $]a,b[$	$\{x : a < x < b\}$	
$[a,b]$	$\{x : a \le x \le b\}$	
$[a,b)$ or $[a,b[$	$\{x : a \le x < b\}$	
$(a,b]$ or $]a,b]$	$\{x : a < x \le b\}$	
$(a,+\infty)$ or $]a,+\infty[$	$\{x : a > x\}$	
$[a,+\infty)$ or $[a,+\infty[$	$\{x : a \ge x\}$	
$(-\infty,b)$ or $]-\infty,b[$	$\{x : x < b\}$	
$(-\infty,b]$ or $]-\infty,b]$	$\{x : x \le b\}$	
$(-\infty,+\infty)$ or $]-\infty,+\infty[$	$\{x : \mathbb{R}\}$ (Set of all real numbers)	

Answers for application activity 1.4.3

1) Closure Property under division for real numbers is not satisfied. Note that 0 itself is a rational number (0 = 0/1).

   So 3 ÷ 0 is a "rational being divided by a rational". But the result violates the definition of rational form $\frac{p}{q}$, $where\, q \neq 0$.

2) A field having width of $60\,m$ out $160\,m$ of length has the area of $60\,m \times 160\,m = 96{,}000\,m^2$ while that one having the width of $100\,m$ as it is its length has the area of $100m \times 100m = 100{,}000m^2$.

   Thus, the biggest field is that one having the width of $100\,m$ as it is its length.

   Even if the field having width of $60\,m$ out $160\,m$ of length has larger perimeter than that one having the width of $100\,m$ as it is its length, the area of second field is the largest(Not confuse perimeter from area).

1.6 Summary of the unit

Numbers help us to count and to measure out quantities of different items. For instance, in catering you may have to ask the client how many sandwiches they need for the event. Certainly, those working in accounts and other financial related jobs may use real numbers mostly. Even when relaxing at the end of the day in front of the television flicking from one channel to the next you are using real numbers.

The numbers we use in counting including zero, are called **Natural numbers**. The set of natural numbers is denoted by N= {0, 1, 2, 3, 4, …}.

There are several subsets of natural numbers:

- **Even numbers** are numbers which are divisible by 2 or numbers which are multiples of 2; the set of even numbers is $E = \{0,2,4,6,8,\ldots\}$
- **Odd numbers** are numbers which leave a remainder of 1 when divided by 2. The set of odd numbers is $O = \{1,3,5,7,\ldots\}$.
- **Prime number** is a number that has only two divisors 1 and itself. The set of prime numbers is $P = \{2,3,5,7,11,13,19,\ldots\}$.

Integers are whole numbers which have either negative or positive sign and include zero. The set of integers is represented by $\mathbb{Z}$.

The set of integers is represented using Carly brackets as follow: $\mathbb{Z}=\{\ldots,-5,-4,-3,-2,-1,1,2,3,4,5,\ldots\}$. Some **of special subset** of integers are the following:

- The set of **non-negative integers** denoted $\mathbb{Z}_0^+$ and $\mathbb{Z}_0^+=\{0,1,2,3,\ldots\}$; this set is also called a set of **whole integers**.
- The set of **positive integers denoted** as $\mathbb{Z}^+=\{1,2,3,\ldots\}$
- The set of **negative integers** denoted as $\mathbb{Z}^-=\{\cdots,-4,-3,-2,-1\}$

From any two integers a and b, we deduce fractions expressed in the form $\frac{a}{b}$, where b is a non-zero integer. **A rational number is a number that can be expressed as a fraction** where both the numerator and the denominator in the fraction are integers, thus set of fractions is known as a set of rational numbers.

From the concept of subset and definition of a set of rational numbers we can establish some subsets of rational numbers; among them we have:

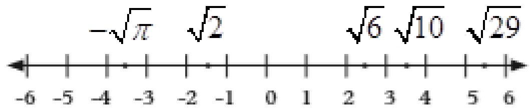
- Integers and its subsets: $\mathbb{Z},\mathbb{Z}_0^+,\mathbb{Z}^-,\mathbb{Z}^+,...$
- Natural numbers and its subsets: $\mathbb{N},\mathbb{N}^+$, prime numbers, odd numbers, even numbers, …
- Counting numbers and its subsets: $\mathbb{N}^+$, square numbers, prime numbers, odd numbers, even numbers …

There are some numbers which do not have exact values and they cannot be expressed as fractions, for example $\pi,\sqrt{2},\sqrt[3]{7},\ldots$. These numbers are called **irrational numbers**.

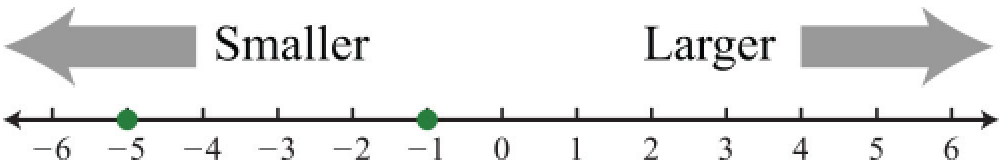
An irrational number can be written as an unlimited decimal, but not as a fraction.

Although irrational numbers are not often used in daily life, they do exist on the number line. In fact, between 0 and 1 on the number line, there are an infinite number of irrational numbers!

The set of rational numbers and the set of irrational numbers combined together, form the set of **real numbers**. The set of real numbers is denoted by $\mathbb{R}$. Real numbers are represented on a number line as infinite points or they are set of decimal numbers found on a number line. This is illustrated on the number line below



The real numbers are ordered. We say is a ***less than*** b and write $a < b$ if $b - a$ is positive. **Geometrically** this means that a lies to the left of b on the number line. Equivalently, we say b is ***greater than*** a and write $b > a$. The symbol $a \le b$ $\left(\text{or}\, b \ge a\right)$ means that either $a < b$ or $a = b$ and is read a ***less than or equal to*** b. In fact, when comparing real numbers on a number line, the larger number will always lie to the right of the smaller one.



Operations and properties on set of real numbers and its subset

Operation and Properties \ Sets	Set of natural numbers $\mathbb{N}=\{0,1,2,3,4,\ldots\}$	Set of integers $\mathbb{Z}=\{\ldots,-2,-1,0,1,2,3,4,\ldots\}$	Set of rational numbers $(\mathbb{Q})$	Set of irrational numbers (I)	Set of real numbers $(\mathbb{R})$
Addition					
Closure	is defined	is defined	is defined	is not defined	is defined
Commutativity	is satisfied	is satisfied	is satisfied		is satisfied
Associativity	is satisfied	is satisfied	is satisfied		is satisfied
Subtraction					
Closure	is not defined	is defined	is defined	is not defined	is defined
Commutativity		Not satisfied	Not satisfied		Not satisfied
Associativity		Not satisfied	Not satisfied		Not satisfied
Multiplication					
Closure	is defined	is defined	is defined	is not defined	is defined
Commutativity	is satisfied	is satisfied	is satisfied		is satisfied
Associativity	is satisfied	is satisfied	is satisfied		is satisfied
Division					
Closure	is not defined	is not defined	is defined except for 0	is not defined	is defined except for 0
Commutativity			Not satisfied		Not satisfied
Associativity					Not satisfied

Notice: After finding out that a set is not closed under the given operation, we do not verify other properties.

1.7 Additional Information for tutors

By the end of this unit the tutor has to inform student teachers about sub sets of the set of real numbers $\mathbb{R}$: $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ and I . Intersection of the set of rational numbers and irrational numbers is empty set. Tutor is required to emphasize to the fact that 0 is natural number or $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$ and 1 is not a prime number as it has only one divisor while a prime is a natural number having two divisors 1 and its self.

Irrational numbers are not closed under addition, subtraction, multiplication or division as described in the following table.

Operation	Example	Description
Addition	$\pi + (-\pi) = 0$	π and $-\pi$ are both irrationals but 0 is not irrational, thus irrational numbers are not closed under addition.
Subtraction	$\sqrt{3} - \sqrt{3} = 0$	$\sqrt{3}$ and $-\sqrt{3}$ are both irrationals but 0 is not irrational, thus irrational numbers are not closed under subtraction.
Multiplication	$\sqrt{7} \times \frac{1}{\sqrt{7}} = 1$	$\sqrt{7}$ and $\frac{1}{\sqrt{7}}$ are both irrationals but 1 is not irrational, thus irrational numbers are not closed under multiplication.
Division	$\frac{\sqrt{125}}{\sqrt{5}} = 5$	$\sqrt{125}$ and $\frac{1}{\sqrt{5}}$ are both irrationals but 5 is not irrational, thus irrational numbers are not closed under division.

We have pairs of values for each operation which yields in a non-irrational result, therefore the set of irrational numbers is not closed under any of the 4 operations.

1.8 End unit assessment

1) To list three rational numbers between 0 and 1, answers may vary as they are infinite rational numbers between 0 and 1.

   Some are $\left\{\frac{1}{4},\frac{1}{3},\frac{1}{2}\right\}$ or $\left\{\frac{3}{4},\frac{4}{5},\frac{5}{6}\right\}$ or $\left\{\frac{2}{3},\frac{5}{7},\frac{11}{13}\right\}$ and so on.

2) The sets to which each of the following numbers belong

	Number	Counting	Natural	Integers	Rational	Irrational	Real
1	-1			✓	✓		✓
2	$\frac{1}{7}$				✓		✓
3	0		✓	✓	✓		✓
4	$-\sqrt{7}$					✓	✓
5	$\sqrt{0.03}$					✓	✓
6	$\frac{\sqrt{4}}{2}$	✓	✓	✓	✓		✓
7	$\frac{1}{0}$						
8	$\frac{\pi}{2}$					✓	✓
9	$\frac{\sqrt{27}}{\sqrt{3}}$	✓	✓	✓	✓		✓
10	$\sqrt[3]{-27}$			✓	✓		✓

Note: By means of a number line, tutor can guide student teachers to present approximately these numbers by a point on a number line.

3) $Cost = \frac{126,000Frw}{3\times 24} = 1,750Frw$

4) Coordinate is -3

1.9 Additional activities

1.9.1 Remedial activities

For question 1 and 2, discuss your ideas with your classmates and then write a summary of your findings in your notebook.

1) How do you find the product of two numbers when
   a) Both are positive?
   b) One is positive and the other is negative?
   c) Both are negative?
   d) One is zero?
2) How do you find the quotient of two numbers when
   a) Both are positive?
   b) One is positive and the other is negative?
   c) Both are negative?
   d) Numerator is zero?
3) Work out the following:
   a) (-10) × (-5) × (-6) :(-3) × (-2);
   b) (-30) × (+2) × (-10): (-5) × (+2)
4) Find the square root of each of the following.
   a) i) $\sqrt{6}$ ii) $\sqrt{8}$;
   b) Find the values of

      i) $\{9+((-2)\times(-15))\}\times(-2+7)\div 3$;

      ii) $-6+(-5)+8\times(-2)-4+(-2)$

      iii) $(-3\times 23)+((-5)\times(-1))+((-8)\times(-4))$;

      iv) $9\times(-2)-4-(-2)$

Answers:

1) The product of two numbers when
   a) both are positive is **positive**
   b) one is positive and the other is negative, is **negative**
   c) both are negative is **positive**

d) One is zero, is **zero**

2) The quotient of two numbers when

   a) both are positive is **positive**

   b) One is positive and the other is negative, is **negative**

   c) Both are negative is **positive**

   d) Numerator is zero, is **zero**

3) a) -50; b) -60;

4) a) i) 2.449489743 ii) 2.828427125

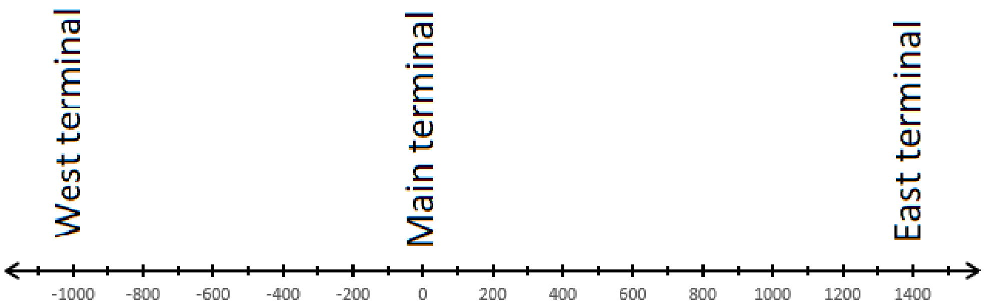
   b) i) 65 ii) -33 iii) -32 iv) -20

1.9.2 Consolidation activities

1) Use the digits 2,5 and 9 to write a fraction with the greatest value. Then write the fraction as a mixed number.

2) When Keza gets home from school, $\frac{3}{4}$ of a sandwich is left in the refrigerator. She cuts the parts remaining into three equal parts and eats two of them. What fraction of the whole sandwich did she eat?

3) At some international airports, trains carry passengers between the separate terminal buildings (in meters). Suppose that one such train system moves along a track like the one below



   a) A train leaves the main terminal going east at 10 meters per second. Where will it be in 10 seconds? Illustrate your solution by using number line. When will it reach the east terminal?

   b) A train passes the main terminal going east at 10 meters per second. Where was that train 20 seconds ago? Illustrate your solution by using number line. When was it at west terminal?

   c) A train leaves the main terminal going west at 10 meters per second. Where will it be in 50 second? Illustrate your solution by using number line. When will it reach the west terminal?

d) A train passes the main terminal going west at 10 meters per second. When was it at east terminal? Where was that train 50 seconds ago? Illustrate your solution by using number line.

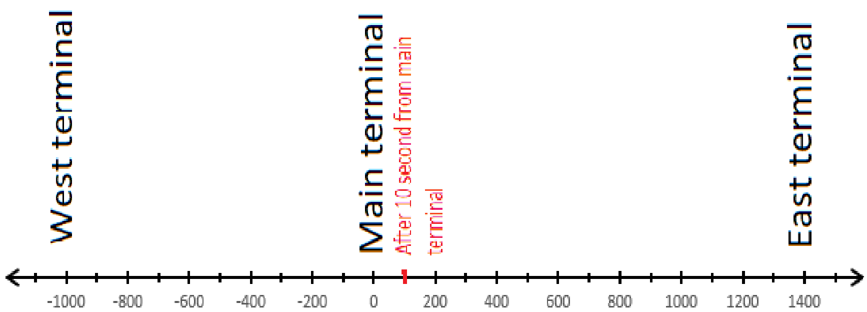
Solution:

1) $\frac{95}{2};47\frac{1}{2}$

2) Keza ate $\frac{2}{3}$ of $\frac{3}{4}$; that is $\frac{2}{3}\times\frac{3}{4}=\frac{2}{4}=\frac{1}{2}$.

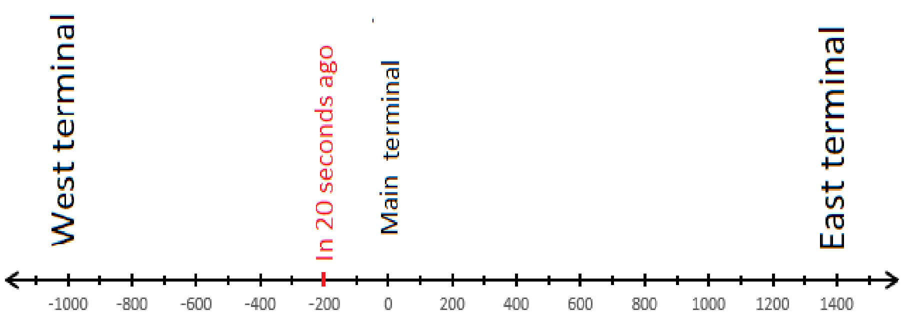
3)

a) In 10 seconds, it will be at $10\times10m=100m$ from the main terminal (on the right side).



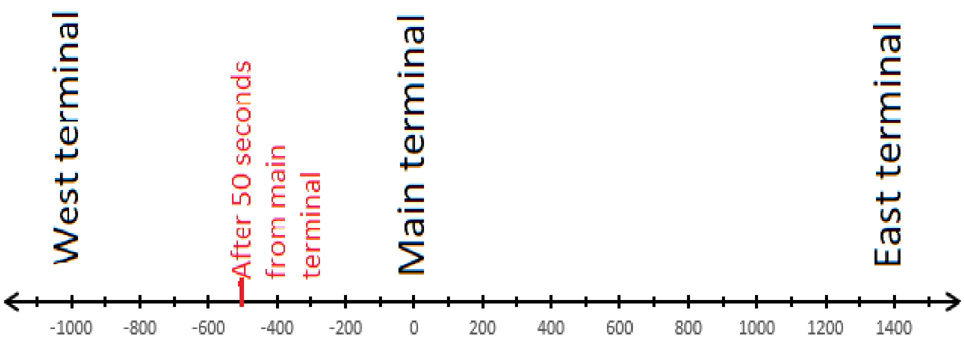
It will reach the east terminal after $\frac{1,400}{10}s=140$ *seconds* (or *2minutes and 20 seconds* from the main terminal).

b) In 20 seconds ago, the train was at $10\times20m=200m$ on the left of the main terminal.



The train was at west terminal before $\frac{1,000}{10}s=100$ *seconds* (or *1 minute and 40 seconds*) from the main terminal.

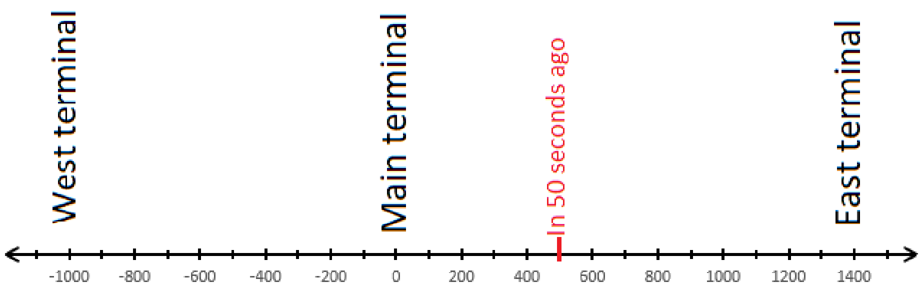
c) In 50 seconds, the train will be at $10 \times 50\,m = 500\,m$ from the main terminal (on the left side).



The train will reach the west terminal after $\frac{1,000}{10}s = 100$ *seconds* (or *1 minute and 40 seconds*) from the main terminal.

d) A train was at east terminal before $\frac{1,400}{10}s = 140$ *seconds* (or *2minutes and 20 seconds)* before passing through the main terminal.

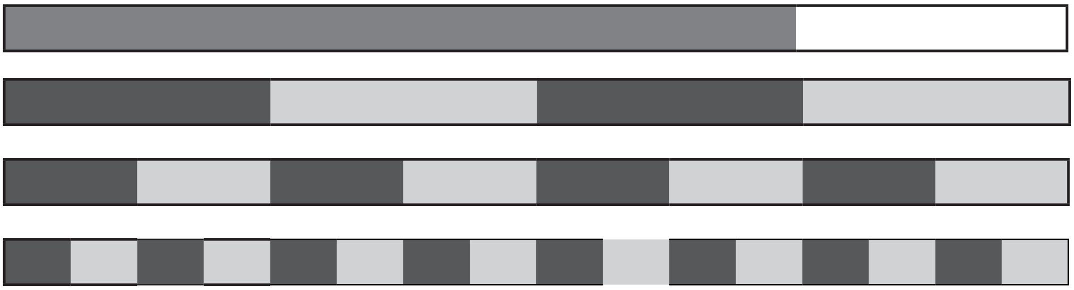
That train in 50 seconds ago, was at $10 \times 50\,m = 500\,m$ on the right of the main terminal.



1.9.3 Extended activities

When you save or download a file, load a program, or open a page on the internet, a status bar is displayed on the computer screen to let you watch the progress. Use the fraction strips shown to find three fractions that describe the status of the work in progress. Find out its equivalent decimal.

Downloading file…



Solution:

Fractions describing the status of the work in progress, are $\frac{3}{4}, \frac{6}{8}, \frac{12}{16}$ respectively.

Equivalent decimal is 0.75.

UNIT: 2

SET THEORY

2.1. Key unit competence

Solve problems that involve Sets operations using Venn diagrams.

2.2 .Prerequisites

Student-teachers will perform better in this unit if they have background on: Expressing mathematical problem set using a Venn diagram. Representing a mathematical problem using a Venn diagram; Using Venn diagram to represent a mathematical problem set. Interpreting, modelling, and solving a mathematical problem set.

2.3. Cross-cutting issues to be addressed

a) Inclusive education

Promote the participation of all student-teachers while teaching

b) Peace and value Education

During group activities, the tutor will encourage student-teachers to help each other and to respect opinions of colleagues.

c) Gender

Give equal opportunities to all student-teachers (girls and boys) to present their findings. Encourage them to participate actively in all learning and teaching activities from the beginning to the end of the teaching and learning process.

2.4. Guidance on introductory activity 2

- Form groups of student-teachers and invite them to work on introductory activity to understand the concept of set theory;
- Give time to student-teachers to analyse the activity and provide pieces of advice and adequate facilitations where necessary.
- Invite group to present their findings and try to harmonize them;
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, use different questions to arouse their curiosity on what is going to be

leant in this unit.

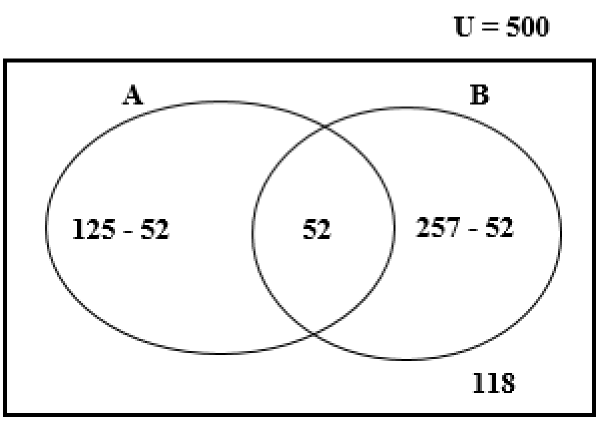
Answers for introductory activity 2

At a TTC school of 500 student-teachers- teachers, there are 125 student-teachers enrolled in Mathematics club, 257 student-teachers who play sports and 52 student-teachers that are enrolled in mathematics club and play sports.

Complete the following table

Symbol	**Description**	**Value for this problem**
$n(M)$	The number of elements in set M (Math club)	125
$n(S)$	The number of elements in set S(Sport club)	257
$n(M \cap S)$	The number of elements in the intersection of sets M and S (all the elements that are in both sets-the overlap)	52
$n(M \cup S)$	The number of elements in the union of sets M and S (all the elements that are in one or both of sets)	382

A Venn diagram to illustrate the information in the table above is the following:



2.5. List of lessons

No	Lesson title	Learning objectives	Periods
0	Introduction activity	To arouse the curiosity of student teachers on the content of unit 2	1
1	Sets and Venn diagram: Analysis and interpretation of a problem using set language (intersection, union…)	Interpret mathematical problem using a Venn diagram.	1
2	Operations on sets (Set difference, symmetric difference, complement of set, set union and intersection).	Perform operations on set (union, intersection, difference, and symmetrical difference on sets).	2
3	Representation of a problem using Venn diagram	Use Venn diagram to illustrate a mathematical problem.	3
4	Modelling and solving problems involving Venn diagrams	Interpret, model, and solve a mathematical problem on set using Venn diagrams.	3
5	End unit assessment	Appreciate the importance of using Venn diagrams to represent and solve a mathematical problem involving sets.	2
	Total number of periods		**12**

Lesson 1: Sets and Venn diagrams

a) Learning objective:

Interpret mathematical problem using a Venn diagram.

b) Teaching resources:

Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, textbooks and wall charts and wall maps, Mathematical models and Internet connection (where available).

c) Prerequisites / Revision / Introduction:

Student-teachers will perform well in this unit if they have a good background on:

Identifications of sets of numbers (natural, integers, rational and real) and relationship among them.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by asking them to use the student teacher's book to discuss the **activity 2.1**
- Move around to each group and ensure all student-teachers participate actively;
- Call upon group representatives to present their findings in a whole class discussion;
- Harmonize their findings insisting on how to illustrate a situation using Venn diagrams;
- Guide student teachers to perform individually the **application activity 2.1 and then,** assess the knowledge and skills acquired.

Answers for activity 2.1

Given the set $A=\{1,3,5,7,9\}$ and $B=\{1,2,3,4,5\}$

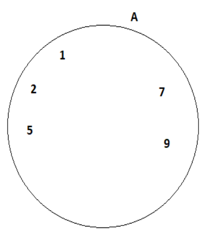
a) Common elements for both sets make $\{1,3,5\}$

b) Elements of set A which are not in set B make $\{7,9\}$

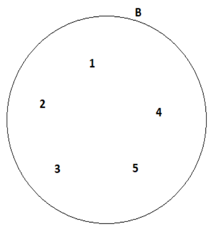
c) Elements of set B which are not in set A make $\{2,4\}$

d) All elements in set A and set B make $\{1,2,3,4,5,6,7\}$

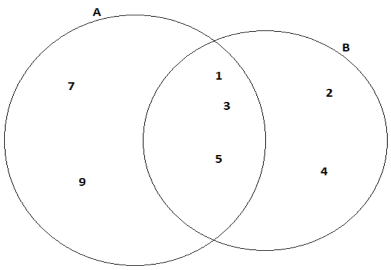
e) Set A in a Venn diagram



f) Set B in a Venn diagram:



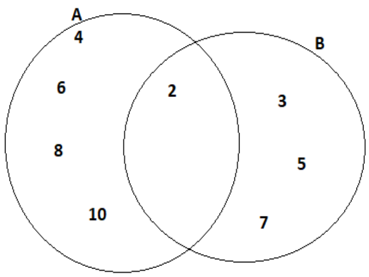
g) Sets A and B using one Venn diagram



e) Application activities

Answers for application activity 2.1

$$A=\{2,4,6,8,10\}\ while\ B=\{2,3,5,7\}$$



By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation of this lesson.

Lesson 2: Operations on sets and Venn diagrams

a) Learning objective:

Apply operation of set: Perform union, intersection, difference–and symmetrical difference on sets.

b) Prerequisites/Revision/Introduction:

Student-teachers will perform better in this lesson if they refer to: operation of sets learnt in S1.

c) Teaching resources:

They include: Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, wall charts and wall maps, Mathematical models and Internet connection where applicable.

d) Learning activities:

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Ask student-teachers to use the student teacher's book to discuss the **activity 2.2;**
- Move around to each group to ensure all student-teachers participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and guide student-teachers to highlight the union of sets, intersection on sets and their differences.
- Use different probing questions to guide student-teachers to explore examples and the content given in the student-teacher's book to the use of operation on sets: intersection, union and difference and guide them to highlight the corresponding properties;
- Guide all student-teachers to explore the symmetric difference and the complement of set;
- After this step, guide student-teachers to do the application activity 2.2**, assess their competences,** and evaluate whether lesson objectives were achieved.

Answers for activity 2.2

a) All student-teachers who were absent in the English class: A'

b) All student-teachers who were present in at least one of the two classes: $A \cup B$

c) All the student-teachers who were present for both English as well as History classes: $A \cap B$

d) All the student-teachers who have attended only the English class and not the History class: $A - B$

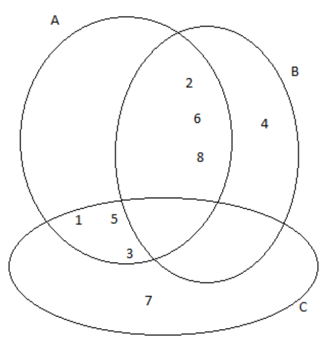
The tutor helps or guides student teacher to discover their own content about: classification of sets.

After the tutor tells the student teacher to continue the other examples found in the student teacher book entitled application activities 2.1.2.

e) Application activities

Answers for application activity 2.2

1) $A = \{1,2,3,5,6,8\}$; $B = \{2,4,6,8\}$; $C = \{1,3,5,7\}$



a). $B \cap C = \{\phi\}$;

b). $A \cap C = \{1,3,5\}$;

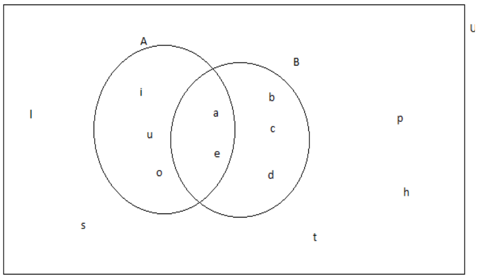
c). $A \cap B = \{2,6,8\}$;

d). $B \cup C = \{1,2,3,4,5,6,7,8\}$;

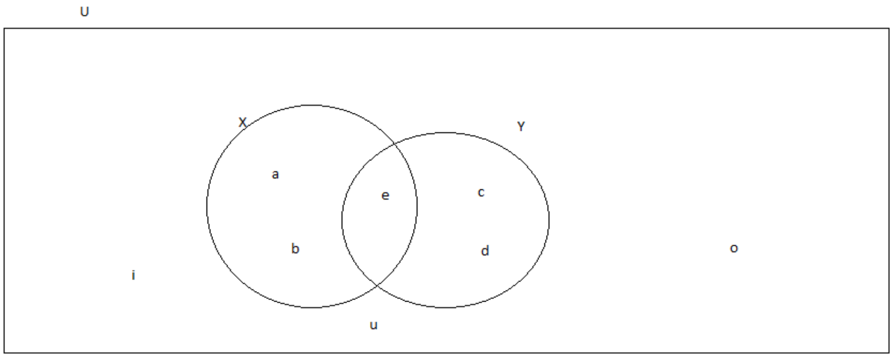
e). $A \cup B = \{1,2,3,4,5,6,8\}$

2) a) $A = \{i,u,o,a,e\}$; b) $B = \{a,b,c,d,e\}$; c) $A - B = \{i,u,o\}$;

d) $B - A = \{b,c,d\}$; e) $A \Delta B = \{i,u,o,b,c,d\}$; f) $U = \{e,l,p,h,\text{a},\text{t},\text{i},\text{s}\}$



3) $U = \{a,\ e,\ i,\ o,\ u,\ c,\ d\}$; $X = \{a,\ b,\ e\}$ *and* $Y = \{c,\ d,\ e\}$



a). $X \cap Y = \{e\}$;

b). $(X \cap Y)' = \{a, b, c, d\}$;

c). $X \cup Y = \{a, b, \mathrm{c}, \mathrm{d}, e\}$;

d). $(X \cup Y)' = \{i, u, o\}$

At the end of this lesson, you can give other many possible exercises as remedial and consolidation activities of this lesson.

Lesson 3: Use Venn diagram to represent a mathematical problem

a) Learning objective

Represent problems using Venn diagrams

b) Teaching resources

Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, wall charts and wall maps.

c) Prerequisites / Revision / Introduction

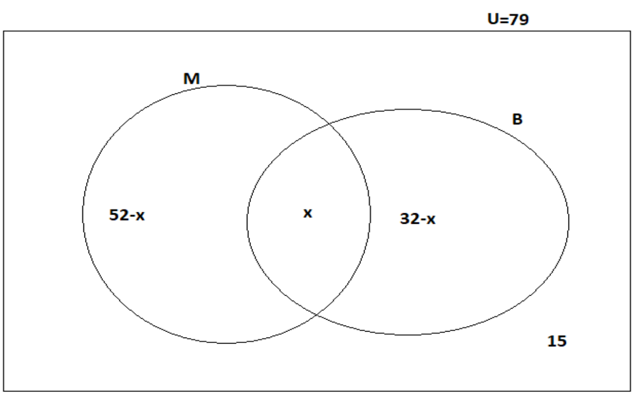
Student-teachers will perform well in this lesson if they revise the Representation of sets using Venn diagrams.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers.
- Ask student-teachers to use the student teacher's book to discuss all questions of the **activity 2.3;**
- Move around to each group to ensure all student-teachers participate actively;
- Call upon groups to present their findings and harmonize their answers;
- Use different probing questions to guide student-teachers to explore examples and the content given in the student-teacher's book on how to illustrate and solve some practical word problems by using Venn diagrams;
- After this step, guide student-teachers to do the application activity **2.3** and assess their competences and evaluate whether lesson objectives were achieved

Answers for activity 2.3

1) a) and b) let M represent those who bought milk and B represent those who bought bread: $n(\mathrm{M}) = 52$; $\mathrm{n(B = 32)}$; $\mathrm{n(U)} = 79$ let those who bought both milk and bread be represented by x that is $x = n(\mathrm{M} \cap \mathrm{B})$;



$52 - x + 32 - x + x + 15 = 79$

$x = 20$

i). Those who bought milk and bread are 20

ii). Those who bought bread only are= 32-20=12 iii). Those who bought milk only are= 52-20=32

c). The easiest method is to use in (a) and (b) above, is the Venn diagram approaches to represent the situation, because it clarifies the situation in a simple way.

2) As 55 student-teachers like Mathematics; $x+x+5+20=55 \Rightarrow 2x=30 \Rightarrow x=15$

a) 15 student-teachers like all subjects

b) The total number of senior one student-teachers is = $U=10+60+20+15+15+5=125$

c) Student-teachers who like Physics and Kinyarwanda only are =0 (no one)

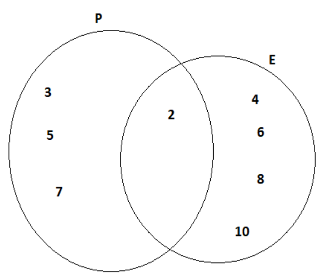
e) Application activities

Answers for application activity 2.3

a) $P=\{2,3,5,7\}$ and $E=\{2,4,6,8,10\}$;

i). $P \cap E=\{2\}$; ii). $P \cup E=\{2,3,4,5,6,7,8,10\}$

b) The Venn diagram representing the situation is:



By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation activity in this lesson.

Lesson 4: Modelling and solving problems involving Venn diagrams

a) Learning objective:

Interpret, model, and solve a mathematical problem using set.

b) Teaching resources:

Ruler, T-square, Manila paper, Calculators, Student-teacher's book and other Reference textbooks to facilitate research, the classroom, textbooks and wall charts and wall maps.

c) Prerequisites / Revision / Introduction:

Student-teachers will perform well in this lesson if they learnt well the previous lesson.

d) Learning activities

- Organize the student-teachers into small groups;
- Provide clear instructions and introduce the activity by guiding student-teachers to use the student teacher's book to discuss the **activity 2.4;**
- Move around to each group to ensure all student-teachers participate actively;
- Call upon groups to present their findings and harmonize their answers in a whole class discussion.
- Use different probing questions to guide student-teachers to explore examples and the content given in the student-teacher's book on how to model word problems by using Venn diagrams;
- Guide student-teachers to perform individually **application activity 2.4** to assess their knowledge and skills.

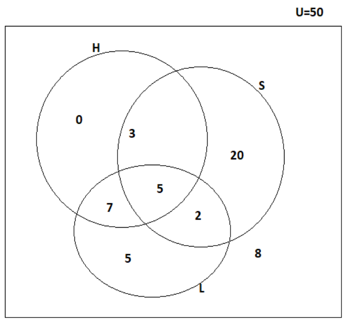
Answers for activity 2.4

a) Let H = Hill top Hotel; S = Serena Hotel; L = Lemigo Hotel

$H = \{15\}$; $S = \{30\}$; $L = \{19\}$;

$H \cap S = \{8\}$; $H \cap L = \{12\}$; $S \cap L = \{7\}$;

$(S \cap L) \cap H = \{5\}$



a) The people ate at Hilltop = 0
b) Hilltop and Serena but not at Lemigo:23
c) People who did not eat from any of these three hotels are 8

e) Application activities

Answers for application activity 2.4

a) Let x be the number of student - teacher . The student-teachers who like the three subjects=15
b) The total number of year one student-teachers in ECLPE= $60+10+15+20+20=125$
c) Student-teachers who like Physics and Kinyarwanda only = 0.

By the end of this lesson the tutor should be able to give other many possible exercises as remedial and consolidation activity of this lesson.

2.6. Summary of the unit

a) **Set:** It is a group of items with a common feature.
b) **Member of a set:** It is an object or item in a set.
c) **Subset:** It is a set which is formed by obtaining some elements in all the elements from a given set.
d) **Venn diagram**: It is a circular or rectangular pattern used to represent sets and its elements.
e) **Intersection of sets:** It is the set formed by common elements which appear in two or more sets.
f) **Union of sets:** It is the set formed by putting together elements of two or more sets.
g) **Complement of a set:** It is a set of all elements in the universal set that are not members of a given set.
h) **Difference between sets A and B (A-B):** It is a set formed by the

elements appearing in set A but not in set B.

2.7. Additional information for tutor

Here the tutor has to focus on general problems in sets using Venn diagrams.

2.8. End Unit assessment

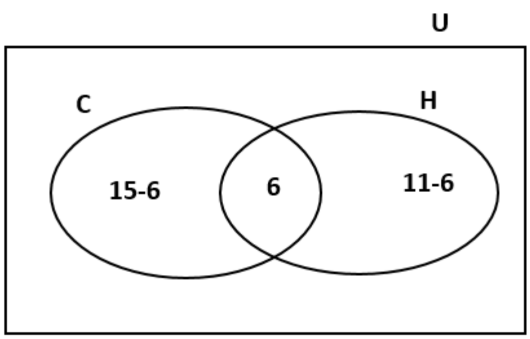
Student teacher should be able to solve all the exercises found in end assessment of unit 2.

Answers for end unit assessment

1) Let the total number of student teacher in that class be U

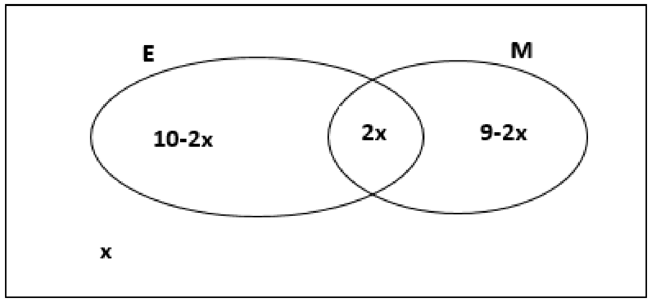
   n (Cricket) = 15; n (Hockey) = 11; $n(C \cup H) = 6$

   The Venn diagram, is then:



   $n(\text{U}) = 15 - 6 + 6 + 11 - 6 = 20$ so Number of pupils in the class is 20.

2) Let x be the number of those who teach none



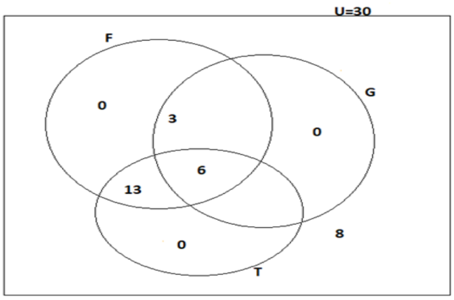
   $n(\text{U}) = 17; \text{n(E)} = 10; \text{n(M} = 9)$

   From Venn diagram: $10 - 2x + 2x + 9 - 2x + x = 17$; $x = 2$

a). $E \cap M = 2\times 2 = 4$; b). none=2; c). only one

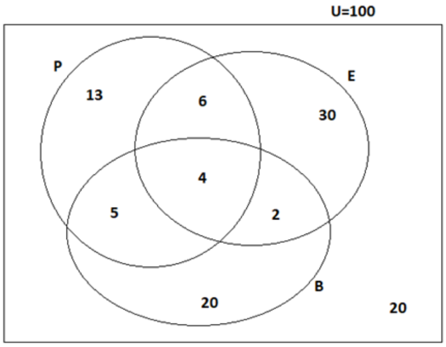
Subject= $10-4=6$ (Economics); $9-4=5$ (Mathematics); total : $6+5=11$

3) Student-teachers did not participate in any of the sports.



The student-teachers did not participate in any of the sports are 8

4)



a) Student-teachers took none of the three subjects: 20
b) Student-teachers took PE, but not BIO or ENG=13
c) Student-teachers took BIO and PE but not ENG=5

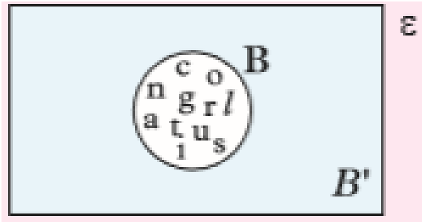
2.9. Additional activities

2.9.1 Remedial activities

1) Given that: ε = {a, b, c, d,…,z} ; and B = {letters in the word congratulations} Describe the set B′ and show set B and B′ in a Venn diagram.
2) Given that ε= {a, b, c, d, e} and A= {a, b}, find A’

Answers:

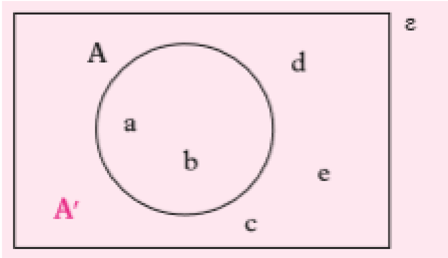
1) Draw a rectangle and label it with ε to show the universal set. Within the rectangle draw set B as a subset of ε



Shade the region within the universal set but outside set B and label it B′. B′ is the set of all the alphabets except those in set B

2) ε= {a, b, c, d, e} A= {a, b} thus,

A' = {c, d, e}. This can be represented on a Venn diagram as shown.



2.9.2. Consolidation activities

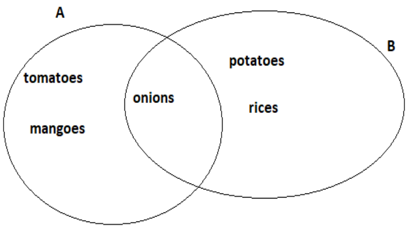
1. If set A = {tomatoes, onions, mangoes} and B = {onions, potatoes, rice}, by usingVenn diagram, find:

   a) $A \cap B$;   b) $A \cup B$;   c) A – B ;   d) B – A;   e) $A \Delta B$

2. Consider this Venn diagram, find the number of universe if: n(P)= 8; n(E)=8; n(B)=7

Answers:

1)



a) $A \cap B = \{onions\}$;

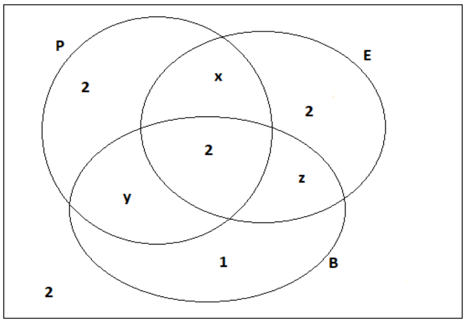
b) $A \cup B = \{tomatoes, mangoes, onions, potatoes, rices\}$;

c) $A - B = \{tomatoes, mangoes\}$;

d) $B-A=\{potatoes, rices\}$;

e) $A\Delta B=\{tomatoes, mangoes, potatoes, rices\}$

2)

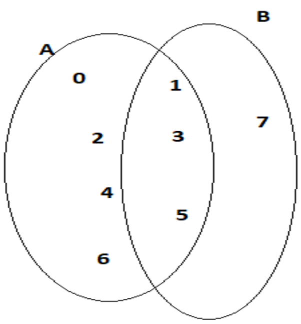


3) $\begin{cases}2+y+2+x=8\\x+2+z+2=8\\2+y+1+z=7\end{cases}\Rightarrow\begin{cases}x=2\\y=2\\z=2\end{cases}\Rightarrow n(\text{U})=2+2+2+2+2+2+2+1=15$

2.9.3. Extended activities

Let $A=\{0,1,2,3,4,5,6\}$ *and* $B=\{1,3,5,7\}$, represent the elements of A and B on a Venn Diagram and find the following sets: $A\cap B$, $A\cup B$, Universal set U, $A-B$, $A\Delta B$, A' and B'

Answers



i) Intersection: $A\cap B=\{1,3,5\}$

ii) Union set : $A\cup B=\{0,1,2,3,4,5,6,7\}$

iii) Universal sets: $U=\{\{0,1,2,3,4,5,6\},\{1,3,5,7\}\}$

iv) Simple difference of sets: $A-B=\{0,2,4,6\}$; $B-A=\{7\}$

v) Symmetric difference of sets: $A\Delta B=\{(\text{A}-\text{B})\cup(\text{B}-\text{A})\}=\{0,2,4,6,7\}$

vi) Complements of sets: $A'=\{7\}$; $B'=\{0,2,4,6\}$

UNIT: 3

PROBLEM ON RATIOS AND PROPORTIONS

3.1. Key unit competence

Apply ratios, proportions and multiplier proportion change to solve real life related problems.

3.2. Prerequisites (Knowledge, Skills and Values)

Student teacher will perform well in this unit if they have a good background on:

- Ratio and proportion (Senior1: unit8),
- Multiplier for proportional change (S2 unit 4),
- Properties of algebraic fractions (Senior 3: unit 3).

3.3. Cross-Cutting issues to be addressed

a) Inclusive education

Promote the participation of all student-teachers while teaching;

b) Peace and value Education

During group activities, the teacher will encourage student-teachers to help each other and to respect opinions of colleagues.

c) Gender

- Give equal opportunities to all student-teachers (girls and boys) to present their findings.
- Encourage them to participate actively in all learning and teaching activities from the beginning to the end of the teaching and learning process.

d) Financial education

Guide student-teachers to discuss how to invest money into a common project. This should be addressed via problems that imply the way of using money that encourage learner to deal with money financially.

3.4. Guidance on introductory activity

- Invite student teachers to form groups as heterogeneous as possible and guide them to work on the introductory activity.
- Give time to student-teachers to analyse the activity;
- Invite group representatives to present findings in a whole class discussion;
- Harmonize student-teachers' answers enhancing that equal share can be done in different ways;
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, use different questions to arouse their curiosity on what is going to be leant in this unit.

Answers for introductory activity 3

A ratio is a comparison between quantities.

A ratio is an ordered pair of numbers written in the order $a:b$ where b cannot be zero (0).

Ratios may be used while calculating things which are compared proportionally.

a) It depends on the total number of the student teachers that are studying in your class. For example: your class can contain 50 student-teachers including 20 boys and 30 girls. In terms of ratio boys can be expressed as: 2:5 and girls can be expressed by 3:5

b) Yes, it is possible to share equally a certain number of mathematics textbooks to different groups in your classroom and then figure out the ratio of Mathematics textbooks per learner. Here it depends on the number of textbooks of mathematics and the number of the student teachers' groups. For example, if we consider that there are 60 textbooks of mathematics and 50 student-teachers grouped into 10 groups, we see that each group will have 6 textbooks. The ratio of mathematics textbooks per learner is 6:10

c) In real life ratio are used for example in Restaurant, in schools, in business, in bank, (student teachers can give other applications)

3.5. List of lessons

No.	Lesson title	Learning objectives	Periods
0	Introduction activity	To arouse the curiosity of student teachers on the content of unit 3.	1
1.	Equal and unequal share, Ratio and proportion	Share quantities in a given ratio.	2
2.	Direct and indirect proportion	Compare two quantities by using a given proportion.	2
3.	Calculation of proportional change using multiplier and compound proportional change or continued proportions	Solve problems involving multiplier and compound proportional change;	3
4	Problems involving direct and indirect proportions	Solve problems involving direct and indirect proportions.	2
5	End assessment		2
Total number of periods			**12**

Lesson 1: Equal and unequal share, Ratio and proportion

a) Learning objective:

Share quantities in a given ratio

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators, Internet connection where applicable.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better in this lesson if they have a good understanding on concepts of shares, ratio and proportion learnt in senior 1.

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask student-teachers to use their books to discuss the **activity 3.1.1** and motivate the**m** to determine how to share money respecting a given ratio. Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts of sharing quantities using ratios.
- Use different probing questions to guide student-teachers to explore examples and the content related to ratios given in the student-teacher's book;
- Guide student-teachers to perform individually **application activity 3.1.1** to assess their competences.

Answers for activity 3.1.1

1) i) Sharing would be done in the ratio of $2:3$

   ii) 7000 FRW is to be shared in the ratio $2:3$.

   It is spilt into 5 equal parts i.e 2+3=5 equal parts

   The first old man gets $\frac{2}{5}\times 7000 = 2800\ FRW$

   The second old man gets $\frac{3}{5}\times 7000 = 4200\ FRW$

2) i) A ratio is a mathematical statement which shows how two or more quantities or numbers are compared.

   ii) a) Their contributions were in the ratio of 800:120

   The simplest ratio is 20:3

   b) First $7.5\,l = 7500\,ml$

   Their milk sales are in ratio of $4500:7500$. The simplest ratio is $3:5$

e) Application activities

Answers for iapplication activity 3.1.1

Ingabire, Mugenzi and Gahima, after investing in buying and selling of shares in the Rwanda stock exchange market, they realised a gain of 1 080 000 Frw and intend to uniquely share it in the ratio $2:3:4$ respectively. The task is to find the share of Mugenzi, as follows:

Mugenzi's share

$$= \frac{3}{2+3+4} \text{ of } 1\ 080\ 000 \text{ FRW}$$

$$= \frac{3}{9} \times 1\ 080\ 000 \text{ FRW}$$

$$= 360\ 000 \text{ FRW}$$

Lesson 2: Direct and indirect proportion

a) Learning objective:

Compare two quantities by using a given proportion

b) Teaching resources:

Student teacher's book, Reference books; Ruler, T-square, Manila paper, Scientific calculators.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they make a short revision on concepts of shares, ratio and proportion learnt in senior 1.

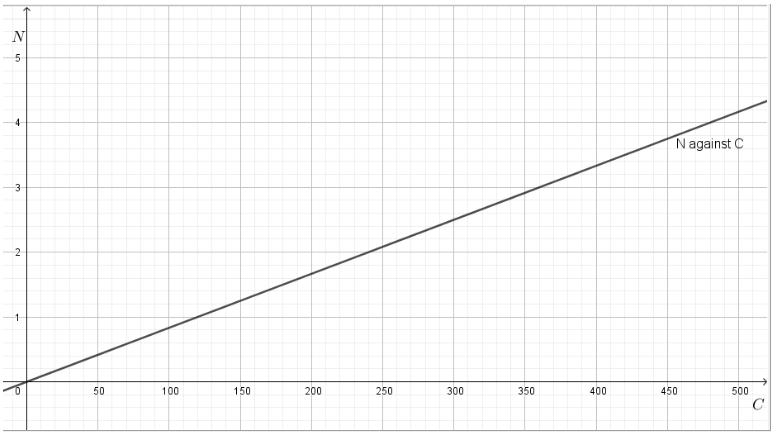
d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask student-teachers to use their books to discuss the **activity 3.1.2** and motivate the**m** to differentiate direct from indirect proportions, determine how to distribute objects respecting the direct or indirect proportions . Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts of distributing objects using direct or indirect proportions;

- Use different probing questions to guide student-teachers to explore examples and the content related to direct or indirect proportions given in the student-teacher's book;
- Guide student-teachers to perform individually **application activity 3.1.2** to assess their competences.

Answers for activity 3.1.2

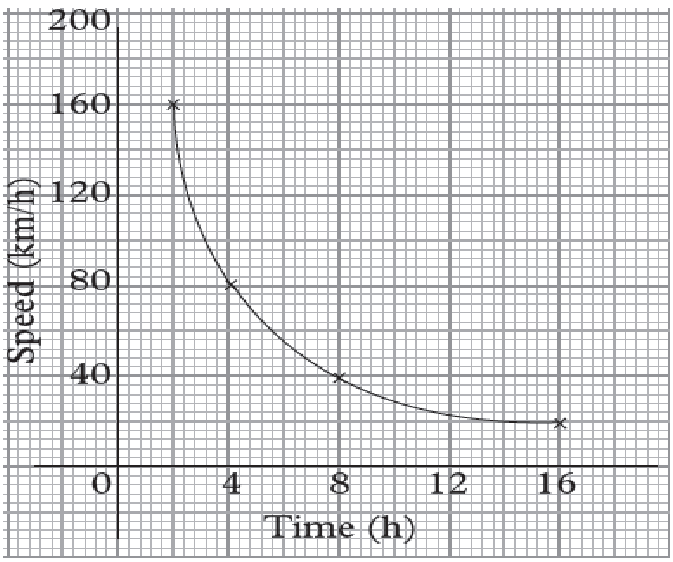
1) i) The graph of the number of pens (N) against cost (C)



ii) The graph of (N) against C is a straight line passing through the origin

2) i) We notice that when the speed is doubled the time is decreasing

ii)



iii). From the above graph you notice that the speed increases in the ratio $1:2$, the time decreases in the ratio $2:1$ and vice versa. Thus the two quantities are said to be **inversely proportional**.

It means that the car is decelerating with acceleration:

$$a=\frac{\Delta v}{\Delta t}=\frac{v_f-v_i}{t_f-t_i}=\frac{20-160}{16-2}=-10\ kmh^{-2}$$

e) Application activities

Answers for application activity 3.1.2

1) Expand the products $(x+9)(y-2)=(x+3)(y-6)$

$$xy-2x+9y-18=xy-6x+3y-18$$
$$9y-2x=3y-6x$$
$$4x=-6y$$
$$\frac{4x}{4y}=\frac{-6y}{4y}$$
$$\frac{x}{y}=\frac{-3}{2}\quad or\quad x:y=-3:2$$

2) $(ap+aq):(bp+bq)$

a). $a(p+q):b(p+q)$

$$a:\text{b}\quad \text{or}\quad \frac{a}{b}$$

b). $(p^2-q^2):(p+q)$

$$(p-q)(\text{p}+\text{q}):\text{p}+\text{q}$$
$$p-q:1\quad or\quad \frac{p-q}{1}$$

3) $If\quad b\propto c^2\ then\ b=k\,c^2$

$$72=k\times 12^2$$
$$72=144k$$
$$k=\frac{72}{144}$$
$$k=\frac{1}{2}$$

4)

a) $mn = k$ or $m = \frac{k}{n}$, where k is the constant of variation, $k \neq 0$

$25 = \frac{k}{2} \Rightarrow k = 50$

b) $p \times q = k \Rightarrow 16 \times 4 = k$

$k = 64$.

Lesson 3: Calculation of proportional change using multiplier and compound proportional change or continued proportions

a) Learning objective:

Solve problems involving multiplier and compound proportional change.

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding on concepts of shares, ratio, multiplier and proportional change learnt in senior 2, in senior 3 and 2 previous lessons;

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask student-teachers to use their books to discuss the activity 3.2 and motivate them to determine the proportional change using multiplier. Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize the concepts of proportional change;
- Use different probing questions to guide student-teachers to explore examples and the content related to Proportional change using multiplier and compound proportional change or continued proportions given in the student-teacher's book;

- Guide student-teachers to perform individually **application activity 3.2** to assess their competences.

Answers for activity 3.2

1)

- Multiplier is a quantity by which a given number is to be multiplied.
- If the shirt is at 20% discount, then the selling price is $100\% - 20\% = 80\%$ of the original price.
- The 80% converted to fraction gives $\frac{80}{100} = 0.80$
- 0.80 is the multiplier price of the shirt.

2) It is evident that the shirt has been sold at a reduced price compared to the initial buying price. The marked price is reduced proportional by 10% which translates to 50FRW. Therefore the customer bought the shirt at 50 FRW less. It means that the new price was $500FRW - 50FRW = 450FRW$

3) **1st alternative:**

If 3 people take 3 days to cultivate 2 acres, the number of the people is not increased but days are increased to 5 days.

So, ratio of acres to days is $\frac{2}{3}$ *or* $2:3$

$For\ 5\ days,\ we\ have\ \frac{2}{3} \times 5 = 3.33\ acres$

If days are increased to 5, then 3.33 acres are cultivated.

2nd alternative:

$3\,people\ in\ 3days\ cultive\ 2acres$

$3\,people\ in\ 1day\ cultivate\ \frac{2}{3}acres$

$3\,people\ in\ 5days\ cultivate\ \frac{2}{3} \times 5acres = 3.33acres$

e) Application activities

Answers for application activity 3.2

1) A 45% decrease means the final percentage for the quantity will be $100\% - 45\% = 55\%$

   55% as a decimal $\frac{55}{100} = 0.55$

   0.55 is the multiplier.

2) The salary increase was $\frac{20}{100}$ *of* $15000 FRW$

   $= \frac{20}{100} \times 15000$

   $= 3000 FRW$

   The new salary

   $= 15\ 000\ FRW + 3\ 000\ FRW$

   $= 18\ 000\ FRW$

   Tonnes processed in $2004 = 800$

   Percentage decreased $= 30\%$

   $= 30\%$ *of* 800 *tonnes*

   Amount decreased $= \frac{30}{100} \times 800$

   $= 240$ *tones*

   Amount produced in 2005:

   $= (800 - 240)$ *tonnes*

   $= 560$ *tonnes*

3) Five men will use $\frac{4 \times 10}{5} = 8$ *days*

Lesson 4: Problems involving direct and indirect proportions

a) Learning objective

Solve problems involving direct and indirect proportions

b) Teaching resources

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

c) Prerequisites / Revision / Introduction

Student teachers will learn better this lesson if they have a good understanding on concepts of shares, ratio and proportion learnt in senior1, multiplier and proportional change learnt in senior 2 , senior 3 and the previous lessons.

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask student-teachers to use their books to discuss the activity 3.3 and motivate them to solve problems involving direct and indirect proportions. Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize how to solve a problem involving direct or indirect proportion;
- Use different probing questions to guide student-teachers to explore examples and the content related to how to solve a problem involving direct or indirect proportion given in the student-teacher's book;
- Guide student-teachers to perform individually **application activity 3.3** to assess their competences.

Answers for activity 3.3

1) $F = k \times x \qquad \Rightarrow 6 = 4k$

$$\Rightarrow k = \frac{6}{4} \qquad \Rightarrow F = \frac{6}{4} \times 5 \qquad \Rightarrow F = \frac{30}{4}$$

2) $A = k \times B^2 \qquad \Rightarrow 10 = 4k$

$$\Rightarrow k = \frac{10}{4} = \frac{5}{2} \qquad \Rightarrow A = \frac{5}{2} \times B^2 \qquad \Rightarrow A = \frac{5}{2} \times 3^2 = \frac{45}{2}$$

The tutor harmonizes findings of student-teachers and check if the following steps are included

i). Understand the problem: Think what information is given and what information is required ii). Decide on a strategy: List the strategies with which you think the solution can be found

iii). Apply the strategy: Find the solution using the strategy you have chosen

iv). Look back:

- Have you verified your solution?
- Are there other solutions?
- Can you solve a simpler problem?

Have you answered the question as it was initially stated?

e) Application activities

Answers for application activity 3.3

1) $A = kP^2 \quad \Rightarrow \quad 4 = 8^2 k \quad \Rightarrow \quad k = \frac{4}{64} = \frac{1}{16}$

$A = \frac{1}{16} P^2 \quad \Rightarrow \quad P = \sqrt{16A} = 4\sqrt{A}$

2) a) $D = \frac{k}{A} \Rightarrow 120 = \frac{k}{40} \Rightarrow k = 4800 \quad \Rightarrow \quad A = \frac{k}{D} = \frac{4800}{D}$

b) $A = \frac{4800}{150} = 32 cm^2$

c) $D = \frac{k}{A} = \frac{4800}{60} = 80\ cm^2$

3.6. Summary of the unit

- **A ratio**: It is a relation that compares two or more quantities of the same kind, such as lengths, using division giving one quantity as a fraction of another.
- **Simplifying ratios**: This is where two quantities of a ratio may be multiplied or divided by the same number without changing the value ratio.
- **Sharing** :To share a quantity into two parts in the ratio $a:b$ is where the quantity is split into $a+b$ equal parts and the required parts become $\frac{a}{a+b}$ and $\frac{b}{a+b}$ of the quantity

- If two ratios have the **same value** then they are **equivalent**, even though they may look different.
- **A proportion:** It is a mathematical statement of the equality of two ratios
- **A direct proportion**: It is the proportion in which two quantities are such that, when one quantity increases through a particular ratio, the other quantity increases in the same ratio and vice versa.
- **An inverse proportion**: It is the proportion in which two quantities are such that, when one quantity increases in the ratio $\frac{a}{b}$, the other quantity decreases in the ratio $\frac{b}{a}$
- **A decreasing multiplier** is a factor that reduces the proportion of a given quantity. To calculate the new price, we proceed as

  $New\ price = initial\ price \times multiplier$, where,

  $Multiplier = \frac{(100-x)}{100}$ and x is the percentage decrease on the cost price.
- **An increasing multiplier** is a factor that increases the proportion of a given quantity. To calculate the new price, we proceed as

  $\text{New price} = \text{initial} \times multiplier$, where

  $Multiplier = \frac{(100+x)}{100}$ and x is the percentage increase on the cost price.

3.7. Additional information for tutors

It is important for the tutors to understand that student teachers have to be equipped with skills of sharing. For this reason, student teachers have to be engaged in many practical activities of sharing to understand these concepts very easily.

3.8. End unit assessment

1) a). $F = \frac{k}{d^2} \Rightarrow 0.006 = \frac{k}{2^2} \Rightarrow k = 0.05N$

$$F = \frac{0.024}{d^2}$$

b). $F = \frac{0.024}{(2.5)^2} = 0.00384N$

c). $d^2 = \frac{0.024}{0.001} \Rightarrow \quad d = 4.90m$

2) a). $2:5 = 4:x \quad i.e \frac{2}{5} = \frac{4}{x} \Rightarrow 2x = 20 \Rightarrow x = 10$

b). $8:4 = 20:4x \quad i.e \quad \frac{8}{4} = \frac{20}{4x} \Rightarrow 32x = 80 \Rightarrow x = \frac{80}{32}$

c). $4:3x = 45:63 \; i.e \frac{4}{3x} = \frac{45}{63} \Rightarrow 4 \times 63 = 45 \times 3x \Rightarrow 252 = 135x \Rightarrow x = \frac{252}{135}$

3) Express each of the following in lowest terms:

a) $-16xy:32y \Rightarrow -x:2$

b) $(8k-2k):2k \Rightarrow 6k:2k \Rightarrow 3k:k$

3.9. Additional activities

3.9.1. Remedial activities (Questions and answers)

1) Given the ratio $a:b = 5:2$ and ratio $b:c = 3:4$ find the ratio $a:b:c$

Answer:

Rewrite the ratio $a:b:$c as follows

$a:b:c$

$5:2$

$3:4$

Since b is a quantity in both ratios, we make the value of b the same in both ratios so as to join the two ratios as one.

We do this by multiplying the first ratio with the value of b in the second ratio and then we multiply the second ratio with the value of b in the first ratio

$a:b:c$

$(5:2)\times 3$

$2\times(3:4)$

$a:b:c$

$15:6$

$6:8$

Since b is the same, $\begin{matrix} a:b:c \\ 15:6:8 \end{matrix}$

3.9.2. Consolidation activities

1) 150 cm^3 of water are contained in 15 litres of a chemical. Find the simplest ratio of water to the chemical.

Answer:

Since quantities of a ratio must be of the same unit, we need to convert liters to cubic centimeters

$1liter = 1000cm^3$

$15liters = 15\times 1000cm^3 = 15000cm^3$

$\text{Re}quired\ ratio = \frac{150cm^3}{15000cm^3} = \frac{1}{100}$

The ratio is $1:100$

3.9.3. Extended activities

The following table shows the time taken to cover a distance of 120*km* at various speeds

Speed, $x(km/h)$	20	30	40	60	80	120
Time, t(h)	6	4	3	2	1.5	1.0

a) Draw a graph of speed against time taken

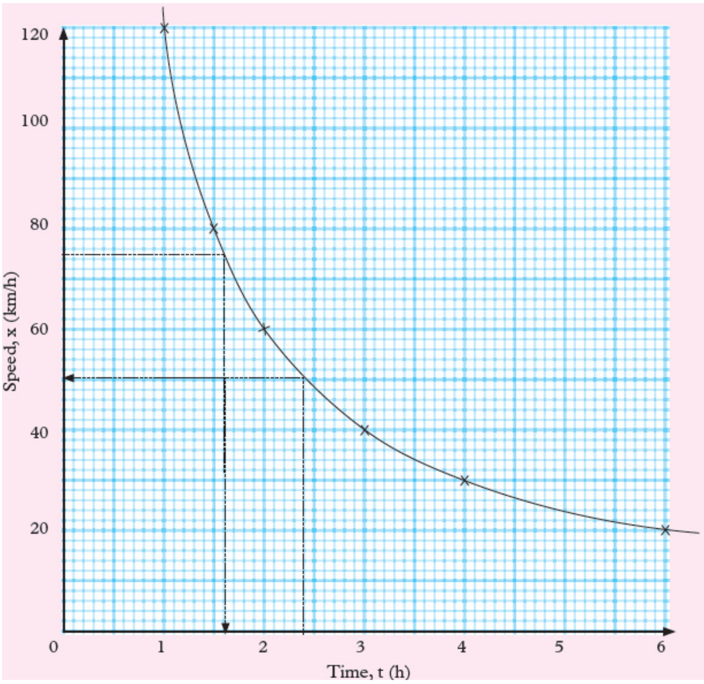
b) Use your graph to determine:

i) The time taken to cover the same distance at a speed of $75km/h$

ii) The speed required to cover the same distance in $2.4h$

Answers:

a)



b) From the graph:

i) The time taken to cover the same distance at a speed of 75 km/h is 1.6 h.

ii) The speed required to cover the same distance in 2.4 h is 50 km/h.

UNIT: 4

PROPOSITIONAL AND PREDICATE LOGIC

4.1. Key unit competence

Use Mathematical logic as a tool of reason and argumentation in daily situation.

4.2. Prerequisite

Student-teachers will perform well in this unit if they have a good background on types of sentences as learnt in English grammar; and Set theory as in unit two of this syllabus.

4.3. Cross-cutting issues to be addressed

a) Inclusive education

Promote the participation of all student-teachers while teaching)

b) Peace and value Education

During group activities, the tutor will encourage student-teachers to help each other and to respect opinions of colleagues.

c) Gender

During group activities try to form heterogeneous groups (with boys and girls) or when student-teachers start to present their findings encourage both (boys and girls) to present.

4.4. Guidance on introductory activity

- Form small groups of student-teachers and guide them to work on the introductory activity 4.
- Visit all groups to provide pieces of advice where necessary.
- After a given time, invite student-teachers to present their findings and harmonize them.
- Use discussions to help student-teachers differentiate an argument which is valid from an argument which is not valid and arouse student-teachers' curiosity on the content for unit 4.

Answers for introductory activity

1)

- If you give a child an orange and another child an orange. Children got an orange.
- Kigali is in Rwanda and $2^n > 0,\ n \in \mathbb{Z}$
- If you do not attend class, then either you read a book or you will not pass the exam.
- All dogs are meat eaters, Cat is a meat eater, therefore, cat is a dog.
- Some investors are wealthy. All wealthy people are happy, therefore, some investors are happy.

2)

- Some rules are unfair. All unfair rules should be eliminated, therefore, . . .
- No team which plays in X- stadium has ever won the Super cup. Some teams that wear red uniforms have won the Super cup, therefore, . . .

4.5. List of lessons or sub-headings

#	Lesson title	Learning objectives	Periods
0	Introduction activity	To arouse the curiosity of student teachers on the content of unit 4	1
1	Simple statement and compound statements	Give example of a logical statement Convert into logical formula composite propositions and vice versa.	1
2	Truth tables	– Draw the truth table of a proposition – Draw the truth table of a composite proposition.	1
3	Negation	Use correctly negation of logical statements in daily life	1
4	Conjunction	Use correctly conjunction in logical statements in daily life	1
5	Disjunction	Use correctly disjunction in logical statements of daily life	1
6	Conditional statement	Use correctly conditional statements in daily life	1

7	Bi-conditional statements	Use correctly bi-conditional statements in daily life statements	1
8	Tautologies and Contradictions	Show that a given logical statement is a tautology or a contradiction	1
9	Predicates	Use correctly predicates, propositions, connectives and quantifiers in daily life	1
10	Quantifiers	Use correctly quantifiers in logical statements of daily life	1
11	Negation of quantifiers	Use correctly the negation of quantifiers in logical statements from ordinal and official dialogue.	1
End unit assessment			1
Total number of periods			**15**

Lesson 1: Simple statement and compound statements

a) Learning objective:

- Give example of a logical statement;
- Convert into logical formula the composite propositions and vice versa.

b) Teaching resources:

Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have back ground on the correct forms of sentences with examples as learnt in English.

d) Learning activities

- Organize the student-teachers into small groups and introduce the activity to be done;
- Ask student-teachers to use their books to discuss the activity 4.1.1 and motivate them to differentiate simple statement from compound statement. Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings and facilitate them to summarize how to form a compound statement;

- Use different probing questions to guide student-teachers to explore examples and the content related to **simple statement** and compound statements given in the student-teacher's book and help them to define the **truth-value of a statement.**
- Guide student-teachers to perform individually **application activity 4.1.1** to assess their competences.

Answers for activity 4.1.1

1) T
2) F
3) Neither true nor false
4) F
5) Neither true nor false
6) Neither true nor false
7) T

e) Application activities

Answers for application activity 4.1

1) Answers of question 1:
   a). A statement, whose true value is true.
   b). Not a statement. It is an imperative sentence.
   c). A statement, whose true value is false
   d). A statement, whose true value is true.
   e). A statement, whose true value is true.
   f). Not a statement. It is an exclamative sentence.
2) a). F
   b). T
   c). T
   d). F
   e). T

Lesson 2: Truth tables

a) Learning objective:

- Draw the truth table of a proposition
- Draw the truth table of a composite proposition.

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding concept of logical statement as learnt in 1st lesson of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.1.2** from student-teacher's book and introduce the concept logical statement.
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings,
- Use different probing questions to guide student-teachers to explore examples and the content related to values of a compound statement and then to complete the truth table**.** Student-teachers can also give their own examples of statements.
- Guide student-teachers to perform individually **application activity 4.1.2** to assess their competences.

Answers for activity 4.1.2

a) Possibilities for the truth-values of *p* and of q.

b) Using a table,

i)

p	q
T	T
T	F
F	T
F	F

ii)

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

There are 8 possibilities, for the triples of truth-values of three statements:

$$\{(T,T,T),(T,T,F),(T,F,T),(T,F,F),(F,T,T),(F,T,F),(F,F,T),(F,F,F)\}.$$

e) Application activities

Answers for application activity 4.1.2

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Lesson 3: Logical connectives "Negation"

a) Learning objective:

Use correctly negation of logical statements in daily life

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good back ground of negative sentence, the concept of logical statement and truth table as learnt in the previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.2.1** from student-teacher's book and introduce the concept of negation of a statement,
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings,
- Use different probing questions to guide student-teachers to explore examples and the content related to the negation of a simple and compound statement and related truth table**.** Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.2.1** to assess their competences.

e) Application activities

Answers for activity 4.2.1

1)

a). Today is not raining.

b). Sky is not blue

c). My native country is not Rwanda.

d). Benimana is not smart and not healthy.

2)

p	q	r	$\neg p$	$\neg q$	$\neg r$
T	T	T	F	F	F
T	T	F	F	F	T
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	T
F	f	F	T	T	F
F	F	F	T	T	T

Lesson 4: Logical connectives "Conjunction"

a) Learning objective

Use correctly conjunction of logical statements in daily life

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding in the concept of compound statement, truth values and truth table as learnt in the 1st and 2nd lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.2.2** from student-teacher's book and introduce the concept of conjunction of statements,
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings,
- Use different probing questions to guide student-teachers to explore examples and the content related to the conjunction of two statements, its negation and related truth table. Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.2.2** to assess their competences.

Answers for activity 4.2.2

i). True

ii). False

iii). False

iv). False.

v). True

vi). False

vii). False (weight is different from mass)

e) Application activities

Answers for application activity 4.2.2

1) If p stands for the statement " *It is cold*" and q stands for the statement " *It is raining*", then what does $\neg q \wedge \neg q$ stands for " *It is not cold and it is not raining*"

Truth table of $\neg q \wedge \neg q$

p	q	$\neg p$	$\neg q$	$\neg q \wedge \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Lesson 5: Logical connectives "Disjunction"

a) Learning objective:

Use correctly disjunction in logical statements of daily life

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good background on concept of compound statement, truth values and truth table as learnt in previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.2.3** from student-teacher's book and introduce the concept of disjunction of statements,

- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings,
- Use different probing questions to guide student-teachers to explore examples and the content related to the disjunction of two statements, its negation and related truth table. Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.2.3** to assess their competences.

Answers for activity 4.2.3

1. True
2. False
3. True
4. True
5. True
6. True

e) Application activities

Answers for application activity 4.2.3

1) Translation in symbolic form
   i). Let p: Bwenge reads News Paper; q: Bwenge reads Mathematics book, Bwenge reads News Paper or Mathematics book is translated symbolically as $p \vee q$
   ii). Let p: Rwema is a student-teacher, q: Rwema is a book seller, thus Rwema is a student-teacher or not a book seller is translated symbolically as $p \vee \neg q$
2) If p is a false statement, and q is a true statement.
   a). The truth-value of the compound statement $\neg p \vee q$ is true
   b). The truth-value of the compound statement $p \vee \neg q$ is false
   c). The truth-value of the compound statement $p \vee q$ is true; the truth-value of the compound statement $\neg p \vee \neg q$ is true.

3)

				a	b	c
p	q	$\neg p$	$\neg q$	$p \vee q$	$p \vee \neg q$	$p \wedge (p \vee \neg q)$
T	T	F	F	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Lesson 6: Conditional statement

a) Learning objective:

Use correctly conditional statements in daily life

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on concept of truth values, truth table, negation, conjunction and disjunction connectives as learnt in the previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.2.4** from student-teacher's book and introduce the concept of conditional statements,
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings,
- Use different probing questions to guide student-teachers to explore examples and the content related to conditional statements and related truth table. Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.2.4** to assess their competences.

Answers for activity 4.2.4

There several answers. Some of them are:

1) I will go to school whenever you buy me a pen
2) The earth is flat implies that the mars is flat
3) You will be a member of our volleyball team if you are tall
4) You buy me these shoes unless I will not go with you.
5) Whenever you pay school fees, you will not get your school report.

e) Application activities

Answers for activity 4.2.4

1) Let p: Mico is fat

   q: Mico is happy

   a). If Mico is fat then she is happy is written in symbolic form as $p \Rightarrow q$

   b). Mico is unhappy implies that Mico is thin is written in symbolic form as $\neg q \Rightarrow \neg p$.

2)

   a) Let p be “n is prime”, q be “n is odd” and r be “n is 2”. We have $p \to (q \vee r)$

   b) Let p be “x is non negative”, q be “x is positive” and r be “x is 0”. We have $p \to (q \vee r)$

   c) Let p be “Tom is Ann’s father”, q be “Jim is her uncle” and r be “Sue is her aunt”. We have $p \to (q \wedge r)$

Lesson 7: Bi-conditional statement

a) Learning objective:

Use correctly bi-conditional statements in daily life statements.

b) Teaching resources:

T-square, ruler, Student-teacher’s book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they refer to the concepts of conjunction connective and conditional statement as learnt in the previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.2.5** from student-teacher's book and introduce the concept of bi-conditional statements,
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, insisting on the use of "if close", and "if …. and only if ….” expressions;
- Use different probing questions to guide student-teachers to explore examples and the content related to bi-conditional statements and related truth table**.** Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.2.5** to assess their competences.

Answers for activity 4.2.5

1) a). $r \Rightarrow s$ is False b). $s \Rightarrow r$ is True c). $(r \Rightarrow s) \wedge (s \Rightarrow r)$ is False

2) a). $r \Rightarrow s$ is True b) $s \Rightarrow r$ is True c). $(r \Rightarrow s) \wedge (s \Rightarrow r)$ is True

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

e) Application activities

Answers for application activity 4.2.5

1) If r is a false statement, s a true statement, then

   a). the truth-value of the compound statement $(\neg r) \Leftrightarrow s$ is **True**

   b). the truth-value of the compound statement $r \Leftrightarrow (\neg s)$ is **True**

   c). the truth-value of the compound statement $r \Leftrightarrow s$ is **False**

   d). the truth-value of the compound statement $\neg(r \Leftrightarrow (\neg s))$ is **False**

2) Construct the truth table for

a) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$

p	q	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

b) $p \leftrightarrow q$ and $(\neg p \vee q) \wedge (\neg q \vee p)$

p	q	$p \Leftrightarrow q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \wedge (\neg q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

c) $\neg(p \leftrightarrow q)$ and $(p \vee q) \wedge \neg(p \wedge q)$

p	q	$\neg(p \Leftrightarrow q)$	$p \vee q$	$\neg(q \wedge p)$	$(p \vee q) \wedge \neg(q \wedge p)$
T	T	F	T	F	F
T	F	T	T	T	T
F	T	T	T	T	T
F	F	F	F	T	F

From these results, help student-teachers to note that :

a) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are equivalent

b) $p \leftrightarrow q$ and $(\neg p \vee q) \wedge (\neg q \vee p)$ are equivalent

c) $\neg(p \leftrightarrow q)$ and $(p \vee q) \wedge \neg(p \wedge q)$ are equivalent

d) $\neg(p \leftrightarrow q)$ and $(p \wedge \neg q) \vee (\neg p \wedge q)$ are equivalent

Lesson 8: Tautologies and Contradictions

a) Learning objective:

Show that a given logical statement is a tautology or a contradiction

b) Teaching resources:

T-square, ruler, Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will learn better this lesson if they have a good understanding on all logical connectives learnt in the previous lessons.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.2.6** from student-teacher's book and introduce the concepts "tautology" and "contradiction",
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, insisting on the compound statement that is always true and the compound statement that is always **false** regardless of the truth values of the individual statements substituted for statement variables.
- Use different probing questions to guide student-teachers to explore examples and the content related to tautology and contradiction and related truth tabl**e.** Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.2.6** to assess their competences.

Answers for activity 4.2.6

1.

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

2.

p	$\neg p$	$p \wedge \neg p$
T	F	**F**
F	T	**F**

3.

p	q	$\neg p$	$p \wedge q$	$\neg p \wedge (p \wedge q)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

4.

p	q	$\neg(p \wedge q)$	$p \vee q$	$\neg(p \wedge q) \vee (p \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	T

e) Application activities

Answers for activity 4.2.6

1.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$p \wedge \neg(p \wedge q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	F
F	F	F	T	F

$p \wedge \neg(p \wedge q)$ is neither tautology nor contradiction

2.

q	r	$\neg q$	$q \wedge r$	$\neg q \wedge (q \wedge r)$
T	T	F	T	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

$\neg q \wedge (q \wedge r)$ is a contradiction

Lesson 9: Predicates

a) Learning objective:

Use correctly logical statements, propositions, connectives and quantifiers in daily life

b) Teaching resources:

Student-teacher's book and other Reference textbooks to facilitate research

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they have a good back ground on concept of logical proposition and truth values learnt in the 1^{st} lesson of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.3.1** from student-teacher's book and introduce the concept "predicate",
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, insisting on definition and clear examples of predicates;
- Use different probing questions to guide student-teachers to explore examples and the content related to predicates **.** Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.3.1** to assess their competences.

Answers for activity 4.3.1

1. Neither
2. Neither
3. True
4. Neither
5. False
6. Neither
7. False
8. False

e) Application activities

Answers for application activity 4.3.1

1) Truth set is $\{2,4,6,8,10\}$
2) a). true b). false

Lesson 10: Quantifiers

a) Learning objective:

Use correctly quantifiers in logical statements of daily life.

b) Teaching resources:

Student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they have a good concept of predicates as learnt in the previous lesson of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.3.2** from student-teacher's book and lead them to use terms that can introduce the concept "quantifiers",
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, insisting on definition and clear examples of the use of two types of quantifiers;
- Use different probing questions to guide student-teachers to explore examples and the content related to quantifiers. Student-teachers can also give their own examples.
- Guide student-teachers to perform individually **application activity 4.3.2** to assess their competences.

Answers for activity 4.3.2

1. False. For example $\frac{1}{2}$ is a real number but $\frac{1}{\frac{1}{2}} = 2 > \frac{1}{2}$
2. True
3. False. For example Ostrich is a bird but it cannot fly.
4. True
5. False. February has either 28 or 29 days
6. False. For example 1 and 2 are natural numbers and $1+1>1,\ 2+1>2$

e) Application activities

Answers for application activity 4.3.2

1) a). $(\exists x)\left[p(x)\wedge q(x)\right]$, where $p(x)$ - x cried out for help and $q(x)$ -x called the police

   b). $(\forall x)\left[\neg p(x)\right]$, where $p(x)$ - x can ignore her.

2) a). True, for example for $x=9$, $r(9)=9-7=2$

   b). False, counter example: $y=10$

Lesson 11: Negation of quantifiers

a) Learning objective:

Use correctly the negation of quantifiers in logical statements from ordinal and official dialogue.

b) Teaching resources:

T-square, ruler, student-teacher's book and other Reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this lesson if they have well learnt the concept of quantifiers in the previous lessons of this unit.

d) Learning activities

- Organize the student-teachers into small groups and ask them to attempt the **Activity 4.3.3** from student-teacher's book and lead them to be able to negate quantifiers using some examples;
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, guiding them to the negation for the two two types of quantifiers they learnt: *"Negating a universally quantified formula changes it into an existentially quantified formula and vice-versa with the part of the formula after the quantifier becoming negated."*
- Use different probing questions to guide student-teachers to explore examples and the content related to negation of quantifier**s.** Student-teachers can also give their own examples.

Answers for activity 4.3.3

1) Some grapefruit are not pink
2) No celebrities are modest.
3) Some people weigh more than five hundred kg.
4) No one is more than ten metres tall.
5) Some snakes are not poisonous.
6) No mammals can stay under water for two days without surfacing for air.
7) Some birds cannot fly

e) Application activities

Answers for application activity 4.3.3

1) No student is math major (or All student-teachers are not math major)

   $\forall x\, p(x)$, where $p(x) \sim x$ is not math major

2) Some real numbers are not positive, negative or zero

   $\exists x\, p(x)$, where $p(x) \sim x$ is not positive, negative or zero

3) Some good boys does not do fine

   $\exists x\, p(x)$, where $p(x) \sim x$ is does not do fine

4) All desk in our classroom are not broken (or No desk in our classroom is broken)

   $\forall x\, p(x)$, where $p(x) \sim x$ is not broken

5) Some lockers must not be turned in by the last day of class

   $\exists x\, p(x)$, where $p(x) \sim x$ is not be turned in by the last day of class

6) Some haste do not make waste

   $\exists x\, p(x)$, where $p(x) \sim x$ does not make waste.

4.6. Summary of the unit

A proposition

A proposition (statement or verbal assertion) is a sentence which is either true or false but not both.

Truth table

We can always summarize **the truth values** of compound statement in a table called truth table.

If the compound statement contains n distinct components, we need to consider 2^n possible combinations of truth values in order to obtain the truth table.

Connectives

Given statements p and q we can combine them with various connectives. The most five useful logical connectives are negation, conjunction, disjunction, conditional and bi-conditional.

The negation of a statement by introducing the word "not" denoted by prefixing the statement has opposite truth value from the statement. It is denoted by $\neg p$ or $\bar{p}$ or $\sim p$.

The conjunction (AND) of two statements p and q is denoted $p \wedge q$. It has the truth value true whenever both p and q have the truth value true; otherwise it has the truth value false.

The disjunction (OR) of two statements p and q is denoted $p \vee q$. It has the truth value false only when p and q have truth value false, otherwise it has the truth value true.

The conditional statement $p \Rightarrow q$ (read $"p \, implies \, q"$) has the truth value false when q has the truth value false while p has truth value true, otherwise it has the truth value true.

The biconditional statement $p \Leftrightarrow q$, which we read "p if and only if q" or "p is equivalent to q" is true if both p and q have the same truth values and false if p and q have opposite truth values.

A tautology is a statement formula that is always true regardless of the truth values of the individual statements substituted for its statement variables.

A contradiction is a statement formula that is always false regardless of the truth values of the individual statements substituted for its statement variables.

A predicate or a declarative sentence is an open statement if:

It contains one or more variables, and It is not a statement, but It becomes a statement when the variables in it are replaced by certain allowable choices.

The quantifiers help decide the frequencies with which a predicate becomes true, whether it is satisfied by no element of a domain, or one element, or some elements, or all elements.

There are **two basic quantifiers**:

The existential quantifiers $\exists$ ("there exist"), and

The universal quantifier $\forall$ ("for all")

These quantifiers are negated as follows

$$\neg\left[\forall x p(x)\right] \equiv \exists x\left[\neg p(x)\right]$$

$$\neg\left[\exists x p(x)\right] \equiv \forall x\left[\neg p(x)\right]$$

4.7. Additional information for the tutor

Converse, inverse and contrapositive statements

These statements are related to the conditional statement as follows: If you have the statement $P \Rightarrow Q$,

The Converse is **Q**$\Rightarrow$P: When the consequent becomes the antecedent

The Inverse is ~P$\Rightarrow$**~Q: the negation of P is an antecedent and the negation of Q is the consequent**

The contrapositive is ~Q$\Rightarrow$~P: It is the converse of the inverse.

These statements are important because they are used in creating valid arguments. It can be shown that if a statement is true, then its contrapositive is always true, and if a statement is false, then its contrapositive is also false.

Examples

1) Write the converse, inverse, and contrapositive of the true conditional statement below. Determine whether each of the statements is true or false.

   "*If it is a laptop, then it is a computer*"

Answer:

Let P = It is a laptop, and Q = It is a computer

$P \Rightarrow Q$ the given conditional statement,

The converse is $Q \Rightarrow P$: It is a computer, then it is a laptop (**False**)

The inverse is $\neg P \Rightarrow \neg Q$: It is not a laptop, then it is not a computer (**False**)

The contrapositive is $\neg Q \Rightarrow \neg P$: If it is not a computer, then it is not a laptop (**True**).

2) Write the converse, the inverse and the contrapositive of the false conditional statement below. Determine whether each of the statements is true or false.

"If x is an even number, then the last digit of x is 2".

Answer:

Let P = x is an even number, Q = the last digit of x is 2

$P \Rightarrow Q$ the given conditional statement,

The **converse** is $Q \Rightarrow P$: If the last digit of x is 2, then x is an even number (**true**),

The *inverse* is $\neg P \Rightarrow \neg Q$: If x is not an even number, then the last digit of x is not 2 (**True**)

The *contrapositive* is $\neg Q \Rightarrow \neg P$: If the last digit of x is not 2, then x is not an even number (**False**).

3) Write the converse, inverse, and contrapositive of the true conditional statement below. Determine whether each of the statements is true or false.

"If two lines are perpendicular, then the two lines form a right angle"

Answer:

Let P = Two lines are perpendicular, Q = two lines form a right angle.

$P \Rightarrow Q$ the given conditional statement,

Converse is $Q \Rightarrow P$: If two lines form a right angle, then two lines are perpendicular (**True**).

Inverse is $\neg P \Rightarrow \neg Q$: If two lines are not perpendicular, then two lines do not form a right angle (**True**)

Contrapositive is $\neg Q \Rightarrow \neg P$: If two lines do not form a right angle, then two lines are not perpendicular (**True**).

The true value of the conditional statement is deduced from the definition that $P \Rightarrow Q$ has the same meaning as $\neg[P \wedge (\neg Q)]$

P	Q	$\neg$P	$\neg$Q	$P \wedge (\neg Q)$	$\neg[P \wedge (\neg Q)]$
1	1	0	0	0	1
1	0	0	1	1	0
0	1	1	0	0	1
0	0	1	1	0	1

The conditional statement $P \Rightarrow Q$ is true unless P is true and Q is false.

Note: In the **converse**, $P \Rightarrow Q$ does not necessarily imply $Q \Rightarrow P$

In the **inverse**, $P \Rightarrow Q$ does not imply $\neg P \Rightarrow \neg Q$

4.8. End unit assessment

1) a) proposition b) not a proposition c) proposition
   d) proposition e) not a proposition
2) The negation is "Today is not Monday".
3) The conjunction of these propositions is the proposition "Today is Sunday and the moon is made of cheese"
4) The disjunction of these propositions is the proposition "Today is Sunday or the moon is made of cheese".
5) For a compound statement made from n statements, 2^n rows, not counting the top one, are needed to construct the truth table.
6)

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \Rightarrow q$	$p \Leftrightarrow q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	T	T

7) Make a truth table for this proposition:

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

8) The proposition is TRUE, since it is composed of two propositions each of which is FALSE.

9) The truth table is

p	q	r	$\neg p \to q$	$(\neg p \to q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

10) a) $(p \wedge \neg q) \wedge r$ b) $(p \wedge q) \to r$

11) To show that $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

Therefore, $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is a tautology.

12) To show that $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

Therefore, $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ is a tautology.

13) $p(1,2)$ is the proposition " 2 = 1 + 3" which is false. The statement $p(0,3)$ is the proposition " 3 = 0 + 3" which is true.

14) Let $q(x)$ be the predicate " x has seen a computer". Then the statement "Every student in this class has seen a computer" can be written as $\forall x\, q(x)$, where the universe of discourse consists of all student-teachers in this class. Also, this statement can be expressed as $\forall x(p(x) \rightarrow q(x))$, where $p(x)$ is the predicate " x is in this class" and the universe of discourse consists of all student-teachers.

15) $p(x)$ is not true for all real numbers x ; for instance, $p\left(\frac{1}{2}\right)$ is false. Thus, the proposition $\forall x\, p(x)$ is false.

16) a) The quantification $\forall x \forall y\, p(x,y)$ denotes the proposition "For every pair $x, y\, p(x,y)$ is TRUE". Clearly, this proposition is FALSE.

b). The quantification $\forall x \exists y\, p(x,y)$ denotes the proposition "For every x there is an y such that $p(x,y)$ is TRUE". Given a real number x, there is a real number y such that $x+y=3$, namely $y=3-x$. Hence, the proposition $\forall x \exists y\, p(x,y)$ is TRUE.

17) a). Let $l(x)$ be the predicate " x likes mathematics", where the universe of discourse is the set of student-teachers in this class. The original statement is $\forall x\, l(x)$ and its negation is $\exists x \neg l(x)$. In English, it reads "Some student in this class does not like mathematics".

b). Consider the predicates $p(z,y)$: "room z is in building y" and $q(x,z)$: "student x has been in room z ". Then the original statement is $\exists x \forall y \exists z(p(z,y) \wedge q(x,z))$. To form the negation, we change all the quantifiers and put the negation on the inside, then apply De Morgan's law. The negation is therefore $\forall x \exists y \forall z(\neg p(z,y) \vee \neg q(x,z))$, which is also equivalent to $\forall x \exists y \forall z(p(z,y) \rightarrow \neg q(x,z))$. In English, this could be read "For every student there is a building on the campus such that for every room in that building, the student has not been in that room".

4.9. Additional activities

4.9.1. Remedial activities

1) Find out which of the following sentences are statements and which are not.
   - i). The moon revolves around the sun.
   - ii). A triangle has four sides.
   - iii). $\sqrt{3}$ is an irrational number.
   - iv). What is your age?
   - v). Africa is a continent.
   - vi). $x+7=10$.
   - vii). The earth is a planet.
   - viii). $7+9>12$.
2) Write down the truth value of the following statements.
   - i). 1 is a prime number.
   - ii). Every square is a rectangle.
   - iii). All real numbers are rational.
   - iv). Karongi is in western province.
   - v). Uganda is north-west of Rwanda.

Answers:

1) To find out which of the following sentences are statements and which are not.
   - i) "The moon revolves around the sun". This is false and so it is a statement.
   - ii) A triangle has four sides. This is false and so it is a statement.
   - iii) $\sqrt{3}$ is an irrational number. This is true and so it is a statement.
   - iv) What is your age? This is not a statement as it is an interrogative sentence.
   - v) Africa is a continent. This is true and so it is a statement
   - vi) The earth is a planet. This is true and so it is a statement
   - vii) $7+9>12$. This is true and so it is a statement.

8) The truth values

   i). The statement has truth value False

   ii). True

   iii). False.

   iv). True.

   v). False.

4.9.2. Consolidation activities

If the statements p,q,r all have truth value "True" and u,v,w are false statements, which of the following are true and which are false?

1. $(r \vee w) \wedge (v \vee q)$
2. $(p \wedge q) \vee (u \wedge v)$
3. $\neg(q \vee u) \wedge \neg(v \vee w)$
4. $\neg(r \vee q) \vee \neg(\neg u \wedge v)$
5. $\neg q \vee r$
6. $\neg(q \vee u)$
7. $\neg u \vee p$
8. $\neg(u \vee v)$
9. $\neg[(q \vee p) \vee (\neg p \vee q)]$
10. $\neg[(\neg v \vee w) \vee (\neg w \vee v)]$
11. $[p \wedge (q \vee r)] \wedge [(p \wedge q) \vee (p \wedge r)]$
12. $\neg[u \wedge (\neg p \vee w)] \vee [(u \wedge \neg p) \vee (u \wedge w)]$
13. $u \Rightarrow (v \Rightarrow r)$

14. $(p \Rightarrow q) \Rightarrow w$

15. $[(u \Rightarrow v) \Rightarrow q] \Rightarrow w$

16. $[(q \Rightarrow w) \Rightarrow q] \Rightarrow w$

17. $u \Rightarrow (q \Rightarrow w)$

18. $[(u \Rightarrow p) \Rightarrow u] \Rightarrow u$

19. $[u \Rightarrow (v \Rightarrow w)] \Rightarrow [(u \Rightarrow v) \Rightarrow w]$

20. $\{p \Rightarrow (q \Rightarrow r) \equiv \neg u\} \Rightarrow \{x \Rightarrow [(p \wedge q) \Rightarrow r]\}$

21. $\{[u \Rightarrow (v \Rightarrow w)] \Rightarrow [(u \wedge v) \Rightarrow w]\} \equiv [(u \Rightarrow p) \Rightarrow (q \Rightarrow v)]$

22. $(q \Rightarrow p) \equiv (\neg p \Rightarrow \neg q)$

23. $\neg(p \wedge q) \equiv (\neg p \vee \neg q)$

24. $[p \Rightarrow (q \Rightarrow r)] \equiv [(p \wedge q) \Rightarrow r]$

4.9.3. Extended activities

Determine which of the pairs of statements in the following are logically equivalent

a). $\neg[(p \wedge q) \wedge r]$ and $\neg[p \wedge (q \wedge r)]$

b). $(p \vee q) \vee r$ and $p \vee (q \wedge r)$

c). $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$ and $(p \vee q) \wedge c$, where c is a contradictory

d). $[(\neg p \vee q)] \wedge [p \wedge (\neg q)]$ and $(p \wedge q) \vee t$, where t is a tautology

e). $(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$ and $[(\neg q) \wedge q] \vee t$, where t is a tautology

Answers:

a) Truth table of $\neg[(p \wedge q) \wedge r]$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$\neg[(p \wedge q) \wedge r]$
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

Truth table of $\neg[p \wedge (q \wedge r)]$

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$	$\neg[p \wedge (q \wedge r)]$
T	T	T	T	T	F
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	F	T
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

As the truth values in the last column of both tables are the same, the statement $\neg[(p \wedge q) \wedge r]$ is logically equivalent to $\neg[p \wedge (q \wedge r)]$

b) Truth table of $(p \vee q) \vee r$

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Truth table of $p \vee (q \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Since the truth values in the last column of both tables are not the same, the statement $(p \vee q) \vee r$ is not logically equivalent to $p \vee (q \wedge r)$.

c) Truth table of $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$

p	q	$\neg p$	$\neg q$	$\neg q \wedge p$	$p \wedge (\neg p)$	$[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

Truth table of $(p \vee q) \wedge c$, where c is a contradictory

p	q	c	$p \vee q$	$(p \vee q) \wedge c$
T	T	F	T	F
T	F	F	T	F
F	T	F	T	F
F	F	F	F	F

Getting that the truth values in the last column of both tables are the same, then statement $[(\neg q \wedge p)] \wedge [p \wedge (\neg p)]$ is logically equivalent to $(p \vee q) \wedge c$, where c is a contradictory.

d) Truth table of $[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \wedge (\neg q)$	$[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

Truth table of $(p \wedge q) \vee t$, where t is a tautology

p	q	t	$p \wedge q$	$(p \vee q) \wedge c$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	T	F	T

As the truth values in the last column of both tables are not the same, the statement $[(\neg p \wedge q)] \wedge [p \wedge (\neg q)]$ is not logically equivalent to $(p \wedge q) \vee t$, where t is a tautology.

e) Truth table of $(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \wedge (\neg q)$	$(\neg p) \vee [p \wedge (\neg q)]$	$(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T

Truth table of $[(\neg p) \wedge q] \vee t$, where t is a tautology

p	q	t	$\neg p$	$(\neg p) \wedge q$	$[(\neg p) \wedge q] \vee t$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	T	T	F	T

As the truth values in the last column of both tables are the same, the statement $(p \wedge q) \vee \{(\neg p) \vee [p \wedge (\neg q)]\}$ is logically equivalent to $[(\neg p) \wedge q] \vee t$, where t is a tautology.

UNIT: 5

OPERATION ON POLYNOMIALS

5.1. Key unit competence

Perform operations on polynomials and solve related problems.

5.2. Prerequisite

Student teachers will perform better in this unit if they have well learnt the following: Definition of polynomials and linear equation (S1 unit 3); factoring algebraic expressions (S2).

5.3. Cross-cutting issues to be addressed

a) Inclusive education

Promote education for all while teaching

b) Peace and value Education

Respect others view and thoughts during class discussions

c) Gender

Equal opportunity of boys and girls in the lesson participation and making groups

5.4. Guidance on introductory activity

- Form small groups of student teachers and guide them to work on the introductory activity 5;
- Walk around all groups to provide pieces of advice where necessary.
- After a given time invite student-teachers to present their findings and harmonize them;
- Basing on the answers provided for the question 3, use different probing questions to guide student-teachers to discover that they are going to deal with polynomials and their characteristics; and then try to arouse their curiosity on the content of the unit 5.

Answers for introductory activity 5

1) Lead student teachers to know that in the question 1, from their own researches to **Polynomial** comes from **poly**- meaning "**many**" and -**nomial** meaning "**term**".

2) i). $P = 2(\text{L} + \text{W}) = 2(4x)\left(x^2 + 3x + 2\right) = 8x^3 + 24x^2 + 16x$;

ii). $A = L \times W = 4x(x^2 + 3x + 2) = 4x^3 + 12x^2 + 8x$

3) $A = 3x\left(2x + 3\right) = 6x^2 + 9x$ or you find the sums of the areas of two rectangles: $A_1 = 3x(2x) = 6x^2$ *and* $A_2 = 3x(3) = 9x$ and their sum become: $A = 6x^2 + 9x$

5.5. List of lessons and sub-headings

No	Lesson title	Learning objectives	Periods
0	Introductory activity	Arouse the curiosity of student teachers on the content for this unit.	1
1.	Definition and comparison of polynomials	Define and compare polynomials	3
2.	Operations on polynomials	Perform operations on polynomials	3
3.	Factoring polynomials	Factor a given algebraic expression using appropriate methods.	3
4.	Expansion of polynomials	- Expand algebraic expressions by removing brackets and collecting like terms - Appreciate the role of numerical value of polynomial and algebraic identities in simplifying mathematical expressions.	3
5.	End assessment		2
Total periods			**15**

Lesson 1: Definition and comparison of polynomials

a) Learning objectives

Define and compare polynomials

b) Prerequisite

Student teachers will learn better this lesson if they have a good understanding on concepts of quadratic equation, linear equation studied in previous year (S3 unit 5)

c) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible, computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

d) Learning activities:

- Organize the student-teachers into small groups and ask them to attempt the **Activity 5.1** from student-teacher's book and lead them to use terms that can introduce the concept of polynomials learnt in S3;
- Move around to ensure all student-teachers in groups participate actively.
- Call upon groups to present their findings.
- Harmonize their findings, insisting on the characteristics of polynomials;
- Use different probing questions to guide student-teachers to explore examples and the content related to polynomials and their comparison**.** Student-teachers can also give their own examples.
- Guide student-teachers to note that two polynomials are equal if the corresponding monomials have the same coefficients. Example: $4x^3 - x + 3 = ax^3 + bx - c - 1$ if $a = 4; b = -1 \; and (-c-1) = 3$
- Guide student-teachers to perform individually **application activity 5.1** to assess their competences.

Answers for activity 5.1

The tutor guides the student teachers to discuss about the classification of polynomials basing on the number of terms.

The **degree** of a polynomial is determined by the **highest exponent** of that variable occurring in the polynomial.

For example, in $4x^3 - x + 3$ the degree is **3** which is the highest exponent of x.

We are ready to answer the activity 5.1

1) a) 1; b) 2; c) 3; d) 4; e) 5 number of terms for polynomials
2) a) 1; b) 1; c) 2; d) 3; e) 4 highest degree of polynomials
3) Consider the following table to classify the polynomials.
4) The following table identifies the types of polynomial.

Number of terms	**Type of polynomial**	**Example**
One term	Monomial	$2x$
Two terms	Binomial	$5x-1$
Three terms	Trinomial	$3a+7b+c$
....	....	....
n terms	Polynomial	$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ where $a_n \neq 0$ and the degree $n \geq 0$

e) Application activities

Answers for application activity

1) a) Polynomial known as quadric-nomial; b) binomial; c) monomial; d) binomial; e) binomial
2) As polynomials containing two or more variables are said to be **homogenous** if every term is of the same degree.

   For example, $xy^2 + x^2y + 3x^3$ *and* $3x + 2y - 4z$ are homogenous polynomials of degree 3 and 1 respectively.

a) Homogeneous degree 1 ;
b) non homogeneous degree 2;
c) Homogeneous degree 3;
d) Homogeneous degree2.

Lesson 2: Operations on polynomials

a) Learning objective:

Perform operations on polynomials

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible, computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction

Student teachers will learn better this lesson if they make a short revision on operation of mathematics expressions learnt in ordinary level.

d) Learning activities

- Organize the student-teachers into small groups and ask them to do the **Activity 5.2** from student-teacher's book and lead them to extend the addition, subtraction, division and multiplication of mathematics expressions to polynomials;
- Move around to ensure all student-teachers participate actively in their groups.
- Call upon groups with different working steps to present their findings.
- Harmonize their findings, insisting on how to deal with operations on polynomials;
- Use different probing questions to guide student-teachers to explore examples and the content related to addition, subtraction, division and multiplication of polynomials and related properties;
- Guide student-teachers to perform individually **application activity 5.2** to assess their competences.

Answers for activity 5.2

1) $\begin{cases} 3x^3 \\ -13x^2+x^2+5x^2 \\ 4x+3x-3x \\ -2+4+3 \end{cases} \Rightarrow 3x^3-7x^2+4x+5$

2) 2. if x=2 and y=3

   a). $2^2+3+1=8$;

   b). $3(2)^2+2(3)-3=15$

3) a). $4a+5+3a=7a+5$;

   b). $4a-5+3a=7a-5$;

   c). $4a-10-6a=-2a-10$;

   d). $4a+6a+10=10a+10$

4) The division is done as follows:

$$\begin{array}{r|l} & x+6 \\ \hline x+3 & x^2+9x+18 \\ & -(x^2+3x) \downarrow \\ \hline & 6x+18 \\ & -(6x+18) \\ \hline & 0 \end{array}$$

e) Application Activities

Answers for application activity 5.2

1) $a)-9-6+12+16=+13$; b) $54+27-18-8=55$

2) a) $x^2-xy-2x+xy-y^2-2y \Rightarrow x^2-y^2-2x-2y$;

   b) $6x^3-6x^2+3x-4x^2+4x-2 \Rightarrow 6x^3-10x^2+7x-2$

3) After using long division we get:

   a) quotient: $3x^2+4x+8$; b) remainder: 12

Lesson 3: Factoring polynomials

a) Learning objectives

Factorizing a given polynomial using appropriate methods

b) Prerequisites/Revision/Introduction

Student-teachers will perform better in this lesson if they refer to: Factorizing quadratic expressions (S2 unit 2); Quadratic equations by factorisation method (S3 unit 5).

c) Teaching resources

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators, Internet (in case the connection is available).

d) Learning activities:

- Organize the student-teachers into small groups and ask them to do the **Activity 5.3** from student-teacher's book and lead them to extend the factorization of mathematics expressions to the factorization of polynomials;
- Move around to ensure all student-teachers participate actively in their groups.
- Call upon groups with different working steps to present their findings.
- Harmonize their findings, insisting on different methods of factoring polynomials;
- Use different probing questions to guide student-teachers to explore examples and the content related to different methods of factoring, as a tutor you can refer to other reference books;
- Guide student-teachers to perform individually **application activity 5.3** to assess their competences.

Answers for activity 5.3

Factorization is writing an expression as the product of its prime factors. In factorization the operation is used depending on the polynomial to be factorized.

a). $2(a+b)$; b). $3r(1+2r)$; c). $xy(1+a)$; d) $3xy(3x+5y)$

e) Application activities

Answers for activity 5.3

a). $2(ab+2c)$; b). $-3b(b+3)$; c). $3x(x^2+2x-3)$;

Lesson 4: Expansion of polynomials

a) Learning objectives

- Expand algebraic expressions by removing brackets and collecting like terms
- Appreciate the role of numerical value of polynomial and algebraic identities in simplifying mathematical expressions.

b) Prerequisites/Revision/Introduction

Student-teachers will perform better in this lesson if they refer to methods of factorizing quadratic expressions (S2 unit 2); Quadratic equations by factorization method (S3 unit 5).

c) Teaching resources

Student teacher's book, internet and other Reference textbooks to facilitate research.

d) Learning activities

- Organize the student-teachers into small groups and ask them to do the **Activity 5.4** from student-teacher's book and lead them to extend the expansion of mathematics expressions to the expansion of polynomials;
- Move around to ensure all student-teachers participate actively in their groups.
- Call upon groups with different working steps to present their findings.
- Harmonize their findings, insisting on different properties applied when expanding polynomials;
- Use different probing questions to guide student-teachers to explore examples and the content related to the expansion of polynomials and the determination of the value of a polynomial when the values of unknown variables were given;
- Guide student-teachers to perform individually **application activity 5.4** to assess their competences.

Answers for activity 5.4

1) $x^2-2x+4x-8=x^2+2x-8$ now let us factorize this polynomial: $m+n=2 \text{ and } m\times n=-8$; let $m=4 \text{ and } n=-2$ so $(x+m)(x+n)=(x+4)(x-2)=x^2+2x-8$

2) i). $x^2+8x+16$; ii) x^2-2x+1; we see that all the final polynomials both have 3 terms for each. And

   - If a trinomial is a perfect square,
   - The first term must be a perfect square.
   - The last term must be a perfect square.
   - The middle term must be twice the product of numbers that were squared to give the first and last terms.

e) Application activities

Answers for application activity 5.4

1) i). $x^2+x+3x+3\Rightarrow x^2+4x+3$;

   ii). $6x+3-2x^2-x\Rightarrow -2x^2+5x+3$;

   iii). $8a^2-12a-6a+9\Rightarrow 8a^2-18a+9$;

   iv). $4b^2+24b-b-6\Rightarrow 4b^2+23b-6$;

   v). $12y-3y^2+24-6y\Rightarrow -3y^2+6y+24$

2) i) $\begin{cases} x^2=x\times x \\ 16=4\times 4 \\ 8x=2(4\times x) \end{cases} \Rightarrow (x+4)(x+4)$;

   ii) $\begin{cases} x^2=x\times x \\ 36=6\times 6 \\ 12x=2(6\times x) \end{cases} \Rightarrow (x+6)(x+6)$

5.6. Summary of the unit:

Definitions related to polynomials

i) **Unlike terms:** These are terms which have different variable parts. **For example**: $2x$ and $3y$ are unlike terms.

ii) **Like terms:** These are terms which have exactly the same variable(s) to the same power. For example: $4n$ and $2n$ are like terms.

iii) **Monomial:** A monomial is an algebraic expression which consists of only one term. **For example** : $2x$.

iv) **Binomial:** A binomial is an algebraic expression which contain (or is made up of) two terms only. **For example:** $3x^2 - 4$.

v) **Trinomial:** A trinomial is an algebraic expression which is made up of three terms. **For example** : $4xy - 3x + 8$.

vi) **Polynomial:** A polynomial is an algebraic expression containing more than two terms of different powers of the same variable or variables. The general form of a polynomial is

$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2}...$

vii) **vii) Degree or order of the variable:** It is defined by the highest power of the variables in a polynomial

Methods for factoring polynomials:

- Factor out the greatest common factor (GCF) from all of the terms;
- If there are two terms, use the Difference of Two Squares pattern;
- If there are three terms in which the first and third terms are squares of numbers or expressions, it may be a perfect square trinomial;
- Use of the Master Product Method;
- The Grouping Method;
- Use of synthetic division;
- Decomposition of independent term;
- Use of Remainder Theorem in which if a polynomial $f(x)$ is divided by $(x-k)$, then the remainder is $r = f(k)$.
- Use of factor theorem: A polynomial function $f(x)$ has a factor $(x-k)$ if and only if $f(\mathrm{k}) = 0$.

5.7. Additional information for the tutor

- The tutor has to inform student-teachers that in factorization one sometimes uses perfect squares like $:a^2+2ab+b^2$; where we see that: $\begin{cases} a^2 = a\times a \\ b^2 = b\times b \\ 2ab = 2(a\times b) \end{cases} \Rightarrow (a+b)(a+b)$

- Help student to avoid common errors; example: $(a+b)^2 = a^2+2ab+b^2$ and not a^2+b^2

  Similarly, $(a-b)^2 = a^2-2ab+b^2$ and not a^2-b^2.

- **Fundamental Connections for Polynomial Functions**

  For a polynomial function *f* and a real number *k*, the following statements are equivalent:

1) $x=k$ is a solution (or root) of the equation $f(x)=0$
2) k is a zero of the function $f(x)$
3) k is an x-intercept of the graph of $y=f(x)$
4) $x-k$ is a factor of $f(x)$

5.8. End unit Assessment

1. $a)$ *binomial*; $b)$ *monomial*; $c)$ *trinomial*; $d)$ *trinomial*

2. $a)$ *homogeneous*; *degree* 1; $b)$ *homogeneous*, *degree* 1; $c)$*homogeneous*; *degree* 3; $d)$ *homogeneous*; *degree* 2; $e)$ *homogeneous*; *degree* 3

3. a) $(2x-y+3)(2x-y+3) = 4x^2-2xy+6x-2xy+y^2-3y+6x-3y+9$

   $\Rightarrow 4x^2+y^2+12x-6y-4xy+9$

   b) $(a-2b)(3a^2-2ab+b^2) = 3a^3-2a^2b+ab^2-6a^2b+4ab^2-2b^3$

   $\Rightarrow 3a^3-2b^3-8a^2b+5ab^2$

   $c)-x(2x-3x-1) = -2x^2+3x^2+x = x^2+x$

d) $(x-y-2)(2x-y+3)=2x^2-xy+3x-2xy+y^2-3y-4x+2y-6$

$\Rightarrow 2x^2+y^2-x-y-3xy-6$

4. a). $a(x+y)$;

   b). $3(x+z)$;

   c). $3x(7y-2x)$;

   d). $3x(2x+5y)$;

   e). $9x^2(1-5xy^2)$

   f). $2x(2+7x)$

5) i). $\begin{cases} 9p^2=3p\times 3p \\ 16q^2=4q\times 4q \\ 24pq=2\times(3p\times 4q) \end{cases} \Rightarrow (3p+4q)(3p+4q)$

   ii). $\begin{cases} 4x^2=2x\times 2x \\ 9=3\times 3 \\ 12x=2\times(2x\times 3) \end{cases} \Rightarrow (2x+3)(2x+3)$

5.9. Additional activities

5.9.1 Remedial activities

Simplify:

$a)\ 2x-4y+5x-3y$

$b)\ x^2-3x-2+4x^2-2x+5$

$c)\ 3y^2-4y-6-3-2y-3y^2$

Answer:

$a)\ 7x-7y=7(x-y)$

$b)\ 5x^2-5x+3$

$c)\ -6y-9=-3(2y+3)$

5.9.2. Consolidation activities:

Remove brackets and simplify:

$a)\ (2x^2-3x)+(5x-8)-(7x^2-4)$
$b)\ 2(3x-y)+4(x+2y)-3(2x-3y)$
$c)\ \{3y-(x-2y)\}-\{5x-(y+3x)\}$

Answer:

$a)\ -5x^2+2x-4;\quad b)\ 4x+15y;\quad c)\ 6y-3x$

5.9.3 Extended activities:

Expand these expressions after you substitute in the final answers when $x=-2$ *and* $y=+2$:

i). $(2x^3-5y^3)(x^2+xy+y^2)$;

ii). $(x-y)^3$

Answer:

i). $i)\ 2x^5+2x^4y+2x^3y^2-5x^2y^3-5xy^4-5y^5$;

when we replace x and y by their values we get: -224

ii). $x^3-3x^2y+3xy^2-y^3$; again we get : -64

UNIT:6

LINEAR AND QUADRATIC EQUATIONS OR INEQUALITIES

6.1. Key unit competence

Solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities

6.2 Prerequisite

Student teachers will perform better in this unit if they have a good background on: Solving equations and inequalities in one unknown (S2 unit 3); Solving simultaneous equations in two unknowns (S2 unit 3).

6.3. Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others' view and thoughts during class discussions)
- Gender (equal opportunity of boys and girls in the lesson participation and when making groups)

6.4. Guidance on introductory activity

- Form small groups of student teachers and guide them to work on the introductory activity 6;
- Walk around all groups to provide pieces of advice where necessary.
- After a given time invite groups with different working steps to present their findings and harmonize them;
- Basing on the answers provided, use different probing questions to arouse their curiosity on the content of the unit 6 and to discover that they are going to solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities.

Answers for introductory activity 6

1) An equation is a statement where values of two mathematical expressions are equal. Consider the statement $ax+b=0$, where $a,b \in \mathbb{R}$

and $a \neq 0$. This statement is true when $x = -\frac{b}{a}$ (the solution or the root of the equation $ax + b = 0$). Thus, to find a solution to the given equation is to find the value that satisfy that equation.

- Linear equation in one unknown: equation of the form $ax + b = 0$, where $a, b \in \mathbb{R}$ and $a \neq 0$.
- Inequality in one unknown: inequality of the form: $ax + b > 0; or\ ax + b < o; or\ ax + b \geq 0; or\ ax + b \leq 0\ where\ a \neq 0$ and $a, b \in \mathbb{R}$
- Quadratic Equation : Equations of the type $ax^2 + bx + c = 0\ (a \neq 0)$

2) We know how to solve the inequality product like $(ax + b)(cx + d) > 0$. If we find the product of the left hand side, the result will be a quadratic expression of the form $ax^2 + bx + c$. Then to solve the inequality of the second degree like $ax^2 + bx + c > 0$ we need to put the expression $ax^2 + bx + c$ in factor form and use the method used to solve inequality product. And this lesson can be applied in economics (price and quantity); in geometry (rectangle building),...

i) $\begin{array}{l} x + 1 - 1 = 5 - 1 \Rightarrow x = 4 \\ s = \{4\} \end{array}$;

ii) $\begin{cases} x + 1 < 0 \\ x - 1 < 0 \end{cases} \Rightarrow \begin{cases} x < -1 \\ x < 1 \end{cases} \Rightarrow s =]-1, 1[$;

iii) $\begin{array}{l} x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm\sqrt{1} \Rightarrow x = \pm 1 \\ s = \{-1, 1\} \end{array}$

6.5. List of lessons or sub-headings

No	Lesson title	Learning objectives	Periods
0	Introductory activity	To arouse the curiosity of student teachers on the content of unit 6	1
1	Linear equations	Solve linear equations	1
2	Quadratic equations	Solve quadratic equations	2
3	Equations reducible to quadratic	Solve equations reducible to quadratic equations	2
4	Linear inequalities (algebraically and graphically)	Solve algebraically and graphically linear inequalities	2
5	Quadratic inequalities (algebraically and graphically)	Solve algebraically and graphically quadratic inequalities	2
6	Word problems involving linear or quadratic equations	List and clarify the steps in modelling a problem by linear or quadratic equations and inequalities. Solve mathematical problems involving linear and quadratic equations	2
7	Parametric equations and inequalities	Solve parametric equations and inequalities and discuss solution basing on the values of the parameter.	2
8	End unit assessment		1
Total periods			**15**

Lesson 1: Linear equations

a) Learning objective

Solve linear equations

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction

Student teachers will learn better this lesson if they have a good understanding on:

- Representation and interpretation of graphs of linear functions
- Solving problems involving linear functions and interpretation of the graphs of quadratic functions.

d) Learning activities

- Invite student-teachers to work in groups and answer questions for activity 6.1.1;
- Ask each group to share their answers with another group and ask them to support each other where they become more challenged in solving that activity.
- Request the group representative to share their findings to the whole class during a class discussion;
- As a tutor, harmonize the work done on activity 6.1.1 through presentation and insist on recalling the Following: increasing function, value of a function at a point, initial value for a function, solving an equation, the solution set for an equation or inequality.
- Use different questions and examples from student book and guide student-teachers on how to solve different types of equations.
- Let student-teachers go through the application activity 6.1.1 and evaluate whether the objectives of the lesson were achieved.

Answers for activity 6.1.1

The tutor guides the student teachers to discuss about polynomials and how to the classification of polynomials is based on the number of terms. The following table identifies the types of polynomial.

1) $x+1=5$ $x+1-1=5-1$ $x=4$	2) $2x-4=0$ $2x-4+4=0+4$ $2x=4$ $\frac{2x}{2}=\frac{4}{2}$ $x=2$	3) $2x+1=-5$ $x+1-1=-5-1$ $x=-6$	4) $x-4=10$ $x-4+4=10+4$ $x=14$

e) Applications activities

Answers for activity 6.1.1

1) $x+5=9 \Rightarrow x+5-5=9-5 \Rightarrow x=4$
2) $6x+5=5 \Rightarrow 6x+5-5=5-5 \Rightarrow 6x=0 \Rightarrow x=0$
3) $x+5=9x+1 \Rightarrow -8x=-4 \Rightarrow x=\frac{1}{2}$
4) $-6x-5=9 \Rightarrow -6x=14 \Rightarrow x=-14/6=\frac{-7}{3}$
5) $6x-51=9 \Rightarrow 6x=9+51 \Rightarrow 6x=60 \Rightarrow x=10$

Lesson 1: Quadratic equations

a) Learning objective:

Solve quadratic equations

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding on concepts learnt in S3:

- Representation and interpretation of graphs of quadratic functions
- Solving problems involving quadratic functions and interpretation of the graphs of quadratic functions.

d) Learning activities

- Invite student-teachers to work in group discussions and do activity 6.1.2 found in their Mathematics books.
- Move around in the class for facilitating various groups during their work, verify their working steps and intervene with probing questions for orienting student-teachers where necessary. For example, you can ask them to recall what the equation product AB = 0 means leading them to guess towards the factorization of the quadratic equation before solving;
- Ask the neighbouring groups of student-teachers to share their answers and compare them where necessary.
- Invite one member from groups with different working steps to present their answers to the whole class;
- Harmonize the findings highlighting how to solve algebraically the quadratic equations;
- Use different types of equations and some examples given in the student-teacher's book and guide student-teachers to identify a type of equation and discuss how they can solve them: Use of different methods of factorization, use of discriminant, ploting graphs, etc. In each case, help them to discover 3 main cases: equation with two real roots, equation with one double root and equation without root in the set of real numbers:

The quadratic equation has the form $ax^2+bx+c=0$. The discriminant is given by $\Delta=b^2-4ac$

The two values of unknown x are generated as follows:

$x_1=\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ And $x_2=\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

Note that, when the $\Delta=0$ two equal roots are generated as $x_1=x_2=\dfrac{-b}{2a}$.

When $\Delta<0$ the equation has no root in the set of real numbers.

- Assign student-teachers to do the application activity 6.1.2 and verify whether the lesson's objective was achieved.

Answer of activity 6.1.2

1) $x^2+2x-24=0$

$\left.\begin{array}{l} Sum=2 \\ \text{Product}=-24 \end{array}\right.$ for $-4;6$

$x^2+2x-24=0$

$x^2-4x+6x-24=0$

$\left(x^2-4x\right)+\left(6x-24\right)=0$

$x\left(x-4\right)+6\left(x-4\right)=0$

$(x-4)(x+6)=0$

$x=4$ and $x=-6$

2) If the length is twice the width and suppose that width is x, then;

For $W=x$, $L=2x$, $A=L\times W$

$900=2x\times x$

$900=2x^2$

$x=\pm\sqrt{\frac{900}{2}}$

$x=\pm\sqrt{450}$

$x=\pm\sqrt{225\times 2}=\pm 15\sqrt{2}$

$\therefore x=+15\sqrt{2}=W; L=15\sqrt{2}\times 2=30\sqrt{2}$

e) Answers of application activity 6.1.2

1) $S=\{-2,-4\}$

2) $S=\{11,1\}$

3) $S=\{-8,3\}$

Lesson 3: Equations reducible to quadratic

a) Learning objective:

Solve equations reducible to quadratic equations

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding on matter learnt in Unit 1 &3 of S1, Unit 2 of S2. This means that they should be skilled on:

- Representation and interpretation of graphs of linear functions
- Solving problems involving linear or quadratics functions
- Performing operations on polynomials and factorizing them.

d) Learning activities

- Invite student-teachers to work in group discussions and do activity 6.2 found in their Mathematics books.
- Move around in the class for facilitating various groups during their work, verify their working steps and intervene with probing questions for orienting student-teachers where necessary. For example, you can ask them what can happen if they let $y = x^2$ and solve as usual an obtained quadratic equation;
- Ask the neighbouring groups of student-teachers to share their answers and compare them where necessary.
- Invite one member from groups with different working steps to present their answers to the whole class;
- Harmonize the findings highlighting how to solve algebraically the equation reducible to quadratic equations, **the number of roots** and how to represent them;
- Use different types of equations and some examples given in the student-teacher's book and guide student-teachers to identify a type of equation and discuss how they can solve them;
- Assign student-teachers to do the application activity 6.2 and verify whether the lesson's objective was achieved.

Answers for activity 6.2

Answers to activity 6.2.

1) For $u = x^2$; then $x^4 - 2x^2 + 4 = 0$ becomes $u^2 - 2u + 4 = 0$.
2) For $u = x^2$; then $6x^4 + 5x^2 + 1 = 0$ becomes $6u^2 + 5u + 1 = 0$.

e) Answers for application activity 6.2

1) $S = \{-3, -2, 2, 3\}$
2) $S = \{-3, -1, 1, 3\}$

Lesson 4: Linear inequalities

a) Learning objective:

Solve algebraically and graphically linear inequalities

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding on taught matter in Unit 1 &3 of S1, Unit 2 of S2. This means that they should to be skilled on:

- Representation and interpretation of graphs of linear functions
- Solving problems involving linear or quadratics functions

d) Learning activities

- Invite student-teachers to work in groups on questions of activity 6.3.1 and 6.3.2;
- Ask student-teachers to share their answers with another group and ask them to support each other where they become more challenged.
- With the use of different questions and examples given in the

student-teacher’s book, guide student-teachers to establish a good algebraic and graphical method of solving different types of inequalities: involving simple expression, product, quotient and absolute values. Insist on different ways of presenting the solution set of an inequality.

- Guide student-teachers to brainstorm on real life problems that involve inequalities and invite them to explore the examples given in the student-teachers’ book.
- After the lesson development, invite student-teachers to do the application activity 6.3.1 and 6.3.2 and verify if the objectives of the lesson were achieved.

Answers to activity 6.3.1

1) $\mathbf{0, 1, 2, 3, 40, 1, 2, 3, 4}$
2) $\mathbf{1, 2, 3, 4, 51, 2, 3, 4, 5}$
3) $\mathbf{-4, -3, -2, -1, 0-4, -3, -2, -1, 0}$
4) $\mathbf{-1, 0, 1, 2, 3-1, 0, 1, 2, 3}$

Answers for activity 6.3.2

Solve inequalities in the set of real numbers

1) Start by solving $(x+1)(x-1)=0$

   $x+1=0$ $\qquad$ $x-1=0$

   $\Rightarrow x=-1$ or $\Rightarrow x=1$

The next is to find the sign table.

x	$-\infty$		-1		1		$+\infty$
$x+1$		-	0	+		+	
$x-1$		-		-	0	+	
$(x+1)(x-1)$		+	0	-	0	+	

Since the inequality is $(x+1)(x-1)<0$; we will take the interval where the product is negative. Thus, $S=]-1,1[$

2) Start by solving

$$\frac{x+2}{x-1}=0; x-1\neq 0 \quad \text{or} \quad x-1=0$$
$$x+2=0 \qquad \Rightarrow x=1$$
$$\Rightarrow x=-2$$

The next is to find the sign table.

x	$-\infty$		-2		1		$+\infty$
$x+2$		-	0	+		+	
$x-1$		-		-	0	+	
$\frac{x+2}{x-1}$		+	//	-	0	+	

Since the inequality is $\frac{x+2}{x-1}\leq 0$; we will take the interval where the quotient is negative. Thus, $S=\left]-2,1\right]$.

e) Answers for application activities

Answers for application activity 6.3.1

1) $S=\left]-\infty,9\right[$
2) $S=\left]-\infty,10\right[$
3) $S=\left]-\infty,5\right]$
4) $S=\left[\frac{26}{3},+\infty\right[$
5) $S=\left]-\infty,\frac{5}{2}\right[$

Answers for application activity 6.3.2

1) $S=\left]-\infty,-3\right[\cup\left]3,+\infty\right[$
2) $S=\left]-\infty,-2\right]\cup\left[3,+\infty\right[$

Lesson 5: Quadratic inequalities

a) **Learning objective:**

Solve algebraically and graphically quadratic inequalities

b) **Teaching resources:**

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) **Prerequisites / Revision / Introduction:**

Student teachers will learn better this lesson if they have a good understanding on taught matter in Unit 1 &3 of S1, Unit 2 of S2. This means that they should to be skilled on:

- Representation and interpretation of graphs of quadratic functions
- Solving problems involving linear or quadratics functions

d) **Learning activities**

- Invite student-teachers to work in groups on questions of activity 6.3.3;
- Ask student-teachers to share their answers with another group and ask them to support each other for improvement;
- Invite group representatives to present their findings and harmonize answers for the activity 6.3.3;
- With the use of different questions and examples given in the student-teacher's book, guide student-teachers to establish a good algebraic and graphical method of solving different types of quadratic inequalities by use of factorization and the table of variation. Insist on different ways of presenting the solution set of an inequality.
- Guide student-teachers to brainstorm on real life problems that involve inequalities and invite them to explore the examples and content given in the student-teachers' book;
- After the lesson development, invite student-teachers to do the application activity 6.3.3 and verify if the objectives of the lesson were achieved.

Answers for activity 6.3.3

$1)2 \le x \le 6:\{2,3,4,5,6\}$

$2)none$

$3)1 \le x < 2:\left\{1,\frac{3}{2},\frac{5}{3},\frac{7}{4},\frac{9}{5}\right\}$

e) Application activities

Answers for activity 6.3.3

1) $x^2 - 10x - 20 > 0 \Rightarrow x_1 = 5+3\sqrt{5}; x_2 = 5-3\sqrt{5}$

$(x-5-3\sqrt{5})(x-5+3\sqrt{5}) = 0$

x	$-\infty$ $\quad 5-3\sqrt{5} \quad 5+3\sqrt{5} \quad +\infty$
$(x-5+3\sqrt{5})$	- - - - **0** + + + + + + +
$(x-5-3\sqrt{5})$	- - - - - - - - **0** + + + + ++ +
$(x-5+3\sqrt{5})(x-5-3\sqrt{5})$	+ + + + + **0** - - - **0** + + + + +

$s = \left]-\infty, 5-3\sqrt{5}\right[\cup \left]5+3\sqrt{5}, +\infty\right[$

2) $6x^2 - 5x + 1 < 0 \Rightarrow x_1 = \frac{1}{2}; x_2 = \frac{1}{3}$

x	$-\infty$ $\quad +\frac{1}{3} \quad +\frac{1}{2} \quad +\infty$
$(x-\frac{1}{2})$	- - - - - - **0** + + + + + + + + + +
$(x-\frac{1}{3})$	- - - - **0** + + + + + + + + + + +
$(x-\frac{1}{2})(x-\frac{1}{3})$	+ + + + **0** - - - **0** + + + + + + + +

$$s = \left]\frac{1}{3}, \frac{1}{2}\right[$$

Lesson 6: Word problems involving linear or quadratic

equations

a) **Learning objective:**

List and clarify the steps in modelling a problem by linear or quadratic equations and inequalities.

Solve mathematical problems involving linear and quadratic equations

b) **Teaching resources:**

Student teacher's book, Reference books, Ruler, T-square, Manila paper, Scientific calculators.

Note: If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) **Prerequisites / Revision / Introduction:**

Student teachers will learn better this lesson if they have a good understanding on:

- Represent and interpret graphs of linear functions,
- Solve linear equations and inequalities, appreciate the importance of checking their solution, and represent the solution.

d) **Learning activities**

- Invite student-teachers to work in groups on questions of activity 6.4;
- Ask student-teachers to share their answers with another group and ask them to support each other for improvement.
- Invite group representatives to present their findings and harmonize answers for the activity 6.4;
- With the use of different questions and examples given in the student-teacher's book, guide student-teachers to establish a good method of solving a given word problem involving linear or quadratic. Insist on different ways of concluding on the solution of the problem;
- After the lesson development, invite student-teachers to do the application activity 6.4 and verify if the objectives of the lesson were achieved.

Answers for activity 6.4

1) The answer is obvious or optional, verify answers and organize a session for feedback;

2)

a) The steps to proceed:

Let the number be x*: then*

Two times the number: $2x$

Six less than two times a number: $2x-6$

Six less than two times a number is equal to nine: $2x-6=9$

b) The steps to proceed:

Let the cost of the shoes be y*; then*

The cost of clothes is $y+2100$

The total cost of both shoes and clothes $(y)+(y+2100)=22100$

Solving the equation:

$$(y)+(y+2{,}100)=22{,}100$$
$$\Rightarrow 2y=20{,}000$$
$$\Rightarrow y=10{,}000$$

Therefore, the cost of the shoes was 10,000Frw.

3) The price of the demand drop to 1000 units: $P_2=23.69$

$$D=2000+100P-6P^2 \Rightarrow 1000=2000+100P-6P^2$$
$$-6P^2+100P+1000=0 \Rightarrow \Delta=10000+24000=34000$$
$$\sqrt{\Delta}=20\sqrt{85}$$

$$P_1=\frac{-100+20\sqrt{85}}{-12}=-7.03$$

$$P_2=\frac{-100-20\sqrt{85}}{-12}=23.69$$

As the price can't be negative, P_1 is rejected and we consider only

$$P_2 = \frac{-100-20\sqrt{85}}{-12} = 23.69$$

e) Application activities

Answers for application activity 6.4

$420-0.2q = 60+0.4q \Rightarrow 0.6q = 360 \Rightarrow q = 600$

$420-0.2q = P \Rightarrow P = 420-0.2(600) = 300$ we see that the equilibrium quantity is 600 and the equilibrium price is 300 .

Lesson 5: Solving and discussing parametric equations

a) Learning objective:

Solve parametric equations and inequalities and discuss solution basing on the values of the parameter.

b) Teaching resources:

Student teacher's book, Reference books, Ruler, T-square, Manila paper, scientific calculators.

<u>**Note:**</u> If possible computers with mathematical software such as Geogebra, Microsoft Excel, Math-lab or Graph-Calc and internet can be used.

c) Prerequisites / Revision / Introduction:

Student teachers will learn better this lesson if they have a good understanding to:

- Perform operations on linear or quadratic polynomials
- Solve quadratics functions.

d) Learning activities

- Invite student-teachers to work in groups on questions of activity 6.5;
- Ask every group of student-teachers to share their answers with another group and ask them to support each other for improvement;

- Invite group representatives to present their findings and harmonize answers for the activity 6.5;
- With the use of different questions and examples given in the student-teacher's book, guide student-teachers to establish a good method of solving parametric equations: solve an equation for the unknown, discuss the solution in different values of the parameters . Insist on different ways of concluding on the solution of the equation;
- After the lesson development, invite student-teachers to do the application activity 6.5 and verify if the objectives of the lesson were achieved.

Answers for activity 6.5

The tutor guides the student teacher to discuss on the given parameter λ in the equation $\lambda x^2+(\lambda-1)x+2=0$

$$x_1, x_2=\frac{-(\lambda-1)\pm\sqrt{(\lambda-1)^2-8\lambda}}{2\lambda}$$

If $\lambda=0$, there is no root

If $(\lambda-1)^2=8\lambda$, there is one double root

If $(\lambda-1)^2<8\lambda$, there is no real root

If $(\lambda-1)^2>8\lambda$, there are two distinct real root

In small groups, facilitate student-teachers to do examples in the student book.

Call student-teachers to do **Application activity 6.5** to master the content.

e) Answers for Application activity 6.5

Summary table for $(10-\lambda)x^2-6x+\lambda=0$

λ	Δ	p	s	Conclusion
$]-\infty,0[$	+	-	+	Two distinct real roots with different signs
0	+	**0**	+	Two distinct real roots: $x_1=0,\ x_2=\frac{3}{5}$
$]0,1[$	+	-	-	Two distinct positive real roots
1	0	+	+	One positive double root $x=\frac{1}{3}$
$]1,9[$	-	+	+	No real roots
9	**0**	+	+	One positive double root: $x=3$
$]9,10[$	+	+	+	Two distinct positive real roots
10	+	||	||	Equation of first degree. $x=\frac{5}{3}$
$]10,+\infty[$	+	-	-	Two distinct real roots with different signs

6.6. Summary of the unit

- In case certain coefficients of the equation contain one or several letter variables, the equation is called parametric and the letters are called real parameters. In this case, we solve and discuss the equation (for parameters only).
- If at least one of the coefficients a, b and c depends on the real parameter which is not determined, the root of the parametric quadratic equation depends on the values attributed to that parameter.

6.7. Additional Information for tutors

- Emphasize on the discriminant method when solving quadratic equation:
  - If $\Delta>0$, there are two real distinct roots.
  - If $\Delta=0$, double root.
  - If $\Delta<0$, there are no real root.

6.8. End unit assessment

Solution for the related questions:

1. a) $x = 13 \Rightarrow s = \{13\}$;

   b) $x \geq 4 \Rightarrow s = [4, +\infty[$;

   c) $s =]-3, 2[$;

   d) $s = \left]-\frac{7}{3}, 2\right[$;

   e) $s = \left[\frac{1}{3}, \frac{1}{2}\right]$;

   f) $s = \left\{5 - 2\sqrt{6}, 5 + 2\sqrt{6}\right\}$;

   g) $s = \{1\}$

2 The minutes on the bill are 200. This number is obtained by solving the following equation $3\,000 + 20x = 7\,000$

6.9 Additional activities

6.9.1 Remedial activities

Find the solution of these inequalities:

1) $(x-2)(x+5) \geq 0$

2) $(-x+1)(x-3) \leq 0$

Solutions:

1) $s =]-\infty, -5] \cup [2, +\infty[$;

2) $s =]-\infty, 1] \cup [3, +\infty[$

6.9.2 Consolidation activities:

Solve in real set

1) $x^4 - 5x^2 + 4 = 0$

2) $x^4 - 25x^2 + 144 = 0$

Solution :

1) $x^4-5x^2+4=0 \Rightarrow u^2-5u+4=0$

$\Delta = 25-16=9 \Rightarrow \sqrt{\Delta}=\pm 3$

$u_1=\frac{5+3}{2}=4$

$u_2=\frac{5-3}{2}=1$

$u=x^2 \Rightarrow x^2=4 \Rightarrow x_1=2; x_2=-2$

$x^2=1 \Rightarrow x_3=1; x_4=-1$

$s=\{-2,-1,1,2\}$

2) $x^4-25x^2+144=0 \Rightarrow u^2-25u+144=0$

$\Delta = 625-576=49 \Rightarrow \sqrt{\Delta}=\pm 7$

$u_1=\frac{25+7}{2}=16$

$u_2=\frac{25-7}{2}=9$

$u=x^2 \Rightarrow x^2=16 \Rightarrow x_1=4; x_2=-4$

$x^2=9 \Rightarrow x_3=3; x_4=-3$

$s=\{-4,-3,3,4\}$

6.9.3 Extended activities:

Solve in set of real numbers

1) $2x^3-3x^2-3x+2=0$
2) $3x^3-13x^2+13x-3=0$
3) $6x^4-5x^3-38x^2-5x+6=0$

Solutions:

1) $s=\left\{-1,\frac{1}{2},2\right\}$;

2) $s=\left\{\frac{1}{3},1,3\right\}$;

3) $s=\left\{-2,-\frac{1}{2},\frac{1}{3},3\right\}$

UNIT:7

PROBLEMS ON POWERS, INDICES RADICALS AND COMMON LOGALITHMS

7.1 Key unit competence

Solve problems related to powers, indices, radicals and common logarithms.

7.2 Prerequisite

Student teacher will perform well in this unit if they have a good background on:

- Apply laws of indices and surds to simplify mathematical expressions as taught in S2 (Unit1).
- Represent very small numbers or very big numbers in standard form.
- Use the conjugates of surds to rationalize the denominator on surds.
- Solve simple equations involving indices and surds.
- Appreciate the importance of indices and surds in solving mathematical problems.

7.3 Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching);
- Peace and value Education (respect others' view and thoughts during class discussions);
- Gender (equal opportunity of boys and girls in the lesson participation);

7.4 Guidance on introductory activity

- Facilitate student-teachers to work in groups, read and do the introductory activity 7 from Student -teacher's book;
- Facilitate discussions to avoid noise or other unnecessary conversation,
- Be aware of straggling groups and provide assistance where necessary;

- Call them to present their findings and promote gender into presentation;
- Through question-answer, facilitate Student-teachers to realize that introductory activity stimulates them to get idea on the content of Unit 7;

Expected answer for introductory activity 7

1) Lead student-teachers to know that in the question1, the area of painted region is equal to the difference between outer rectangle of design and inner rectangle.

Outer area = $\sqrt{256}\times\sqrt{100}$ square unit

Lead student-teachers to simplify radicals;

Outer area = 16 × 10 square length unit = 160 square length unit

Then, Inner area = $\sqrt{16}\times\sqrt{64}$ square length unit

Simplifying radicals:

Inner area = 4 × 8 square length unit = 32 square length unit

Therefore: painted area= (160 - 32)sq. length unit= 128 sq. length unit

2) Lead student-teachers to do question 2, and help them to provide properties of powers and radicals.

a) If $P=b^p$ and $Q=b^q$, then

i) $P.Q = b^p.b^q = b^{p+q}$

ii) $\frac{P}{Q}=\frac{b^p}{b^q}=b^{p-q}$

iii) $P^n = (b^p)^n = b^{pn}$

iv) $\sqrt[n]{P}=\sqrt[n]{b^p}=b^{\frac{p}{n}}$

b) Substituting the expression by $n=3$, $b=2$, $p=3$ and $q=7$

i) $b^{p+q} = 2^{3+7}=2^{10}=1024$

ii) $b^{p-q}=2^{3-7}=2^{-5}=\frac{1}{32}$

iii) $b^{pn} =2^{3(3)}=512$

iv) $b^{\frac{p}{n}}=2^{\frac{2}{2}}=2$

7.5 List of lessons

No	Lesson title	Learning objectives	Periods
0	Introductory activity	To arouse the curiosity of student teachers on the content of unit 7	1
1	Definition of powers and radicals	Define powers/ exponents or indices, and radicals.	2
2	Properties of indices and radicals	Identify and use the properties of powers/ exponents or indices, radicals in mathematics expressions	3
3	Operations on indices and radicals.	Perform operations on indices and radicals and simplify the result.	2
4	Definition of decimal logarithm	Define decimal logarithms	1
5	Properties of decimal logarithms	– Explore the properties of decimal logarithms; – Perform operations on logarithms and use them to solve some exponential equations.	3
6	Application of powers and decimal in real life	– Solve real life problems that involve decimal logarithms.	4
7	End unit assessment		2
Total periods			**18**

Lesson 1: Definition of powers and radicals

a) Learning objective

Define powers / exponents or indices and radicals.

b) Teaching resources

Student-teacher's book and other reference books to facilitate research, notebooks, etc.

c) Prerequisites/Revision/Introduction

Through examples, let student-teachers discuss how to simplify powers, and radicals in real life situations.

d) Learning activities

- Form groups of student-teachers and invite them to do read and do the **activity 7.1.1.** from student-teacher's book;
- Make sure that all student-teachers (boys and girls) participate actively in their groups;
- Visit ever group, facilitate where possible and remind them to show their working steps;
- After a given time, ask randomly some groups to present their findings to the whole class;
- After the presentation, harmonize their answers and guide student-teachers discover how to write a power where they will differentiate a base from exponent and the result;
- Use different probing questions to guide student-teachers to explore examples and the content for explaining powers and radicals;
- Guide student-teachers to work individually **application activity 7.1.1**, given in **student-teacher's book,** to assess their competences.

Answer for activity 7.1.1

1.a) $5\times5\times5\times5\times5\times5\times5=5^7$

b) $3\times3\times3\times3\times3\times t\times t\times s\times s\times s\times s\times s=3^5t^2s^5$

c) $(x+2)(x+2)(x+2)=(x+2)^3$

2. a) $\sqrt{4}=\sqrt{2^2}=2$

b) $\sqrt[3]{8}=\sqrt[3]{2^3}=2$

c) $\sqrt[4]{16}=\sqrt[4]{2^4}=2$

d) $\sqrt[5]{32}=\sqrt[5]{2^5}=2$

e) $\sqrt[6]{64}=\sqrt[6]{2^6}=2$

f) $\sqrt[7]{128}=\sqrt[7]{2^7}=2$

e) Answers to application activity 7.1.1

1. a) $10\sqrt{7}$ b) $2\sqrt[3]{3}$ c) $-2\sqrt[5]{2}$ d) $6\sqrt{2}$ e) $\frac{5}{12}$ f) $\frac{\sqrt{10}}{4}$

2. $side = 150cm$

Lesson 2: Properties of indices and radicals

a) Learning objective

Identify and use the properties of powers/ exponents or indices, radicals in mathematics expressions

b) Teaching resources

Student-teacher's book and other reference textbooks to facilitate research. notebooks, etc.

c) Prerequisites/Revision/Introduction

Student-teachers will perform well if they make a short revision on how they should obtain the general form of powers and how the radicals are simplified as it was learnt in S1.

d) Learning activities

- Remind all student-teachers that they are full of potentials to perform the given activities.
- Organize the student-teachers into groups and invite them to read and do the **activity 7.1.2** from student-teacher's book;
- Circulate to each group and facilitate where possible, and remind them to prove their answers using their own words and to try to deduce the general form
- After a given time, ask selected groups with different working steps to present their findings to the whole class;
- After the presentation, harmonize their answers and guide student-teachers to establish properties for powers and radicals where they can be asked to give their own examples;
- Use different probing questions to guide student-teachers to explore examples and the content on properties of powers and radicals and motivate them to use their own words to explain those properties;
- Guide student-teachers to work individually the application activity 7.1.2 , given in the student-teacher's book, to assess student-

teachers' competences.

Answer for activity 7.1.2

1. Work out:

a) $10^2\cdot 10^3 = 10^{2+3} = 10^5$ or $10^2\cdot 10^3 = 10\times 10\times 10\times 10\times 10 = 10^5$

b) $(2^5)^2 = 2^{5\times 2} = 2^{10}$ or $(2^5)\cdot(2^5) = 2^{5+5} = 2^{10}$

c) i) $\frac{5^4}{5^3} = \frac{5\times5\times5\times5}{5\times5\times5} = 5$ or $5^{4-3} = 5$

ii) The answer is 1.

iii) $\frac{3^6}{3^8} = \frac{3x3x3x3x3x3}{3x3x3x3x3x3x3x3} = \frac{1}{3^2}$

2. True or false:

a) $\sqrt{36\cdot 81} = \sqrt{36}\cdot\sqrt{81}$ this is true because $\sqrt{36} = 6$; $\sqrt{81}$=9 then $9\times 6 = 54$ while $36\times 81 = 2916$ then $\sqrt{2916} = 54$.

b) $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$ the statement is true.

e) Answers to application activity 7.1.2

1. a) $ab\sqrt[10]{a^5}$ or $ab\sqrt{a}$ b) $ab^2c\sqrt{abc}$ c) $\frac{s}{3}$

2. the error of student is that he/she confused $(-5)^0 and -5^0$.

Since we have $(-5)^0 = 1$ while $-5^0 = -1$.

3. The answer is negative, because $-3^{-2} = -\left(\frac{1}{3^2}\right) = -\frac{1}{9}$

4. Volume of a cube is S^3 , so $V = 3^{12}\cdot 2^{18}m^3$

5. $(u^3)^4 = u^{3\times4} = u^{12}$ while $u^{3^4} = u^{3\times3\times3\times3} = u^{81}$, so the expressions are totally different.

6.a) No, because $-(2^3)^2 = -2^6$ while $(-2^3)^2 = 2^6$

b) No, because $(3y)^7 = (3)^7(y)^7$ while $3(y)^7$

7.2^{30} because $2^{15}\times 2^{15}$ or $(2^{15})^2$

Lesson 3: Operations on indices and radicals

a) Learning objective

Perform operations on indices and radicals and simplify the result.

b) Teaching resources

Student-teacher's book and other reference books to facilitate research.

c) Prerequisites/Revision/Introduction

Student-teachers will learn well in this lesson if they refer to the lesson on "evaluating and simplifying powers and radicals" learnt in S2.

d) Learning activities

- Remind all student-teachers that they are full of potentials to perform the given activity;
- Organize the student-teachers into groups and invite them to read and do the **activity 7.2** from student-teacher's book;
- Circulate to each group and facilitate where possible and remind them to use properties of powers and radicals to get the answer and to prove their answers using their own words;
- After a given time, ask selected groups with different working steps to present their findings to the whole class;
- After the presentation, harmonize their answers and guide student-teachers to discover the good method of performing operations on for powers and radicals;
- Use different probing questions to guide student-teachers to explore examples and the content on addition and subtraction of indices and radicals, Multiplication and division of indices and radicals, and then Rationalizing the denominator containing the radicals.
- Motivate student-teachers to use their own words to explain how to deal with operations with powers and radicals;
- Guide student-teachers to work individually the application activity 7.2 , given in the student-teacher's book, to assess student-teachers'

competences.

Answers to activity 7.2

1) a) $x^3x^2 = x^{2+3} = x^5$ b) $\frac{6xy^2}{3xy} = 2y$ c) $\frac{xy}{4yx} = \frac{1}{4}$

2) (1). $\sqrt{18} + \sqrt{2} = \sqrt{9 \times 2} + \sqrt{2} = 3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$

(2). $\sqrt{12} - 3\sqrt{3} = \sqrt{4 \times 3} - 3\sqrt{3} = 2\sqrt{3} - 3\sqrt{3} = -\sqrt{3}$

(3). $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

(4). $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$

e) Answers to application activity 7.2

1) a) $5x^2y^6$ **b)** 0

2) a) $3\sqrt{5}$ **b)** $3\sqrt{7}$ **c)** $2\sqrt{3}$ **d)** 12 **e)** $13\sqrt{5}$

3) a) $\frac{\sqrt{2}}{2}$ **b)** $\frac{2\sqrt{5}-\sqrt{15}}{10}$ **c)** $\frac{-2-2\sqrt{6}}{5}$ **d)** $\frac{-3-\sqrt{6}+\sqrt{10}+\sqrt{15}}{2}$

Lesson 4: Definition of Decimal logarithm

a) Learning objective

Define decimal logarithms

b) Teaching resources

Student-teacher's book and other reference textbooks to facilitate research.

c) Prerequisites/Revision/Introduction

Student-teachers will learn well in this lesson if they refer to the previous lessons related to powers and radicals especially the **standard notation form** ($e.g$: $10{,}000{,}000 = 10^7$).

d) Learning activities

- Let student-teachers work in groups and do the activity 7.3.1 from the student-teachers' book;
- As student-teachers are working, circulate to each group and ask some questions which can lead to the objectives of this lesson;

- Ask groups to share their answers with other groups and allow them to share the challenging points they faced in their groups.
- Invite group representative to present their answers to the whole class;
- Try to harmonize student-teachers' findings;
- Ask them different questions leading them to discover the meaning of decimal logarithm of a number written in the power of 10.
- After attempting different examples, help them to formulate the decimal logarithm of a number and establish how to find it.
- Let student-teachers discover that the **decimal logarithm** of a positive real number x is defined to be a real number y for which 10 must be raised to obtain x means $y = 10^x$. Hence, $\forall x > 0$, $y = \log x$ **or $y = \log_{10} x$.**

In general notation we do not write this base for decimal logarithm.

In the notation $y = \log x$, xx is said to be the **antilogarithm of yy**.

Therefore $y = \log x$ means $10^y = x$

Co-logarithm of a number is the logarithm of the reciprocal of that number, equal to the negative of the logarithm of the number itself, $\text{colog}\, x = \log\left(\frac{1}{x}\right) = -\log x$

- After this step, invite student-teachers to do application activity 7.3.1 to assess the competences developed.

Answers of activity 7.3.1

1) 0 because $10^0 = 1$

2) 1 because $10^1 = 10$

3) 2 because $10^2 = 100$

4) 3 because $10^3 = 1$

5) 4 because $10^4 = 10000$

6) 5 because $10^5 = 100000$

e) Answers to application activity 7.3.1

1. a) $\log_7 49 = 2$ b) $\log_6 \frac{1}{6} = -1$ c) $\log_0 1 = -1$ d) $\log_8 2 = \frac{1}{3}$

2. a) -2 b) -1.6 c) -1.17

3. a) $3^4 = 81$ b) $10^1 = 10$ c) $5^{-4} = \frac{1}{625}$ d) $9^{\frac{3}{2}} = 27$

4. a) -3 b) -3 c) $\frac{1}{3}$ d) -2 e) 5 f) 36 g) -5

Lesson 5: Operations on decimal logarithms, their properties and application in real life

Note: This is a longue lesson which can be taught in two main lessons:

- Operations on decimal logarithms and their properties;
- Application of powers and decimal logarithms in real life.

a) Learning objective

- Explore the properties of decimal logarithms;
- Perform operations on logarithms and use them to solve some exponential equations.
- Solve real life problems that involve decimal logarithms.

b) Teaching resources

Student-teachers' books and other Reference textbooks to facilitate research. calculators, notebooks, etc.

c) Prerequisites/Revision/Introduction

Student-teachers will learn well in this lesson if they refer to the previous lessons related to powers and radicals.

d) Learning activities

- Remind all student-teachers that they are full of potentials to perform the given activities.

- Invite them to work in groups and do the **activity 7.3.2** from student-teacher's book.
- Circulate to each group and facilitate where necessary;
- After a given time, ask randomly some groups to present their findings to the whole class;
- Harmonize student-teachers' answers and guide them discover a rule (property) that can facilitate the calculation of logarithm requested;
- Ask them different probing questions leading student-teachers to discover properties or rules that can facilitate them to determine the decimal logarithm of an expression where some of them are the following:

$\forall a, b \in]0, +\infty[$

a) $\log ab = \log a + \log b$

b) $\log \frac{1}{b} = -\log b$

c) $\log \frac{a}{b} = \log a - \log b$

d) $\log a^n = n \log a$

e) $\log \sqrt{a} = \log a^{\frac{1}{2}} = \frac{1}{2} \log a$

f) $\log \sqrt[m]{a^n} = \log a^{\frac{n}{m}} = \frac{n}{m} \log a$

g) $\text{colog}\, x = \log\left(\frac{1}{x}\right) = -\log x$

h) Change of base formula: If u ($u > 0$) and if a and b are positive real numbers different from 1, $\log_b u = \frac{\log_a u}{\log_a b}$ This means that if you have a logarithm in any other base, you can convert it in the decimal logarithm in the following way where $a = 10$:

$$\log_b u = \frac{\log_{10} u}{\log_{10} b} = \frac{\log u}{\log b}.$$

i) There is another special logarithm called natural logarithm which has the base a number $e \approx 2.71828$. This logarithm is written as: $\log_e x = \ln x$.

- After this session, invite student-teachers to go back to their groups and search in the library, or on internet more examples from real life that can be solved with the intervention of powers and logarithms.
- Invite groups to come back to the whole class discussion where some groups can be invited to present their findings;
- Harmonize student-teachers' answers and guide them to explore examples given in the student-teacher's book and note a summary;
- Guide student-teachers to work individually application activity 7.3.2, given in Student-teacher's book, to assess student-teachers' competences.

Answer to activity 7.3.2

1. Calculating logarithms:

a) $\log 100 = \log 10^2 = 2\log 10 = 2$

b) $\log 1000 = \log 10^3 = 3\log 10$ 17^2

2. Calculating logarithms one can find:

$$\begin{aligned}\log(100)\times(1000) &= \log 100 + \log 1000\\ &= \log 10^2 + \log 10^3\\ &= 2\log 10 + 3\log 10\\ &= 2\log 10 + 3\log 10\\ &= 2+3=5\end{aligned}$$

3. Comparing the results in 2 and the sum $\log 100 + \log 1000$. One can notice that the result is 5 and from this, one can deduce that $\log(a\times b) = \log a + \log b$, when a *and* b are positive real numbers

e) Answers to application activity 7.3.2

1. a) $a > b$ b) $a = b$ c) $a < b$

2. a) 2.17 b) 0.66 c) 0.30

3. $a)\ 2^{x-1} = 16 \Leftrightarrow 2^{x-1} = 2^4 \Rightarrow x-1 = 4 \Leftrightarrow x = 5, \quad S = \{5\}$

$b)\ 3^x = \frac{1}{81} \Leftrightarrow 3^x = 3^{-4} \Rightarrow x = -4, \quad S = \{-4\}$

$$\frac{A}{P} = (1+r)^n \Leftrightarrow \frac{£56,711.25}{£50,000} = (1+r)^2 \Leftrightarrow 1.134225 = (1+r)^2 \Leftrightarrow 1.2864663506 = 1+r$$

$r = 0.2864663506 \approx 28\%$

$$\text{or } 2 = \frac{\log\left(\frac{56,711.25}{50,000}\right)}{\log(1+r)} \Leftrightarrow 2\log(1+r) = \log\left(\frac{56,711.25}{50,000}\right) \Leftrightarrow \log(1+r)^2 = \log\left(\frac{56,711.25}{50,000}\right)$$

$$(1+r)^2 = \frac{56,711.25}{50,000} \Rightarrow r = \left(\frac{56,711.25}{50,000}\right)^2 - 1 \Rightarrow r = 0.2864663506 \approx 28\%$$

7.6 Unit summary

Properties of powers

- **Multiplying powers with the same base**

To multiply numbers or variables with the same base, add their powers.

$a^m . a^n = a^{m+n}$; for positive integers m and n.

- **Finding a power of a power**

To find a power of a power, multiply those powers.

$(a^m)^n = a^{m.n}$; for positive integers m and n.

- **Raising a product to a power**

For every nonzero number a and b and positive integer n then,

$(ab)^n = a^n b^n$

- **Dividing powers with the same bases**

To divide numbers or variables with the same base, subtract their powers.

$\frac{a^m}{a^n} = a^{m-n}$; for $a \neq 0$ and positive integers m and n.

- **Raising a ratio to a power**

For every nonzero number a and b and positive integer n then,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

- **Zero as a power**

If a power of a nonzero number or variable is zero, then $a^0 = 1$; for $a \neq 0$.

- **Negative powers**

$a^{-n} = \frac{1}{a^n}$; for $a \neq 0$.

- **Fractional index/power property and its converse**

For every nonzero number a and positive integers m and n

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ and } \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and } \sqrt[n]{a} = a^{\frac{1}{n}}$$

Properties of n^{th} roots.

- **Multiplication property of radicals and its converse**

For every number $a \geq 0$ and $b \geq 0$ and positive integer n

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \text{ and } \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

- **Division property of radicals and its converse**

For every number $a \geq 0$ and $b > 0$ and positive integer n

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \text{ and } \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

- **A radical of a radical property its converse**

a) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = a^{\frac{1}{nm}}$

In fact,

$$\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{m} \times \frac{1}{m}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$$

But it must be carefully noted that:

For nonzero a and b : $\sqrt[n]{a \pm b} \neq \sqrt[n]{a} \pm \sqrt[n]{b}$

1. We call n^{th} power of a real number a that we note a^n, the product of n factors of a. that is

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}} \quad \begin{cases} n \text{ is an exponent} \\ a \text{ is the base} \end{cases}$$

2. The n^{th} root of a real number is $\frac{1}{n}$ power of that real number.

It is noted by $\sqrt[n]{b}$, $b \in \mathbb{R}, n \in \mathbb{N} \setminus \{1\}$.

$$\forall a, b \in \mathbb{R}, \sqrt[n]{b} = a \Leftrightarrow b^{\frac{1}{n}} = a \Leftrightarrow b = a^n$$

$$\begin{cases} n \text{ is called the index} \\ b \text{ is called the base or radicand} \\ \sqrt[n]{\ } \text{ is called the radical sign} \end{cases}$$

3. Rationalizing is to convert a fraction with an irrational denominator to a fraction with rational denominator. To do this, if the denominator involves radicals we multiply the numerator and denominator by the conjugate of the denominator.

The decimal logarithm of a positive real number x is defined to be a real number y for which 10 must be raised to obtain x. We write $\forall x > 0, \quad y = \log x$

The compound interest formula

The accumulated amount of money A after the time t (number of years P is invested) is given by:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ where n is the number of interest periods per year, r is the interest rate expressed as decimal, A is the amount after t years.

When the interest rate is compounded monthly, $A = P\left(1 + \frac{r}{12}\right)^{12.t}$ where t is the number of years and r expressed as a decimal.

If we take $t = 1$, a general formula to solve for n can be derived as follows from the final sum formula:

$A = P(1+r)^n$, $\frac{A}{P} = (1+r)^n$ and $n = \frac{\log(A/P)}{\log(1+r)}$.

7.7 Additional Information for tutors

Some misconceptions:

1. $\log_a x \pm y \neq \log_a(x \pm y)$ because $\log_a x \pm y = (\log_a x) \pm y$

2. $\log_a(x \pm y) \neq \log_a(x) \pm \log_a(y)$

3. $\log_a x^{-1} \neq \frac{1}{\log_a x}$ because $\log_a x^{-1} = \log_a \frac{1}{x}$ while $(\log_a x)^{-1} = \frac{1}{\log_a x}$

4. $(\log_a x)^n \neq \log_a x^n$ because $(\log_a x)^n = \log_a x \times \log_a x \times \log_a x \times \ldots \times \log_a x$ while

$\log_a x^n = n \log_a x$

5. $\sqrt{x \pm y} \neq \sqrt{x} \pm \sqrt{y}$

The compound interest formula for different interest periods

To find the new amount of principal after one year if the interest is compounded quarterly, monthly, weekly, daily hourly and each minute, one can use the compound interest formula (and a calculator).

Interest period	Quarter	Month	Week	Day	Hour	Minute
n	4	12	52	365	8760	525,600

As the time is one year, this formula is $A = P\left(1+\frac{r}{n}\right)^n$.

Table below summarize on how the formula applied in the above interest period.

Interest period	**Amount after one year**	**Amount after t years**
Quarter	$A = P\left(1+\frac{r}{4}\right)^4$	$A = P\left(1+\frac{r}{4}\right)^{4t}$
Month	$A = P\left(1+\frac{r}{12}\right)^{12}$	$A = P\left(1+\frac{r}{12}\right)^{12t}$
Week	$A = P\left(1+\frac{r}{52}\right)^{52}$	$A = P\left(1+\frac{r}{52}\right)^{52t}$

Day	$A=P\left(1+\frac{r}{365}\right)^{365}$	$A=P\left(1+\frac{r}{365}\right)^{365t}$
Hour	$A=P\left(1+\frac{r}{8760}\right)^{8760}$	$A=P\left(1+\frac{r}{8760}\right)^{8760t}$
Minute	$A=P\left(1+\frac{r}{525600}\right)^{525600}$	$A=P\left(1+\frac{r}{525600}\right)^{525600t}$

7.8 End unit assessment

Answers:

1. a) ab^2c b) abc c) $\frac{2}{3}$ d) x e) $\frac{xy^4}{2}$

2. a) $\log_b X+\log_b Y-\log_b Z$ b) $\log_b X-\log_b Y-\log_b Z$

c) $2\log_b P+\frac{1}{3}\log_b Q$ d) $\frac{1}{2}\log_b P+\frac{3}{2}\log_b Q-\frac{1}{4}\log_b R-\frac{1}{2}\log_b S$

3. a) $\log_b\frac{xy}{z^2}$ b) $\log_b 2\pi\sqrt{\frac{l}{g}}$

4. a) $c=10cm$ b) $b=4cm$ c) $a=5cm$

7.9 Additional activities

7.9.1 Remedial activities

1) Suppose that \$1000 is invested at an interest rate of 9% compounded monthly. Find the new amount of principal after 5 years, after 10 years, and after 15 years, calculate the amount after those periods of time.

Solutions: we find that the amount after time t is given by $A=P\left(1+\frac{r}{4}\right)^{4t}$

After 5 years: $A=\$1000\left(1+\frac{0.09}{12}\right)^{12\times5}=\$1000(1.0075)^{60}=\$1565.68$

After 10 years, $A=\$1000\left(1+\frac{0.09}{12}\right)^{12\times10}=\$1000(1.0075)^{120}=\$2451.36$

After 15 years, $A = \$1000\left(1+\dfrac{0.09}{12}\right)^{12\times 15} = \$1000(1.0075)^{180} = \$3838.04$

2) Simplify the following

a) $\sqrt{46656} = \sqrt{6^6} = 6^3 = 216$

b) $\sqrt[3]{ab}\times\sqrt[3]{a^2b^2} = \sqrt[3]{a^3b^3} = \sqrt[3]{(ab)^3} = ab$

c) $\sqrt{\dfrac{36}{81}} = \dfrac{\sqrt{36}}{\sqrt{81}} = \dfrac{6}{9} = \dfrac{2}{3}$

d) (i) $(x-1)(x-1)$ ii) $x.x.x.x.x$

iii) $\dfrac{x^{-3}y^5}{y^{-2}x^3}$ iv) $\sqrt[3]{\sqrt{\dfrac{x^6y^{12}}{z^3}}}$

e) Expand and simplify the following logarithms in terms of their composites.

i) $\log_2\frac{1}{2}$ ii) $\log_3 3xyz$ iii) $\log_p P^2$ iv) $\log_b\sqrt{\frac{Q^3}{S}}$

f) Express each expression below as simple logarithm

i) $\log_a b + 2\log_a a - \log_a c$ ii) $\log_b 2 + \log_b \pi$

7.9.2 Consolidation activities

1) The number N of bacteria present in a culture at time t (in hours) 0.01t obeys the function N (t) = $1000e^{0.01t}$.

a) Determine the number of bacteria at $t = 0$ *hour*

b) What is the growth rate of the bacteria?

c) What is the population after 4 hours?

d) When will the number of bacteria reach 1700?

e) When will the number of bacteria double?

Solution

a) at $t = 0$, N (0) = 1000 bacteria's

b) Growth rate of the bacteria is 0.01

c) $N(4) = 1000e^{0.04} = 1040.8\ bacterias$

d) $1700 = 1000e^{0.01t} \Rightarrow 1.7 = 0.01t$

$\Rightarrow t = \dfrac{ln1.7}{0.01} = 53\ hours$

e) $2000 = 1000e^{0.01t} \Rightarrow\ t = \frac{ln2}{0.01} = 69.3\ hours$

2) Rationalise $\dfrac{\sqrt{3}+\sqrt{7}}{4\sqrt{2}}$

Solution: $\dfrac{\sqrt{3}+\sqrt{7}}{4\sqrt{2}} = \dfrac{\left(\sqrt{3}+\sqrt{7}\right)\sqrt{2}}{\left(4\sqrt{2}\right)\sqrt{2}} = \dfrac{\sqrt{6}+\sqrt{14}}{8}$

3) Expand the following logarithms in terms of their composites.

a) $\log_b \frac{bt}{z}$ b) $\log_b \frac{x}{yb}$ c) $\log_b b^2 \cdot \sqrt[3]{b}$ d) $\log_b \sqrt{\frac{b^3}{b^{\frac{1}{2}}}}$

Solution:

a) $\log_b \dfrac{bt}{z} = \log_b b + \log_b t - \log_b z = 1 + \log_b t - \log_b z$

b) $\log_b \dfrac{x}{yb} = \log_b x - \log_b y - 1$

c) $\log_b b^2 \sqrt[3]{b} = \log_b b^2 + \log_b \sqrt[3]{b}$

$= \log_b b^2 + \log_b b^{\frac{1}{3}}$

$= 2\log_b b + \dfrac{1}{3}\log_b b$

$= 2 + \dfrac{1}{3} = \dfrac{7}{3}$

$d)\log_b \sqrt{\dfrac{b^3}{b^{\frac{1}{2}}}} = \log_b \left(\dfrac{b^3}{b^{\frac{1}{2}}}\right)^{\frac{1}{2}}$

$= \dfrac{1}{2}\log_b \dfrac{b^3}{b^{\frac{1}{2}}}$

$= \dfrac{1}{2}\left(\log_b b^3 - \log_b b^{\frac{1}{2}}\right)$

$= \dfrac{3}{2}\log_b b - \dfrac{1}{4}\log_b b$

$= \dfrac{3}{2} - \dfrac{1}{4} = \dfrac{5}{4}$

4) Express each expression below as simple logarithm

a) $\log_b xy^3 - 2\log_b xy + \log_b z$ b) $\log_b 2 + \log_b \pi + \frac{4}{5}\log_b k - \frac{2}{5}\log_b gh$

Solution:

$$a)\log_b xy^3 - 2\log_b xy + \log_b z$$

$$= \log_b xy^3 - \log_b x^2y^2 + \log_b z$$

$$= \log_b \frac{xy^3}{x^2y^2} + \log_b z$$

$$= \log_b \frac{y}{x} + \log_b z$$

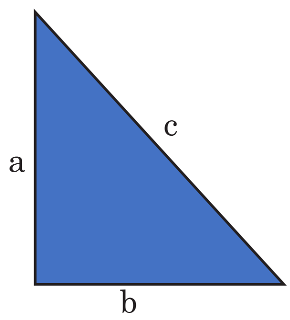
$$= \log_b \frac{yz}{x}$$

$$b)\log_b 2 + \log_b \pi + \frac{4}{5}\log_b k - \frac{2}{5}\log_b gh = \log_b 2\pi k^{\frac{4}{5}} - \log_b (gh)^{\frac{2}{5}}$$

$$= \log_b \frac{2\pi k^{\frac{4}{5}}}{(gh)^{\frac{2}{5}}}$$

$$= \log_b \frac{2\pi\sqrt[5]{k^4}}{\sqrt[5]{(gh)^2}}$$

5) Use the right triangle below. Find the missing length(keep your answer in cm)



a) a=12cm, b = 8cm

b) a = 0.36dm, c = 4.4cm

$a)\,c = \sqrt{208}cm$

$b)\,b = \sqrt{6.4}cm$

7.9.3 Extended activities

1) Rationalise

a) $\dfrac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

Solution:

$$\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{2}\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)} = \frac{\sqrt{10}+\sqrt{6}}{5-3} = \frac{\sqrt{10}+\sqrt{6}}{2}$$

2) Expand and simplify the following logarithms in terms of their composites.

a) $\log_b \dfrac{\sqrt[3]{xy^6z^{10}}}{\sqrt[3]{zx^4y^3}}$ b) $\log_b \sqrt{\left(\frac{P}{Q}\right)^3} \cdot \sqrt[3]{\frac{P}{Q}}$ c) $\log_b \sqrt[3]{\dfrac{P\cdot Q^3}{R^{\frac{1}{2}}S}}$

Solution:

$a)\,\log_b \dfrac{yz^{3}}{x}$ $b)\log_b P^{\frac{11}{6}}$ $c)\,P^{\frac{1}{3}}Q^{1}R^{-\frac{1}{6}}S^{-\frac{1}{3}}$

3) Evaluate and simplify

a) $\sqrt{3}\log_b b^2 - 2(\log_b b^5 + \log_z z^{\sqrt{3}})$ b) $\sqrt{3}\log_b b^2 - \frac{1}{\sqrt{3}}(\log_b b^5 + \log_z z^{\sqrt{3}})$

Solution:

$a)\,-10$ $b)\dfrac{\sqrt{3}}{3}-1$

4) Express each of the expressions in simplest form.

a. $4\sqrt{2}+3\sqrt{2}-2\sqrt{2}$

b. $6\sqrt{3}-\sqrt{27}$

c. $\sqrt{3}\cdot\sqrt{15}$

d. $2\sqrt[3]{5}-\sqrt[3]{135}+4\sqrt[6]{25}$

e. $\sqrt[3]{18}\cdot\sqrt[3]{5}$

f. $\sqrt[4]{64}-5\cdot\sqrt[6]{\frac{1}{8}}$

g. $2\frac{1}{\sqrt{7}}+3\sqrt{28}-\sqrt{63}$

Solution:

$a)\,5\sqrt{2}\quad b)3\sqrt{3}\quad c)\,3\sqrt{5}\quad d)\,3\sqrt{5}\quad e)\,\sqrt[3]{90}\quad f)-\frac{\sqrt{2}}{2}\quad g)\,\frac{23\sqrt{7}}{7}$

5) Write each answer as a power of 2.

a) Computer capacity is often measured in bits and bytes. A bit is the smallest unit, a 1 or 0 in the computer's memory. A byte is 2^3 bits. A megabyte (MB) is 2^{20} bytes. How many bits are in a megabyte?

b) A gigabyte (GB) is 2^{10} megabytes. How many bytes are there in a gigabyte? How many bits are there in a gigabyte?

Solution:

$a)\,1MB \quad \to 2^{23}\,bits$

$b)\,1GB \quad \to 2^{33}\,bits$

6) Explain why $x^8\cdot x^2$ has the same value as $x^5\cdot x^5$.

Solution:

$x^8.\,x^2 = x^{8+2} = x^{10}$

$x^5.\,x^5 = x^{5+5} = x^{10}$

7) Mugisha thinks that e^3+e^3 simplifies to $2e^3$. Rukundo thinks that e^3+e^3 simplifies to e^6. Which result is correct? Explain

Solution:

Mugisha is right because

$e^3+e^3=2e^3 \quad \Rightarrow e^3(1+1)=2e^3$

8) Explain why $(-xy)^2=(xy)^2$.

Solution:

$(-xy)^2 = (-1)^2 (xy)^2 = (xy)^2$

9) Rationalize the denominator

a) $\frac{5}{\sqrt{7}}$ b) $\frac{3-2\sqrt{2}}{1-\sqrt{2}}$ c) $\frac{2\sqrt{2}}{4+3\sqrt{3}}$

d) $\frac{a-\sqrt{b}}{\sqrt{d}}$ e) $\frac{3\sqrt{3}+2\sqrt{2}}{1+2\sqrt{2}}$

Solution:

$a)\frac{5\sqrt{7}}{7}$ $b)1-\sqrt{2}$ $c)\frac{6\sqrt{6}-8\sqrt{2}}{11}$ $d)\frac{a\sqrt{d}-\sqrt{bd}}{d}$ $e)\frac{8+6\sqrt{6}-2\sqrt{2}-3\sqrt{3}}{7}$

10) Let $P = b^p$ and $Q = b^q$, then $\log_b P = p$ and $\log_b Q = q$. Show that:

a) $P \cdot Q = b^{p+q}$ and $P \cdot Q = \log_b P + \log_b Q$

b) $\frac{P}{Q} = b^{p-q}$ and $\frac{P}{Q} = \log_b P - \log_b Q$

c) $P^n = b^{np}$ and $P^n = n \log_b P$

d) $\sqrt[n]{P} = b^{\frac{p}{n}}$ and $\sqrt[n]{P} = \frac{1}{n} \log_b P$

Solution : these are properties of logarithms

UNIT: 8

PARAMETERS OF CENTRAL TENDENCES AND DISPERSION

8.1 Key unit competence

Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to the standard deviation.

8.2 Prerequisites

Student-teachers will easily learn this unit if they refer to the elementary statistics learnt in ordinary level (unit 8 of senior 1).

8.3 Cross-cutting issues to be addressed

- Inclusive education (promote education for all while teaching)
- Peace and value Education (respect others' view and thoughts during class discussions)
- Gender (provide equal opportunity to boys and girls in the lesson)

8.4 Guidance on introductory activity

- Invite student-teachers to work in groups and give them instructions on how they can do the introductory activity 8 found in unit 8 of student-teacher's book;
- Guide student-teachers to read and analyse the questions insisting on the analysis of data, the more repeated value and how they can find the mean;
- Invite some group members to present groups' findings, then try to harmonize their answers;
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, use different questions to facilitate them to give their predictions and ensure that you arouse their curiosity on what is going to be leant in this unit.

Answers for the introductory activity:

The table below shows the types and the number of sold fruits in one week.

Type of fruit	A (Banana)	B (Orange)	C (Pineple)	D (Avocado)	E (Mango)	F (apple)
Number of fruits sold	1100	962	1080	1200	884	900

a) The highest number of fruits sold is 1200 (Avocadoes)

b) The least number of fruits sold is 884 (mangoes)

c) The total number of fruits sold during the week is 6126 fruits

d) The average number of fruits sold per day is $\frac{6126}{6}=1021$

2.

a) The mean mark of the class is $\frac{3+5+6+3+8+7+8+4+8+6}{10}=\frac{58}{10}=5.8$.

b) The mark that was obtained by many student-teachers is 8

c) Comparing the mean mark of the class and the mark for every student-teacher, one can find that 4 student-teachers have the marks (3 , 4 and 5) below the mean, 2 student-teachers scored the mark near the mean while 4 student-teachers have scored higher marks than the mean. Mathematics tutor should prepare remedial activities for student-teachers whom their marks are below and near the mean.

8.5 List of lessons and sub-headings

#	**Lesson title**	**Learning objectives**	**Number of periods**
0	Introductory activity	To arouse the curiosity of student teachers on the content of unit 8	1
1	Collection and organization of ungrouped data	Collect and organize ungrouped data	2
2	Collection and organization of grouped data	Collect and organize data in groups or classes with frequency distribution	2

3	Central tendencies (mean, median, mode)	Determine the measures of dispersion of a given statistical series.	3
4	Graphical representation of grouped and ungrouped data	Represent data using graphs, interpret them critically and take conclusion	4
5	Measure of dispersion (range, variance, Standard Deviation and coefficient of variation)	Define the variance, standard deviation and the coefficient of variation and their role during the interpretation of data	6
6	Application of statistics in daily life	Discover the importance of statistics in personal or national daily life planning.	4
7	End unit assessment.		2
Total			**24**

Lesson 1: Collection and organization of ungrouped data

a) Learning objectives

Collect and organize ungrouped data.

b) Teaching resources

Student-teacher's book and other Reference books to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils…..

c) Prerequisites/Revision/Introduction

Student-teachers will learn better in this lesson if they refer to the lesson on data presentation learnt in unit 8 of senior 1.

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 8.1.1 found in their Mathematics books;
- Move around in the class for facilitating student-teachers where necessary and give more clarification on eventual challenges they may face during their work;
- Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to

present their work where they must explain the working steps;

- As a tutor, harmonize the findings from presentation;.
- Use different probing questions and guide them to explore the content and examples given in the student-teacher's book and lead them to discover how to organize ungrouped statistical data: frequency distribution, Cumulative frequency, Stem and Leaf Plots, etc.
- After this step, guide student-teachers to do the application activity 8.1.1 and evaluate whether lesson objectives were achieved.

Answer for activity 8.1.1

Question1:

Age	15	16	17	18	19	20	21	22	23	24
Number of student-teachers	2	3	1	2	1	1	2	2	0	1

Question2:

Cathegorical data	Numerical data
Arm span Dominant hand reaction time Favourite sport Language mostly spoken at home Opinions on environmental conservation State/Territory live in Travel method to school Year level	Amount of money earned last week Birthdate Height Hours slept per night Foot length School post code Travel time to school

e) Answer of Application activity 8.1.1.

1. Below is frequency distribution table

Age	8	9	10	11	12	13	14	15	16	17	18
Number of student-teachers	2	9	11	8	2	2	1	3	4	4	4

2. Below is the table that shows some examples of qualitative and quantitative data

Qualitative data	Quantitative data
• Product rating, • Basketball team classification.	• Number of student-teachers in the classroom, • Weight, • Age, • Number of rooms in a house, • Number of teachers in school.

Lesson 2: Collection and organization of grouped data

a) Learning objectives

Collect and organize data in groups or classes with frequency distribution.

b) Teaching resources

Student-teacher's book and other reference books to facilitate research, Mathematical set, calculator, Manila paper, markers, pens, pencils, etc.

c) Prerequisites/Revision/Introduction

Student-teachers will perform well in this lesson if they refer to the lesson on data representation learnt in unit 10 of S2.

d) Learning activities

- Invite student-teachers to work in groups and do the activity 8.1.2 found in their Mathematics books;
- Move around in the class for facilitating student-teachers where necessary and give more clarification on eventual challenges they may face during their work;
- Let student-teachers discover the reason why it is necessary to group data and how data are grouped into classes;
- Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work where they must explain working steps.
- Use different probing questions and guide them to explore the content and examples given in the student-teacher's book and lead them to discover how to organize a frequency distribution for grouped statistical data;

- After this step, guide student-teachers to do the application activity 8.1.2 and evaluate whether lesson objectives were achieved.

Answer of activity 8.1.2

The table below shows a frequency distribution table of mass of tomatoes.

Mass (g)	84.5-89.5	89.5-94.5	94.5-99.5	99.5-104.5	104.5-109.5	109.5-114.5	114.5-119.5
Number of tomatoes	4	7	6	13	10	5	5

e) Answers of application activity 8.1.2

The table below shows a frequency distribution table of distances are between department store and employees 'homes.

Distance(km)	1-3	4-6	7-9	10-12	13-15	16-18
Number of tomatoes	10	14	10	6	5	5

Lesson 3: Measures of Central tendency for grouped and ungrouped data: mean, median, mode

a) Learning objective

Determine the measures of central tendency for grouped and ungrouped statistical data.

b) Teaching resources

Student-teacher's book and other reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, spreadsheet (Microsoft excel).

Prerequisites/Revision/Introduction

Student-teachers should have basic knowledge and skills on how to present simple data on a frequency distribution and other content knowledge learnt in senior one unit8, senior two unit 10, and senior three unit13.

Learning activities

- In invite student-teachers to work in groups and do the activity 8.2;
- Organize a whole class discussion where representative of groups present and explain their findings;
- As a tutor harmonize the findings from presentation of student-teachers and guide them to highlight the meaning of Mean, Mode, Median and their role when interpreting ungrouped statistical data;
- Use different probing questions and guide them to explore examples given in the student-teacher's book and lead them to discover the different ways finding measures of central tendency for ungrouped data and grouped data;
- After this step, guide student-teachers to do the application activity 8.2 and evaluate whether lesson objectives were achieved.

Answer of activity 8.2

Question 1

Marks (x)	Frequency (f)	f x
3	2	6
4	3	12
5	2	10
6	1	6
7	1	7
10	1	10
Total	10	41

i) $Mean = \frac{\sum xf}{\sum f} = \frac{41}{10} = 4.1$

ii) $Median = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2} = \frac{x_5 + x_6}{2} = \frac{4+5}{2} = 4.5$

iii) $Mode = 4$

iv) $Range = 10 - 3 = 7$

Question2:

a) $3marks$ obtained by 29 student-teachers**.**

b) Mean marks is given by the sum of product of xf divided by sum of frequencies.

$$Mean = \frac{\sum xf}{\sum f}$$

e) Answers for application activity 8.2

1. $Mean = 3.48$ $\quad$ $Mode = 4$ $\quad$ $Median = 3.5$

2. $Mean = 8.2$ $\quad$ $Mode = 10$ $\quad$ $Median = 4$

Lesson 4: Graphical representation of grouped and ungrouped data

a) Learning objective

Represent data using graphs, interpret them critically and take conclusion.

b) Teaching resources

Student-teacher's book and other reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils, …

c) Prerequisites/Revision/Introduction

Student-teachers should have knowledge and skills on how to present data on a frequency distribution and other content knowledge learnt in senior one unit 8, senior two unit 10, and senior three unit 13.

d) Learning activities

- Invite student-teachers to work in groups and do the activity 8.3 found in their Mathematics books;
- Move around in the class for facilitating student-teachers where necessary and give more clarification on eventual challenges they may face during their work;
- Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to

present their work where they must explain the working steps;

- As a tutor, harmonize the findings from presentation;.
- Use different probing questions and guide them to explore the content and examples given in the student-teacher's book and lead them to discover the different ways of presenting graphically ungrouped and grouped data: Pie chart, Bar graph, Histogram and Frequency Polygons;
- After this step, guide student-teachers to do the application activity 8.3 and evaluate whether lesson objectives were achieved

Answer of activity 8.3

1) the number of student-teachers with small size is 4
   the number of student-teachers with medium size is 5
   the number of student-teachers with large size is 8
   the number of student-teachers with extra-large size is 13

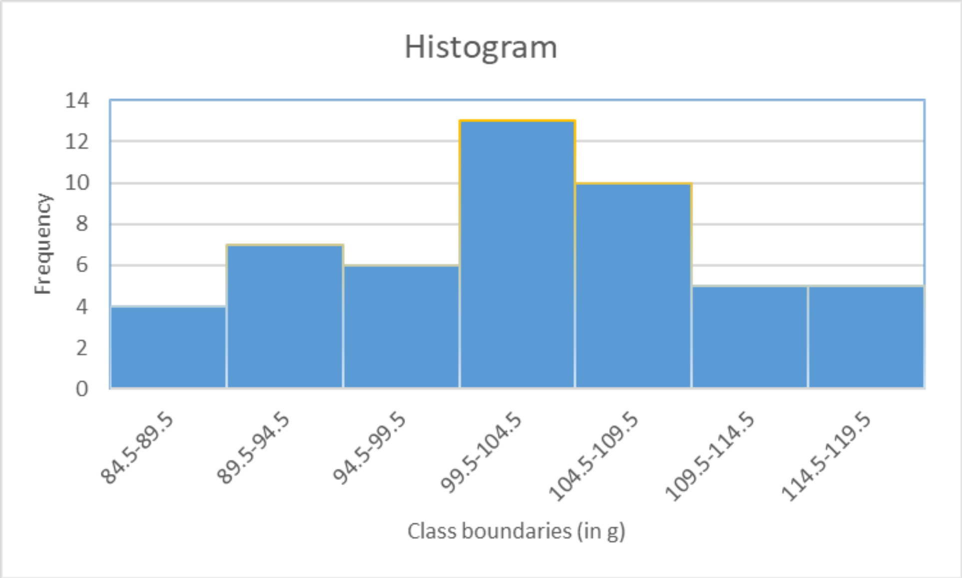
the graph is a Bar chart.

2) The mass of 50 tomatoes

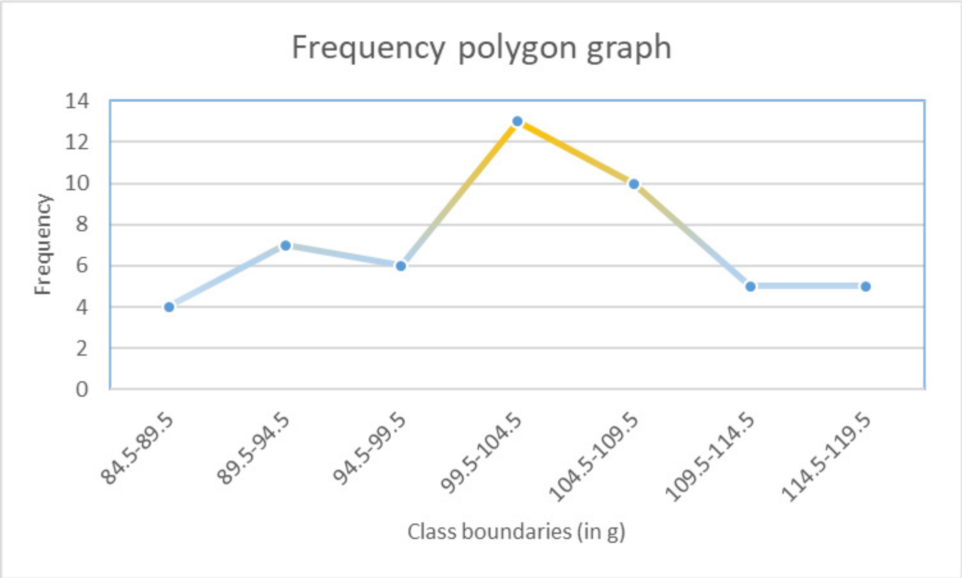
The table below shows cumulative frequency.

Class boundaries (in g)	84.5-89.5	89.5-94.5	94.5-99.5	99.5-104.5	104.5-109.5	109.5-114.5	114.5-119.5
Frequency	4	7	6	13	10	5	5
Cumulative frequency	4	11	17	30	40	45	50

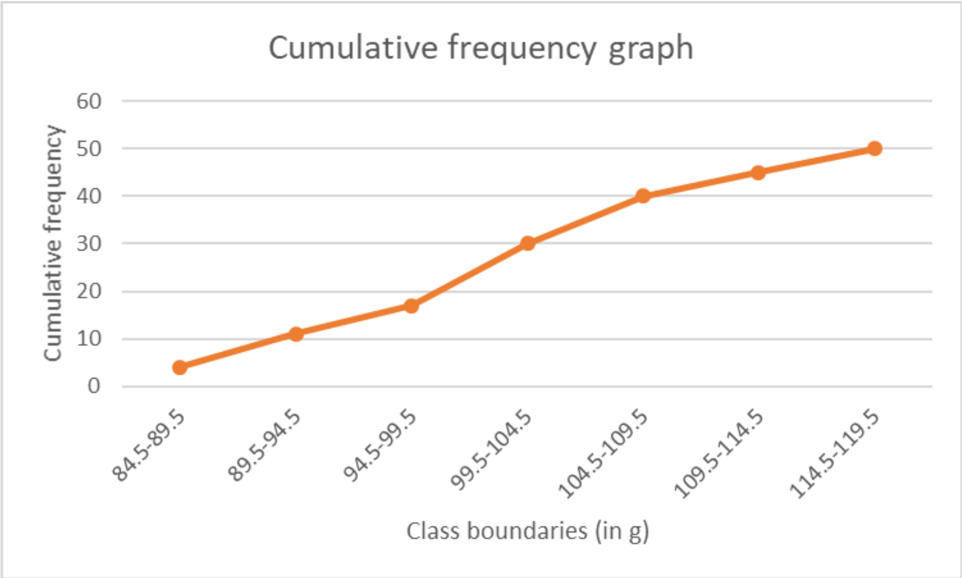
i) Histogram



ii) Frequency polygon

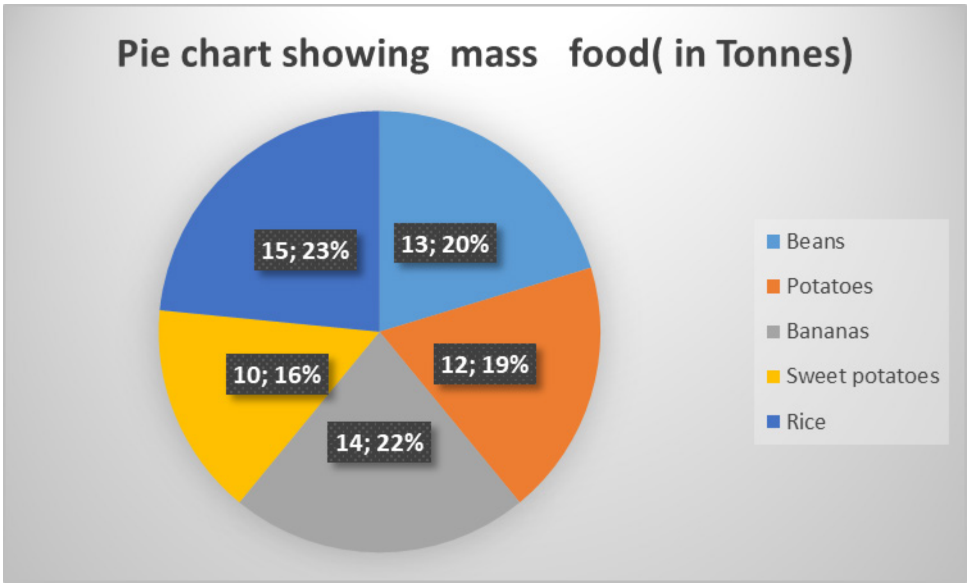


iii) Cumulative frequency polygon



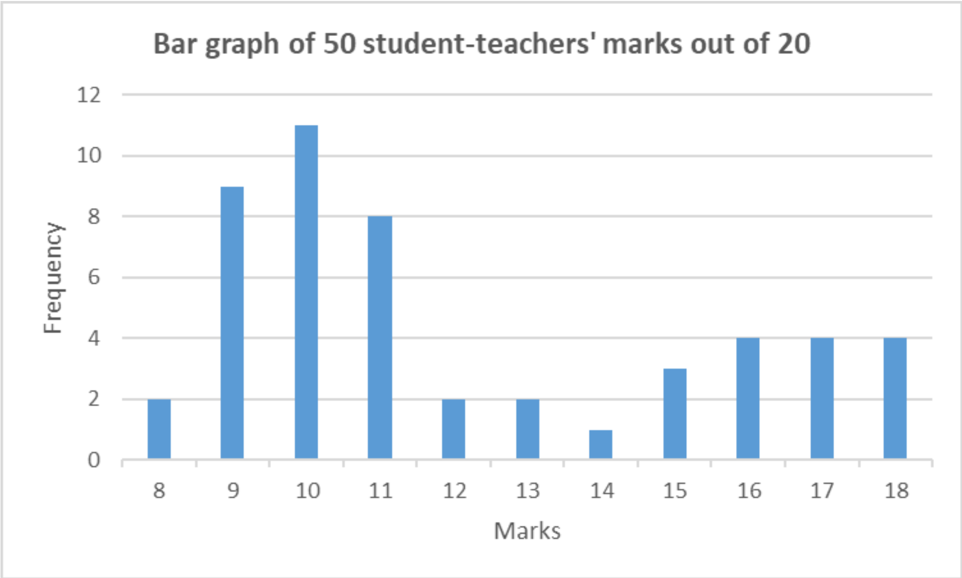
3) Frequency distribution table

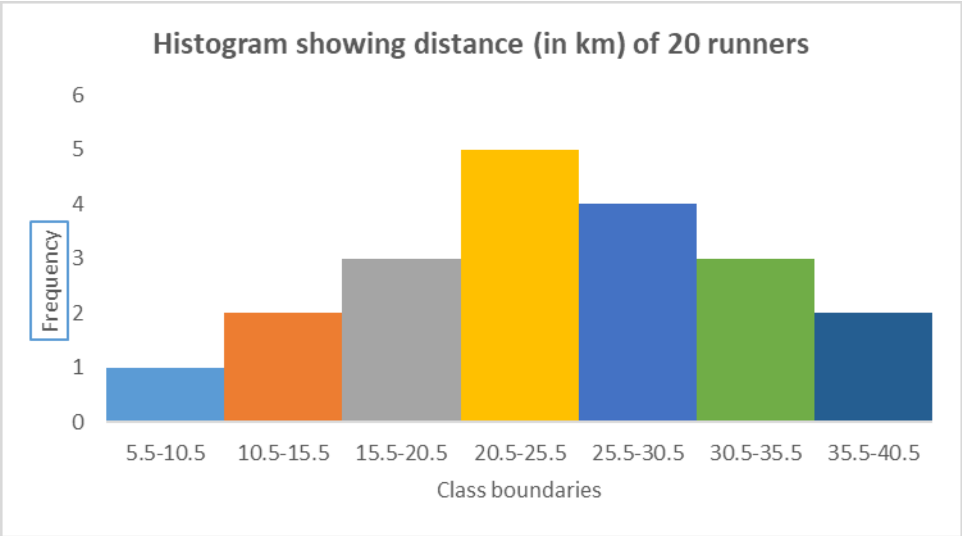
Type of food	Beans	Potatoes	Bananas	Sweet potatoes	Rice
Mass (Tonnes)	13	12	14	10	15

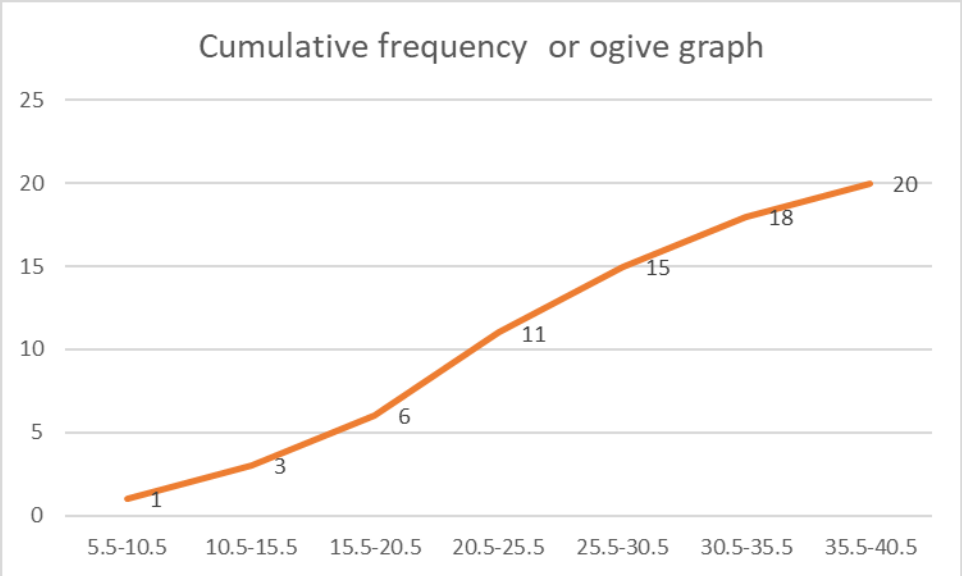


e) Answers of Application activity 8.3

1. **Bar graph of 50 student-teachers' marks out of 20**







Lesson 5: Measure of dispersion: range, variance, standard deviation and coefficient of variation

a) Learning objective

Define the variance, standard deviation and the coefficient of variation and their role during the interpretation of data.

b) Teaching resources

Student's book and other reference textbooks to facilitate research, Mathematical set, calculator, Manila paper, markers, pens, pencils, etc.

c) Prerequisites/Revision/Introduction

Student-teachers should have knowledge and skills on how to present data on a frequency distribution and other content knowledge learnt in senior one unit 8, senior two unit 10, and senior three unit 13.

d) Learning activities

- Invite student-teachers to work in small group discussions and use calculators and to do activity 8.4 in their Mathematics books related to measures of dispersion for ungrouped data and for grouped data;
- Move around in the class for facilitating and give more clarification on eventual challenges they may face during their practices on activity 8.4;
- Ask neighboring groups of student-teachers to share their answers for improvement;
- Invite each group to present their findings and working steps to the

whole class discussion;

- As a tutor, harmonize the findings from presentation highlighting how to avoid misconception when completing the table;
- Through the use of probing questions and examples given in the student-teacher's book, guide student-teachers to use data from the completed tables to determine the variance, the standard deviation for ungrouped data;
- From the table completed above, ask student-teachers to deduce how to complete the table to be used when calculating the variance and the standard deviation for the grouped data;
- Invite student-teachers to brainstorm on other measures of dispersion, how to determine them and guide them to discover the role of each measure when interpreting statistical data.
- After this step, guide student-teachers to do the application activity 8.4 and evaluate whether lesson objectives were achieved.

Answer of activity 8.4

For $\bar{x} = 16.875$

x	f	$x-\bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
12	4	−4.875	23.765625	95.0625
13	2	−3.875	15.015625	30.03125
15	1	−1.875	3.515625	3.515625
19	4	2.125	4.515625	18.0625
21	5	4.125	17.015625	85.078125
	$\sum f = 16$			$\sum f(x-\bar{x})^2 = 231.75$

e) Answers of Application activity 8.4

1) $Me = \dfrac{64+49}{2} = 56.5$

2) For $\overline{X} = \dfrac{54+55+55+56+57+58+59}{7} = 56.285$

x	f	$x-\overline{x}$	$\left(x-\overline{x}\right)^2$	$f\left(x-\overline{x}\right)^2$
54	1	−2.285	5.221225	5.221225
55	2	−1.285	1.651225	3.30245
56	1	−0.285	0.081225	0.081225
57	1	0.715	0.511225	0.511225
58	1	1.715	2.941255	2.941255
59	1	2.715	7.371225	7.371225
	$\sum f = 7$			$\sum f\left(x-\overline{x}\right)^2 = 19.428575$

Therefore, the standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2}{n}}$$

$$\sigma = \sqrt{\frac{19.428575}{7}} \simeq 1.67$$

3) In the classroom of SME the first five student-teachers scored the following marks out of 10 in a quiz of Mathematics.

a) $\overline{X} = 5.8$ $\quad Mode = 5$ $\quad Median = 5.5$

b) $Q_1 = 5$ $\quad Q_2 = 5.5$ $\quad Q_3 = 8.25$

c) $Variance = 3.76$ $\quad \sigma = 1.94$ $\quad C.v = 33.45$

Lesson 6: Application of statistics in daily life

a) Learning objective

Discover the importance of statistics in personal or national daily life planning.

b) Teaching resources

Student-teacher's book and other reference textbooks to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils….

Note: where it is possible, student-teachers can use excel (spreadsheet) to deal with all concepts learnt in previous lessons of this unit.

c) Prerequisites/Revision/Introduction

Student teachers should have knowledge and skills on how to present data on a frequency distribution and other content knowledge learnt in senior one unit 8, senior two unit 10, and senior three unit13.

Student teachers will learn better in this lesson if they mastered the concepts learnt in the previous lessons of this unit.

d) **Learning activities**

- Invite student-teachers to work in group discussions and do activity 8.5 found in their Mathematics books: set a limited number of groups as every group will present its work;
- Move around and visit every group and give more clarification on the eventual challenges they may face when searching on the use of frequency distribution, graphs in statistics, the mean and the variance;
- Invite all groups to present their findings and motivate all student-teachers to be engaged in such a whole class discussion with their constructive comments;
- As a tutor, harmonize the answers and guide student-teachers to enhance the importance of statistics: in their everyday life, in the national planning, in the analysis of student-teachers' results (marks) etc;
- After this step, guide student-teachers to do the application activity 6.7 and evaluate whether lesson objectives were achieved.

Answers for activity 8.5

Answers may vary with the group, as a tutor, try to harmonize them and provide feedback.

1. **Importance of Mean, Mode, Median, Variance , standard deviation**

   A measure of central tendency is a single value that attempts to describe a set of data by identifying the central position within that set of data. The mean, median and mode are all valid measures of central tendency, but under different conditions, some measures of central tendency become more appropriate to use than others. The mean is essentially a model of your data set. It is the value that is most common. You will notice, however, that the mean is not often one of the actual values that you have observed in your data set. However, one of its important properties is that it minimises error in the prediction of any one value in your data set. That is, it is the value that produces the lowest amount of error from all other values in the data set. An important property of the mean is that it includes every value in your data set as part of the calculation. In addition, the mean is the only measure of central tendency where the sum of the deviations of each value from the mean is always zero

   Standard Deviation is a statistical term used to measure the amount of variability or dispersion around an average. Technically it is a measure of volatility. Dispersion is the difference between the actual and the average value. The larger this dispersion or variability is, the higher is the standard deviation.

   Finance and banking is all about measuring and managing risk and standard deviation measures risk (Volatility). Standard deviation is used by all portfolio managers to measure and track risk.

The standard deviation tells those interpreting the data, how reliable the data is or how much difference there is between the pieces of data by showing how close to the average all of the data is.

- *A low standard deviation means that the data is very closely related to the average, thus very reliable.*
- *A high standard deviation means that there is a large variance between*

the data and the statistical average, thus not as reliable.

- *Standard deviation and variance may be basic mathematical concepts, but they play important roles throughout the financial sector, including the areas of accounting, economics, and investing. In the latter, for example, a firm grasp of the calculation and interpretation of these two measurements is crucial for the creation of an effective trading strategy.*
- *Standard deviation and variance are both determined by using the mean of the group of numbers in question. The mean is the average of a group of numbers, and the variance measures the average degree to which each number is different from the mean. The extent of the variance correlates to the size of the overall range of numbers—meaning the variance is greater when there is a wider range of numbers in the group, and the variance is lesser when there is a narrower range of numbers.*

Some examples of situations in which standard deviation might help to understand the value of the data:

- *A class of student-teachers took a math test. Their teacher found that the mean score on the test was an 85%. She then calculated the standard deviation of the other test scores and found a very small standard deviation which suggested that most student-teachers scored very close to 85%.*
- *A class of student-teachers took a test in Language Arts. The teacher determines that the mean grade on the exam is a 65%. She is concerned that this is very low, so she determines the standard deviation to see if it seems that most student-teachers scored close to the mean, or not. The teacher finds that the standard deviation is high. After closely examining all of the tests, the teacher is able to determine that several student-teachers with very low scores were the outliers that pulled down the mean of the entire class's scores.*
- *An employer wants to determine if the salaries in one department seem fair for all employees, or if there is a great disparity. He finds the average of the salaries in that department and then calculates the variance, and then the standard deviation. The employer finds that the standard deviation is slightly higher than he expected, so he examines the data further and finds that while most employees fall within a similar pay bracket, three loyal employees who have been in the department for 20 years or more, far longer than the others, are making far more due to their longevity with the company. Doing the analysis helped the employer to understand the range of salaries of*

the people in the department.

2. Importance of statistical graphs

One goal of statistics is to present data in a meaningful way. That is where graphs can be invaluable, allowing statisticians to provide a visual interpretation of complex numerical stories.

Good graphs convey information quickly and easily to the user. Graphs highlight the salient features of the data. They can show relationships that are not obvious from studying a list of numbers. They can also provide a convenient way to compare different sets of data.

Different situations call for different types of graphs, and it helps to have a good knowledge of what types are available. The type of data often determines what graph is appropriate to use. Qualitative data, quantitative data, and paired data each use different types of graphs.

Some examples of graphs are the following:

- *Bar graph is a way to visually represent qualitative data. Data is displayed either horizontally or vertically and allows viewers to compare items, such as amounts, characteristics, times, and frequency.*
- *Pie chart is helpful when graphing qualitative data, where the information describes a trait or attribute and is not numerical. Each slice of pie represents a different category, and each trait corresponds to a different slice of the pie; some slices usually noticeably larger than others. By looking at all of the pie pieces, you can compare how much of the data fits in each category, or slice.*
- *A histogram in another kind of graph that uses bars in its display. This type of graph is used with quantitative data. Ranges of values, called classes, are listed at the bottom, and the classes with greater frequencies have taller bars.*
- *A stem and leaf plot breaks each value of a quantitative data set into two pieces: a stem, typically for the highest place value, and a leaf for the other place values. It provides a way to list all data values in a compact form.*

Expected answers on application activity 8.5

1. Answers may vary with the group, as a tutor, try to harmonize them and provide feedback.

Examples: To analyze the scores of athletes in a given competition.

These scores may be represented using the frequency distribution table, the mean, median and standard deviation of the scores can be calculated respectively. One can explain the role of these measures

2. Answers will vary from group to another. Try to organize a session where every group will have time to present its findings and others will ask questions and provide constructive feedbacks for learning purpose.

8.6 Summary of the unit

Statistics: The branch of mathematics that deals with the collection, presentation, interpretation and analysis of data.

Qualitative data

Qualitative data is a categorical measurement expressed not in terms of numbers, but rather by means of a natural language description.

Example of qualitative data	Possible categories or variables
- Marital status	- Single, married, divorced
- Gender	- Male, Female
- Pain level	- None, moderated, severe
- Colour	- Red, black, green, yellow

Quantitative data

Quantitative data is a numerical measurement expressed not by means of a natural language description, but rather in terms of numbers.

Discrete data represent items that can be counted; they take on possible values that can be listed out.

Continuous data represent measurements; their possible values cannot be counted and can only be described using intervals on the real number line.

Raw data

Data which have been collected in **original form**, they are called **raw data**

Frequency distribution

A frequency distribution is a table showing how often each value (or set of values) of the collected data occurs in a data set. A frequency table is used to summarize categorical or numerical data. Data presented in the form of a frequency distribution are called grouped data.

Cumulative frequency

The cumulative frequency corresponding to a particular value is the sum of all frequencies up to the last value including the first value. Cumulative frequency can also have defined as the sum of all previous frequencies up to the current point.

Stem and leaf displays

Is a plot where each data value is split into a leaf usually the last digit and a stem the other digit. The stem values are listed down, and the leaf values are listed next to them.

Collection and presentation of grouped data

When the range of data is large, the data must be grouped into classes that are more than one unit in width. In this case a grouped frequency distribution is used. Data in this case are grouped in a frequency distribution using groups **or classes.**

- **Class limits:** The class limits are the lower and upper values of the class
- **Lower class limit:** Lower class limit represents the smallest data value that can be included in the class.
- **Upper class limit:** Upper class limit represents the largest data value that can be included in the class.
- $\textit{class midpoint} = \dfrac{\textit{lower class limit} + \textit{upper class limt}}{2}$

Class boundaries: Class boundaries are the midpoints between the **upper class limit** of a class and **the lower class limit of the next class.** Therefore, each class has a lower and an upper class boundary.

Class width

Class width is the difference between the upper class boundary and lower class boundary

If the classes are presented in the form$[a,b[,[b,c[,[c,d[$

$\boldsymbol{Class\ limit = class\ boundary}$

Example

Classes	Class midpoint	Frequency
[5,10[	7.5	2
[10,15[	12.5	6
[15,20[	17.5	4
[2,25[	22.5	3
[25,30[	27.5	4
[30,35[	32.5	1

For $[10,15[$ $The\ lower\ class\ boundary\ is\ 10$, $The\ upper\ class\ boundary$ is 15

$\boldsymbol{Class\ width = 15 - 10 = 5}$

Measures of central tendency: Mean, mode , median and range of ungrouped data

Thus for any particular set of ungrouped data, it is possible to select some typical values or average to represent or describe the entire set such a descriptive value is referred to as a measure of central tendency or location such as mean, mode, median and range.

The median:

the data is arranged in order from the smallest to the largest, the middle number is then selected. This really the central number of the range and is called the median.

When total observation ($\sum fi = n$) is odd the median is given by

$Me \rightarrow \left(\frac{n+1}{2}\right)^{th}$ or $Me = x_{\frac{n+1}{2}}$ and read the number which located on this position. On the other side when n is even, $Me = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right]$

or $Median = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$ then the median is a half of the sum of number located on those two positions.

Examples

1. Calculate the median of the following numbers: 4, 5, 7,2, 1

Solution: Arrange data from lowest to highest number as 1, 2, 4, 5, 7

$$Me \rightarrow \left(\frac{n+1}{2}\right)^{th}$$

$$Me \rightarrow \left(\frac{5+1}{2}\right)^{th} = 3^{rd} \text{ Then, } Me = 4$$

2. Calculate the median of the following numbers: 4, 5, 7, 2, 1 and 8

Solution: arrange numbers as 1, 2, 4, 5, 7, 8. Total observation $(n) = 6$

since the total observation When n is even then, $Me = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right]$

$$Me = \frac{1}{2}\left[\left(\frac{6}{2}\right)^{th} + \left(\frac{6}{2}+1\right)^{th}\right] = \frac{1}{2}\left[(3)^{rd} + (4)^{th}\right]$$

$$Me = \frac{4+5}{2} = 4.5$$

The mean:

All the data is added up and the total divided by the number of items. This is called the mean and is equivalent to sharing out all data evenly.

Mean is given by $\bar{x} = \frac{1}{n}\sum xfi$

The mode:

The mode is the number that appears the most often from the set of data

The range

In the case of ungrouped data, the range of a set of observations is the difference in values between the largest and the smallest observations in the set.

Example

Calculate the mean, median, mode and range of the following set of numbers: 3, 4, 4, 6, 8, 5, 4, and 8

Solution: $\bar{x} = \frac{1}{n}\sum xfi$

x	fi	xfi
3	1	3
4	3	12
5	1	5
6	1	6
8	2	16
	$\sum fi = n = 8$	$\sum xfi = 42$

- The mean is given by

  $\bar{x} = \frac{1}{n}\sum xfi$, $\bar{x} = \frac{42}{8} = 5.25$

- The median: Arrange data first 3, 4, 4, 4, 5, 6, 8,8.

  Then the median $= \frac{4+5}{2} = 4.5$

- The mode is 4

- The range $8 - 3 = 5$

Mean, mode, median and range of grouped data

for any particular set of grouped data, it is possible to select some typical values or average to represent or describe the entire set such a descriptive value is referred to as a measure of central tendency or location such as mean, mode, median and range.

Mean

Mean is given by $\bar{x} = \frac{1}{n}\sum xfi$, where $n = \sum f$: the sum of frequencies of data.

Mode

$$Mode = L_m + c\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)$$

Specifically, these notation symbols used in above formula;

L_m is lower boundary of modal class.

C is class width: the difference between upper and lower boundary of

modal class ($C = U_m - L_m$)

f_m : frequency of modal class.

$\Delta_1 = f_m - f_b$

$\Delta_2 = f_m - f_a$ f_b is frequency followed by f_m and f_a is frequency follows f_m.

Median

$$Median = L_m + C\left(\frac{\frac{n}{2} - CF_b}{f_m}\right)$$

Specifically, these notation symbols used in above formula;

L_m is lower boundary of modal class.

C is class width: the difference between upper and lower boundary of modal class ($C = U_m - L_m$)

f_m : frequency of modal class

$n = \sum f$: the sum of frequencies of data.

CF_b : cumulative frequency proceeded by cumulative frequency of modal class (cumulative frequency before modal class)

Range

In the case of grouped data the range is defined as the difference between the upper limit of the highest class and the lower limit of the smallest class.

Graphical representation of grouped and ungrouped data

The most commonly used graphs are: Bar graph, Pie chart, Histogram, Frequency polygon, Cumulative frequency graph or Ogive.

A bar graph

A bar chart or bar graph is a chart or graph that presents numerical data with rectangular bars with heights or lengths proportional to the values that they represent. The bars can be plotted vertically or horizontally. A vertical bar chart is sometimes called a line graph.

Pie chart

A pie chart is used to display a set of categorical data. It is a circle, which is divided into segments. Each segment represents a particular category. The area of each segment is proportional to the number of cases in that category.

$$\text{Angle for sector S} = \frac{\text{Frequency of S} \times 360^0}{\text{Total frequency}}$$

Histogram

Histogram is a statistical graph showing frequency of data. The horizontal axis details the class boundaries, and the vertical axis represents the frequency. Blocks are drawn such that their areas (rather than their height, as in a bar chart) are proportional to the frequencies within a class or across several class boundaries. There are no spaces between blocks.

Frequency polygon

In a frequency polygon, a line is drawn by joining all the midpoints of the top of the bars of a histogram.

A frequency polygon gives the idea about the shape of the data distribution. The two end points of a frequency polygon always lie on the x-axis

Measure of dispersion (range, variance, Standard Deviation and coefficient of variation)

The word dispersion has a technical meaning in statistics. The average measures the center of the data. It is one aspect of observations. Another feature of the observations is how the observations are spread about the center. The observation may be close to the center or they may be spread away from the center. If the observation are close to the center (usually the arithmetic mean or median), we say that dispersion, scatter or variation is small. If the observations are spread away from the center, we say that dispersion is large.

The study of dispersion is very important in statistical data. If in a certain factory there is consistence in the wages of workers, the workers will be satisfied. But if some workers have high wages and some have low wages, there will be unrest among the low paid workers and they might go on strikes and arrange demonstrations. If in a certain country some people are very poor and some are very high rich, we say there is economic disparity. It means that dispersion is large.

The extent or degree in which data tend to spread around an average is also called the dispersion or variation. Measures of dispersion help us in studying the extent to which observations are scattered around the average or central value. Such measures are helpful in comparing two or more sets of data with regard to their variability.

Properties of a good measure of dispersion

i) It should be simple to calculate and easy to understand

ii) It should be rigidly defined

iii) Its computation be based on all the observations

iv) It should be amenable to further algebraic treatment

Some measures of dispersion are Quartiles, variance, Range, standard deviation, coefficient of variation

$$Q_1 \to \frac{1}{4}(n+1)^{th} \, or Q_1 = x_{\frac{n+1}{4}}. \qquad Q_2 \to \frac{1}{2}(n+1)^{th} \, or Q_2 = x_{\frac{n+1}{2}} = Me$$

$$Q_3 \to \frac{3}{4}(n+1)^{th} \, or Q_3 = x_{\frac{3(n+1)}{4}}$$

The inter-quartile range is given by the difference between third quartile and the first quartile.

Example 1

Find the first and the second quartiles of the data set: 1, 3, 4, 5, 5, 6, 9, 14, 21

Solution:

$$Q_1 \to \frac{1}{4}(n+1)^{th} = \frac{1}{4}(9+1)^{th} = (2.5)^{th},$$

$$\therefore Q_1 = 4$$

$$Q_2 = Me, \; Q_2 \to \frac{1}{2}(n+1)^{th} = \frac{1}{2}(9+1)^{th} = 5^{th}, \; Q_2 = 5$$

Variance

Variance measures how far a set of numbers is spread out. A variance of zero indicates that all the values are identical. Variance is always non-negative: a small variance indicates that the data points tend to be very close to the mean and hence to each other, while a high variance indicates that the data points are very spread out around the mean and from each other.

The variance is denoted and defined by

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n}$$

Developing this formula we have

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i^2 - 2x_i\overline{x} + \left(\overline{x}\right)^2\right)}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n}2\overline{x}\sum_{i=1}^{n} x_i + \frac{1}{n}\left(\overline{x}\right)^2\sum_{i=1}^{n} 1$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\overline{x}\,\overline{x} + \left(\overline{x}\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\overline{x}\right)^2$$

Thus, the variance is also defined by

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\overline{x}\right)^2$$

Recall that the mean of the set of n values $x_1, x_2, x_3, ..., x_n$ is denoted and defined by

$$\overline{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

$$= \sum_{i=1}^{n} \frac{x_i}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i$$

Example

Calculate the variance of the following distribution: 9, 3, 8, 8, 9, 8, 9, 18

Solution

$$\overline{x} = \frac{9+3+8+8+9+8+9+18}{8} = 9$$

$$\sigma^2 = \frac{(9-9)^2 + (3-9)^2 + (8-9)^2 + (8-9)^2 + (9-9)^2 + (8-9)^2 + (9-9)^2 + (18-9)^2}{8}$$

Standard deviation

The standard deviation has the same dimension as the data, and hence is comparable to deviations from the mean. We define the **standard deviation** to be the square root of the variance.

Thus, the standard deviation is denoted and defined by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n}} \text{ or } \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\overline{x}\right)^2}$$

The above results are used for the grouped data where x_i is the mid-interval value for the i^{th} group.

The following results follow directly from the definitions of mean and standard deviation:

- When all the data values are multiplied by a constant a, the new mean and new standard deviation are equal to a times the original mean and standard deviation. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $a\left(\overline{x}\right)$ and the standard deviation is $a\sigma$.
- When a constant value, b,is added to all data values, then new mean is increased by b. However standard deviation does not change. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $\overline{x}+b$ and the standard deviation is σ.

Coefficient of variation

The coefficient of variation measures variability in relation to the mean (or average) and is used to compare the relative dispersion in one type of data with the relative dispersion in another type of data. It allows us to compare the dispersions of two different distributions if their means are positive. The greater dispersion corresponds to the value of the coefficient of greater variation.

The coefficient of variation is a calculation built on other calculations: the standard deviation and the mean as follows:

$$Cv = \frac{\sigma}{\overline{x}} \times 100$$

Where σ is the standard deviation and $\bar{x}$ is the mean.

Example:

One data series has a mean of 140 and standard deviation 28.28. The second data series has a mean of 150 and standard deviation 24. Which of the two has a greater dispersion?

Solution:

$$Cv_1 = \frac{28.28}{140} \times 100 = 20.2\%$$

$$Cv_1 = \frac{24}{150} \times 100 = 16\%$$

The first data series has a higher dispersion.

Range

In the case of ungrouped data, the range of a set of observations is the difference in values between the largest and the smallest observations in the set. In the case of grouped data the range is defined as the difference between the upper limit of the highest class and the lower limit of the smallest class.

Example

Calculate the range of the following set of the data set: 1, 3, 4, 5, 5, 6, 9, 14 and 21

Solution: From the given series the lowest data is 1 and the highest data is 21

The Range = highest value − lowest value

Range = 21 − 1 = 20

8.7 Additional Information for tutors

- Emphasize on the following results follow directly from the definitions of mean and standard deviation:

- When all the data values are multiplied by a constant a, the new mean and new standard deviation are equal to a times the original mean and standard deviation. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $a\left(\overline{x}\right)$ and the standard deviation is $a\delta$.

- When a constant value, b,is added to all data values, then new mean is increased by b. However standard deviation does not change. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $\overline{x}+b$ and the standard deviation is σ.

• Emphasize on calculating median of ungrouped data. We need to clarify formula

for $n-oddn-odd$ and for $n-evenn-even$ as highlighted in the student-teachers book and paying attention on notation.

- When n is odd, the median is given by

$$Me \to \left(\frac{n+1}{2}\right)^{th} \text{ or } Me = x_{\frac{n+1}{2}} \quad \text{we don't write } Me = \left(\frac{n+1}{2}\right)^{th}$$

- When n is even, the median is given by

$$Me = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right] \text{ or } Median = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$$

• Emphasize on calculating mode and median of grouped data.

$$Mode = L_m + c\left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)$$

$$Median = L_m + c\left(\frac{\frac{n}{2} - CF_b}{f_m}\right)$$

Be specific on this notation symbols used in above formulae

L_m is lower boundary of modal class.

C is class width: the difference between upper and lower boundary of modal class $(C = U_m - L_m)$

f_m : frequency of modal class

$n = \sum f$: the sum of frequencies of data.

CF_b: cumulative frequency proceeded by cumulative frequency of modal class (cumulative frequency before modal class)

$\Delta_1 = f_m - f_b$

$\Delta_2 = f_m - f_a$

f_b is frequency followed by f_m and f_a is frequency follows f_m.

- We need to clarify formulae of quartiles

for $n-odd n-odd$ and for $n-even n-even$

$Q_1 \to \frac{1}{4}(n+1)^{th}$ or $Q_1 = x_{\frac{n+1}{4}}$. We don't write $Q_1 = \frac{1}{4}(n+1)^{th}$

$Q_2 \to \frac{1}{2}(n+1)^{th}$ or $Q_2 = x_{\frac{n+1}{2}} = Me$ we don't write $Q_2 = \frac{1}{2}(n+1)^{th}$

$Q_3 \to \frac{3}{4}(n+1)^{th}$ or $Q_3 = x_{\frac{3(n+1)}{4}}$ we don't write $Q_3 = \frac{3}{4}(n+1)^{th}$

8.8 End unit assessment 8

Answers of questions for the end unit assessment:

1) a) Mode is 70,000 Frw
   b) Range is 60,000 Frw
   c)

Monthly wage(Frw)	40,000	50,000	60,000	70,000	80,000	90,000	100,000
Number of workers	4	10	12	24	12	4	18

2) a) $\textbf{Mean} = 6.6$ $Q_1 = 6$ $Q_2 = 6.5$ $Q_3 = 8$ $interquartile\ range = 2$
   b) $\textbf{Variance} = 2.04 \textbf{Variance} = 2.04$ $Standard\ deviation \approx 1.43$
   c) $Coeff.of\ variance \approx 21.7$

8.9 Additional activities

8.9.1 Remedial activities

In test of French, 17 student-teachers got the following marks out of 17:

6,7,6,7,8,9,5,4, 8, 5, 7, 6, 6, 9, 4, 8,8.

a. Calculate the mean, mode, median, range and quartiles and interquartile range
b. Calculate the variance and standard deviation
c. Calculate the coefficient of variation.

Solution

$$a)\ i)\ \overline{X} = 6.65$$

$$ii)\ Mode = 6\ and\ 8$$

$$iii)\ Median = 7$$

$$iv)\ Q_1 = 6 \qquad Q_2 = 7 \qquad Q_3 = 8$$

$$b)\ i)\ Variance \rightarrow \sigma_X^2 = 2.35$$

$$ii)\ \text{stanndard deviation} \rightarrow \sigma_X = \sqrt{\sigma_X^2} = 1.53$$

$$c)\ \text{coefficient of variation} = CV = \frac{\sigma_X}{\overline{X}}.100 = 23\%$$

8.9.2 Consolidation activities

The six runners in a 200 meters race clocked times (in seconds) of 24.2, 23.7, 25.0, 23.7, 24.0,4.6

a) Find the mean and standard deviation of these times.
b) These readings were found to be 10% too low due to faulty timekeeping. Write down the new mean and standard deviation.
c) draw bar graph of the above information

Solution

a) $$\bar{x} = \frac{24.2+23.7+25.0+23.7+24.0+24.6}{6} = 24.2 \text{ seconds}$$

$$\sigma = \sqrt{\frac{(24.2-24.2)^2+(23.7-24.2)^2+(25.0-24.2)^2+(23.7-24.2)^2+(24.0-24.2)^2+(24.6-24.2)^2}{6}}$$

$$= 0.473 \text{ seconds}$$

b) We must divide each term 0.9 to find the correct time. The new mean is

$\bar{x} = \frac{24.2}{0.9} = 26.9$ sec . The new standard deviation is $\sigma = \frac{0.4726}{0.9} = 0.525$ sec

8.9.3 Extended activities

Hospital records indicated that maternity patients stayed in the hospital for the number of days shown in the distribution:

Number of days stayed	Frequency
3	15
4	32
5	56
6	19
7	5
Total	**127**

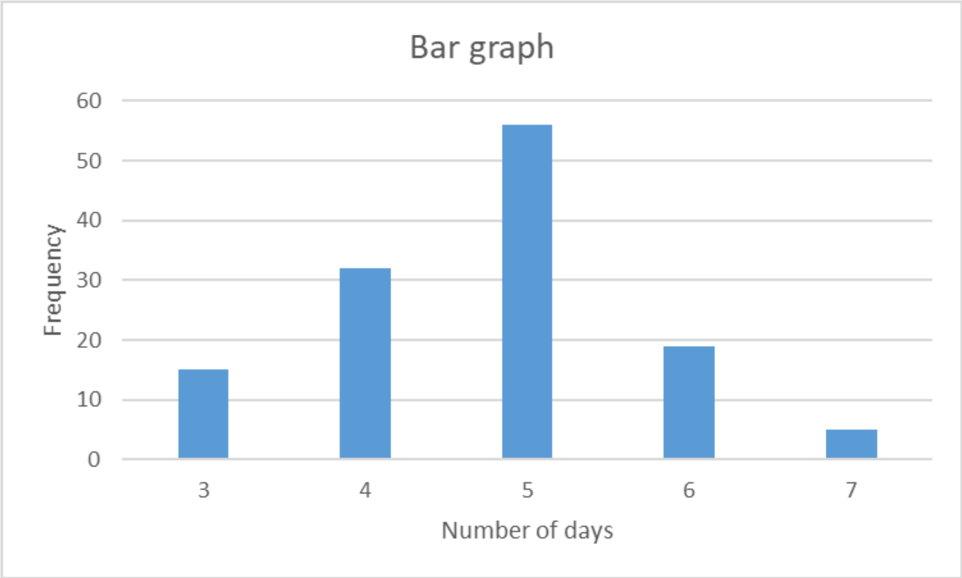
Construct

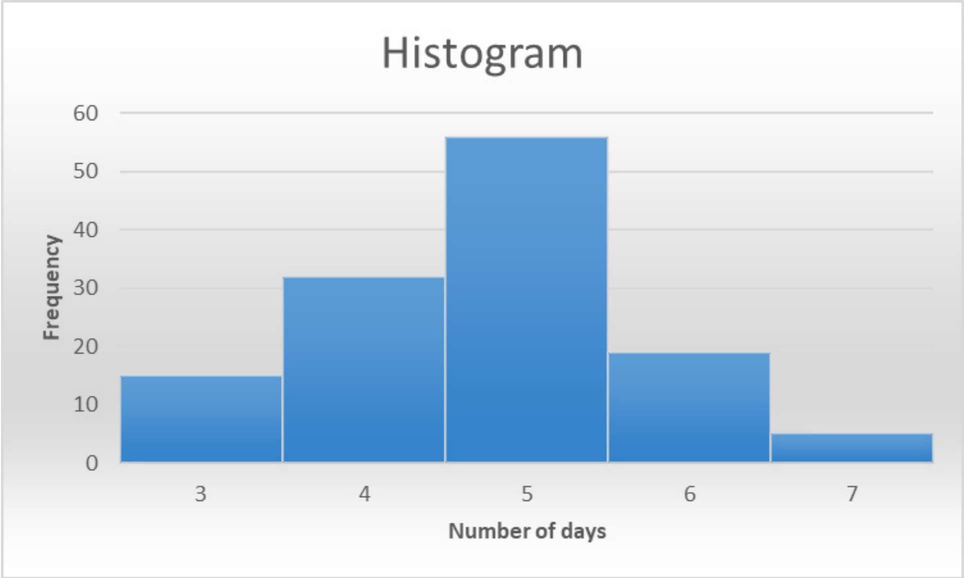
i) Bar graph

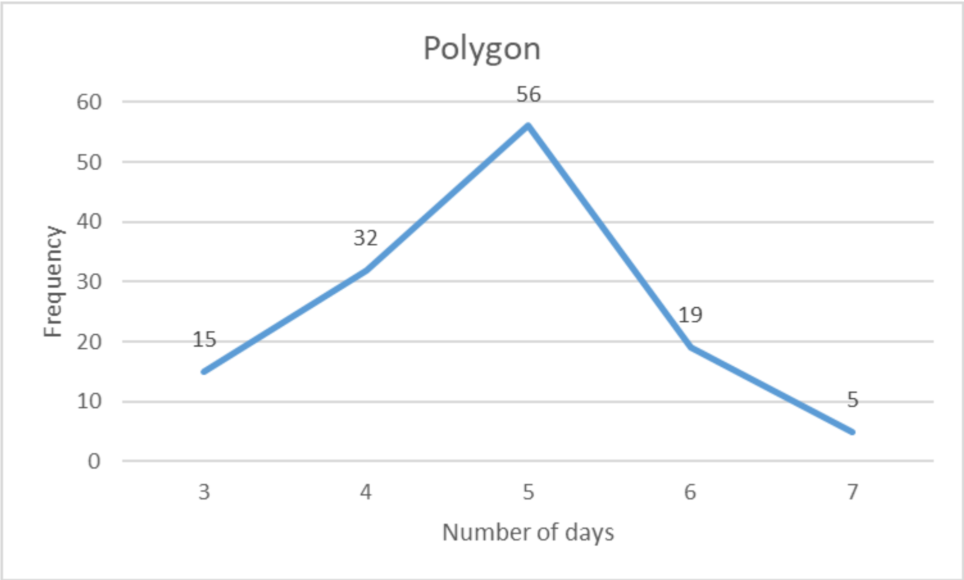
ii) Histogram

iii) Polygon

Solutions




Histogram
Frequency
60
50
40
30
20
10
0
3
4
5
6
7
Number of days


Polygon
Frequency
60
50
40
30
20
10
0
56
32
15
19
5
3
4
5
6
7
Number of days

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