Introduction

In this unit, we will draw motion graphs for objects moving in linear motion, analyses and interpret the motions as represented in the graphs.

In our daily lives, we come across or interact with objects in motion. For example, people, animals and objects are from time to time involved in motion in various directions. The motion in a straight line is called linear motion, also referred to as rectilinear motion.

Graphs of linear motion help us to visualize and analyses various aspects of the motion including distance and displacement covered, speed and velocity, direction of motion and acceleration.

List of lessons

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1.	Uniform and non-uniform linear motion	2
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	Total	8

1.1 Uniform and non-uniform linear motion

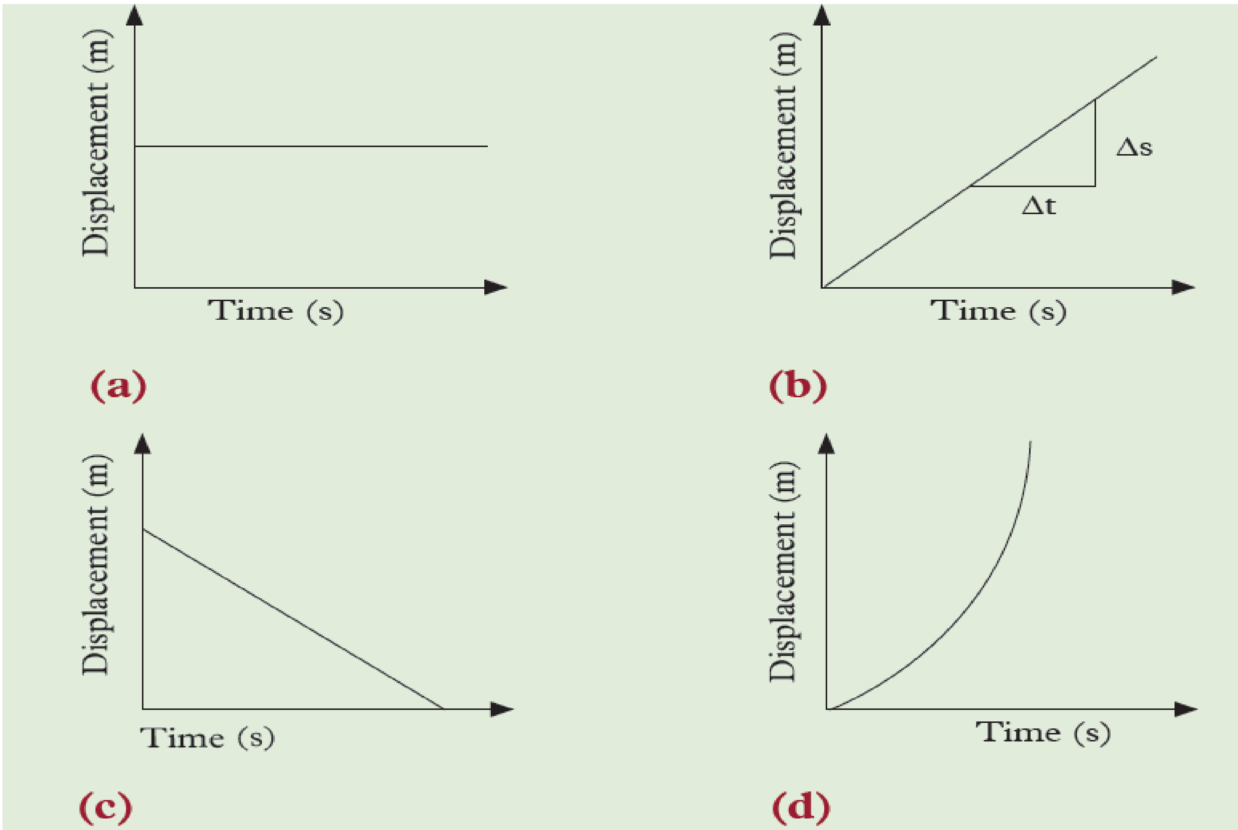
Uniform linear motion is a type of motion in which the body moves with constant velocity. In other words, it moves with zero acceleration or along a straight path with constant speed.

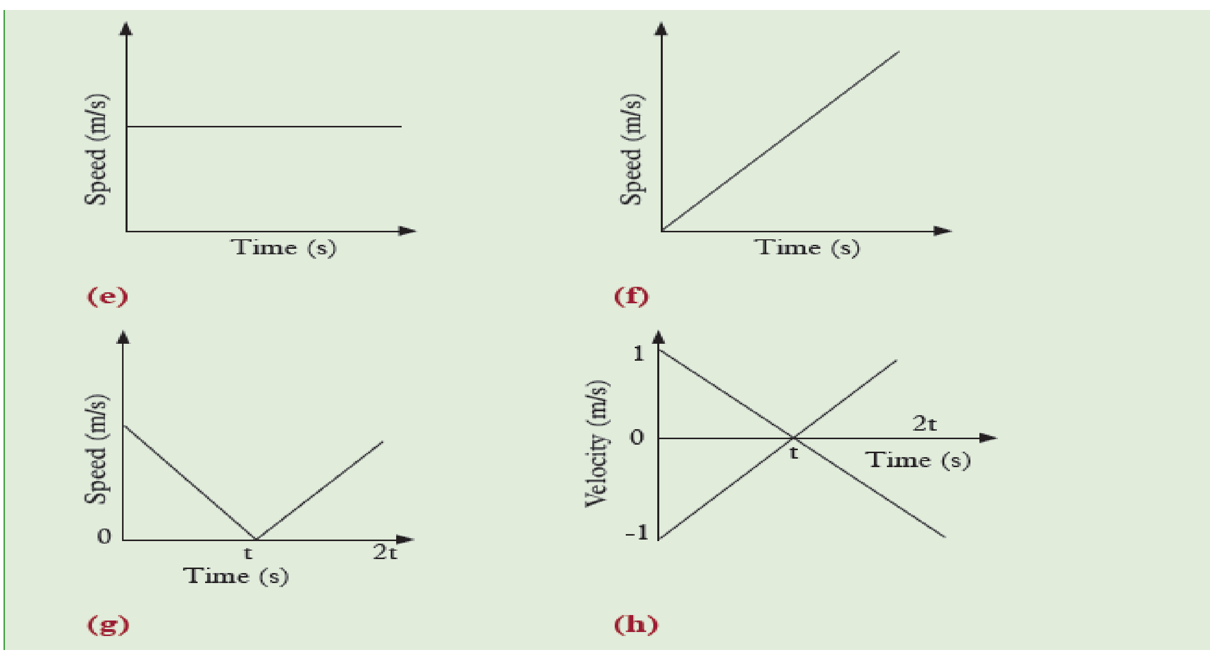
An example of this kind of motion is a car moving along a straight section of road at a constant speed.
Non uniform linear motion is the kind of motion in which a body moves with a varying velocity. An example of a non-linear motion is a bouncing ball.

Uniform motion is the kind of motion in which a body covers equal distances in equal intervals of time. It does not matter how small the time intervals are, as long as the distances covered are equal. If a body is involved in rectilinear motion and the motion is uniform, then the acceleration of the body must be zero.

Example:

Identify **uniform linear motions** and **non-uniform linear motions** from the following graphs. Give a reason in each case to support your decision.





Uniform motion is a type of motion where bodies move with a **constant velocity** on with zero **acceleration** along a strait path.

While non uniform motion is motion in which a body moves with a varying **velocity/speed.**

Uniform motion are **(b)** and **(e)** because the body covers equal distances in equal intervals of time.

Non-uniform motion are **(a), (c), (d), (f) ,(g)** and **(h)** because the body covers unequal distances in equal time interval.

Exercise

1. With the help of a sketch graph, explain how a body undergoes uniform linear motion.
2. By giving an example, differentiate between uniform and non-uniform motion.

1.2. Plotting graphs of linear motion

Graph of motion is the relation graph between position, velocity and acceleration as functions of time.
There are two types of graphs of linear motion

- **Distance (Displacement)-time graph**
- **Velocity (Speed)-time graphs**

1.2.1 Distance-time graphs

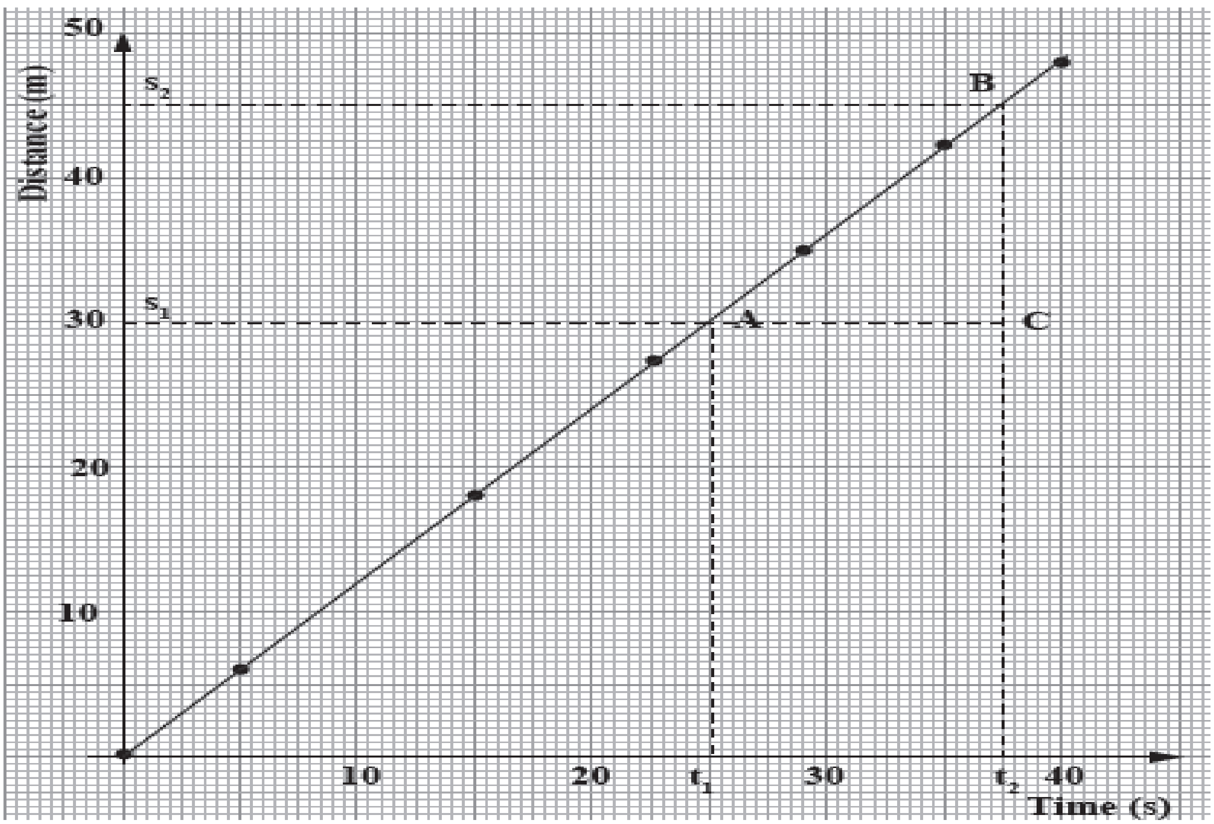
The change in the **position** of an object with **time** can be represented on the **distance-time graph**. Such a graph helps us to visualize and analyses various aspects of the motion, e.g. **the average speed** or **velocity** of the body.

(a) Distance-time graph of an object moving with uniform speed

Let us consider a moving body whose distance changes with time as shown in this table

Distance (km)	0	6	18	27	35	42	48
Time (s)	0	5	15	22.5	29	35	40

The distance-time graph of the body is as shown in this Figure



From the above graph in Fig. the object travels equal distance in equal time intervals. It moves with **uniform speed**. Therefore, for a body moving with uniform speed, it's distance-time graph is a straight line. In other words, the distance is increasing at a uniform rate. The term **velocity** is used instead of **speed** if the direction is specified.

Determination of Speed/Velocity from distance-time graph

In the graph in Fig. above, the distance travelled from **A** to **B** is obtained as follows: **Distance=$S_2 - S_1$ = 45m – 30m = 15m** (where S_1 is the initial distance and S_2 is the final distance.)
The time taken = $t_2 - t_1$ = 37.5s – 25.0s = 12.5s (where t_1 is time at S_1 and t_2 is time at S_2.)

$$Speed = \frac{Distance\ travelled}{time\ taken} = \frac{15}{12{,}5} = 1.2\ m/s$$

∴ We see that,
speed = slope of the graph (see triangle of ABC in Fig.). Since the speed is not changing, this is a case of non-accelerating motion.

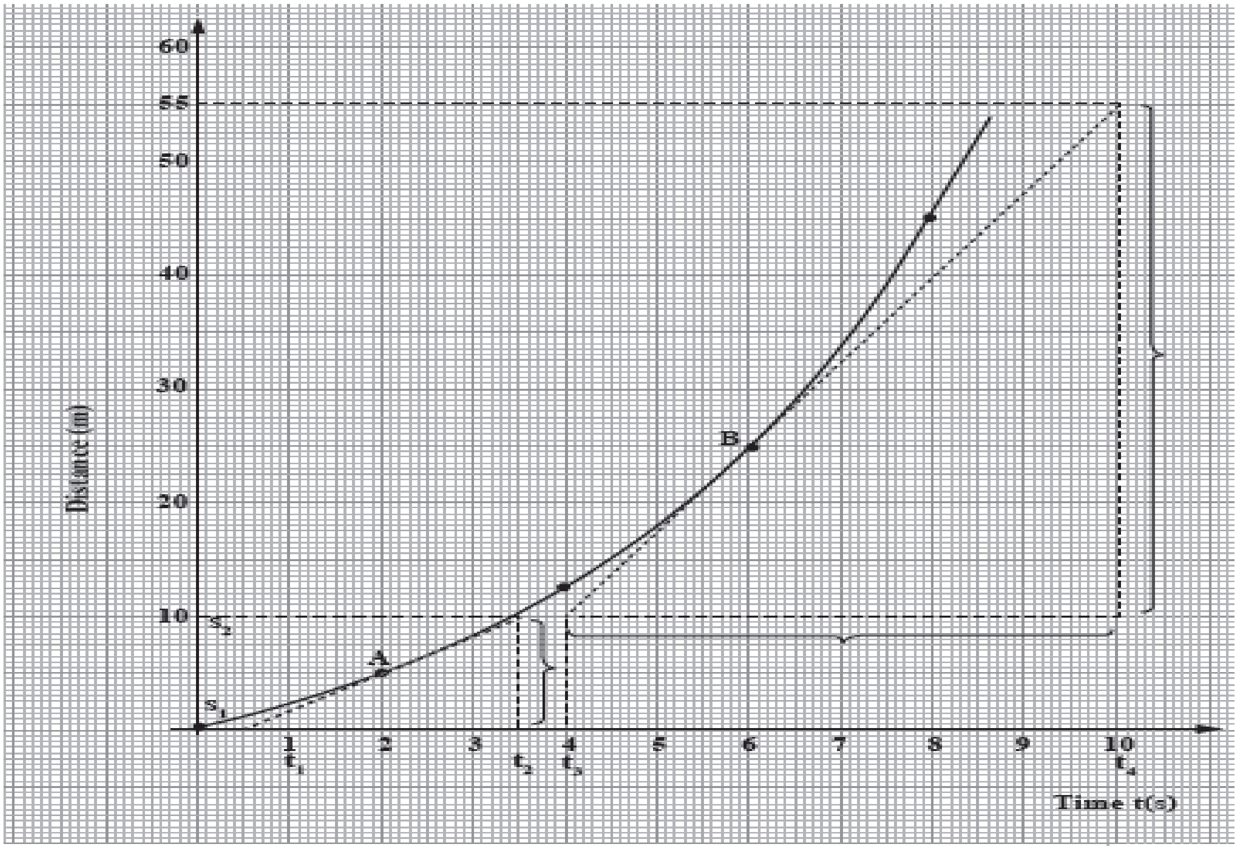
(b) Distance-time graph for an object moving with non-uniform speed

The motion of a body moving with **non-uniform speed** can be presented on a graph. Such a graph helps us to identify time intervals when the body was either at **rest, accelerating** or **moving at a constant speed**. It also helps us to determine its velocity at any instance or average velocity in any given time interval.

Let us consider a case similar to the one above The distance covered with time by a body was recorded as shown in this table.

Distance (m)	0	5	12.5	25	45
Time (s)	0	2	4	6	8

The distance-time graph of the body is as shown in this Fig.



The graph above shows variation in the rate of distance covered in time. This represent motion with non-uniform speed.

Determination of **speed/velocity** from the graph **The slope (gradient)** of the graph

i.e $\frac{Change\ in\ distance}{Change\ in\ time}$ or gradient $=\frac{\Delta s}{\Delta t}$ where Δs is change in distance and Δt is change in time, represents speed/velocity of the object at any given point. Therefore, we can find the speed of the object at any instance by finding the slope of the graph at that point.

To do this, we draw a tangent to the curve at that point i.e a line touching the

graph at only that point, then find the gradient of that line. At point **A,** instantaneous speed

VA $=\frac{S2-S1}{t2 - t1} = \frac{10 - 0}{3.5 - 0.5}$ = 3.3 m/s

At point B, instantaneous speed VB= $\frac{S3-S2}{t4 - t3} = \frac{55 - 10}{10 - 4}$ = 7.5 m/s

We can see that the speed of the object is higher at point B than at point A hence
the body is accelerating.

Exercise

1. What does the gradient from a distance-time graph represent? Explain.
2. Uwimana carried out an experiment with an object moving at different distances in different times and recorded the findings as shown on this table the table

Time(s)	20	30	40	50	60
Distance (m)	12	16	20	24	28

(a) Plot a graph of distance (y-axis) against time.
(b) Find the slope of the graph.
(c) Describe the motion of the object.
(d) Give two reasons why drawing graphs of linear motion is important.

1.2.2 Velocity-time graphs

The change in the **Velocity** of an object with **time** can be represented on the **Velocity-time graph**.

(a) Velocity-time graph for an object moving at a constant velocity

The velocity-time graph for a body moving at constant velocity is a straight line parallel to the x-axis.

Since velocity is constant, then

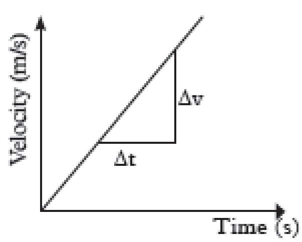
Velocity $= \dfrac{\textit{displacement covered}}{\text{time}}$

∴ Velocity × Time = Displacement

Looking at the graph in Fig. above we see that the distance covered from A to B is (given by velocity × time covered) is equal to the area of the rectangle bounded by AB and the time axis. **Therefore, distance covered by a body moving at constant velocity or speed from a velocity time graph is equal to the area under the graph in the section under consideration.**

(b) Velocity – time graph for an accelerating/decelerating object.

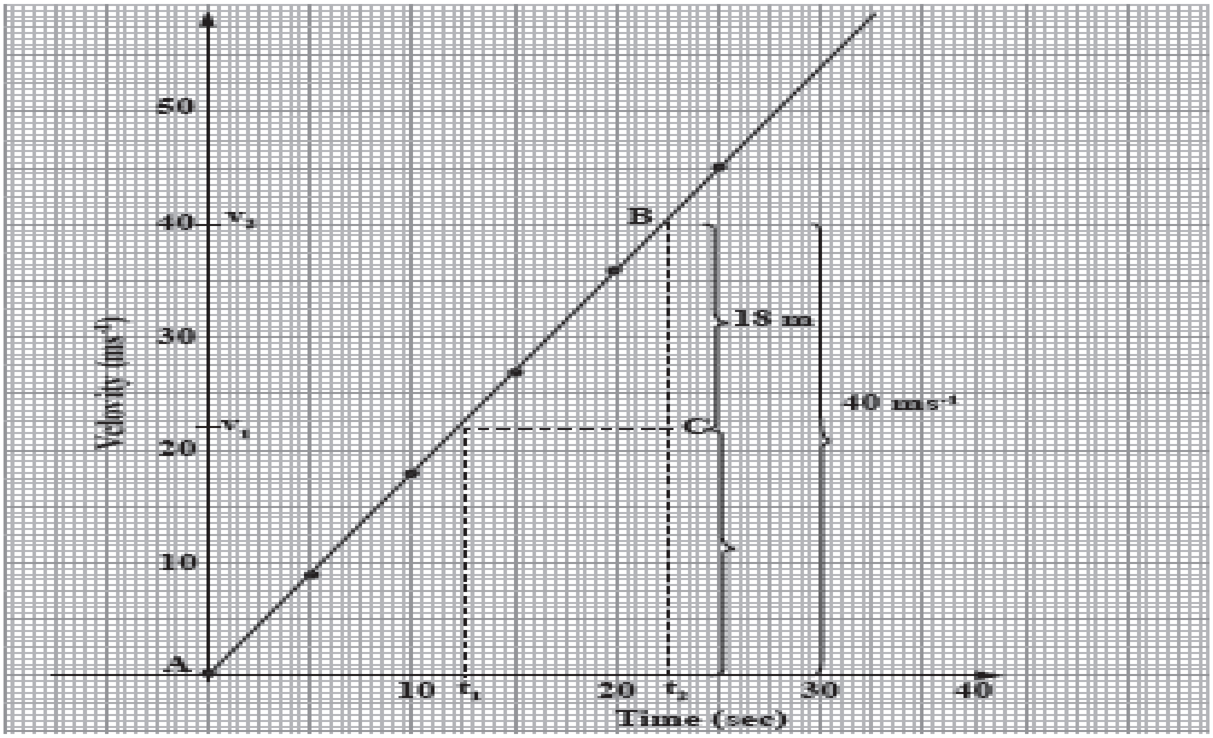
When the velocity of a body is not constant, the body is either accelerating or decelerating. Consider an object whose motion is shown in the velocity-time graph in this Fig.



The gradient $= \dfrac{\text{change in velocity}}{\text{Time taken}} = \dfrac{\Delta v}{\Delta t} =$ **acceleration**

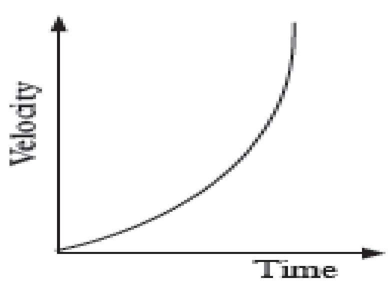
Thus, the gradient of a velocity-time graph gives the acceleration of the object. When the graph has a uniform slope (gradient), the body has **uniform acceleration.**

Consider a motion of a body whose velocity at regular time is as shown in this table.

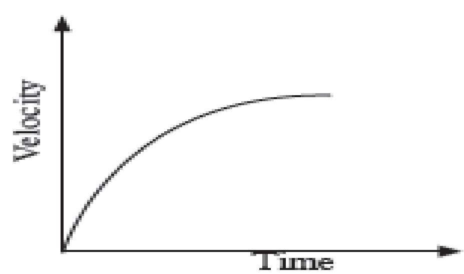


Change in velocity between t1 and t2 = v2 – v1 = 40 m/s – 22 m/s = 18 m/s
Change in time = t2 – t1 = 22.5 s – 12.5 s = 10 s
Slope $= \frac{v2 - v1}{t2 - t1} = \frac{18\ m/s}{10\ s} = 1.8\ m/s2$
The velocity changes at the same rate in unit time. This is a case of **uniformly accelerated motion.**

When a velocity-time graph is a curve, then the body is moving with either increasing or decreasing acceleration as shown in the graphs in this Fig.



Graph of body moving with increasing acceleration



Graph of body moving with decreasing acceleration

The **instantaneous acceleration** of the object at any instant of a body moving with non-uniform speed is given by the gradient of the graph at that point, in the same way we determined the instantaneous speed from the graph of a body moving with non-uniform speed earlier in this unit.

Exercise

1. Differentiate between speed and velocity.
2. In a velocity-time graph, the slope obtained stand for?
3. The velocity of a body increases from 60 km/h to 90 km/h in 20 seconds. Calculate its acceleration.
4. An applied force changes the velocity of an object from 20 m/s to 36 m/s in 0.01 seconds. What is the acceleration produced?