



# **PHYSICS**

## **TTC**

### **STUDENT-TEACHER'S BOOK**

#### **YEAR 2**

#### **OPTION: SME**

**Kigali, 2019**

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## **FOREWORD**

Dear Student- teacher,

Rwanda Education Board is honoured to present to you this Physics book for Year Two of TTC which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of Physics subject. The Rwandan educational philosophy is to ensure that you achieve full potential at every level of education which will prepare you to be well integrated in society and exploit employment opportunities.

The government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate your learning process. Many factors influence what you learn, how well you learn and the competences you acquire. Those factors include the instructional materials available among others. Special attention was paid to the activities that facilitate the learning process in which you can develop your ideas and make new discoveries during concrete activities carried out individually or with peers.

In competence-based curriculum, learning is considered as a process of active building and developing knowledge and meanings by the learner where concepts are mainly introduced by an activity, a situation or a scenario that helps the learner to construct knowledge, develop skills and acquire positive attitudes and values. For effective use of this textbook, your role is to:

- Work on given activities including laboratory experiments which lead to the development of skills;
- Share relevant information with other learners through presentations, discussions, group work and other active learning techniques such as role play, case studies, investigation and research in the library, from the internet or from your community;
- Participate and take responsibility for your own learning;
- Draw conclusions based on the findings from the learning activities.

I wish to sincerely extend my appreciation to the people who contributed towards the development of this book, particularly REB staff who organized the whole process from its inception. Special gratitude goes to teachers, illustrators and designers who diligently worked to successful completion of this book.

**Dr.NDAYAMBAJE Irénée**

**Director General of Rwanda Education Board**

*Experimental version*

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**Joan MURUNGI,**

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## UNIT 1: APPLICATIONS OF THERMODYNAMICS LAWS

**Key unit competence:** Evaluate applications of first and second laws of thermodynamics in real life.

### INTRODUCTORY ACTIVITY



Mutesi is a parent of two children at a certain school. Before she takes them to school, she first makes sure that she prepares food and drinks for them and packs some in flasks so that her children can eat and drink during lunch time.

She then drives them to school before she reports to her working place and then from the school she then diverts to her working place which is about 5 km away from the school.

The parking yard at her work place is a plain place without any shade but she makes sure that her car is parked near a tree that is near the parking yard to prevent it from different damages among which is destruction of tyres of the car.

- Explain why Mutesi makes use of flasks not normal utensils like metallic bowls while parking foods and drinks for her children.
- Is there heat exchange inside the flasks? Explain your reasoning.
- Imagine on a certain day these two children only eat food and leaves, the drink in the flask and by mistake they forget flask in the store and the mother come to pick it the next day. Do you

think the contents in the flask will be at the same temperatures? Explain all scientific phenomena that may lead to either loss or gain in energy of the contents in the flask.

- d) Explain why in most cases the outer covering of a flask is always made of a poor conductor? Explain how quality and efficiency of these flasks can be improved by manufactures.
- e) Based on statements above, Mutesi normally parks her car under a shade to prevent her car from being exposed to sunshine. Explain how during hot days the tyres of a car may burst.
- f) Her Car uses petrol in operation. During operation of her car, the engine draws fuel (Petrol) air mixture from the tank into the engine, explain all the processes that take place in the engine.

## 1.1 Internal energy and total energy of a system

### ACTIVITY 1.1

Suppose you are in a closed small room with one door and no window on a hot day. After a given time, you find and the room is hot and yourself sweating.

- a) Explain what you think was the cause of sweating?
- b) Explain what causes the rise in the temperature in your room?
- c) Suppose you left the door open, do you think the temperatures in the room would be the same? Explain your reasoning.
- d) Select which physical quantities among pressure, volume and temperature changed?

**Thermodynamics** refers to the study of heat and its transformation into mechanical energy.

In thermodynamics, the **internal energy** is one of the two extremely important state functions of the variables of a thermodynamic system. It refers to total energy contained within the system excluding the kinetic energy of motion of the system and the potential energy of the system due to external forces. It keeps account of the gains and losses of energy of the system.

The internal energy of a system may be changed by

- i) heating the system
- ii) doing work on it,

iii) adding or taking away matter.

The **thermal energy** is the portion of internal energy that changes when the temperature of the system changes. Sometimes the term thermal energy is used to mean internal energy. **Heat** is defined as the transfer of energy across the boundary of a system due to a temperature difference between the system and its surroundings.

When you *heat* a substance, you are transferring energy into it by placing it in contact with surroundings that have a higher temperature. For example, when you place a pan of cold water on a stove burner, the burner is at a higher temperature than the water, and so the water gains energy.

In daily life, we recognize the difference between internal energy and heat. The heat transfer is caused by a temperature difference between the system and its surroundings. However, in some systems there are no temperature and pressure gradients, such systems are said to be in **thermodynamic equilibrium**.

#### APPLICATION ACTIVITY 1.1

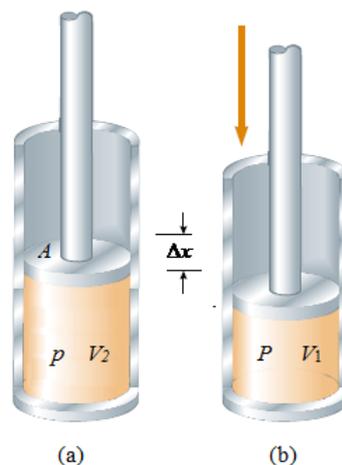
1. Explain the meaning of the following terms as applied in thermodynamics
  - i) Heat
  - ii) Internal energy of a system
2. Explain clearly how the internal energy of a system can be altered.
3. With real life examples, point out areas where there are energy changes. Explain the need for these changes for each stated system.
4. Two bodies at different temperatures are brought into contact. Will temperature be shared between these bodies or each body will remain at its temperature? Explain to support your reasoning in a scientific way.

## 1.2 Work done by an expanding gas

### ACTIVITY 1.2

- Gas is a state of matter with particles that are in constant motion and held together by weak forces. Like any other state of matter, if a certain gas is confined in a container, it can do some work. How do you know that a gas has done some work?
- A gas confined in a room with fixed and strong walls. Suppose the gas particles are allowed to interact, can the particles make the walls be displaced? Basing on your reasoning, do we say that the gas has done work. Comment to support your decision.
- Explain changes that may take place if a gas expands.

Consider a gas contained in a cylinder fitted with a movable piston. At equilibrium, the gas occupies a volume  $V$  and exerts a uniform pressure  $P$  on the cylinder's walls and on the piston. If the piston has a cross-sectional area  $A$ , the force exerted by the gas on the piston is  $F = PA$ . Now let us assume that we push the piston inward and compress slowly to allow the system to remain essentially in thermal equilibrium.



**Fig.1. 1 Work is done on a gas contained in a cylinder at a pressure  $P$  as the piston is pushed downward so that the gas is compressed.**

With work done by the force due to pressure, we find the same relation but having a negative sign. The force is exerted in opposite direction and the final volume is less than the initial one.

Since  $W = -P\Delta V \Leftrightarrow W = -P(V_2 - V_1)$

$$W = -P(V_f - V_i)$$

The total work done on the gas as its volume changes from initial volume ( $V_i$ ) to final volume ( $V_f$ ) is given by the above equation.

If the gas is compressed,  $\Delta V$  is negative and the work done on the gas is positive (Work done by the gas is positive) and if the gas expands,  $\Delta V$  is positive and the work done on the gas is negative (Work done on the gas is negative). If the volume remains constant, the work done on the gas is zero. Thus, no work done. To evaluate this relation, one must know how the pressure varies with volume during the process.

The work done on a gas in a quasi-static process that takes the gas from an initial state to a final state is the negative of the area under the curve on a PV diagram, evaluated between the initial and final states.

Based on the processes of compressing a gas in the cylinder indicated in figure 1.1, the work done depends on the path taken between the initial and final states.

### Example 1.1

a) A ball is pumped at a constant pressure of  $2 \times 10^5$  Pa and the volume of the ball changed from 2 L to 5 L. Calculate the work done by the gas.

b) If for the same system, the volume remained the same. What would be work done by the gas.

#### Solution

(a) From  $W = -P\Delta V \Leftrightarrow W = -P(V_2 - V_1)$

Since  $P = 2 \times 10^5$  Pa,  $V_f = 5$  L,  $V_i = 2$  L,

$$W = 2 \times 10^2 (5 - 2) = 6 \times 10^2 \text{ Pa}$$

(b) From the equation  $W = P\Delta V$

But  $\Delta V = 0$  thus  $W = 0J$

Comment: There is no work done by the gas since the volume remained constant.

## APPLICATION ACTIVITY 1.2

1. Explain the effect of the nature of the walls of a container on work done by the gas.

## 1.3 First law of thermodynamics and its applications.

3. Gas in a container is at a pressure of  $1.6 \times 10^5$  Pa and a volume of  $4.0 \text{ m}^3$ . What is the work done on or by the gas if:
  - a) It expands at constant pressure to twice its initial volume?
  - b) It is compressed at constant pressure to one-quarter of its initial volume?
4. An engine cylinder has a cross-sectional area of  $0.010 \text{ cm}^2$ . How much work can be done by a gas in the cylinder if the gas exerts a constant pressure of  $7.5 \times 10^5 \text{ Pa}$  on the piston, moving it a distance of  $0.040 \text{ m}$ ?

## ACTIVITY 1.3

1. Imagine you are boiling water in a closed container (say a closed saucepan) on a gas stove. Explain all changes that may take place in the saucepan.
2. From your suggestions in 1) above, explain how the applied heat may lead to change in internal energy of water and even displacement of saucepan's cover.
3. From 2) above, create a mathematical relation that relates heat supplied, internal energy of water and work done by water.
4.
  - a) What is the boiling point in  $^{\circ}\text{C}$  of pure water?
  - b) If extra heat is applied to water at its boiling point, can its temperature change? Explain to support your decision.
  - c) Suppose hot water at  $78^{\circ}\text{C}$  is put in a flask and it remains there when the temperature is not changing and even the volume. There is no heat added from the surroundings nor heat removed from the system. Explain all scientific processes that take place in this flask.

### 1.3 1 First law of Thermodynamics.

It states that the change in internal energy of a system is equal to the heat added to the system minus the work done by the system. Therefore, the law stated gives mathematical treatment of internal energy of a system shown below. Hence the first law of thermodynamics.

$$\Delta U = Q - W$$

Where

- $\Delta U$  is the change in internal energy
- $Q$  is the heat supplied to the system
- $W$  is the work done by the system

Note: - The first law of thermodynamics is an extension of the law of conservation of energy.

- A positive  $W$  is the work done on the system.
- When  $n$  moles of a gas are considered, the amount of heat supplied at constant pressure is

$Q = nc_p \Delta T$ , whereas the amount of heat supplied at constant volume would be

$$Q_v = nc_v \Delta T$$

Where

- $C_p$  is the molar heat capacity at a constant pressure
- $C_v$  is the molar heat capacity at a constant volume.

### Example 1.2

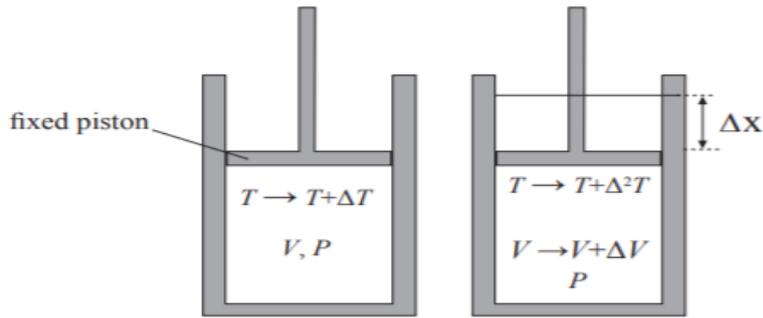
The internal energy of a system is initially 27 J. A total of 33 J of energy is added to the system by heat while the system does 26 J of work. What is the system's final internal energy?

**Answer**

$$Q = W + \Delta U \Leftrightarrow U_f = Q - W + U_i = 33 - 26 + 27 = 34 \text{ J}$$

### 1.3.2 Relationship between $c_p$ and $c_v$

Consider one mole of a gas at a pressure  $P$ , temperature  $T$  and volume  $V$ , heated first at constant volume and secondly at constant pressure to cause the same temperature change,  $\Delta T$ .



**Fig.1. 2 A gas in cylinder heated at constant pressure**

For syringe with a fixed piston, the gas is heated at a constant volume. Therefore

$$\Delta\theta = \Delta U = C_v \Delta T \quad (i)$$

Since the system was heated at constant Volume and the temperature changed by 1K(Molar heat capacity at constant Volume), Heat supplied into the system.

$$\Delta\theta = C_v \Delta T$$

But  $\Delta T = 1\text{K}$  , hence  $\Delta\theta = C_v$

For syringe with a movable piston, the gas is heated such that the volume changes thus work is done.

Therefore,  $\Delta\theta = \Delta U + \Delta W$

Since the gas is heated at a constant pressure (molar heat capacity at constant pressure) ,the heat supplied  $\Delta\theta = C_p$  therefore, the above equation becomes

$$C_p = \Delta U + \Delta W \quad (ii)$$

Substitute i) into ii)

$$C_p = C_v + P(V_2 - V_1) \quad (iii)$$

$$C_p - C_v = P(V_2 - V_1)$$

Before the system was heated,  $PV_1 = RT$  (x)

After warming  $PV_2 = R(T + 1)$  (y)

Because there was a rise in temperature by 1K.

Subtracting x from y

$$PV_2 - PV_1 = R(T + 1 - T)$$

$$P(V_2 - V_1) = R$$

Therefore equation (iii) becomes  $C_p - C_v = R$ , where R is the universal gas constant.

**Note:** - The first law of thermodynamics is a special case of the law of conservation of energy that encompasses changes in internal energy and energy transfer by heat and work.

-It is a law that can be applied to many processes. It is noticed that energy can be transferred between a system and its surroundings.

-One is work done on the system, which requires that there be a macroscopic displacement of the point of application of a force.

- The other is heat, which occurs on a molecular level whenever a temperature difference exists across the boundary of the system.

- Both mechanisms result in a change in the internal energy of the system and therefore usually result in measurable changes in the macroscopic variables of the system, such as the pressure, temperature, and volume of a gas.

- The increase in internal energy of a system is the sum of the work done on the system and the heat supplied to the system.

- One of the important consequences of the first law of thermodynamics is that there exists a quantity known as **internal energy** whose value is determined by the state of the system. The internal energy is therefore a state variable like pressure, volume, and temperature.

- The first law of thermodynamics is an energy conservation equation specifying that the only type of energy that changes in the system is the internal energy  $\Delta U$

### Example 1.3

1. Fill in the boxes with -, +, or 0 for each of the following three terms in the first law of thermodynamics. For each situation, the system to be considered is identified.

Situation	System	Q	W	$\Delta U$
a) Rapidly pumping up a bicycle tire	Air in the pump			
b) Pan of room-temperature water sitting on a hot stove	Water in the pan			
c) Air Quickly leaking out of a balloon	Air originally in the balloon.			

**Answer**

Situation	System	Q	W	$\Delta U$
a) Rapidly pumping up a bicycle tire	Air in the pump	0	+	+
b) Pan of room-temperature water sitting on a hot stove	Water in the pan	+	0	+
c) Air Quickly leaking out of a balloon	Air originally in the balloon.	0	-	-

2. A 20.5 kJ of heat is supplied to a system, and 10.8 kJ of work is done on the system. What is the change in internal energy of the system?

**Answer**

The change in internal energy:  $\Delta U = Q - W$ ,  $\Delta U = 20.5 \text{ kJ} - (-10.8 \text{ kJ})$

Therefore,  $\Delta U = 32.3 \text{ kJ}$

### 1.3.3 Applications of first law of Thermodynamics.

The first law of thermodynamics that we discussed relates the changes in internal energy of a system to transfers of energy by work or heat. In this case, we consider applications of the first law in processes through which a gas is taken as a model.

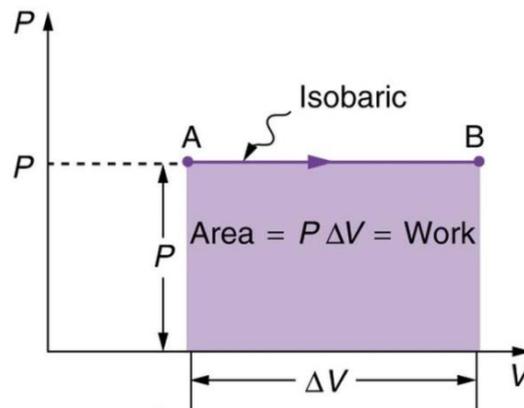
### ISOBARIC PROCESS

A process that occurs at constant pressure is called an **isobaric process**. In such processes, the values of the heat and the work are both usually nonzero. The work done during isobaric process is simply

$$W = P(V_f - V_i)$$

Where

- P is the constant pressure,
- $V_f$  is the final volume
- $V_i$  is the initial volume.



**Fig.1. 3 P-V graph showing isobaric process**

Note: - As indicated from the fig above AB is an Isobaric process

- From first law of thermodynamics,  $\Delta\theta = \Delta U + \Delta W$

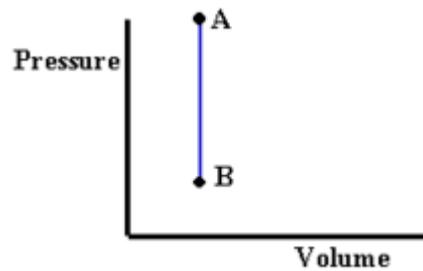
$$\Delta\theta = \Delta U + P(V_f - V_i)$$

- The statement above implies that work done during isobaric process is given by

$$W = P(V_f - V_i)$$

### **ISOVOLUMETRIC PROCESS/ISOCHORIC PROCESS**

A process that takes place at constant volume is called an **isovolumetric process**.



**Fig.1. 4 P-V graph showing isovolumetric process**

From the figure 1.4 above, process AB takes place at a constant Volume (volume doesn't change). In such a process, the value of the work done is zero because the volume does not change. Hence, from the first law we see that in an isovolumetric process,

$$W = 0 \text{ and } \Delta U = Q \text{ (isovolumetric process)}$$

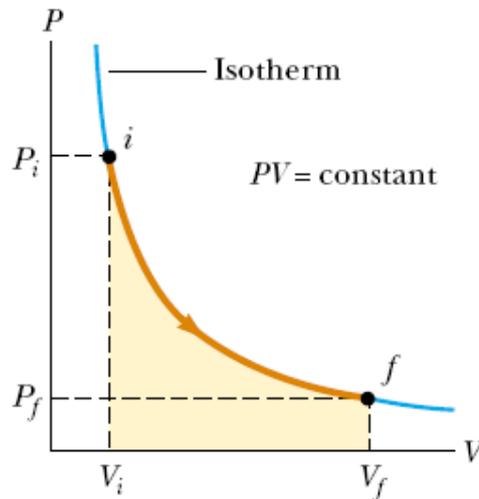
**Note:** - This expression specifies that if energy is added by heat to a system at constant volume, then all of the transferred energy remains in the system as an increase in its internal energy.

- For example, when a can of spray paint is thrown into a fire, energy enters the system (the gas in the can) by heat through the metal walls of the can. Consequently, the temperature, and thus the pressure in the can increases until the can possibly explodes.

### **ISOTHERMAL PROCESS**

A process that occurs at constant temperature is called an *isothermal process*.

A plot of  $P$  versus  $V$  at constant temperature for an ideal gas yields a hyperbolic curve called an *isotherm*.



**Fig.1. 5 The PV diagram for an isothermal expansion (i→f) of an ideal gas from an initial state to a final state. The curve is a hyperbola.**

The internal energy of an ideal gas is a function of temperature only. Hence, in an isothermal process involving an ideal gas,

$$\Delta U = 0.$$

For an isothermal process, then, we conclude from the first law that the energy transfer  $Q$  must be equal to the negative of the work done on the gas, that is,  $Q = -W$

Any energy that enters the system by heat is transferred out of the system by work, as a result, no change in the internal energy of the system occurs in an isothermal process.

It can be noted from the graph that  $PV = \text{constant}$ . This symbolizes that under isothermal, the process obeys Boyle's law where pressure is inversely proportional to the volume at a constant temperature.

Therefore,  $P \propto \frac{1}{\text{Volume}}$  at constant temperature

$$PV = \text{constant.}$$

$$P_1V_1 = P_2V_2 = \dots = P_nV_n$$

From the equation above, it can be deduced that if a gas is at conditions  $P_1V_1$ , the quantities can change to  $P_2V_2$  with temperatures remaining constant. However, still at constant temperature, the system at  $P_2V_2$  can still undergo the same change ( at constant temperature) to  $P_1V_1$ . Such processes are referred to as **Reversible isothermal change**.

## Conditions necessary for an isothermal process to take place

For an isothermal process to take place the following conditions must be satisfied

- i) The gas must be contained in a thin –walled heat conducting vessel/container in good thermal contact with a constant temperature bath.
- ii) The process must be carried out slowly to allow time for heat exchange to take place.

## Work done during isothermal process.

If a gas expands from  $V_i$  to  $V_f$  at a constant temperature as shown in figure 1.5, work is done

Considering  $n$  moles of a gas, the work done by a gas during isothermal process and is given by

$$W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

where

$n$  is the number of moles

$R$  is the universal gas constant =  $8.3145 \text{ J / mol}\cdot\text{K}$

$T$  is the temperature.

$V_f$  is the final Volume.

$V_i$  is the initial Volume.

From the above equation, the following can be drawn;

- (i) When the gas expands (That is  $V_f > V_i$ ), then  $W$  is positive.
- (ii) When the gas is compressed (That is  $V_i > V_f$ ), then  $W$  is negative. It means work is done on the gas in compressing it.

### Example 1.4

A vessel containing  $1.5 \times 10^{-3} \text{ m}^3$  of an ideal gas at a pressure of  $8.7 \times 10^{-2} \text{ Pa}$  and at a temperature  $25^\circ \text{C}$  is compressed isothermally to halve its original volume.

Calculate the work done during this process. Comment on the sign of the answer ( $R = 8.314 \text{ J / mol}\cdot\text{K}$ )

### Answer

Given that  $V_i = 1.5 \times 10^{-3} \text{ m}^3$ ,  $V_f = 0.75 \times 10^{-3} \text{ m}^3$  after compression,  $P = 8.7 \times 10^2 \text{ Pa}$ ,  $T = 298 \text{ K}$  ( $25^\circ \text{C}$ ) and  $R = 8.314 \text{ J / mol.K}$

From  $W = RT \ln\left(\frac{V_f}{V_i}\right)$

$$W = 8.314 \times 298 \ln\left(\frac{0.75 \times 10^{-3}}{1.5 \times 10^{-3}}\right) = -1717.3 \text{ J}$$

Comment.

The answer has a negative value. This shows that the work is done on the gas (compressed).

### APPLICATION ACTIVITY 1.3

1. Explain the meaning of the following terms as applied in thermodynamics and in each case sketch their P-V curves.

a) Isothermal process

b) Isovolumetric process

c) Isobaric process

2. One mole of an ideal gas expands at a constant pressure of  $1.0325 \times 10^5 \text{ Pa}$  from 2 L to 4 L. It then expands isothermally to a volume of 6 L at constant temperature of 298 K.

a) Represent the whole process on a P-V curve.

b) Using the values given in the question, calculate the final pressure reached by the gas.

c) For each case, Calculate the work done.

d) Assuming the gas at 6 L changed its pressure back to the original pressure still at constant temperature. What is the work done by the gas. Comment on the value of work done obtained.

3(a) (i) What is meant by a reversible isothermal change?

(ii) State the conditions for achieving a reversible isothermal change.

## 1.4 Second law of thermodynamics and its applications

### ACTIVITY 1.4



Observe the picture of a refrigerator above and answer the following questions:

- Basing on your observation and experience, explain how food can be cooled by a refrigerator.
- If a hot drink is put into a refrigerator and spends like 30 minutes. It cools down. Where does the heat from the drink go? Explain to support your idea.

Since the first law of thermodynamics states that energy is conserved. There are, however, many processes we can imagine that conserve energy but are not observed to occur in nature. Lets consider an example below of the first law to introduce the second law.

For example, when a hot object is placed in contact with a cold object, heat flows from the hotter one to the colder one, never spontaneously the reverse. If heat were to leave the colder object and pass to the hotter one, energy could still be conserved. Yet it doesn't happen spontaneously the reverse.

There are many other examples of processes that occur in nature but whose reverse does not. To explain this lack of reversibility, scientists in the latter half of the nineteenth century formulated a new principle known as **the second law of thermodynamics**.

The second law of thermodynamics is a statement about which processes occur in nature and which do not. It can be stated in a variety of ways, all of which are equivalent. One statement is that: **Heat**

**can flow spontaneously from a hot object to cold object; heat will not flow spontaneously from a cold object to a hot object”.**

The development of a general statement of the second law of thermodynamics was based partly on the study of heat engines. A **heat engine** is any device that changes thermal energy into mechanical work, such as steam engines and automobile engines.

### 1.4.1 Applications of second law of Thermodynamics

#### Adiabatic process

An adiabatic process is one in which heat is not allowed to enter or leave the system.

Therefore  $Q$  (heat added) is Zero . This means  $\Delta U = -W$

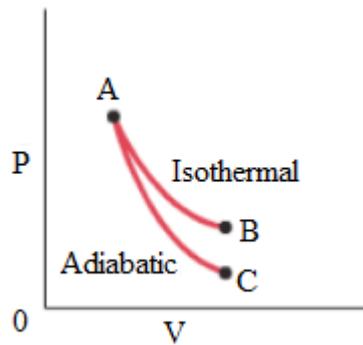
For example, if a gas is compressed (or expanded) very rapidly, very little energy is transferred out of (or into) the system by heat, and so the process is nearly **adiabatic**. Such processes occur in the cycle of a gasoline engine. Another example of an adiabatic process is the very slow expansion of a gas that is thermally insulated from its surroundings.

Suppose that an ideal gas undergoes an adiabatic expansion. At any time during the process, we assume that the gas is in an equilibrium state, so that the equation of state  $PV = nRT$  is valid. As expressed below, the pressure and volume of an ideal gas at any time during an adiabatic process are related by the expression

$$PV^\gamma = \text{constant}$$

where  $\gamma = \frac{C_p}{C_v}$  is assumed to be constant during the process. Thus, we can see that all three variables in the ideal gas law,  $P$ ,  $V$ , and  $T$  change during an adiabatic process.

However, at conditions  $P_1$ , and  $V_1$ , the system may change to,  $P_2V_2$  when there is no heat allowed to enter or leave the system, the process is termed to be reversible adiabatic process. Such processes that are both adiabatic and reversible are termed as **isentropic process**



**Fig.1. 6 Adiabatic and isothermal processes**

### Conditions necessary for an adiabatic change to occur

For an adiabatic process to be achieved,

- i) the gas must be contained in a thick –walled and perfectly insulated isolated container.
- ii) The process must be carried out rapidly to avoid any possible heat exchanges between the gas system and the surroundings.

### Work done during adiabatic process

Since all quantities may change during adiabatic, work is always done and it is calculated from the equation below

$$W = \frac{1}{1-\gamma} (P_f V_f - P_i V_i)$$

Where

- W is the work done during adiabatic
- where  $\gamma = \frac{C_p}{C_v}$ , the ratio of heat capacities
- P is the pressure of the gas
- V is the volume occupied by the gas.

### 1.4.2 Heat engine

Any device that transforms heat into work or mechanical energy is called **heat engine**. All heat engines absorb heat from a source at high temperature, perform some mechanical work, and discard heat at a lower temperature.

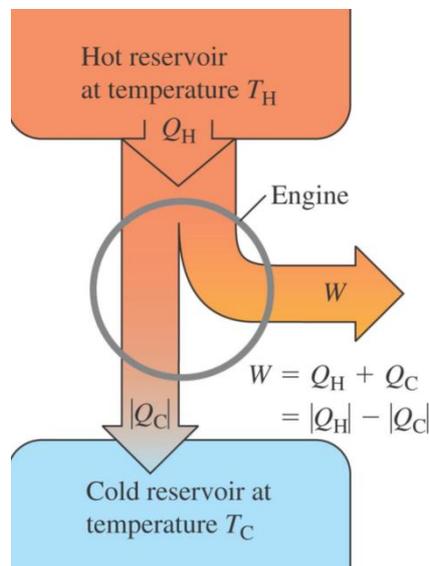
The process in heat engine is **cyclic** and therefore there is no change in **internal energy**. This implies  $\Delta U = 0$ . Thus, from first law of thermodynamics,  $\Delta\theta = \Delta U + \Delta W$  and  $\Delta U = 0$

Thus  $\theta = \Delta W$ . This implies that all heat supplied into the system/engine is used to do work.

**Note:** A **cyclic process** is one which comes back to its initial state. The graph of a cyclic process is always a closed graph.

### 1.4.2.1 Structure of heat engine

A heat engine has a hot reservoir at temperature  $T_H$  and a cold reservoir at temperature  $T_C$ ;  $Q_H$  flows in from the hot reservoir and  $Q_C$  flows out to the cold reservoir.



**Fig.1. 7 Structure of heat engine**

**Note:** The net heat absorbed per cycle is  $\theta = \theta_H + \theta_C$  and it is also the work done:  $W = \theta_H + \theta_C$

### 1.4.2.2 Efficiency of heat engines

The efficiency of the engine as the fraction of the heat input that is converted to work. Ideally, we would like to convert all  $\theta_H$  to work. Then  $W = \theta_H$  and  $\theta_C = 0$

Therefore efficiency  $e$  is calculated from  $e = \frac{W}{\theta_H} = \frac{\theta_H + \theta_C}{\theta_H}$

This equation can also be expressed in terms of temperatures **T** (Must be in Kelvin)

$$e = \frac{T_H - T_C}{T_H}$$

### Example 1.7

A steam engine operates between 500 °C and 270 °C. What is the minimum possible efficiency of this engine?

#### Answer

From  $e = \frac{T_H - T_C}{T_H}$

Converting Temperatures into Kelvins  $T_H = (500 + 273) \text{ K}$ ,  $T_H = 773 \text{ K}$  and  $T_L = (270 + 273) \text{ K}$ ,  $T_L = 543 \text{ K}$ .

$$e = \frac{773 - 543}{773} = 0.298 = 29.8\%$$

### 1.4.2.3 Impact of heat engines on climate

Most of air pollution is caused by the burning of fuels such as oil, natural gas etc. The air pollution has an adverse effect on the climate. Climate change is the greatest environmental threat of our time endangering our health. When a heat engine is running, several different types of gases and particles are emitted that can have detrimental effects on the environment.

Of concern to the environment are carbon dioxide, a greenhouse gas; and hydrocarbons. Engines emit greenhouse gases, such as carbon dioxide, which contribute to **global warming**. Fuels used in heat engines contain carbon. The carbon burns in air to form carbon dioxide.

The Carbon dioxide and other global warming pollutants collect in the atmosphere and act like a thickening blanket and destroy the **ozone layer**. Therefore, the sun's heat from the sun is received direct on the earth surface and causes the planet to warm up.

As a result of global warming, the vegetation is destroyed, ice melts and water tables are reduced. Heat engines especially diesel engines produce Soot which contributes to global warming and its influence on climate.

The findings show that soot, also called black carbon, has a warming effect. It contains black carbon particles which affect atmospheric temperatures in a variety of ways. The dark particles absorb incoming and scattered heat from the sun; they can promote the formation of clouds that can have either cooling or warming impact. Therefore soot emissions have significant impact on climate change.

Similarly, some engines leak, for example, old car engines and oil spills all over. When it rains, this oil is transported by rain water to lakes and rivers. The oils then create a layer on top of the water and prevent free evaporation of the water.

### 1.4.3 Carnot cycle and Carnot engine

In 1824 a French engineer named **Sadi Carnot** described a theoretical engine, now called a ***Carnot engine***, which is of great importance from both practical and theoretical viewpoints. He showed that a heat engine operating in an ideal, reversible cycle—called a ***Carnot cycle***—between two energy reservoirs is the most efficient engine possible.

An ideal engine establishes an upper limit on the efficiencies of all other engines. That is, the net work done by a working substance taken through the Carnot cycle is the greatest amount of work possible for a given amount of energy supplied to the substance at the higher temperature.

Carnot's theorem can be stated that **no real heat engine operating between two energy reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.**

Note: No Carnot engine actually exists, but as a theoretical idea it played an important role in the development of thermodynamics.

The idealized Carnot engine consisted of four processes done in a cycle, two of which are adiabatic ( $Q = 0$ ) and two are isothermal ( $\Delta T = 0$ ). This idealized cycle is shown in figure 1.8.

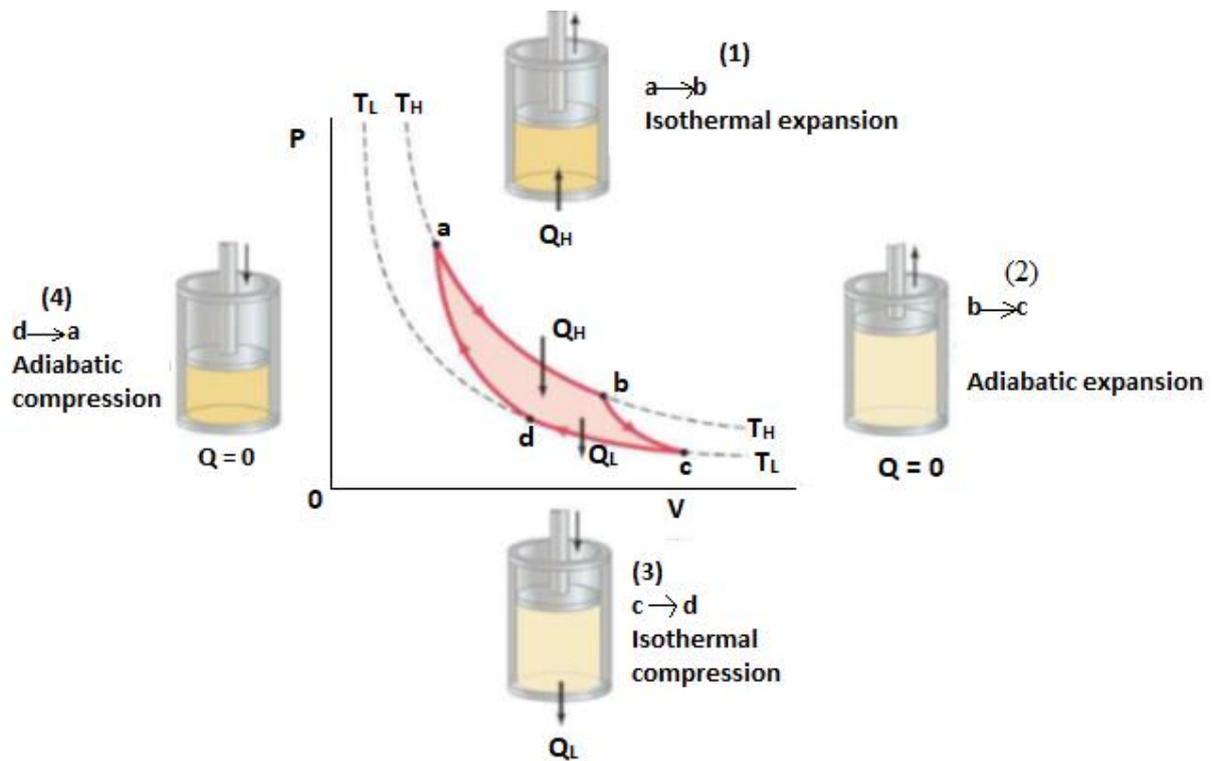


Fig.1. 8 Carnot Cycle.

Each of the processes was considered to be done *reversibly*. That is, each of the processes (say, during expansion of the gases against a piston) was done so slowly that the process could be considered a series of equilibrium states, and the whole process could be done in reverse with no change in the magnitude of work done or heat exchanged.

A real process, on the other hand, would occur more quickly; there would be turbulence in the gas, friction would be present, and so on. Because of these factors, a real process cannot be done precisely in reverse, the turbulence would be different, and the heat lost to friction would not reverse itself. Thus real processes are *irreversible*.

In the figure 1.7 (Carnot cycle), heat engines work in a cycle, and the cycle for the Carnot engine begins at point *a* on the PV diagram.

**Note:**

- The gas is first expanded isothermally, with addition of heat  $Q_H$ , along the path *ab* at temperature  $T_H$ .
- Next the gas expands adiabatically from *b* to *c*, no heat exchanged, but the temperature drops to  $T_L$ .
- The gas is then compressed at constant temperature  $T_L$ , path *cd*, and let  $Q_L$  flows out.

- Finally, the gas is compressed adiabatically, path da, back to its original state.

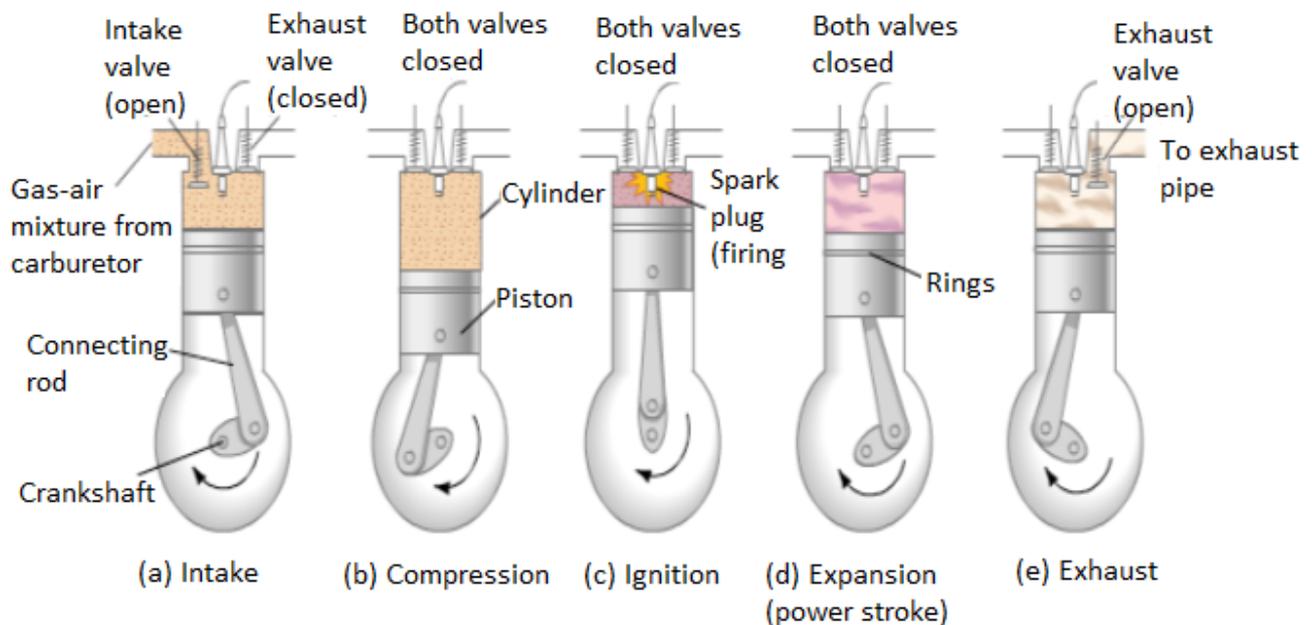
Carnot showed that for an ideal reversible engine, the heats  $Q_H$  and  $Q_L$  are proportional to the operating temperatures  $T_H$  and  $T_L$  (in Kelvin), so the efficiency can be written as:

$$e_{\text{ideal}} = \frac{T_H - T_L}{T_H} \Rightarrow e = 1 - \frac{T_L}{T_H}$$

The equation above gives a Carnot (ideal) efficiency. It expresses the fundamental upper limit to the efficiency. Real engines always have an efficiency lower than this because of losses due to friction. Real engines that are well designed reach 60 to 80% of the Carnot efficiency.

### 1.4.4 Diesel engine

In an internal combustion engine (used in most automobiles), the high temperature is achieved by burning the gasoline-air mixture in the cylinder itself (ignited by the spark plug), as described in figure 1.9.



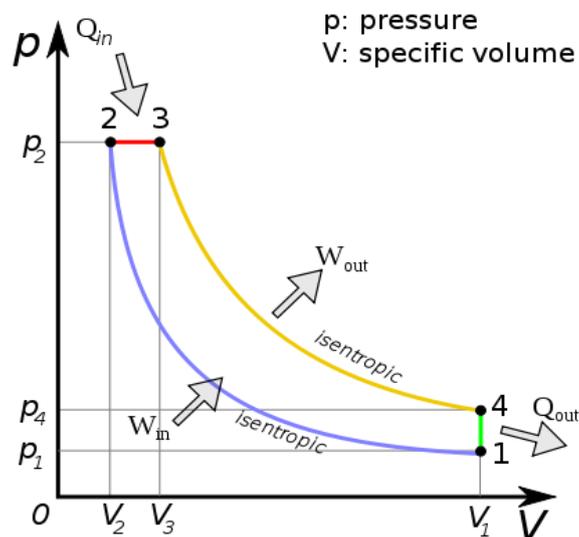
**Fig.1. 9 Four-strokes-cycle internal combustion engine**

On the figure 1.9 above, the following takes place.

(a) the gasoline-air mixture flows into the cylinder as the piston moves down;

- (b) the piston moves upward and compresses the gas;
- (c) the brief instant when firing of the spark plug ignites the highly compressed gasoline-air mixture, raising it to a high temperature.
- (d) the gases, now at high temperature and pressure, expand against the piston in this, the power stroke
- (e) the burned gases are pushed out to the exhaust pipe; when the piston reaches the top, the exhaust valve closes and the intake valve opens, and the whole cycle repeats. (a), (b), (d) and (c) are the four strokes of the cycle.

### 1.4.5 Idealized diesel cycle



**Fig.1. 10 Ideal diesel cycle**

From P-V diagram for the Ideal Diesel cycle, the cycle follows the numbers 1-4 in clockwise direction. The image on the top shows a P-V diagram for the ideal Diesel cycle; where P is pressure and V is specific volume. The ideal Diesel cycle follows the following four distinct processes (the color references refers to the color of the line on the diagram).

- Process 1-2 is isentropic (adiabatic) compression of the fluid (blue color).
- Process 2-3 is reversible (isobaric constant pressure heating (red).
- Process 3-4 is isentropic (adiabatic) expansion (yellow).
- Process 4-1 is reversible constant volume cooling (green).

The Diesel is a heat engine; it converts heat into work. The isentropic processes are impermeable to heat; heat flows into the loop through the left expanding isobaric process and some of it flows back out through the right depressurizing process, and the heat that remains does the work.

**Note:**

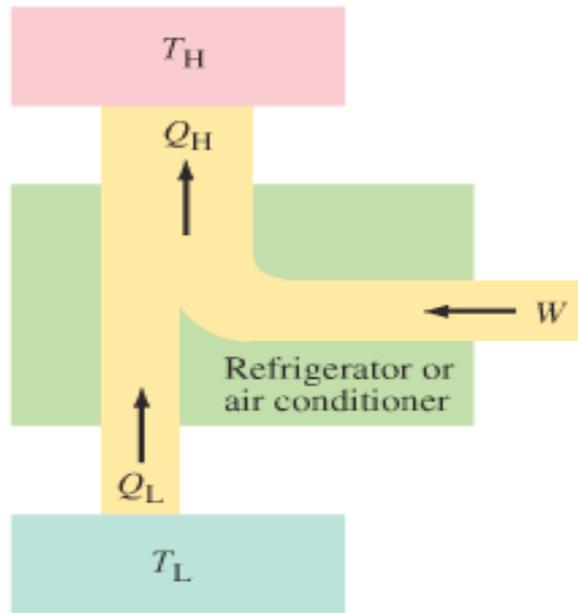
- Work in ( $W_{in}$ ) is done by the piston compressing the working fluid.
- Heat in ( $Q_{in}$ ) is done by the combustion of the fuel.
- Work out ( $W_{out}$ ) is done by the working fluid expanding on to the piston (this produces usable torque).
- Heat out ( $V_{out}$ ) is done by venting the air.

### 1.4.6 Refrigerator

A refrigerator is a device used to cool substances. It cools things by evaporation of a volatile liquid called **Freon**. The coiled pipe around the freezer at the top contains Freon which evaporates and takes latent heat from the surroundings so causing cooling. The electrically driven pump removes the vapor and forces it into the heat exchanger (pipes with cooling fins outside the rear of the refrigerator).

Here the vapor is compressed and liquefies (condenses) giving out latent heat of vaporization to the surrounding air. The liquid returns to the coils around the freezer and the cycle is repeated. An adjustable **thermostat** switches the pump on and off, controlling the rate of evaporation and so the temperature of the refrigerator.

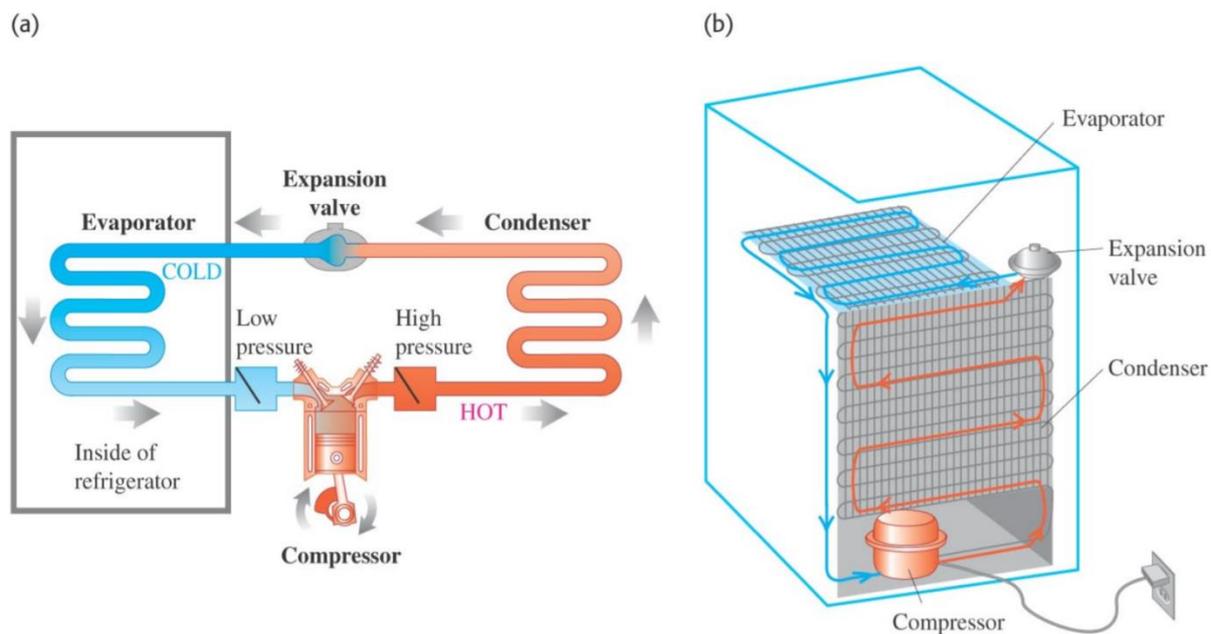
The operating principle of refrigerators is just the reverse of a heat engine. Each operates to transfer heat out of a cool environment into warm environment.



**Fig.1. 11. Schematic diagram of energy transfers for a refrigerator**

By doing work  $W$  as shown in figure 1.11, heat is taken from a low-temperature region,  $Q_L$  (such as inside a refrigerator), and a greater amount of heat is exhausted at a high temperature,  $Q_H$  (the room). You can often feel this heat blowing out beneath a refrigerator.

The work is usually done by an electric compressor motor which compresses a fluid, as illustrated in figure 1.12.



**Fig.1. 12 Typical refrigerator system**

In figure 1.12 (a) above, the electric compressor motor forces a gas at high pressure through a heat exchanger (condenser) on the rear outside wall of the refrigerator. Where  $Q_H$  is given off and the gas cools to become liquid.

The liquid passes from a high-pressure region, via a valve, to low-pressure tubes on the inside walls of the refrigerator; the liquid evaporates at this lower pressure and thus absorbs heat  $Q_L$  from the inside of the refrigerator. The fluid returns to the compressor, where the cycle begins again.

A perfect *refrigerator* is the one in which no work is required to take heat from the low-temperature region to the high temperature region is not possible. This is **Clausius statement of the second law of thermodynamics**, already mentioned can be stated formally as:

**“No device is possible whose sole effect is to transfer heat from one system at a temperature  $T_L$  into a second system at a higher temperature  $T_H$ ”.**

To make heat flow from a low-temperature object (or system) to one at a higher temperature, work must be done. Thus, **there can be no perfect refrigerator.**

The **coefficient of performance (COP)** of a refrigerator is defined as the heat  $Q_L$  removed from the low-temperature area (inside the generator) divided by the work  $W$  done to remove the heat:

$$\text{COP} = \frac{Q_L}{W}$$

This makes sense since the more heat,  $Q_L$ , that can be removed from inside the refrigerator for a given amount of work, the better (more efficient) the refrigerator is. Energy is conserved, so from the first law of thermodynamics we can write

$$Q_L + W = Q_H \text{ or } W = Q_H - Q_L.$$

Then we have: 
$$\text{COP} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

For an ideal refrigerator (not a perfect one, which is impossible), the best one could be:

$$\text{COP}_{\text{ideal}} = \frac{T_L}{T_H - T_L}$$

### Example 1.6

An ideal refrigerator-freezer operates with a  $\text{COP} = 7.0$  in a  $24^\circ\text{C}$  room. What is the temperature inside the freezer?

Solution:

From the equation of COP,

$$\text{COP}_{\text{ideal}} = \frac{T_L}{T_H - T_L}$$

Changing temperatures into Kelvins  $24^\circ\text{C} + 273 = 297\text{ K}$

$$7 = \frac{T_L}{297 - T_L}$$

Solving the Equation  $T_L = 259.875\text{ K}$ , Therefore,  $T_L = -13.125^\circ\text{C}$

#### APPLICATION ACTIVITY 1.4

1. What is the generic name for a cyclical device that transforms heat energy into work?
  - A. Refrigerator
  - B. Thermal motor
  - C. Heat engine
  - D. Carnot cycle
  - E. Otto processor
2. The maximum possible efficiency of a heat engine is determined by
  - A. its design.
  - B. the amount of heat that flows.
  - C. the maximum and minimum pressure.
  - D. the compression ratio.
  - E. the maximum and minimum temperature
3. A heat engine does 9200 J of work per cycle while absorbing 22.0kcal of heat from a high-temperature reservoir. What is the efficiency of this engine? (one calorie, Cal equals 4.186 joules)
4. A heat engine exhausts 8200 J of heat while performing 3200 J of useful work. What is the efficiency of this engine?
5. (a) Explain why the cooling compartment of a refrigerator is always on top.  
  
(b) The refrigerator cools substances by evaporation of a volatile liquid. Explain how evaporation causes cooling.
6. The low temperature of a freezer cooling coil is  $-15^{\circ}\text{C}$ , and the discharge temperature is  $30^{\circ}\text{C}$ . What is the maximum theoretical coefficient of performance?
7. A restaurant refrigerator has a coefficient of performance of 5.0. If the temperature in the kitchen outside the refrigerator is  $29^{\circ}\text{C}$ , what is the lowest temperature that could be obtained inside the refrigerator if it was ideal?

## **SKILLS LAB 1**

Design a working refrigerator, with the aid of the materials indicated below.

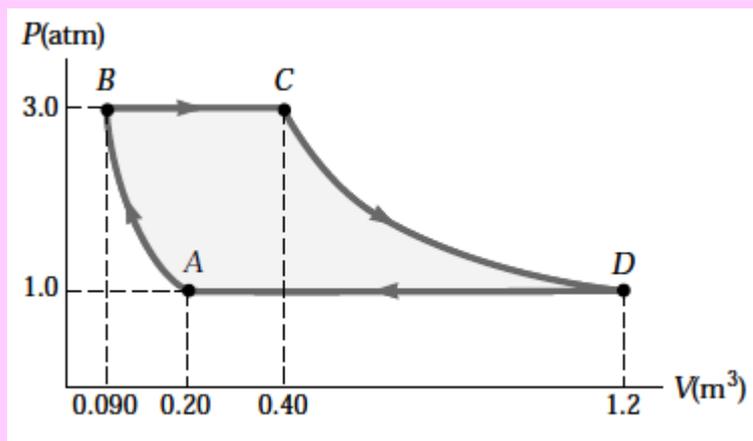
Materials: A pot, Charcoal, Cotton cloth and water.

Orientation tips:

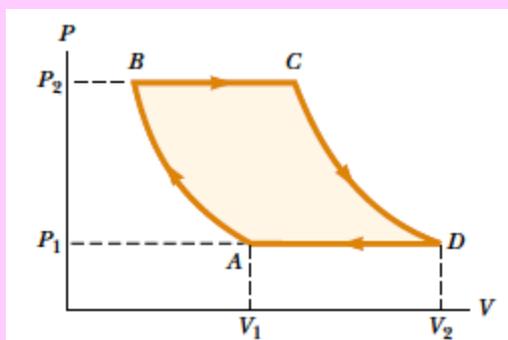
- You can use either a drink or food to test whether your fridge is functioning.
- You are not limited to use only these materials. You can also use/add other materials that can enable you to design a better refrigerator.
- You should make sure that your setup is put in one place for proper inspection and your set up may work after several days ( at least 3 days ).

## END UNIT ASSESSMENT 1

1. A steam engine operates between  $500\text{ }^{\circ}\text{C}$  and  $270\text{ }^{\circ}\text{C}$ . What is the maximum possible efficiency of this engine?
2. An engine manufacturer makes the following claims: the heat input per second of the engine is  $9\text{ kJ}$  at  $375\text{ K}$ . The heat output per second is  $4\text{ kJ}$  at  $225\text{ K}$ . Do you believe these claims?
3.  $2500\text{ J}$  of heat is added to a system, and  $1800\text{ J}$  of work is done on the system. What is the change in internal energy of the system?
4. A sample of an ideal gas goes through the process shown in Figure below. From  $A$  to  $B$ , the process is adiabatic; from  $B$  to  $C$ , it is isobaric with  $100\text{ kJ}$  of energy entering the system by heat. From  $C$  to  $D$ , the process is isothermal; from  $D$  to  $A$ , it is isobaric with  $150\text{ kJ}$  of energy leaving the system by heat. Determine the difference in internal energy  $U_B - U_A$



5. The internal energy of a system is initially  $27\text{ J}$ . A total of  $33\text{ J}$  of energy is added to the system by heat while the system does  $26\text{ J}$  of work. What is the system's final internal energy?
6. An ideal gas is carried through a thermodynamic cycle consisting of two isobaric and two isothermal processes as shown in Figure below. Show that the net work done on the gas in the entire cycle is given by  $W_{net} = P_1(V_2 - V_1) \ln \frac{P_2}{P_1}$

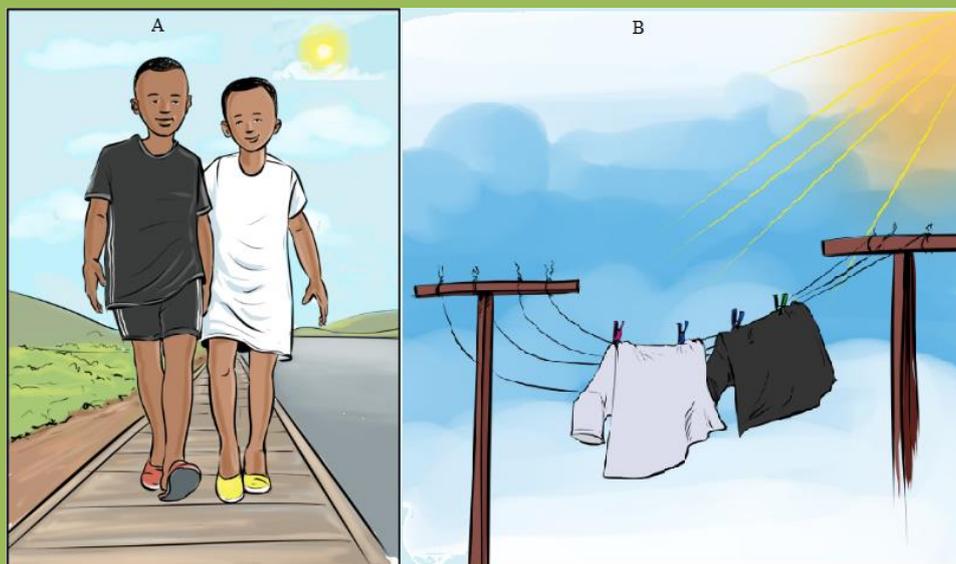




## UNIT 2 WAVE AND PARTICLE NATURE OF LIGHT

**Key unit competence:** Compare the nature of light

### INTRODUCTORY ACTIVITY



Observe the pictures A and B, and answer with scientific explanations the following questions:

1.

- Who will absorb more heat/radiations?
- In the dried clothes, which cloth will dry faster?
- Basing on observations made, explain why in most schools white shirts and blouses are preferred instead of other colours.

2. a) In packaging silvered foils are used to wrap most of fished products. Explain why these foils are preferred instead of darkened foils.

b) Explain why it's not recommended to paint inside one's room with a black paint?

c) Explain the variations in temperatures inside a house that is roofed using black coloured iron sheets and one roofed using white iron sheets.

## 2.1 Nature and properties of light

### ACTIVITY 2.1

- (a) What does it mean to say that a certain quantity is quantized?
- (b) They are three states of matter (solid, liquid and gas). Is light matter? If yes what is its state? If not list three major properties of light.
- (c) We do not notice the wavelength of moving matter in our ordinary experience. Is this because the wavelength is extraordinarily large or extraordinarily small? Explain to support your idea.

### 2.1.1 Concept of light

#### Particle theory of light

The nature and properties of light have been a subject of great interest and speculation since ancient times. Until the time of **Isaac Newton** (1642–1727), the **Greeks** believed that light consisted of tiny particles that either were emitted by a light source or emanated from the eyes of the viewer.

**Newton** the chief architect of the particle theory of light held that light consisted of tiny particles that were emitted from a light source and that these particles stimulated the sense of sight upon entering the eye. By particle theory, he was able to explain reflection and refraction of light.

However, derivation of the law of refraction depend on the assumption that light travels faster in water and in glass than in air, an assumption later shown to be false. Most scientists accepted Newton's particle theory.

#### Wave theory of light

In the mid-seventeenth century, the Jesuit priest **Francesco Grimaldi** (1618–1663) had observed that when sunlight entered a darkened room through a tiny hole in a screen, the spot on the opposite wall was larger than would be expected from geometric rays. He also observed that the border of the image was not clear but was surrounded by colored fringes. Grimaldi attributed this to the **diffraction** of light.

In 1678, one of Newton's contemporaries, the Dutch physicist and astronomer Christian **Huygens** (1629–1695), was able to explain many other properties of light by proposing that light is a **wave**.

By wave theory of light, Huygens was able to explain reflection and refraction of light by assuming that light travels more slowly in water and in glass than in air. Huygens' Principle is particularly useful for analyzing what happens when waves run into an obstacle.

The bending of waves behind obstacles into the “shadow region” is known as **diffraction**. Since diffraction occurs for waves, but not for particles, it can serve as one means for distinguishing the nature of light.

In 1801, the Englishman Thomas **Young** (1773–1829) provided the first clear demonstration of the wave nature of light and showed that light beams can **interfere** with one another, giving strong support to the wave theory. Young showed that, under appropriate conditions, light rays interfere with each other. Such behaviour could not be explained at that time by a particle theory because there was no conceivable way in which two or more particles could come together and cancel one another.

The general acceptance of wave theory was due to the French physicist **Augustin Fresnell** (1788-1827), who performed extensive experiments on interference and diffraction and put the wave theory on a mathematical basis. In 1850, **Jean Foucault** measured the speed of light in water and showed that it is less than in air, thus **ruling out Newton's particle theory**.

### 2.1.2 Planck quantum theory

In 1900, German Physicist **Max Planck (1858–1947)** returned to the particle theory of light to explain the radiation emitted by hot objects. Planck's quantum theory suggests that:

- The matter is composed of a large number of oscillating particles. These oscillators have different frequencies.
- The radiant energy which is emitted or absorbed by the blackbody is not continuous but discontinuous in the form of small discrete packets of energy and each such packet of energy is called a '**quantum**'. In case of light, the quantum of energy is called a '**photon**'.
- The energy of each quantum is directly proportional to the frequency ( $f$ ) of the radiation, the energy of the oscillations of atoms within molecules cannot have just any value; instead each has energy which is a multiple of a minimum value related to the frequency of oscillation by

$$E = hf = \frac{hc}{\lambda}$$

Where

$c = 3.00 \times 10^8 \text{ m/s}$  is the speed of light,

$\lambda$  is the wavelength and  $f$  the frequency of the corresponding wave

$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$  is the Planck's constant and  $E$  refers to the energy of a particle;

- The oscillator emits energy, when it moves from one quantized state to the other quantized state. The oscillator does not emit energy as long as it remains in one energy state. The total amount of energy emitted or absorbed by a body will be some whole number quanta. Hence

$$E = nhf$$

Because all light ultimately comes from a radiating source, this idea suggests that light is transmitted as tiny particles, or **photons** as they are now called, as well as via the waves predicted by **Maxwell's** electromagnetic theory.

**In 1905, Einstein** then used the particle theory to explain how electrons are emitted by a metal exposed to light. This effect is called the **photoelectric effect**.

The quantization model assumes that the energy of light wave is present in particles called **photons**. Massless particle of energy and each carries portion of the energy of the wave  $E$ ; hence, the energy is said to be quantized (in discrete portion).

A photon has no mass and no charge. It is a carrier of electromagnetic energy and interacts with other discrete particles, e.g. electrons, atoms and molecules.

### 2.1.3 Duality nature of light

Today, scientists view light as having a dual nature—that is, light exhibits characteristics of a wave in some situations and characteristics of a particle in other situations.

In view of these developments, light must be regarded as having a dual nature: Wave particle duality postulates that all particles exhibit both **wave properties** and **particle properties**.

- Phenomena of light like interference, diffraction and polarization can be explained by wave theory and not by particle nature of light.
- Energy distribution in perfect black body radiation, photoelectric effect and Compton Effect can be explained by particle nature of light and not by wave theory. The concept of quantum mechanics is applied even to the motion of electrons in an atom in Bohr's atomic model.

## Principle of complementarities

Some experiments indicate that light behaves like a wave; others indicate that it behaves like a stream of particles. These two theories seem to be incompatible, but both have been shown to have validity. Physicists finally came to the conclusion that this duality of light must be accepted as a fact of life. It is referred to as the **wave particle duality**. To clarify the situation, the great Danish physicist **Niels Bohr** (1885–1962) proposed his famous **principle of complementarity**. It states that:

*“To understand an experiment, sometimes we find an explanation using wave theory and sometimes using particle theory. Yet we must be aware of both the wave and particle aspects of light if we are to have a full understanding of light.”*

We need both to complete our model of nature, but we will never need to use both at the same time to describe a single part of an occurrence. Therefore these two aspects of light complement one another. We cannot readily picture a combination of wave and particle. Instead, we must recognize that the two aspects of light are different “faces” that light shows to experimenters.

### 2.1.4 Wave Nature of Matter

In 1924, Louis de **Broglie** (1892–1987) extended the idea of the wave–particle duality. He formulated the hypothesis, claiming that all matter, not just light only, has a wave like nature. He related the wavelength ( $\lambda$ ) and the momentum ( $p$ ) by the equation:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

For a nonrelativistic particle, we have:  $E = \frac{p^2}{2m} = eV$

The momentum of a photon is given by  $p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$

And the wavelength  $\lambda$  of light is given by  $\lambda = \frac{c}{f}$

According to classical mechanics, particle is a point like object having position and momentum, whereas wave is a disturbance in some space

### Example 2.1: Wave nature of matter

What is the energy of an electron with  $\lambda = 550 \text{ nm}$  (like green light)?

**Answer**

$$\text{Energy of electron: } E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

$$E = \frac{(6.63 \times 10^{-34} \text{ m/s})^2}{2(9.1 \times 10^{-31} \text{ kg})(550 \times 10^{-9} \text{ m})^2} = 7.98 \times 10^{-25} \text{ J} = 5.0 \times 10^{-6} \text{ eV}$$

### 2.1.5 Types of photon Interactions

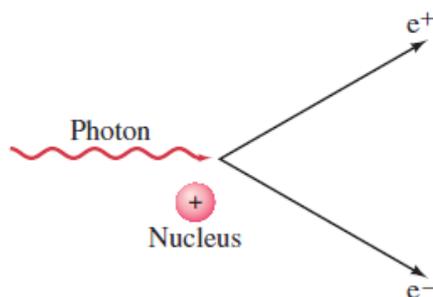
When a photon passes through matter, it interacts with the atoms and electrons. There are four important types of interactions that a photon can undergo:

1. The **photoelectric effect**: A photon may knock an electron out of an atom and in the process the photon disappears. To escape from the surface, an electron must absorb enough energy from the incident light to overcome the attraction of positive ions in the material. These attractions constitute a potential-energy barrier; the light supplies the “kick” that enables the electron to escape. The photoelectric effect provides convincing evidence that light is *absorbed* in the form of photons.
2. The photon may knock an atomic electron to a higher energy state in the atom if its energy is not sufficient to knock the electron out altogether. In this process the photon also disappears, and all its energy is given to the atom. Such an atom is then said to be in an **excited state**.
3. **Compton Effect**: The photon can be scattered from an electron (or a nucleus) and in the process lose some energy; this is the **Compton Effect**. But notice that the photon is not slowed down. It still travels with speed  $c$ , but its frequency will be lower because it has lost some energy.

A single photon of wavelength strikes an electron in some material, knocking it out of its atom. The scattered photon has less energy (some energy is given to the electron) and hence has a longer wavelength (shown exaggerated).

**4. Pair production**: If a gamma-ray photon of sufficiently short wavelength is fired at a target, it may not scatter. Instead, as depicted in Fig.2.2, it may disappear completely and be replaced by two new particles: an **electron** and a **positron** (a particle that has the same rest mass as an electron but has a positive charge rather than the negative charge of the electron). This process, called **pair production**,

was first observed by the physicists (Patrick Blackett and Giuseppe Occhialini). The electron and positron have to be produced in pairs in order to conserve electric charge.



**Fig.2. 1 Pair production: a photon disappears and produces an electron and a positron.**

The inverse process, electron–positron **pair annihilation**, occurs when a positron and an electron collide.

In pair **production**, the photon disappears in the process of creating the electron–positron pair. This is an example of mass being created from pure energy, and it occurs in accord with Einstein’s equation.

### APPLICATION ACTIVITY 2.1

1. Which of the following can be thought of as either a wave or a particle?

- A. Light.
- B. An electron.
- C. A proton.
- D. All of the above.

2. Electrons and photons of light are similar in that

- A. both have momentum given by
- B. both exhibit wave–particle duality.
- C. both are used in diffraction experiments to explore structure.
- D. All of the above.
- E. None of the above.

3. Calculate the energy of a photon of blue light,  $\lambda = 450 \text{ nm}$  in air (or vacuum).

4. A laser pointer with a power output of 5.00 mW emits red light  $\lambda = 650 \text{ nm}$

- a. What is the magnitude of the momentum of each photon?
- b. How many photons does the laser pointer emit each second?

5. Light of a certain orange colour has a wavelength of 589 nm. What is the energy of one photon of this light? Speed of light  $c = 3.00 \times 10^8 \text{ m/s}$ .

## 2.2 Blackbody radiation

### ACTIVITY 2.2

**Investigate with the help of the given materials to realize the nature of black body and its characteristics.**

**Materials:** 1 black cloth and 1 White cloth.

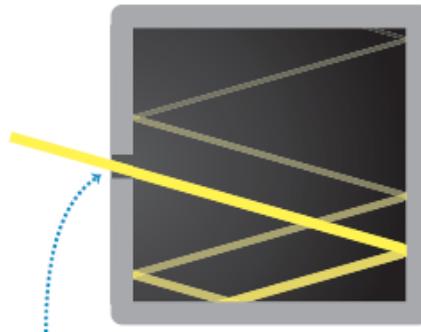
#### **Proceduce**

- Get two clothes of the same kind (similar material) but of different colour black and White
- Soak them in water at the same time
- Display them to places of the same sunlight intensity.
- Check them after like 20 min, 30 min, or 40 min to observe how they are drying up.
- Relate your findings to the concept of black body and try to explain what 'a black body' means.
- Based on your observation, explain what happens when the temperature of a black body increase.

### 2.2.1 Concept of blackbody

A **blackbody** is a body that, when cool, would absorb all the radiation falling on it (and so would appear black under reflection when illuminated by other sources).

A good approximation to a blackbody is a hollow box with a small aperture in one wall (Fig. 2.3). Light that enters the aperture will eventually be absorbed by the walls of the box, so the box is a nearly perfect absorber. Conversely, when we heat the box, the light that emanates from the aperture is nearly ideal blackbody radiation with a continuous spectrum.



Light that enters box is eventually absorbed.  
Hence box approximates a perfect blackbody.

**Fig.2. 2 A hollow box with a small aperture behaves like a blackbody.**

**Note :** When the box is heated, the electromagnetic radiation that emerges from the aperture has a blackbody spectrum.

Our sun, which has a surface temperature of about 6000 K, appears yellow, while the cooler star Betelgeuse has a red-orange appearance due to its lower surface temperature of 2900 K. Our body at 310 K emit electromagnetic radiation in the infra-red region of the spectrum, and these can be detected with infra-red sensitive devices.

### 2.2.2 Stefan–Boltzmann law for a black body

All objects, no matter how hot or cold, emit electromagnetic radiation (thermal radiation) whose total intensity  $I$  emitted from the surface of an ideal radiator is proportional to the fourth power of the Kelvin (absolute) temperature.

$$I = \frac{E}{tA} = \frac{P}{A} = \varepsilon\sigma T^4$$

Where

$\sigma = 5.670400 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$  is called the Stefan–Boltzmann constant.

$\varepsilon$  emissivity,  $\varepsilon = 1$  for perfect radiator (blackbody)

A is radiating area , P radiated power and T temperature of radiator

I is average rate of radiation of energy per unit surface area per unit time or average power per area.

### Example 2.3: Stefan–Boltzmann law for a black body

A metal sphere with a black surface and radius 30 mm, is cooled to  $-73^{\circ}\text{C}$  and placed inside an enclosure at temperature of  $27^{\circ}\text{C}$ . Calculate the initial rate of temperature rise of the sphere, assuming the sphere is a black body. (assume density of metal  $\rho = 8000 \text{ kg/m}^3$ , specific heat capacity of metal  $c = 400 \text{ J/kg}\cdot\text{K}$  and Stefan Boltzmann  $\sigma = 5.670400 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ )

#### Answer

Area:  $A = 4\pi r^2$ ,  $T = -73 + 273 = 200 \text{ K}$ ,  $T = 27 + 273 = 300 \text{ K}$

Since the temperature of the surroundings is given by the sphere, the energy emitted per second,

$$P_e = \sigma A(T^4 - T_0^4) = 4\pi\sigma r^2(T^4 - T_0^4)$$

The mass of the sphere  $m = \rho V = \frac{4\pi r^3 \rho}{3}$

The energy received per second:  $P_r = \frac{cm\Delta T}{t} = \frac{4\pi r^3 \rho c \Delta T}{3t}$

Assuming radiative equilibrium, The power emitted by sphere is equal to the power received by surrounding i.e  $P_e = P_r$

$$\text{Therefore } \frac{4\pi r^3 \rho c \Delta T}{3t} = 4\pi\sigma r^2(T^4 - T_0^4) \Leftrightarrow \frac{\Delta T}{t} = \frac{3\sigma(T^4 - T_0^4)}{r\rho c}$$

$$\frac{\Delta T}{t} = \frac{3 \times 5.7 \times 10^{-8} (300^4 - 200^4)}{30 \times 10^{-3} \times 8000 \times 400} = 0.012 \text{ K/s}$$

### 2.2.3 Wien's displacement law

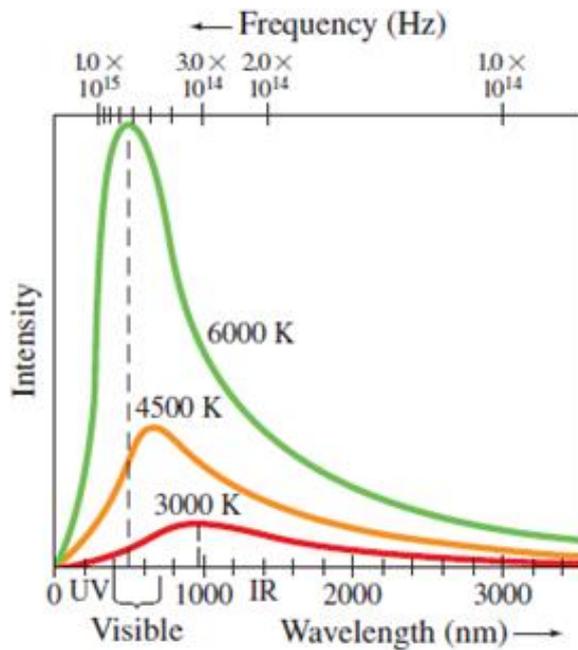
Fig.2.4 shows the measured spectral emittances  $I(\lambda)$  for blackbody radiation at three different temperatures. Each has a peak wavelength  $\lambda_m$  at which the emitted intensity per wavelength interval is largest. Experiment shows that  $\lambda$  is inversely proportional to  $T$ , so their product is constant. This observation is called the **Wien displacement law**.

$$\lambda_m T = 2.90 \times 10^{-3} \text{ m}\cdot\text{K}$$

The experimental value of the constant in expression above is  $2.90 \times 10^{-3} \text{ m.K}$ . The spectrum of radiation depends on the temperature and the properties of the object.

At normal temperatures, we are not aware of this electromagnetic radiation because of its low intensity. At higher temperatures, there is sufficient infrared radiation that we can feel heat if we are close to the object.

At still higher temperatures (on the order of 1000 K), objects actually glow, such as a red-hot electric stove burner or the heating element in a toaster. At temperatures above 2000 K, objects glow with a yellow or whitish color, such as white-hot iron and the filament of a light bulb.



**Fig.2. 3 Spectrum of light emitted by a hot dense object for an idealization blackbody**

The spectrum of light emitted by a hot dense object is shown in Fig. 2.4 for an idealized **blackbody**.

The radiation such an idealized blackbody would emit when hot and luminous, called **blackbody radiation** (though not necessarily black in color), and approximates that from many real objects.

The 6000 K curve in Fig. 2.4, corresponding to the temperature of the surface of the Sun, peaks in the visible part of the spectrum. For lower temperatures, the total intensity drops considerably and the peak occurs at longer wavelengths (or lower frequencies).

This is why objects glow with a red color at around 1000 K. Measured spectra of wavelengths and frequencies emitted by a blackbody at three different temperatures.

### Example 2.4: The Sun's surface temperature and temperature

Estimate the temperature of the surface of our Sun, given that the Sun emits light whose peak intensity occurs in the visible spectrum at around 500 nm.

#### Answer

We assume the Sun acts as a blackbody, and use in Wien's law:

$$T = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{\lambda_p} = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{500 \times 10^{-9} \text{ m}} = 6000 \text{ K}$$

**NOTE** This example helps us to understand why stars have different colors (reddish for the coolest stars; orangish, yellow, white, bluish for "hotter" stars.)

#### APPLICATION ACTIVITY 2.2

1. Electromagnetic radiations are emitted by which of the following?
  - A. Only by radio and television transmitting antennas
  - B. Only bodies at temperature higher than their surrounding
  - C. Only by red-hot bodies
  - D. By all bodies
2. Which of the following statements is true regarding how blackbody radiation changes as the temperature of the radiating object increases?
  - A. Both the maximum intensity and the peak wavelength increase.
  - B. The maximum intensity increases, and the peak wavelength decreases.
  - C. Both the maximum intensity and the peak wavelength decrease.
  - D. The maximum intensity decreases, and the peak wavelength increases.
3. Suppose a star has a surface temperature of 32 500 K. What color would this star appear?

## 2.3 Compton effect

### ACTIVITY 2.3

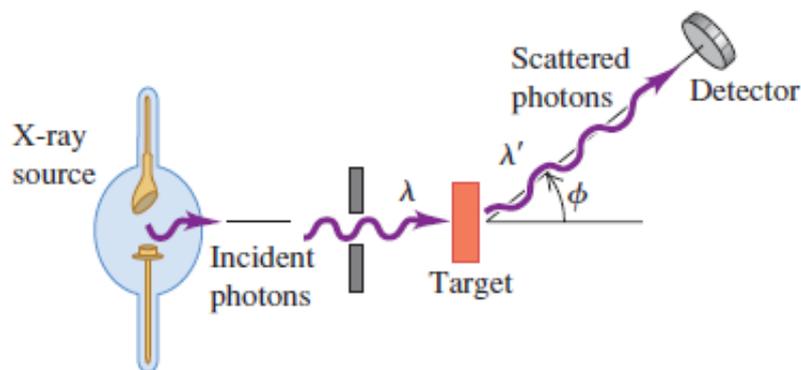
Light is made up of discrete particles called photons. These particles interact.

- Explain what happens to energy of these particles on collision.
- Where is this phenomenon of photon interaction applied in devices we use in real life?
- Discuss why you think there was a need to analyse the behaviour of light

In 1920, **Arthur Holly Compton** investigated the scattering of monochromatic x-rays (electromagnetic radiation) from various materials. In his experiment Compton aimed a beam of x rays at a solid target and measured the wavelength of the radiation scattered from the target (Fig. 2.5). The incident photon would give up part of its energy and momentum to the electron, which recoils as a result of this impact.

The scattered photon that remains can fly off at a variety of angles  $\theta$  with respect to the incident direction, but it has less energy and less momentum than the incident photon (Fig.2.5).

Therefore, in the photon model, the scattered light has a lower frequency and longer wavelength than the incident light. This is precisely what the photon model predicts for light scattered from electrons in the target, a process that is now called **Compton scattering**.



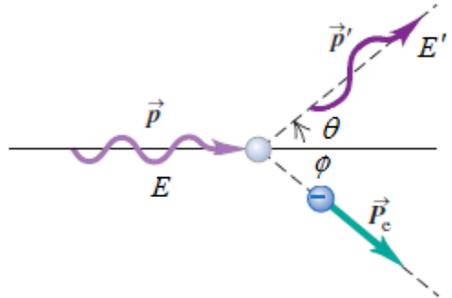
**Fig.2. 4 A Compton-effect experiment**

**Note:** The change in wavelength after striking the target depends on the angle at which the photons are scattered.

Compton showed that Einstein's photon theory, combined with the principles of conservation of energy and conservation of momentum, provides a beautifully clear explanation of his experimental

results. If, on the other hand, the incoming radiation is thought of as a beam of photons (electromagnetic quanta) then the situation becomes that of photons of energy  $E = hf$  scattering from free electrons in the target material.

Energy-momentum conservation, applied to this situation, predicts that the scattered photons will have energy  $E' = hf' < E$ , in complete agreement with Compton's experiments.



**Fig.2. 5 Energy conservation,**

The incident photon has momentum with magnitude  $\vec{p}$  and energy  $E = pc$

The scattered photon has momentum with magnitude  $\vec{p}'$  and energy  $E' = \vec{p}'c$

The electron is initially at rest, so its initial momentum is zero and its initial energy is its rest energy

$$E_0 = mc^2$$

The final electron momentum  $\vec{p}_e$  has magnitude  $p_e$  and the final electron energy is given by

$$E_e^2 = (mc^2)^2 + (p_e c)^2$$

Then energy conservation gives us

$$pc + mc^2 = p'c + E_e \quad (1)$$

Rearranging, we find

$$(pc - p'c + mc^2)^2 = E_e^2 = (mc^2)^2 + (p_e c)^2 \quad (*)$$

We can eliminate the electron momentum by using momentum conservation

$$\vec{p} = \vec{p}' + \vec{p}_e \Leftrightarrow \vec{p}_e = \vec{p} - \vec{p}'$$

By taking the scalar product of each side of this equation with itself or by using the law of cosines, we find

$$p_e^2 = p^2 + p'^2 - 2pp' \cos \phi$$

We now substitute this expression for  $p_e^2$  into eq (\*) and multiply out the left side. We divide out a common factor  $c^2$ , several terms cancel, and when the resultant equation is divided through by  $(pp')$ , the result is

$$\frac{mc}{p'} - \frac{mc}{p} = 1 - \cos \phi$$

Finally, we substitute  $p' = h/\lambda'$  and  $p = h/\lambda$ , then multiply by  $h/mc$  to obtain

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

The Compton wavelength,  $\lambda_c$ , of a particle is given by  $\lambda_c = \frac{h}{mc}$

The **Compton wavelength of electron** is:  $\lambda_c = \frac{h}{m_e c} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.1 \times 10^{-31} \text{ kg})(3 \times 10^8 \text{ m/s})} = 0.0024 \text{ nm}$

The Compton wavelength of a particle is equivalent to the wavelength of a photon whose energy is the same as the rest mass energy of the particle ( $E = mc^2$ ).

For the shift in wavelength of the scattered beam, while converting frequency to energy

$$(E = hf = \frac{hc}{\lambda}) \text{ yields } \frac{1}{E'} = \frac{1}{E} - \frac{1}{mc^2}(1 - \cos\theta)$$

Thus the **Compton energy equation** predicts that the inverse of the energy of the scattered photon varies linearly as  $(1 - \cos\theta)$  where  $\theta$  is the photon scattering angle.

### Example.2.5: X-ray scattering

X-rays of wavelength 0.140 nm are scattered from a very thin slice of carbon. What will be the wavelengths of X-rays scattered at (a)  $0^\circ$ , (b)  $90^\circ$ , (c)  $180^\circ$ ?

#### Answer

(a) For  $\varphi = 0^\circ$ ,  $\cos\varphi = 1$  and . From  $\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$  gives  $\lambda' = \lambda = 0.140 \text{ nm}$

(b) For  $\varphi = 90^\circ$ ; we get  $\lambda' = \lambda + \frac{h}{m_e c} = 0.140 \text{ nm} + \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} = 0.142 \text{ nm}$

(c) For  $\varphi = 180^\circ$  which means the photon is scattered backward, returning in the direction from which it came (a direct “head-on” collision),  $\cos\varphi = -1$  and  $1 - \cos\varphi = 2$

So  $\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\theta) = 0.140 \text{ nm} + \frac{2 \times 6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})} = 0.145 \text{ nm}$

#### APPLICATION ACTIVITY 2.3

1. An incident 71 pm X-ray is incident on a calcite target. Find the wavelength of the X-ray scattered at a  $30^\circ$  angle. What is the largest shift that can be expected in this experiment?
2. A 5.5 MeV gamma ray is scattered at  $60^\circ$  from an electron. What is the energy in mega electron volts of the scattered photon?

## **SKILLS LAB 2**

**Investigate different coloured surfaces to compare the absorption rates using sample materials provided.**

**Materials :** Silvered surfaces, Black coloured surface (painted, red, blue and green) and laboratory thermometers ( Ones responding to temperatures of surface) .

**procedures:**

- a) Measure and note down the initial temperatures of the surfaces using the thermometer.
- b) Display the three surfaces in an open place so that they can be illuminated by radiations from the sun or any source of heat. In case you are using sun's radiation make sure that this experiment is done on a hot/sunny day.
- c) After like 50 minutes, measure and record the temperatures of the different surfaces.
- d) Compare the temperatures of the different surfaces.
- e) Explain the general observation about different surfaces.

## END UNIT ASSESSMENT 2

1. Why do we say that light has wave properties? Why do we say that light has particle properties?
2. In both the photoelectric effect and in the Compton Effect, a photon collides with an electron causing the electron to fly off. What is the difference between the two processes?
3. If an electron and a proton travel at the same speed, which has the shorter wavelength? Explain.
4. Why do we say that electrons have wave properties? Why do we say that electrons have particle properties?
5. What are the differences between a photon and an electron? Be specific: make a list.
6. Blue light of frequency  $7.06 \times 10^{14}$  Hz shines on sodium. Calculate the maximum energy of the photoelectrons released.
7. The range of frequency of ultraviolet rays is  $7.9 \times 10^{14}$  Hz to  $5 \times 10^{17}$  Hz. What is corresponding range of energies of the photons of ultraviolet light? (Planck's constant  $h = 6.62 \times 10^{-34}$  J.s)
8. Electrons in X-rays tube are accelerated by a potential difference of 10.0 kV. If an electron produces one photon of wavelength 0.124 nm in a Compton scattering experiment. At what angle is
  - a) the wavelength of the scattered X-rays 1.0% longer than that of the incident X-rays?
  - b) it 0.050% longer?

## UNIT 3: SIMPLE HARMONIC MOTION

**Key unit competence:** Illustrate and explain energy changes in simple harmonic motion.

### INTRODUCTORY ACTIVITY

During break time in a school ,you happen to see pupils swinging in the child's swing and others enjoying in Merry-Go-Round as shown in figure A and B.



- a) Based on your observation, describe the motion of pupils in
  - i) Child's swing
  - ii) Merry-Go-Round.
- b) How is the kinds of motion described in a) above differ from linear motion?
- c) By using the situation above, state and explain all the energy changes before and after undergoing motion.
- d) How is the study of such kinds of motion in physics significant in real life situations ?

### 3.1 Kinematics of simple harmonic motion

#### ACTIVITY 3.1

Observe carefully the figure 3.1 and answer the questions that follow.



**Fig.3. 1 Motion of a bob.**

- Examine the type of motion undergone by the bob in the pendulum.
- Can you guess a point where the bob moves fast. Explain to support your decision.
- Discuss some of the factors that can make the bob to move faster or slower while in the swing.
- Would the bob continue oscillating indefinitely if displaced? If yes explain why? If not, explain why not?

In simple harmonic motion a body moves periodically such that its acceleration is directed towards a fixed point and directly proportional to the displacement of the body from the fixed point, we say that a body has executed **simple harmonic motion**.

Simple Harmonic motion can be defined as a special type of periodic motion in which acceleration is directed towards a fixed point and directly proportional to the displacement of the body from that fixed point.

### CHARACTERISTICS OF SIMPLE HARMONIC MOTION

- i) It is classified under periodic motion. Periodic motion is the motion of the body which continuously retraces its paths in equal intervals of time.
- ii) Its acceleration is directly proportional to the displacement from a fixed point
- iii) Its acceleration is always directed towards a fixed point
- iv) Mechanical energy is always conserved

**Note:** The motions, which all repeat in a regular cycle, are examples of **periodic motion**. Whenever the object is pulled away from its equilibrium position, the net force on the system becomes nonzero and pulls the object back toward equilibrium.

### KEY TERMS IN SIMPLE HARMONIC MOTION.

- a) **Amplitude, A** : This is the maximum displacement of a particle from its rest point (Equilibrium)
- b) **Period, T** : This is the time take for a particle to complete one cycle
- c) **Frequency, f** : This is the number of complete oscillations made per second. An oscillation is to and fro movement of a particle. It is the reciprocal of period T.

Thus,  $f = \frac{1}{T}$  it is measured in Hertz (Hz)

- d) **Displacement**: This is distance moved by a particle in a specified direction.
- e) **Wavelength,  $\lambda$**  . This is the distance between two successive points that in phase. It is measured in meters.

**Note:** The above terms can be summarized using the graphs below.

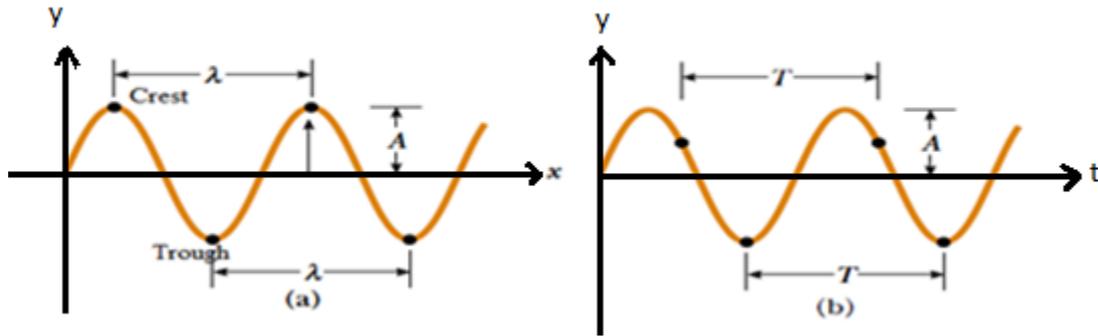


Fig.3. 2 Sinusoidal graphs of SHM

### 3.1.1 Equations of simple harmonic motion

Consider the diagram below to derive the displacement equation. .

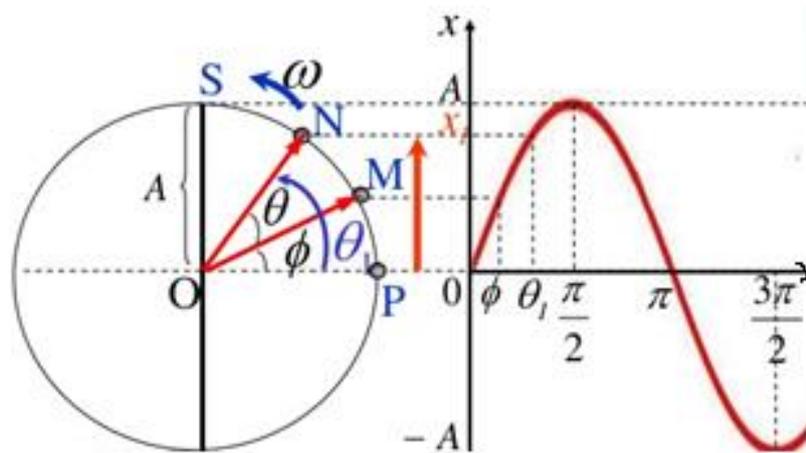


Fig.3. 3 A derivation of sine curve for a body executing simple harmonic motion

The object's movement to and from P, M, S continuing can be regarded as simple harmonic motion.

The maximum displacement  $OS = a$  is known as amplitude.

From the figure,  $OS \approx ON = A$

$$\sin \theta = \frac{x}{A}$$

$$x = A \sin \theta \quad (i)$$

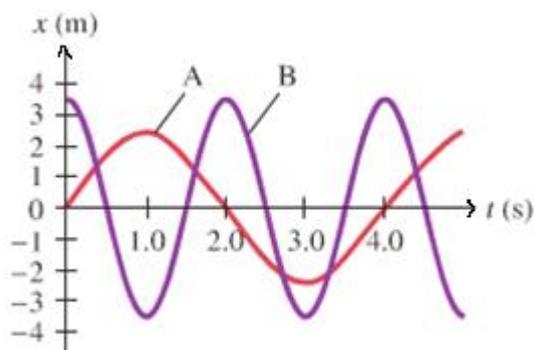
But  $\theta$  changes as time changes  $\theta$  is called angular displacement,  $\frac{\theta}{t}$  is the angular velocity  $\omega$

Thus  $\theta = \omega t$  (ii)

Substituting ii) into i) gives  $x = A \sin \omega t$

Note: It should be noted that the displacement, can be given as a sine curve if the system starts from a certain point. Thus, the equation can also be stated as  $x = a \cos \omega t$

In summary, these equations can be illustrated using the graph below



**Fig.3. 4 Displacement time graph for a body executing simple harmonic motion**

Graph A represents a body executing simple harmonic motion when it starts at rest and its expression is a sine curve.

$$x = A \sin \omega t$$

While Graph B is for a body that is in motion and is given by a cosine curve

$$x = A \cos \omega t$$

### Example 3.1

A body vibrates according to equation  $y_{cm} = 1.60 \sin(1.30t)$ . Calculate the amplitude of the motion, angular frequency and initial displacement

Solution.

Comparing equation  $y_{cm} = 1.60 \sin(1.30t)$ . with  $y_{cm} = A \sin(\omega t)$ .

Amplitude = 1.60 cm

Angular frequency  $\omega = 1.3 \text{ rad/s}$

Initial displacement is when  $t = 0$

Therefore,  $y_{cm} = 1.60 \sin(1.30 \times 0) = 1.60 \text{ cm}$

### 3.1.2 Velocity in simple harmonic motion

Since velocity is the rate of change of displacement of a body. Therefore velocity,  $V$  is given by

$$V = \frac{dx}{dt} \quad \text{and} \quad x = A \sin \omega t$$

$$V = \frac{d(A \sin \omega t)}{dt}$$

$$V = A\omega \cos \omega t$$

From equation of displacement and velocity in simple harmonic motion, we can deduce the general expression of velocity in simple harmonic motion.

$$x = A \sin \omega t$$

$$\frac{x}{A} = \sin \omega t$$

Squaring the above equation gives

$$\sin^2 \omega t = \frac{x^2}{A^2} \dots\dots\dots(i)$$

Also, from equation of velocity

$$V = A\omega \cos \omega t$$

$$\frac{V}{A\omega} = \cos \omega t$$

Squaring the equation above gives

$$\frac{V^2}{A^2 \omega^2} = \cos^2 \omega t \dots\dots\dots(ii)$$

Adding equations i) and ii)

$$\sin^2 \omega t + \cos^2 \omega t = \frac{x^2}{A^2} + \frac{V^2}{A^2 \omega^2} \dots\dots\dots(iii)$$

But from trigonometry,

$$\sin^2 \omega t + \cos^2 \omega t = 1$$

Therefore equation iii) becomes

$$1 = \frac{x^2}{A^2} + \frac{V^2}{A^2 \omega^2}$$

$$V^2 = A^2 \omega^2 - x^2 \omega^2 \dots\dots\dots(iv)$$

Taking square root equation iv) becomes

$$\sqrt{V^2} = \sqrt{A^2 \omega^2 - x^2 \omega^2}$$

$$V = \pm \omega \sqrt{A^2 - x^2} \dots\dots\dots(v)$$

Which is the general expression for velocity in simple harmonic motion. Maximum velocity is attained if the body passes through the equilibrium position, that is  $x=0$

Therefore, for maximum velocity

$$V_{\max} = \pm \omega \sqrt{A^2}$$

$$V_{\max} = \pm \omega A$$

**Example 3.2**

A particle that hangs from a spring oscillates with an angular frequency of 2 rad/s. The spring is suspended from the ceiling of an elevator car and hangs motionless (relative to the car) as the car descends at a constant speed of 1.5 m/s. The car then suddenly stops. Neglect the mass of the spring.

- a. With what amplitude does the particle oscillate?
- b. What is the equation of motion for the particle? (Choose the upward direction to be positive.)

**Answer**

(a) When traveling in the elevator at constant speed, the total force on the mass is zero. The force exerted by the spring is equal in magnitude to the gravitational force on the mass; the spring has the equilibrium length of a vertical spring. When the elevator suddenly stops, the end of the spring attached to the ceiling

stops. The mass, however has momentum,  $p = mv$ , and therefore starts stretching the spring. It moves through the equilibrium position of the vertical spring with its maximum velocity  $v_{\max} = 1.5 \text{ m/s}$ .

Its velocity as a function of time is  $v(t) = -\omega A \sin(\omega t + \varphi)$ .

Since  $v_{\max} = \omega A$  and  $\omega = 2 \text{ rad/s}$ , the amplitude of the oscillations is  $A = 0.75 \text{ m}$ .

(b) The equation of motion for the particle is  $\frac{d^2x}{dt^2} = -\frac{k}{m}x \Leftrightarrow \frac{d^2x}{dt^2} = -\omega^2x$ .

Its solution is  $x(t) = A \cos(\omega t + \varphi) = 0.75 \cos(2t + \varphi)$

If we choose the  $t=0$  to be the time the elevator stops and let the upward direction be positive, then  $x(0) = 0$ , and  $v(0) = -1.5 \text{ m/s}$ .

We therefore need  $\varphi$  to be  $\frac{\pi}{2}$

### 3.1.3 Acceleration in simple harmonic motion

Since the acceleration is the rate of change of velocity. Therefore, the acceleration is given by

$$a = \frac{dV}{dt} = \frac{d(A\omega \cos \omega t)}{dt}$$

Since  $V = A\omega \cos \omega t$ . The acceleration,  $a = -\omega^2 A \sin \omega t$

$$a = -\omega^2 (A \sin \omega t) \text{ but } A \sin \omega t = x$$

Therefore, acceleration in simple harmonic motion is given by  $a = -\omega^2 x$

This implies acceleration  $a$  is directly proportional to displacement  $x$  hence simple harmonic. The negative indicates that the object starts to decelerate as it starts to pass the centre.

### 3.1.4 Force in simple harmonic motion

For a system to change its state there must be force applied. Hence also in Simple harmonic motion, there is a force within the system.

From Newton's second law, force is directly proportional to the rate of change of momentum.

Therefore,  $F = ma$

Since this is force in simple harmonic motion, we use acceleration,  $a = -\omega^2 x$

Thus  $F = m(-\omega^2 x)$

But  $\omega^2 m = k$

Therefore, force in simple harmonic motion is given by

$$F = -kx$$

**Note:**

- The net force's magnitude is  $F = m\omega^2 x = kx$
- $k = m\omega^2$  is called the elastic constant or the force constant of a spring
- The net force occurs when a spring is deformed by an external force, we call it the **force of Hooke**
- Then, the period and the frequency of a SHM can be related to the strength  $k$  of a spring by:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

### Example 3.3

A 0.500 kg mass suspended from a spring and oscillates. The spring has a constant of 15 N/m. What is the period of oscillation?

**Answer**

From  $T = 2\pi\sqrt{\frac{m}{k}}$

$$T = 2\pi\sqrt{\frac{5}{15}} = 3.63 \text{ s}$$

### APPLICATION ACTIVITY 3.1

1. a) What is simple harmonic motion?  
b) State characteristics of a body performing Simple harmonic motion  
c) Write down expressions for
  - i) Displacement
  - ii) Velocity of the systems performing Simple harmonic motion, hence sketch their velocity time graph.
2. Given that the equation of motion of a mass is  $x(m) = 0.20\sin 3.0t$ , find the velocity and acceleration when the object is 5 cm from the equilibrium position.
3. A SHM has a speed of 2.15 m/s when it is 0.23 m from equilibrium. What is its period if its amplitude is 0.47 m?
4. A particle moving with S.H.M has velocities of 4 cm/s and 3 cm/s at distance of 3 cm and 4 cm respectively from its equilibrium position. Find
  - a) Amplitude of oscillation;
  - b) The angular velocity and period;
  - c) The velocity of the particle as it passes through the equilibrium position.
5. A SHM has a mass of 1.83 kg, a frequency of 10.0 Hz, and amplitude of 0.18 m. What is its potential energy when it is 0.13 m from equilibrium?

### 3.2 Simple Harmonic Oscillators

### ACTIVITY 3.2



**Fig.3. 5 Oscillating seat**

One day you went to a picnic on a certain hotel .In the compound, there is a swinging chair that is suspended on a string. When you sit down on the chair, it oscillates vertically. After the oscillations have stopped, you stand up slowly, and the chair rises up a small distance. Your friend also sits in the chair, and you find that the rate at which the chair is oscillating is different.

- Basing on the scenario above,can you predict the kind of oscillator shown above? What other examples of oscillators do you know?
- Explain any two factors you think affects the number of oscillations made by the swinging seat.
- Imagine, the springs are replaced by an elastic rope. Do you think the seat can swing the same way as when there were springs? Explain your reasoning.
- When your friend sat on the same seat, it oscillated with different oscillations. Explain what you think caused the difference?

The following are some of examples of harmonic oscillators that will be discussed in this unit.

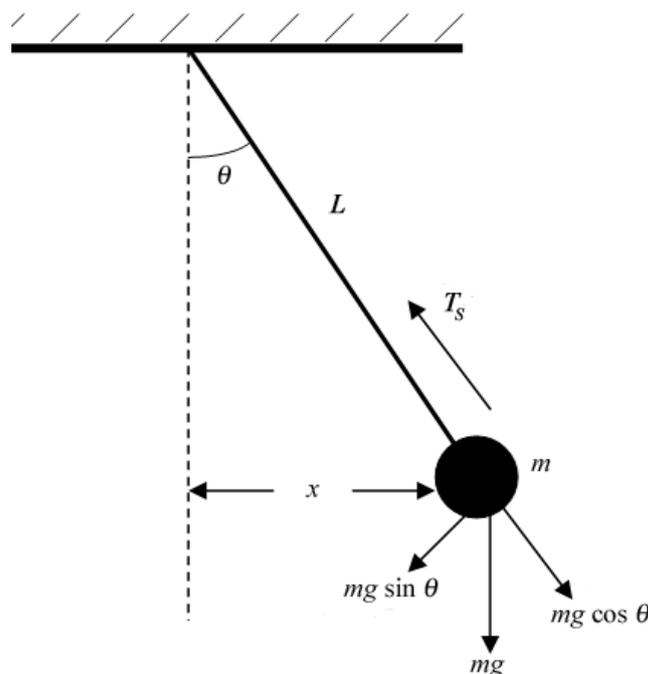
- Simple Pendulum,

- b. Mass on a helical spring (Helical Spring mass system)/ Stretched Spring
- c. Water in a U-tube

### 3.2.1 Simple Pendulum

A pendulum consists of a small mass  $m$  attached to the end of wire/thread of length  $l$  and the other end is attached to the fixed-point  $p$ .

If we displace the mass slightly and release it, we have the oscillation. The arc of a circle of center  $P$  and radius  $l$  whose  $o$  is the equilibrium point.



**Fig.3. 6 Swinging pendulum**

At  $m$ , the pulling force is tangential, and it is towards center, its magnitude restoring force is

$F = -mg \sin \theta$  and we know that  $F = ma$ , then  $a = g \sin \theta$ , the angle  $\theta$  is taken to be very small

Then we assume that  $\sin \theta = \theta$

Also  $\sin \theta = \frac{x}{L}$

Hence  $-mg \sin \theta = mg \theta \Leftrightarrow \frac{-mgx}{L} = ma$

Therefore  $a = \frac{-gx}{L}$

Hence simple harmonic motion since acceleration is directly proportional to the displacement.

From the equation above,

$$\frac{g}{L} = \frac{-a}{x}$$

Thus  $\omega^2 = \frac{-a}{x} = \frac{g}{L}$

From the equation of period in oscillations,

$$T = \frac{2\pi}{\omega} \Leftrightarrow T = \frac{2\pi}{\sqrt{\frac{g}{L}}} = 2\pi\sqrt{\frac{L}{g}} \quad , \quad \text{Therefore, } T = 2\pi\sqrt{\frac{L}{g}}$$

This is the expression for period of a swinging pendulum

### Example 3.4

If a child is suspended on in a swing with its string connected to the fixed point is 1 meter long. Find the period and frequency of oscillations made by a child at a location where  $g = 9.81 \text{ m/s}^2$

Solution:

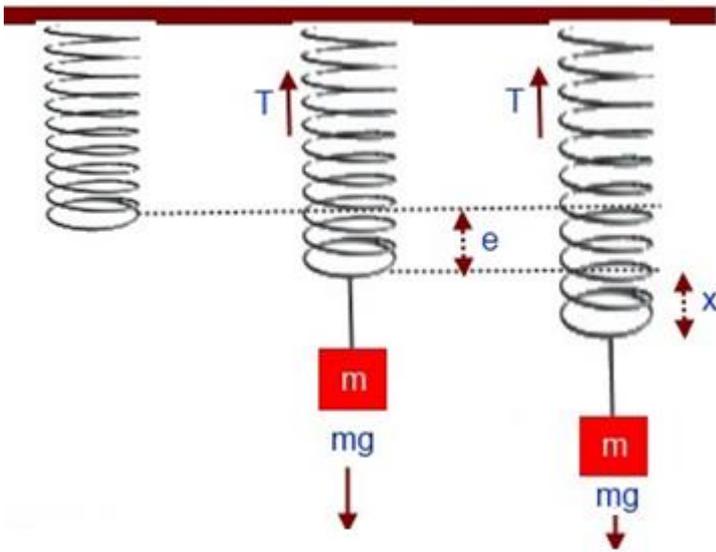
For the equation of Period,  $T = 2\pi\sqrt{\frac{L}{g}}$

$$T = 2\pi\sqrt{\frac{1}{9.81}} = 2.007 \text{ s}$$

For frequency  $f = \frac{1}{T} = \frac{1}{2.007} = 0.498 \text{ Hz}$

### 3.2.2 Mass on a herical spring (helical spring-mass system)

Suppose a spring whose one end is attached to fixed point and other support is a body of mass m, the system is in equilibrium as shown in figure 3.7.



**Fig.3. 7 Mass on a spring swinging**

Assuming that the used spiral spring obeys Hooke's law, the extension or compression of elastic object is directly proportional to the extending force.

In the first instance  $mg = ke$

Suppose the mass is now pulled down a further distance  $x$  below its equilibrium position, so

$$F = k(e + x)$$

Which is also the tension in the spring acting upwards. When the mass is released it oscillates up downwards. Hence the restoring force upward on mass is given by

$$f = k(e + x) - mg = kx \text{ since } (mg = ke)$$

At any displacement  $x$ , the acceleration is such that  $kx = ma$ ,  $a = \frac{kx}{m}$  the comparison with that we have already seen.

Since  $\frac{k}{m} = \omega^2$ , so  $\omega = \sqrt{\frac{k}{m}}$  and the period

$$T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{e}{g}}, \left(\frac{m}{k} = \frac{e}{g}\right)$$

### The effective mass of a spring

When experiencing the study of the spring, one can find the period which is different from the expected value when using mathematical formula. This is due to the effect of the mass of a spring.

This effective mass is denoted as “ $m_s$ ”. For this case, the period becomes:

$$T = 2\pi\sqrt{\frac{m+m_s}{k}}, \text{ and we know that from}$$

$$\text{Hooke's law; } mg = ke \Rightarrow m = \frac{ke}{g}.$$

From this one can find the equation for the period:

$$T = 2\pi\sqrt{\frac{(ke/g) + m_s}{k}} \Rightarrow T^2 = \frac{4\pi^2}{k} \left( \frac{ke}{g} + m_s \right).$$

From the equation above , we can extract the static extension as follows:

$$e = \frac{g}{4\pi^2} \times T^2 - \frac{gm_s}{k}$$

### Example 3.5

A light spiral spring is loaded with a mass of 50 g and it extends by 10 cm . Calculate the period of small vertical oscillations. Use  $g = 10 \text{ m.s}^{-2}$

Solution:

The period  $T$  of the oscillations is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

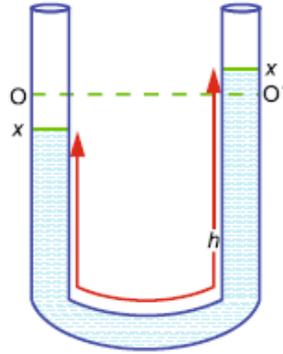
$$\text{Where } m = 50 \times 10^{-3} \text{ kg and from Hooke's law } k = \frac{mg}{e} = \frac{50 \times 10^{-3} \times 10 \text{ N}}{10 \times 10^{-2} \text{ m}} = 5.0 \text{ N} \cdot \text{m}^{-1}$$

$$\therefore T = 2\pi\sqrt{\left(\frac{50 \times 10^{-3}}{5}\right)} \text{ s} = 2\pi \times 10^{-1} \text{ s} = 0.63 \text{ s}$$

### 3.2.3 Oscillations of a liquid in a u-tube

In a U-tube with any other force adding on the liquid a part from the atmospheric pressure, the level is the same.

Let the liquid level in the left limb be depressed by  $x$ , so that it is elevated by the same height in the right limb (Fig. 3.8).



**Fig.3. 8 Liquid oscillating in U-tube**

If  $\rho$  is the density of the liquid,  $A$  the cross-section of the tube,  $M$  the total mass, and  $m$  the mass of liquid corresponding to the length  $2x$ , which provides the unbalanced force.

By applying a force on the liquid with the aid of balloon, water depends on one branch ascends: releasing the balloon water descends and oscillations take place. The oscillation is due to the difference of levels, i.e. the liquid that occupies the distance  $2x$  of the second branch.

This level difference produces a force  $F$  responsible for oscillations.  $F = -mg$  where  $m$  mass of the liquid in level difference  $2x$ .

At mass of the liquid,  $m = \text{Density of the liquid} \times \text{Volume of the liquid}$ . Therefore,  $m = \rho \times (2xA)$ .

$$M \frac{d^2x}{dt^2} = -mg \Leftrightarrow M \frac{d^2x}{dt^2} = -2xA\rho g$$

$$\frac{d^2x}{dt^2} = -\frac{2A\rho g}{M}x = -\frac{2Ag\rho x}{hA\rho}, \text{ Since } V = Ah$$

$$\frac{d^2x}{dt^2} = -\frac{2gx}{h} = -\omega^2 x$$

In SHM ,  $F = -kx$  . This means  $-kx = -2xA\rho g \Leftrightarrow k = 2\rho Ag$  which is the elastic constant of the liquid. The period of oscillations in U-tube is given by  $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{2g}}$

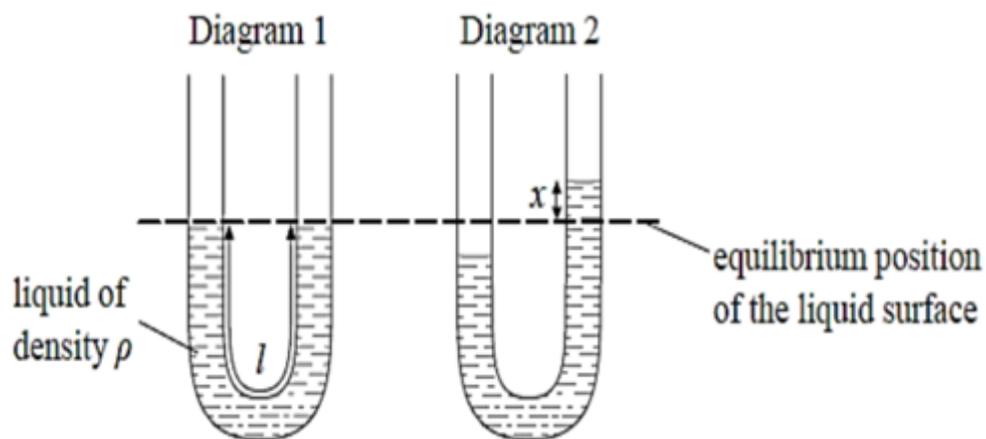
The equation of a period,T becomes

$T = 2\pi\sqrt{\frac{h}{2g}}$  and it is used if you are given full length of the liquid. The equation changes to

$T = 2\pi\sqrt{\frac{h}{g}}$  if a half-length of the liquid is given.

### Example 3.5

The pressure on the liquid in one side of the tube is increased so that the liquid is displaced as shown in diagram below. When the pressure is suddenly released the liquid oscillates.



**Fig.3. 9 Example on liquid in a U-tube.**

The total length of the liquid column in the tube is 0.32 m. Determine the period of oscillation.

Solution:

From.

$$T = 2\pi\sqrt{\frac{h}{2g}} \Leftrightarrow T = 2\pi\sqrt{\frac{l}{2g}}$$

$$T = 2\pi\sqrt{\frac{0.32}{2 \times 9.81}}$$

$$T = 0.8 \text{ s}.$$

### APPLICATION ACTIVITY 3.2

1. A mass  $M$  hangs in equilibrium on a spring.  $M$  is made to oscillate about the equilibrium position by pulling it down 10 cm and releasing it. The time for  $M$  to travel back to the equilibrium position for the first time is 0.50 s. Which line, A to D, is correct for these oscillations?

	Amplitude /cm	Period /s
A	10	1.0
B	10	2.0
C	20	2.0
D	20	1.0

2. Which one of the following statements is true when an object performs simple harmonic motion about a central point  $O$ ?

A	The acceleration is always away from $O$
B	The acceleration and velocity are always in opposite directions
C	The acceleration and the displacement from $O$ are always in the same direction.
D	The graph of acceleration against displacement is a straight line.

3. A simple pendulum and a mass-spring system are taken to the Moon, where the gravitational field strength is less than on Earth. Which line, A to D, correctly describes the change, if any, in the period when compared with its value on Earth?

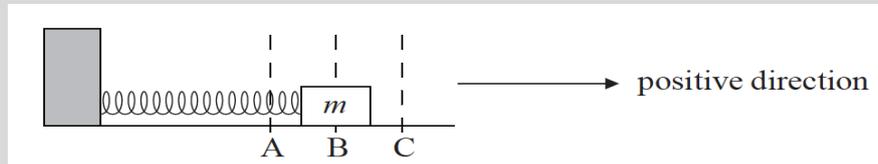
	Period of pendulum	Period of mass-spring system
A	Decrease	Decrease
B	Increase	Increase
C	no change	decrease

D

increase

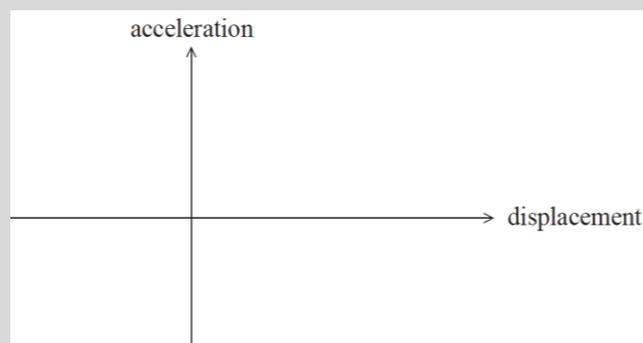
no change

4. An object of mass  $m$  is placed on a frictionless surface and attached to a light horizontal spring. The other end of the spring is fixed.



The equilibrium position is at B. The direction B to C is taken to be positive. The object is released from position A and executes simple harmonic motion between positions A and C.

- Define simple harmonic motion.
- On the axes below, sketch a graph to show how the acceleration of the mass varies with displacement from the equilibrium position B.
- On your graph, label the points that correspond to the positions A, B and C.

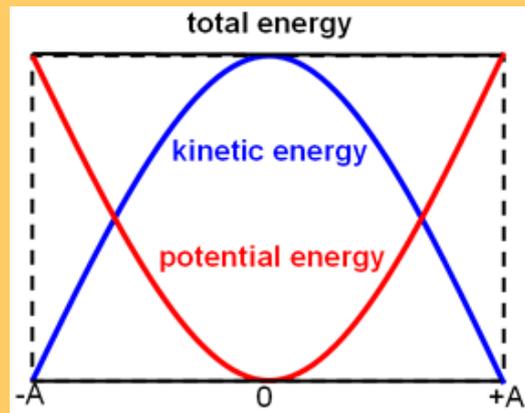


5. A 50 g mass vibrates in a SHM at the end of a spring. The amplitude of the motion is 12 cm, and the period is 1.70 s. Find:

- the frequency; the spring constant;
- the maximum speed of the mass;
- the maximum acceleration of the mass;
- the speed when the displacement is 6.0 cm;
- the acceleration when the displacement is 6.0 cm

### 3.3 Energy changes and conservation in oscillating systems

### ACTIVITY 3.3



**Fig.3. 10 Energy variation in oscillating systems**

It is known that every system in mechanics, there is variation of kinetic energy and potential energy.

Explain what causes the changes of energies in a given system.

#### 3.3.1 Kinetic energy of a particle in simple harmonic motion

Kinetic energy of a particle for the translation motion with the speed  $v$  is given by  $E_k = \frac{1}{2}mv^2$ ,

$$\text{But } v = \pm\omega\sqrt{A^2 - x^2} \text{ and } E_k = \frac{1}{2}m\omega^2(\sqrt{A^2 - x^2})^2$$

So the kinetic energy of the particle for the SHM will be  $E_k = \frac{1}{2}m\omega^2(A^2 - x^2)$ . It will be zero at each end of the particle's path where  $X = \pm X_m$  and will be maximum when  $X = 0$

#### 3.3.2 Potential energy of particle in simple harmonic motion

We know that for a force that derives on a potential  $F = -\frac{dE_p}{dx}$

$$F = -k.x \Leftrightarrow dE_p = k.x.dx,$$

So the **potential energy** is  $E_p = \frac{1}{2}k x^2 = \frac{1}{2}m\omega^2 x^2$

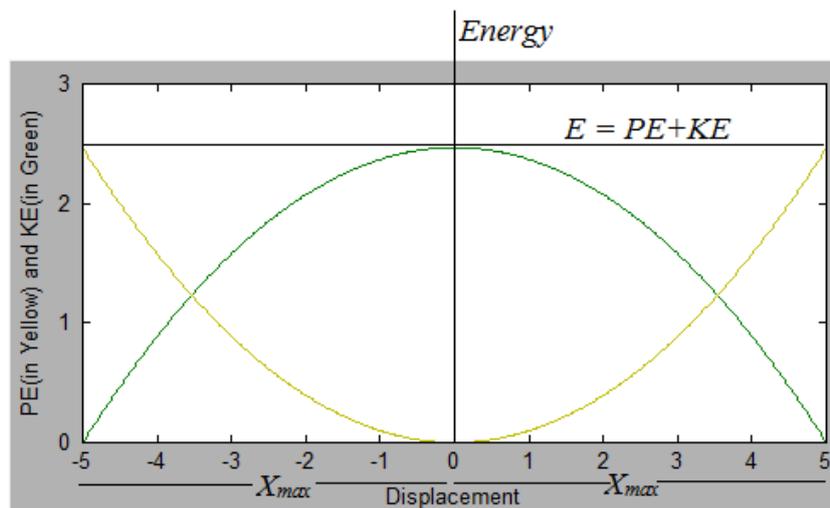
### 3.3.3 Mechanical energy in simple harmonic motion

Mechanical energy is the sum of potential energy and kinetic energy in the system.

Therefore,  $E = E_p + E_k = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2$

**Mechanical Energy**  $= \frac{1}{2}m\omega^2 A^2$

As  $m$ ,  $\omega$  and  $a$  are constant, the total mechanical energy  $E$  for a particle in SHM is also constant; we say that  $E$  is conserved.



**Fig.3. 11 A graph energy against displacement**

In this graph we have considered  $m = 0.5 \text{ kg}$  ,  $\omega = 0.2\pi \text{ rad / s}$  ,  $A = 5 \text{ m}$

If the force that restores the object to its equilibrium position is directly proportional to the displacement of the object, the motion that results is called **simple harmonic motion**.

### Example 3.6

A 0.500 kg mass is vibrating in a system in which the restoring constant is 100 N/m; the amplitude of vibration is 0.200 m. Find

- the mechanical energy of the system
- the maximum velocity
- the PE and KE when  $x = 0.100$  m

#### Solution:

From *Mechanical Energy*,  $E = \frac{1}{2} m \omega^2 a^2$

and  $K = m \omega^2$

$$E = \frac{1}{2} K a^2 = \frac{1}{2} \times 100 \times (0.2)^2 = 2 \text{ J}$$

b) the maximum velocity  $V_{\max} = \omega a$

$$\text{Finding } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{0.5}} = 14.14 \text{ rad / s}$$

Therefore.  $V_{\max} = 14.14 \times 0.2 = 2.83 \text{ m / s}$

c) The PE and KE when  $x = 0.100$  m

Potential energy  $E_p = \frac{1}{2} k \cdot x^2 = \frac{1}{2} m \omega^2 \cdot x^2$

$$E_p = \frac{1}{2} \times 100 \times 0.1^2 = 0.5 \text{ J}$$

Kinetic Energy,  $E_k = \frac{1}{2} m \omega^2 (a^2 - x^2)$

$$E_k = \frac{1}{2} \times 0.5 \times (14.14)^2 (0.2^2 - 0.1^2) = 1.5 \text{ J}$$

### APPLICATION ACTIVITY 3.3

1. Read the passage below and answer the questions that follow

In Simple harmonic motion, a body's acceleration is directly proportional to the displacement of a particle and is directed toward a fixed point. As it is oscillating it stores both kinetic and potential energy which together results into the mechanical energy of the system.

The loss of this energy in the system is characterized by reduction in the amplitude of the system. It is important to know that the system's velocity is maximum as it passes through the Centre. However at maximum points the system stores all of its energy as potential energy hence its velocity is zero.

#### questions

a. From the passage, acceleration is inversely proportional to displacement.

A True

B False

b. The oscillating system stores both kinetic and potential energy

A True

B False

c. The velocity of the system when it passes via the centre is minimum

A True

B False

d. Mechanical Energy = Kinetic Energy - Potential energy

A True

B False

e. From the passage the loss of energy is characterized by amplitude increase

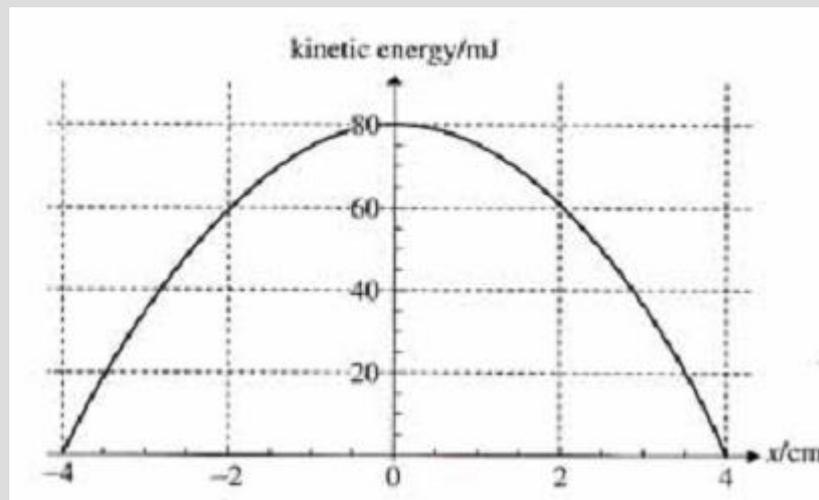
A True

B False

2. The period of a simple pendulum is 2 s. what will be the period if the mass and the length of pendulum string are both doubled? Use  $g = 9.81 \text{ m/s}^2$

3. A mass undergoes SHM with amplitude A. What fraction of the energy is kinetic energy when  $x = A/2$ .

4. The graph in fig. below shows the variation with displacement of the kinetic energy with displacement of a particle of mass 0.40 kg performing SHM.



**Fig.3. 12 Variation of kinetic energy with displacement**

Use the graph to determine:

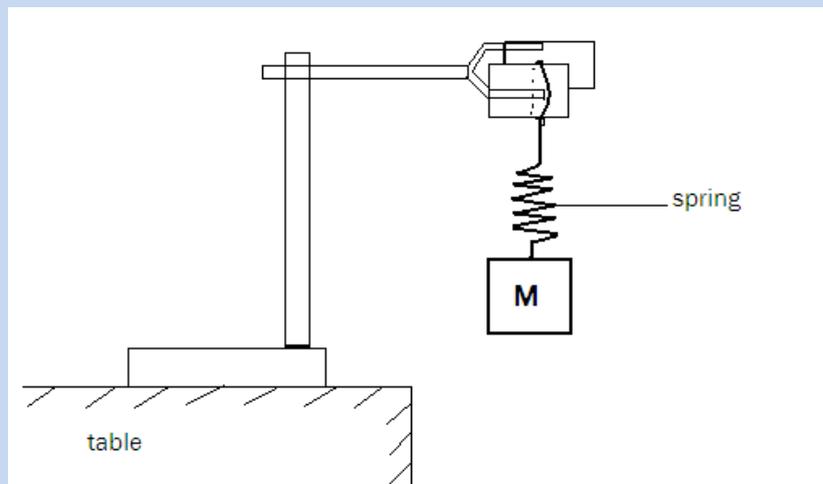
- i) The total energy of the particle;
- ii) The maximum speed of the particle;
- iii) The amplitude of the motion;
- iv) The potential energy when the displacement is 2.0 cm;
- v) The period of the motion.

### SKILLS LAB 3

In this experiment you will determine the force constant  $K$  of the spring.

#### Materials required.

- 1 Full set of retort stand
- 4 masses of 50 g each.
- 1 spring.



**Fig.3. 13 Mass spring system on a retort stand.**

- Clamp one end of a spring to a retort stand.
- Suspend a mass  $M=0.05$  Kg from the free end of the spring as shown above.
- Pull  $M$  vertically downwards through a small distance and release.
- Determine the time for 20 oscillations.
- Calculate the period  $T$ .
- Repeat procedures (b) to (e) for  $M=0.10, 0.15, 0.20$  Kg
- Record your results in a table including values of  $T^2$ .
- Plot a graph of  $T^2$  against  $M$ .
- Find the slope  $S$ , of the graph.
- Calculate the force constant  $K$  from the expression  $K = \frac{4\pi^2}{S}$ .
- Comment on the value of  $K$  obtained in j) above.

### END UNIT ASSESSMENT 3

1. A SHM has a speed of 2.15 m/s when it is 0.23 m from equilibrium. What is its period if its amplitude is 0.47 m?
2. Assume an object attached to a spring exhibits simple harmonic motion. Let one end of the spring be attached to a wall and let the object move horizontally on a frictionless table. What is the total energy of the object?
3. An object oscillates with simple harmonic motion along the  $x$  axis. Its displacement from the origin varies with time according to the equation  $x = (4.00 \text{ m}) \cos(\pi t + \frac{\pi}{4})$  where  $t$  is in seconds and the angles in the parentheses are in radians.
  - (a) Determine the amplitude, frequency, and period of the motion.
  - (b) Calculate the velocity and acceleration of the object at any time  $t$ .
  - (c) Using the results of part (b), determine the position, velocity, and acceleration of the object at  $t = 1.00 \text{ s}$ .
  - (d) Determine the maximum speed and maximum acceleration of the object.
  - (e) Find the displacement of the object between  $t = 0$  and  $t = 1.00 \text{ s}$ .
  - (f) What is the phase of the motion at  $t = 2.00 \text{ s}$ ?
4. A particle oscillates with simple harmonic motion, so that its displacement varies according to the expression  $x = (5 \text{ cm}) \cos(2t + \frac{\pi}{6})$  where  $x$  is in centimeters and  $t$  is in seconds. At  $t = 0$  find
  - a) The displacement of the particle,
  - b) Its velocity, and Its acceleration.
  - c) The period and amplitude of the motion.
5. Estimate the length of the pendulum in a grandfather clock that ticks once per second. What would be the period of a clock with a 1.0 m long pendulum?
6. A 1kg mass attached to a spring of force constant 25 N/m oscillates on a horizontal frictionless track. At  $t = 0$  the mass is released from rest at  $x = -3 \text{ cm}$ , that is the spring is compressed by 3 cm. Neglect the mass of the spring. Find
  - (a) The period of its motion,

(b) The maximum value of its speed and acceleration,  $a$  and

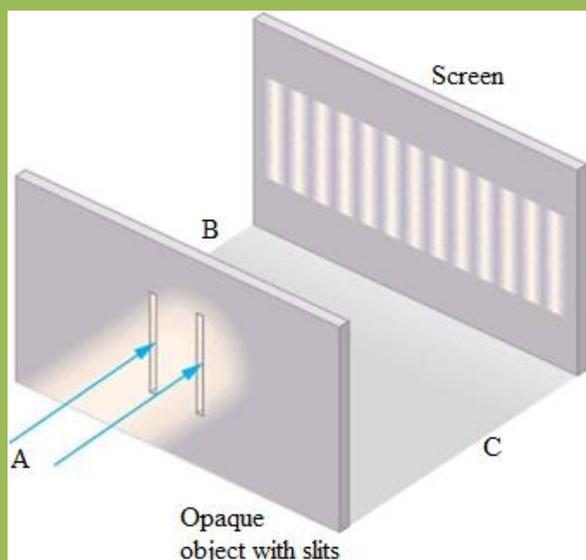
(c) The displacement, velocity and acceleration as a function of time

## UNIT 4: PROPAGATION OF MECHANICAL WAVES

**Key unit competence:** Evaluate the propagation of mechanical waves.

### INTRODUCTORY ACTIVITY

Consider the arrangement of the illustration below to investigate what happens on the screen when light passes through narrow slits.



**Materials:** Two torches of the same intensity, Screen, material with two small slits and material with big slits.

#### Procedures:

- Arrange the materials as shown in the illustration above following the procedures to complete the investigation:
- The first student at position A switched on the torch and light passed through one slit. What do you think is the nature of image(s) observed on the screen by second student at position B or C?
- Explain what causes the nature of the image(s) observed on the screen.
- Assuming the first Student at position A used two torches giving light of same intensities torching on two slits simultaneously, would image(s) on the screen be identical as observed in (a) above. Explain to justify your observation.
- Now, if small slits are replaced with ones of big holes (widened slits). Explain what this change will have on the images formed on the screen.
- Explain why do we not ordinarily observe wave behaviour for light, such as observed in Young's double slit experiment?
- Explain how this experiment is significant in real life

## 4.1 Interference of waves

### ACTIVITY 4.1

It is possible to induce ripples at the surface of water due to different disturbances including wind, aquatic animals in water, boats sailed on top of water and other.

Materials: Water, basin and small piece of stone.

Procedures:

- a. Fill the basin with water and leave it to settle.
- b. Apply a single disturbance at the center and note down your observations.
- c. Apply disturbances on different points on the surface and note down your observations.
- d. Distinguish the two cases (when there was a single disturbance and multiple disturbance)
- e. Explain the scientific phenomenon that explain what happened in (c) above.
- f. Explain other scenarios where similar phenomenon stated in (e) above occur.

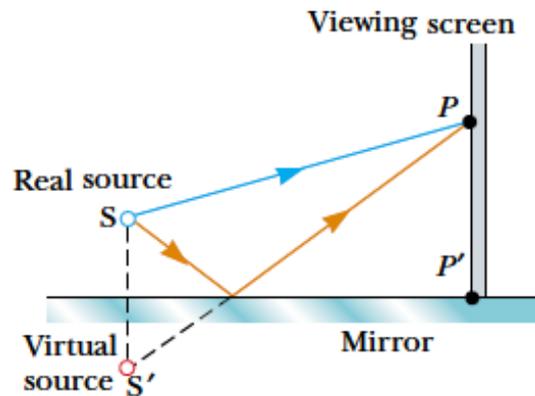
### 4.1.1 Coherent sources

**Coherent sources** are those which emit light waves of the same wavelength or frequency which are always in phase with each other or have a constant phase difference. Two coherent and monochromatic sources can together produce the phenomenon of interference.

When light passes through a slit with a size that is close to the light's wavelength, the light will **diffract**, or **spread out** in waves.

Interference is a phenomenon in which two waves superpose(meet) to form a resultant wave of greater, lower, or the same amplitude.

Young's method for producing two coherent light sources involves illuminating a pair of slits with a single source. Another arrangement for producing an interference pattern with a single light source is known as *Lloyd's mirror*.



**Fig.4. 1 Lloyd's mirror.**

A point light source is placed at point  $S$  close to a mirror, and a viewing screen is positioned some distance away and perpendicular to the mirror. Light waves can reach point  $P$  on the screen either directly from  $S$  to  $P$  or by the path involving reflection from the mirror.

An interference pattern is produced at point  $P$  on the screen as a result of the combination of the direct ray (blue) and the reflected ray (brown). The reflected ray undergoes a phase change of  $180^\circ$ .

In order to observe interference in light waves, the following conditions must be met:

- The sources must be coherent—that is, they must maintain a constant phase with respect to each other.
- The sources should be monochromatic—that is, of a single wavelength.
- The interfering waves Must Obey the Principal of superposition.

As an example, single-frequency sound waves emitted by two side-by-side loudspeakers driven by a single amplifier can interfere with each other because the two speakers are coherent—that is, they respond to the amplifier in the same way at the same time.

If two light bulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two light bulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a light bulb undergo random phase changes in time intervals less than a nanosecond. Such light sources are said to be incoherent.

When light passes through two or slits, the waves from one slit will **interfere** with the waves from the other:

- **Constructive interference** occurs when two crests or two troughs meet forming a wave with a larger crest or lower trough.
- **Destructive interference** occurs when a crest meets a trough cancelling each other to produce a smaller wave or no wave at all.

#### 4.1.2 Principle of superposition

The principle of superposition states that when two or more waves meet at a point, the resultant displacement at that point is the vector sum of the individual displacement of each wave

Consider the displacement  $y_1 = a \sin(\omega t - \Phi)$  of a progressive sinusoidal wave at time  $t$  and at a distance  $x$  from the origin and moving to right.

Consider also the displacement  $y_2$  of an identical wave travelling in opposite direction given by

$$y_2 = a \sin(\omega t + \Phi)$$

By principal of superposition, the resultant  $Y$  is got from  $Y = y_1 + y_2$

$$Y = a \sin(\omega t - \Phi) + a \sin(\omega t + \Phi)$$

$$Y = 2a \sin(\omega t) \cos \Phi$$

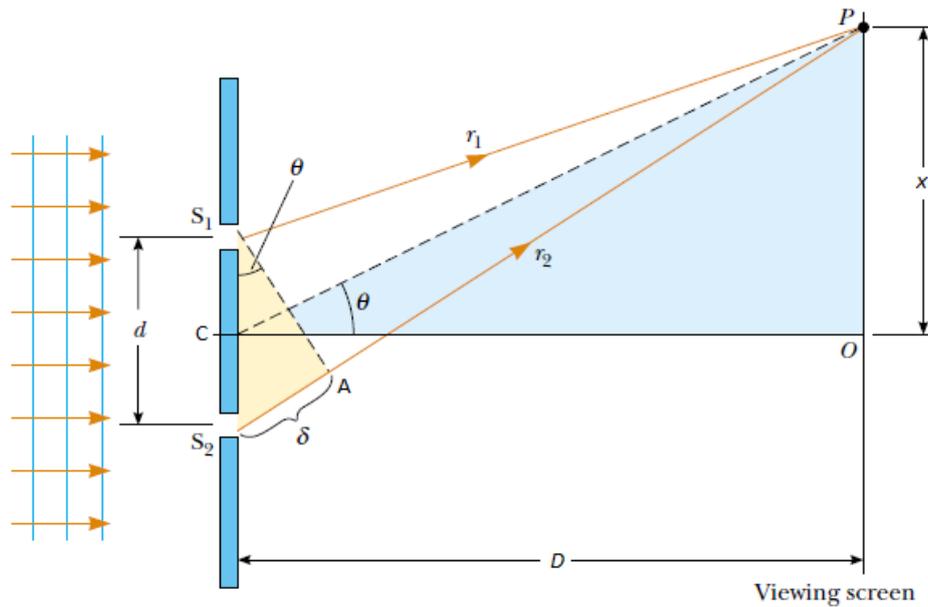
$$\text{but } \Phi = kx$$

$$Y = 2a \sin(\omega t) \cos kx$$

The only variable part of equation is  $\sin \omega t$ . This means that the amplitude of the resultant amplitude  $A$  is given by equation  $A = 2a \cos kx$

#### 4.1.3 Young's double-slit experiment

In Young's experiment, two very narrow parallel slits, separated by a distance  $d$ , are cut into a thin sheet of metal. Monochromatic light, from a distant light source, passes through the slits and eventually hits a screen a comparatively large distance  $D$  from the slits. The experimental setup is sketched in Fig.4.2



**Fig.4. 2 Measurement Ray of wavelength from Young's experiment**

According to Huygens' principle, each slit radiates spherical light waves. The light waves emanating from each slit are superposed on the screen. If the waves are  $180^\circ$  out of phase then **destructive interference** occurs, resulting in a dark patch on the screen. On the other hand, if the waves are completely in phase then **constructive interference** occurs, resulting in a light patch on the screen.

The light from  $S_1$  and  $S_2$  produces on a viewing screen a visible pattern of bright and dark parallel bands called **fringes**. When the light from  $S_1$  and that from  $S_2$  both arrive at a point on the screen such that constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen, a dark fringe results.

For any given point on the other side of the incident wave, we have the following geometry:

If we assume that  $r_1$  and  $r_2$  are parallel, which is approximately true if  $L$  is much greater than  $d$ , then the difference in length  $\delta$  of the two rays from the slits to their point of intersection is

$$S_2P - S_1P = \lambda = d \sin \theta$$

If we know the distance between adjacent slit centers or grating space ( $d$ ) and measure  $\theta$ ,  $\lambda$  can be calculated. In fig.4.3  $S_1$  and  $S_2$  are the two slits;  $O$  is the position of the central bright band. The path difference between waves reaching  $O$  from  $S_1$  and  $S_2$  is zero, i.e.  $S_1O = S_2O$ , they therefore arrive in phase and so there is a bright fringe at  $O$ .

P is the position of the first bright band and C is the midpoint of  $S_1S_2$ .  $S_2A$  must be one wavelength longer than  $S_1P$ . If  $S_1A$  is drawn perpendicular to  $S_2P$  then  $S_2A$  will be approximately equal to wave length.

If P is near O then  $S_1P$  and  $S_2P$  are approximately parallel to CP so the triangles  $S_1S_2P$  and CPO will be similar.

$$\text{From triangle } S_1S_2P: \sin \theta = \frac{S_1P}{S_2S_1} = \frac{n\lambda}{d}$$

$$\text{From triangle OCP: } \tan \theta = \frac{PO}{CO} = \frac{x_n}{D} \text{ where } x_n \text{ is the distance of the } n^{\text{th}} \text{ fringe from the centre O.}$$

Now the angle  $\theta$  is very small so  $\tan \theta = \sin \theta$  from trigonometry.

$$\text{Then } \frac{n\lambda}{d} = \frac{x_n}{D} \Leftrightarrow x_n = \frac{n\lambda D}{d}$$

$$\text{For } (n-1)^{\text{th}} \text{ before } n \text{ fringe } x_n = \frac{(n-1)\lambda D}{d}$$

$$\text{To find the distance between two fringes (separation of fringe) : } x_n - x_{n-1} = \frac{\lambda D}{d}$$

Assigning the fringe separation, the letter  $x$ ,

$$x = \frac{\lambda D}{d} \Leftrightarrow \lambda = \frac{dx}{D}$$

$$\text{Or in words, wavelength } \lambda = \frac{\text{slit separation} \times \text{bright band separation}}{\text{distance of slits from screen}}$$

### Example 4.1

A viewing screen is separated from a double-slit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe ( $m = 2$ ) is 4.5 cm from the center line.

- Determine the wavelength of the light.
- Calculate the distance between adjacent bright fringes.

**Solution:**

a) From equation:  $x_n = \frac{n\lambda D}{d}$  with  $n = 2$ ,  $x_{\text{bright}} = 4.5 \times 10^{-2} \text{ m}$ ,  $L = 1.2 \text{ m}$  and  $d = 3.0 \times 10^{-5}$

We find  $\lambda = \frac{\lambda_{\text{bright}} d}{nD} = 5.6 \times 10^{-7} \text{ m}$

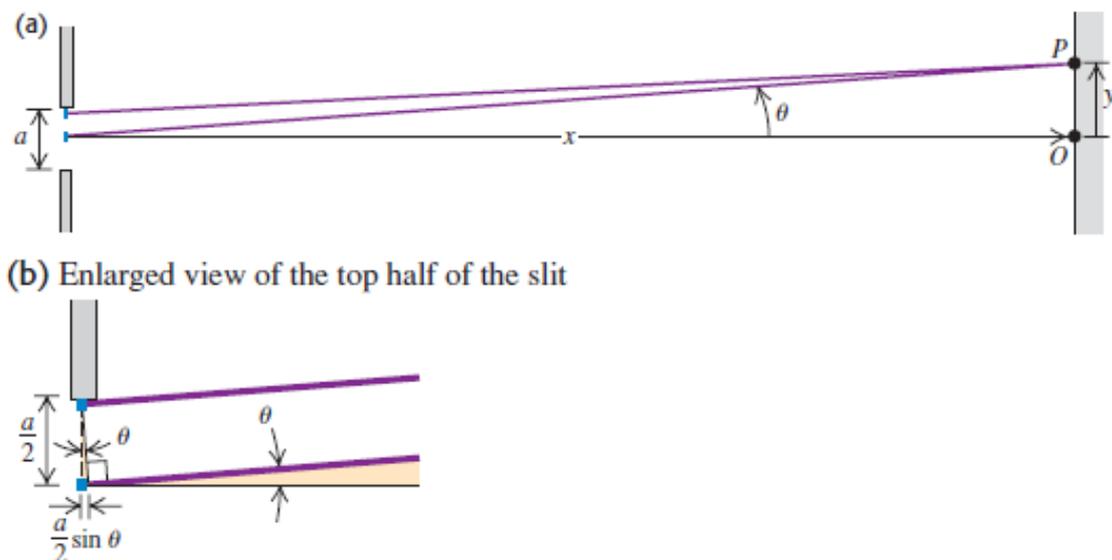
b) From equation:  $x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d} = \frac{5.6 \times 10^{-7} \times 1.2}{3.0 \times 10^{-5}} = 2.2 \text{ cm}$

**Predicting the location of interference fringes**

We can derive quite simply the most important characteristics of the Fraunhofer diffraction pattern from a single slit. First consider two narrow strips, one just below the top edge of the drawing of the slit and one at its center, shown in end view in Fig. 4.4. The difference in path length to point P is

$\frac{a}{2} \sin \theta$  where  $a$  is the slit width and  $\theta$  is the angle between the perpendicular to the slit and a line

from the center of the slit to P.



**Fig.4.3 Side view of a horizontal slit.**

**Note:** (a) the path difference to P is  $\Delta x = \frac{a}{2} \sin \theta$  When the distance to the screen is much greater than the slit width the rays from a distance apart may be considered parallel.

(b)  $\theta$  is usually very small, so we can use the approximations  $\sin \theta = \theta$  then the condition for dark

$$\text{band is } y_m = \frac{m\lambda}{a}$$

The **condition for bright fringes** (constructive interference) is given by the following:

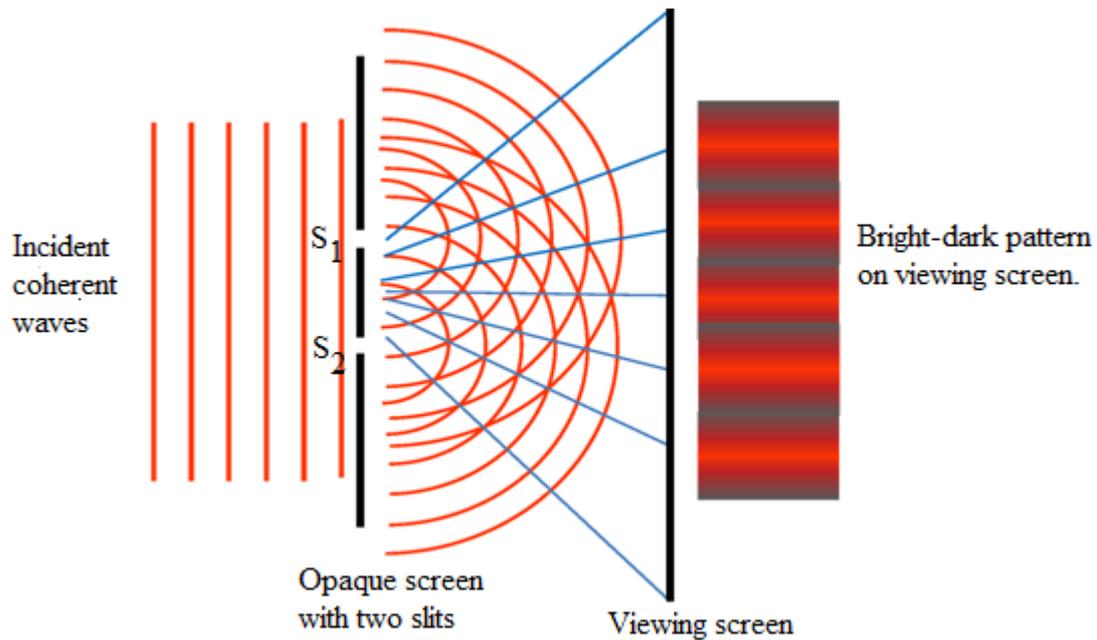
$$d \sin \theta = m\lambda$$

In this equation,  $m$  is the order number of the fringe. The central bright fringe at  $\theta = 0$  is called the zeroth-order maximum, or the central maximum, the first maximum on either side of the central maximum, which occurs when  $m = \pm 1$  is called the first order maximum, and so forth.

Similarly, when the path difference is an odd multiple of  $\frac{\lambda}{2}$ , the two waves arriving at screen are  $180^\circ$  out of phase, giving rise to destructive interference, the **condition for dark fringes**, or destructive interference, is given by the following equation:

$$d \sin \theta = (m + \frac{1}{2})\lambda$$

If  $m = 0$  in this equation, the path difference is  $d \sin \theta = \frac{\lambda}{2}$  which is the condition under which the first dark fringe forms on either side of the bright central maximum. Likewise, if  $m = \pm 1$ , the path difference is  $\frac{3\lambda}{2}$ , which is the condition for the second dark fringe on each side of central maximum, and so forth.



**Fig.4. 4 Showing interference fringes leading to dark and bright fringes.**

**Example 4.2**

1. A light source emits visible light of two wavelengths:  $\lambda = 430\text{nm}$  and  $\lambda = 510\text{nm}$ . The source is used in a double-slit interference experiment in which  $D = 1.5\text{m}$  and  $d = 0.0250\text{mm}$ . Find the separation distance between the third-order bright fringes.

**Solution:**

From equation: 
$$x_n = \frac{n\lambda D}{d} = \frac{4 \times 430 \times 10^{-9} \times 1.50}{0.0250 \times 10^{-3}} = 7.74 \times 10^{-2} \text{ m}$$

$$x'_n = \frac{5 \times 10 \times 10^{-9} \times 1.50}{0.0250 \times 10^{-3}} = 9.18 \times 10^{-2} \text{ m}$$

Hence, the separation distance between the two fringes is

$$\Delta x = x'_n - x_n = 1.44 \times 10^{-2} \text{ m}$$

2. Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

**Solution:**

The two dark fringes that flank the central bright fringe correspond to  $m = \pm 1$  in Equation

$$\sin \theta = m \frac{\lambda}{d}$$

Hence, we find that  $\sin \theta = \pm \frac{580 \times 10^{-9}}{0.300 \times 10^{-3}} = \pm 1.93 \times 10^{-3}$

From the triangle in Figure 4.3, note that  $\tan \theta = \frac{x_1}{D}$

Because  $\theta$  is very small, we can use the approximation  $\sin \theta = \tan \theta = \frac{x_1}{D}$

Therefore, the positions of the first minima measured from the central axis are given by

$$x_1 = \pm D \sin \theta = \pm D \frac{\lambda}{d} = \pm 2.00 \times 1.93 \times 10^{-3} = 3.87 \times 10^{-3} \text{ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe.

Hence, the width of the central bright fringe is equal to

$$2x_1 = 2 \times 3.87 \times 10^{-3} = 7.74 \times 10^{-3} \text{ m}$$

Note that this value is much greater than the width of the slit.

### **Applications of interference of waves in real life**

The applications of interference in real life include the following:

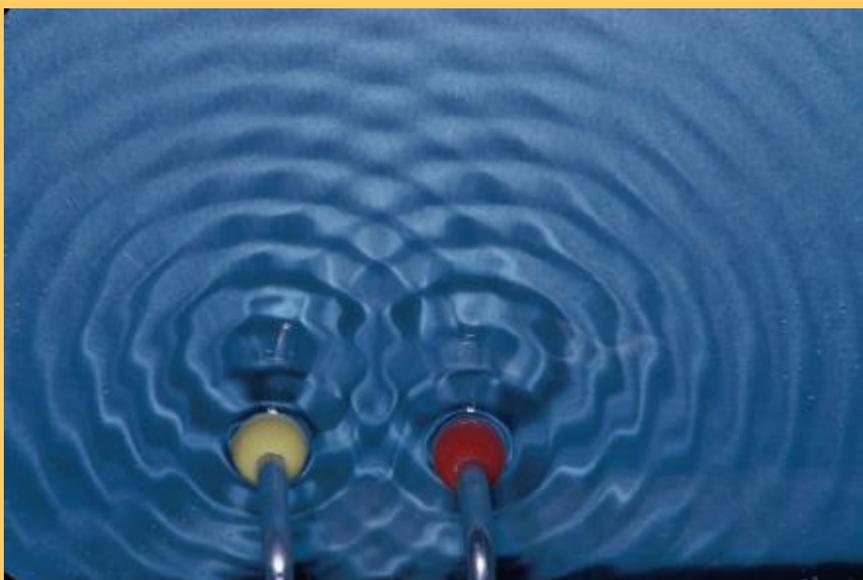
- a) Signal processing: reference signal is modulated by a sinusoidal waveform, and dynamic displacement is derived from the envelope curves of the interference signal.
- b) In laser production. In laser processing a beam is made to interfere within a cavity leading to amplification of light.
- c) Light Amplification. Light can be amplified by making light to interfere constructively.

### APPLICATION ACTIVITY 4.1

1. What is the necessary condition on the path length difference between two waves that interfere (a) constructively and (b) destructively?
2. If Young's double-slit experiment were performed under water, how would the observed interference pattern be affected?
3. In Young's double-slit experiment, why do we use monochromatic light? If white light is used, how would the pattern change?
4. The distance between the two slits is 0.030 mm. the second-order bright fringe ( $m = 2$ ) is measured on a viewing screen at an angle of  $2.15^\circ$  from the central maximum. Determine the wavelength of the light.
5. A 2-slit experiment is set up in which the slits are 0.03 m apart. A bright fringe is observed at an angle  $10^\circ$  from the normal. What is wavelength of electromagnetic radiation being used?
6. In Young's double slit experiment the separation between the 1<sup>st</sup> and 5<sup>th</sup> bright fringes is 2.5 mm . When the wavelength used is  $6.2 \times 10^{-7} \text{ m}$  . The distance from the slits to screen is 0.8 m. Calculate the separation of the slits.

### 4.2 Stationary or standing waves

## ACTIVITY 4.2



**Fig.4. 5 Interference of ripples in water waves**

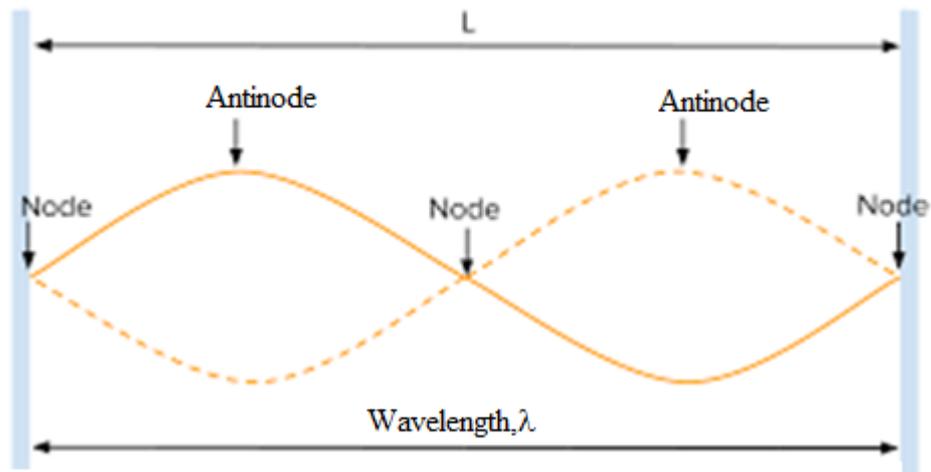
The figure above shows ripples of water waves interact after meeting.

- Account for the causes of different shapes shown on the surface of water
- Explain the appearance of surface of water if one circular dipper was used instead of two.
- There are points at which the amplitude of resultant wave after meeting is maximum, and points where the amplitude is zero. Suggest the name of these points.
- Comment on the energy and displacement of resultant wave when the different waves meet...

### 4.2.1 Concept of stationary wave

Standing wave also known as a **stationary wave**, is wave pattern that results when two waves of the same frequency; wavelength and amplitude travelling in opposite directions in the same medium interfere or meet.

The point at which the two waves cancel are called **node**. There is no motion in the string at the nodes, but midway between two adjacent nodes, the string vibrates with the largest amplitude. These points are called **antinodes**. At points between successive nodes the vibrations are in **phase**.



**Fig.4. 6 Standing wave with 3 nodes**

Standing-wave produced at various times by two waves of equal amplitude traveling in opposite directions. For the resultant wave  $y$ , the nodes points of zero displacement, and the antinodes are points of maximum displacement.

The standing wave has 3 nodes: One at either end and one in the middle, in that case there are two loops, corresponding to a crest and trough. Thus, this standing wave has a wavelength equal to the string length. i.e.

$$\lambda = L$$

A single loop corresponds to either a crest or trough alone, while two loops correspond to a crest and trough together, or one wave length.

Because a single loop corresponds to either a crest or trough alone, this standing wave corresponds to one half of a wavelength. Thus, the wavelength in this case is equal to twice the string length i.e.

$$L = \frac{\lambda}{2} \Leftrightarrow \lambda = 2L$$

Stationary waves are present in the vibrating strings of musical instruments. A violin string, for instance, when bowed or plucked, vibrates as a whole, with nodes at the ends, and also vibrates in halves, with a node at the center, in thirds, with two equally spaced nodes, and in various other fractions, all simultaneously. The vibration as a whole produces the fundamental tone, and the other vibrations produce the various harmonics.

## 4.2.2 Stationary wave equations

A stationary wave can be considered as a produced by superposition of two progressive waves, of the same amplitude and frequency, travelling in opposite directions and the extreme below is free .



**Fig.4. 7 progressive waves of the same amplitude and frequency, travelling in opposite directions.**

Suppose  $y_1 = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$  is a plane progressive wave travelling in one direction along x axis.

Then  $y_2 = A \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$  represents a wave of the same amplitude and frequency travelling in

opposite direction so the resultant displacement, y, is given by  $\sin a + \sin b = 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)$

$$y_1 + y_2 = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + A \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$y_1 + y_2 = A\left[\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

Let *let*  $y_1 + y_2$  be  $y$

$$Y = 2A \sin \frac{2\pi t}{T} \cos \frac{2\pi x}{\lambda}$$

$$Y = 2A \cos kx \sin \omega t$$

Therefore the equation ,  $Y = 2A \cos kx \sin \omega t$  represents the wave function of a standing wave where

$$k = \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = \frac{2\pi}{T}$$

### Position of antinodes

The element with the *greatest* possible displacement from equilibrium has an amplitude of  $2A$ , and we define this as the amplitude of the standing wave. The positions in the medium at which this maximum displacement occurs are called **antinodes**. The antinodes are located at positions for which the coordinate  $x$  satisfies the condition

$$2A \cos kx = \pm 2A$$

$$\cos kx = \pm 1$$

$$kx = n\pi$$

$$\frac{2\pi}{\lambda} x = n\pi \Leftrightarrow x = \frac{n\lambda}{2}$$

The distance between 2 successive antinodes is  $\frac{\lambda}{2}$  and the distance between a node and an adjacent

antinode is  $\frac{\lambda}{4}$

### Position of nodes

The maximum amplitude of an element of the medium has a minimum value of zero when  $x$  satisfies the condition:

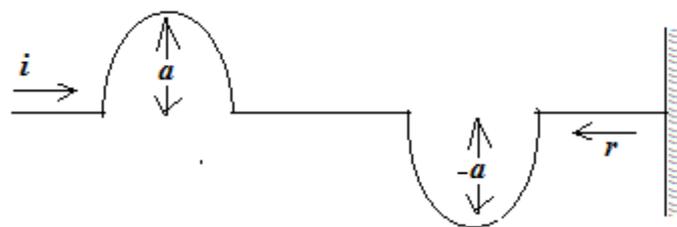
$$2A \cos kx = 0 \Leftrightarrow \cos kx = 0$$

$$kx = \frac{2n+1}{2} \pi$$

$$\frac{2\pi x}{\lambda} = \frac{(2n+1)\pi}{2} \Leftrightarrow x = \frac{(2n+1)\lambda}{4}$$

These points of zero amplitude are called nodes.

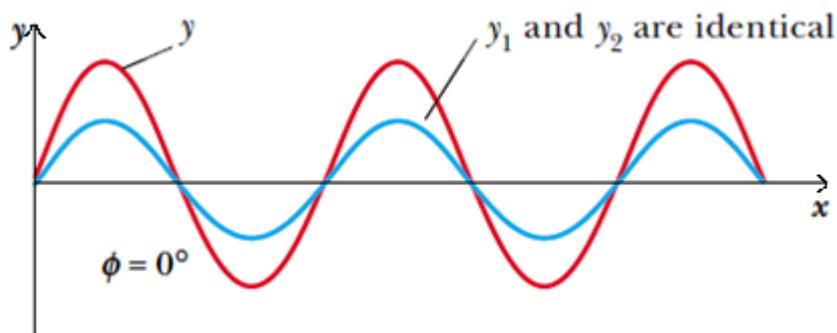
**Note:** The extreme is fixed and the reflected wave is opposite phase with the incident wave



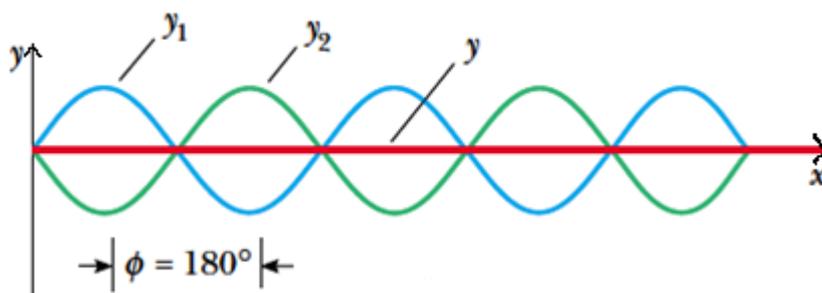
**Fig.4. 8 Reflected wave is opposite phase with the incident wave**

$$y_1 = a \sin(\omega t - kx) \quad \text{and} \quad y_2 = a \sin(\omega t + kx)$$

$$y_1 + y_2 = 2a \sin kx \cos \omega t$$



**Fig.4. 9** The superposition when  $y_1$  and  $y_2$  are in phase, the result is constructive interference.



**Fig.4. 10** The superposition when  $y_1$  and  $y_2$  are out of phase, the result is destructive interference.

**Note:** The superposition of two identical waves  $y_1$  and  $y_2$  (blue and green) to yield a resultant wave (red).

Difference between stationary waves and progressive waves

STATIONARY WAVES/STANDING WAVE	TRAVELLING WAVE/PROGRESSIVE WAVE
The wave shape does move	The wave shape progresses
Neighboring points have different amplitudes	Neighboring points have the same amplitude
Neighboring points have the same phase	Neighboring points have different phase
It store energy	It transmits energy

## Applications of stationary waves

Stationary/Standing waves are applied in the following :

- i) In musical instruments (strings and pipes) in production of musical notes
- ii) In the manufacture of laser beam.
- iii) In production of sound from the vocal cord

### Example 4.3

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are  $y_1 = 4.0 \sin(3.0x - 2.0t)$  and  $y_2 = 4.0 \sin(3.0x + 2.0t)$  where  $x$  and  $y$  are measured in centimeters.

a) Find the amplitude of the simple harmonic motion of the element of the medium located at  $x = 2.3$  cm

Solution:

The standing wave is described by Equation  $y = 2A \cos kx \sin \omega t$  where  $k = \frac{2\pi}{\lambda}$ ,  $\omega = \frac{2\pi}{T}$  in this problem, we have  $A = 2.0$  cm,  $k = 3.0$  rad/cm, and  $\omega = 2.0$  rad/s.

Thus,  $y = 2A \cos kx \sin \omega t = 8.0 \cos 3.0x \sin 2.0t$

Thus, we obtain the amplitude of the simple harmonic motion of the element at the position  $x = 2.3$  cm by evaluating the coefficient of the cosine function at this position:

$$y_m = 8.0 \cos(3.0 \times 2.3) \text{ rad} = 6.5 \text{ cm}$$

b) Find the positions of the nodes and antinodes if one end of the string is at  $x = 0$ .

Solution:

With  $k = \frac{2\pi}{\lambda} = 3 \text{ rad/s}$ , we see that the wavelength is  $\lambda = \frac{2\pi}{3}$ .

Therefore, from Equation  $x = \frac{2n+1}{4} \lambda$  we find that the nodes are located at  $x = \frac{(2n+1)}{4} \lambda$

it follows that  $x = \frac{(2n+1)\pi}{6}$

and from Equation  $x = \frac{n\lambda}{2}$  we find that the antinodes are located at  $x = \frac{n\lambda}{2} = \frac{n\pi}{3}$

c) What is the maximum value of the position in the simple harmonic motion of an element located at an antinode?

Solution:

The maximum position of an element at an antinode is the amplitude of the standing wave, which is twice the amplitude of the individual traveling waves:

$$Y_{\max} = 2A \cos kx = 8.0 \cos 3.0x = 8.0x(\pm 1) = \pm 8.0 \text{ cm}$$

where we have used the fact that the maximum value of  $\cos kx = \pm 1$

#### APPLICATION ACTIVITY 4.2

1. Explain the meaning of displacement node and antinodes.
2. Discuss at least 5 the characteristics of stationary waves.
3. Show how the standard travelling wave equation can be varied by substitution to get:

a)  $y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ .

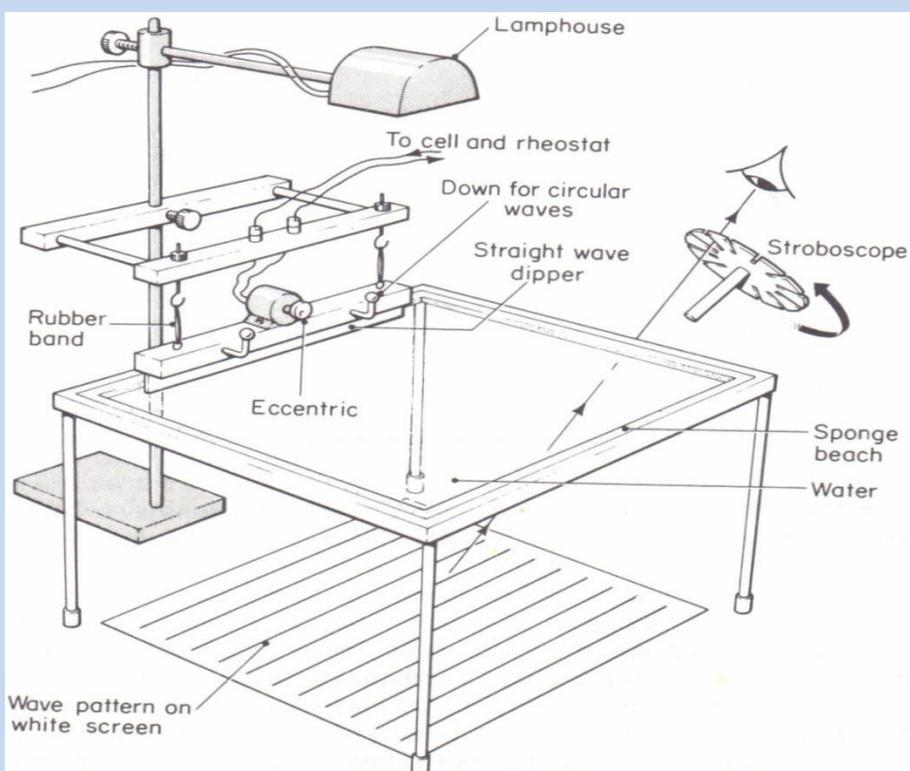
b)  $y = A \sin 2\pi(\omega t - kx)$ ,  $f = \frac{1}{T}$ ,  $V = \lambda f$  and  $k = \frac{2\pi}{\lambda}$  may be useful.

4. A travelling wave is represented by the equation  $y = 8 \sin(40\pi t - 0.8\pi x)$  where  $y$  is in cm. State a wave that would superimpose to the one given to give rise to a stationary wave. Hence calculate the amplitude and velocity of the stationary wave.

## SKILLS LAB 4

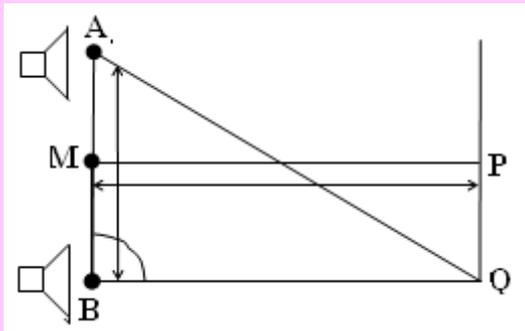
In this activity you will construct a ripple tank. A ripple tank is a device that is majorly made of glass, wood (timber), source of light like a torch, stroboscope, source of electricity, motor, White sheet of paper to act as a screen and others. It is used to study all properties of water waves.

Make a research about its working and then using the described materials, construct a functioning ripple tank. Your structure may appear as one shown below.



## END UNIT ASSESSMENT 4

1. Two small loudspeakers A and B 1.00 m apart, are connected to the same oscillator so that both emit sound waves of frequency 1700 Hz in phase. A sensitive detector, moving parallel to the line AB along PQ 2.40 m away as shown in Fig. below, detects a maximum wave at P on the perpendicular bisector MP of AB and another maximum wave when it first reaches a point Q directly opposite to B. Calculate the speed  $c$  of the sound waves in air from these measurements.



2. White light passes through two slits 0.5 mm apart and an interference pattern is observed on screen 2.5 m away. The first-order fringe resembles a rainbow with violet and red light at either end. The violet light falls about 2.0 mm and the red 3.5 mm from the center of the central white fringe. Estimate the wavelength of the violet and red lights.
3. In young's double-slit experiment the distance between the slits and the screen is 1.60 m, using light of wavelength  $5.89 \times 10^{-7}$  m the distance between the centre of the interference pattern and the fourth bright fringe on either side is 16 mm. what is the slit separation?
4. Light, with a wavelength of 500 nm, is shone through 2 slits, which are 0.05 m apart. What are the angles to the normal of the first three dark fringes?
5. Does the vertical speed of a segment of a horizontal taut string, through which a wave is traveling, depend on the wave speed?

## UNIT 5: FOSSIL, NON FOSSIL FUEL AND POWER PRODUCTION

**Key unit competence:** Justify the effects of fossil and non fossil fuel in power production

### INTRODUCTORY ACTIVITY

In our homes, we need different sources of energy while cooking, ironing and lightning. Cars, motorcycles need fuels for their engines. Plants use energy during photosynthesis.

- List and explain the origin of all possible forms of energy used in our homes
- Distinguish energy used in photosynthesis and energy used by cars in their engines
- Explain the difference between renewable and non renewable energies.
- According to their formation and origin, classify different forms of energy into fossil and non fossil energies.
- If you were asked to choose one category of energy classified above, which class of energy would you like to use in your daily life? Explain why.

Most of the energy that we consume comes from fossil fuels. Coal, petroleum and natural gas are called fossil fuels. Millions of years ago, during the carboniferous age, due to the change in atmospheric conditions and other changes, the forests were destroyed and they were fossilized.

With the action of bacteria and other microorganisms on the surface of the earth, these trees and other vegetations were decayed and disintegrated. Years after these trees were available in solid, liquid and gaseous state. The solid form is coal. It is the most widely used form of fossil fuel for domestic purposes.

### 5.1 Fossil fuel and non fossil fuel

#### ACTIVITY 5.1

In our daily life, we use different forms of energy at home when cooking, lightning, driving cars and motorcycles etc. Try to answer to the questions that follow:

1. What kind of energy you use at home when
  - i) Cooking food?
  - ii) Lightning in your bed room?
  - ii) Ironing your clothes?
2. What is the name of fuel used in vehicles in your region? Where does this form of energy found?
3. Green plants need energy to synthesize their foods. Where do they find energy used in photosynthesis?
4. Some of the energies given in above questions can be exhausted in nature others should be used indefinitely. Use one example for each category to explain why.

### 5.1.1 Fossil fuel

#### History of usage of Fossil Fuel

Before steam engines were invented, heavy industry depended on mechanical water power to grind flour, saw wood, and so forth. Industrialization led to a higher rate of energy usage. Fossil fuel led to development and it played a crucial role as energy sources, inputs for agriculture, and feed stocks for chemical manufacture. The Industrial Revolution marked a big change for people of the world.

Many of the agriculture based societies that used human and animal labor forces switched to use machines to do work. Coal was commonly used in the early era of industrialization until internal combustion engine and the automobile were invented. Oil and gas became the most common fossil fuel people used.

Fossil fuels are hydrocarbons, primarily coal, fuel oil or natural gas, formed from the remains of dead plants and animals. In common dialogue, the term 'fossil fuel' also includes hydrocarbon-containing natural resources that are not derived from animal or plant sources.

Coal, oil and natural gas are called '**fossil fuels**' because they have been formed from the fossilized remains of prehistoric plants and animals. Fossil fuels are non-renewable energy source since they take millions of years to form. They ultimately get their energy from the sun.

## **Types of Fossil Fuels**

### **➤ Coal**

Coal is a hard, black colored rock-like substance formed when dead plants were subjected to extreme heat and pressure for millions of years. Coal is formed through coalification. Coal is made of decomposed plant matter in conditions of high temperature and pressure. Its formation is similar to oil's but it takes less time to form.

It is made up of carbon, hydrogen, oxygen, nitrogen and varying amounts of sulphur. There are two ways to mine coal: surface mining and underground mining.

### **➤ Natural Gas**

Natural gas is formed from the remains of tiny sea animals and plants that died millions of years ago. The gas then became trapped in layers of rock-like water in a wet sponge. Raw natural gas is a mixture of different gases. Its main ingredient is methane. The strange smell of natural gas (like rotten eggs) comes from a chemical added by the companies.

### **➤ Oil (Petroleum)**

Oil is formed from the remains of animals and plants that died millions of years ago. The organic material was then broken down into hydrogen and carbon atoms and a sponge-like rock was formed, full of oil.

Oil cannot be used as it is when it is drawn from the ground. Oil refineries clean and separate the oil into various fuels and by products. The most important of these is gasoline.

## **Uses of Fossil Fuels**

The main systems of fossil fuels are the steam cycle and the gas turbine cycle. Fossil fuels are used to generate electrical energy in a series of energy transformations. The following is an example:



**Fig.5. 1: Energy transformations**

### **Advantages of Fossil Fuels**

1. Can be easily transported via pipelines, railroads, trucks and ships.
2. They are easily available. More and more extractions are occurring all over the world and therefore resulting in a large amount of readily available energy sources.
3. Oil refineries close to the sea have easy access to shipping.
4. Fossil fuels are easily combustible. In other words, they produce larger amounts of energy.
5. Creates infrastructure jobs for the surrounding communities.
6. Much of our infrastructure is designed to run using fossil fuels.
7. Although fossil fuels are considered as a relatively new energy source, in reality they have been around for hundreds of years.
8. Every machine that is not run by electricity uses fossil fuels. Vehicles, machines, devices, etc. are powered by coal, petroleum or natural gas.
9. They are considered to be very stable.
10. They are easy to set up. Since fossil fuels are easily available, their power plants can be constructed anywhere in the world. They are also easier to extract and process, as well as capable of producing large amounts of energy at a single location.
11. Fossil fuels are easy to store and transport because they are so stable. They are easily distributed.
12. Easy transportation allows countries around the world to enjoy affordable power.
13. The price of fossil fuels is inexpensive compared to other sources of energy.

### **Disadvantages of Fossil Fuels**

Fossil fuels, for all their pros, have many cons that have major concerns for human being, animals and the environment.

The biggest disadvantage of fossil fuels is the air pollution that many are claiming is causing global warming. It is claimed that with global warming, the Earth's climates are changing. Below is a list of the disadvantages of fossil fuels.

1. Air Pollution and its effects on the Earth and environment. This includes the concepts of global warming and climate change.
2. They are non-renewable sources of energy. As fossil fuels are extracted to an unlimited level, they would surely deplete one day. They are non-renewable, so it is likely that when fuel reserves have been completely used up, there is nothing more left. It would take millions of years to replace them. They are on a limited amount, and we are not actually sure where that limit is.
3. Pipelines transporting fossil fuels spoil the natural beauty.
4. They affect marine life through oil spills. Fossil fuels, being needed to be transported to their processing plants via land, air and water poses a threat to the environment. The process can involve leaks in oil tankers or ships getting drowned deep under the sea. The crude oil contains some toxic substances that, when mixed up with water, pose serious hazards to marine life.
5. Risk of political issues and terrorism
6. Most facilities that are powered by coal require large quantities of coal to have on hand for use. Storage facilities for the coal are required, this can be pricey.
7. Coal mining is a very dangerous and many workers have been killed in the mines as well as becoming ill with lung diseases after working the coal mines.
8. While fossil fuels are relatively inexpensive, the prices are rising due to Middle Eastern countries holding large reserves of oil such as petroleum.
9. Coal mining has created destroyed lands and the mines are creating hazards in the event of natural disasters.
10. They need huge amounts of reserves. Coal power plants for example need regular and huge supply of resources to produce large amounts of electricity on a constant basis, which means they need reserves to carry out their operations.
11. The extraction of natural gas is leaving large craters within the Earth's surface.

### **5.1.2 Non fossil fuel**

Non fossil fuels are alternative sources of energy or renewable source of energy that do not rely on burning up limited supply of coal, oil or natural gas. They should generate power that can be utilized indefinitely. They include sun light, wind, hydro, tidal and waves from water, geothermal all of them generate energy.



**Fig.5. 2 Solar panel and wind Energy sources of energy.**

Non-fossil fuels are considered to be extremely important for power creation. This is because they are usually renewable energy sources that could be tapped for hundreds of years and not run out. In addition, energy production using non fossil-based fuels usually generates much less pollution than fossil-based energy sources.

#### **APPLICATION ACTIVITY 5.1**

1. Fossil fuel can be used to generate other forms of energy. Use appropriate examples to support the idea above.
2. Non-fossil fuels are alternative sources of energy. They are renewable sources of energy that do not rely on burning up limited supplies of coal, oil or natural gas.
  - (i) Outline examples of non fossil fuels you know
  - (ii) Discuss the benefits of non fossil fuel as alternative of sources of energy in the world.
3. Explain the formation of fossil fuels
4. Discuss in pairs the historical and geographical reasons for the widespread use of fossil fuels.

#### **5.2 Transportation and storage of fossil fuels.**

## ACTIVITY 5.2

- a) Do you think fossil fuels exist in Rwanda? If yes, which one?
- b) Give the names of fossil fuels used in Rwanda.
- c) Rwanda is a land locked country. However we use fossil fuels as sources of energy. Where these energies come from? By which means of transport do we use to get them? Is this method costless for underdeveloped country? Explain.
- d) We need to store fossils fuels we use in Rwanda namely oil and gas.
  - (i) Give advantages of storage of oil as a source of energy in Rwanda
  - (ii) Outline dangers with the storage of fossil fuels

It is easy to think that the advantages of fossil fuels outweigh their disadvantages. All over the World, Fossil fuels are gaining popularity as energy sources because they are relatively inexpensive and look like clean. Remember that fossil fuels are comprised of three substances: coal, oil and gas. In the following lines we are going to discuss some of the common advantages and disadvantages of fossil fuels transportation and storage.

### 5.2.1 Advantages associated with transportation and storage of fossil fuels

- The majority of oil transported by maritime means reaches their destination. Normally there are no serious oil spillages. In fact, as soon as the pipeline is damaged by accident or sabotage, pumping is stopped and pollution remains limited.
- Oil depots are usually situated close to oil refineries or in locations where marine tankers containing products can discharge their cargo.
- The long life of the permanent assets, relatively trouble-free operation with minimum maintenance, the large-volume shipments that are possible, the high mechanical efficiencies that are obtained with low rolling resistances.
- The total costs of moving slurry during the life of the line do not increase in proportion to inflation. The advantage over rail and truck transport is clear, as the costs of these latter modes escalate with inflation.

- Taller and wider stockpiles reduce the land area required to store a set tonnage of coal. Larger coal stockpiles have a reduced rate of heat lost, leading to a higher risk of spontaneous combustion.
- Waterways are usually circuitous, resulting in slow delivery times. However, transport of coal on barges is highly cost-efficient.
- Transportation by gas pipelines are less costly and are thus more common.

### 5.2.2 Disadvantages associated with transportation of fossil fuels

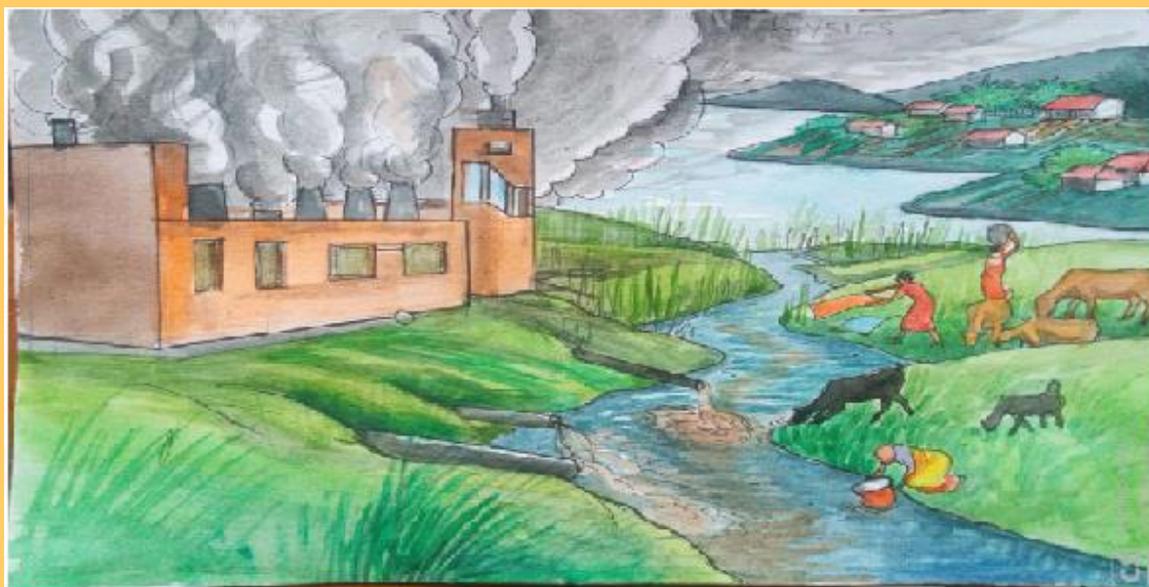
- At sea, the relative disadvantages derive from the possibilities of oil spills and discharging of polluting products such as the residue from tank and bilge cleaning.
- Oil is always corrosive to a greater or lesser extent, because it contains acidic gases. The pipes deteriorate from the inside and if they are not changed in time, they finish by leaking.
- The construction of major pipelines crossing several countries requires intense negotiation.
- On the other hand, slurry pipelines involve potential environmental problems. Water requirements are substantial: almost one ton of water is needed to move one ton of coal.
- Even though pipelines are useful, in certain cases the construction of gas pipelines is technically impossible or too expensive.

#### APPLICATION ACTIVITY 5.2

1. Give any advantages associated with the transportation and storage of fossil fuels.
2. Explain means of transport of crude oil
3. The transportation of natural gas is difficult compared with that of Oil. Explain why.

### 5.3 Environmental problems of fossil fuels

### ACTIVITY 5.3



Observe the picture clearly and answer the following questions

- 1) Name all elements observed on the picture
- 2) What do you think is the source of energy to run the factory shown in the picture
- 3) From your observations, do you think environment is safe? Explain your reasoning.
- 4) How do such activities affect human life, aquatic life and any other creature in the ecosystem?
- 5) Explain any three reasons, why such industries should be situated in isolated places.

Fossil fuels have been formed from the organic matter: these are remains of long-dead plants and animals. They contain a high percentage of carbon and hydrocarbons. Primary sources of energy we are using in our country and around the world in particular include petroleum, coal, and natural gas, all fossil fuels. With the needs increase of energy, the production and use of these fossil fuels create serious environmental concerns. Until a global movement for renewable energy is successful, the negative effects of fossil fuel will continue.

#### 5.3.1 Climate Change and Global Warming

Global warming occurs when carbon dioxide is accumulated in the atmosphere. Carbon monoxide is produced by the combustion of fossil fuels and converted into carbon dioxide. These gases trap more sunlight; therefore, less light is reflected back into space. They are called **Greenhouse Gases**,

because the effect is like being in a plant glasshouse, or in a car with the windows wound up. As a result, the surface temperature of the earth is increasing drastically.

If the increase is enough it will distress the ecological systems. The consequences are: severe weather, droughts, floods, drastic temperature changes, heat waves, and more severe wildfires. Food and water supplies are also threatened. Tropical regions will expand; allowing disease-carrying insects to expand their ranges.

### **5.3.2 Hole in the Ozone Layer**

Ozone is a gas in the Earth's upper atmosphere whose chemical formula is  $O_3$ . Ozone acts to block out much of the sun's ultraviolet radiation which causes skin cancer and contributes to the fluctuations of global climatic conditions that affect the environment.

However, the World is facing a serious confrontation as the emissions of chlorofluorocarbons and other destructive gases are causing ozone holes to appear in the stratospheric ozone layer. As a consequence, the concentration of detrimental ultraviolet radiation is increasing at ground level and jeopardizing humans, crops and ecosystems.

### **5.3.3 Acid rain**

Acidic rain, which is made up of several acidic compounds, forms when sulfur dioxide and nitrogen dioxide react in the air with water, oxygen and other chemicals. The wind carries the acidic compounds into the air, and they later fall to the ground in either dry or wet form.

They form an acidic 'rain' which can destroy vegetation. Some of these gases are from natural sources, such as lightning, decomposing plants and volcanoes. However, much of these gases are the result of emissions from cars, power stations, smelters and factories.

The effects of acid rain are as follows :

- Acidification of lakes, streams, and soils .
- Direct and indirect effects (release of metals, for example: Aluminum which washes away plant nutrients).
- Killing of wildlife (trees, crops, aquatic plants, and animals).
- Decay of building materials and paints, statues, and sculptures .
- Health problems (respiratory, burning- skin and eyes)

### 5.3.4 Air Pollution

Air pollution is the release of excessive amounts of harmful gases (e.g. methane, carbon dioxide, sulphur dioxide, nitrogen oxides) as well as particles (e.g. dust of tyre, rubber, and lead from car exhausts) into the atmosphere. Areas of high air pollution indexes have populations with higher rates of asthma than cleaner environments do.

### 5.3.5 Changes in Food Supply

Changing weather affects the agricultural industry and the human food supply. Carbon emissions contribute to increasing temperatures and decreasing precipitation, changing the growing conditions for food crops in many areas. Major changes in crop yield will cause food prices to rise around the world.

In addition, climate change influenced by carbon emissions forces animals, many of which are hunted as food, to migrate to higher altitudes or northern habitats as the climate warms.

### 5.3.6 Water Pollution

1. Sewage is the household waste water. Many detergents contain phosphates which act as plant fertilizers. When these phosphates and the sewerage reach rivers, they help water plants to grow in abundance, reducing the dissolved oxygen in the river water.
2. Biodegradable detergents are more environment-friendly because they are readily broken down to harmless substances by decomposing bacteria.
3. Suspended solids in water, such as silt reduce the amount of light that reaches the depths of the water in lakes and rivers. This reduces the ability of aquatic plants to photosynthesise and reduce the plant and animal life. **Turbidity** is the measure of 'cloudiness' or the depth to which light can reach in water.

### 5.3.7 Population Explosion

It is the rapid increase in population in developing countries causing famine, and also in developed countries causing more demand for energy and with that, it increases pollution and destruction of the environment.

### APPLICATION ACTIVITY 5.3

Regardless their eminent role in industrialization era, use appropriate examples, to show how fossil fuels are harmful to human being and environment in general when they are badly handled.

### 5.4 Problems associated with the production of nuclear power

#### ACTIVITY 5.4



**Fig.5. 3 Production of nuclear power**

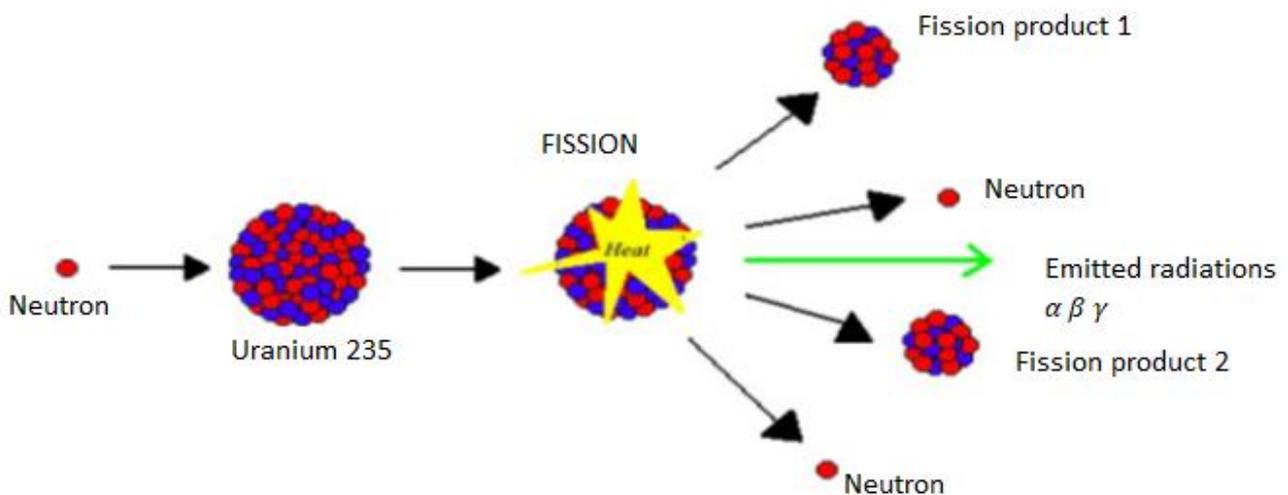
Study the figure 5.4 and try to respond to the following questions:

- i) Predict and write down what is observed in the picture above.
- ii) Do you think that the picture above produces food? Explain your reasoning.
- iii) The power Plant ejects big amount of smoke in the atmosphere. What kind of combustibles do you think are used there?
- iv) Apart from the gaseous smoke ejected, discuss other problems met during the production of energy using fossil fuel.

### 5.4.1 Nuclear fuel and nuclear fission

**Nuclear fuel** is any material that can be consumed to derive nuclear energy. The nuclear fuel can be made to undergo nuclear fission chain reactions in a nuclear reactor. The most common nuclear fuels are  $^{235}\text{U}$  (uranium 235) and  $^{239}\text{Pu}$  (plutonium 239). Not all nuclear fuels are used in fission chain reactions.

**Nuclear fission** is a process, by which a heavy nucleus splits into two or more simpler pieces. This process releases a lot of energy.

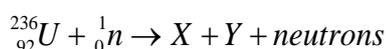


**Fig.5. 4 Chain reaction of uranium 235.**

When a neutron strikes an atom of uranium, the uranium nucleus splits into two lighter atoms and releases heat simultaneously. Fission of heavy elements is an exothermic reaction which can release large amounts of energy both as electromagnetic radiation and as kinetic energy of the fragments.

A chain reaction refers to a process in which neutrons released in fission produce an additional fission in at least one further nucleus. This nucleus in turn produces neutrons, and the process continues. If the process is controlled it is used for nuclear power or if uncontrolled it is used for nuclear weapons.

The fission of  $^{235}\text{U}$  by thermal neutrons can be represented by the reaction



The following is an example



#### 5.4.2 Controlled fission (power production) and uncontrolled fission (nuclear weapons)

Nuclear fission is based upon the release of neutrons during the reaction. If more than one neutron is released for every fission reaction it will accelerate, less than one it will decelerate.

Of the three neutrons, liberated during a fission reaction, only one triggers a new reaction and the others are simply captured. The system is in equilibrium. One fission reaction leads to one new fission reaction, which leads to one more, and so on. This is known as controlled fission.

In an uncontrolled fission reaction (weaponry) the appropriate amount of  ${}^{235}\text{U}$  is simply mixed with a moderator, making the reaction go out of control. As the reaction is out of control, the exponential acceleration of the reactions creates massive amounts of energy.

This can be kept from going off by keeping the moderator and the  ${}^{235}\text{U}$  separate, both below critical mass, until the desired time of explosion. In a controlled reaction there is a higher proportion of  ${}^{238}\text{U}$  to ease the reaction.

However, this is difficult to control as the reaction becomes faster or slower. To counteract this, control rods, made of neutron absorbing materials (i.e. Boron) are added or removed between each fuel rod.

#### 5.4.3 Problems associated with the production of nuclear power

- The *problem of radioactive waste* is still unsolved. The waste from nuclear energy is extremely dangerous and it has to be carefully looked after for several thousand years.
- *High risks:* Despite a generally high security standard, accidents can still happen. It is technically impossible to build a plant with 100% security. A small probability of failure will always last. The consequences of an accident would be absolutely devastating both for human beings and the nature.

The more nuclear power plants (and nuclear waste storage shelters) are built, the higher is the probability of a disastrous failure somewhere in the world.

- During the operation of nuclear power plants, radioactive waste is produced, which, in turn, can be used for the production of *nuclear weapons*.
- Nuclear power plants could be preferred *targets for terrorist attacks*. Such a terrorist act would have catastrophic effects for the whole world.

- The energy source for nuclear energy is Uranium. *Uranium is a scarce resource*; its supply is estimated to last only for the next 30 to 60 years depending on the actual demand.
- The timeframe needed for formalities, planning and building of a new nuclear power generation plant, is in the range of 20 to 30 years in the western democracies. In other words, it is an *illusion to build new nuclear power plants* in a short time.

#### APPLICATION ACTIVITY 5.4

- 1) Define (i) nuclear fuel (ii) nuclear fission
- 2) List any problems associated with the production of nuclear power.

### 5.5 Safety issues and risks of nuclear power

#### Activity 5.5:

The government of Rwanda in collaboration with other Countries wishes build a nuclear power plant that will be more productive in addressing issues about nuclear power.

1. Explain why you think Rwanda has a need of using nuclear power sources of energy?
2. Do you think that nuclear power is fossil fuel or non fossil fuel? Explain to support your idea.
3. What are the negative impacts of nuclear power plant?
4. You know negative impacts of radioactives wastes of fossil fuels and nuclear power in general on environment. Suppose that you have a chance of meeting the ministers of environment and infrastructures. Make a list of Proposal of safety measures you should hand them to avoid environmental pollution in Rwanda today and in the future.

#### 5.5.1 Nuclear Meltdown

A nuclear meltdown is an informal term for a severe nuclear reactor accident that results in core damage from overheating.

A nuclear meltdown occurs when a nuclear power plant system or component fails so the reactor core becomes overheat and melts. Usually, this occurs due to the lack of coolant that decreases the temperature of the reactor. The commonly used coolant is water but sometimes a liquid metal, which is circulated past the reactor core to absorb the heat, is also used.

In another case, a sudden power surge that exceeds the coolant's cooling capabilities causes an extreme increase in temperature which leads to a meltdown. A meltdown releases the core's highly radioactive and toxic elements into the atmosphere and environment.

The causes of a meltdown occur due to:

**A loss of pressure control:** The loss of pressure control of confined coolant may be caused by the failure of the pump or having resistance or blockage within the pipes. This causes the coolant to cease flow or insufficiency flow rate to the reactor; thus the heat transfer efficiency decreases.

**A loss of coolant:** A physical loss of coolant, due to leakage or insufficient provision, causes a deficit of coolant to decrease the heat of the reactor. A physical loss of coolant can be caused by leakages. In some cases, the loss of pressure control and the loss of coolant are similar because of the systematic failure of the coolant system.

#### **An uncontrolled power excursion**

A sudden power surge in the reactor is a sudden increase in reactor reactivity. It is caused by an uncontrolled power excursion due to the failure of the moderator or the control that slows down the neutron during chain reaction. A sudden power surge will create a high and abrupt increase of the reactor's temperature, and will continue to increase due to system failure. Hence, the uncontrollable increase of the reactor's temperature will ultimately lead to a meltdown.

### **5.5.2 Nuclear (Radioactive) Wastes**

Nuclear wastes are radioactive materials that are produced after the nuclear reaction. Nuclear reactors produce high-level radioactive wastes. The wastes must be isolated from human contact for a very long time in order to prevent radiation.

Short- and long-term storage of spent nuclear fuel has been a challenge for the industry and policymakers. Spent fuel, if not disposed of properly, could contaminate water supplies or be used by terrorists to create a dirty bomb. In the short-term, spent fuel is stored in pools on-site--but they only need to stay there a few months until they are cool enough to move to dry storage (either

on site or in a long-term storage facility). Still, at some plants, fuel rods are packed in pools in numbers well above design specifications and stay in the pools long after they are ready to be moved

Efforts to reprocess nuclear waste are expensive and come with associated environmental and security risks. Yet a growing number of countries--including Japan and Russia--have begun fuel recycling projects.

### **5.5.3 Security Issues.**

Most countries either pursuing nuclear power or currently using it have signed on to the Nuclear Nonproliferation Treaty and have agreed to comply with rules that ensure that they will not use nuclear technologies toward making weapons. However, any country with nuclear technology is considered a proliferation risk

#### **APPLICATION ACTIVITY 5.5**

1. State and explain the risks of nuclear power.
2. Suggest the safety issues of nuclear power.

## SKILLS LAB 5

This activity will be done as a project work aimed at protecting environment against pollutants. You are asked to make a study of all sources of pollutants in your area. They may due to different energy sources used in different activities in your region. Prepare a presentation to be addressed to local government and people during Umuganda day. In your presentation, you should include the following:

- i) Targeted activities
- ii) Different energy sources used in these activities polluting environment
- iii) Dangers of these energy sources if any.
- iv) Other forms of pollutants you know
- v) How to protect our environment from pollutants listed above
- vi) Role of local leaders and sponsors
- vii) Role of the people and individual person in community to ensure that your environment is protected.

## END UNIT ASSESSMENT 5

### 1. Answer by true or false

- a) In modern cities most people are carrying solar power devices
- b) At the present rate of usage of energy, without changing the source of energy, it is likely that amount of carbon dioxide in the atmosphere will double in less than 100 years.
- c) Large fraction of the energy used by the world comes from fossil fuels.
- d) Modern oil wells have been around for less than 200 years.
- e) The price of crude oil takes into accounts the time and effort that went into creating it in nature.

### 2. Matching questions

1. Describe how a nuclear power plant generates electricity	A. Oil and natural gas are formed from marine organisms that accumulate in the ocean floor and are buried in sediment. These organisms break down the released oil into the sediment which depending on the type of sediment traps the oil
2. What are the products of the nuclear disintegration of a radioactive isotope?	B. A nuclear power plant generates electricity when the nuclei of radioactive atoms disintegrate and release energy which is used to heat water and produce steam. The steam turns a turbine that generates electricity
3. Describe three factors that can cause the amount of an oil reserve to increase	C. Time, distance, and shielding are the basic principles of radiation protection
4. Describe the processes that resulted in the formation of oil and natural gas	D. Coal is formed when large quantities of plant material from swamps is trapped in sediment and subjected to heat and pressure
5. Describe the differences between lignite, bituminous, and anthracite coal	E. Radiation is the product of the disintegration of a radioactive isotope

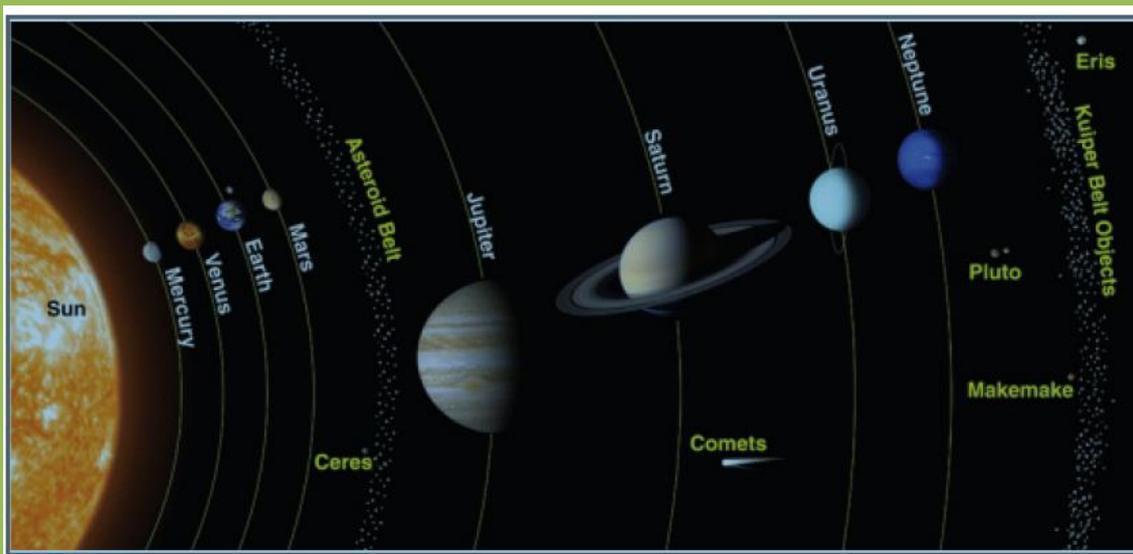
6. List the three primary methods of protecting people from damaging radiation	F. Depending on the amount of time the organic plant material is trapped in the earth determines the type of coal. Lignite is the lowest quality because of its high moisture content. Bituminous or soft coal has the low moisture content but is abundant whereas anthracite has the highest carbon content (energy) however is rare
7. Describe the geologic processes that resulted in the formation of coal	G. Economic conditions, technological advancement, and new deposits discovered changes the amount of oil reserves

3. Which of the following is true about solar energy?
- a) It is becoming cheaper to produce photovoltaic cells
  - b) Solar energy can currently replace all of the energy created by fossil fuels
  - c) Most solar panels convert more than 25% of the light that strikes them
4. Identify three technological challenges that limit the use of solar power in Rwanda.
- A. Weight, cost, toxicity
  - B. Aesthetics, toxicity, efficiency
  - C. Storage, weight, fragility
  - D. Cost, storage, efficiency
5. Which of the following is NOT utilized in the process of harnessing solar energy?
- A. Gas
  - B. Mirrors
  - C. Steam
  - D. Photovoltaic cells
6. Which issues would better energy storage technologies help solve?
- A. Inconsistent energy demands
  - B. Inconsistent power production
  - C. The need to keep inefficient power plants on standby
  - D. All of the above
7. a) Design and explain advantages of non fossil fuel.  
b) Suggest disadvantages of non fuel energy if any.
8. Evaluate different ways used to eradicate environment pollution in Rwanda.

## UNIT 6: MOTION IN ORBITS

**Key unit competence:** Apply Newton’s law of gravitation and Kepler’s laws in explaining planetary motion

### INTRODUCTORY ACTIVITY



People have always enjoyed viewing stars and planets on clear, dark nights. It is not only the beauty and variety of objects in the sky that is so fascinating, but also the search for answers to questions related to the patterns and motions of those objects.

Until the late 1700s, Jupiter and Saturn were the only outer planets identified in our solar system because they were visible to the naked eye. Combined with the inner planets the solar system was believed to consist of the Sun and six planets, as well as other smaller bodies such as moons. Some of the earliest investigations in physical science started with questions that people asked about the night sky.

- i) Based on the scenario above and the observation from the picture. Briefly summarize what is illustrated in the picture.
- ii) What is the name of belt separating the largest and smallest planets?
- iii) Explain why you think the moon doesn't fall on the earth.
- iv) Why don't we fly off into space rather than remaining on the Earth's surface? Explain your idea.
- v) Explain why planets move across the sky.

## Introduction

A natural phenomenon by which all things with mass or energy including satellites, planets, stars, galaxies, and even light, are brought toward (or *gravitate* toward) one another is referred to as **gravity or gravitation**. On Earth, gravity gives weight to all physical objects around it.

Gravity is very important to our everyday lives. Without Earth's gravity we would fly right off it. If you kicked a ball, it would fly off forever. While it might be fun to try for a few minutes, we certainly can't live without gravity. Gravity also is important on a larger scale.

It is the Sun's gravity that keeps the Earth in orbit around the Sun. Life on Earth needs the Sun's light and warmth to survive. Gravity helps the Earth to stay at just the right distance from the Sun, so it's not too hot or too cold.

### 6.1 Newton's law of gravitation

#### ACTIVITY 6.1

Two big stones are separated by a small distance as shown in the figure below



**Fig.6. 1 Two stones at a distance apart.**

- Discuss the interactions between two stones.
- Can the two stones attract one another? Explain your reasoning.
- Make a general conclusion about small bodies close to one another.
- Imagine whether we have two bodies which are massive (too big), explain the difference in interactions of massive bodies and small bodies. Give any examples

From the time of Aristotle, the circular motions of heavenly bodies were regarded as natural. The ancients believed that the stars, planets, and Moon moved in divine circles, free from any impressed forces. Newton, however, recognized that a force must be acting on the Planets; otherwise, their paths would be straight lines.

And whereas others of his time, influenced by Aristotle, said that any such force would be directed along the planets' motion, Newton reasoned it must be perpendicular to their motion, directed toward the center of their curved paths- toward the sun. This was the force of gravity, the same force that pulls apples off trees.

**The Newton's law of universal gravitation** states that

“Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them”.

If the particles have masses  $m_1$  and  $m_2$  and are separated by a distance  $r$ , the magnitude of this gravitational force is

$$F_g = G \frac{m_1 m_2}{r^2}$$

Where  $G$  is a constant, called the *universal gravitational constant*, that has been measured experimentally. Its value in SI units is  $G = 6.673 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$

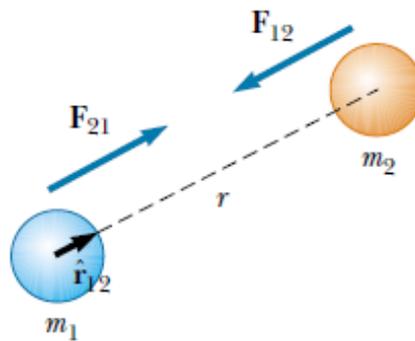
Another important point that we can show from Equation (i) is that the gravitational force exerted by a finite-size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distribution were concentrated at the center. For example, the magnitude of the force exerted by the Earth on a particle of mass  $m$  near the Earth's surface is

$$F_g = G \frac{M_E m}{R_E^2}$$

Where  $M_E$  is the Earth's mass and  $R_E$  its radius. This force is directed toward the center of the Earth.

The space surrounding the Earth where the mass of an object experiences a gravitational pull, or force due to gravity, is gravitational field of the Earth. On the surface of the earth the value of gravitational field is  $g = 9.8 \text{ m/s}^2$ , on the surface on the moon's surface is only about  $g = 1.6 \text{ m/s}^2$

- The constant of proportionality  $G$  is known as the "universal gravitational constant" because it is thought to be the same at all places and all times, and thus universally characterizes the intrinsic strength of the gravitational force.
- $M$  is the mass of the Earth,  $M_E = 5.98 \times 10^{24} \text{ kg}$
- We assume the Earth to be spherical and neglect the radius of the object relative to the radius of the Earth in this discussion.  $R_E = 6380 \text{ km}$
- The law of universal gravitation is applied in analyzing the motions of bodies in the universe, such as planets in the solar system. (This analysis can lead to the discovery of other celestial bodies.)
- The measured gravitational acceleration at the Earth's surface is found to be about  $g = 9.8 \text{ m/s}^2$
- $$\vec{F}_{12} = -\frac{Gm_1m_2}{r^3}\vec{r}_{12}$$



**Fig.6. 2 The gravitational force between two particles**

Remember two objects exert equal and opposite force of gravitation on each other.

The gravitational attraction of object  $m_2$  pulling on object  $m_1$  designated  $F_{21}$  is in a direction opposite to the force exerted by particle 1 on particle 2 by Newton's third law, is equal in magnitude to  $F_{12}$  and in the opposite direction.

That is, these forces form an action–reaction pair, and  $F_{12} = -F_{21}$  and hence the force  $F_{12}$  must be directed toward particle 1. In addition, the direction is along the line connecting the centers of the objects.

The gravitational force is a field force that always exists between two particles, regardless of the medium that separates them. Because the force varies as the inverse square of the distance between the particles, it decreases rapidly with increasing separation.

### Example 6.1

1. Find the correct answer and give a short explanation in the following question:

A planet has two moons of equal mass. Moon 1 is in a circular orbit of radius  $r$ . Moon 2 is in a circular orbit of radius  $2r$ . The magnitude of the gravitational force exerted by the planet on moon 2 is

- (a) four times as large as that on moon 1
- (b) twice as large as that on moon 1
- (c) equal to that on moon 1
- (d) half as large as that on moon 1
- (e) one fourth as large as that on moon 1.

**Solution: (e)**

The gravitational force follows an inverse-square behavior, so doubling the distance causes the force to be one fourth as large.

2. Calculate the force of attraction between 90kg spheres of metal space so that their centers are 40cm apart. In SI units the gravitational constant has the value  $G = 6.67 \times 10^{-11} \text{ N.m}^2 / \text{kg}^2$ .

**Solution:** 
$$F = G \frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}) \frac{90 \times 90}{(0.40)^2} = 3.38 \times 10^{-6} \text{ N}$$

### Properties of Gravitational Force

- It is always attractive in nature while electric and magnetic force can be attractive or repulsive.
- It is independent of the medium between the particles while electric and magnetic forces depend on the nature of the medium between the particles.

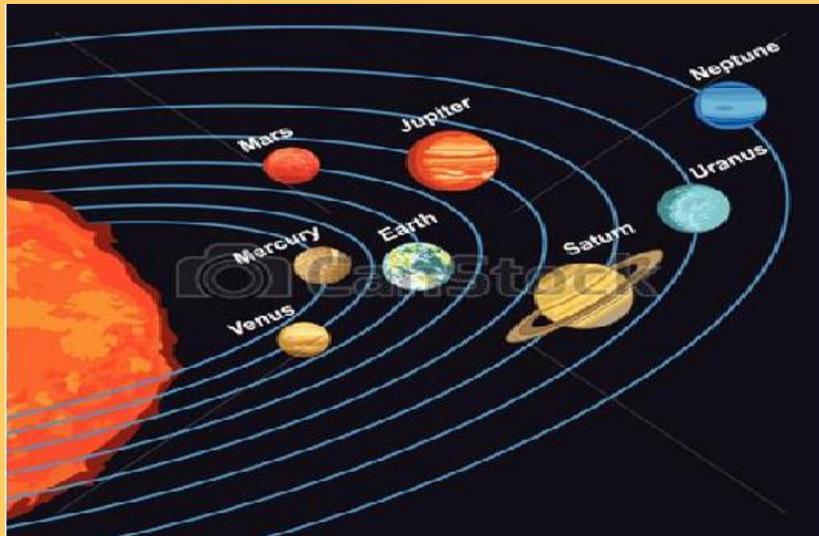
- It holds well over a wide range of distances. It is found true for interplanetary to inter-atomic distances.
- It is a central force which means it acts along the line joining the centers of two interacting bodies.
- It is a two-body interaction, where gravitational force between two particles is independent of the presence or absence of other particles; so, the principle of superposition is valid, and on the contrary, nuclear force is a many-body interaction.
- It is the weakest force in nature.
- It is a conservative force, where work done by it is path independent or work done in moving a particle round a closed path under the action of gravitational force is zero.
- It is an action reaction pair, where the force with which one body (say, earth) attracts the second body (say, moon) is equal to the force with which moon attracts the earth. This is in accordance with Newton's third law of motion.

#### **APPLICATION ACTIVITY 6.1**

1. Calculate the force of gravity between two bowling balls each having a mass of 8.0kg, when they are 0.50m apart.
2. What is the gravitational attraction between an object with a mass of 10 kg and another object with a mass of 20 kg if they are separated by 0.01 m? (Assume both objects are on same surface and not suspended in midair.)
3. At the surface of a certain planet, the gravitational acceleration  $g$  has a magnitude of  $2.0 \text{ m/s}^2$ . A 4.0kg brass ball is transported to this planet. Give:
  - a) The mass of the brass ball on the earth and on the planet
  - b) The weight of the brass ball on the earth and on the planet.

## 6.2 Kepler's laws of gravitational motion

### ACTIVITY 6.2



Critically observe the picture above and answer the questions that follow.

- Describe the motion of bodies (planets) as indicated in the picture.
- Do the different planets pass through the same path? Explain to support your decision.
- With clear observations, which body is the largest. Explain to support your selection.
- Basing on the knowledge from Newton's law of gravitation, is there any force of attraction between the body stated in above and other bodies? Explain your reasoning.
- If yes, how does the force affect the motion of the bodies as they are in their paths?

### 6.2.1 Kepler's First Law

It states that the orbits of the planets are ellipses, with the Sun at one focus of the ellipse.

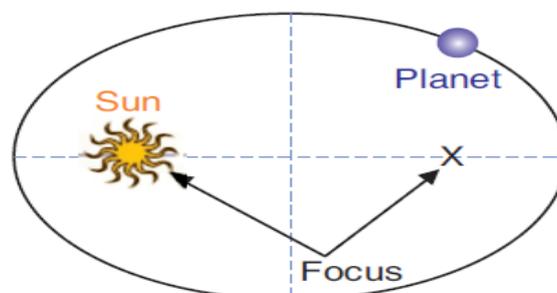


Fig.6. 3 Sun at one focus of the ellipse

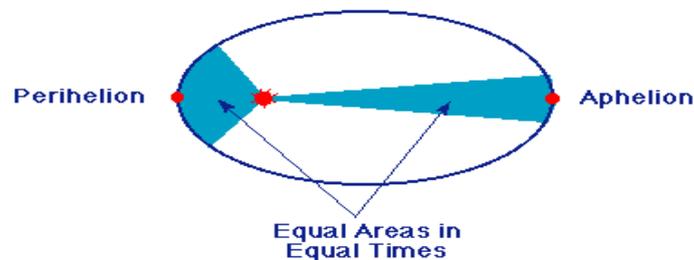
The Sun is not at the center of the ellipse, but is instead at one focus (generally there is nothing at the other focus of the ellipse).

The planet then follows the ellipse in its orbit, which means that the Earth-Sun distance is constantly changing as the planet goes around its orbit.

For purpose of illustration we have shown the orbit as rather eccentric; remember that the actual orbits are much less eccentric than this.

### 6.2.2 Kepler's Second Law

It states that the line joining the planet to the Sun sweeps out equal areas in equal times as the planet travels around the ellipse.

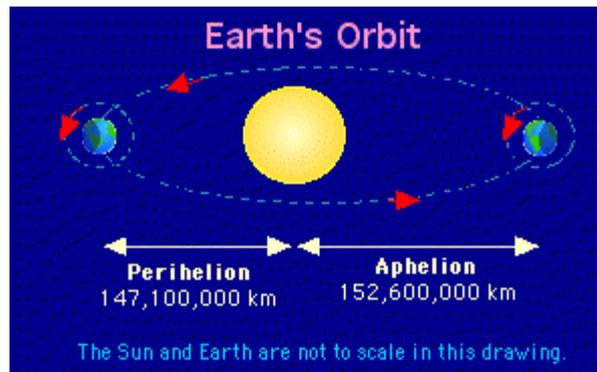


**Fig.6. 4 Law of areas**

Kepler's second law is illustrated in the preceding figure. The line joining the Sun and planet sweeps out equal areas in equal times, so the planet moves faster when it is nearer the Sun. Thus, a planet executes elliptical motion with constantly changing angular speed as it moves about its orbit.

What happens is best understood in terms of **energy**. As the planet moves away from the Sun (or the satellite from Earth), it loses energy by overcoming the pull of gravity, and it slows down, like a stone thrown upwards. And like the stone, it **regains** its energy as it comes back

The point of nearest approach of the planet to the Sun is termed **perihelion**; the point of greatest separation is termed **aphelion**. Hence, by Kepler's second law, the planet moves fastest when it is near perihelion and slowest when it is near aphelion.



**Fig.6. 5 Aphelion and Perihelion**

### 6.2.3 Kepler's Third Law

It states that the ratio of the squares of the revolutionary periods for two planets is equal to the ratio of the cubes of their semi major axes .Therefore, the law is summarized in the expression below.

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Note: - In this equation T represents the period of revolution for a planet and R represents the length of its semi major axis. The subscripts "1" and "2" distinguish quantities for planet 1 and 2 respectively. The periods for the two planets are assumed to be in the same time units and the lengths of the semi major axes for the two planets are assumed to be in the same distance units.

- Kepler's Third Law implies that the period for a planet to orbit the Sun increases rapidly with the radius of its orbit. Thus, we find that Mercury, the innermost planet, takes only 88 days to orbit the Sun but the outermost planet (Pluto) requires 248 years to do the same.

- Kepler's 3<sup>rd</sup> law applies only to objects orbiting the same attracting center. Do not use to compare, say the Moon's orbit around the Earth to the orbit of Mars around the Sun because they depend on different attracting centers.

#### a) Verification of Kepler's third law

There is only one speed that a planet can have if the planet is to remain in an orbit with a fixed radius. Since the gravitational force acting on the planet of mass m in the radial direction, it alone provides the centripetal force. Therefore, using Newton's law of gravitation, we have:

$$F_G = F_C \Leftrightarrow \frac{GMm}{r^2} = \frac{mv^2}{r} \Leftrightarrow v = \sqrt{\frac{GM}{r}}$$

The mass  $m$  of planet does not appear in equation consequently, for a given orbit, a planet with a large mass has exactly the same orbital speed as a planet with a small mass.

The radius  $r$  of the orbit (distance from the center of planet to the center of the sun) is in the denominator in equation. This means that the closer the planet is to Sun, the smaller is the value for  $r$  and the greater the orbital speed must be.

The period  $T$  of a planet is the time required for one orbital revolution. The period is related to the speed of the motion by

$$v = \frac{2\pi r}{T}$$

Substituting  $v$  in the equation below:

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T} \Leftrightarrow T = 2\pi \sqrt{\frac{r^3}{GM}}$$

Therefore, the preceding expression becomes  $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

### Example 6.2

A satellite travels at a height from the centre of Earth twice the radius of the Earth. Determine the period of revolution of the satellite. (The radius of Earth =  $6.4 \times 10^6 \text{ m}$ , mass of Earth =  $6.0 \times 10^{24} \text{ kg}$ )

#### Solution:

We have seen that  $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

$$r^3 = GM \left( \frac{T}{2\pi} \right)^2$$

$$T^2 = \left( \frac{4\pi^2}{GM} \right) (2R)^3$$

Where  $R$  is the radius of the Earth and  $\frac{4\pi^2}{GM} = \frac{4\pi^2}{(6.7 \times 10^{-11})(6 \times 10^{24})} = 0.982 \times 10^{-13}$

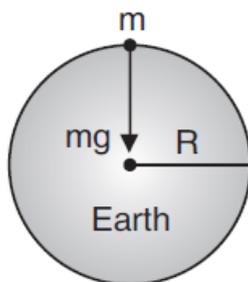
$$T^2 = (0.982 \times 10^{-13}) (2 \times 6.4 \times 10^6)^3 = 2060 \times 10^5 = 206 \times 10^6$$

$$T = 14.4 \times 10^3 \text{ s} = 4.0 \text{ h}$$

### b) Verification of acceleration due to gravity at the surface of the earth

The force of attraction exerted by the earth on a body is called gravitational pull or gravity. We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate. The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity ( $g$ ).

Consider a body of mass  $m$  lying on the surface of earth.



**Fig.6. 6 A body on earth surface**

Then gravitational force on the body is given by:

$$F = \frac{GMm}{R^2}$$

Where  $M$  = mass of the earth and  $R$  = radius of the earth. If  $g$  is the acceleration due to gravity, then the force on the body due to earth is given by Force = mass  $\times$  acceleration

Or  $F = m \times g$

We have  $mg = \frac{GMm}{R^2}$  ,  $g = \frac{GM}{R^2}$  and

$$\therefore g = \frac{G}{R^2} \left( \frac{4}{3} \pi R^3 \rho \right)$$

Because  $M = \text{Volume} \times \text{Density}$  . Since,  $M = \frac{4}{3} \pi R^3 \rho$  and

$$\therefore g = \frac{4}{3} \pi \rho G R$$

**Note:**

- 1) From the expression  $g = \frac{GM}{R^2} = \frac{4}{3} \pi \rho G R$ , it is clear that its value depends upon the mass, radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet. i.e. a given planet (reference body) produces same acceleration in a light as well as heavy body.
- 2) The greater the value of  $\frac{M}{R^2}$  or  $\rho R$ , greater will be the value of  $g$  for that planet.
- 3) Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.
- 4) Dimensions of  $[g] = [LT^{-2}]$
- 5) Average value of  $g$  is taken as  $g = 9.81 \text{ m/s}^2$  on the surface of the earth at mean sea level.
- 6) In general, the value of acceleration due to gravity vary due to the following factors:
  - (a) Shape of the earth,
  - (b) Height above the earth surface,
  - (c) Depth below the earth surface and
  - (d) Axial rotation of the earth.

**Example 6.3**

Acceleration due to gravity on moon is  $(1/6)^{th}$  of the acceleration due to gravity on earth. If the ratio of densities of earth  $\rho_e$  and moon  $\rho_m$  is  $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$  find the radius of moon  $R_m$  in terms of radius of the earth  $R_e$

**Solution:**

Acceleration due to gravity,  $g = \frac{4}{3} \pi \rho G R \Rightarrow g \propto R$

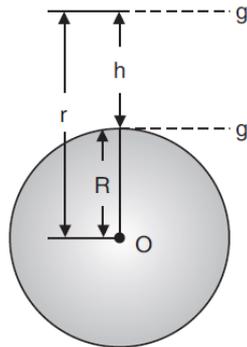
Or  $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e} \times \frac{R_m}{R_e}$

As  $\frac{g_m}{g_e} = \frac{1}{6}$  and  $\frac{\rho_e}{\rho_m} = \frac{5}{3}$  give  $\frac{R_e}{R_m} = \left(\frac{g_e}{g_m}\right) \left(\frac{\rho_e}{\rho_m}\right) = \frac{6}{1} \times \frac{5}{3} = \frac{30}{3} \Rightarrow R_m = \frac{1}{10} R_e$

### 3. Variation of gravity above and below the earth surface.

#### (i) Variation of gravity above the earth surface

Consider a particle placed at a height  $h$  above the surface of the earth where acceleration due to gravity is  $g'$  as shown on the figure below:



**Fig.6. 7 A particle placed at a height  $h$  above the surface of the earth**

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} \quad (1)$$

Acceleration due to gravity at height  $h$  from the surface of the earth

$$g' = \frac{GM}{(R+h)^2} \quad (2)$$

From equations (1) and (2) we should write also the following

$$g' = g \left( \frac{R}{R+h} \right)^2 \quad (3)$$

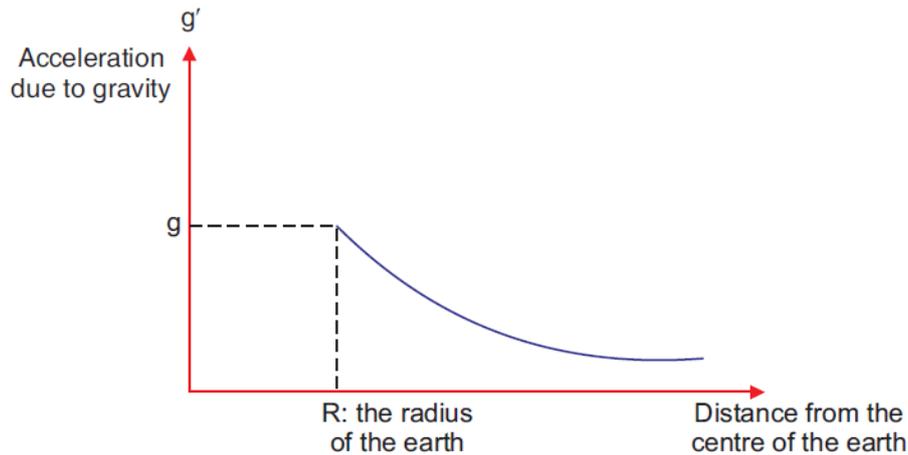
If  $r = R+h$

$$\text{Then equation (3) becomes } g' = g \frac{R^2}{r^2} \quad (4)$$

**Note:**

As we go above the surface of the earth, the value of  $g$  decreases because  $g \propto \frac{1}{r^2}$

This expression can be plotted on the graph as:



**Fig.6. 8: Curve of variation of acceleration due to gravity with height.**

### Example 6.3

The International Space Station operates at an altitude of 350 km. When final construction is completed, it will have a weight (measured at the Earth's surface) of  $4.22 \times 10^6 \text{ N}$ . What is its weight when in orbit?

#### Solution

We first find the mass of the space station from its weight at the surface of the Earth:

$$m = \frac{F_g}{g} = \frac{4.22 \times 10^6 \text{ N}}{9.80 \text{ m/s}^2} = 4.31 \times 10^5 \text{ kg}$$

This mass is fixed—it is independent of the location of the space station. Because the station is above the surface of the Earth, however, we expect its weight in orbit to be less than its weight on the Earth. Using Equation (3) with  $h = 350 \text{ km}$  we obtain:

$$g' = g \left( \frac{R}{R+h} \right)^2$$

$$g' = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m} + 0.350 \times 10^6 \text{ m})^2}$$

$$g' = 8.83 \text{ m/s}^2$$

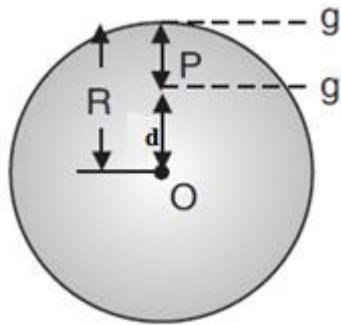
Because this value is about 90% of the value of  $g$  at the Earth surface, we expect that the weight of the station at an altitude of 350 km is 90% of the value at the Earth's surface.

Using the value of  $g'$  at the location of the station, the station's weight in orbit is given by:

$$mg = (4.31 \times 10^6 \text{ kg})(8.83 \text{ m/s}^2) = 3.80 \times 10^6 \text{ N}$$

(ii) **Variation of gravity below the earth surface.**

Consider a mass placed at point P below the surface of the earth as shown below:



**Fig.6. 9: A mass placed at P below the Earth's surface**

Acceleration due to gravity at the surface of the earth is  $g = \frac{GM}{R^2} \Leftrightarrow g = \frac{4}{3} \pi \rho GR$  (5)

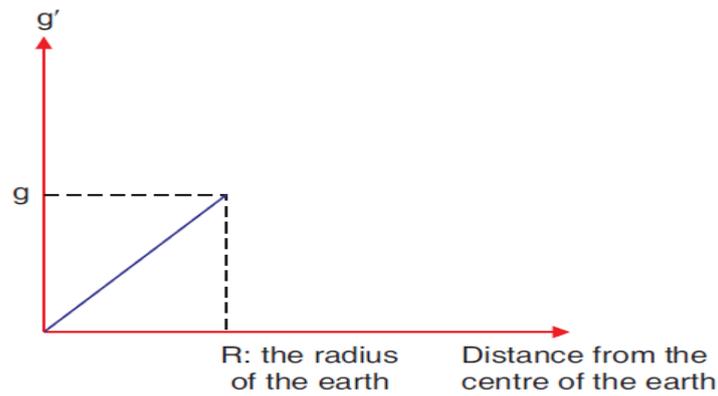
Acceleration due to gravity at depth  $d$  from the surface of the earth of radius  $R$  is given by

$$g' = \frac{4}{3} \pi \rho G (R - d) \quad (6)$$

Dividing equations (6) by (5), we get  $g' = g \left| 1 - \frac{d}{R} \right|$  (7)

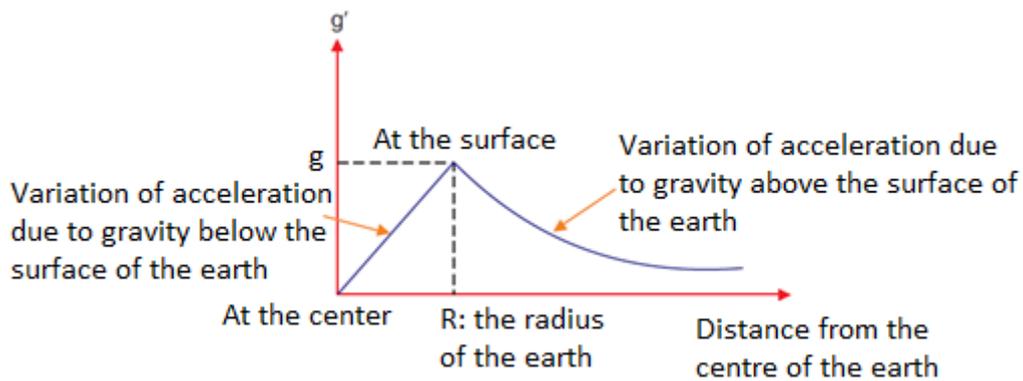
**Note:**

The value of  $g$  decreases ongoing below the surface of the earth. From equation (6), we get  $g' \propto (R - d)$ . So it is clear that if  $d$  increases the value of  $g$  decreases.



**Fig.6. 10: Variation of gravity with depth**

In summary, combining the graphs for variation of acceleration due to gravity below and above the surface of the earth will give the graph as shown below:



**Fig.6. 11: Curve of variation of gravity with depth and height**

Note: At the centre of earth  $g' = 0$ , it means that the acceleration due to gravity at the centre of earth becomes zero. Decrease in the value of  $g$  with depth

-Absolute decrease,  $\Delta g = g - g' = \frac{gd}{R}$

-Fractional decrease,  $\frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{d}{R}$ . Therefore, the percentage decrease,  $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} \times 100\%$

The rate of decrease of gravity outside the earth ( $h \ll R$ ) is double to that of inside the earth.

### Example 6.4

Determine the height from the surface of the earth, which an object must travel through so that the gravitational field strength changes by 1.0%. (Radius of the Earth is 6400km).

#### Solution:

On Earth's surface,

$$g = \frac{GM}{R^2} \text{ where } R \text{ is the radius of the Earth.}$$

At a height  $r$  from the centre of the Earth,  $g' = \frac{GM}{r^2}$

$$\frac{\Delta g}{g} = \frac{g - g'}{g}$$

$$= 1 - \frac{g'}{g} = 1 - \left(\frac{R}{r}\right)^2$$

$$= \left(\frac{R}{r}\right)^2 = 1 - \frac{1}{100} = 0.990$$

$$\frac{R}{r} = 0.995 \Rightarrow r = \frac{R}{0.995}$$

But  $r = R + h$  where  $h$  is the required height.

$$R + h = \frac{R}{0.995} \Rightarrow h = \frac{0.005(6400 \text{ km})}{0.995} = 32 \text{ km}$$

### APPLICATION ACTIVITY 6.2

1. The planet is revolving around the sun as shown in elliptical path.

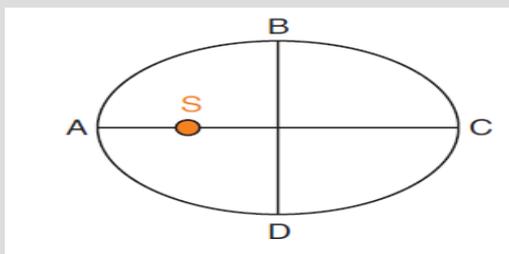


Fig.6. 12 : Revolution of a planet around the sun

The correct option is:

- (a) The time taken in travelling  $DAB$  is less than that for  $BCD$ .
- (b) The time taken in travelling  $DAB$  is greater than that for  $BCD$ .
- (c) The time taken in travelling  $CDA$  is less than that for  $ABC$ .
- (d) The time taken in travelling  $CDA$  is greater than that for  $ABC$ .

2. a) Calculate the magnitude of the gravitational field strength on the surface of Mars where the mass of Mars is  $M = 6.37 \times 10^{23} \text{ kg}$  and the radius of Mars is  $r = 3.4 \times 10^6 \text{ m}$ .

b) What is the ratio of the magnitude of the gravitational field strength on the surface of Mars to that on the surface of Earth?

3. Determine the mass of Earth using the magnitude of the gravitational field strength at the surface of the Earth, the distance  $r$  between Earth's surface and its centre  $6.38 \times 10^6 \text{ m}$  and the universal gravitation constant

4. Determine the magnitude of the gravitational field strength at a point

- (i) 1000 km above the surface of the Earth
- (ii) On the surface of the Earth

$$(G = 6.7 \times 10^{-11} \text{ N.kg}^{-2}.\text{m}^{-2}, M_E = 6.0 \times 10^{24} \text{ kg}, R_e = 6.7 \times 10^6 \text{ m})$$

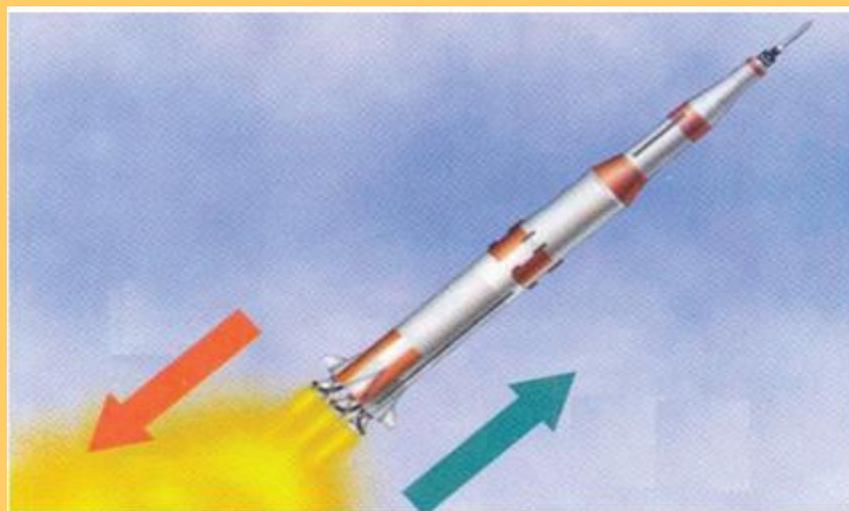
5. Estimate the effective value of  $g$  on the top of Mt. Everest, 8850 m above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

6 a) Calculate the escape speed from the Earth for a 5 000 kg spacecraft, and determine the kinetic energy it must have at the Earth's surface in order to move infinitely far away from the Earth.

b) What if we wish to launch a 1 000 kg spacecraft at the escape speed? How much energy does this require?

### 6.3 Rockets and satellites.

### ACTIVITY 6.3



**Fig.6. 13 Propulsion rocket.**

1. Hold a balloon and fill it with air. Then let it go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled?
2. If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.
3. Based on the observations made on (a) and (b) above, analyze the movement of the rocket shown in the figure above.
4. Artificial satellites are machines launched in the atmosphere to move around the Earth.
  - (i) What is the instrument do you think is used to launch them in the atmosphere?
  - (ii) Discuss any roles of artificial satellites.

#### 6.3.1 Rockets

A rocket is a missile, spacecraft, aircraft or other vehicle that obtains thrust from a rocket engine. A rocket is a device that produces thrust by ejecting stored matter (fuel). A rocket moves forward when gas expelled from the rear of a rocket pushes it in the opposite direction. From Newton's laws of motion, for every action, there is an equal and opposite reaction.

#### Basic principle of Rocket propulsion

Rocket propulsion is based on Newton's laws of motion:

- Momentum conservation law
- Newton's third law

In a rocket, fuel is burned to make a hot gas and this hot gas is forced out of narrow nozzles in the back of the rocket, propelling the rocket forward.

### **Factors Affecting a Rocket's Acceleration**

- The greater the exhaust velocity of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

### **Spacecraft Propulsion**

**Spacecraft** is a vehicle designed to operate, with or without a crew, in a controlled flight pattern above Earth's lower atmosphere. The spacecraft typically either is placed into an orbit around Earth or, if given sufficient velocity to escape Earth's gravity, continues toward another destination in space. The spacecraft itself often carries small rocket engines for maneuvering and orienting in space.

Spacecraft Propulsion is characterized in general by its complete integration within the spacecraft (e.g. satellites). Its function is to provide forces and torques in (empty) space to:

- transfer the spacecraft: used for interplanetary travel
- position the spacecraft: used for orbit control
- orient the spacecraft: used for altitude control

The jet propulsion systems for launching rockets are also called primary propulsion systems. Spacecrafts, e.g. satellites, are operated by secondary propulsion systems.

### **Characteristics of Spacecraft Propulsion Systems**

In order to fulfill altitude and orbit operational requirements of spacecraft, spacecraft propulsion systems are characterized by:

- Very high velocity increment capability ( km/s)
- Low thrust levels (1 mN to 500 N) with low acceleration levels
- Continuous operation mode for orbit control
- Pulsed operation mode for altitude control
- Predictable, accurate and repeatable performance (impulse bits)
- Reliable, leak-free long time operation (storable propellants)

- Minimum and predictable thrust exhaust impingement effects

### **Classification of Propulsion Systems**

Spacecraft propulsion can be classified according to the source of energy utilized for the ejection of propellant:

- ***Chemical propulsion*** use heat energy produced by a chemical reaction to generate gases at high temperature and pressure in a combustion chamber. These hot gases are accelerated through a nozzle and ejected from the system at a high exit velocity to produce thrust force.
- ***Electric propulsion*** uses electric or electromagnetic energy to eject matter at high velocity to produce thrust force.

***Nuclear propulsion*** uses energy from a nuclear reactor to heat gases which are then accelerated through a nozzle and ejected from the system at a high exit velocity to produce thrust force.

### **6.3.2 Satellites**

A satellite is an artificial or a natural body placed in orbit round the earth or another planet in order to collect information or for communication. Communication satellites are satellites that are used specifically to communicate. The payload of communication satellite consists of huge collection of powerful radio transmitters and or a big dish, to enable it to exchange information with the ground. We use them to transmit TV signals, to transmit radio signals, and in some cases, it transmits internet signals.

There is only one main force acting on a satellite when it is in orbit, and that is the gravitational force exerted on the satellite by the Earth. This force is constantly pulling the satellite towards the centre of the Earth.

A satellite doesn't fall straight down to the Earth because of its velocity. Throughout a satellite's orbit there is a perfect balance between the gravitational force due to the Earth and the centripetal force necessary to maintain the orbit of the satellite.

Satellites are natural or artificial bodies describing orbit around a planet under its gravitational attraction. Moon is a natural satellite while INSAT-1B is an artificial satellite of the earth. Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth.

### **Orbital Velocity of Satellite.**

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth. For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

As  $GM = gR^2$  and  $r = R+h$  and Then  $v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$

Note:

- Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit, i.e. satellites of different masses have the same orbital velocity, if they are in the same orbit.
- Orbital velocity depends on the mass of central body and radius of orbit.
- For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite ( $v \propto 1/r$ ).
- Orbital velocity of the satellite when it revolves very close to the surface of the planet:

$$v = \sqrt{\frac{GM}{R+h}} \text{ Becomes } v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \text{ since } h=0 \text{ and } GM = gR^2$$

For the earth  $v = \sqrt{9.8 \times 6.4 \times 10^6} = 7.9 \text{ km/s} = 8 \text{ km/s}$

**Escape velocity** is a function of the orbital velocity of an object. If you take the velocity required maintaining orbit at a given altitude and multiplying it by the square root of 2 (which is approximately 1.414), you will derive the velocity required to escape orbit and the gravitational field controlling that orbit.

Close to the surface of planet, the orbital velocity is  $v = \sqrt{\frac{GM}{R}}$

Then the escape velocity is  $v_e = \sqrt{\frac{2GM}{R}}$  i.e.  $v_e = \frac{v}{\sqrt{2}} \Rightarrow v = \sqrt{2} v_e$

### Example 5.5

1. A satellite is moving around the earth with speed  $v$  in a circular orbit of radius  $r$ . If the orbit radius is decreased by 1%, what is its speed?

**Solution:**

$$\text{Orbital velocity, } v = \sqrt{\frac{MG}{r}}$$

$$\therefore v \propto \frac{1}{r} \text{ [If } r \text{ decreases, then } v \text{ increases]}$$

$$\text{Percentage change in } v = \frac{1}{2} (\text{percentage change in } r) = \frac{1}{2} (1\%) = 0.5\%$$

$\therefore$  Orbital velocity increases by 0.5%.

2. Two satellites  $A$  and  $B$  go round a planet  $P$  in circular orbits having radii  $4R$  and  $R$  respectively. If the speed of the satellite  $A$  is  $3V$ , what is the speed of the satellite  $B$ ?

**Solution:**

$$\text{Orbital velocity of satellite } v = \sqrt{\frac{GM}{r}}$$

$$v \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} \Rightarrow \frac{v_B}{3V} = \sqrt{\frac{4R}{R}} \Rightarrow v_B = 6V$$

### Time Period of Satellite

It is the time taken by satellite to go once around the earth.

$T = \text{circumference of the orbit divided by the orbit velocity}$

$$T = \frac{2\pi r}{v} = 2\pi r \sqrt{\frac{r}{GM}} \Rightarrow T = 2\pi \sqrt{\frac{r^3}{GM}} = 2\pi \sqrt{\frac{(R+h)^3}{gR^2}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{R}{g} \left(1 + \frac{h}{R}\right)^{\frac{3}{2}}}$$

**Note:**

- From  $T = 2\pi\sqrt{\frac{r^3}{GM}}$ , it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit.

$T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T^2 = \frac{4\pi^2}{GM}r^3$  i.e  $T^2 \propto r^3$ . This is in accordance with Kepler's third law of planetary motion.

### Example 4.6

1. A satellite is launched into a circular orbit of radius 'R' around earth while a second satellite is launched into an orbit of radius 1.02 R. What is the percentage difference in the time periods of the two satellites?

**Solution:**

Orbital radius of second satellite is 2% more than the first satellite.

So from  $T \propto (r)^{3/2}$ , percentage increase in time period =  $\frac{3}{2}$  (Percentage increase in orbital radius)

$$= \frac{3}{2}(2\%) = 3\% \quad \therefore 23(2\%) = 3\%.$$

2. What is the periodic time of a satellite revolving above Earth's surface at a height equal to R, where R is the radius of Earth?

**Solution:**

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+R)^3}{gR^2}} = 2\pi\sqrt{\frac{8R}{g}} = 4\sqrt{2}\pi\sqrt{\frac{R}{g}}$$

### 6.3.3 Applications of satellites

Satellites that are launched in to the orbit by using the rockets are called man-made satellites or artificial satellites. Artificial satellites revolve around the earth because of the gravitational force of attraction between the earth and satellites. Unlike the natural satellites (moon), artificial satellites are used in various applications. The various applications of artificial satellites include:

Weather forecasting, Navigation, Astronomy, Satellite phone, Satellite television, Military satellite, Satellite internet and Satellite radio.

## **1. Weather forecasting**

Weather forecasting is the prediction of the future of weather. The satellites that are used to predict the future of weather are called weather satellites. Weather satellites continuously monitor the climate and weather conditions of earth. They use sensors called radiometers for measuring the heat energy released from the earth surface. Weather satellites also predict the most dangerous storms such as hurricanes.

## **2. Navigation**

Generally, navigation refers to determining the geographical location of an object. The satellites that are used to determine the geographic location of aircrafts, ships, cars, trains, or any other object are called navigation satellites. GPS (Global Positioning System) is an example of navigation system. It allows the user to determine their exact location at anywhere in the world.

## **3. Astronomy**

Astronomy is the study of celestial objects such as stars, planets, galaxies, natural satellites, comets, etc. The satellites that are used to study or observe the distant stars, galaxies, planets, etc. are called astronomical satellites. They are mainly used to find the new stars, planets, and galaxies. Hubble space telescope is an example of astronomical satellite. It captures the high-resolution images of the distant stars, galaxies, planets etc.

## **4. Satellite phone**

Satellite phone is a type of mobile phone that uses satellites instead of cell towers for transmitting the signal or information over long distances.

Mobile phones that use cell towers will work only within the coverage area of a cell tower. If we go beyond the coverage area of a cell tower or if we reach the remote areas, it becomes difficult to make a voice call or send text messages with the mobile phones. Unlike the mobile mobiles, satellite phones have global coverage. Satellites phones uses geostationary satellites and low earth orbit (LEO) satellites for transmitting the information.

When a person makes a call from the satellite phone, the signal is sent to the satellite. The satellite will receives that signal, processes it, and redirects the signal back to the earth via a gateway. The gateway then send the signal or call to the destination by using the regular cellular and landline networks. The usage of satellite phones is illegal in some countries like Cuba, North Korea, Burma, India, and Russia.

## 5. Satellite television

Satellite television or satellite TV is a wireless system that uses communication satellites to deliver the television programs or television signals to the users or viewers.

TV or television mostly uses geostationary satellites because they look stationary from the earth. Hence, the signal is easily transmitted. When the television signal is sent to the satellite, it receives the signal, amplifies it, and retransmits it back to the earth. The first satellite television signal was sent from Europe to North America by using the Telstar satellite.

## 6. Military satellite

Military satellite is an artificial satellite used by the army for various purposes such as spying on enemy countries, military communication, and navigation.

Military satellites obtain the secret information from the enemy countries. These satellites also detect the missiles launched by the other countries in the space.

Military satellites are used by armed forces to communicate with each other. These satellites also used to determine the exact location of an object.

## 7. Satellite internet

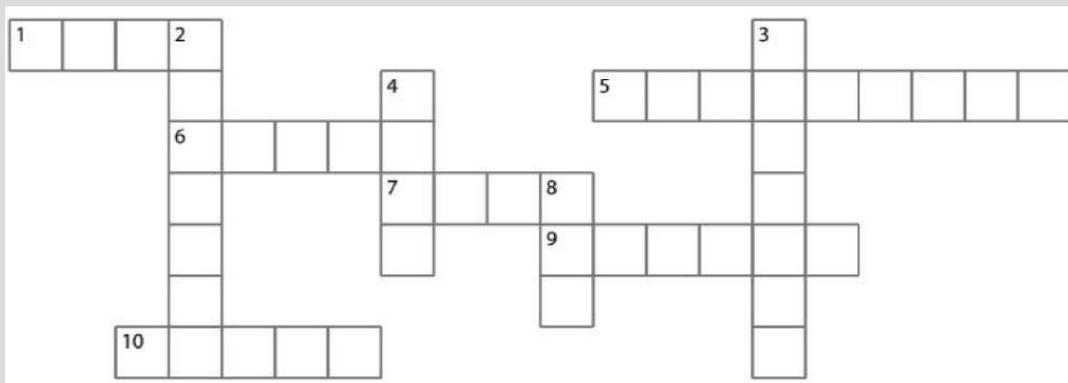
Satellite internet is a wireless system that uses satellites to deliver the internet signals to users. High-speed internet is the main advantage of satellite internet. Satellite internet does not use cable systems, but instead it uses satellites to transmit the information or signal.

## 8. Satellite radio

Satellite radio is a wireless transmission service that uses orbiting satellites to deliver the information or radio signals to the consumers. It is primarily used in the cars. When the ground station transmits signal to the satellite that is revolving around the earth, the satellite receives the signal, amplifies it, and redirects the signal back to the earth (radio receivers in the cars).

### APPLICATION ACTIVITY 6.3

1) Using the across and Down clues, write the correct words in the numbered grid below.



### ACROSS

1. The only natural satellite of Earth.
5. An object in orbit around a planet.
6. The smallest planet and farthest from the Sun.
7. This planet probably got this name due to its red color and is sometimes referred to as the Red Planet.
9. This planet's blue color is the result of absorption of red light by methane in the upper atmosphere.
10. It is the brightest object in the sky except for the Sun and the Moon.

### DOWN

2. Named after the Roman god of the sea.
  3. The closest planet to the Sun and the eighth smallest.
  4. A large cloud of dust and gas which escapes from the nucleus of an active comet.
  8. The largest object in the solar system.
- 2) (i) Define astronomical satellite  
(ii) What does astronomical satellite used for? Give one example of it.
- 3) For a satellite to be in a circular orbit 780 km above the surface of the earth, (a) what orbital speed must it be given, and (b) what is the period of the orbit (in hours)?

## SKILLS LAB 6

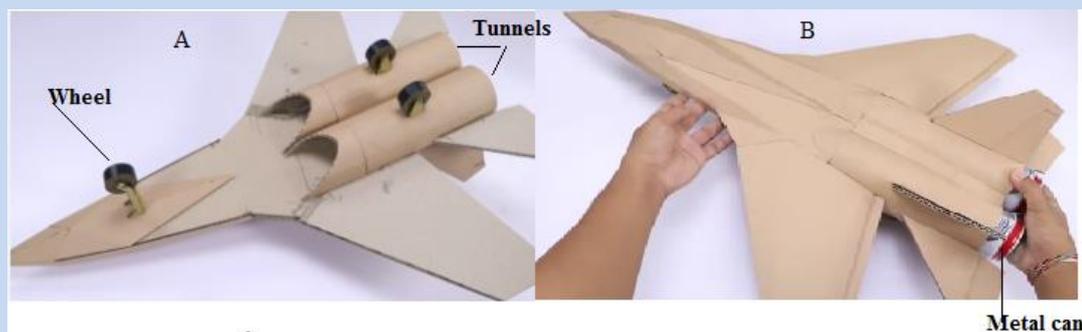
In this activity you will design a match powered cardboard jet.

### Materials required.

Match boxes; Card board/box type; A pair of scissors and a razor blade; 2 empty light metallic cans ; Glue; 3 Plastic bottle tops for wheels; and Smooth surface ( a smooth table)

### Procedures.

- a) Cut different shapes of card board so that if joined they can make a jet as shown in the figure below. You can make any shape of your choice.



**Fig.6. 14 Pictures of a match powered cardboard jet.**

- b) Join them to make the desired shape using glue. Make sure that your shape has 2 tunnels which will act as propellers.
- c) Insert the two cans inside the back tunnels. Make sure that the can is open one end and closed the other end.
- d) Fix the 3 wheels made of plastic bottle tops.
- e) Place the whole setup on a smooth surface as if it is an airplane ready to fly.
- f) Put the matches inside the metal cans and light them. Make sure you put a lot of matches to make the set up move.

### REPORT

Make a comprehensive report and in your report include the following.

- i) Why your set up was or wasn't able to move.
- ii) Recommendations to have a better functioning jet.

### CAUTIONS

- i) Make sure that at a time if lighting your matches, The setup is outside (in an open environment without dry matter)
- ii) The whole of your set up may burn. Make sure that you remove all the burnt materials and put them in the dust bin

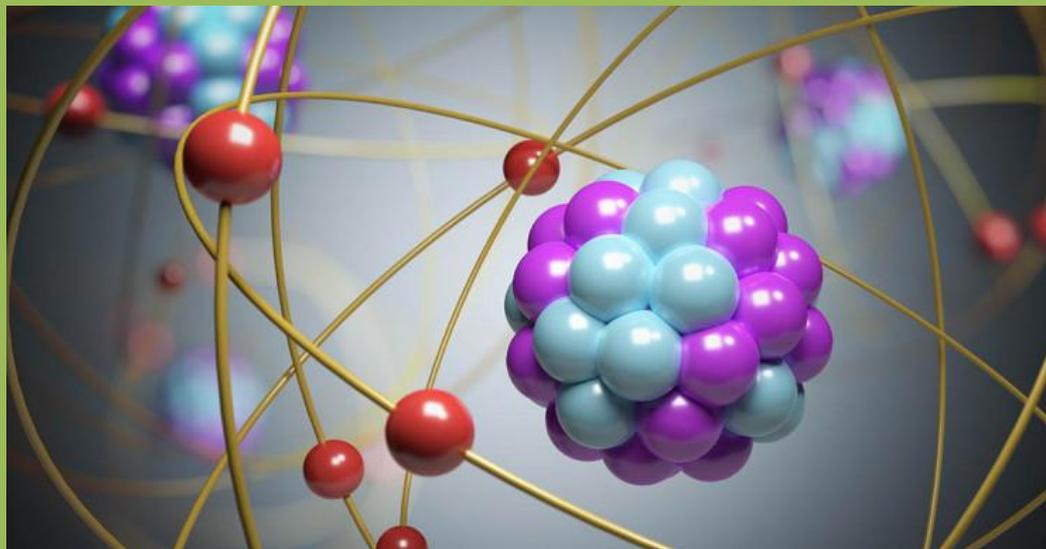
## END UNIT ASSESSMENT 6

1. If the gravitational force on an object is directly proportional to its mass, why don't objects with large masses fall with greater acceleration than small ones?
2. What is the gravitational attraction between an object with a mass of 10 kg and another object with a mass of 20 kg if they are separated by 0.01 m? (Assume both objects are on some surface and not suspended in midair.)
3. What is the gravitational attraction between an object with a mass of 10 kg and the earth if the object is on the Earth's surface? (Assume the 10 kg object is a very tiny but dense ball. In other words, assume it is a point.)
4. A 50kg person and a 75kg person are sitting on a bench 0.50 m apart. Estimate the magnitude of the gravitational force each exerts on the other
5. During a solar eclipse, the Moon, Earth, and Sun all lie on the same line, with the Moon between the Earth and the Sun (The Sun-Earth distance is  $1.496 \times 10^{11}$  and the Earth-Moon distance is  $3.84 \times 10^8$  m, the mass of the Sun, Earth, and Moon are  $M_s = 1.99 \times 10^{30}$  kg  $M_e = 5.98 \times 10^{24}$  kg and  $M_m = 7.36 \times 10^{22}$  kg
  - (a) What force is exerted by the Sun on the Moon?
  - (b) What force is exerted by the Earth on the Moon?
  - (c) What force is exerted by the Sun on the Earth?
6. Compute the mass of the earth assuming it to be a sphere of radius 6370km
7. A mass  $m_1 = 1$ kg weighs one-sixth as much on the surface of the moon as on the earth. Calculate the mass  $m_2$  of the moon. The radius of the moon's  $1.738 \times 10^6$  m
8. The magnitude of the total gravitational field strength at a point in interstellar space is  $g = 5.42 \times 10^{-9}$  N / kg what is the magnitude of the gravitational force at this point on an object (a) of mass 1.00 kg and (b) of mass  $8.91 \times 10^5$  kg ?

## UNIT 7 ATOMIC MODELS AND PHOTOELECTRIC EFFECT

**Key unit competence:** Interpret the atomic model and photoelectric effect and solve related problems

### INTRODUCTORY ACTIVITY



1. Basing on the figure above,

- How is the structure/arrangement of balls shown in the figure related to an atom? You can use chemistry knowledge from O'level.
- Relate the arrangement of electrons in an atom to how the balls in the figure above are arranged.
- Explain how movement of particles in an atom leads to release or absorption of energy

2. It is important to realise that a lot of what we know about the structure of atoms has been developed over a long period of time. This is often how scientific knowledge develops, with one person building on the ideas of someone else. In attempt to explain an atom, different scientists suggest different models. An *atomic model* represents what the structure of an atom *could* look like, based on what we know about how atoms behave. It is not necessarily a true picture of the exact structure of an atom.

- Why did these scientists use the word Model not exact structure of an atom?
- Can you explain some of the scientific models that tried to explain the structure of an Atom?

## 7.1 Bohr model of the atom and energy levels

### ACTIVITY 7.1

In year 1, you discussed about Rutherford model and this was a great step in understanding atomic structure of an atom but it still had some limitations that are listed below.

- Why doesn't the electron fall into the nucleus since it revolves around the nucleus
- Rutherford's model could not explain the observed line spectra of elements. As electrons spiraled towards the nucleus with increasing speed, they should emit all frequencies of radiation not just one. Thus, the observed spectrum of the element should be a continuous spectrum not a line spectrum.
- The model did not explain the distribution of electrons outside the nucleus.

- a) Basing on Rutherford's limitations above, suggest some of corrections that would be made
- b) Talk about the energy possessed by these electrons as they are in energy levels.
- c) Does an electron remain with same energy if it
  - i) Jumps
  - ii) Drops from one energy level to another?

### 7.1.1 Bohr's atomic model

In 1900 **Max Planck (1858–1947)** investigated the relationship between the intensity and frequency of the radiation emitted by very hot objects. **Planck** showed that the radiation from a hot body was emitted **only in discrete quantities** or “**packets**” called **quanta**. The energy,  $E$ , of each quantum was shown to be proportional to the frequency,  $f$ , of the radiation emitted:

$$E = hf$$

where  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  Planck's constant

Planck's assumption suggests that the energy of any molecular vibration could be only a whole number multiple of  $hf$ :

$$E = nhf$$

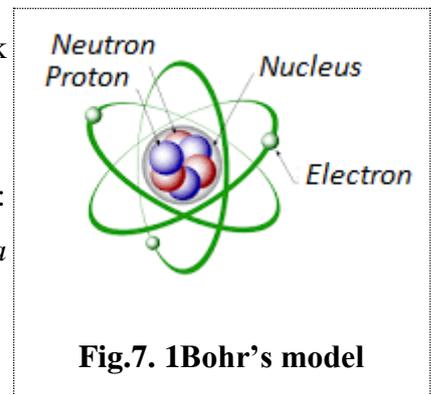
Where  $n$  is called the **principal quantum number** for the level of energy  $E_n$

“Quantum” means “discrete amount” as opposed to “continuous”. This idea is often called **Planck’s quantum hypothesis**. i.e. atoms could only absorb or emit energy in discrete quanta.

Albert Einstein use of Planck’s quantization idea to successfully explain the photoelectric effect added great support to this belief. So, Bohr was convinced that a successful atomic model had to incorporate this energy quantization phenomenon.

Bohr’s thinking on a new atomic model was also guided by the work that had been done on the **spectrum of hydrogen**.

In 1913 he develops a theory of the atom in which he assumes that: “Electrons are arranged in definite shells, or quantum levels, at a considerable distance from the nucleus”.



Bohr postulated that:

-Electrons were present in orbits outside the nucleus and execute circular motion around the nucleus under the influence of the Coulomb attraction between the electron and nucleus and in accordance with the laws of classical physics.

-The electron can occupy only certain **allowed orbits** or **stationary states** for which the orbital angular momentum,  $L$ , of the electron is an integral multiple of Planck’s constant divided by  $2\pi$ .

$$L_n = mv_n r_n = \frac{nh}{2\pi} \text{ that is } L_n = mv_n r_n = n\hbar$$

$$\text{where } \hbar = \frac{h}{2\pi} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s}$$

-An electron in such a stationary state **does not radiate electromagnetic energy**.

-Energy is emitted or absorbed by an atom when an electron moves from one stationary state to another. The difference in energy between the initial and final states is equal to the energy of the emitted or absorbed photon and is quantized according to the Planck relationship:

$$\Delta E = E_f - E_i = hf$$

### Example 7.1

Calculate the energy in joules and electron volts for the following photon wavelengths:

$\lambda_x = 1 \times 10^{-10} \text{ m}$  (X-ray),  $\lambda_{uv} = 200 \times 10^{-9} \text{ m}$  (body), and  $\lambda_{yel} = 500 \times 10^{-9} \text{ m}$  (Yellow light near the peak of the sun's spectrum),  $\lambda_{IR} = 10 \times 10^{-6} \text{ m}$  (Infra red radiated by your body), and

$$\lambda_{AM} = 200 \times 10^{-6} \text{ m}$$

#### Solution:

From Photon energy:  $E = hf = \frac{hc}{\lambda}$ , and using  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

We get  $E_x = 2.0 \times 10^{-15} \text{ J} = 12.4 \text{ keV}$ ,  $E_{uv} = 9.9 \times 10^{-19} \text{ J} = 6.2 \text{ eV}$ ,

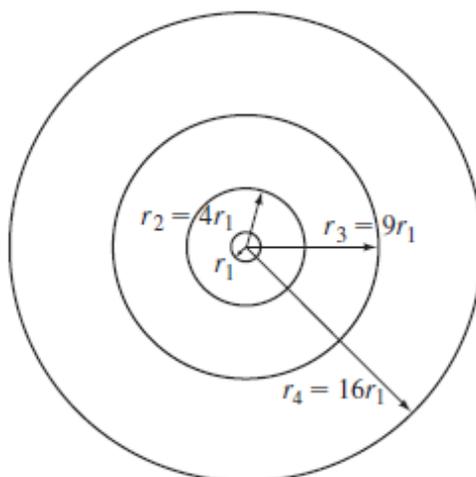
$E_{yel} = 4.0 \times 10^{-19} \text{ J} = 2.5 \text{ eV}$  and  $E_{AM} = 9.9 \times 10^{-28} \text{ J} = 6.2 \text{ eV}$

### 7.1.2 Orbital radii, orbital speed and Energy level

Starting with these four postulates and using a mixture of Classical and Quantum Physics, Bohr derived equations for:

- **The radii of the various stationary states and the velocity of an electron in a particular stationary state;**

Let's assume the electron's orbit is a circular orbit with radius  $r$  which is approximately the size of the hydrogen atom. Since the electron in Fig.7.2 is moving in a circle, there must be a force directed toward the center of the circle.



**Fig.7. 2 The four smallest orbits in the Bohr model of hydrogen**

Since the proton is much heavier than the electron, assume the proton mass is infinite and that the proton remains fixed while the electron circles it in a circular orbit then

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

We solve this for  $r_n = \frac{ke^2}{mv^2}$  and then substitute for  $v$  from  $v = \frac{nh}{2\pi mr}$  and we solve for  $r_n$

$$r_n = \frac{n^2 h^2}{4\pi^2 kme^2}$$

This Equation gives the radii of all possible orbits.

The smallest orbit for hydrogen has the value  $r_0 = r_1$  and is sometimes called the **Bohr radius**

$$\text{Where: } r_0 = \frac{h^2}{4\pi^2 kme^2} = \frac{(6.63 \times 10^{-34})^2}{4\pi^2 (9.0 \times 10^9)(9.1 \times 10^{-31})(1.6 \times 10^{-19})^2} = 5.29 \times 10^{-11} \text{ m}$$

$$r_n = n^2 r_0 \text{ where } n = 1, 2, 3, 4, \dots$$

Thus the larger the value of  $n$ , the further the electron from the nucleus. The energy of the electron depends on the orbit it occupies.

The **orbital speed**:  $v_n = \frac{2\pi ke^2}{hn} = \frac{e^2}{2\varepsilon_0 nh}$

This Equation shows that the orbital speed is inversely proportional to n. Hence the greater the value of n, the larger the orbital radius of the electron and the slower its orbital speed.

The speed in the orbit n = 1, which is the **greatest possible speed** of the electron in the hydrogen atom is

$$v_o = \frac{e^2}{2\varepsilon_0 h} = 2.19 \times 10^6 \text{ m/s}$$

This orbiting electron has quite a high speed. Therefore the orbital speed:  $v_n = \frac{v_o}{n}$

### Example 7.2

What is the radius of the orbit for which n = 3

**Solution:**

The radius of the orbits:  $r_n = n^2 r_o = 3^2 \times 0.53 \times 10^{-10} = 4.77 \times 10^{-10} \text{ m}$

### The energy of an electron in a particular stationary state

Kinetic energy:  $K_n = \frac{1}{2} m v_n^2 = \frac{m e^4}{8 \varepsilon_0^2 h^2 n^2}$

Potential energy:  $U_n = -\frac{e^2}{4\pi\varepsilon_0 r_n} = -\frac{m e^4}{4\pi\varepsilon_0^2 h^2 n}$

The potential energy has a negative sign because we have taken the electric potential energy to be zero when the electron is infinitely far from the nucleus.

The energy  $E = K + U$  therefore  $K_n = \frac{1}{2} m v_n^2 - \frac{ke^2}{r} = \frac{ke^2}{2r} - \frac{ke^2}{r} = -\frac{ke^2}{2r}$

Combine this with  $r_n = n^2 r_0$  then  $E_n = \frac{ke^2}{2r_0} \left(\frac{1}{n^2}\right) = -\frac{E_0}{n^2}$

$$\text{Where } E_0 = \frac{ke^2}{2r_0} = \frac{(9.0 \times 10^9)(1.6 \times 10^{-19})^2}{2(5.30 \times 10^{-10})} = 2.18 \times 10^{-18} \text{ J}$$

$$\text{Or } E_0 = \frac{2.18 \times 10^{-18} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} = 13.6 \text{ eV}$$

The lowest **energy level** or **energy state** has energy  $E_1 = E_0 = -13.6 \text{ eV}$  and is called the **ground state**. The **higher states**,  $E_2 = -13.40 \text{ eV}$ ,  $E_3 = -1.51 \text{ eV}$  and so on, are called **excited states**. The fixed energy levels are also called **stationary state**

Notice that although the energy for the larger orbits has a smaller numerical value, all the energies are less than zero. Thus,  $E_2 = -13.40 \text{ eV}$  is a higher energy than  $E_1 = E_0 = -13.6 \text{ eV}$ .

Hence the orbit closest to the nucleus has the lowest energy (the most negative).

The energy of the atom is least when  $n=1$  and has its most negative value. This is the *ground level* of the hydrogen atom; it is the level with the smallest orbit, of radius  $r_0$ .

For  $n = 2, 3, \dots$  the absolute value of  $E_n$  is smaller and the energy is progressively larger (less negative).

### Example 7.3 Wavelength of an emission line

Find the kinetic, potential, and total energies of the hydrogen atom in the first excited level, and find the wavelength of the photon emitted in a transition from that level to the ground level.

**Solution:**

$$K_2 = \frac{13.60}{2^2} \text{ eV} = 3.40 \text{ eV}$$

$$U_2 = \frac{-13.60 \times 2}{2^2} \text{ eV} = -6.80 \text{ eV}$$

$$E_2 = \frac{-13.60}{2^2} \text{ eV} = -3.40 \text{ eV}$$

The energy of the emitted photo:  $\Delta E = E_2 - E_1 = -3.40 - (-13.60) = 10.20 \text{ eV}$

The wavelength:  $\lambda = \frac{hc}{\Delta E} = 1.22 \times 10^{-7} \text{ m} = 122 \text{ nm}$

### The energy difference between any two stationary states and The ionization energy of hydrogen;

The atom cannot have any energy values lying between these quantum states. If the electron receives enough energy to remove it from the attraction of the nucleus completely, the atom is ionized. The energy difference  $E_\infty - E_1$  is called the **ionization energy (binding energy)**.

That is the minimum energy required to remove an electron from the ground state of an atom. For hydrogen the ionization energy has measured to be 13.6 eV, and corresponds precisely to removing an electron from the lowest state,

$$E_1 = -13.6 \text{ eV up to } E_\infty = 0 \text{ where it will be free.}$$

The frequency of a spectral line is given by:  $\Delta E = \left| -13.6 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right| = hf$

#### Example 7. 4

A hypothetical atom (Fig. 7.3a) has energy levels at 0.00 eV (the ground level), 1.00 eV, and 3.00 eV.

- (a) What are the frequencies and wavelengths of the spectral lines this atom can emit when excited?
- (b) What wavelengths can this atom absorb if it is in its ground level?

#### Solution:

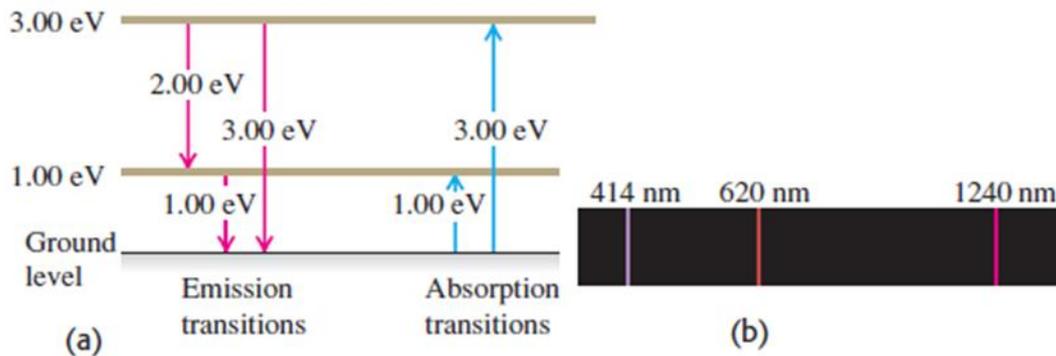
Energy is conserved when a photon is emitted or absorbed. In each transition the photon energy is equal to the difference between the energies of the levels involved in the transition.

- (a) The possible energies of emitted photons are  $\Delta E_1 = 1.00 \text{ eV}$ ,  $\Delta E_2 = 2.00 \text{ eV}$  and  $\Delta E_3 = 3.00 \text{ eV}$

For  $\Delta E_1 = 1.00 \text{ eV}$  gives the frequency:  $f = \frac{\Delta E}{h} = \frac{(1.00 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = 2.42 \times 10^{14} \text{ Hz}$

$$\text{And } \lambda_1 = \frac{c}{f_1} = \frac{3.00 \times 10^8 \text{ m/s}}{2.42 \times 10^{14} \text{ Hz}} = 124 \text{ nm}$$

This is in the infrared region of the spectrum (Fig.7.3b)



**Fig.7. 3: Energy-level diagram for the hypothetical atom**

For  $\Delta E_2 = 2.00 \text{ eV}$  gives  $f_2 = 4.82 \times 10^{14} \text{ Hz}$  and  $\lambda_2 = 620 \text{ nm}$  (red)

For  $\Delta E_3 = 3.00 \text{ eV}$  gives  $f_3 = 7.25 \times 10^{14} \text{ Hz}$  and  $\lambda_3 = 414 \text{ nm}$  (violet)

(b) From the ground level, only a 1.00 eV or a 3.00 eV photon can be absorbed (Fig. 7.3b); a 2.00 eV photon cannot be absorbed because the atom has no energy level 2.00 eV above the ground level. Passing light from a hot solid through a gas of these hypothetical atoms (almost all of which would be in the ground state if the gas were cool) would yield a continuous spectrum with dark absorption lines at 1240 nm and 414 nm.

- The Rydberg constant; and the Rydberg equation for the wavelengths of hydrogen emission spectral lines.

A photo is emitted when an electron makes a transition from a higher level to a lower level. When an electron jumps from a lower level to a higher level, a photon is absorber.

According to Niels Bohr, for an electron to move from an orbit of energy  $E_i$  to one of energy  $E_f$ , the light absorbed must have a frequency given by Planck's equation

$$\Delta E = hf = \frac{hc}{\lambda}$$

Substituting the expression for the energy of the electron in Planck's equation, we have for hydrogen atom:

$$\frac{1}{\lambda} = \frac{\Delta E}{hc} = -\frac{E_o}{hc} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = -R_h \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where

- $n_i$  and  $n_f$  represent the quantum numbers for the initial and final states, respectively
- $R_h$  is the Rydberg constant

$$R_h = \frac{E_o}{hc} = \frac{(13.6 \text{ eV})(1.6 \times 10^{-19} \text{ J / eV})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m / s})} = 1.097 \times 10^7 \text{ m}^{-1}$$

The negative sign in the equation above denotes stability relative to some reference state. In other words, the more negative the value for energy, the more stable the system is. This is known as **Moseley's equation**.

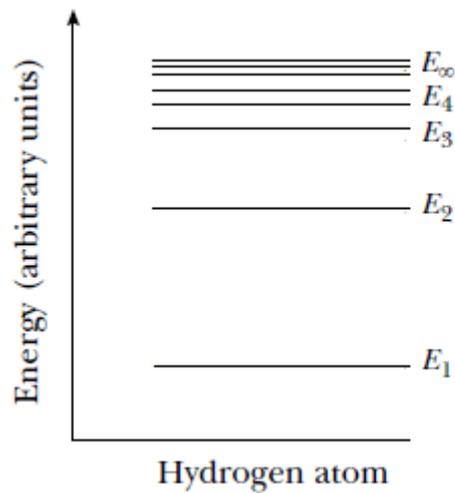
Complete removal of the electron from a hydrogen atom, corresponding to a transition from  $n = 1$  (ground state) to the  $n = \infty$   $n = \infty$  state, is known as **ionization**. This is represented as



The energy required for ionization from the ground state is called **ionization energy**.

Bohr assigned **quantum numbers** to the orbits. He gave the orbit of lowest energy (nearest to the nucleus) the quantum number 1 with  $n = 1$  is known as the **ground state** and electron is in its ground state. When the electron is in a higher energy orbit, that is,  $n = 2$  or higher, the atom is said to be in an electronically **excited state**.

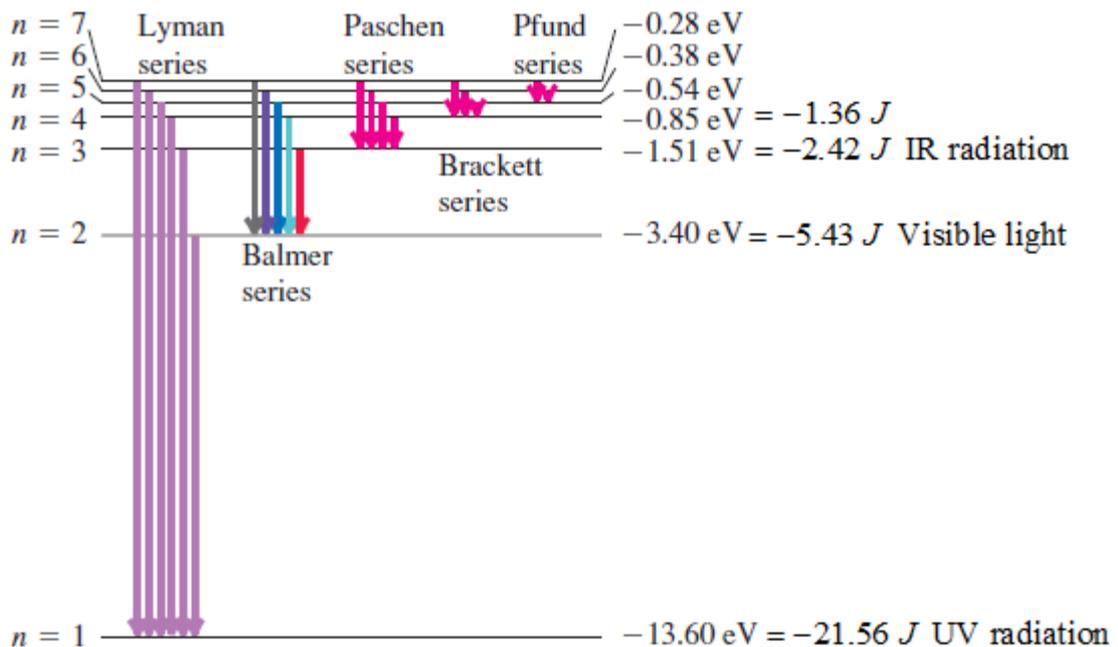
The highest energy on the scale shown in Fig.7.4,  $E_\infty = 0$ , corresponds to the energy of the atom when the electron is completely removed from the proton. The atom can occupy only certain energy levels, called **quantum states**, as shown in Fig. 7.4.



**Fig.7. 4: Energy level diagram**

Note that the energy levels get closer together at the high end of the scale.

Fig.7.5 depicts the orbits and energy levels and also shows some of the possible transitions from one electron to an orbit of lower energy.



**Fig.7. 5: Energy transition in hydrogen atom**

When light coming from a discharge tube containing hydrogen gas is passed through a prism, a series of lines is observed in the visible part of the spectrum: this is termed Balmer series. Some are

found in the IR (Infra-Red) and UV (ultraviolet) regions. Those lines detected in the UV are known as Lyman series and those detected in the IR were discovered by Paschen, Brackett, and Pfund.

From 1884 to 1886, **Johann Balmer**, a Swiss school teacher, suggested a mathematical formula to fit the known wavelengths of the hydrogen emission spectrum:

$$\lambda = b \left( \frac{m^2}{m^2 - 2^2} \right)$$

where

- m is an integer with a different value for each line (m = 3, 4, 5, 6)
- b is a constant with a value of 364.56 nm.

This formula produces wavelength values for the hydrogen emission spectral lines in excellent agreement with measured values. This series of lines has become known as the **Balmer series**.

Balmer predicted that there should be other series of hydrogen spectral lines and that their wavelengths could be found by substituting values higher than the 2 shown on the right hand side of the denominator in his formula.

In 1890, **Johannes Rydberg** produced a generalized form of Balmer's formula for all wavelengths emitted from excited hydrogen gas:

$$\frac{1}{\lambda} = \frac{\Delta E}{hc} = -\frac{E_o}{hc} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = -R_h \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where

- R = Rydberg's constant =  $1.097 \times 10^7 \text{ m}^{-1}$ ,
- $n_f$  = an integer specific to a spectral series (eg for the Balmer series  $n_i = 2$ )
- $n_f = 2, 3, 4, \dots$

The **Lyman series** in the emission spectrum arise when the electron moves to the n = 1 orbit (the ground state) from any of the other orbits of hydrogen atom:

$$\frac{1}{\lambda} = -R_h \left( \frac{1}{n_f^2} - \frac{1}{1^2} \right) \text{ where } n_f = 2, 3, 4, \dots$$

The **Balmer series** arises from transitions to the  $n = 2$  orbit,  $n = 3$ ,  $n = 4$ , etc. orbits.

$$\frac{1}{\lambda} = -R_h \left( \frac{1}{n_f^2} - \frac{1}{2^2} \right) \text{ where } n_f = 3, 4, 5, \dots$$

The **Paschen, Brackett** and **Pfund** series arise from transition to the  $n = 3$ ,  $n = 4$ , and  $n = 5$  orbits from higher orbits.

$$\text{Paschen series: } \frac{1}{\lambda} = -R_h \left( \frac{1}{n_f^2} - \frac{1}{3^2} \right) \text{ where } n_f = 4, 5, 6, \dots$$

$$\text{Brackett series } \frac{1}{\lambda} = -R_h \left( \frac{1}{n_f^2} - \frac{1}{4^2} \right) \text{ where } n_f = 5, 6, 7, \dots$$

$$\text{Pfund series } \frac{1}{\lambda} = -R_h \left( \frac{1}{n_f^2} - \frac{1}{5^2} \right) \text{ where } n_f = 6, 7, 8, \dots \text{ } n_f = 6, 7, 8, \dots$$

In order to explain this, Bohr made the following assumptions

- i) The atom consists of stationary orbital. In this orbital, electrons do not emit radiations.
- ii) An electron may suddenly jump from one of its specified non- radiating orbits to higher energy level. Radiation is emitted when an electron jumps from a higher energy level to a lower energy level so that

$$\Delta E = E_n - E_{n-1} = -E_0 \left( \frac{1}{n^2} - \frac{1}{n_{n-1}^2} \right) = hf = \frac{hc}{\lambda}$$

Where  $E_n$  and  $E_{n-1}$  are two successive energy levels

### Example 7.5

Calculate the energy required for ionization of an electron from the ground state of the hydrogen atom.

#### Solution:

The ionization energy may be written as the difference between the final and initial state energies.

We have  $n_f = \infty$ ,  $n_i = 1$  therefore  $\Delta E = E_f - E_i = -E_0 \left( \frac{1}{\infty^2} - \frac{1}{1^2} \right) = E_0 = 2.18 \times 10^{-18} \text{ J} = 13.6 \text{ eV}$

It is often useful to express this energy on a molar basis by multiplying by Avogadro's number:

$$\Delta E = (2.18 \times 10^{-18} \text{ J / atom})(6.02 \times 10^{23} \text{ atoms / mol}) \left( \frac{1 \text{ kJ}}{1000 \text{ J}} \right) = 1.31 \times 10^3 \text{ kJ / mol}$$

The **great success of Bohr's model** is that:

- It gives an explanation for why atoms emit line spectra, and accurately predicts the wavelengths of emitted light for hydrogen.
- It explains absorption spectra: photons of just the right wavelength can knock an electron from one energy level to a higher one. To conserve energy, only photons that have just the right energy will be absorbed.

This explains why a continuous spectrum of light entering a gas will emerge with dark (absorption) lines at frequencies that correspond to emission lines.

- It ensures the stability of atoms. It establishes stability by decree: the ground state is the lowest state for an electron and there is no lower energy level to which it can go and emit more energy.
- It accurately predicts the ionization energy of 13.6 eV for hydrogen. However, the Bohr model was not so successful for other atoms

#### Limitations of the Bohr model:

As with any scientific model, however, there were limitations. The problems with the Bohr model can be summarized as follows:

- Bohr used a mixture of classical and quantum physics, mainly the former. He assumed that some laws of classical physics worked while others did not.

- The model could not explain the relative intensities of spectral lines. Some lines were more intense than others.
- It could not explain the hyperfine structure of spectral lines. Some spectral lines actually consist of a series of very fine, closely spaced lines.
- It could not satisfactorily be extended to atoms with more than one electron in their valence shell because it does not account for the electrostatic force that one electron exerts on another.
- It could not explain the “Zeeman splitting” of spectral lines under the influence of a magnetic field. The Zeeman Effect is the splitting of atomic energy levels and the associated spectral lines when the atoms are placed in a magnetic field.
- It could not explain the Stark effect (splitting up in electric field)

### APPLICATION ACTIVITY 7.1

1. State the four postulates used by Bohr to explain the nature of the atom.
2. The electron in the hydrogen atom makes a transition from the  $n = 2$  energy state to the ground state (corresponding to  $n = 1$ ). Find the wavelength and frequency of the emitted photon. What is the wavelength of the photon emitted by hydrogen when the electron makes a transition from the  $n = 3$  to  $n = 1$  state?
3. Determine the de Broglie wavelength for the following:
  - (a) a moving golf ball ( $m = 0.05 \text{ kg}$ ,  $v = 40 \text{ m/s}$ ),
  - (b) an orbiting electron in the ground state of hydrogen ( $13.6 \text{ eV}$ ),
  - (c) an electron accelerated through 100 kV in an electron microscope.
4. If the momentum of an electron were doubled, how would its wavelength change?
 

A. No change.	D. It would be quadrupled.
B. It would be halved.	E. It would be reduced to one-fourth.
C. It would double.	
5. An electromagnetic radiation was emitted in the Balmer’s series as a result of electron transitions between  $n = 2$  and  $n = 5$ . Calculate the
  - i) Energy of the radiation in  $\text{kJ} \cdot \text{mol}^{-1}$

- ii) Frequency of the radiation in hertz
- iii) Wavelength of the radiation in metres.

## 7.2 Photoelectric effect

### ACTIVITY 7.2

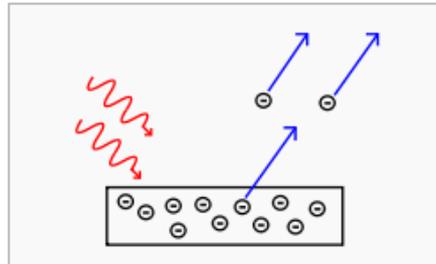


The picture above is a section of solar panels that were installed in Rwamagana district to supplement power in the region.

- a) From your experience what do you know about solar panels.
- b) Using the knowledge of black bodies, explain how a solar panel operates.
- c) Explain why the same project may not work well in areas like Musanze and Gicumbi.
- d) Basing on your answers provided in the above questions, does light carry energy?

### 7.2.1 Photoelectric Effect

**Photoelectric effect** is the emission of electrons from the surface of metal when illuminated with electromagnetic radiation of sufficient frequency. A material that exhibits photoelectric effect is said to be **Photosensitive**.



**Fig.7. 6: Incoming photons on the left strike a metal plate (bottom), and eject electrons, each with energy  $E = hf$  depicted as flying off to the right.**

### 7.2.2 Einstein's equation of photoelectric emission

Since electrons are held in the metal by attractive forces, some minimum energy is required just to get an electron out through the surface. The minimum energy required to release the photoelectron from the metal surface is called the **work function**,  $\phi$ , of the metal. An electron that has received this minimum energy has no kinetic energy once outside the metal. For photoelectric emission to occur, the energy of the photon must be equal to or greater than the work function. If the photon's energy,  $E$ , is *just* enough to release a photoelectron, then its frequency is called the **threshold frequency**,  $f_0$ :

$$E = hf_0 = \phi$$

Where  $h$  is the Planck constant  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

If the energy of the photon is greater than the work function, then the photoelectron can acquire some kinetic energy. By energy conservation:

*Photon energy = Work done in releasing the electron + Kinetic energy of electron.*

This can be written by **Einstein's equation** of photoelectric emission as

$$E = \phi + K_{\max}$$

Where

- $K_{\max} = \frac{1}{2}mv_{\max}^2$  is the maximum kinetic energy of the photoelectron
- $E = hf$  Photon energy or energy of the absorbed photons
- $\phi = hf_0$  Work function or Work done in releasing the electron

When the photon energy just equals the work function,  $\phi = hf_0$  and therefore  $\frac{1}{2}mv_{\max}^2 = 0$

In such a case, no photoelectric effect is observed. This happens when the frequency is at threshold level,  $f_0$  and therefore  $\phi = hf_0$

Work function of several metals are shown in Table 7.1

Meta l	Work function (J)	Work function (eV)	Meta l	Work function (J)	Work function (eV)
Na (Sodium)	$3.8 \times 10^{-19}$	2.4	Fe (iron)	$6.9 \times 10^{-19}$	4.3
Al	$3.6 \times 10^{-19}$	4.3	Ag (Silver)	$6.9 \times 10^{-19}$	4.3
Ca (Calcium)	$4.5 \times 10^{-19}$	2.8	Pb	$1.0 \times 10^{-19}$	6.4
Cu (copper)	$7.0 \times 10^{-19}$	4.4	Lead	$6.4 \times 10^{-19}$	4.0

**Table 7.1 Work function of several metals**

**Example 7.7**

Antimony-caesium has a threshold wavelength of 700 nm. What is its work function in joules?

**Solution:**

Using the equations  $\phi = hf_0$  and  $c = \lambda_0 f_0$  and substituting in the resulting equation for  $\phi$  gives:

$$\phi = \frac{hc}{\lambda_0} = \frac{(6.63 \times 10^{-34} \text{ J.s})(3 \times 10^8 \text{ m/s})}{700 \times 10^{-9} \text{ m}} = 2.84 \times 10^{-19} \text{ J}$$

If the reversed voltage is increased, a point is reached where the current reaches zero—no electrons have sufficient kinetic energy to reach C. This is called the *stopping potential, or stopping voltage*,

and from its measurement, can be determined using conservation of energy (loss of in potential energy = gain in kinetic energy):

$$K_{\max} = eV_{co}$$

The minimum potential (or maximum negative potential) for which photoelectric effect is observed is **Cut-off potential**. It is given the symbol  $V_{co}$ .

At the cut-off potential, the p.d. across the cell is reversed to stop the ejected photoelectrons since energy is given by:

$$E = eV = eV_{co}$$

Where  $e$  is the charge on an electron and  $V$  is the cut-off potential, this is the energy required to stop the fastest photoelectrons.

A graph of  $V_o$  as a function of  $f$  turns out to be a straight line, verifying **Einstein's equation for photoelectric effect** Equation,

$$V_o = \frac{hf}{e} - \frac{\phi}{e}$$

### Example 7.8

What is the shortest-wavelength X-ray photon emitted in an X-ray tube subjected to 50 kV?

#### Solution:

The electrons striking the target will have a KE of 50 keV. The shortest-wavelength photons are due to collisions in which all of the electron's KE is given to the photon so

$$K = \phi = \frac{hc}{\lambda} \Leftrightarrow \lambda_o = \frac{hc}{eV} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^8 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})(5.0 \times 10^4 \text{ V})} = 2.5 \times 10^{-11} \text{ m} = 0.025 \text{ nm}$$

### 7.2.3 Factors affecting photoelectric emission

Assuming monochromatic light, the two important properties of a light wave are:

- Its **intensity** of light is the rate of energy flow per unit area when the radiation is directed normally to the cross section area:

$$I = \frac{W}{tA} = \frac{P}{A}$$

- Its frequency (or wavelength)

When these two quantities are varied, the **experimental** results of **photon theory** makes the following predictions:

### 1. Light intensity

If the light intensity is increased (means more photons are incident), the number of electrons ejected increases and their maximum kinetic energy of the ejected electrons remains the same, provided the light frequency remains the same.

In a monochromatic beam, all photons have the same energy  $E = hf$ . Increasing the intensity of the light beam means increasing the number of photons in the beam, but does not affect the energy of each photon as long as the frequency is not changed.

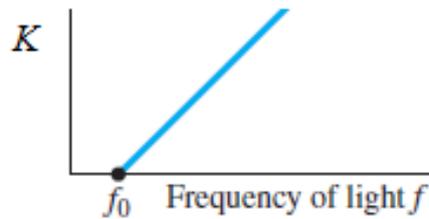
### 2. Characteristic of the material being illuminated

The minimum frequency (or maximum wavelength) for which photoelectric effect is observed is called **Threshold frequency**. It is also called **cut-off frequency** and given the symbol  $f_{co}$  or  $f_o$ . Threshold frequency varies for different materials.

Within the region of effective frequencies, that is higher than the threshold frequency, the maximum kinetic energy ( $K_{max}$ ) of the photoelectrons is directly proportional to the frequency of the incident radiation according to Einstein's equation for photoelectric effect

$$K = hf - \phi \text{ where } \phi = hf_o$$

This relationship is plotted in Fig.7.7



**Fig.7. 7 The maximum kinetic energy of ejected electrons increases linearly with the frequency of incident light.**

If the frequency  $f$  is less than the “cut-off” frequency  $f_0$  where  $f_0 h = \phi$  no electrons will be ejected, no matter how great the intensity of the light

That is characteristic of the material being illuminated. For a given material, monochromatic light with a frequency below a minimum **threshold frequency** produces *no* photocurrent, regardless of intensity. For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths between 200 nm and 300 nm), but for other materials like potassium oxide and caesium oxide it is in the visible spectrum (between 380 nm and 750 nm).

### 3. Time of released

Electrons are emitted from the surface of the metal instantaneously even at low light intensities.

The incident light energy arrives at the surface in small packets and there is a one-to-one interaction between photons and photoelectrons. In this interaction, the photon’s energy is imparted to an electron that then has enough energy to leave the metal. This is in contrast to the wave theory in which the incident energy is distributed over a large area of the surface metal.

### 4. Stopping voltage

The stopping potential does not depend on intensity, but does depend only on frequency.

The only effect of increasing the intensity is to increase the number of electrons per second and hence the photocurrent  $i$ . If the intensity of light is held constant but the frequency is increased, the stopping potential also increases.

In other words, Greater intensity at a particular frequency means a greater number of photons per second absorbed, and thus a greater number of electrons emitted per second and a greater

photocurrent. The greater the light frequency is, the higher the energy of the ejected photoelectrons is.

### Example 7.9: Photoelectron speed and energy

(a) What is the kinetic energy and the speed of an electron ejected from a sodium surface whose work function is  $\phi = 2.28 \text{ eV}$  when illuminated by light of wavelength

- i. 410 nm,
- ii. 550 nm

(b) Determine the lowest frequency or the longest wavelength needed to emit electrons from sodium.

#### Solution:

(a) We first find the energy of the photons  $E = hf = \frac{hc}{\lambda}$

If the energy is greater than  $\phi$  then electrons will be ejected with varying amounts of  $K$  with a maximum of  $K = hf - \phi$

$$(i) \text{ For } \lambda = 410 \text{ nm then } E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^8 \text{ m/s})}{410 \times 10^{-9} \text{ m}} = 4.85 \times 10^{-19} \text{ J} = 3.03 \text{ eV}$$

The maximum kinetic energy an electron can have is given by

$$K = 3.03 - 2.28 = 0.75 \text{ eV} = (0.75 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = 1.2 \times 10^{-19} \text{ J}$$

$$\text{Since } K = \frac{1}{2}mv^2 \Leftrightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.2 \times 10^{-19} \text{ J})}{9.1 \times 10^{-31} \text{ kg}}} = 5.1 \times 10^5 \text{ m/s}$$

Most ejected electrons will have less  $K$  and less speed than these maximum values.

$$(ii) \text{ For } \lambda = 550 \text{ nm then } E = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3 \times 10^8 \text{ m/s})}{550 \times 10^{-9} \text{ m}} = 3.61 \times 10^{-19} \text{ J} = 2.26 \text{ eV}$$

Since this photon energy is less than the work function, no electrons are ejected.

$$(b) \phi_0 = hf_0 \rightarrow f_0 = \frac{\phi_0}{h} = \frac{2.28 \times 1.6 \times 10^{-19} J}{6.63 \times 10^{-34} J \cdot s} = 5.502 \times 10^{14} Hz$$

### 7.2.4 Applications of photoelectric effect

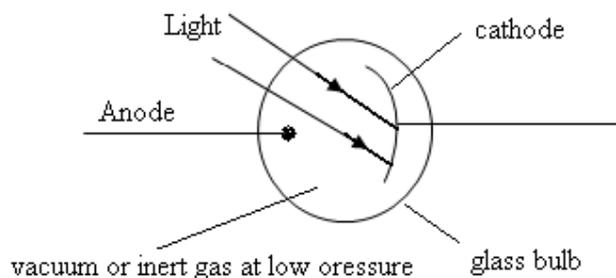
The photoelectric effect has a number of applications. Digital cameras, studying nuclear processes, chemically analyzing materials based on their emitted electrons, image intensifiers and night-vision scopes use it to convert light energy into an electric signal that is reconstructed into an image.

On the moon, sunlight striking the surface causes surface dust to eject electrons, leaving the dust particles with a positive charge. The mutual electric repulsion of these charged dust particles causes them to rise above the moon's surface, a phenomenon that was observed from lunar orbit by the Apollo astronauts.

It led physicists to think about the nature of light and the structure of atoms in an entirely new way

#### (a) Photo emissive cells

These are used in reproduction of sound in a film sound track and also in controlling lift doors. Photo emissive cells are also used in security alarms. The symbol for a photo emissive cell is shown below. Light falling on the cathode ejects electrons which are attracted to the anode and a current flow.



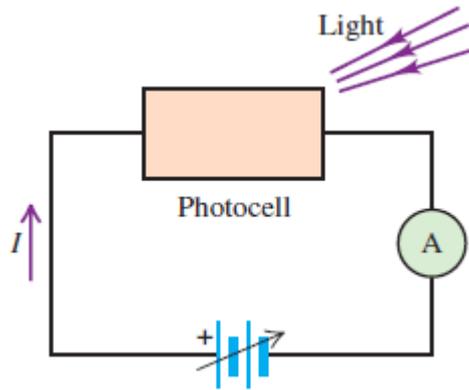
**Fig.7. 8 Photo emissive cells**

#### (b) Photovoltaic cells

In photovoltaic cells, the ejected electron travels through the emitting material to enter a solid electrode in contact with the photo emitter (instead of travelling through a vacuum to the anode) leading to the direct conversion of radiant energy to electrical energy. The more intense the light

falling on the photocell, the greater the conductivity of the photocell and the greater the current measured by the ammeter (A).

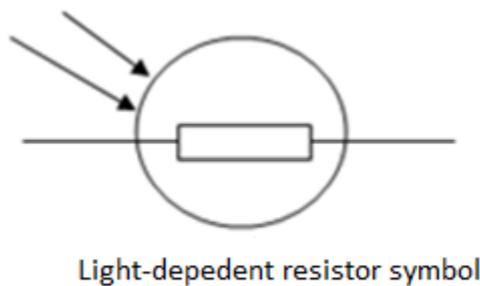
Photovoltaic cells are used in calculators and light exposure metres in cameras. They can also drive small machines.



**Fig.7. 9 A semiconductor photocell in a circuit**

**(c) Photoconductive cells.**

Examples of photoconductive cells are photodiodes, photo resistors (light-dependent resistors, LDR) and phototransistors. These work on the principle that light reduces the resistance of some semiconductor materials such as calcium sulphide.



**Fig.7. 10 Photoconductive cells**

## APPLICATION ACTIVITY 7.2

1. Sodium has a work function of 2.3 eV. Calculate:

- Its threshold frequency,
- The maximum velocity of the photoelectrons produced when the sodium is illuminated by light of wavelength  $5 \times 10^{-7}$  m,
- The stopping potential with light of this wavelength.
- The longest-wavelength light that can cause photoelectron emission from sodium?

( $h = 6.63 \times 10^{-34}$  J.s,  $c = 3 \times 10^8$  m/s,  $1 \text{ eV} = 1.6 \times 10^{-19}$  J, mass of electron  $m = 9.1 \times 10^{-31}$  kg.)

2. Estimate how many visible light photons a 100 W light bulb emits per second. Assume the bulb has a typical efficiency of about 3% (that is, 97% of the energy goes to heat).

3. Compute the energy of a photon of blue light of wavelength 450 nm.

4. As red light shines on a piece of metal, no electrons are released. When the red light is slowly changed to shorter wavelength light (basically progressing through the rainbow), nothing happens until yellow light shines on the metal, at which point electrons are released from the metal. If this metal is replaced with a metal having a higher work function, which light would have the best chance of releasing electrons from the metal?

- |          |  |
|----------|--|
| A. Blue. | C. Yellow would still work fine.                   |
| B. Red.  | D. We need to know more about the metals involved. |

5. A beam of red light and a beam of blue light have equal intensities. Which statement is true?

- There are more photons in the blue beam.
- There are more photons in the red beam.
- Both beams contain the same number of photons.
- The number of photons is not related to intensity.

## 7.3 Thermo electronic emission

### ACTIVITY 7.3

Good conductors of heat and electricity contain electrons that are evenly distributed within the metal and even on its surface. These particles gain energy and start to move within the metal when they acquire energy from the surroundings.

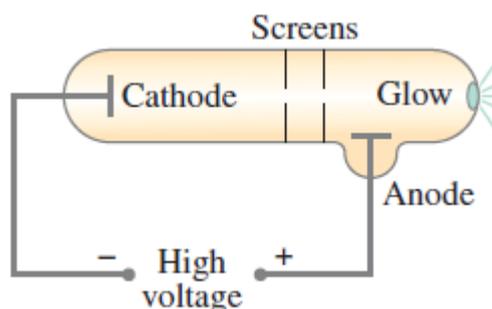
- a) Basing on the statement, discuss what happens when a surface of good conductor is connected to electricity.
- b) Basing on your conclusion in (a) above what do you think are characteristics of the emitted particles?
- c) What would happen to the process if energy source is disconnected from the metal surface

### 7.3.1 Production of Cathode rays

Thermoelectric emission is the process by which electrons are emitted from a metal surface when it has been electrically heated. The least energy an electron requires to break away from the surface is called the work function and this value varies from one metal to another. Substance with low work functions emits electrons at lower temperatures compared with metals with higher work function.

Cathode rays are stream of electrons that are moving at high speed. Cathode rays are produced in a discharge tube which is a long (about 30 cm or more) hard glass tube with two electrodes attached at its two ends. The electrodes are made of any metal which is a good conductor such as copper, aluminium or platinum, and are connected externally to a high voltage source. The discharge tube has a facility to connect it to a vacuum pump. Gaseous discharge takes place between the two electrodes and hence the name of the tube.

If the temperature of the metal is raised, the thermal velocities of the electrons will be increased. The chance of electrons escaping from the attraction of the positive ions, fixed in the lattice, will then also be raised. Thus by heating a metal such as tungsten to a high temperature, electrons can be boiled off. This called thermionic emission.



**Fig.7. 11 Discharge tube.**

The electrode connected to the negative terminal is known as the cathode and the terminal connected to the positive terminal is known as the anode. As the discharge tube is evacuated, various changes take place.

- The air inside the discharge tube is a non-conductor of electricity and therefore initially tube looks intact.
- As the air pressure inside reduces, the gas starts ionizing. Since a potential difference is maintained inside the tube, when one gas atom is ionized, the electrons escaping from it ionize other gas atoms. This creates a stream of positive ions and negative electrons. These start moving towards the cathode and the anode respectively and generate a current.
- When the pressure is not very low, the gas movement looks like bluish streaks. As the pressure reduces further, the gas inside looks pink.
- When the discharge tube is evacuated to a high degree, the inside will start looking black, as there is no gas inside to conduct a current. This dark space is called Faraday's dark space. A small glow can be observed at the cathode and the anode. This is due to residual gases.
- As the vacuum is reduced further, there will be a greenish glow behind the anode. The rays or particles come from the cathode towards the anode. Some of them overshoot the anode and reach the inner surface of the tube. This causes the glow. These rays are called cathode rays. Since the cathode rays come towards the anode, they must be negatively charged.

It has been proved that the cathode rays are nothing but electrons. As the discharge tube is evacuated, the electrons at the cathode get attracted to the anode due to the high potential difference. Cathode rays are not seen when the potential difference is low or if the gas pressure is high.

### **7.3.2 Properties of cathode rays.**

Cathode rays are moving electrons and have the following properties:

- They travel in straight lines and They carry negative charge.
- They are deflected by electric and magnetic fields.
- Cathode rays cause fluorescence on striking certain materials.
- They have energy and momentum.
- Cathode rays are capable of ionizing gas atoms if the potential difference is large and the gas pressure is not high.
- Depending on their energy, cathode rays can penetrate thin sheets of paper or metal foils.

- When cathode rays are stopped suddenly, they produce X-rays.
- They affect photographic plates.

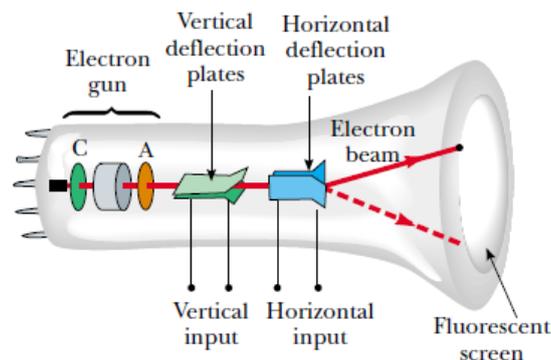
### 7.3.3 Applications of cathode rays

#### a) Cathode ray oscilloscope

A Cathode Ray Oscilloscope (CRO) also called Oscillograph is an instrument generally used in a laboratory to display, measure and analyze various waveforms of electrical circuits. A cathode ray oscilloscope is a very fast X-Y plotters that can display an input signal versus time or other signal. Cathode ray oscilloscopes use luminous spots which are produced by striking the beam of electrons and this luminous spot moves in response variation in the input quantity.

Nowadays, with the help of transducers it is possible to convert various physical quantities like current, pressure, acceleration etc to voltage thus it enable us to have a visual representations of these various quantities on cathode ray oscilloscope.

The main part of cathode ray oscilloscope is cathode ray tube (CRT) which is also known as the heart of cathode ray oscilloscope. The CRT is a vacuum tube in which a beam of electrons is accelerated and deflected under the influence of electric or magnetic fields. The electron beam is produced by an assembly called an *electron gun* located in the neck of the tube. These electrons, if left undisturbed, travel in a straight-line path until they strike the front of the CRT, the “screen,” which is coated with a material that emits visible light when bombarded with electrons. Electrons leaving the hot cathode C are accelerated to the anode A. The beam of electrons produced is called Cathode rays. In addition to accelerating electrons, the electron gun is also used to focus the beam of electrons, and the plates deflect the beam.



**Fig.7. 11 Parts of a CRO**

This tube is commonly used to obtain a visual display of electronic information in oscilloscopes, radar systems, television receivers, and computer monitors.

### **Functions of a Cathode Ray Oscilloscope**

A cathode ray oscillograph is essentially an electrostatic instrument which consists of a high evacuated glass tube. The features of a CRO (Cathode ray oscilloscope) can be split into 3 main sections: The **electron gun**, the **deflection system** and the **fluorescent screen**.

- **Electron Gun:** The role of this section is to produce electrons at a high, fixed, velocity and focus them on the screen. This is done through a process known as thermionic emission. A filament in the cathode is heated to the point where its electrons become loose.

An anode with a high voltage applied to it accelerates the electrons towards the screen due to electrostatic attraction. On the way, the electrons pass through a series of control grids which control the brightness of the image produced. The more negative the grid, the darker the image and vice versa.

- **Deflection system:** The role of the deflection system is to control the image produced by controlling the position that the electrons hit the screen. It consists of Two perpendicular sets of Electric/Magnetic fields. This allows control over both horizontal and vertical axes. By controlling the Voltage applied to the fields, it is possible to vary the deflection through Electrostatic force/Motor effect.

- **Fluorescent screen:** The role of this part is to display where the electrons are hitting the CRT. It is a screen coated with a material that emits light when struck by electrons. Zinc sulfide or Phosphorus are two commonly used materials. The CRO is a perfect voltmeter as its input resistance is very high. It is usually placed in parallel with a component.

The voltage is measured on the vertical axis, which is controlled by the Y-plates. It can also be used as an ammeter by placing it across a resistor of known resistance. The CRO is used to analyze waveforms. It can be used to determine the peak voltage of an a.c. waveform and the period, which in turn allows one to work out its frequency.

### **b) Televisions**

A CRT TV works by having the electron beam "**scan**" the screen at a rate faster than our eyes can perceive. This means that it shoots across the screen like a machine gun, and the images we see are actually made from many fluorescent dots.

The fluorescence caused by the beam striking the screen lasts a bit longer so that the next scan can be made without the previous image disappearing. It scans twice each time, first filling in the odd "holes" then the even ones. Each scan is about 1/50 of a second.

Colour CRT TVs has electron guns rather than a single one, a shadow mask, and a modified fluorescent screen. The 3 electron guns are needed as there are three primary colours (Red, Green and Blue) that can be adjusted in different amounts to create any colour.

The colours are formed as a result of the shadow mask, which is a layer with holes in it that controls the angle of the incoming electron beams. This is because the fluorescent screen is separated into multi-coloured phosphors that are placed adjacent to each other at small intervals. Thus it isn't actually a single coloured pixel, but rather 3 very small pixels that join together to form a larger dot.

The vertical sensitivity defines the voltage associated with each vertical division of the display or the amplitude of the displayed signal. Virtually all oscilloscope screens are cut into a crosshatch pattern of lines separated by 1 cm in the vertical and horizontal directions.

This section carries a Volts-per-Division (Volts/Div) selector knob, an AC/DC/Ground selector switch and the vertical (primary) input for the instrument. Additionally, this section is typically equipped with the vertical beam position knob.

### 7.3.4 Fluorescence and Phosphorescence

When an atom is excited from one energy state to a higher one by the absorption of a photon, it may return to the lower level in a series of two (or more) transitions if there is at least one energy level in between. The photons emitted will consequently have lower energy and frequency than the absorbed photon. When the absorbed photon is in the UV and the emitted photons are in the visible region of the spectrum, this phenomenon is called **fluorescence**.

The wavelength for which fluorescence will occur depends on the energy levels of the particular atoms. Because the frequencies are different for different substances, and because many substances fluoresce readily, fluorescence is a powerful tool for identification of compounds. It is also used for determining how much of a substance is present and for following substances along a natural *metabolic pathway* in biological organisms.

For detection of a given compound, the stimulating light must be monochromatic, and solvents or other materials present must not fluoresce in the same region of the spectrum.

Sometimes the observation of fluorescent light being emitted is sufficient to detect a compound. In other cases, spectrometers are used to measure the wavelengths and intensities of the emitted light.

**Fluorescent light bulbs** work in a two-step process. The applied voltage accelerates electrons that strike atoms of the gas in the tube and cause them to be excited. When the excited atoms jump down to their normal levels, they emit UV photons which strike a fluorescent coating on the inside of the tube. The light we see is a result of this material fluorescing in response to the UV light striking it.

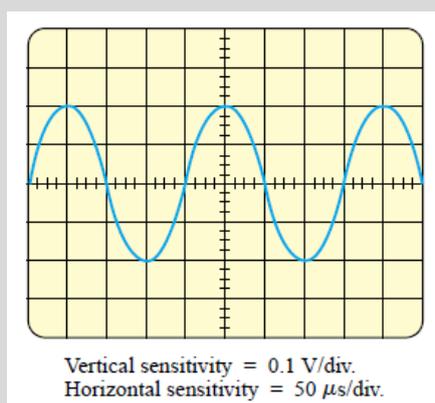
Materials such as those used for luminous watch dials, and other glow-in-the-dark products, are said to be **phosphorescent**. When an atom is raised to a normal excited state, it drops back down within about .

In phosphorescent substances, atoms can be excited by photon absorption to energy levels called **metastable**, which are states that last much longer because to jump down is a “forbidden” transition. Metastable states can last even a few seconds or longer.

In a collection of such atoms, many of the atoms will descend to the lower state fairly soon, but many will remain in the excited state for over an hour. Hence light will be emitted even after long periods.

### APPLICATION ACTIVITY 7.3

1. For the pattern of Fig. below and the indicated sensitivities, determine the period, frequency, and peak value of the waveform.



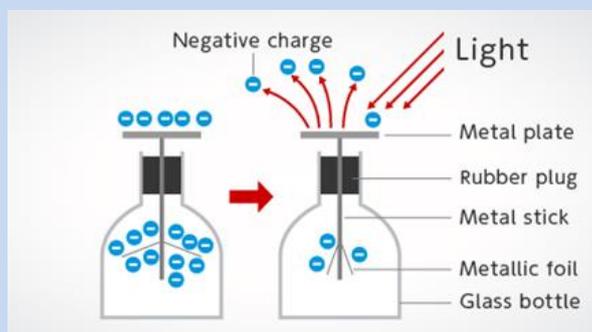
**Fig.7. 12 Oscilloscope**

2. Describe how a simple C.R.O. is adjusted to give
  - a. a spot trace,
  - b. a continuous horizontal trace on the screen, explaining the functions of the various controls

## SKILLS LAB 7

### Demonstrating the Photo-electric Effect

Using a zinc plate with a gold leaf electroscope (or a coulomb meter)...

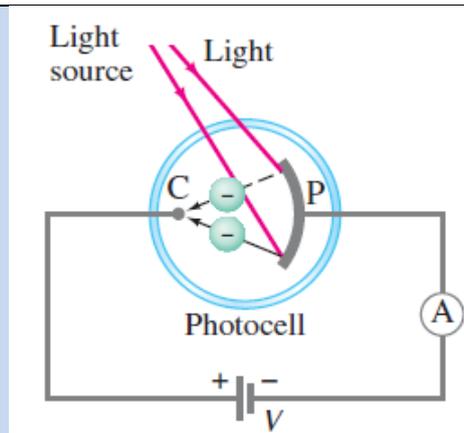


**Fig.7. 13 Zinc plate with a gold leaf electroscope**

- Clean a zinc plate with fine emery paper or steel wool.
- Attach the plate to the top disc on a gold leaf electroscope, so there is good electrical contact.
- Charge the zinc plate and inner assembly of the electroscope negatively, e.g. by rubbing the zinc plate with a polythene rod which has been rubbed with wool or fur. [Charging by induction using a perspex rod is more reliable, but might be considered too confusing!]
- The leaf should now be raised, because the leaf and the back plate are both charged negatively and repel each other. The leaf should temporarily rise further if the charged polythene rod is brought near the zinc plate.
- Place an ultraviolet lamp near the zinc plate. Switch it on. The leaf should be seen to fall.
- *Safety note:* Don't look at the ultraviolet lamp (when it's turned on!)] Clearly the plate (and inner assembly of electroscope) is losing charge.
- Repeat the procedure, but charging the zinc plate and inner assembly of the electroscope *positively*, e.g. by rubbing the plate with a charged perspex rod.
- Observe what happen

This time the ultraviolet does not affect the leaf. Charge is not lost. The simplest explanation is the correct one... The ultraviolet causes electrons to be emitted from the zinc plate. If the plate is charged positively, the electrons are attracted back again. If the plate is charged negatively the emitted electrons are repelled and lost from the plate for ever.

### Using a vacuum photocell



**Fig.7. 14 Photoelectric effect**

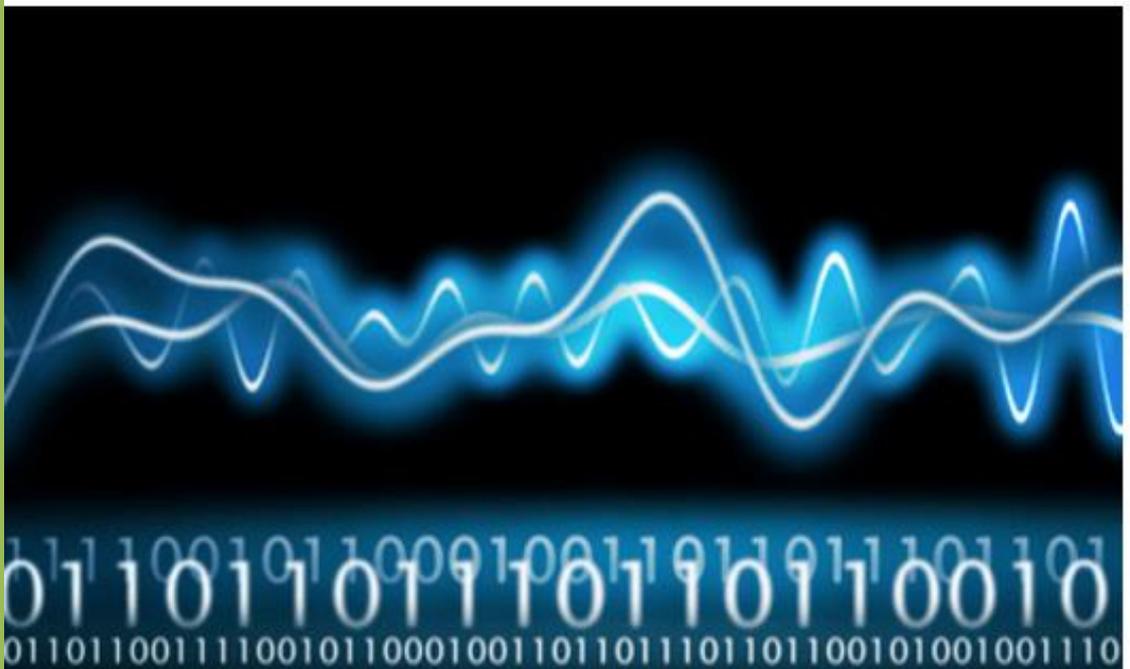
- A metal plate P made in caesium and a smaller electrode C (collecting electrode) are placed inside an evacuated glass tube, called a **photocell**. The two electrodes are connected to an ammeter and a source of emf, as shown Fig.7.14. Note the polarity of the power supply.
- Any electrons emitted from the caesium surface will be collected by the ‘collecting electrode’.
- If the photocell is covered the current is zero; if light falls on the caesium electrode there is current.



## UNIT 8: ANALOG AND DIGITAL SIGNALS IN TELECOMMUNICATION SYSTEMS

**Key unit competence:** Evaluate the application of analog and digital signals in telecommunication systems

### INTRODUCTORY ACTIVITY



There has been a move by the government of Rwanda to make her citizens to change from using analog devices to digital devices. Analog devices transmit and receive signals in analog form whereas digital devices transmit and receive signals digitally.

- What are different forms of signals you know that you normally use in daily life communication?
- Why do you think there is a need to change from analog to digital signal transmission?
- Mutesi communicates to her brother Ndayisenga who studies abroad using Facebook. Is the flow of information analog or digital? Explain your argument.
- Using information gained in above questions, discuss different signals shown in the illustration.

## 8.1 Telecommunication Terms and Concepts

### ACTIVITY 8.1

Take the case of two people talking on telephone (see Fig.8.1). Use it to answer the questions that follow.



**Fig.8. 1** *People talking on telephone*

1. Discuss what the two people are doing.
2. In telecommunication between two people, one is a sender the other is a receiver. Using the figure above, identify the sender and the receiver.
3. Do you think the message being transmitted in the figure above is good or bad? Explain why.
4. When we communicate, we offer or receive different forms of information. Discuss different forms of information you know.
5. Describe the telephone being used in the figure.

### 8.1.1 Classification of types of Information

**Information** is any entity or form that resolves uncertainty or provides the answer to a question of some kind. It is thus related to data and knowledge, as data represents values attributed to parameters, and knowledge signifies understanding of real things or abstract concepts.

Buck (1983) provides a useful classification of types of information that can be displayed to users. These are: Instructions, Command, Advisory, Answers, Historical, and Predictive.

Each of these types of information can, in theory, be provided on most types of displays. However, some lend themselves better to one form of display rather than another. The characteristics of each of these types can now be briefly discussed.

**1. Instructions:** refer to information that guides behavior in a particular way. In other words, it supports performance to carry out a task by prompting on what to do and when to do it. A simple sign telling people to enter or not enter a door would be one example. Other simple cases include the dialogue messages that are provided on automated cash machines (ACM). More complex instructions will appear in printed form on the packaging or the instructional manuals for pieces of equipment.

**2. Command :** messages give a very straightforward statement on what is or what is not permitted. 'Do not enter', 'do not smoke', 'do not eat or drink', are examples of command messages. Sometimes they are similar to instructions, but are much more focused on simple statements that refer to high priority items.

**3. Advisory:** messages are somewhat watered down versions of command messages. In some cases, these will be recommendations to avoid a situation, at other times they would be information allowing for the preparation or planning of particular activities. For example, we might be advised that our train is late by a spoken message and we might, possibly, be given an accurate time estimate for when the train will be available.

**4. Answers:** information may be provided in response to a particular enquiry that has been made. This is typical of an interactive information-handling situation, where we have a particular question in mind or degree of uncertainty and we seek information from a source with regard to removing that uncertainty.

It turns out that most of the information that is sought from displays is of the answer kind. If we want to know what the time of day is, we look at our watches and clocks to find the answer.

**5. Historical:** displays are used to look back at the state of a variable over a period of minutes, hours, days or even years. A graphical representation of road accidents over the last century would be a historical display of information. If we want to know what the temperature fluctuation has been in an office on a daily basis, then specialist devices can be brought in and placed in the office that will give a pen recording over a fixed period of time.

It is much easier to see if there is a trend in information if it is displayed in this way; the alternative is to hold in memory a general impression of what the temperature readings have been at a number

of points during the day or record them manually on a chart. Gauging the temperature in an office concerns a relatively low risk situation.

However, if the concern is with the temperature in a critical vessel in a chemical process, then the temperature trends exhibited over the time are quite important.

6. **Predictive:** displays are much more specialized, but increasingly found in complex processes. In the same way that historical data support performance in making a judgment based on the current value, predictive information enables examination of the current value and indicates any likely change in the future.

Predictor displays enable better control over vehicles, typically at sea or airborne, and enable smoother transitions from one state to another. They are used in slow response systems where it is difficult to see the immediate effect of an action that has been carried out.

Predictive displays will enable a variable to be plotted into the future. The same graphs that are used as historical displays can also be used as predictive displays.

**Telecommunication** in real life is the **transmission** of signals and other types of data of any nature by wire, radio, optical or other electromagnetic systems of communication.

Telecommunication occurs when the exchange of information between communicating participants includes the use of signs or other technologically based materials such as telephone, TV set, radio receiver, radio emitter, computer, and so on. All can be done either mechanically, electrically or electronically.

**Message:** A **message** is a term standing for information put in an appropriate form for transmission. Each message contains information. A message can be either **analog message** (a physical time variable quantity usually in smooth and continuous form) or a **digital message** (an ordered sequence of symbols selected from finite set of elements)

**-Analog message:** a physical time-variable quantity usually in smooth and continuous form.

**-Digital message:** ordered sequence of symbols selected from finite set of elements.

A **signal** is a mathematical function representing the time variation of a physical variable characterizing a physical process and which, by using various models, can be mathematically represented.

In telecommunication, the message is also known as a signal and the signal is transmitted in an electrical or voltage form.

### **8.1.2 Elements of Communication**

Communication is the process of sharing the message through continuous flow of Symbols. It is composed by the following elements:

#### **Sender**

The sender is a party that plays the specific role of initiating communication. To communicate effectively, the sender must use effective verbal as well as nonverbal techniques such as:

- Speaking or writing clearly.
- Organizing your points to make them easy to follow and understand.
- Maintaining eye contact.
- Using proper grammar.
- Giving accurate information.

All the above components are essential in the effectiveness of your message. One will lose the audience if it becomes aware of obvious oversights on ones part. The sender should have some understanding of who the receiver is, in order to modify the message to make it more relevant.

#### **Receiver**

The receiver means the party to whom the sender transmits the message. A receiver can be one person or an entire audience of people. In the basic communication model, the receiver is directly connected with the speaker. The receiver can also communicate verbally and nonverbally. The best way to receive a message is:

- To listen carefully.
- Sitting up straight.
- Making eye contact.
- Don't get distracted or try to do something else while you're listening.
- Nodding and smiling as you listen.
- Demonstrate that you understand the message.

#### **Message**

The message is the most crucial element of effective communication which includes the content a sender conveys to the receiver. A message can come in many different forms, such as an oral presentation, a written document, an advertisement or just a comment.

In the basic communication model, the way from one point to another represents the sender's message travelling to the receiver. The message isn't necessarily what the receiver perceives it to be. Rather, the message is what the sender intends the message to be. The sender must not only compose the message carefully, but also evaluate the ways in which the message can be interpreted.

## **Channel**

The channel is a medium through which a message travels from the sender to the receiver. The message travels from one point to another via a channel of communication. The channel is a physical medium stands between the sender and receiver.

Many channels or types of communication exist, such as

- The spoken word,
- Radio or television,
- An Internet site or
- Something written, like a book, letter or magazine.

Every channel of communication has its advantages and disadvantages. For example, one disadvantage of the written word, on a computer screen or in a book, is that the receiver cannot evaluate the tone of the message. For this reason, effective communicators should make written word communications clear so receivers don't rely on a specific tone of voice to convey the message accurately.

The advantages of television as a channel for communication include its expansive reach to a wide audience and the sender's ability to further manipulate the message using editing and special effects.

## **Feedback**

This describes the receiver's response or reaction to the sender's message. The receiver can transmit feedback through asking questions, making comments or just supporting the message that was delivered.

Feedback helps the sender to determine how the receiver interpreted the message and how it can be improved. The signal normally, must be raised at a level that will permit it to reach its destination. This operation is accomplished by **amplifiers**

### **8.1 3 Modes of transmission**

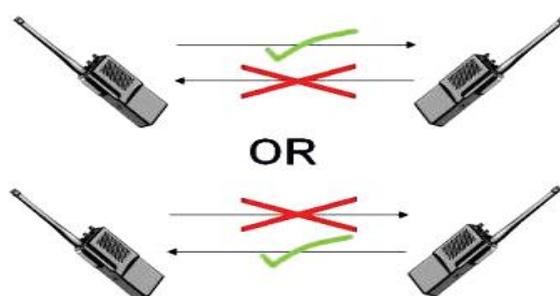
#### **1) Simplex transmission**

Simplex transmission is a single one-way base band transmission. Simplex transmission, as the name implies, is simple. It is also called unidirectional transmission because the signal travels in only one direction. An example of simplex transmission is the signal sent from the TV station to the home television.

Data in a simplex channel is always one way. Simplex channels are not often used because it is not possible to send back error or control signals to the transmit end.

## 2) **Half-duplex communications**

Half-duplex transmission is an improvement over simplex transmission because the traffic can travel in both directions. Unfortunately, the road is not wide enough to accommodate bidirectional signals simultaneously. This means that only one side can transmit at a time. Two-way radios, such as police or emergency communications mobile radios, work with half-duplex transmissions. If people at both ends try to talk at the same time, none of the transmissions get through.



**Fig.8. 2 Two way radio transmitters**

## 3) **Full-duplex communications**

Full-duplex transmission operates like a two-way, two-lane street. Traffic can travel in both directions at the same time. A land-based telephone conversation is an example of full-duplex communication. Both parties can talk at the same time, and the person talking on the other end can still be heard by the other party while they are talking. Although when both parties are talking at the same time, it might be difficult to understand what is being said.

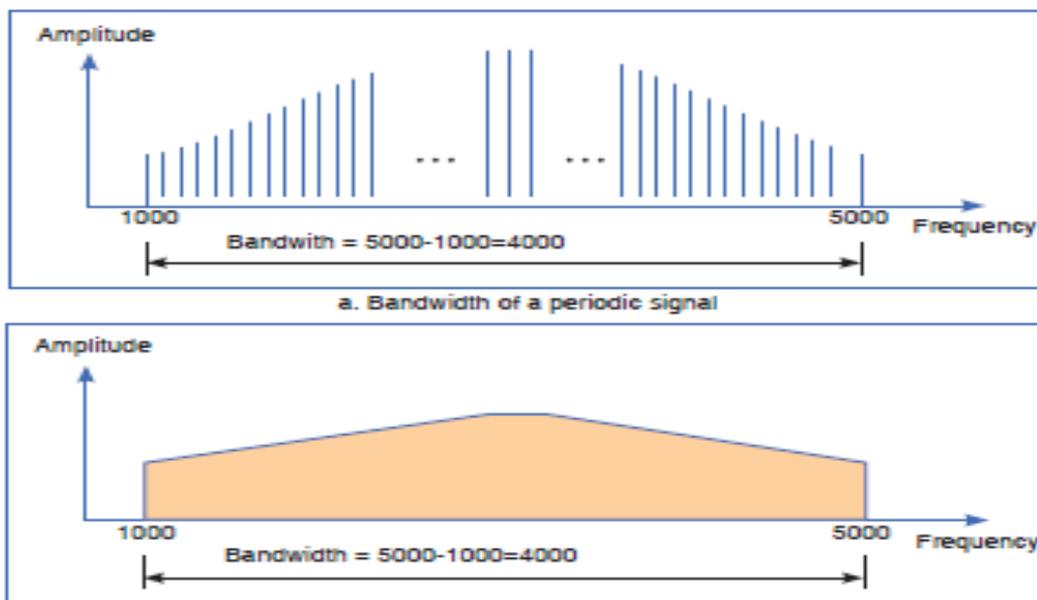
Full-duplex networking technology increases performance because data can be sent and received at the same time. Digital subscriber line (DSL), two-way cable modem, and other broadband technologies operate in full duplex mode. With DSL, for example, users can download data to their computer at the same time they are sending a voice message over the line.

### 8.1.4 Frequency and bandwidth

Frequency is a parameter that determines how often the sinusoidal signal goes through a cycle. It is usually represented with the symbol  $f$ , and it has the unit hertz.

$$f = \frac{1}{T}$$

Where  $T$  is a periodic time and is measured in seconds. The bandwidth of a composite signal is the difference between the highest and the lowest frequencies contained in that signal.



**Fig.8. 3 Graph of amplitude against frequency**

Mathematically, the bandwidth is given by;

$$BW = \Delta f = f_{usb} - f_{lsb}$$

Where  $f_{usb}$  and  $f_{lsb}$  stand for upper side band and lower side band respectively.

#### **APPLICATION ACTIVITY 8.1**

1. A cordless telephone using separate frequencies for transmission in base and portable units is known as

- A. Duplex arrangement
- B. Half duplex arrangement
- C. Either (a) or (b)

D. Neither (a) nor (b)

2. A telephone channel requires a band width of about

A. 1 kHz

B. 3 kHz

C. 10 kHz

D. 50 kHz

3. Full-duplex operation permits transmission in both directions at the same time

A. True

B. False

4. Give the difference between simplex, half-duplex, and full-duplex

5. What is the difference between coherence bandwidth and coherence time?

## 8.2 Analog and digital signal systems

### ACTIVITY 8.2

Investigating the function of wireless microphone

#### Materials:

-Wireless microphone set

-Connecting wires

-Amplifier and mixer

-Speaker

#### Procedure:

Connect the full sound system such that the signal will be transmitted to the speakers using wireless microphone.

#### Questions:

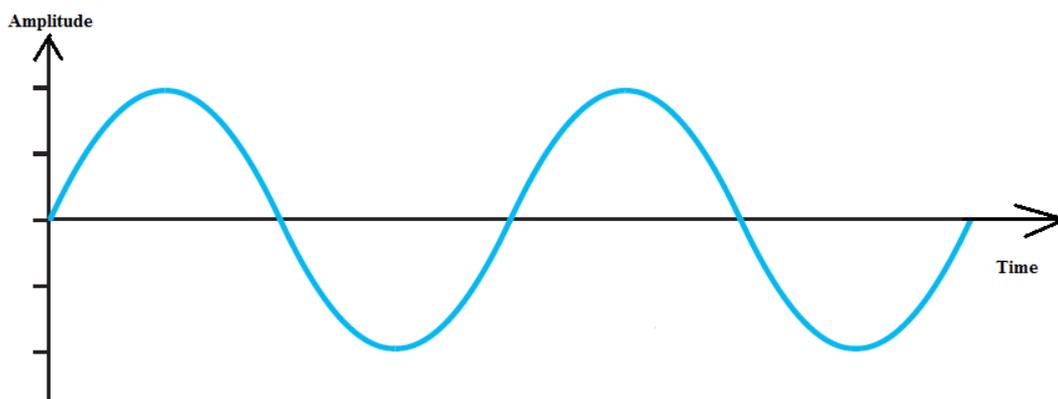
1. While connecting a sound system, you interact with your classmate. How is your voice getting to the speaker?
2. After connecting the system, you use wireless microphone to talk to your audience. How is your voice getting to the audience? Where else does this system used?
3. Using a case study of communication using telephones, discuss how the information flows from the caller to the receiver?

4. Imagine there was a need to have our mobile phones be connected to others using physical wires. Would this be bad or good? Explain your reasoning.

## 8.2.1 Analog signal system

### Analog signals

Analog signal is a continuous signal that contains time varying quantities. An analog signal is a continuous wave denoted by a sine wave and may vary in signal strength (amplitude) or frequency (time). The sine wave's amplitude value can be seen as the higher and lower points of the wave, while the frequency (time) value is measured in the sine wave's physical length from left to right.



**Fig.8. 4 Analog signal**

Analog signal can be used to measure changes in physical phenomenon such as light, sound, pressure, or temperature. For instance, microphone can convert sound waves into analog signal. Even in digital devices, there is typically some analog component that is used to take in information from the external world which will then get translated into digital form –using analog to digital converter.

A system is a physical set of components that take a signal and produces a signal. In terms of engineering, the input is generally some electrical signal and the output is another electrical signal.

Analog systems operate with values that vary continuously and have no abrupt transitions between levels. For a long time, almost all electronic systems were analog, as most things we measure in nature are analog. For example, your voice is analogous; it contains an infinite number of levels and

frequencies. Therefore, if you wanted a circuit to amplify your voice, an analog circuit seems a likely choice.

## **Example of analog electronic systems**

### **A public address system**

A **public address system (PAS)** is an electronic sound amplification and distribution system with a microphone, amplifier and loudspeakers, used to allow a person to address a large public, for example for announcements of movements at large and noisy air and rail terminals or a sports stadium.

### **Advantages of analog signals**

- Uses less bandwidth than digital sounds.
- More accurate representation of sound.
- It is the natural form of sound.
- Because of editing limitations, there is little someone can do to tinker with the sound, so what you are hearing is the original sound.

### **Disadvantages**

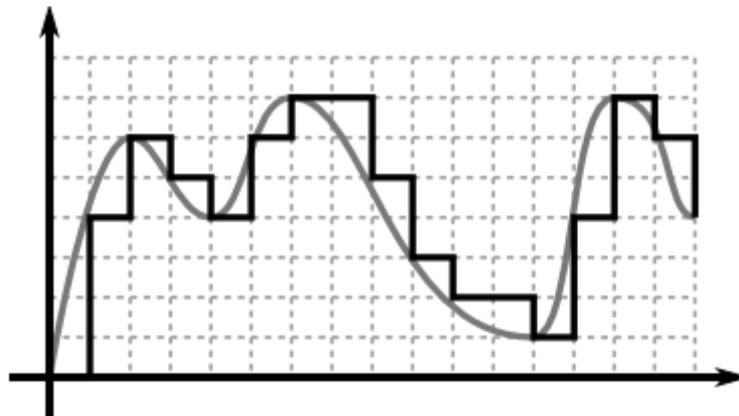
- There are limitations in editing.
- Recording analog sound on tape is expensive.
- It is harder to synchronize analogous sound.
- Quality is easily lost if the tape becomes ruined.
- A tape must always be wound and rewound in order to listen to specific part of sound which can damage it.
- Analog is susceptible to clipping where the highest and lowest notes of a sound are cut out during recording.

In Rwanda recently analog systems were replaced by digital systems that provide greater capacity of data transfer and increased reliability and security.

### 8.2.2 Digital Signal system

A digital signal refers to an electrical signal that is converted into a pattern of bits. Unlike an analog signal, which is a continuous signal that contains time-varying quantities, a digital signal has a discrete value at each sampling point.

The precision of the signal is determined by how many samples are recorded per unit of time. For example, the illustration of fig.8.5 below shows an analog pattern (represented as the curve) alongside a digital pattern (represented as the discrete lines).

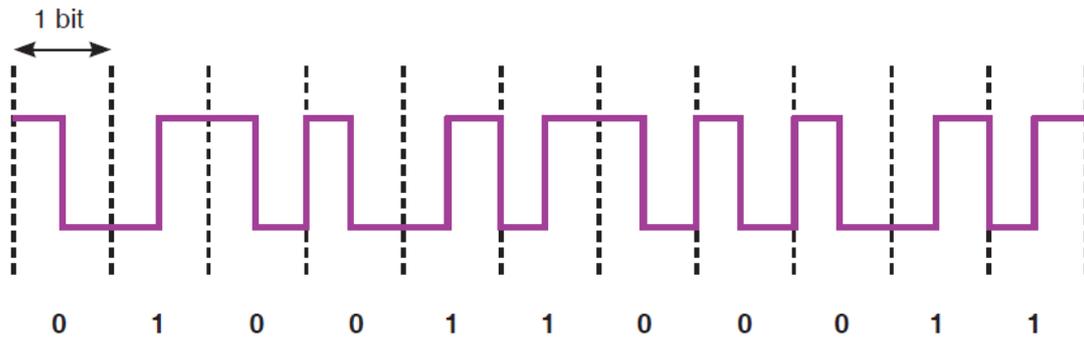


**Fig.8. 5 Analog pattern alongside digital pattern**

A digital signal is easily represented by a computer because each sample can be defined with a series of bits that are either in the state 1 (on) or 0 (off). Digital signals can be compressed and can include additional information for error correction.

A radio signal, for example, will be either on or off. Digital signals can be sent for long distances and suffer less interference than analog signals.

Unlike analog technology which uses continuous signals, digital technology encodes the information into discrete signal states. When only two states are assigned per digital signal, these signals are termed binary signals. One single binary digit is termed a bit - a contraction for binary digit.



**Fig.8. 1 Digital signal**

In electronic signal and information processing and transmission, digital technology is increasingly being used because, in various applications, digital signal transmission has many advantages over analog signal transmission.

Numerous and very successful applications of digital technology include the continuously growing number of Personal Computers, the communication network ISDN as well as the increasing use of digital control stations (Direct Digital Control: DDC).

### **Advantages of digital signals**

- **More capacity from the same number of frequencies;** that is, they provide superior Spectral Efficiency. This is a result of the modulation methods used, and the fact that, in many cases more than one ‘conversation’ can be accommodated within a single radio channel.

- **Consistent voice clarity at low received signal levels near the edge of coverage.** The general consensus is that digital radios provide better audio quality than analog ones. With analog FM radios, the audio quality steadily declines as the received signal strength gets weaker.

Digital radios however, will have a consistent audio quality throughout the full service area. The edges of the coverage area in a digital radio system are similar to those experienced with cellular telephones.

- **Data is defined in the standard.** This means data implementations are no longer proprietary, there are a wide variety of data mechanisms and inter operability can extend into the data domain. With the accepted increase of efficiency by using data communications over voice, this will further increase the usability and effectiveness of digital radio systems.

- Secure transmissions:** In digital technologies, data and voice can be secured using encryption without impacting voice quality using industry standard encryption techniques.

## Comparing digital and analog signals

	Analog	Digital
Signal	Analog signal is a continuous signal which represents physical measurements.	Digital signals are discrete time signals generated by digital modulation.
Waves	Denoted by sine waves	Denoted by square waves
Representation	Uses continuous range of values to represent information	Uses discrete or discontinuous values to represent information
Example	Human voice in air, analog electronic devices	Computers, CDs, DVDs, and other digital electronic devices.
Data transmissions	Subjected to deterioration by noise during transmission and write/read cycle.	Can be noise-immune without deterioration during transmission and write/read cycle.

### 8.2.3 Principle of digital signal systems

Digital systems process digital signals which can take only a limited number of values (discrete steps), usually just two values are used: the positive supply voltage (+Vs) and zero volts (0V).

Digital systems contain devices such as **logic gates**, flip-flops, shift registers and counters. A computer is an example of a digital system.

A logic gate is a building block of a digital circuit. Most logic gates have two inputs and one output and are based on Boolean algebra. At any given moment, every terminal is in one of the two binary conditions false (high) or true (low). False represents 0, and true represents 1. Depending on the type of logic gate being used and the combination of inputs, the binary output will differ. A logic gate can be thought of like a light switch, wherein one position the output is off—0, and in another, it is on—1. Logic gates are commonly used in integrated circuits (IC).

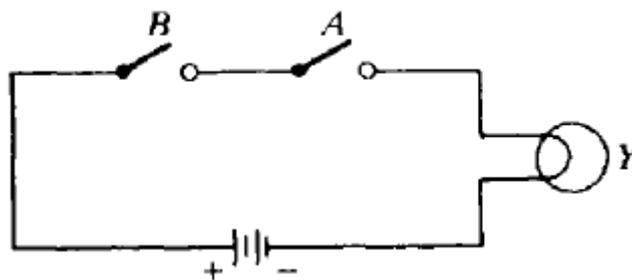
Boolean functions may be practically implemented by using electronic gates. The following points are important to understand.

- Electronic gates require a power supply.
- Gate **INPUTS** are driven by voltages having two nominal values, e.g. 0 V and 5 V representing logic 0 and logic 1 respectively.
- The **OUTPUT** of a gate provides two nominal values of voltage only, e.g. 0 V and 5 V representing logic 0 and logic 1 respectively. In general, there is only one output to a logic gate except in some special cases.
- There is always a time delay between an input being applied and the output responding.

Truth tables are used to help to show the function of a logic gate. Digital systems are said to be constructed by using logic gates. These gates are the AND, OR, NOT, NAND, NOR, EXOR and EXNOR gates. The basic operations are described below with the aid of truth tables.

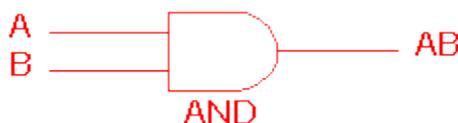
### AND gate and Truth Tables

The AND gate is called the “all or nothing” gate. The graph of fig.8.8 shows the idea of the AND gate. The lamp (  $Y$  ) will light only when both input switches (  $A$  and  $B$  ) are closed. The truth table shows that the output (  $Y$  ) is enabled (lit) only when both inputs are closed.



**Fig.8. 7 Electric circuit of AND gate**

The standard logic symbol for the AND gate is drawn in Fig.8.9. This symbol shows the inputs as  $A$  and  $B$



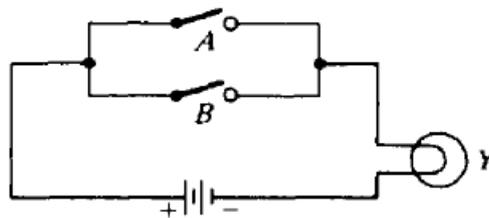
**Fig.8.8 AND gate symbol**

2 Input AND gate		
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

The AND gate is an electronic circuit that gives a **high** output (1) only if **all** its inputs are high. A dot (.) is used to show the AND operation i.e. A.B. Bear in mind that this dot is sometimes omitted we write AB.

### OR gate and truth tables

The OR gate is called the “any or all” gate. The schematic Fig.8.10 shows the idea of the OR gate. The lamp ( *Y* ) will glow when either switch *A* **or** switch *B* is closed. The lamp will also glow when both switches *A* and *B* are closed. The lamp (*Y*) will *not glow* when both switches ( *A* and *B* ) are open. The truth table details the *OR function* of the switch and lamp circuit are shown in fig. 8.10. The output of the OR circuit will be enabled (lamp lit) when any or all input switches are closed.



**Fig.8. 9 Electric circuit of OR gate**

The standard logic symbol for an OR gate is drawn in Fig.8.11. Note the different shape of the OR gate. The OR gate has two inputs labeled *A* and *B*. The output is labeled *Y*. The OR gate is an electronic circuit that gives a high output (1) if **one or more** of its inputs are high. A plus (+) is used to show the OR operation.



**Fig.8. 10 OR logic gate symbol**

2 Input OR gate		
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

### NOT gate and truth table

A NOT gate is also called an inverter. A NOT gate, or inverter, is an unusual gate. The NOT gate has only one input and one output as shown in fig.8.12. If the input variable is A, the inverted output is known as NOT A. This is also shown as A', or A with a bar over the top, as shown at the outputs.

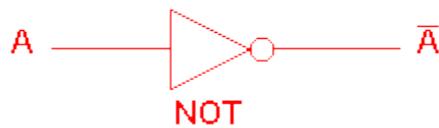


Fig.8. 11 NOT logic gate symbol

NOT gate	
A	$\bar{A}$
0	1
1	0

The diagrams below show two ways that the NAND logic gate can be configured to produce a NOT gate. It can not also be done using NOR logic gates in the same way.



Fig.8. 22 NAND logic gate



Fig.8. 13 2 input NAND logic gate

2 Input NAND gate		
A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

This is a NOT-AND gate which is equal to an AND gate followed by a NOT gate. The outputs of all NAND gates are high if any of the inputs are low. The symbol is an AND gate with a small circle on the output. The small circle represents inversion.



**Fig.8. 14 NOR gate**

2 Input NOR gate		
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

This is a NOT-OR gate which is equal to an OR gate followed by a NOT gate. The outputs of all NOR gates are low if any of the inputs are high.

The symbol is an OR gate with a small circle on the output. The small circle represents inversion.



**Fig.8. 15 EXOR gate**

2 Input EXOR gate		
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

The 'Exclusive-OR' gate is a circuit which will give a high output if either, but not both, of its two inputs are high. An encircled plus sign ( $\oplus$ ) is used to show the EOR operation.

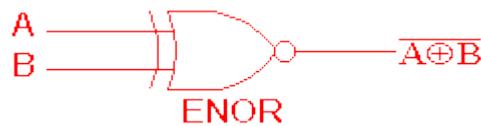


Fig.8. 16 EXNOR gate

2 Input EXNOR gate		
A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

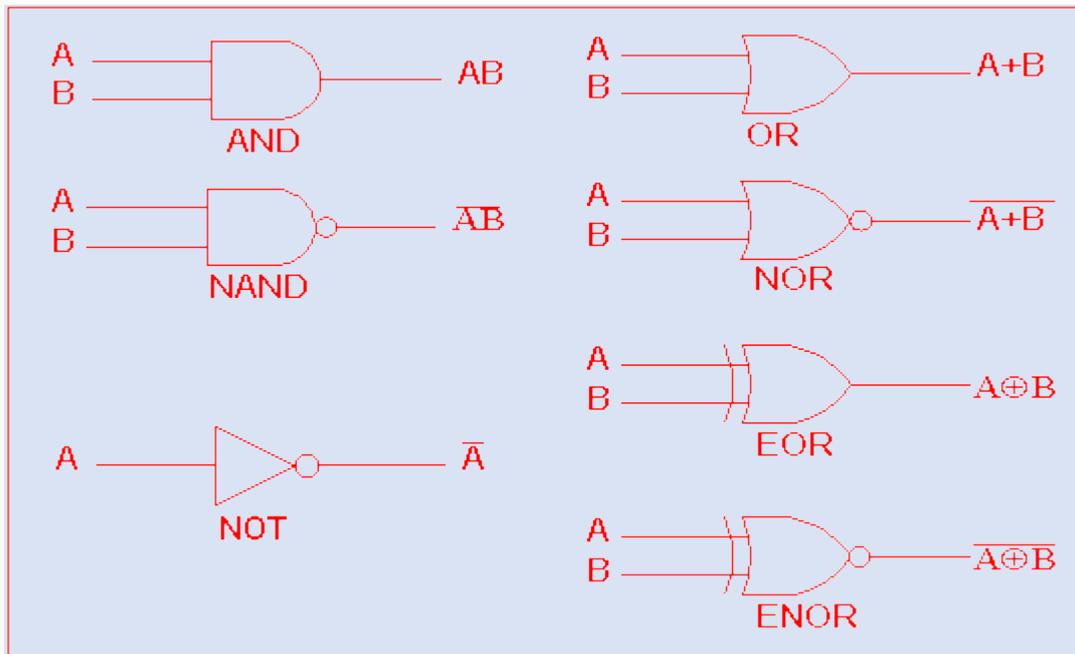
The 'Exclusive-NOR' gate circuit does the opposite to the EOR gate. It will give a low output if either, but not both, of its two inputs are high. The symbol is an EXOR gate with a small circle on the output. The small circle represents inversion.

The NAND and NOR gates are called *universal functions* since with either one the AND and OR functions and NOT can be generated.

**Note:**

A function in *sum of products* form can be implemented using NAND gates by replacing all AND and OR gates by NAND gates.

A neither function in *product of sums* form can be implemented using NOR gates by replacing all AND and OR gates by NOR gates.



**Fig.8. 37 Logic gate symbols**

Table 8.18 is a summary truth table of the input/output combinations for the NOT gate together with all possible input/output combinations for the other gate functions. Also note that a truth table with 'n' inputs has  $2^n$  rows.

You can compare the outputs of different gates.

		INPUTS		OUTPUTS					
		A	B	AND	NAND	OR	NOR	EXOR	EXNOR
<b>NOT gate</b>		0	0	0	1	0	1	0	1
A	$\overline{A}$	0	1	0	1	1	0	1	0
0	1	1	0	0	1	1	0	1	0
1	0	1	1	1	0	1	0	0	1

Who invented the idea?

This logical way of comparing numbers to make decisions that produce either a yes or no, 1 or 0, true or false is called Boolean algebra after its discoverer, English mathematician George Boole (1815–1864), who set out the idea in an 1854 book titled *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities*. His objective was to show how complex human reasoning could be represented in a logical, mathematical form.

### APPLICATION ACTIVITY ACTIVITIES 8.2

1. Digital data refers to information that is

A. Continuous

B. Discrete

C. Bits

D. Bytes

2. In data communications, non periodic signals are:

A. Sine wave

B. Digital Signals

C. Analog Signals

D. None of the above

3. Completion of one full pattern is called a:

A. Period

B. Cycle

C. Frame

D. Segment

4. Term that refers to infinite number of values in range is

A. Peak

B. Analog Signal

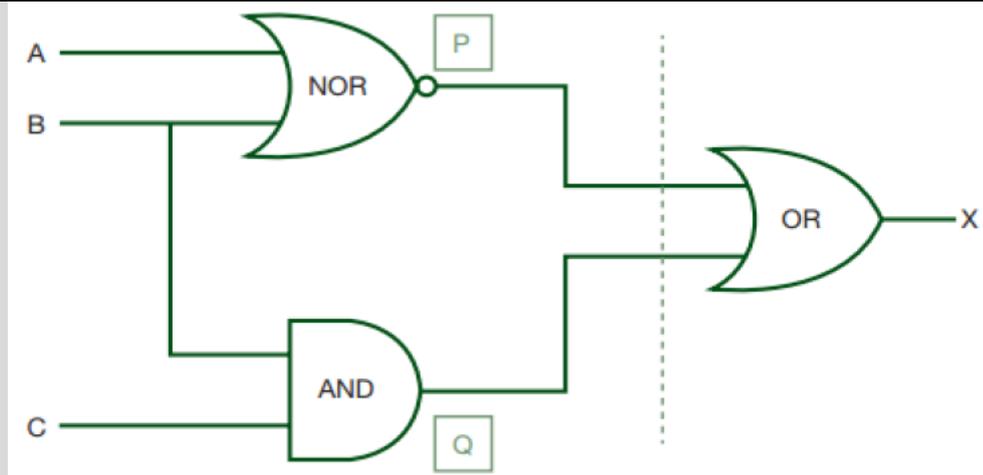
C. Digital Signal

D. None of the above

5. Design electrical circuits for AND gate, OR gate and NOT gate.

Draw the circuit diagrams to show how a NOR gate can be made into a NOT gate.

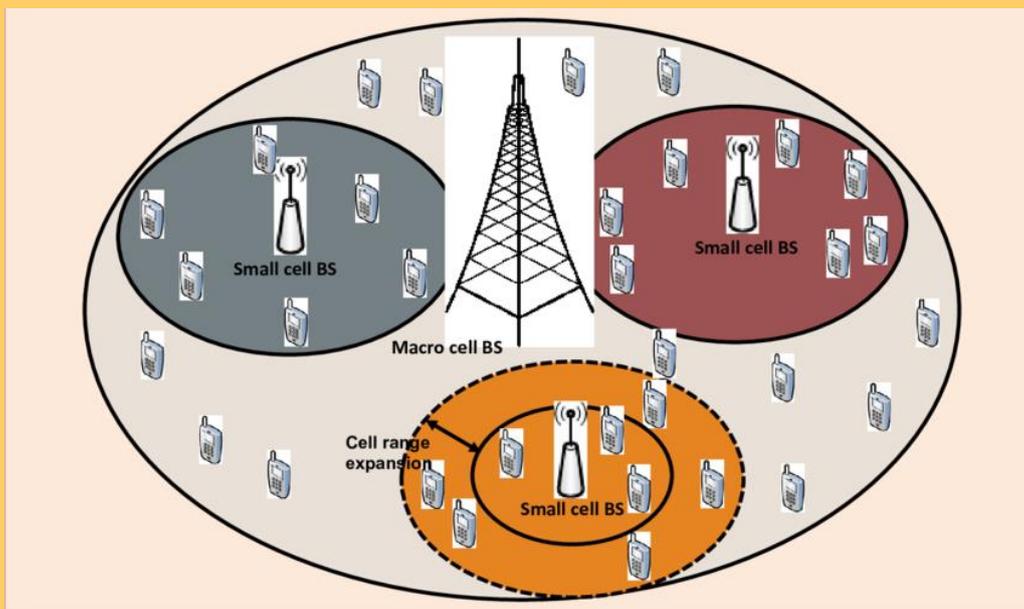
6. Produce a truth table from the following logic circuit (network)



**Fig.8. 18 logic gate circuit**

### 8.3 Mobile communication systems

#### ACTIVITY 8.3



**Fig.8. 19 Network transmission**

The figure above shows how network for a certain telecommunications company in Rwanda. Study it carefully and answer the following questions.

- How many cells are shown on the figure above? Give their respective names.
- Identify different masts shown on the figure.
- In regard to the figure, what is the importance of masts in those different cells?

- d) Why do you think in transmission of network, the targeted area is divided into small portions?
- e) Compare the number of cells that should be allocated for urban areas to those for rural areas.

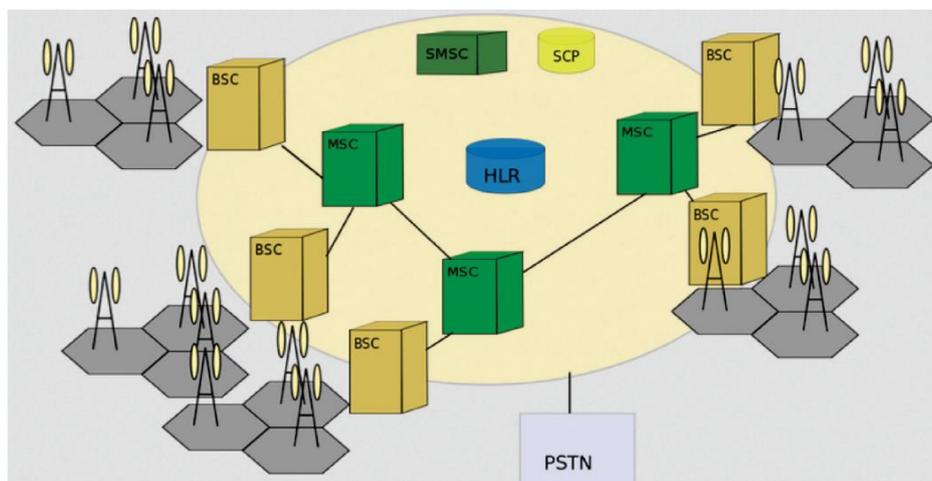
### 8.3.1 Structure of cellular network

An overall cellular network contains a number of different elements from the **base transceiver station** (BTS) itself with its antenna back through a **base station controller** (BSC), and a **mobile switching centre**(MSC) to the location registers (HLR and VLR) and the link to the public switched telephone network (PSTN).

Of the units within the cellular network, the BTS provides the direct communication with the mobile phones. There may be a small number of base stations linked to a base station controller. This unit acts as a small centre to route calls to the required base station, and it also makes some decisions about which base station is the best suited for a particular call.

The links between the BTS and the BSC may use either land lines or even microwave links. Often the BTS antenna towers also support a small microwave dish antenna used for the link to the BSC. The BSC is often co-located with a BTS.

The BSC interfaces with the mobile switching centre. This makes more widespread choices about the routing of calls and interfaces to the land line based PSTN as well as the location registers.



**Fig.8. 20 Structure of Cellular network**

### 8.3.2 Principle of cellular network

The increase in demand and the poor quality of existing service led mobile service providers to research ways to improve the quality of service and to support more users in their systems. Because the amount of frequency spectrum available for mobile cellular use was limited, efficient use of the required frequencies was needed for mobile cellular coverage.

In modern cellular telephony, rural and urban regions are divided into areas according to specific provisioning guidelines.

Deployment parameters, such as amount of cell-splitting and cell sizes, are determined by engineers experienced in cellular system architecture. Provisioning for each region is planned according to an engineering plan that includes cells, clusters, frequency reuse, and handovers.

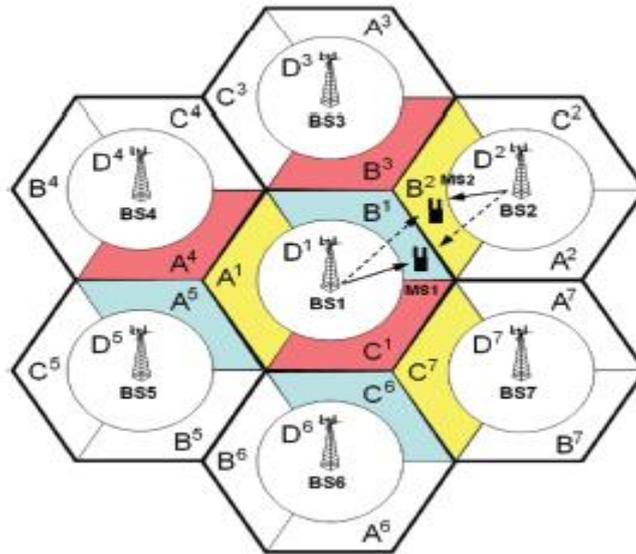
#### Cells

A cell is the basic geographic unit of a cellular system. The term cellular comes from the honeycomb shape of the areas into which a coverage region is divided. **Cells** are base stations transmitting over small geographic areas that are represented as hexagons. Each cell size varies depending on the landscape. Because of constraints imposed by natural terrain and man-made structures, the true shape of cells is not a perfect hexagon.

#### Clusters

A cluster is a group of cells. No channels are reused within a cluster.

Fig.8.23 illustrates a seven-cell cluster. In clustering, all the available frequencies are used once and only once. As shown on fig.8.24, each cell has a base station and any mobile user moving remains connected due to hand-offs between the stations.



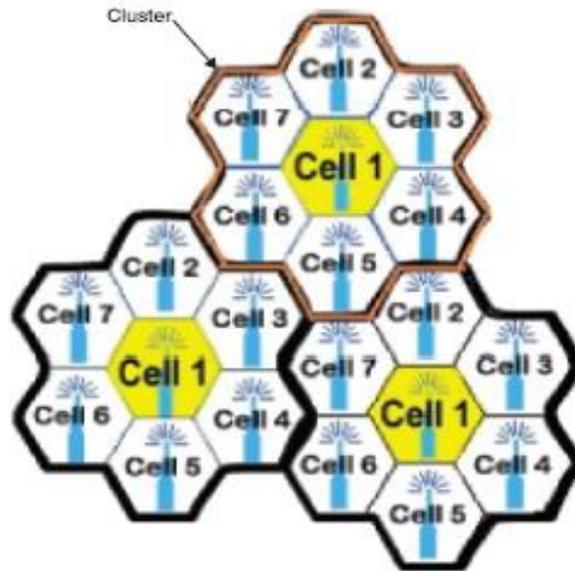
**Fig.8. 21 Cluster**

### **Frequency Reuse**

Because only a small number of radio channel frequencies were available for mobile systems, engineers had to find a way to reuse radio channels in order to carry more than one conversation at a time. The solution was called **frequency planning** or **frequency reuse**. Frequency reuse was implemented by restructuring the mobile telephone system architecture into the cellular concept.

The concept of frequency reuse is based on assigning to each cell a group of radio channels used within a small geographic area. Cells are assigned a group of channels that is completely different from neighboring cells.

The coverage areas of cells are called the **footprint**. This footprint is limited by a boundary so that the same group of channels can be used in different cells that are far enough away from each other so that their frequencies do not interfere.

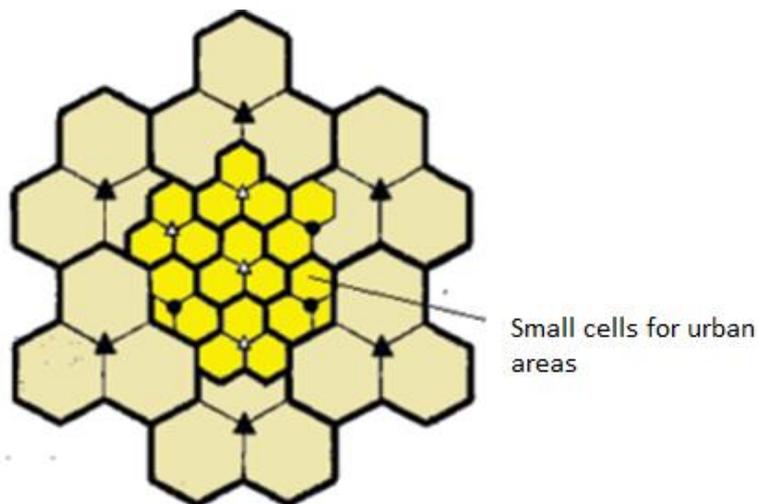


**Fig.8. 22 Frequency reuse.**

Cells with the same number have the same set of frequencies. Here, because the number of available frequencies is 7, the frequency reuse factor is  $1/7$ . That is, each cell is using  $1/7$  of available cellular channels.

### Cell Splitting

Unfortunately, economic considerations made the concept of creating full systems with many small areas impractical. To overcome this difficulty, system operators developed the idea of cell splitting.



**Fig.8. 23 Cell splitting**

As a service area becomes full of users, this approach is used to split a single area into smaller ones. In this way, urban centers can be split into as many areas as necessary in order to provide acceptable

service levels in heavy-traffic regions, while larger, less expensive cells can be used to cover remote rural regions.

### **Handoff**

The final obstacle in the development of the cellular network involved the problem created when a mobile subscriber travelled from one cell to another during a call. As adjacent areas do not use the same radio channels, a call must either be dropped or transferred from one radio channel to another when a user crosses the line between adjacent cells.

Because dropping the call is unacceptable, the process of handoff was created. **Handoff** occurs when the mobile telephone network automatically transfers a call from radio channel to radio channel as mobile crosses adjacent cells.



**Fig.8. 244 Handoff**

During a call, two parties are on one voice channel. When the mobile unit moves out of the coverage area of a given cell site, the reception becomes weak. At this point, the cell site in use requests a handoff. The system switches the call to a stronger-frequency channel in a new site without interrupting the call or alerting the user. The call continues as long as the user is talking, and the user does not notice the handoff at all.

### **Conclusion**

We can say that mobile communication system is a high capacity communication system arranged to establish and maintain continuity of communication paths to mobile stations passing from the coverage of one radio transmitter into the coverage of another radio transmitter.

A control center determines mobile station locations and enables a switching center to control dual access trunk circuitry to transfer an existing mobile station communication path from a formerly

occupied cell to a new cell location. The switching center subsequently enables the dual access trunk to release the call connection to the formerly occupied cell.

### APPLICATION ACTIVITIES 8.3

1. Write in full the following terms:

- a) BTS
- b) BSC
- c) PSTN

as they are used in network telecommunication

2. Explain the following terms:

- a) cell
- b) handoff
- c) frequency reuse

### 8.4 Radio transmission and reception (AM, FM, PM)

#### ACTIVITY 8.4



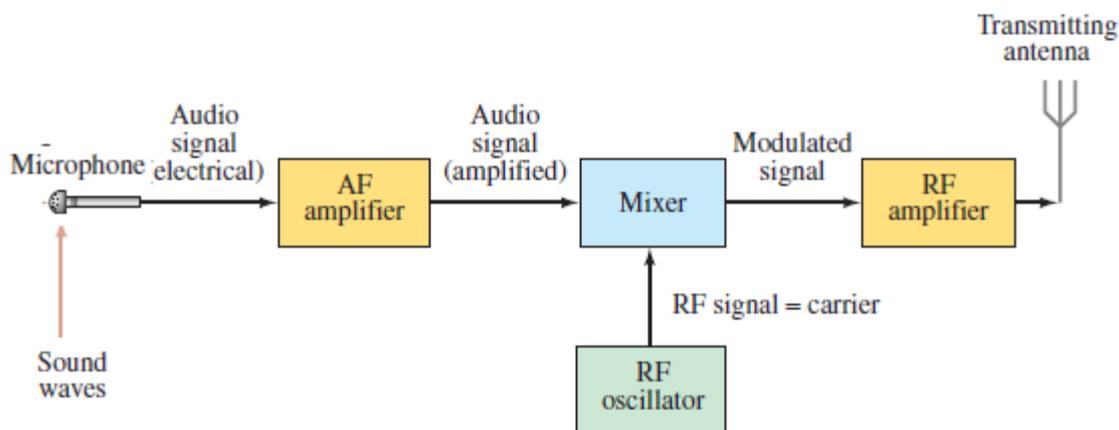
**Fig.8. 25 Radio receiver**

While listening to radio on one of the evening, Mukamisha heard that the tuned channel was on FM at 100.7 MHz But her radio works efficiently when she pulls up the antenna.

- a) What do you think is the significance of the antenna on her radio?
- b) Hoping you has ever used/played a radio. Where do you think the information/sound from the radio come from?
- c) Explain the mode of transmission of information as suggested in b) above to the receiving radio.
- d) While going to sleep, her radio fell down and the speaker got problems. Do you think she was able to listen to late night programs on the same channel?
- e) As indicated on the radio, what does FM, MW, and SW mean?

### 8.4.1 Simple radio transmitter

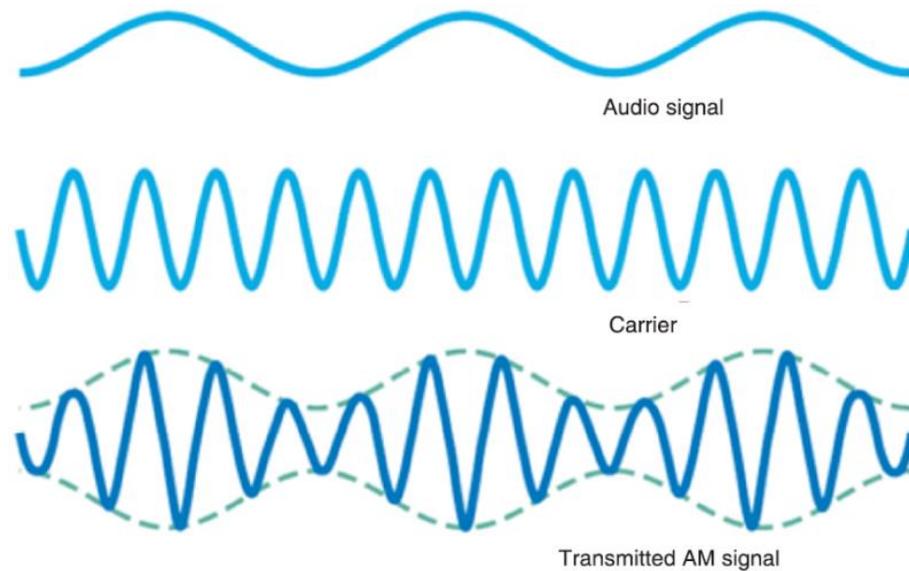
A radio transmitter consists of several elements that work together to generate radio waves that contain useful information such as audio, video, or digital data. The process by which a radio station transmits information is outlined in Fig. 8.29.



**Fig.8. 56 Block diagram of a radio transmitter**

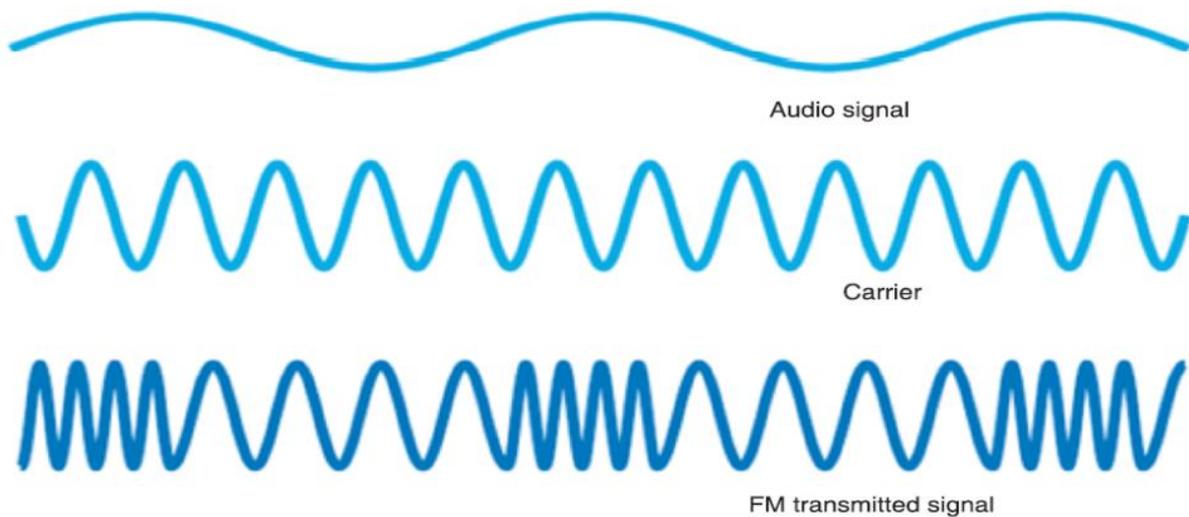
- **Power supply:** Provides the necessary electrical power to operate the transmitter.
- The audio (sound) information is changed into an electrical signal of the same frequencies by, say, a microphone, a laser, or a magnetic read write head. This electrical signal is called an **audio frequency (AF) signal**, because the frequencies are in the audio range (20 Hz to 20 000 Hz).
- The signal is amplified electronically in AF amplifier and is then mixed with a radio-frequency (RF) signal called its **carrier frequency**, which represents that station. AM radio stations have carrier frequencies from about 530 kHz to 1700 kHz. Today’s digital broadcasting uses the same frequencies as the pre-2009 analog transmission.
- The **Modulator or Mixer** adds useful information to the carrier wave. The mixing of the audio and carrier frequencies is done in two ways.

In **amplitude modulation (AM)**, the amplitude of the high-frequency carrier wave is made to vary in proportion to the amplitude of the audio signal, as shown in Fig.8.30. It is called “amplitude modulation” because the *amplitude* of the carrier is altered (“modulate” means to change or alter).



**Fig.8. 27 Amplitude Modulation**

**In frequency modulation (FM)**, the *frequency* of the carrier wave is made to change in proportion to the audio signal’s amplitude, as shown in Fig.8.31. The mixed signal is amplified further and sent to the transmitting antenna of fig.8.29 where the complex mixture of frequencies is sent out in the form of electromagnetic waves.

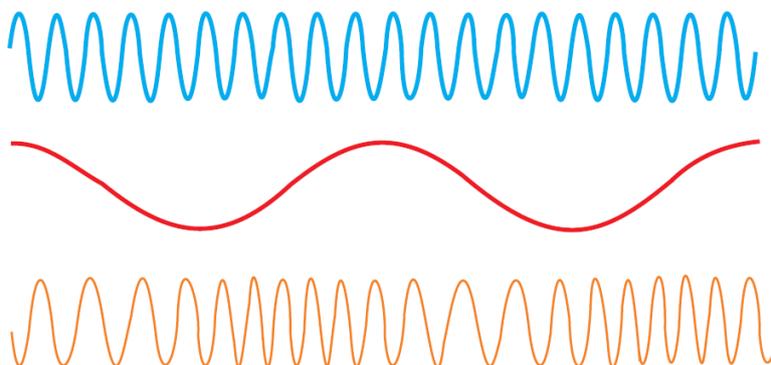


**Fig.8. 68 Frequency Modulation**

**Phase modulation (PM)**

Phase modulation is a form of modulation that encodes information as variations in the instantaneous phase of the carrier wave. It is widely used for transmitting radio waves and is an integral part of many

digital transmission coding schemes that underlie a wide range of technologies like Wi-Fi, GSM and satellite television. In this type of modulation, the amplitude and frequency of the carrier signal remains unchanged after PM.



**Fig.8. 297 Phase modulation**

The modulating signal is mapped to the carrier signal in the form of variations in the instantaneous phase of the carrier signal. Phase modulation is closely related to frequency modulation and is often used as intermediate step to achieve FM.

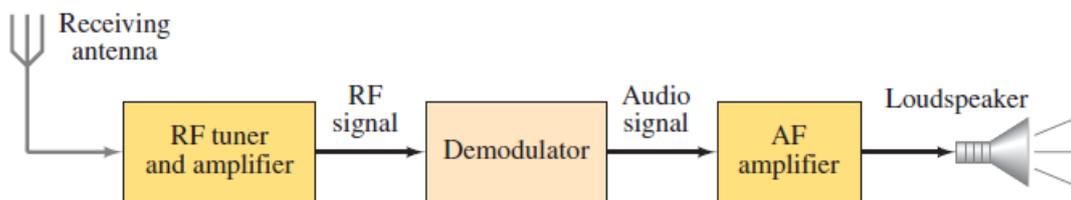
**Amplifier:** Amplifies the modulated carrier wave to increase its power. The more powerful the amplifier, the more powerful the broadcast.

In digital communication, the signal is put into digital form which modulates the carrier. A television transmitter works in a similar way, using FM for audio and AM for video; both audio and video signals are mixed with carrier frequencies.

#### **8.4.2 Simple radio receiver**

A radio receiver is the opposite of a radio transmitter. It uses an antenna to capture radio waves, processes those waves to extract only those waves that are vibrating at the desired frequency, extracts the audio signals that were added to those waves, amplifies the audio signals, and finally plays them on a speaker.

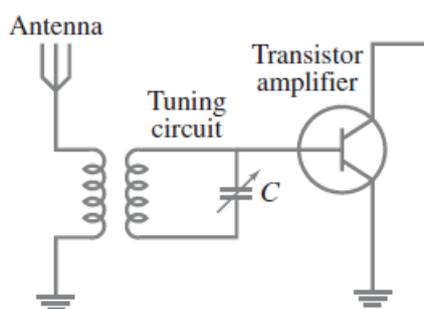
Now let us look at the other end of the process, the reception of radio and TV programs at home. A simple radio receiver is graphed in Fig. 8.33. The EM waves sent out by all stations are received by the antenna.



**Fig.8. 30 Block diagram of a simple radio receiver**

The signal **antenna** detects and sends the radio waves, to the receiver is very small and contains frequencies from many different stations. The receiver uses a resonant  $LC$  circuit to select out a particular RF frequency (actually a narrow range of frequencies) corresponding to a particular station.

A simple way of tuning a station is shown in Fig.8.34. When the wire of antenna is exposed to radio waves, the waves induce a very small alternating current in the antenna.



**Fig.8. 31 Simple tuning stage of a radio.**

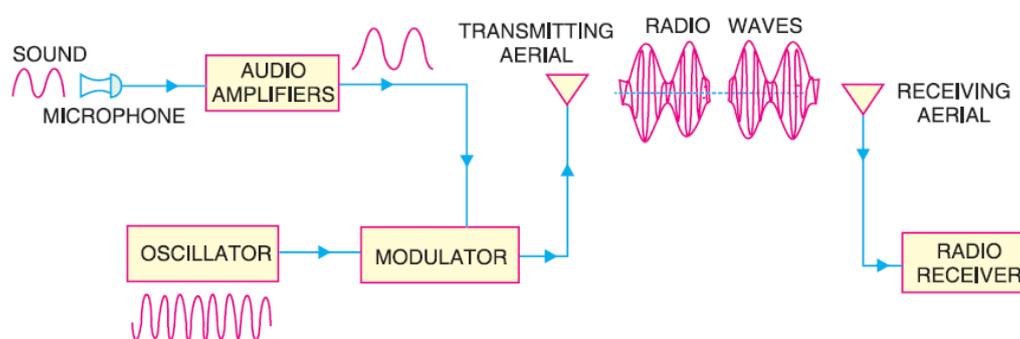
A particular station is “tuned in” by adjusting the capacitance  $C$  and/or inductance  $L$  so that the resonant frequency of the circuit equals that of the station’s carrier frequency.

**R.F. Amplifier:** A sensitive amplifier that amplifies the very weak radio frequency (RF) signal from the antenna so that the signal can be processed by the tuner.

**R.F. Tuner:** A circuit that can extract signals of a particular frequency from a mix of signals of different frequencies. On its own, the antenna captures radio waves of all frequencies and sends them to the RF amplifier, which dutifully amplifies them all. Unless you want to listen to every radio channel at the same time, you need a circuit that can pick out just the signals for the channel you want to hear. That’s the role of the tuner.

The tuner usually employs the combination of an inductor (for example, a coil) and a capacitor to form a circuit that resonates at a particular frequency. This frequency, called the *resonant frequency*, is determined by the values chosen for the coil and the capacitor. This type of circuit tends to block any AC signals at a frequency above or below the resonant frequency. The fig.8.35 shows a combination of a radio transmitter and a radio receiver.

You can adjust the resonant frequency by varying the amount of inductance in the coil or the capacitance of the capacitor. In simple radio receiver circuits, the tuning is adjusted by varying the number of turns of wire in the coil. More sophisticated tuners use a variable capacitor (also called a *tuning capacitor*) to vary the frequency.

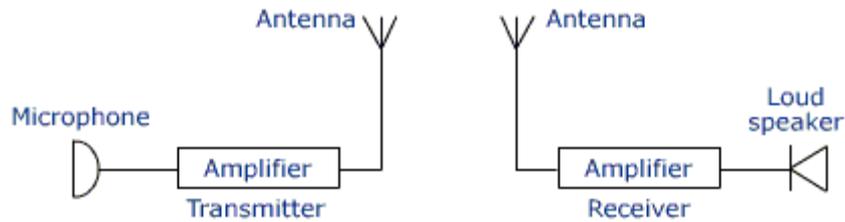


**Fig.8. 32 Bloc diagram of a radio transmitter and a radio receiver**

### 8.4.3 Wireless Radio Communication

Let us now discuss the basic principles of wireless radio communications. We shall mainly concentrate on the principle of amplitude modulation and demodulation.

The simplest scheme of wireless communication would be to convert the speech or music to be transmitted to electric signals using a microphone, boost up the power of the signal using amplifiers and radiate the signal in space with the aid of an antenna. This would constitute the transmitter. At the receiver end, one could have a pick-up antenna feeding the speech or music signal to an amplifier and a loud speaker.



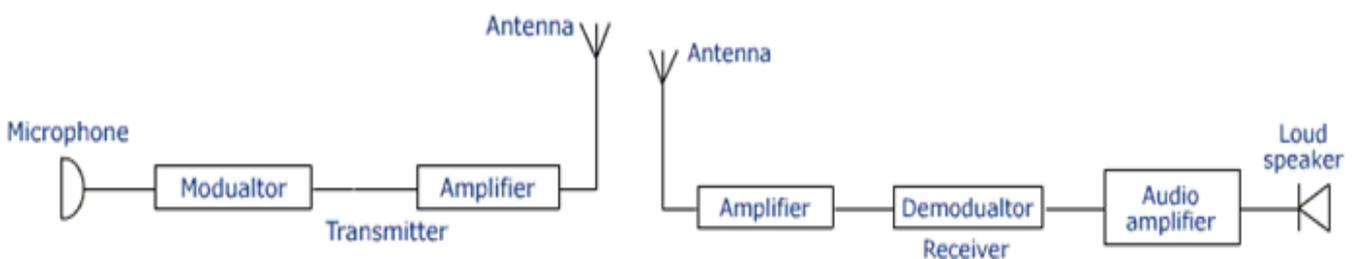
**Fig.8. 33 Wireless radio communication**

The above scheme suffers from the following drawbacks:

- (i) Electromagnetic waves in the frequency range of 20 Hz to 20 kHz (audio-frequency range) cannot be efficiently radiated and do not propagate well in space.
- (ii) Simultaneous transmission of different signals by different transmitters would lead to confusion at the receiver.

In order to solve these problems; we need to devise methods to convert or translate the audio signals to the radio-frequency range before transmission and recover the audio-frequency signals back at the receiver. Different transmitting stations can then be allotted slots in the radio-frequency range and a single receiver can then tune into these transmitters without confusion.

The frequency range 500 kHz to 20 MHz is reserved for amplitude-modulated broadcast, which is the range covered by most three band transistor radios. The process of frequency translation at the transmitter is called **modulation**. The process of recovering the audio-signal at the receiver is called **demodulation**. A simplified block diagram of such a system is shown in the below figure.



**Fig.8. 34 Block diagram of radio transmitter and receiver**

### **APPLICATION ACTIVITIES 8.4**

1. Distinguish between Modulation and demodulation.
2. What is the importance of power amplifier in simple radio transmitter
3. What are different types of analog modulation?
4. Give any differences between AM and FM

### **SKILLS LAB 8**

In this activity you will make a survey in any selected Village (Umudugudu) of your choice. The survey is about the daily usage of Digital and Analog devices by local people in the specified area.

#### **Procedures**

- a) Select a Village of your interest either around your school or where you stay.
- b) Move around asking people (family) whether they have/use the following gadgets:  
Phones, Computers, Radios, Televisions, tablets and others
- c) Ask them whether these devices operate by wireless or they need to be connected to wires that come from serving centers.
- d) Make a suitable chart indicating all the data recorded or attained
- e) Make a general comment about your findings.

## END UNIT ASSESSMENT 8

### Multiple choice questions

1. Modulation is done in .....

  - (i) transmitter
  - (ii) radio receiver
  - (iii) between transmitter and radio receiver
  - (iv) none of the above

2. In India, ..... modulation is used for radio transmission.

  - (i) frequency
  - (ii) amplitude
  - (iii) phase
  - (iv) none of the above

3. In an AM wave, useful power is carried by.....

  - (i) carrier
  - (ii) sidebands
  - (iii) both sidebands and carrier
  - (iv) none of the above

4. In amplitude modulation, bandwidth is..... the audio signal frequency

  - (i) thrice
  - (ii) four times
  - (iii) twice
  - (iv) none of the above

5. In amplitude modulation, the ..... of carrier is varied according to the strength of the signal.

  - (i) amplitude
  - (ii) frequency
  - (iii) phase

(iv) none of the above

6. Overmodulation (amplitude) occurs when signal amplitude is ..... carrier amplitude.

- (i) equal to
- (ii) greater than
- (iii) less than
- (iv) none of the above

7. In an AM wave, the majority of the power is in .....

- (i) lower sideband
- (ii) upper sideband
- (iii) carrier
- (iv) none of the above

8. Overmodulation results in .....

- (i) weakening of the signal
- (ii) excessive carrier power
- (iii) distortion
- (iv) none of the above

9. If modulation is 100 %, then signal amplitude is ..... carrier amplitude.

- (i) equal to
- (ii) greater than
- (iii) less than
- (iv) none of the above

10. As the modulation level is increased, the carrier power .....

- (i) is increased
- (ii) remains the same
- (iii) is decreased
- (iv) none of the above

11. Demodulation is done in .....

- (i) receiving antenna
- (ii) transmitter
- (iii) radio receiver
- (iv) transmitting antenna

12. A high  $Q$  tuned circuit will permit an amplifier to have high .....

- (i) fidelity
- (ii) frequency range
- (iii) sensitivity
- (iv) selectivity

13. In radio transmission, the medium of transmission is .....

- (i) space
- (ii) an antenna
- (iii) cable
- (iv) none of the above

14. If level of modulation is increased ..... power is increased.

- (i) carrier
- (ii) sideband
- (iii) carrier as well as sideband
- (iv) none of the above

15. Man made noises are ..... variations.

- (i) amplitude
- (ii) frequency
- (iii) phase
- (iv) both phase and frequency

16.If a radio receiver amplifies all the signal frequencies equally well, it is said to have high .....

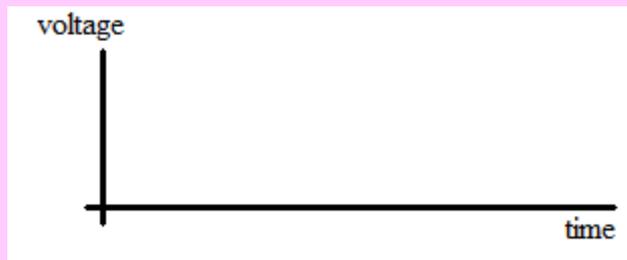
- (i) sensitivity (ii) selectivity
- (ii) distortion (iv) fidelity

17.The major advantage of FM over AM is.....

- (i) reception is less noisy
- (ii) higher carrier frequency
- (iii)smaller bandwidth
- (iv)small frequency deviation

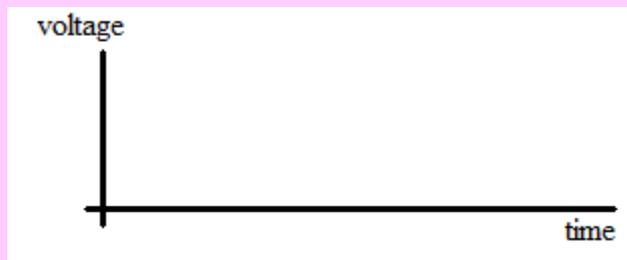
18. What is meant by telecommunication system?

19. (i) On the axes below, draw an example of an analogue signal.



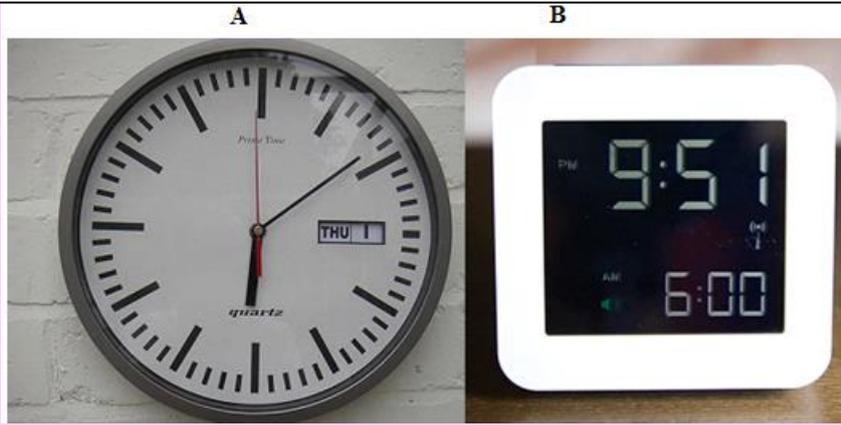
(ii) State two examples of sources of analogue signals.

b).(i) On the axes below, draw an example of a digital signal.

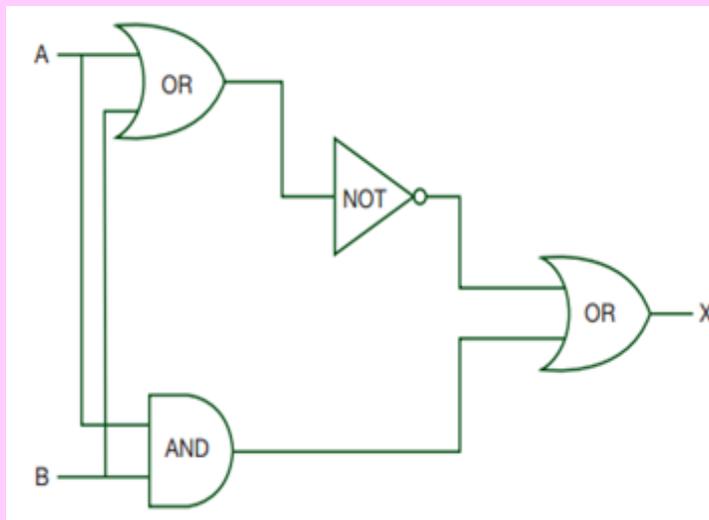


(ii)State two examples of sources of digital signals.

20. (a) The following are graphs of watches. Choose which one is digital and which is analogue device.



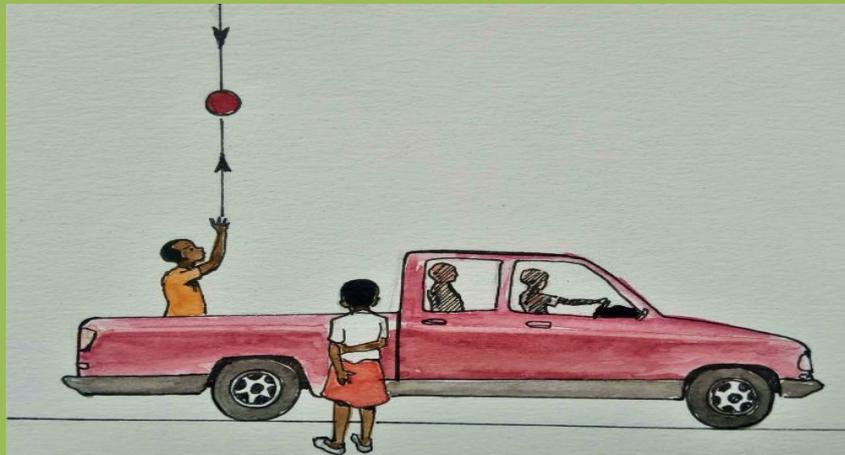
(b) Construct a truth table of the following logic circuit



## UNIT 9 RELATIVITY CONCEPTS AND POSTULATES OF SPECIAL RELATIVITY

**Key unit competence:** *Explain relativity Concepts and postulates of special relativity*

### INTRODUCTORY ACTIVITY



**Fig.9.1** Observation of a ball thrown by a student teacher in a moving car

Yves a student teacher in year two once was moving in a pick up as shown in the figure above. She had a small ball that she projected upwards when the car was moving at a speed of 60km/h.

Basing on the statement and figure above, answer the following questions.

- Do you think Yves was able to catch the ball 3 seconds later after projection assuming the car continued at a steady speed of 60 km/h?
- What do you think was the shape of the path described by the ball as observed by Yves while in the car?
- Diana a stationary observer at the banks of the road observes the projected ball right at a time when Yves projected it. Do you think the path of the ball as observed by Diana was similar to that of Yves? If not, can you describe what you think would be the observed path by her.
- While still in the moving car, Yves moves at 5 km/h with respect to the car. Do you think as observed by Diana, Yves was moving at 5 km/h? If not, what is the estimation of speed of Yves as observed by Diana?

## 9.1 Concept of Relativity

### ACTIVITY 9.1

On the first day of traveling in a car, Shyaka observed trees, stones, mountains and all stationary saw them moving in the direction where the car was coming from.

- Were the trees, stones and mountains actually moving?
- If No, why did Shyaka see them moving?
- As Shyaka and friends in the same car tried to take over another speeding vehicle that was travelling in the same direction with the same speed, Shyaka observed that the car they were trying to overtake seemed to be stationary. Explain the cause of this effect

### 9.1.1 Introduction to special relativity

Physics as it was known at the end of the nineteenth century is referred to as **classical physics**:

-**Newtonian mechanics** beautifully explained the motion of objects on Earth and in the heavens. Furthermore, it formed the basis for successful treatments of fluids, wave motion, and sound.

-**Kinetic theory** explained the behavior of gases and other materials.

-**Maxwell's theory of electromagnetism** developed in 1873 by James Clerk Maxwell, a Scottish physicist embodied all of electric and magnetic phenomena,

Soon, however, scientists began to look more closely at a few inconvenient phenomena that could not be explained by the theories available at the time. This led to birth of the new Physics that grew out of the great revolution at the turn of the twentieth century and is now called **Modern Physics** (the *Theory of Relativity and Quantum Theory*).

Most of our everyday experiences and observations have to do with objects that move at speeds much less than the speed of light. Newtonian mechanics was formulated to describe the motion of such objects, and this formalism is still very successful in describing a wide range of phenomena that occur at low speeds. It fails, however, when applied to particles whose speeds approach that of light.

Experimentally, the predictions of Newtonian theory can be tested at high speeds by accelerating electrons or other charged particles through a large electric potential difference. For example, it is

possible to accelerate an electron to a speed of  $0.99c$  (where  $c$  is the speed of light) by using a potential difference of several million volts.

According to Newtonian mechanics, if the potential difference is increased by a factor of 4, the electron's kinetic energy is four times greater and its speed should double to  $1.98c$ . However, experiments show that the speed of the electron—as well as the speed of any other particle in the Universe—always remains less than the speed of light, regardless of the size of the accelerating voltage. Because it places no upper limit on speed, Newtonian mechanics is contrary to modern experimental results and is clearly a limited theory.

In 1905, at the age of only 26, Einstein published three papers of extraordinary importance:

-One was an analysis of **Brownian motion**;

-a second (for which he was awarded the Nobel Prize) was on the **photoelectric effect**.

-In the third, Einstein introduced his **special theory of relativity**.

Although Einstein made many other important contributions to Science, the **special theory of relativity** alone represents one of the greatest intellectual achievements of all time.

With this theory, experimental observations can be correctly predicted over the range of speeds from to speeds approaching the speed of light. At low speeds, Einstein's theory reduces to Newtonian mechanics as a limiting situation (**principle of correspondence**).

It is important to recognize that Einstein was working on Electromagnetism when he developed the special theory of relativity. He was convinced that Maxwell's equations were correct, and in order to reconcile them with one of his postulates, he was forced into the bizarre notion of assuming that **space and time are not absolute**.

In addition to its well-known and essential role in theoretical Physics, the Special Theory of Relativity has practical applications, including the design of nuclear power plants and modern global positioning system (GPS) units. These devices do not work if designed in accordance with non-relativistic principles.

### 9.1.2 Galilean transformation equation

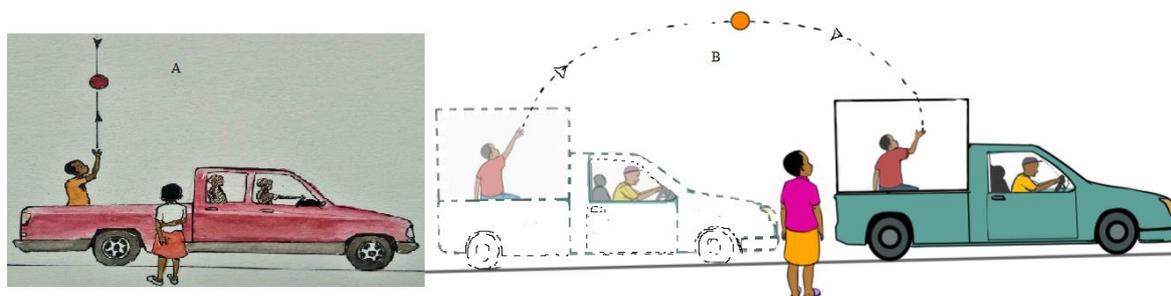
#### (a) Principle of Galilean relativity

You've no doubt observed how a car that is moving slowly forward appears to be moving backward when you pass it. In general, when two observers measure the velocity of a moving body, they get different results if one observer is moving relative to the other. The velocity seen by a particular observer is called the velocity *relative* to that observer, or simply **relative velocity**.

To describe a physical event, it is necessary to establish a **frame of reference**. You should recall from Mechanics that Newton's laws are valid in all inertial frames of reference. Because an **inertial frame** is defined as one in which Newton's first law is valid, we can say that an inertial frame of reference is one in which an object is observed to have no acceleration when no forces act on it. Furthermore, any system moving with constant velocity with respect to an inertial system must also be an inertial system.

There is no preferred inertial reference frame. This means that the results of an experiment performed in a vehicle moving with uniform velocity will be identical to the results of the same experiment performed in a stationary vehicle. The formal statement of this result is called the **principle of Galilean relativity**: "The laws of Physics must be the same in all inertial frames of reference."

Let us consider an observation that illustrates the equivalence of the laws of Mechanics in different inertial frames. A pickup truck moves with a constant velocity, as shown in Fig. 9.2a.



**Fig.9. 2(a) The observer in the truck sees the ball move in a vertical path when thrown upward.  
(b) The Earth observer sees the path of the ball as a parabola.**

If a passenger in the truck throws a ball straight up, and if air effects are neglected, the passenger observes that the ball moves in a vertical path. The motion of the ball appears to be precisely the same as if the ball were thrown by a person at rest on the Earth. The law of gravity and the equations of motion under constant acceleration are obeyed whether the truck is at rest or in uniform motion.

Now consider the same situation viewed by an observer at rest on the Earth. This stationary observer sees the path of the ball as a parabola, as illustrated in Fig. 9.2b. Furthermore, according to this observer, the ball has a horizontal component of velocity equal to the velocity of the truck.

Although the two observers disagree on certain aspects of the situation, they agree on the validity of Newton's laws and on such classical principles as conservation of energy and conservation of linear momentum. This agreement implies that no mechanical experiment can detect any difference between the two inertial frames.

The only thing that can be detected is the relative motion of one frame with respect to the other. That is, the notion of absolute motion through space is meaningless, as is the notion of a preferred reference frame.

### Example 9.1

A child sits upright in a wagon which is moving to the right at constant speed as shown in Fig.9.3. The child extends her hand and throws an apple straight upward (from her own point of view), while the wagon continues to travel forward at constant speed.

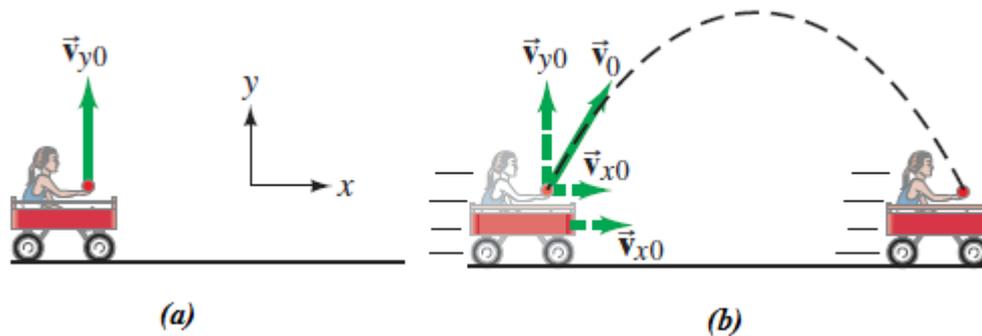
If air resistance is neglected will the apple land:

- a) Behind the wagon
- b) In the wagon
- c) Or in front of the wagon?

#### **Solution:**

The child throws the apple straight up from her own reference frame with initial velocity  $v_{y0}$  (Fig.9.3a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the speed of the wagon,  $v_{x0}$ . Thus, to a person on the ground, the apple will follow the path of a projectile as shown in Fig.9.3b. The apple experiences no horizontal acceleration, so  $v_{x0}$  will stay constant and equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal

velocity. When the apple comes down, it will drop right into the outstretched hand of the child. The answer is (b).



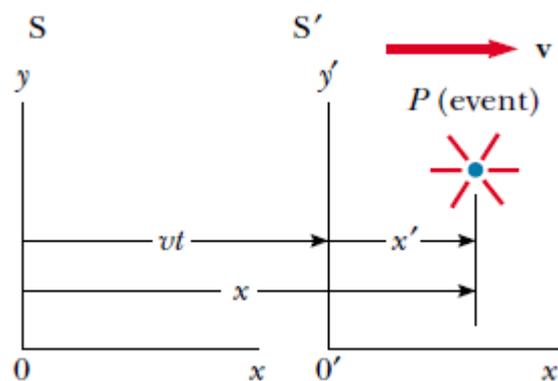
**Fig.9. 3(a)Wagon reference frame, (b) Ground reference frame**

**(b) Galilean space–time transformation equations**

Suppose that some physical phenomenon, which we call an *event*, occurs in an inertial system. The event’s location and time of occurrence can be specified by the four coordinates  $(x, y, z, t)$ . We would like to be able to transform these coordinates from one inertial system to another one moving with uniform relative velocity.

Consider two inertial systems  $S$  and  $S'$  (Fig. 9.4). The system  $S'$  moves with a constant velocity  $v$  along the  $xx'$  axes, where  $v$  is measured relative to  $S$ .

We assume that an event occurs at the point  $P$  and that the origins of  $S$  and  $S'$  coincide at  $t = 0$



**Fig.9. 4 An event occurs at a point  $P$ . The event is seen by two observers in inertial frames  $S$  and  $S'$ , where  $S'$  moves with a velocity  $v$  relative to  $S$ .**

An observer in S describes the event with space–time coordinates  $(x, y, z, t)$ , whereas an observer in S' uses the coordinates  $(x', y', z', t')$  to describe the same event.

As we see from Fig. 9.4, the relationships between these various coordinates can be written

$$\begin{cases} x' = x - ut \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

These equations are the **Galilean space–time transformation equations**.

Note that time is assumed to be the same in both inertial systems. That is, within the framework of classical mechanics, all clocks run at the same rate, regardless of their velocity, so that the time at which an event occurs for an observer in S is the same as the time for the same event in S'. Consequently, the time interval between two successive events should be the same for both observers.

Although this assumption may seem obvious, it turns out to be incorrect in situations where  $v$  is comparable to the speed of light.

### ***Length and time intervals are absolute***

*Galilean–Newtonian relativity* assumed that the lengths of objects are the same in one reference frame as in another, and that time passes at the same rate in different reference frames.

In classical mechanics, then, space and time intervals are considered to be **absolute**: their measurement does not change from one reference frame to another. The mass of an object, as well as all forces, are assumed to be unchanged by a change in inertial reference frame.

### **(c) Galilean-Newton Relative velocity**

Now suppose that a particle moves a distance  $dx$  in a time interval  $dt$  as measured by an observer in S. It follows that the corresponding distance  $dx'$  measured by an observer in S' is

$$\begin{cases} dx' = dx - d(ut) \\ dy' = dy \\ dz' = dz \\ dt' = dt \end{cases} \Leftrightarrow v'_x = v_x - u$$

Where  $v_x$  and  $v'_x$  are the  $x$  components of the velocity relative to  $S$  and  $S'$ , respectively. This is the **Galilean velocity transformation equation or relative velocity**. It is used in everyday observations and is consistent with our intuitive notion of time and space. As we shall soon see, however, it leads to serious contradictions when applied to electromagnetic waves.

### Example 9.2

A girl on a 10-speed bicycle travels at 9 m/s relative to the ground as she passes a little boy on a tricycle going on opposite direction. If the boy is traveling at 1 m/s relative to the ground, how fast does the boy appear to be moving relative to the girl?

#### Answer

The velocity of the girl relative to the ground  $S$ :  $u = 9 \text{ m/s}$

The velocity of the boy relative to the ground:  $v = -1 \text{ m/s}$

The velocity of the boy relative to the girl:  $v' = v - u \Leftrightarrow u = -1 - 9 = -10 \text{ m/s}$

#### *Position and velocity are different in different reference frames, but length is the same*

The position of an object is different when specified in different reference frames, and so is velocity. For example, a person may walk inside a bus toward the front with a speed of  $v' = 2 \text{ m/s}$ .

But if the bus moves with  $u = 10 \text{ m/s}$  respect to the Earth, the person is then moving with a speed of

$$v = v' + u = 10 + 2 = 12 \text{ m/s} \text{ with respect to the Earth.}$$

The acceleration of an object, however, is the same in any inertial reference frame according to classical mechanics. This is because the change in velocity, and the time interval, will be the same.

For example, the person in the bus may accelerate from 0 to  $2 \text{ m/s}$  in  $1.0 \text{ s}$ ,

$$\text{So } a' = \frac{2 \text{ m/s} - 0 \text{ m/s}}{1.0 \text{ s}} = 2.0 \text{ m/s}^2 \text{ in the reference frame of the bus}$$

With respect to the Earth, the acceleration is  $a = \frac{14 \text{ m/s} - 12 \text{ m/s}}{1.0 \text{ s}} = 2.0 \text{ m/s}^2$  which is the same.

Since neither  $F$ ,  $m$ , nor  $a$  changes from one inertial frame to another, Newton's second law,  $F = ma$  does not change. Thus Newton's second law satisfies the **relativity principle**. The other laws of mechanics also satisfy the relativity principle. That the laws of mechanics are the same in all inertial reference frames implies that no one inertial frame is special in any sense. We express this important conclusion by saying that **all inertial reference frames are equivalent** for the description of mechanical phenomena.

No one inertial reference frame is any better than another. A reference frame fixed to a car or an aircraft traveling at constant velocity is as good as one fixed on the Earth. When you travel smoothly at constant velocity in a car or airplane, it is just as valid to say you are at rest and the Earth is moving as it is to say the reverse. There is no experiment you can do to tell which frame is "really" at rest and which is moving. Thus, there is no way to single out one particular reference frame as being at **absolute rest**.

### 9.1.3 Einstein's principle of relativity

The special theory of relativity has made wide-ranging changes in our understanding of nature, but Einstein based his special theory of relativity on two **postulates**:

1. The **principle of relativity**: The laws of Physics must be the same in all inertial reference frames. The first postulate can also be stated as: there is no experiment you can do in an inertial reference frame to determine if you are at rest or moving uniformly at constant velocity.
2. The **constancy of the speed of light**: The speed of light in vacuum has the same value  $c = 3.0 \times 10^8 \text{ m/s}$ , in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.
3. **Uniform motion is invariant**: A particle at rest or with constant velocity in one inertial reference frame will be at rest or have constant velocity in all inertial reference frames.

The first postulate asserts that all the laws of Physics dealing with Mechanics, Electromagnetism, Optics, Thermodynamics, and are the same in all reference frames moving with constant velocity relative to one another. This postulate is a sweeping generalization of the principle of Galilean relativity, which refers only to the laws of mechanics.

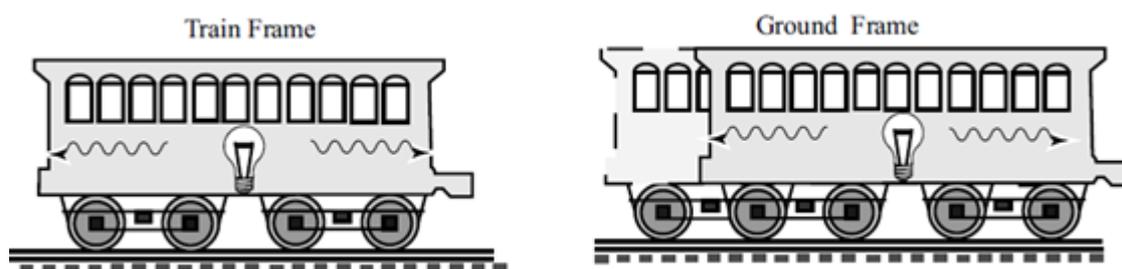
Einstein's second postulate immediately implies the following result: **It is impossible for an inertial observer to travel at  $c$ , the speed of light in vacuum.**

Note that postulate 2 is required by postulate 1: If the speed of light were not the same in all inertial frames, measurements of different speeds would make it possible to distinguish between inertial frames; as a result, a preferred, absolute frame could be identified, in contradiction to postulate 1.

These innocent-sounding propositions have far-reaching implications. Here are three:

1. **Events that are simultaneous for one observer may not be simultaneous for another.**

Two events occurring at different points in space which are simultaneous to one observer are not necessarily simultaneous to a second observer. The central point of relativity is this: Any inertial frame of reference can be used to describe events and do Physics. There is no preferred inertial frame of reference. However, observers in different inertial frames always measure different time intervals with their clocks and different distances with their meter sticks. A light flash goes off in the center of a moving train (Fig.9.5).



**Fig.9. 5A light flash goes off in the center of a moving train**

In the train's frame, the light hits the front and back of the car simultaneously. In the ground frame, the train is moving with velocity  $v$ , so the light strikes the rear of the car before reaching the front. Two events that are **simultaneous** in one frame are not simultaneous in another frame.

Nevertheless, all observers agree on the forms of the laws of Physics in their respective frames because these laws must be the same for all observers in uniform motion. For example, the relationship

$F = ma$  in a frame  $S$  has the same form  $F = ma'$  in a frame  $S'$  that is moving at constant velocity relative to frame  $S$ .

2. When two observers moving relative to each other measure a time interval or a length, they may not get the same results.
3. For the conservation principles for momentum and energy to be valid in all inertial systems, Newton's second law and the equations for momentum and kinetic energy have to be revised.

#### APPLICATION ACTIVITY 9.1

1. A 2 000 kg car moving at 20.0 m/s collides and locks together with a 1 500 kg car at rest at a stop sign. Show that momentum is conserved in a reference frame moving at 10.0 m/s in the direction of the moving car.
2. A ball is thrown at 20.0 m/s inside a boxcar moving along the tracks at 40.0 m/s. What is the speed of the ball relative to the ground if the ball is thrown (a) forward (b) backward (c) out the side door?
3. In a laboratory frame of reference, an observer notes that Newton's second law is valid. Show that it is also valid for an observer moving at a constant speed, small compared with the speed of light, relative to the laboratory frame
4. You drive north on a straight two-lane road at a constant 88 km/h . A truck in the other lane approaches you at a constant 104 km/h . Find
  - (a) the truck's velocity relative to you
  - (b) Your velocity relative to the truck.
  - (c) How do the relative velocities change after you and the truck pass each other?

## 9.2 Relativistic Dynamics

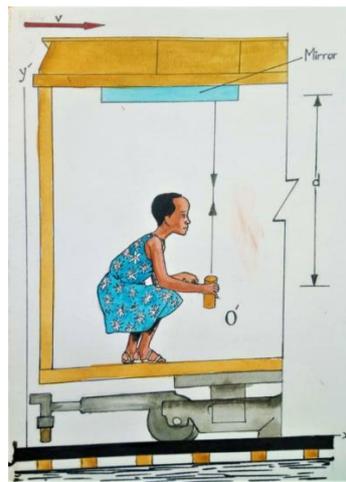
### ACTIVITY 9.2

The expressions "Time flies" and "This has been the longest day of my life" or "a pleasant day may fly past", while "unpleasant hour may seem to last forever" suggest that time do not flow equally in all situations.

Can you describe any case in which it is actually true that the flow of time is in some sense variable?

### 9.2.1 Time dilation: Moving clocks run slowly

We can illustrate the fact that observers in different inertial frames always measure different time intervals between a pair of events by considering a vehicle moving to the right with a speed  $v$ , as shown in Fig.9.6.



**Fig.9. 6 A mirror is fixed to a moving vehicle, and a light pulse is sent out by observer  $O'$  at rest in the vehicle.**

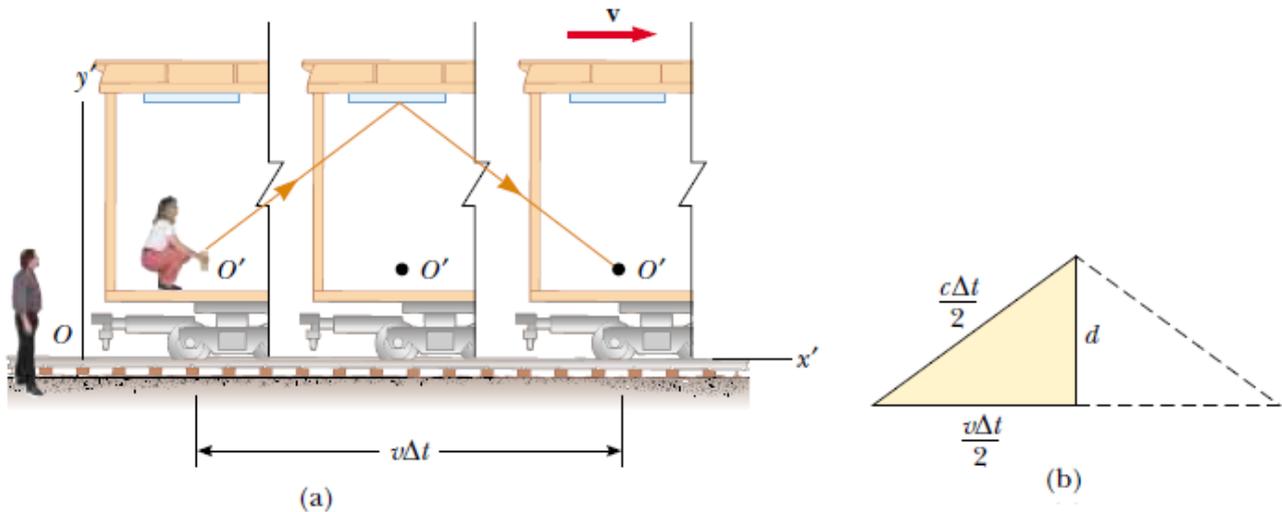
A mirror is fixed to the ceiling of the vehicle, and observer  $O'$  at rest in this system holds a laser a distance  $d$  below the mirror. At some instant, the laser emits a pulse of light directed toward the mirror (event 1), and at some later time after reflecting from the mirror, the pulse arrives back at the laser (event 2). Observer  $O'$  carries a clock  $C'$  and uses it to measure the time interval  $\Delta t_p$  between these two events. (The subscript  $p$  stands for *proper*, as we shall see in a moment.)

Because the light pulse has a speed  $c$ , the time it takes the pulse to travel from  $O'$  to the mirror and back to  $O'$  is

$$\Delta t_p = \frac{2d}{c}$$

This time interval  $\Delta t_p$  measured by  $O'$  requires only a single clock  $C'$  located at the same place as the laser in this frame.

Now consider the same pair of events as viewed by observer  $O$  in a second frame, as shown in Fig.9.7a.



**Fig.9.7 (a) Relative to a stationary observer  $O$  standing alongside the vehicle, the mirror and  $O'$  move with a speed  $v$ . Note that what observer  $O$  measures for the distance the pulse travels is greater than  $2d$ . (b) The right triangle for calculating the relationship between  $\Delta t$  and  $\Delta t_p$ .**

According to this observer, the mirror and laser are moving to the right with a speed  $v$ , and as a result the sequence of events appears entirely different. By the time the light from the laser reaches the mirror, the mirror has moved to the right a distance

$$\Delta t_p = \frac{v\Delta t}{c},$$

where  $\Delta t$  is the time it takes the light to travel from  $O'$  to the mirror and back to  $O'$  as measured by  $O$ .

In other words,  $O$  concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the laser at an angle with respect to the vertical direction. Comparing Fig 9.7 and Fig.9.7a, we see that the light must travel farther in (Fig.9.7b) than in (Fig.9.7).

Note that neither observer “knows” that he or she is moving. Each is at rest in his or her own inertial frame.

According to the second postulate of the special theory of relativity, both observers must measure  $c$  for the speed of light. Because the light travels farther in the frame of  $O$ , it follows that the time interval  $\Delta t$  measured by  $O$  is longer than the time interval  $\Delta t_p$  measured by  $O'$ . To obtain a relationship between

these two time intervals, it is convenient to use the right triangle shown in Fig.9.7b. The Pythagorean Theorem gives

$$\left(\frac{c\Delta t}{2}\right)^2 = \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

Solving for  $\Delta t$  gives 
$$\Delta t = \frac{2d}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Because  $\Delta t_p = \frac{2d}{c}$ , we can express this result as 
$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma\Delta t_p$$

Where

- $\Delta t$  represents the time interval according to the stationary system. Time interval measured by an observer who is in motion with respect to the events and who views the events as occurring at different places.
- $\Delta t_p$  represents the time interval according to the moving system (proper time). Time measured by an observer who is at rest with respect to the events and who views the events as occurring at the same place.
- $v$  represents the relative speed between the two systems
- $c$  speed of light in vacuum

A clock in motion relative to an observer would seem to be slowed down, by the beta factor

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Fig.9.7 shows a graph of  $\gamma$  as a function of the relative speed  $u$  of two frames of reference. When  $u$  is very small compared to  $\frac{u^2}{c^2}$  is much smaller than 1 and  $\gamma$  is very nearly *equal* to 1.

Because  $\gamma$  is always greater than unity, this result says that the time interval  $\Delta t$  measured by an observer moving with respect to a clock is longer than the time interval  $\Delta t_p$  measured by an observer at rest with respect to the clock. That is,  $\Delta t > \Delta t_p$  this effect is known as **time dilation**.

In general, **proper time** is the time interval between two events measured by an observer who sees the events occur at the same point in space. Proper time is always the time measured with a single clock (clock  $C'$  in our case) at rest in the frame in which the events take place.

### Example 9.3: Life time of Muon

A stationary muon decays in  $2.2 \times 10^{-6} \text{ s}$ . What is its lifetime if it is moving at  $0.97 c$  relative to laboratory clocks? How far does it travel before it decays?

**Solution:**

$$\text{The proper time } \Delta t_p = 2.2 \times 10^{-6} \text{ s} \quad \text{then } \Delta t = \frac{\Delta t_p}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1 - \left(\frac{0.97 c}{c}\right)^2}} = 9 \mu\text{s}$$

During this time it travels a distance  $d = vt_p = (0.97 \times 3 \times 10^8 \text{ m/s})(9 \times 10^{-6} \text{ s}) = 2.6 \text{ km}$

Thus, the dilated, or expanded, lifetime provides sufficient time for the muon to reach the surface of the Earth.

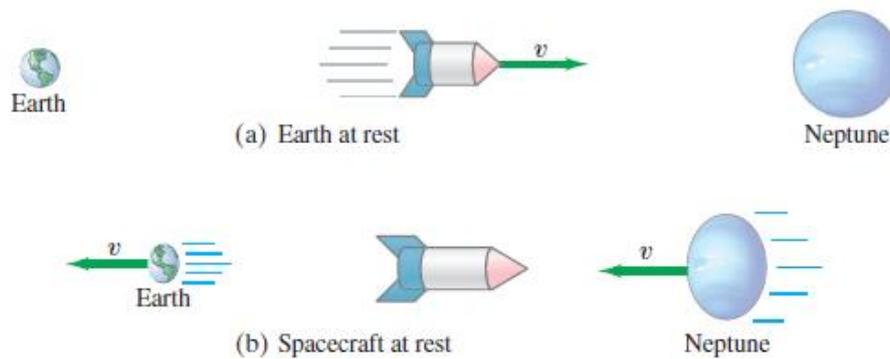
If its lifetime were only  $2.2 \times 10^{-6} \text{ s}$ , muon would travel only a distance  $S$  before disintegrating and could never reach the Earth:

$$S = vt = (0.97 \times 3 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 640 \text{ m}.$$

### 9.2.2 Length contraction

The measured distance between two points also depends on the frame of reference according to the special theory of relativity, and we illustrate this with a thought experiment. Observers on Earth watch a spacecraft traveling at speed  $v$  from Earth to, say, Neptune, Fig. 9.8a. The distance between the planets, as measured by the Earth observers, is  $L_p$ . The time required for the trip, measured from Earth, is

$$t = \frac{L_p}{v}$$



**Fig.9. 8 State of earth and spacecraft**

**Note:** (a) A spaceship traveling at very high speed from Earth to the planet Neptune, as seen from Earth’s frame of reference.

(b) According to an observer on the spaceship, Earth and Neptune are moving at the very high speed  $v$ : Earth leaves the spaceship, and a time later  $t_0$  Neptune arrives at the spaceship. (Douglass & Giancoli, 1980)

In Fig. 9.8b we see the point of view of observers on the spacecraft. In this frame of reference, the spaceship is at rest; Earth and Neptune move with speed  $v$ . The time between departure of Earth and arrival of Neptune, as observed from the spacecraft, is the “**proper time**  $t_p$ ” because these two events occur at the same point in space (i.e., at the spacecraft). That is, because of time dilation, the time for the trip as viewed by the spacecraft is

$$t_p = \frac{t}{\gamma}$$

Therefore the time interval is less for the spacecraft observers than for the Earth observers. Because the spacecraft observers measure the same speed but less time between these two events, they also measure the distance as less. If we let  $L$  be the distance between the planets as viewed by the spacecraft observers, then

$$L = vt_p = v \frac{t}{\gamma} = \frac{L_p}{\gamma}$$

The length of an object moving relative to an observer is measured to be shorter along its direction of motion than when it is at rest. It is important to note that length contraction occurs only along the direction of motion

**Example 12.4: Length contraction**

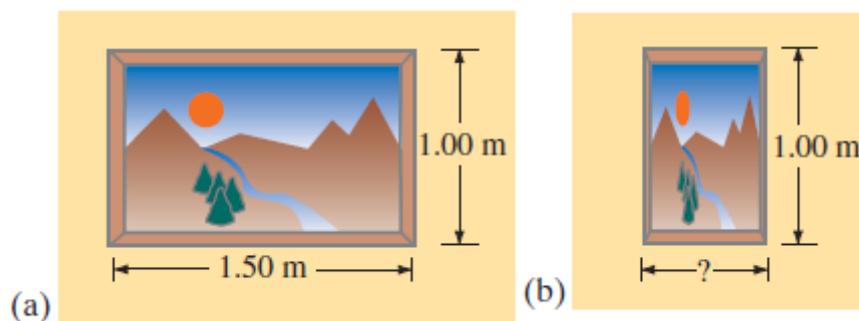
1. A spaceship flies past earth at a speed of  $0.990 c$ . A crew member on board the spaceship measures its length, obtaining the value  $400 \text{ m}$ . What length do observers measure on earth?

**Solution:**

The spaceship’s  $400 \text{ m}$  length is the *proper* length because it is measured in the frame in which the spaceship is at rest. Our target variable is the length  $l$  measured in the earth frame, relative to which the spaceship is moving at  $0.990 c$ .

$$L = L_p \sqrt{1 - \left(\frac{u}{c}\right)^2} = 400 \sqrt{1 - \left(\frac{0.990 c}{c}\right)^2} = 56.4 \text{ m}$$

2. A rectangle painting measures  $1.00 \text{ m}$  tall and  $1.50 \text{ m}$  wide see Fig.12.15. It is hung on the side wall of a spaceship which is moving past the Earth at a speed of  $0.90 C$ .



**Fig.9. 9 Painting’s contraction**

- a) What are the dimensions of the picture according to the captain of the spaceship?
- b) What are the dimensions as seen by an observer on the Earth?

**Answer**

a) The painting (as well as everything else in the spaceship) looks perfectly normal to everyone on the spaceship, so the captain sees a 1.00 m by 1.50 m painting.

b) Only the dimension in the direction of motion is shortened, so the height is unchanged at 1.00 m.

The length, however, is contracted to

$$L = L_p \sqrt{1 - \left(\frac{u}{c}\right)^2} \Leftrightarrow L = 1.50 \sqrt{1 - \left(\frac{0.90c}{c}\right)^2} = 0.65 \text{ m}$$

so the picture has dimensions 1.00 m  $\times$  0.65 m.

### APPLICATION ACTIVITY 9.2

1. A passenger on a fictional high-speed spaceship traveling between Earth and Jupiter at a steady speed of  $0.75c$  reads a magazine which takes 10.0 min according to her watch.

a. How long does this take as measured by Earth-based clocks?

b. How much farther is the spaceship from Earth at the end of reading the article than it was at the beginning?

2. An astronaut on a spaceship traveling at  $0.75c$  relative to Earth measures his ship to be 23 m long. On the ship, he eats his lunch in 28 min. (a) What length is the spaceship according to observers on Earth? (b) How long does the astronaut's lunch take to eat according to observers on Earth?

## SKILLS LAB 9

One day you happen to see aeroplane moving in space about 2 km from point of view. It is moving past your point of view.

Use relativistic knowledge to explain why

- a) i) The aeroplane appears too small as if even one man cannot fit.
- ii) Imagine at that instant the aeroplane started landing (coming down to the observer). Describe all changes in the size of the aeroplane as seen by you.
- c) Mutoni and Kalisa are all in a taxi from Remera heading to Nyabugogo. As they are seated and observe one another to be at rest. Explain why someone observing them from outside sees them moving at the same speed of the car.

**Multiple choices 1-6**

1. The principle of relativity can be best stated as
  - a) The laws of physics differ only by a constant in all reference frames differing by a constant acceleration.
  - b) The laws of physics change from one inertial reference frames to another.
  - c) The laws of physics are the same in all inertial reference frames.
  
2. An inertial reference frame is best described by
  - a. one that moves with constant acceleration
  - b. a frame that is subject to constant forces
  - c. a frame that moves with constant velocity
  - d. a frame that is subject to Galilean transformations
  
3. The most significant revolution in Physics in the 20<sup>th</sup> century has been
  - A. Bohr's theory of hydrogen atom
  - B. Nuclear fusion
  - C. Plank's quantum theory and Einstein's theory of relativity
  - D. Quantum mechanics
  
4. Relativistic formulas for time dilation, length contraction, and mass are valid
  - A. only for speeds less than  $0.10 c$ .
  - B. only for speeds greater than  $0.10 c$ .
  - C. only for speeds very close to  $c$ .
  - D. for all speeds.
  
5. Which of the following will two observers in inertial reference frames always agree on?  
(Choose all that apply.)
  - A. The time an event occurred.
  - B. The distance between two events.
  - C. The time interval between the occurrence of two events.
  - D. The speed of light.
  - E. The validity of the laws of Physics.
  - F. The simultaneity of two events.
  
6. Two observers in different inertial reference frames moving relative to each other at nearly the speed of light see the same two events but, using precise equipment, record different time intervals between the two events. Which of the following is true of their measurements?

## UNIT 10 STELLAR DISTANCE AND RADIATION

**Key unit competence:** Explain stellar radiation and stellar distance

### INTRODUCTORY ACTIVITY

From junior science classes we learn that different frames of Bunsen burner have different colours as observed in the figure below



**Fig.10. 1** Flames from a Bunsen burner

These colours are still significant in describing the temperatures of earthly objects. These objects produce energy through thermonuclear fusion in their cores. Despite their complexity and size we can analyse some of their properties by adopting a simpler model. One such useful concept is that of a *blackbody*.

Use the knowledge gained in statements above to answer the following questions:

- a) From the figure above, which flame is hotter? Explain your reasoning.
- b) In universe, we have different objects including star, planets, comets, meteorites to mention but a few. These bodies have a range of temperatures and their temperatures differ depending on different factors.
- i) What do you think would be the colour of hotter earthly objects? Base on your deductions in (a) above.
- ii) What do you think are the factors on which the temperatures of these bodies depend on?
- c) Basing your personal experience, between hot and cold objects, which ones are brighter?
- d) From your deductions made from (a) to (c) describe briefly our sun.

## 10.1 Sun's atmosphere and interior

### ACTIVITY 10.1

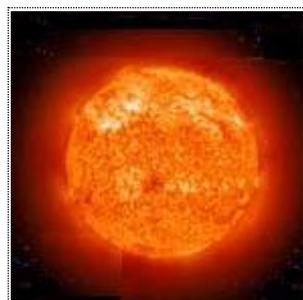
Make a comprehensive research on the interior and atmosphere of the sun.

In your research, include all parts (layers) of the sun and their temperatures and how they evolved

### 10.1.1 Sun's composition

The sun is the source of heat energy and light for the maintenance of life on Earth. It is made up of about  $2 \times 10^{30}$  kg of gas held together by its own gravity and powered by nuclear fusion at its center. Its radius is  $R_s = 6.96 \times 10^8$  m.

It is composed of about 75% hydrogen and 25% helium. About 0.1% is metals (made from hydrogen via nuclear fusion). This ratio is changing over time (very slowly), as the nuclear reactions continue, converting smaller atoms into more massive ones. And trace amounts of other elements — oxygen, carbon, neon, nitrogen, magnesium, iron and silicon.



**Fig.10. 2 Our Sun**

The particles in a gas are moving in random directions, with speeds that depend on the temperature. Hotter gases have faster particles, and cooler gases have slower particles, on average. The average speed depends on the mass,  $m$ , of the particles:

$$v = \sqrt{\frac{8kT}{\pi m}}$$

Where  $k = 1.38 \times 10^{-23} \text{ J / K}$  Boltzmann constant

There will be some particles moving faster than this speed, and some moving slower. If this average speed is greater than 1/6 the escape velocity of a planet, the gas will eventually escape, and the planet will no longer have an atmosphere. This equation only holds for an ideal gas under equilibrium conditions, where it is neither expanding nor contracting

### Example 10.1

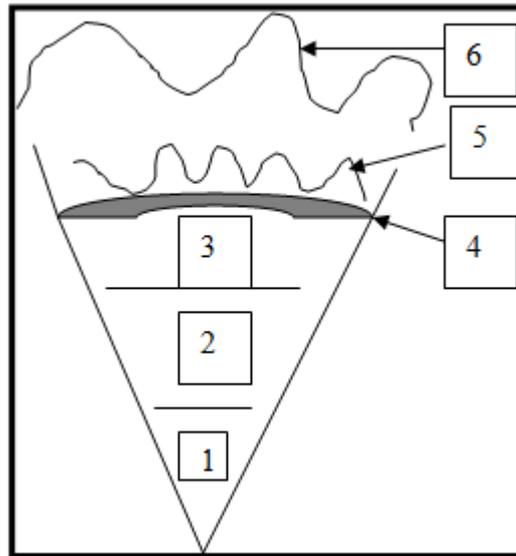
What is the average speed of nitrogen molecules ( $m = 4.7 \times 10^{-26} \text{ kg}$ ) at  $75^\circ\text{F}$ ?

**Solution:**

First, convert  $75^\circ\text{F}$  to kelvin  $v = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8(1.38 \times 10^{-23} \text{ J / K})(297 \text{ K})}{\pi(4.7 \times 10^{-26} \text{ kg})}} = 470 \text{ m / s}.$

#### 10.1.2 Main layers of Sun's atmosphere

Scientists who study the Sun usually divide it up into three main regions: the Sun's **interior**, the solar **atmosphere**, and the visible "**surface**" of the Sun which lies between the interior and the atmosphere.



**Fig.10.3 1 – core (nuclear reactions) ; 2 – radiative zone; 3 – convective zone; 4 – photosphere; 5 – chromosphere; 6 – corona**

The atmosphere of the sun is composed of several layers, mainly the **photosphere**, the **chromosphere** and the **corona**. It's in these outer layers that the sun's energy, which has bubbled up from the sun's interior layers, is detected as sunlight.

### **Photosphere**

The boundary between the Sun's interior and the solar atmosphere is called the **photosphere (light emitting layer)**. It is what we see as the visible "surface" of the Sun. The temperature in the photosphere varies between about 6500 K at the bottom and 4000 K at the top. The photosphere is not like the surface of a planet; even if you could tolerate the heat you couldn't stand on it.

It is the lowest layer of the sun's atmosphere and it is about 500 *km* thick. This layer is where the sun's energy is released as light. Because of the distance from the sun to Earth, light reaches our planet in about eight minutes.

The emitted energy per second per meter square that arrives from the sun at the top of the Earth's atmosphere is called the **solar constant**. It has been measured to be

$$b_o = 1.36 \times 10^3 \text{ W} / \text{m}^2$$

This quantity for stars other than the Sun is called the **apparent brightness b**.

## Sunspot

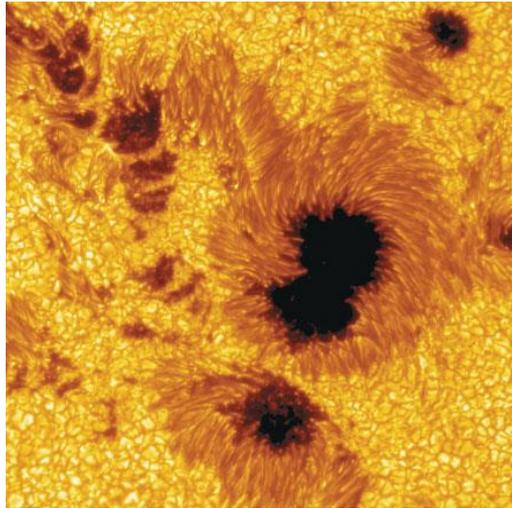
Sunspots are dark marks on the photosphere of the Sun. Often, groups of small spots will merge to form large spots, which can be as large as 50 000 km across (four times the diameter of the Earth), and last for months. The inner, dark part of a sunspot is called the **umbra**. Around this is the **penumbra**. The region surrounding a cluster of sunspots is called an **active region**.

Sunspots appear to move across the sun's disk. Observing this motion led astronomers to realize that the sun rotates on its axis. Since the sun is a ball of gas with no solid form, different regions rotate at different rates. The **sun's equatorial** regions **rotate** in about **24 days**, while the Polar Regions take more than **30 days** to make a complete rotation. The photosphere is also the source of solar flares: tongues of fire that extend hundreds of thousands of kilometres above the sun's surface. Solar flares produce bursts of X-rays, ultraviolet radiation, electromagnetic radiation and radio waves.

Sunspots are **dark** (in visible light) because the umbra is about 2 000 K cooler than the rest of the photosphere. However, even though they appear dark, they are still very hot. Sunspots have temperatures around 6 300 Fahrenheit (~3 500 Celsius) while the surrounding surface of the sun has a temperature of about 10 000 Fahrenheit (5 500 Celsius). If a sunspot was alone in space, it would glow brightly. A sunspot against the night sky would appear about 10 times as bright as the full moon. Sunspots only seem dim because the rest of the Sun is so bright.

Sunspots are regions of strong magnetic fields. These magnetic fields trap the particles that have bubbled up to the surface by convection. These particles can't cross the magnetic field lines, and so they can't move out of the way to allow new hot material to bubble up. The particles cool off, and therefore darken, but remain in place.

The magnetic field comes out of the Sun at one location, and goes back into the Sun at another. Figure below. This means that sunspots always come in pairs. One spot will have north polarity like the north pole of a bar magnet, and the other will have south polarity.



**This close-up view of the sun's surface shows two dark sunspots.** Their temperature is about 4000 K, while the surrounding solar material is at 5800 K.

From the Stefan–Boltzmann law, the intensity from a given area of sunspot is only  $\left(\frac{4\,000}{5\,800}\right)^4 = 0.23$  as great as the intensity from the same area of the surrounding material—which is **why sunspots appear dark**.

The Sun's surface is mottled with upwelling convection currents seen as hot, bright granules. Solar wind particles stream out into the solar system through coronal holes. When such particles reach the vicinity of Earth, they produce **auroras**, which are strongest near Earth's magnetic poles.

## **Chromosphere**

The next layer is the **chromosphere** which is a layer in the Sun between about 400 km and 2100 km above the solar surface (the photosphere). The temperature in the chromosphere varies between about 4 000 K at the bottom (the so-called temperature minimum) and 8 000 K at the top.

So in this layer (and higher layers) it actually gets hotter if you go further away from the Sun, unlike in the lower layers, where it gets hotter if you go closer to the center of the Sun. The chromosphere emits a reddish glow as super-heated hydrogen burns off. But the red rim can only be seen during a total solar eclipse. At other times, light from the chromosphere is usually too weak to be seen against the brighter photosphere.

**Transition Region** - The transition region is a very narrow 100 km layer between the chromosphere and the corona where the temperature rises abruptly from about 8 000 K to about 500 000 K.

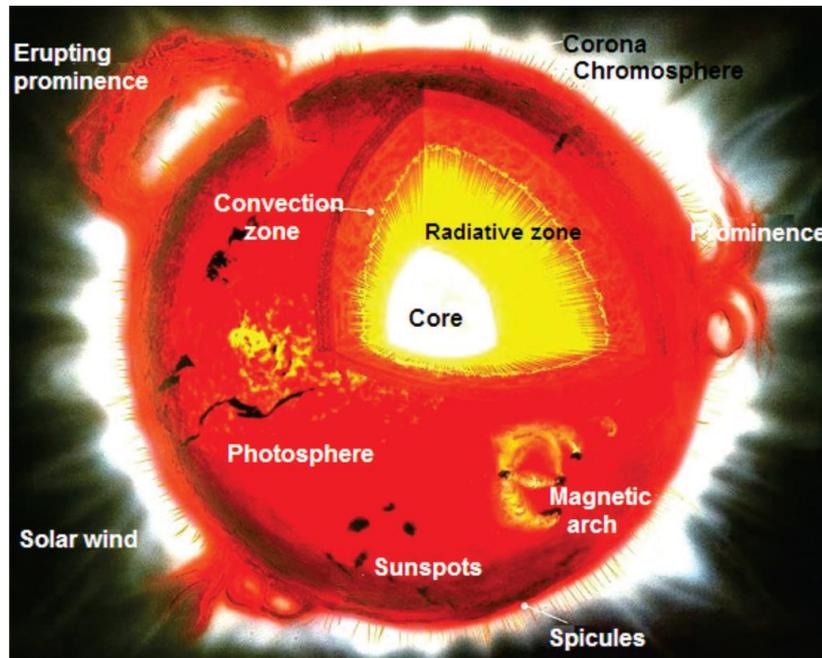
## Corona

The third layer of the sun's atmosphere is the **corona**. The corona is the outermost layer of the Sun, starting at about 2100 km above the solar surface (the photosphere). The temperature in the corona is 500 000 K or more, up to a few million K. The corona cannot be seen with the naked eye except during a total solar eclipse, or with the use of a **coronagraph**.

It appears as white streamers or plumes of ionized gas that flow outward into space. The corona does not have an upper limit. As the gases cool, they become the solar wind.

### 10.1.3 Main layers of Sun's interior

There are three main parts to the Sun's interior: the core, the radiative zone, and the convective zone.

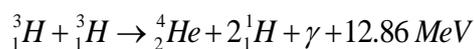
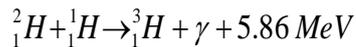
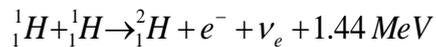


## Core

The core is at the center. It is the **hottest region**, where the nuclear fusion reactions that power the Sun occur. The core of the Sun extends from the center to about 20–25% of the solar radius (200 000 km thick). It has a density of up to  $150 \text{ g/cm}^3$  and a temperature of close to  $15.7 \times 10^6 \text{ K}$ .

By contrast, the **Sun's surface temperature** is approximately **5 800 K**. Through most of the Sun's life, energy has been produced by nuclear fusion in the core region through a series of steps called the p–p (proton–proton) chain; this process converts hydrogen into helium.

Only 0.8% of the energy generated in the Sun comes from the CNO cycle, though this proportion is expected to increase as the Sun becomes older.



The core is the only region in the Sun that produces an appreciable amount of thermal energy through fusion; 99% of the power is generated within 24% of the Sun's radius, and by 30% of the radius, fusion has stopped nearly entirely. The remainder of the Sun is heated by this energy as it is transferred outwards through many successive layers, finally to the solar photosphere where it escapes into space as sunlight or the kinetic energy of particles.

### **Radiative zone**

Moving outward, next comes the radiative (or radiation) zone. Its name is derived from the way energy is carried outward through this layer, carried by photons as thermal radiation.

From the core out to about 0.7 solar radii, thermal radiation is the primary means of energy transfer. The temperature drops from approximately 7 million to 2 million kelvins with increasing distance from the core. This temperature gradient is less than the value of the adiabatic lapse rate and hence cannot drive convection, which explains why the transfer of energy through this zone is by radiation instead of thermal convection. Ions of hydrogen and helium emit photons, which travel only a brief distance before being reabsorbed by other ions.

The density drops a hundredfold (from 20 g/cm<sup>3</sup> to 0.2 g/cm<sup>3</sup>) from 0.25 solar radii to the 0.7 radii, the top of the radiative zone.

### **Tachocline**

The radiative zone and the convective zone are separated by a transition layer, the tachocline. This is a region where the sharp regime change between the uniform rotation of the radiative zone and the differential rotation of the convection zone results in a large shear between the two .

A condition where successive horizontal layers slide past one another. Presently, it is hypothesized that a magnetic dynamo within this layer generates the Sun's magnetic field

### **Convective zone**

The Sun's convection zone extends from 0.7 solar radii to near the surface (thick 200 000 km). In this layer, the solar plasma is not dense enough or hot enough to transfer the heat energy of the interior outward via radiation. Instead, the density of the plasma is low enough to allow convective currents to develop and move the Sun's energy outward towards its surface.

Material heated at the tachocline picks up heat and expands, thereby reducing its density and allowing it to rise. As a result, an orderly motion of the mass develops into thermal cells that carry the majority of the heat outward to the Sun's photosphere above.

Once the material diffusively and radiatively cools just beneath the photospheric surface, its density increases, and it sinks to the base of the convection zone, where it again picks up heat from the top of the radiative zone and the convective cycle continues. At the photosphere, the temperature has dropped to 5 700 K and the density to only 0.2 g/m<sup>3</sup>.

#### **APPLICATION ACTIVITY 10.1**

1. What are sunspots?

- A. Not Sure
- B. Spots you see after staring at the Sun too long
- C. Optical illusions caused by the Sun's atmosphere
- D. Cooler areas on the surface of the Sun that appear darker

2. The average speed of atoms in a gas is 5 km/s. How fast will they move if the temperature increases by a factor of four?

3. What part of the sun do we see from Earth?

4. State three layers of the sun's atmosphere

5. Make a sketch of the Sun's atmosphere showing the locations of the photosphere, chromosphere, and corona. What is the approximate temperature of each of these regions?

6. Why does Mercury have no significant atmosphere

## 10.2 Brightness and Magnitude Scale

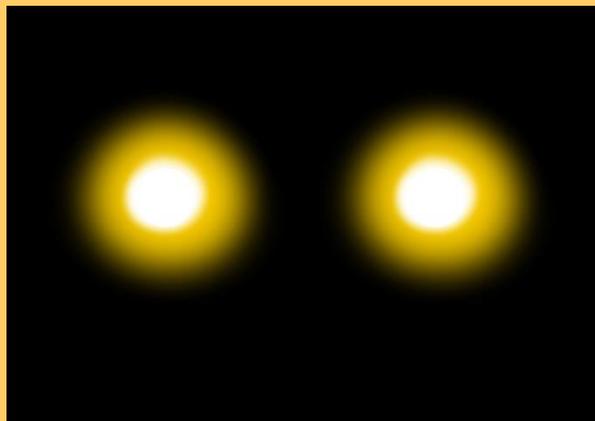
### ACTIVITY 10.2

#### Apparatus required

3 Light torches with two identical torches, meter rule, dry cells

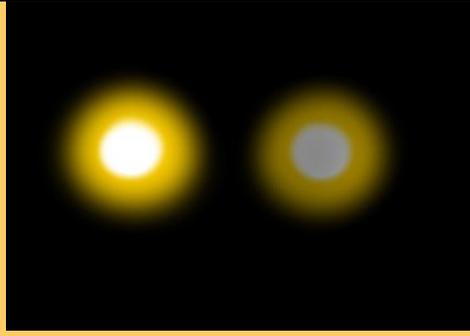
#### Procedures

- Take two identical torches with the same brightness
- move one torch to a distance of say 50 m,
- Observe the brightness of the torches. Is the brightness still the same? Why?



**Fig.10. 4**Two identical torches with the same brightness

- Repeat with two torches with different brightness, one bright and the other one not so bright
- Leave to a distance of say 100 m with the bright one,
- observe the brightness of the torches compare their brightness.



**Fig.10. 5 Two torches at the same distance with brightest torch (right)**

### 10.2.1 Brightness and luminosity

Some stars look bright only because they're near Earth. Astronomers call the true, intrinsic brightness of a star its **intrinsic luminosity**,  $L$  (or simply **luminosity**), which is its total amount of energy per unit time (total power) radiated by a **star**, galaxy, or other astronomical object in a given spectral region. In SI units, **luminosity** is measured in joules per second or watts.

**Apparent brightness  $b$**  is a term used to indicate how bright a star appears to an observer on Earth. It is defined as the power crossing unit area at the Earth perpendicular to the path of the light.

Given that energy is conserved, and ignoring any absorption in space, the total emitted power  $L$  when it reaches a distance  $r$  from the star will be spread over a sphere of surface area  $4\pi r^2$ , then the **luminosity  $L$ , brightness (flux) and distance  $r$**  are related by the inverse square law:

$$b = \frac{L}{4\pi r^2}$$

Where

- $b$  apparent brightness (power per unit area at Earth), for the sun

$$b = b_o = 1.36 \times 10^3 \text{ W} / \text{m}^2$$

- $L$  luminosity, for the sun

$$L = b_o A = (1.36 \times 10^3 \text{ W} / \text{m}^2)(4\pi)(1.50 \times 10^{11} \text{ m})^2 = 3.85 \times 10^{26} \text{ W}$$

The **luminosity** of any star is the product of the surface area times the surface temperature raised to the fourth power using the Stefan-Boltzmann law.

$$L = 4\pi\sigma R^2 T^4$$

Where

- L luminosity, the rate at which an object radiates energy
- $\sigma = 5.67 \times 10^{-8} \text{ W / m}^2 \cdot \text{K}$  is Universal constant called the **Stefan-Boltzmann constant**
- T is absolute temperature
- The factor  $\varepsilon$  called the **emissivity**, is a number† between 0 and 1 that is characteristic of the surface of the radiating material

The intensity (power per unit area) radiated by black body in thermal equilibrium is given by

$$I = \sigma T^4$$

Three factors control the apparent brightness of a star as seen from earth: how **big** it is, how **hot** it is, and how **far away** it is. For two stars of magnitudes  $m_1$  and  $m_2$  and apparent brightness  $b_1$  and  $b_2$ , respectively, we have

$$\frac{b_1}{b_2} = 2.5^{m_2 - m_1} = 100^{\frac{m_2 - m_1}{5}}$$

Each increase in magnitude corresponds to a decrease in brightness by a factor of  $100^{\frac{1}{5}} \approx 5$  (so that an increase of five magnitudes corresponds to a decrease in brightness of 100). In other words, a 1<sup>st</sup>-magnitude star is 100 times brighter than a 6<sup>th</sup>-magnitude star – or conversely, a 6<sup>th</sup>-magnitude star is 100 times dimmer than a 1<sup>st</sup>-magnitude star. The fifth root of 100 approximately equals 2.512, so a difference of one magnitude corresponds to a brightness factor of about 2.512 times.

- 1 $m$ : brightness factor of 2.512
- 2 $m$ : brightness factor of  $2.512 \times 2.512 = 6.31$
- 3 $m$ : brightness factor of  $2.512 \times 2.512 \times 2.512 = 15.84$
- 4 $m$ : brightness factor of  $2.512 \times 2.512 \times 2.512 \times 2.512 = 39.81$
- 5 $m$ : brightness factor of  $2.512 \times 2.512 \times 2.512 \times 2.512 \times 2.512 = 100$

## Example 10.2

1. Given a star whose radius is 3 solar and a surface temperature that's 2 solar, calculate its luminosity.

**Solution:**

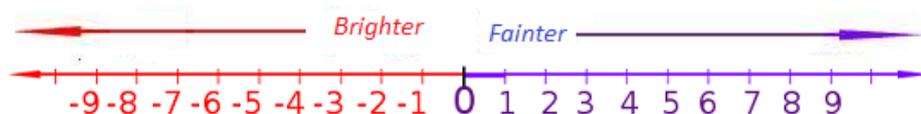
$$\text{Luminosity of the sun } L_s = 4\pi\sigma R^2 T^4$$

$$\text{Luminosity of star: } L = 4\pi\sigma(3R)^2 (2T)^4 = 144(4\pi\sigma R^2 T^4) = 144L_s$$

### 10.2.2 Stellar Magnitude

**Stellar magnitude (magnitude of a star or Apparent magnitude)** abbreviated as **m** is a measure of brightness of celestial body or a **star** as seen by an observer on Earth. Lower numbers represent brighter objects than higher numbers; very bright stars are 1<sup>st</sup> magnitude, less bright stars are 2<sup>nd</sup> magnitude, etc. as shown in Fig.10.10.

In ancient times, **stars** were ranked in six **magnitude** classes, the first **magnitude** class containing the brightest **stars**. The star **Vega** has magnitude 0, by definition. Stars fainter than Vega have positive magnitude. Stars brighter than Vega have negative magnitudes.



**Fig.10. 6 lower numbers represent brighter stars than higher numbers. Fainter stars have larger magnitudes**

The full moon has an apparent magnitude of -12.6; the sun's is -26.8. We can see objects up to 6<sup>th</sup> magnitude without a telescope. This system of rating the brightness of celestial objects was developed by the Greek astronomer **Hipparchus** in 120 B.C.

**Absolute magnitude M** is the measure of intrinsic brightness of a celestial object. It is the hypothetical apparent magnitude of an object at a standard distance of exactly 10 parsecs (**32.6** light years) from the observer, assuming no astronomical extinction of starlight. Our Sun has absolute magnitude of 4.8. The lower the number, the brighter the object. Negative numbers indicate extreme brightness.

The relationship between apparent magnitude,  $m$ , absolute magnitude,  $M$ , and distance,  $d$ , in parsecs, is given by the **distance modulus**

$$m - M = 5 \log \frac{d}{10}$$

### Example 10.3

What is the absolute magnitude of (a) the Sun? (The apparent magnitude is  $-26.72$ .), (b) Deneb (apparent magnitude of  $1.26$  and is  $490$  pc away)

**Solution:**

$$m = M + 5 \log \frac{d}{10} \Leftrightarrow M = m - 5 \log \frac{d}{10} = -26.72 - 5 \log \frac{206265 \text{ pc}}{10 \text{ pc}} = 4.85$$

### APPLICATION ACTIVITY 10.2

1. How do you find the absolute magnitude of a star?
2. What are the two factors that determine the brightness of a star?
3. Why do stars have different levels of brightness?
4. What is the luminosity of a star that is twice as big as the sun but has the same surface temperature?
5. What is the luminosity of a star that is half as big as the sun and twice as hot?
6. Estimate the size of a star that has the same surface temperature as the sun but produces 100 times as much energy.
7. A Cepheid variable star in the Virgo cluster has an absolute magnitude of  $-5$ , and an apparent magnitude of  $26.3$ . How far away is the Virgo cluster?

## 10.3 Types of stars and spectra of stars

### ACTIVITY 10.3

Stars are born and stars die.

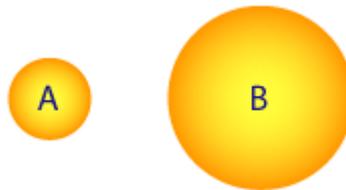
- a) Can you describe all the changes that may take place right from the time of formation to when it dies?

b) Do you think their sizes change as they develop? If yes explain what causes the change. If Explain why not?

### 10.3.1 Classification by size and mass

#### The size (radius) of stars

As the **size** of a star increases, luminosity increases. Large stars are brighter than small stars of the same temperature. If you think about it, a larger star has more **surface area**. That increased **surface area** allows more light and energy to be given off.



**Fig.10. 7 Stars A and B have the same temperature. B is bigger than A and has greater surface. B has greater intrinsic luminosity**

#### Mass of stars

The luminosity for most stars depends on the mass: the more massive the star, the greater its luminosity. The smallest stars have masses of about  $0.08 M_{\text{Sun}}$ . Objects less massive than this never begin hydrogen fusion, and so are never technically considered stars.

Objects just below the cutoff often emit energy produced by gravitational collapse, and are called **brown dwarfs**. These objects are brightest in the infrared. The most massive stars known (for example, Eta Carinae) are as large as  $150 M_{\text{Sun}}$ .

In general, more massive main-sequence stars are larger (this rule does not necessarily apply to stars that are not main-sequence stars). Also, more massive stars have higher gravity, causing higher temperature and pressure in the core, which speeds the hydrogen fusion reaction.

More massive stars produce more energy and are hotter. Since the radius is related to the luminosity, and the mass of the star governs the rate of energy production, there is a mass–luminosity relationship for main-sequence stars:

$$\frac{L}{L_{\text{sun}}} = \left(\frac{M}{M_{\text{sun}}}\right)^{3.5}$$

where

- $L$  is the luminosity of the main-sequence star,  $L_{\text{Sun}}$  is the luminosity of the Sun,
- $M$  is the mass of the main-sequence star, and  $M_{\text{Sun}}$  is the mass of the Sun.

### Example 10.4 Estimate the star's radius

The giant star Betelgeuse emits radiant energy at a rate  $10^4$  times greater than our Sun, whereas its surface temperature is only half (2900 K) that of our Sun. Estimate the radius of Betelgeuse, assuming  $\varepsilon = 1$ . The Sun's radius is  $R_s = 7 \times 10^8 \text{ m}$

#### Solution:

We assume both stars are spherical, with surface area  $A = 4\pi R^2$

From Stefan-Boltzmann law:  $L = \varepsilon\sigma AT^4 \Leftrightarrow A = \frac{L}{\varepsilon\sigma T^4}$

$$\text{Then } \frac{4\pi R_B^2}{4\pi R_s^2} = \frac{L}{\varepsilon\sigma T_s^4} \times \frac{\varepsilon\sigma T_B^2}{L} \Leftrightarrow R_B = \frac{T_s^2}{T_B^2} R_s = 400 \times 7 \times 10^8 = 28 \times 10^{10} \text{ m}$$

If Betelgeuse were our Sun, it would envelop us (Earth is  $150 \times 10^9 \text{ m}$  from the Sun)

### The main-sequence lifetime of stars

Once hydrogen burning begins in the core of a protostar, it becomes a main sequence star. The main-sequence (hydrogen-burning) phase is by far the largest fraction of a star's lifetime, and therefore most stars observed in the sky are main sequence stars.

During this time, stars burn quietly, and change slowly. The length of time a star remains on the main-sequence depends on the mass. High-mass stars burn hot and die young. This is because when the mass is higher, there is more gravity, and the pressures and temperatures inside the star are higher, which enhances the rate of hydrogen fusion.

The relationship between main-sequence lifetime,  $t$ , and mass,  $M$ , of a star can be expressed mathematically as

$$\frac{t}{t_s} = \left(\frac{M}{M_s}\right)^{-2.5}$$

where

- $t_s = 10^{10}$  yr is the main-sequence lifetime of the Sun
- $M_{\text{Sun}}$  is the mass of the Sun.

Sun's main-sequence lifetime is estimated to be 10 billion years. A star 10 times as massive as the Sun has a main-sequence lifetime of only 30 million years. A star 0.1 times as massive as the Sun has a main-sequence lifetime of 3 trillion years.

As stars begin to use up their hydrogen, they increase in size; however, the surface temperature does not change nearly as much. As the radius increases, the luminosity increases as the square of the radius. For example, during the next 5 billion years, the Sun will increase in luminosity by about 60%.

### Example 10.5 Estimate the star's radius

Globular clusters are groups of stars all born at the same time out of the same cloud, the highest mass star in the globular cluster still on the main sequence is about  $2 M_{\text{Sun}}$ . How old is this globular cluster?

#### Solution:

Since, the age of the globular cluster must be the main-sequence lifetime of a  $2 M_{\text{Sun}}$  star:

$$t = t_s \left( \frac{M}{M_s} \right)^{-2.5} = 10 \times 10^9 (2)^{-2.5} = 1.8 \times 10^9 \text{ yr}$$

This imaginary globular cluster is about 2 billion years old

The luminosity class is a way of talking about the radius. Stars are classified into five main luminosity classes, labelled by Roman numerals. These are the five luminosity classes:

#### I Super-giants

- Very massive and luminous stars near the end of their lives. Stars with larger radii more than 100 times that of the sun
- They are sub classified as Ia or Ib, with Ia representing the brightest of these stars.
- These stars are very rare one star in a million stars is a supergiant
- The nearest supergiant star is Canopus (F0Ib) 310 light years away.
- Some other examples are Betelgeuse (M2Ib), Antares (M1Ib) and Rigel (B8Ia).

## II Bright Giants

- Stars which have luminosity between the giant and supergiant stars.
- Some examples are Sargas (F1II) and Alphard (K3II).

## III Normal Giants

- These are mainly low-mass stars at the end of their lives that have swelled to become a giant star.
- This category also includes some high mass stars evolving on their way to supergiant status.
- Some examples are Arcturus (K2III), Hadar (B1III) and Aldebaran (K5III).

## IV Sub-giants

- Stars which have begun evolving to giant or supergiant status.
- Some examples are Alnair (B7IV), Muphrid (G0IV), Procyon (F5IV-V).

## V Main-sequence Dwarfs

- All normal hydrogen-burning stars, Stars spend most of their lives in this category before evolving up the scale.
- Class O and B stars in this category are actually very bright and luminous and generally brighter than most Giant stars.
- Some examples are the Sun (G2V), Sirius (A1V), and Vega (A0V).
- **White dwarf** stars (luminosity class **D**) are the final evolutionary stage of low to intermediate mass stars, and are found in the bottom left of the HR diagram. These stars are very hot but have low luminosities due to their small size.

### 10.2.2 Classification of stars according to their spectral lines

#### Temperature

**Temperature** also affects a star's luminosity. Hot stars are brighter than cool stars of the same size and radiate blue or blue-white.

As the temperature of a body increases, the spectrum shifts from predominantly lower frequencies (and longer wavelengths, such as red) to higher frequencies (and shorter wavelengths such as blue).

We can use color in a more precise way to measure a star's temperature with **Wien's law**. It states that a body's temperature,  $T$ , in Kelvin is given by the following:

$$T = \frac{2.90 \times 10^{-3}}{\lambda_m}$$

Where  $\lambda_m$  is the wavelength at the peak of the spectrum of light emitted by a black body

Thus the longer the wavelength of the maximum emitted energy, the lower is the temperature of the radiating body. The surface temperatures of stars typically range from about 3 000 K (reddish) to about 50 000 K (UV). Wien's law tell us that the hotter the object, the bluer its radiation.

### Example 10.4

Suppose that the distances from Earth to two nearby stars can be reasonably estimated, and that their measured apparent brightnesses suggest the two stars have about the same luminosity,  $L$ . The spectrum of one of the stars peaks at about 700 nm (so it is reddish). The spectrum of the other peaks at about 350 nm (bluish). Use Wien's law to determine the surface temperature of each star.

#### Solution:

The temperature of the reddish star  $T_a = \frac{2.90 \times 10^{-3}}{\lambda_m} = \frac{2.90 \times 10^{-3} \text{ m.K}}{700 \times 10^{-9} \text{ m}} = 4\,140 \text{ K}$

The temperature of the bluish star:  $T_b = \frac{2.90 \times 10^{-3}}{\lambda_m} = \frac{2.90 \times 10^{-3} \text{ m.K}}{350 \times 10^{-9} \text{ m}} = 8\,280 \text{ K}$

Stars that are fusing hydrogen into helium in their cores are called **main-sequence stars**. Because stars spend most of their lifetimes fusing hydrogen into helium, most stars are main-sequence stars. The Sun, for example, is a main-sequence star.

The most common method of classification is known as the **Morgan–Keenan (MK)** system, which classifies stars based on decreasing temperature using the letters **O** (blue; surface temperatures 30 000 K-40 000 K), **B** (blue), **A** (blue-white), **F** (white), **G** (yellow), **K** (red-orange), **M** (red; surface temperatures around 3 000 K), Each spectral class is then divided into 10 subclasses, with 0 being hottest and 9 being coolest. An O0 star is the hottest, followed by an O1, O2, etc. O9 is just a little bit hotter than a B0, and so on. Most stars can be classified in this way as shown in Table 10.2

An easy mnemonic for remembering these is: "Oh be a fine guy/girl, kiss me."

Star class	Color	Surface Temperature	Mass ( $m_s = 1$ )	Radius ( $R_s = 1$ )	Luminosity ( $L_s = 1$ )	Examples
<b>O</b>	Blue	over 25 000 K	<b>60</b>	<b>15</b>	<b>1 400 000</b>	Lacertra
<b>B</b>	Blue	11 000 – 25 000 K	<b>18</b>	<b>7</b>	<b>20 000</b>	Rigel (B8Ia), Spica
<b>A</b>	Blue-White	7 500 – 11 000 K	<b>3.2</b>	<b>2.5</b>	<b>80</b>	Sirius (A1V), Vega (A0V)
<b>F</b>	Blue to White	6 000 – 7 500 K	<b>1.7</b>	<b>1.3</b>	<b>6</b>	Canopus (F0IV), Procyon(F5IV)
<b>G</b>	White to Yellow	5 000 – 6 000 K	<b>1.1</b>	<b>1.1</b>	<b>1.2</b>	Sun (G2V), Capella ( K0III, G1 III )
<b>K</b>	Orange to Red	3 500 – 5 000 K	<b>0.8</b>	<b>0.9</b>	<b>0.4</b>	Arcturus (K2III), Aldebaran (K5III)
<b>M</b>	Red	under 3 500 K	<b>0.3</b>	<b>0.4</b>	<b>0.04</b> (very faint)	Betelgeuse (M2Ib), Antares (M1Ib)

**Table 10.1 Characteristics of stars of different spectral classes**

The **color** and the **temperature** of a star depend on its surface temperature. But a blue **star** doesn't emit only blue light, nor does a red **star** emit only red light. They emit visible light of all **colors** to some degree. It's just that their spectrum peaks at a particular **color**.

- **The Hottest stars** (i.e., Spectral Type **O** and **B**) emit mostly at blue and ultra-violet wavelengths, making them appear blue or white dwarfs. In astronomy, a **blue giant** is a hot **star** with a luminosity class of III (giant) or II (bright giant). e.g., Sirius B, the small companion of the brightest star Sirius).
- **Cool stars** (i.e., **Spectral Type K** and **M**) radiate most of their energy in the red and infrared region of the electromagnetic spectrum and thus appear red. The temperature of the coolest stars (**red dwarfs**) is around 5 000 °C and its color is **orange** followed by **yellow** (like our **Sun**, a G star with temperature of around 6 000 °C and glows orange/yellow).
- **The coolest stars** are red and the smallest, they are called **red dwarfs** and are called M-type (Red giants e.g., Aldebaran, the brightest star in the constellation of Taurus).

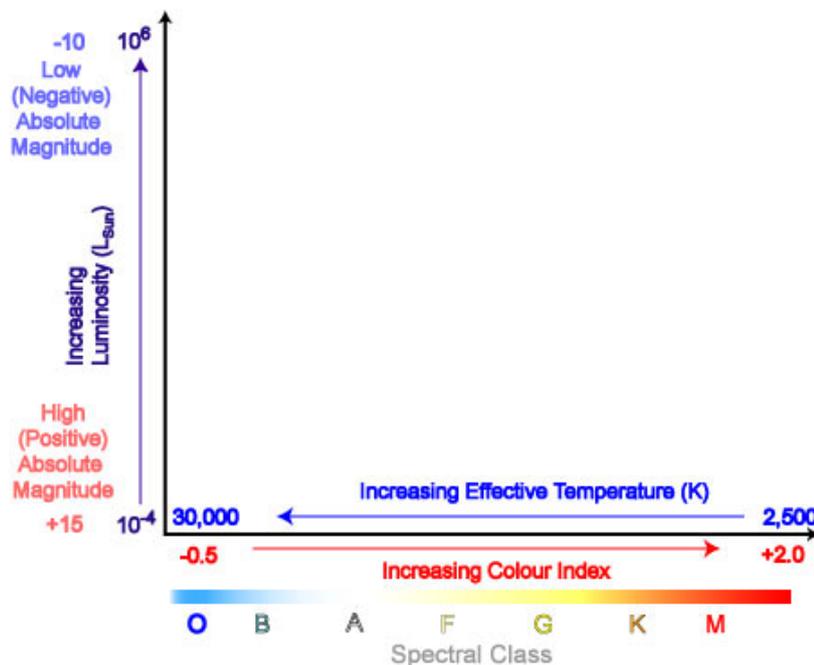
The reason most **stars** appear **white** to us is because we have two different kind of light sensors in our eyes. Sensors called "rods" detect brightness, while sensors called "cones" detect color. The cones are not very sensitive, so if a light is too dim they are not activated, and we perceive the color as **white**. Since **blue** light is higher frequency than red light, that means that things that glow red are cooler than things that glow **blue**. So: red **stars** are cooler!

Red dwarfs are small stars in terms of their mass. Large red dwarfs have a mass have about half the mass of our **Sun**, but they can be as small as 0.075 solar masses (a bit more than 78 **Jupiter** masses).

### 10.2.3 Star types on a Hertzsprung–Russell (H-R) diagram

The **Hertzsprung - Russell** diagram (Fig.10.7) is a graphical tool that astronomers use to **classify stars** according to their luminosity, spectral type, color, temperature and evolutionary stage.

Developed independently in the early 1900s by Ejnar **Hertzsprung** and Henry Norris **Russell** (**Fig.10.7**), it plots the temperature of stars against their luminosity (the theoretical **HR diagram**), or the colour of stars (or spectral type) against their absolute magnitude (the observational **HR diagram**, also known as a colour-magnitude ...

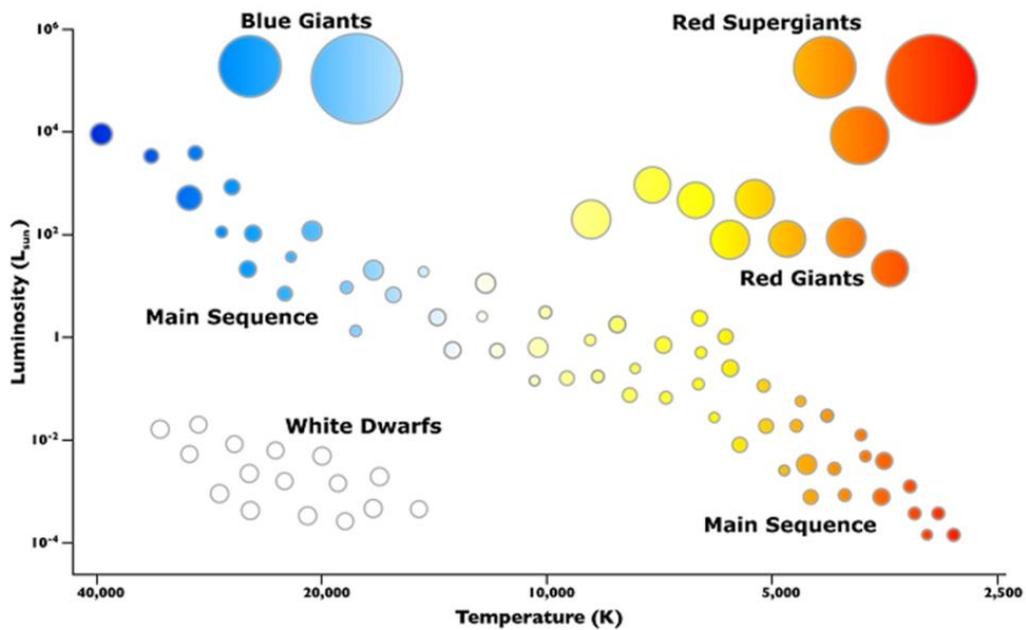


**Fig.10. 8 Axes for H-R diagram**

There are 3 main regions (or evolutionary stages) of the HR diagram:

### a) Main sequence stars

Stars that are fusing hydrogen into helium in their cores as their primary source of energy are called **main-sequence stars** (Stars that fall on this diagonal band). Because stars spend most of their lifetimes fusing hydrogen into helium, most stars are main-sequence stars. The Sun, for example, is a main-sequence star.



**Fig.10. 9 Hertzsprung–Russell (H–R) diagram.**

**Note:** The figure 10.9 is a logarithmic graph of luminosity vs. surface temperature  $T$  of stars according to their luminosity, spectral type, color, temperature and evolutionary stage. Stars in the stable phase of hydrogen burning lie along the Main Sequence according to their mass.

Starting at the lower right we find the coolest stars approximately 2 500 K (cool, faint stars) their light output peaks at long wavelengths, so they are reddish in color and dim (absolute magnitudes of about 13, only about 1/10 000 the luminosity of our Sun).

They are also the least luminous and therefore of low mass. Barnard's star and Proxima Centauri are examples. These are both cool. Farther up toward the left we find hotter (hot, luminous stars) and more luminous stars that are whitish, like our Sun and Alpha Centauri. Still farther up we find even more luminous and more massive stars, bluish in color such as Sirius A, Vega, Altair and Procyon A.

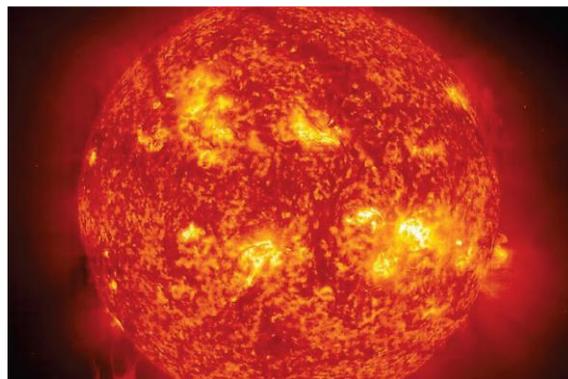
## Turnoff Point

The point at which the stars deviate from the main sequence after using up most of their fuel is known as the **turnoff point**. This is useful as a dating mechanism for globular clusters, since once stars become red giants, their lifespan is practically over on a universal standpoint.

### b) Red Giants

Above and to the right we find extremely large stars, with high luminosities but with low (reddish) color temperature: these are called **red giants**. Aldebaran (the brightest star in the constellation of Taurus) and Mira are red giant.

The nearest star to the Sun, Proxima Centauri, is a Red Dwarf star. It is a star with a diameter (width) less than half the diameter of the Sun, a surface temperature about 2 000 °C to 3 000 °C cooler. The Sun is also about 10 000 times brighter than Proxima Centauri.



### Supergiants

An old Blue-white star becomes a **Supergiant**. They expand, just like average-sized stars expand to become Giant stars. Because they are beginning to run out of hydrogen, they cool down and glow a more orange colour. A star called Betelgeuse is extremely old, but also extremely big.

In fact, it is 500 times wider than the Sun and would, if it was at the centre of the Sun's Solar System, be big enough to stretch nearly to Jupiter. This giant star will one day collapse in a huge explosion called a supernova and will become a neutron star or maybe even a Black Hole

Supergiants such as Betelgeuse, Deneb, Rigel and Antares are some of the most prominent stars in our sky and visible over vast distances due to their extreme luminosities (almost  $10\,000\times$  as luminous as the Sun). These high-mass stars are rare and have very short lifespans relative to lower-mass stars.

### c) White Dwarfs

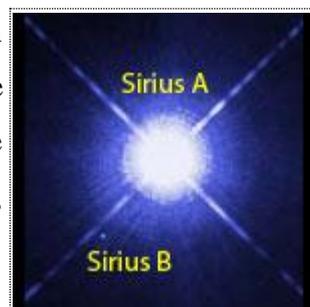
At the lower left, there are a few stars of low luminosity but with high temperature ( $10\,000\text{ K}$ ) composed mainly of carbon and helium: these are the **white dwarfs**. Their luminosity is low because their surface is small. The electrons in white dwarfs are degenerate. The more mass in the white dwarf, the smaller the radius.

White dwarfs are much smaller than main sequence stars and are roughly the size of Earth. Sirius B (the small companion of the brightest star Sirius A) and Procyon B are examples.

White Dwarfs are similar to Red Dwarfs, except that their surface temperatures are much higher, and shine white instead of red. When the Sun comes to the end of its life, it will become a White Dwarf. It will be much smaller than it is now, not quite as bright but twice as hot. Its matter (particles) will be more densely-packed together.

There are also **Black Dwarfs**. These are stars that we cannot see which have used up their energy for producing light, but are still closely-packed but still have a strong gravitational pull.

Stars with masses similar to our Sun will end up as white dwarfs. These stellar remnants have unusual properties. Firstly, they are very small but the more massive white dwarfs are actually smaller than less massive ones. With their fuel used up no fusion takes place so there is no outward radiation pressure to withstand gravitational collapse. More massive stellar cores experience stronger gravitational force so actually compress more. A 0.5 solar-mass white dwarf has a radius  $1.9\times$  that of Earth, a 1.0 solar-mass one is only 1.5 earth radii whilst a 1.3 solar-mass white dwarf 1.4 earth radii. A white dwarf is composed of carbon and oxygen ions mixed in with a sea of degenerate electrons. It is the degeneracy pressure provided by the electrons that prevents further collapse.



The white dwarf, Sirius B and its main sequence companion, Sirius A.

### APPLICATION ACTIVITY 10.3

1. What is the main-sequence lifetime of a  $1.5 M_{\odot}$  star?
2. The luminosity of the Sun is  $3.8 \times 10^{26} \text{ W}$ . How much energy is released after 10 billion years?
3. How far from earth is main sequence star if the wavelength at the peak of the intensity spectrum is 450 nm and its apparent brightness is  $1.0 \times 10^{-11} \text{ W / m}^2$ ? (its absolute luminosity is  $L = 10^{27} \text{ W}$ )
4. Place the following spectral types in order of temperature: A, B, F, G, K, M, O
5. Copy and complete: temperature information, luminosity, distance
  - a. The spectral type (class) of a star gives us ....., but we don't know its .....
  - b. To get the luminosity, we must know the .....

## 10.4 Stellar Distance

### ACTIVITY 10.4

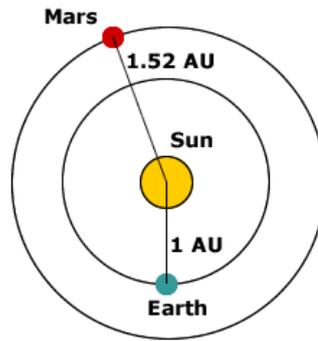
How do astronomers know anything about objects far from Earth? How do we obtain detailed information about any Planets, Stars, or galaxy too distant for a personal visit or any kind of controlled experiment?

#### 10.4.1 Astronomical unit (AU)

The unit of measurement that is convenient for stating the large distances within our Solar System is the astronomical unit (abbreviated as AU). An **astronomical unit** is a unit of length now defined as exactly 149 597 870 700 m (92 955 807.3 mi), or roughly the average Earth–Sun distance. That distance is approximately 150 million kilometers

$$1 \text{ AU} = 150 \times 10^9 \text{ m}$$

The distance between planets depends on their orientation in their orbits. Mars can be between 2.52 AU and 0.53 AU from Earth, depending on their relative positions.



**Fig.10. 10 Distances from Sun to Mars and distance from sun to Earth**

Although the astronomical unit is fine for our Solar System, it is not sufficient to designate the greater distances to other stars and galaxies. Instead, the light year is used as a unit of measurement.

A **lightyear** is the distance light travels in one year and can be calculated as

$$1 \text{ ly} = ct$$

Where

- $c$  is the speed of light in m/s and is approximately equal to  $c = 3 \times 10^8 \text{ m/s}$
- $t$  times  $t = (365 \text{ d} \times 24 \text{ h} / \text{d} \times 3\,600 \text{ s} / \text{h}) = 31\,536\,000 \text{ s}$  the number of seconds in a year

The time it takes light to travel from the Sun to the Earth (1 AU) is approximately 499 s or 8.32 minutes. You could say that 1 AU equals 8.33 light minutes.

Thus, a light year is about  $1 \text{ ly} = ct = (3 \times 10^8 \text{ m/s})(31\,536\,000 \text{ s}) = 9.5 \times 10^{15} \text{ m}$

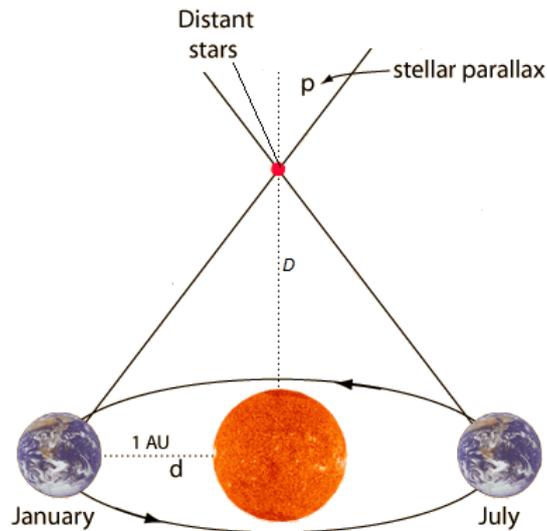
A light year is also equals  $1 \text{ ly} = \frac{9.5 \times 10^{15} \text{ m}}{150 \times 10^9 \text{ m/AU}} = 63\,270 \text{ AU}$

Common large distances in space, measured in light years, include:

- Proxima Centauri (Alpha centauri), the nearest star (excluding the Sun) in our Milky Way galaxy, is 4.22 light years away. The next is Barnard 6.0 ly
- The Milky Way galaxy is about 100 000 light years across.
- The Andromeda Galaxy is approximately 2 500 000 light years away.

### 10.4.2 Stellar parallax

If we know the distance  $d$  from Earth to Sun, we can reconstruct the right triangles shown in Fig. 14.35 and can then determine the distance  $D$  to the star.



**Fig.10. 11 Determining the distance  $D$  to a relatively nearby star using parallax. Horizontal distances are greatly exaggerated: in reality  $\phi$  is a very small angle (less than  $\frac{1}{3\,600} = 1''$ )**

by “triangulation”  $\tan p = \frac{d}{D}$

Where

- $D$  is distance from Earth to star in pc
- $d = 1 \text{ AU}$  distance from Earth to Sun
- $p$  parallax of one arc second

The largest parallax known is that of Proxima Centauri, which has a parallax of  $p = 0.76''$  which means that it is about 1.3 pc away-about 270 00 AU, or 4.3 ly. It is the closest star to the Earth after Sun. The next nearest neighbour to the Sun beyond the Alpha Centauri is Barnard; its parallax is  $p = 0.55''$  so it lies at a distance of  $d = 1.8 \text{ pc}$

By **parallax** we mean the apparent motion of a star, against the background of much more distant stars, due to the Earth’s motion around the Sun. As shown in Fig. 10.10, we can measure the angle  $p$  that the star appears to shift, relative to very distant stars, when viewed 6 months apart.

Parallax can be used to determine the distance to stars as far away as about 100 light-years from Earth, and from an orbiting spacecraft perhaps 5 to 10 times farther. Beyond that distance, parallax angles are too small to measure. For greater distances, more subtle techniques must be employed.

We might compare the apparent **brightness** of two stars, or two galaxies, and use the *inversesquarelaw* (apparent brightness drops off as the square of the distance) to roughly estimate their relative distances. We can't expect this technique to be very precise because we don't expect any two stars, or two galaxies, to have the same intrinsic luminosity.

Astronomers use an effect called parallax to measure distances to nearby stars. Parallax is the apparent displacement of an object because of a change in the observer's point of view.

### Example 14.7

Estimate the distance  $D$  to a star if the angle  $p$  in Fig. 14.10 is measured to be  $p = 0.0006^\circ$

#### Solution:

$$\text{From trigonometry } \tan p = \frac{d}{D} \Leftrightarrow D = \frac{d}{\tan p} = \frac{150 \times 10^9}{\tan 0.0006} = 1.5 \times 10^{14} \text{ km} = 15 \text{ ly}$$

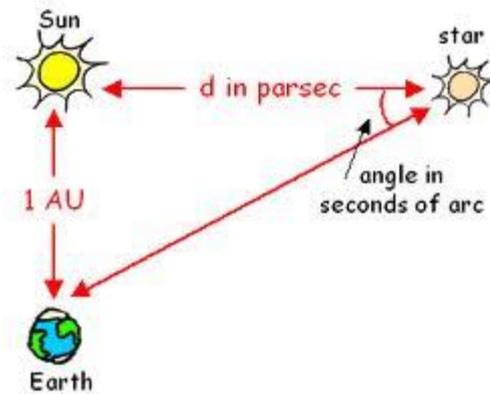
### 10.4.3 Parsec

The method of parallax suggests a natural distance unit that astronomers call the **parsec** (which we shall abbreviate as **pc**). The parsec is defined to be the distance at which a star would have a parallax angle  $p$  equal to one second of arc.

$$1 \text{ pc} = \frac{1 \text{ AU}}{1 \text{ arc second}}$$

$$\text{This distance is equal to } 1 \text{ pc} = \frac{1 \text{ AU}}{\tan\left(\frac{1}{3600}\right)} = 206\,265 \text{ AU}$$

The parsec is also equivalent to a distance:  $1 \text{ pc} = 3.26 \text{ ly} = 206\,265 \text{ AU} = 3.09 \times 10^{16} \text{ m}$



**Fig.10. 12 One parsec is the distance at which there is a one-arc second ( $1'' = \frac{1^\circ}{3600}$ ) angular separation between two objects apart  $11.50 \times 10^{11} m$  (the average distance from the earth to the sun).**

### Limitations

If the stars are too far away, the parallax can be too small to measure accurately. In general, the greater the distance, the smaller the parallax, and so the less precise the distance measurement will be. The smallest parallax measurable from the ground is about 0.01 arcsec.

A **minute of arc, arc minute** (arcmin), **arc minute**, or **minute arc** is a unit of angular measurement equal to 1/60 of one degree.

$$\text{arc min} = \frac{1^\circ}{60}$$

A **second of arc, arc second** (arcsec), or **arc second** is 1/60 of an arc minute, 1/3600 of a degree,

$$1'' = \frac{1}{3600} \text{ arc sec} = \frac{1^\circ}{3600}$$

These units originated in Babylonian astronomy as sexagesimal subdivisions of the degree; they are used in fields that involve very small angles, such as astronomy, optometry, ophthalmology, optics, navigation, land surveying, and marksmanship.

### Example 14.8

A star has a parallax angle of 0.723 arc seconds. What is the distance to the star?

**Solution:**

For small angle:  $\tan p = p \Leftrightarrow d = \frac{1}{p} = \frac{1 \text{ AU}}{0.723''} \times \frac{3600''}{1^\circ} \times \frac{180^\circ}{\pi \text{ rad}} = 1.38 \text{ pc}$

### SKILLS LAB 10

Having identified the existence of different types of stars based on measurable properties, make a comprehensive report (in a table form) showing different examples of stars.

In your report, you can also include

- i) Variations of masses of stars
- ii) Factors that affect the life span of these stars
- iii) What will happen to these stars when they explode?
- iv) Their average distance in Astronomical units and Light years from the earth.

Rigel (beta Orionis)	0.0042	
Sirius (alpha Canis Majoris)	0.379	
Altair (alpha Aurigae)	0.194	

## END UNIT ASSESSMENT

Multiple choices: chose the best answer 1-8

1. How do gamma rays at the core become visible light at the surface?

- A. Not Sure
- B. Radiation is absorbed and re-emitted at a lower frequency
- C. Visible light rays are the same as gamma rays
- D. The amplitude of the radiation drops as it moves to the surface

2. Time taken by the sun to complete one orbit around the galactic centre is called

- A. One parsec
- B. One astronomical unit
- C. One light year
- D. One cosmic year

3. Light years is the unit of

- A. Distance
- B. Weight
- C. time
- D. Intensity of light

4. The time taken by light to travel from the sun to the earth is

- A. 15.2 min
- B. 8.33 min
- C. 4.66 min
- D. 1.5 min

5. What invention revolutionized our study of the sky?

- A. Not Sure
- B. The microscope
- C. The airplane
- D. The telescope

**6-9. Choose the word or phrase that complete the sentence**

6. The star of a constellation -----

- a. Are in the same cluster
- b. Are equally bright
- c. Are all giants
- d. form a pattern



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37. [http://en.wikipedia.org/wiki/Newton%27s\\_laws\\_of\\_motion](http://en.wikipedia.org/wiki/Newton%27s_laws_of_motion)
38. <http://phet.colorado.edu/en/simulations/category/new>
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