

Subsidiary Mathematics

For Associate Nursing Program

Teacher's Guide-Senior 4

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FOREWORD

Dear Teacher,

Rwanda Basic Education Board is honoured to present the teacher's guide for S4 Mathematics in Associate Nursing Program which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of the mathematics content. The Rwandan educational philosophy is to ensure that learners achieve full potential at every level of education which will prepare them to be well integrated in society and exploit employment opportunities.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate their learning process. Many factors influence what they learn, how well they learn and the competences they acquire. Those factors include the relevance of the specific content, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials.

The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market has necessitated the shift to a competence-based curriculum.

This book provides active teaching and learning techniques that engage student teachers to develop competences.

In view of this, your role is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize group discussions for students considering the importance of social constructivism suggesting that learning occurs more effectively when the students works collaboratively with more knowledgeable and experienced people.
- Engage students through active learning methods such as inquiry methods, group discussions, research, investigative activities and group and individual work activities.

- Provide supervised opportunities for students to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation.
- Support and facilitate the learning process by valuing students' contributions in the class activities.
- Guide students towards the harmonization of their findings.
- Encourage individual, peer and group evaluation of the work done in the classroom and use appropriate competence-based assessment approaches and methods.

To facilitate you in your teaching activities, the content of this book is self-explanatory so that you can easily use it. It is divided in 3 parts:

The part I explains the structure of this book and gives you the methodological guidance;

The part II gives a sample lesson plan;

The part III details the teaching guidance for each concept given in the student book.

Even though this Teacher's guide contains the guidance on solutions for all activities given in the learner's book, you are requested to work through each question before judging student's findings.

I wish to sincerely express my appreciation to the people who contributed towards the development of this book, particularly, REB staff, Teachers from general education for their technical support and the Human Resources for Health Secretariat (HRHS) for their financial support.

Dr. MBARUSHIMANA Nelson

Director General, REB

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Table content

<i>FOREWORD</i>	<i>iii</i>
<i>ACKNOWLEDGEMENT</i>	<i>v</i>
<i>PART I. GENERAL INTRODUCTION</i>	<i>1</i>
<i>PART III: UNIT DEVELOPMENT</i>	<i>24</i>
Unit 1. Set of Real Numbers.....	25
Unit 2. Fundamentals of Trigonometry	45
Unit 3. LINEAR, QUADRATIC EQUATIONS AND INEQUALITIES	63
Unit 4. POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS.....	93
Unit 5. LIMITS OF POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS ... 113	
Unit 6. DIFFERENTIATION OF POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS.....	133
Unit 7. Vector Space of Real numbers.....	149
Unit 8. MATRICES AND DETERMINANTS OF ORDER TWO	155
Unit 9. Measures of Dispersion.....	163
Unit 10. ELEMENTARY PROBABILITY.....	171

PART I. GENERAL INTRODUCTION

1.1 The structure of the guide

The teacher's guide of S4 Mathematics is composed of three parts:

The Part I concerns general introduction that discusses methodological guidance on how best to teach and learn Mathematics, developing competences in teaching and learning, addressing cross-cutting issues in teaching and learning and Guidance on assessment.

Part II presents a sample lesson plan. This lesson plan serves to guide the teacher on how to prepare a lesson in Mathematics.

The Part III is about the structure of a unit and the structure of a lesson. This includes information related to the different components of the unit and these components are the same for all units. This part provides a list of lessons for each unit as well as the information and guidelines on how to facilitate students while working on learning activities. More other, application activities from the textbook have answers in this part.

1.2 Methodological guidance

1.2.1 Developing competences

Since 2015 Rwanda shifted from a knowledge based to a competence-based curriculum for pre-primary, primary and secondary education. This called for changing the way of learning by shifting from teacher centred to learner centred approach. Teachers are not only responsible for knowledge transfer but also for fostering learners' learning achievement and creating safe and supportive learning environment. It implies also that learners have to demonstrate what they are able to transfer the acquired knowledge, skills, values and attitude to new situations.

The competence-based curriculum employs an approach of teaching and learning based on skills rather than dwelling on only knowledge or the cognitive domain of learning. It focuses on what learner can do rather than what learner knows. Learners develop competences

through subject unit with specific learning objectives broken down into knowledge, skills , attitudes and values through learning activities.

In addition to the competences related to Mathematics, student teachers also develop generic competences which should promote the development of the higher order thinking skills and professional skills. Generic competences are developed throughout all units of Mathematics as follows:

Generic competences	Ways of developing generic competences
Critical thinking	All activities that require learners to calculate, convert, interpret, analyse, compare and contrast, etc. have a common factor of developing critical thinking into learners
Creativity and innovation	All activities that require learners to plot a graph of given mathematical data, to organize and interpret statistical data collected and apply skills in solving problems have a common character of developing creativity into learners
Research and problem solving	All activities that require learners to make a research and apply their knowledge to solve problems from the real-life situation have a character of developing research and problem solving into learners.
Communication	During Mathematics class, all activities that require learners to discuss either in groups or in the whole class, present findings, debate ...have a common character of developing communication skills into learners.
Co-operation, interpersonal relations and life skills	All activities that require learners to work in pairs or in groups have character of developing cooperation and social life skills among learners.

Lifelong learning	All activities that are connected with research have a common character of developing into learners a curiosity of applying the knowledge learnt in a range of situations. The purpose of such kind of activities is for enabling learners to become life-long learners who can adapt to the fast-changing world and the uncertain future by taking initiative to update knowledge and skills with minimum external support.
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The generic competences help learners deepen their understanding of Mathematics and apply their knowledge in a range of situations. As students develop generic competences they also acquire the set of skills that are needed on the job market.

1.2.2 Addressing cross cutting issues

Among the changes brought by the competence-based curriculum is the integration of cross cutting issues as an integral part of the teaching learning process-as they relate to and must be considered within all subjects to be appropriately addressed. The eight cross cutting issues identified in the national curriculum framework are: *Comprehensive Sexuality Education, Environment and Sustainability, Financial Education, Genocide studies, Gender, Inclusive Education, Peace and Values Education, and Standardization Culture.*

Some cross-cutting issues may seem specific to particular learning areas/subjects but the teacher need to address all of them whenever an opportunity arises. In addition, learners should always be given opportunities during the learning process to address these cross-cutting issues both within and out of the classroom.

Below are examples of how crosscutting issues can be addressed:

Cross-Cutting Issue	Ways of addressing cross-cutting issues
<p>Comprehensive Sexuality Education: The primary goal of introducing Comprehensive Sexuality Education program in schools is to equip children, adolescents, and young people with knowledge, skills and values in an age appropriate and culturally gender sensitive manner so as to enable them to make responsible choices about their sexual and social relationships, explain and clarify feelings, values and attitudes, and promote and sustain risk reducing behaviour.</p>	<p>Using different charts, graphs and their interpretation, Mathematics teachers should lead students to discuss the following situations: “Alcohol abuse and unwanted pregnancies” and advise student teachers on how they can instil learners to fight those abuses.</p> <p>Some examples can be given when learning statistics, powers, logarithms and their properties.</p>
<p>Environment and Sustainability: Integration of Environment, Climate Change and Sustainability in the curriculum focuses on and advocates for the need to balance economic growth, society well-being and ecological systems. Learners need basic knowledge from the natural sciences, social sciences, and humanities to understand and interpret principles of sustainability.</p>	<p>Using Real life models or students’ experience, Mathematics Teacher should lead students to illustrate the situation of “population growth” and discuss its effects on the environment and sustainability.</p>

<p>Financial Education:</p> <p>The integration of Financial Education into the curriculum is aimed at a comprehensive Financial Education program as a precondition for achieving financial inclusion targets and improving the financial capability of Rwandans so that they can make appropriate financial decisions that best fit the circumstances of one's life.</p>	<p>Through different examples and calculations on interest rate problems, total revenue and total cost, Mathematics teacher can lead students to discuss how to make appropriate financial decisions.</p>
<p>Gender: At school, gender will be understood as family complementarities, gender roles and responsibilities, the need for gender equality and equity, gender stereotypes, gender sensitivity, etc.</p>	<p>Mathematics teacher should address gender as cross-cutting issue through assigning leading roles in the management of groups to both girls and boys and providing equal opportunities in the lesson participation and avoid any gender stereotype in the whole teaching and learning process.</p>
<p>Inclusive Education:</p> <p>Inclusion is based on the right of all learners to a quality and equitable education that meets their basic learning needs and understands the diversity of backgrounds and abilities as a learning opportunity.</p>	<p>Firstly, Mathematics Teachers need to identify/recognize students with special needs. Then by using adapted teaching and learning resources while conducting a lesson and setting appropriate tasks to the level of students, they can cater for students with special education needs. They must create opportunity where students can discuss how to cater for learners with special educational needs.</p>

<p>Peace and Values Education: Peace and Values Education is defined as education that promotes social cohesion, positive values, including pluralism and personal responsibility, empathy, critical thinking and action in order to build a more peaceful society.</p>	<p>Through a given lesson, a teacher should:</p> <ul style="list-style-type: none"> • Set a learning objective which is addressing positive attitudes and values, • Encourage students to develop the culture of tolerance during discussion and to be able to instil it in colleagues and cohabitants; • Encourage students to respect ideas for others.
<p>Standardization Culture: Standardization Culture in Rwanda will be promoted through formal education and plays a vital role in terms of health improvement, economic growth, industrialization, trade and general welfare of the people through the effective implementation of Standardization, Quality Assurance and Testing.</p>	<p>With different word problems related to the effective implementation of Standardization, Quality Assurance and Testing, students can be motivated to be aware of health improvement, economic growth, industrialization, trade and general welfare of the people.</p>

1.2.3 Guidance on how to help students with special education needs in classroom

In the classroom, students learn in different way depending to their learning pace, needs or any other special problem they might have. However, the teacher has the responsibility to know how to adopt his/ her methodologies and approaches in order to meet the learning need of each student in the classroom. Also teachers need to understand that

student with special needs, need to be taught differently or need some accommodations to enhance the learning environment. This will be done depending on the subject and the nature of the lesson.

In order to create a well-rounded learning atmosphere, teachers need to:

- Remember that learners learn in different ways so they have to offer a variety of activities (e.g. role-play, music and singing, word games and quizzes, and outdoor activities);
- Maintain an organized classroom and limits distraction. This will help learners with special needs to stay on track during lesson and follow instruction easily;
- Vary the pace of teaching to meet the needs of each child. Some learners process information and learn more slowly than others;
- Break down instructions into smaller, manageable tasks. Learners with special needs often have difficulty understanding long-winded or several instructions at once. It is better to use simple and concrete sentences in order to facilitate them understand what you are asking.
- Use clear consistent language to explain the meaning (demonstrate or show pictures) if you introduce new words or concepts;
- Make full use of facial expressions, gestures and body language;
- Pair a learner who has a disability with a friend. Let them do things together and learn from each other. Make sure the friend is not over protective and does not do everything for the one with disability. Both learners will benefit from this strategy;
- Use multi-sensory strategies. As all learners learn in different ways, it is important to make every lesson as multi-sensory as possible. Learners with learning disabilities might have difficulty in one area, while they might excel/shine in another. For example, use both visual and auditory cues/prompts.

Below are general strategies related to each main category of

disabilities and how to deal with every situation that may arise in the classroom. However, the list is not exhaustive because each child is unique with different needs and that should be handled differently.

Strategy to help learners with developmental impairment:

- Use simple words and sentences when giving instructions;
- Use real objects that learners can feel and handle. Rather than just working abstractly with pen and paper;
- Break a task down into small steps or learning objectives. The learner should start with an activity that she/he can do already before moving on to something that is more difficult;
- Gradually give the learner less help;
- Let the learner with disability work in the same group with those without disability.

Strategy to help learners with visual impairment:

- Help learners to use their other senses (hearing, touch, smell and taste) and carry out activities that will promote their learning and development;
- Use simple, clear and consistent language;
- Use tactile objects to help explain a concept;
- If the learner has some sight, ask him/her what he/she can see;
- Make sure the learner has a group of friends who are helpful and who allow him/her to be as independent as possible;
- Plan activities so that learners work in pairs or groups whenever possible;

Strategy to help learners with hearing disabilities or communication difficulties

- Always get the learner 's attention before you begin to speak;
- Encourage the learner to look at your face;
- Use gestures, body language and facial expressions;
- Use pictures and objects as much as possible.
- Keep background noise to a minimum.

Strategies to help learners with physical disabilities or mobility difficulties:

- Adapt activities so that learners who use wheelchairs or other mobility aids, can participate.
- Ask parents/caregivers to assist with adapting furniture e.g. the height of a table may need to be changed to make it easier for a learner to reach it or fit their legs or wheelchair under;
- Encourage peer support when needed;
- Get advice from parents or a health professional about assistive devices if the learner has one.

Adaptation of assessment strategies:

At the end of each unit, the teacher is advised to provide additional activities to help students achieve the key unit competence. These assessment activities are for remedial, consolidation and extension designed to cater for the needs of all categories of students; slow, average and gifted students respectively. Therefore, the teacher is expected to do assessment that fits individual student.

Remedial activities	After evaluation, slow students are provided with lower order thinking activities related to the concepts learnt to facilitate them in their learning. These activities can also be given to assist deepening knowledge acquired through the learning activities for slow students.
Consolidation activities	After introduction of any concept, a range number of activities can be provided to all students to enhance/ reinforce learning.

Extended activities	After evaluation, gifted and talented students can be provided with high order thinking activities related to the concepts learnt to make them think deeply and critically. These activities can be assigned to gifted and talented students to keep them working while other students are getting up to required level of knowledge through the learning activity.
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1.2.4. Guidance on assessment

Assessment is an integral part of teaching and learning process. The main purpose of assessment is for improvement of learning outcomes. Assessment for learning/ Continuous/ formative assessment intends to improve students' learning and teacher's teaching whereas assessment of learning/summative assessment intends to improve the entire school's performance and education system in general.

Continuous/ formative assessment

It is an on-going process that arises during the teaching and learning process. It includes lesson evaluation and end of sub unit assessment. This formative assessment should play a big role in teaching and learning process. The teacher should encourage individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods.

Formative assessment is used to:

- Determine the extent to which learning objectives are being achieved and competences are being acquired and to identify which students need remedial interventions, reinforcement as well as extended activities. The application activities are developed in the learner book and they are designed to be given as remedial, reinforcement, end lesson assessment, homework or assignment
- Motivate students to learn and succeed by encouraging students to read, or learn more, revise, etc.

- Check effectiveness of teaching methods in terms of variety, appropriateness, relevance, or need for new approaches and strategies. Mathematics teachers need to consider various aspects of the instructional process including appropriate language levels, meaningful examples, suitable methods and teaching aids/ materials, etc.
- Help students to take control of their own learning.

In teaching Mathematics, formative or continuous assessment should compare performance against instructional objectives. Formative assessment should measure the student's ability with respect to a criterion or standard. For this reason, it is used to determine what students can do, rather than how much they know.

Summative assessment

The assessment can serve as summative and informative depending to its purpose. The end unit assessment will be considered summative when it is done at end of unit and want to start a new one.

It will be formative assessment, when it is done in order to give information on the progress of learners and from there decide what adjustments need to be done.

The assessment done at the end of the term, end of year, is considered as summative assessment so that the teacher, school and parents are informed of the achievement of educational objective and think of improvement strategies. There is also end of level/ cycle assessment in form of national assessment or examinations.

When carrying out assessment?

Assessment should be clearly visible in lesson, unit, term and yearly plans.

- Before learning (diagnostic): At the beginning of a new unit or a section of work; assessment can be organized to find out what students already know / can do, and to check whether the students are at the same level.

- During learning (formative/continuous): When students appear to be having difficulty with some of the work, by using on-going assessment (continuous). The assessment aims at giving students support and feedback.
- After learning (summative): At the end of a section of work or a learning unit, the Mathematics Tutor has to assess after the learning. This is also known as Assessment of Learning to establish and record overall progress of students towards full achievement. Summative assessment in Rwandan schools mainly takes the form of written tests at the end of a learning unit or end of the month, and examinations at the end of a term, school year or cycle.

Instruments used in assessment.

- **Observation:** This is where the Mathematics teacher gathers information by watching students interacting, conversing, working, playing, etc. A teacher can use observations to collect data on behaviours that are difficult to assess by other methods such as attitudes, values, and generic competences and intellectual skills. It is very important because it is used before the lesson begins and throughout the lesson since the teacher has to continue observing each and every activity.
- **Questioning**
 - (a) Oral questioning: a process which requires a student to respond verbally to questions
 - (b) Class activities/ exercise: tasks that are given during the learning/ teaching process
 - (c) Short and informal questions usually asked during a lesson
 - (d) Homework and assignments: tasks assigned to students by their tutors to be completed outside of class.

Homework assignments, portfolio, project work, interview, debate, science fair, Mathematics projects and Mathematics competitions are also the different forms/instruments of assessment.

1.2.5. Teaching methods and techniques that promote active learning

The different learning styles for students can be catered for, if the teacher uses active learning whereby learners are really engaged in the learning process.

The main teaching methods used in mathematics are the following:

- **Dogmatic method** (the teacher tells the students what to do, What to observe, How to attempt, How to conclude)
- **Inductive-deductive method:** Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples.
- **Analytic-synthetic method:** Analytic method proceeds from unknown to known, 'Analysis' means 'breaking up' of the problem in hand so that it ultimately gets connected with something obvious or already known. Synthetic method is the opposite of the analytic method. Here one proceeds from known to unknown.
- **Laboratory method:** Laboratory method is based on the maxim "learning by doing." It is a procedure for stimulating the activities of the students and to encourage them to make discoveries through practical activities.
- **Problem solving method, Project method and Seminar Method.**

The following are some active techniques to be used in Mathematics:

- Group work
- Research
- Probing questions
- Practical activities (drawing, plotting, interpreting graphs)
- Modelling
- Brainstorming
- Quiz Technique
- Discussion Technique
- Scenario building Technique

What is Active learning?

Active learning is a pedagogical approach that engages learners in doing things and thinking about the things they are doing. Learners play the key role in the active learning process. They are not empty vessels to fill but people with ideas, capacity and skills to build on for effective learning. Thus, in active learning, learners are encouraged to bring their own experience and knowledge into the learning process.

The role of the teacher in active learning	The role of learners in active learning
<ul style="list-style-type: none"> • The teacher engages learners through active learning methods such as inquiry methods, group discussions, research, investigative activities, group and individual work activities. • He/she encourages individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods. • He provides supervised opportunities for learners to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation. • Teacher supports and facilitates the learning process by valuing learners' contributions in the class activities. 	<ul style="list-style-type: none"> • A learner engaged in active learning: • Communicates and shares relevant information with peers through presentations, discussions, group work and other learner-centred activities (role play, case studies, project work, research and investigation); • Actively participates and takes responsibility for his/her own learning; • Develops knowledge and skills in active ways; • Carries out research/investigation by consulting print/online documents and resourceful people, and presents their findings; • Ensures the effective contribution of each group member in assigned tasks through clear explanation and arguments, critical thinking, responsibility and confidence in public speaking • Draws conclusions based on the findings from the learning activities.

Main steps for a lesson in active learning approach

All the principles and characteristics of the active learning process highlighted above are reflected in steps of a lesson as displayed below. Generally, the lesson is divided into three main parts whereby each one is divided into smaller steps to make sure that learners are involved in the learning process. Below are those main part and their small steps:

1) Introduction

Introduction is a part where the teacher makes connection between the current and previous lesson through appropriate technique. The teacher opens short discussions to encourage learners to think about the previous learning experience and connect it with the current instructional objective. The teacher reviews the prior knowledge, skills and attitudes which have a link with the new concepts to create good foundation and logical sequencings.

2) Development of the new lesson

The development of a lesson that introduces a new concept will go through the following small steps: discovery activities, presentation of learners' findings, exploitation, synthesis/summary and exercises/application activities.

- Discovery activity

Step 1

- The teacher discusses convincingly with learners to take responsibility of their learning
- He/she distributes the task/activity and gives instructions related to the tasks (working in groups, pairs, or individual to instigate collaborative learning, to discover knowledge to be learned)

Step 2

- The teacher let learners work collaboratively on the task;
- During this period the teacher refrains to intervene directly on the knowledge;

- He/she then monitors how the learners are progressing towards the knowledge to be learned and boosts those who are still behind (but without communicating to them the knowledge).
- Presentation of learners' findings/productions
 - In this episode, the teacher invites representatives of groups to present their productions/findings.
 - After three/four or an acceptable number of presentations, the teacher decides to engage the class into exploitation of learners' productions.
- Exploitation of learner's findings/ productions
 - The teacher asks learners to evaluate the productions: which ones are correct, incomplete or false
 - Then the teacher judges the logic of the learners' products, corrects those which are false, completes those which are incomplete, and confirms those which are correct.
- Institutionalization or harmonization (summary/conclusion/ and examples)
 - The teacher summarizes the learned knowledge and gives examples which illustrate the learned content.
- Application activities
 - Exercises of applying processes and products/objects related to learned unit/sub-unit
 - Exercises in real life contexts
 - Teacher guides learners to make the connection of what they learnt to real life situations. At this level, the role of teacher is to monitor the fixation of process and product/object being learned.

3) Assessment

In this step the teacher asks some questions to assess achievement of instructional objective. During assessment activity, learners work individually on the task/activity. The teacher avoids intervening directly.

In fact, results from this assessment inform the teacher on next steps for the whole class and individuals. In some cases, the teacher can end with a homework/ assignment. Doing this will allow learners to relay their understanding on the concepts covered that day. Teacher leads them not to wait until the last minute for doing the homework as this often results in an incomplete homework set and/or an incomplete understanding of the concept.

NOTE TO THE TEACHER ON THE IMPORTANCE OF MATHEMATICS IN THE MEDICINE FIELD

Some students become nervous when they hear the word “Mathematics.” This worried feeling is called Mathematics anxiety. Because Mathematics is important to all areas of life, it is important for students to overcome such anxiety.

In this regard, before the learning of a new Mathematical concept, the teacher should help learners to mentally assess what they already know about the topic. He/she will explain them that the best learning takes place when they participate and engage their mind and take notes during class so that they remember and practice important concepts.

Even though this book contains the mathematics concepts for Advanced levels, the teacher is advised to start by what the learners know and their ability to perform some skills. From the prerequisites, he/she will use learning activities to help learners build knowledge and develop skills, attitudes, and values.

The discussion on the new concept will lead students to discover new knowledge and relate it to their real-life experiences. The teacher tries to help learners to highlight the meanings and applications of the concept they developed. When giving application activities and examples, the teacher select those related to nursing, medicine, biology, chemistry, or other real-life experiences for students of Associate Nursing Program. When such examples are missing, for example in the unit of Complex numbers, the teacher can give some examples from other sciences or professions (such as engineering) but he/she will not task them to do

such activities during the assessment.

As a conclusion, the teacher must regularly motivate students to discuss the important applications of Mathematics in real life situations. Such applications are summarised below.

Professionals in the medical field use Mathematics to determine proper doses for patients' medicine, read results from computed tomography (CAT) scans, Magnetic Resonance Imaging (MRIs) and X-rays and to evaluate body mass index. Physicians, nurses, and others in the medical field use mathematics daily in hospitals and offices and when performing research. Mathematics plays a crucial role in health, as it allows for the safe administration of painkillers and antibiotics and ensures appropriate treatment and diagnoses.

The role of Arithmetic is very important in medicine as it leads to clinical calculations or medication dosage. Health care workers must be careful when making mathematical calculations. It is possible for errors to have harmful effects. For example, miscalculating a medication dosage for a patient can lead to further sickness or even death. Even simple errors, such as copying a wrong number or placing a decimal point incorrectly, may create serious problems.

Many medication dosages are calculated according to the patient's body weight. This prevents giving too much medication to a smaller patient or too little medication to a larger patient. Dosage by weight is typically given in terms of milligrams per kilogram per day.

Statistics plays a huge role in the everyday activities of a medical laboratory. They use quality control to ensure analysis are accurate and precise. Statistics could also determine how to handle patient results and whether corrected reports need to be made. Statistics is also a major player in determining the reference ranges for tests. Data is compiled from all the healthy patients, and they can be setup by age and gender. Any results below or above these ranges can be considered abnormal which would require immediate attention.

Calculus as the study of both the infinitesimally small and the infinitely

large object, it forces us to consider what kinds of changes happen on an infinitesimally small scale. These kinds of changes are common in biology and medicine; tiny changes in medicine (such as pH, drug concentration, etc.) can effect large changes in the health of a patient. In medicine, Integrals lead to the ability to think of the body as the sum of many smaller pieces (namely the different systems, organs, tissues, etc) and to consider their effects. Overall, the study of variations and how different variables changing can change an overall system has useful applications in medicine.

In the areas of Research, medical professionals draw charts, analyse graphs and study statistical trends to understand societal ailments, such as the prevalence of chronic illnesses and diseases. When reading results from CAT scans and X-rays, doctors use geometric concepts to convert the two-dimensional images on the screens into three-dimensional body parts. They determine abnormalities by looking at body parts from many different angles.

Clearly, Mathematics is vital to the medical profession. A deep understanding of Mathematics will improve a student's performance in science classes and on the Medical Admission Tests. Additionally, having a familiarity with arithmetic, Calculus, Geometry, Statistics, and Probability will improve his or her practice of nurse and physician.

PART II: SAMPLE LESSON

Lesson: Domain of definition of polynomial and rational functions

School Name: ...XXXXX..... Teachers name.....YYYYY...

Term	Date	Subject	Class	Unit N°	Lesson N°	Duration	Class size
2	.../.../...	Mathematics		4		40 minutes	
Types/category/number of special Educational need to be catered for in this lesson.				<ul style="list-style-type: none"> - 4 low vision learners will be assisted by availing big printed documents - 5 Gifted learners will be encouraged to explain and help their classmates. 			
Unit title				POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS			
Key Unit Competence				Use concepts and definitions of polynomial, rational and irrational functions to determine their domain, represent and their graphs.			
Title of the lesson				Domain of definition of polynomial and rational functions			
Instructional Objective				Given polynomial or rational function, the learner should be able to accurately determine their domain of definition.			
Plan for this Class (location: in the classroom / outside)				In the Classroom , Learners are organized into small groups and do activities			
Learning Materials (for all learners)				Calculator			
References				Subsidiary Mathematics for Associate Nursing Program, Student 's Book, Senior 4.			

Timing for each step	Description of teaching and learning activity		Generic competences and Cross cutting issues to be addressed + a short explanation
	<p>In groups of 6, learners will do activity 4.3 and each group will report their findings to the class. Teacher will facilitate the learners' presentations and help them to harmonize by making a summary from their presentations. At the end of the lesson, an assignment to be discussed as an activity of the next lesson "Finding domain of definition of irrational function" will be given to the learners as a homework.</p>		
	Teacher activities	Learner activities	
Introduction 5 minutes	Ask questions on previous lesson Question: Give three examples of; <ol style="list-style-type: none"> 1. polynomial function 2. rational function 	Learners respond to questions Answers: Answers may vary. Ex: 1. $f(x) = x + 1$ $g(x) = x^2 + 3x - 3$ 2. $f(x) = \frac{x+1}{3x+1}$ $g(x) = \frac{x^2 + 3x - 3}{x - 2}$	Students are developing communication skills when they are explaining and sharing ideas

Development of the lesson	10 minutes	Step 1: - Form groups of at least 6 learners each, request the learners to do activity 4.3	In their groups, learners will do activity 4.3	Cooperation and interpersonal management developed through working in groups
	5 minutes	- Goes round to check the progress of the discussion, and intervenes where necessary.	Reporter represents the work. Learners interact through questions and comments. Answers: 1. No value 2. 0 3. 1	
	2 minutes	Step 2: Request a reporter from each group to present the work on the chalkboard.	Take notes in their note books. Given that $f(x)$ is polynomial, then the domain of definition is the set of real numbers. That is $Domf = \mathbb{R}$	Communication skills developed when learners communicate and convey information and ideas through speaking and presentation of their work. Peace and values education are developed through respecting one another while learners are conveying ideas with agreements.
	5 minutes	Step 3: Capture the main points from the presentation of the learners and summarise them. Step 4: Request some learners to answer to the questions 1 and 3 of application activity 4. 3 in groups	Given that $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials, then the domain of definition is the set of real numbers excluding all values where the denominator is zero. That is $Domf = \{x \in \mathbb{R} : h(x) \neq 0\}$ Work in groups and make a presentation to the whole class.	

Conclusion/ summary/ 3 minutes	Request learners to give the main points of the learned lesson.	Summarize the learned lesson	Communication skills: This is developed through discussions and sharing of answers when they are summarizing the lesson.
Assessment/ 7 minutes	Request learners to do questions 2, 4 and 5 of the application activity 4. 3.	Individually Learners do activity	Problem solving skills: This skill is developed when learners are extending knowledge of finding domain of polynomial and rational functions to irrational functions.
Homework 3 minutes	Give to the learners the homework which will be an activity for the next lesson	Write the home work in their notebooks and do it at home individually. Find the domain of definition for each of the following functions: 1. $f(x) = x + 1$ 2. $f(x) = \frac{x + 1}{x - 2}$ 3. $f(x) = \frac{x + 1}{\sqrt{x - 2}}$	Problem solving skills: This skill is developed when learners are extending knowledge of finding domain of polynomial and rational functions to irrational functions.
Teacher self-evaluation	Ensures that the learners understood the learned lesson and decide to repeat the lesson or to continue with new lesson next time. Example of a decision taken by the teacher The lesson was successfully delivered because learners are able to find the domain of polynomial and rational functions.		

PART III: UNIT DEVELOPMENT

Unit 1

Set of Real Numbers

1.1 Key Unity Competency

Think critically to understand and perform operations on the set of real numbers.

1.2 Objectives

By the end of this unit, the learners will:

- define a power, an exponential, a radical, a logarithm, the absolute value of a real number
- illustrate each property of a power, an exponential, a radical, a logarithm, the absolute value of a real number.
- use logarithm and exponentials to model simple problems about growth, decay, compound interest, magnitude of an earthquake...

1.3 List of lessons

Week	Lesson number	Content	Number of Periods
1	1	Set of real numbers: Real numbers, operations on the set \mathbb{R} , Arithmetic of integers and whole numbers.	1
	2	- Rounding and estimating decimal numbers. - Equivalent fractions, ratios and proportions, and rates.	2

2&3	3	Absolute value and its properties	1
	4	Powers and radicals	1
	5	Operation on radicals	2
	6	Decimal logarithms and properties	2
4	7	Model simple problems about growth and decay, compound interest and magnitude of an earthquake	2
	8	End unit assessment and Remediation	1

1.4 Vocabulary

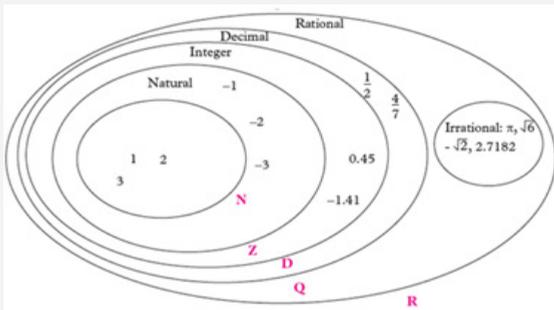
Logarithm

1.5 Guidance on introductory activity 1.0

- Form small groups of learners and guide them to work on the introductory activity.
- Provide learning materials accordingly to the given activities and give clear guidance and instructions to perform the activities.
- Give time to learners to read and analyse the given activity and let them discuss about different possible solutions of the problem.
- Walk around in different groups to provide advice and facilitations where necessary and remind them to justify and support their answer / findings.
- Lead learners to recognize that the given activity should get different answers depending on the considered set of number.
- Basing on learners' experience, prior knowledge and abilities shown in answering the questions for the introductory activity, use different questions to prompt them to give their predictions and ensure that you arouse their curiosity on what is going to be learnt in the unit 1
- After presenting their finding, the teacher harmonizes and guides class discussions and interventions.

Answer of introductory activity 1.0

1. Lead learners to know that in the question1, set of numbers they already know from senior one (S1) in secondary schools, are:
 $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \dots$
2. The numbers we use in counting plus zero are called Natural numbers; integers are numbers which have either negative or positive sign and includes zero. The set of integers is represented by \mathbb{Z} ; the set of **rational numbers** \mathbb{Q} and the set of irrational numbers \mathbb{I} form the set of real numbers. The set of real numbers is denoted by \mathbb{R}
3. Some examples of numbers in each set:



4. The relationship between set of numbers is as follows: Natural numbers are part of integers, integers are part of rational numbers, rational numbers and irrational numbers are parts of real numbers. Therefore, $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

1.6. Contents and activities

1.6.1 Set of real numbers: Real numbers, operations on the set \mathbb{R} , Arithmetic of integers and whole numbers.

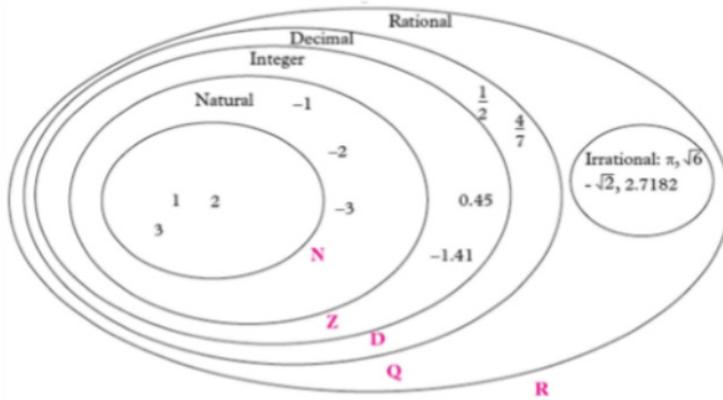
This section introduces the generalities about sets and their

relationship. $\mathbb{N} = \{0, 1, 2, 3, 4, 5, \dots\}$ of natural numbers

, $\mathbb{Z} = \{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ of

integers, $D = \{\dots, -5.0, -4.4, 1.2, 5.7, \dots\}$ of decimals and

$\mathbb{Q} = \left\{ \dots, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \dots \right\}$ of rational numbers are subsets of set \mathbb{R} of real numbers. Therefore, $\mathbb{N} \subset \mathbb{Z} \subset D \subset \mathbb{Q} \subset \mathbb{R}$. All subsets of \mathbb{R} are illustrated in the Venn diagram below.



Learning activity 1.1

Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

- Facilitate learners in Group, in answering the activity1.1
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to explore set of real numbers and its subsets to make accurate interconnection between them and solve related problems.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.

- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application activity 1.1 and evaluate whether lesson objectives were achieved.

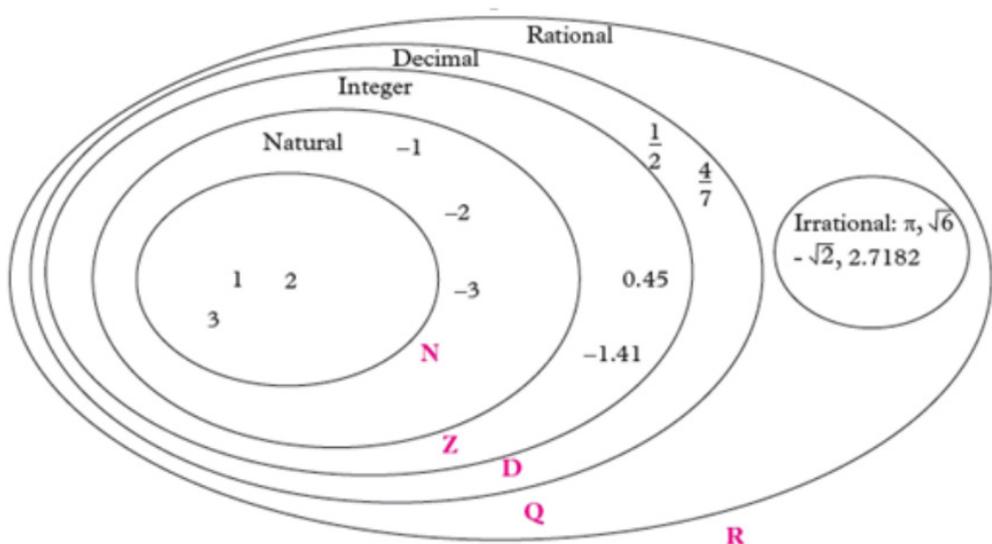
Answers for activity 1.1

a) Students will give different answers on the definitions requested. As a tutor, support those who eventually can provide a wrong answer.

In the list of numbers, $0; 1; -5; 6; \frac{3}{4}; 3.146; 1.3333\dots; \pi; \sqrt{8}$, we have:

Natural numbers	Integers	Rational numbers	Irrational numbers
0, 1, 6	0, 1, -5, 6	0, 1, -5, 6, $\frac{3}{4}$, 3.146, 1.333...	$\pi; \sqrt{8}$

b) Different answers will be given on the definition of the set \mathbb{R} , verify whether stipulates that the set \mathbb{R} includes the union of rational and irrational numbers.



c) Given any 3 real numbers a, b and c of your choice,

$a + b = b + a$: with an example, they will show that addition of two real numbers is commutative

$ab = ba$, with an example, they will show that the multiplication of two real numbers is commutative

$a(bc) = (ab)c$, with an example, they will show that the multiplication of real numbers is associative.

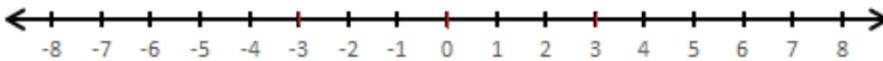
d. $\frac{a}{b}$ is a real number only when the number $b \neq 0$ because we do not

divide by 0 in \mathbb{R} . $\frac{5}{0} \notin \mathbb{R}$

Answers to Application activity 1.1

1) Appropriate scale and graph the following sets of real numbers on a number line

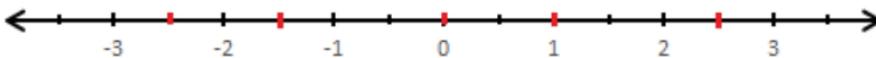
a) $\{-3, 0, 3\}$ on a number line



b) $\{-2, 2, 4, 6, 8, 10\}$ on number line



c) $\{-2.5, -1.5, 0, 1, 2.5\}$



2. a. $3 + 5 = 8$ and $5 + 3 = 8$ therefore $3 + 5 = 5 + 3 = 8$

b. $13 - 4 = 9$ and $-4 + 13 = 9$ therefore $13 - 4 = -4 + 13$

c. $12 + (33 + 25) = 70$ and $(12 + 33) + 25 = 70$

Therefore $12 + (33 + 25) = (12 + 33) + 25 = 70$

d. $0 + 5 = 5$ and $5 + 0 = 5$, therefore $0 + 5 = 5 + 0$

e. $18 + -18 = 0$ and $-18 + 18 = 0$, therefore $18 + -18 = -18 + 18 = 0$

f. $\frac{1}{5} \times 5 = 1$ and $5 \times \frac{1}{5} = 1$ $\therefore \frac{1}{5} \times 5 = 5 \times \frac{1}{5} = 1$

g. $13 \times (3 + 7) = 130$ and $(3 + 7) \times 13 = 130$

$\therefore 13 \times (3 + 7) = (3 + 7) \times 13 = 130$

1.6.2 Arithmetic of integers and whole numbers.

Learning activity 1.2

Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

- Facilitate learners in Group work, in answering the activity 1.2
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to explore set of real numbers and its subsets and solve related problems.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 1.2** and evaluate whether lesson objectives were achieved.

Answers for activity 1.2

- 1) a) At 9:00 where temperature is 37°C
b) from 21:00, temperature is 37.2°C , from 6:00, temperature is 37.5°C . Difference is -0.3°C
c) from 9:00, temperature is -37°C , from 12:00, temperature is 37.25°C . Difference is 0.25°C
d) $37.2^{\circ}\text{C} + 0.25^{\circ}\text{C} = 37.45^{\circ}\text{C}$
- 2) yes it can happen, in Macheria, Algeria -13.8°C on 28th January 2005 and in Ifrane, Morocco, -23.9°C on 11th 1935.

Answers to Application activity 1.2

- 1) b. Lower than 0°C
- 2) Starting with the lowest.
 $-10, -4, 0, 2, 5, 6$
- 3) a) Kigali with 24°C
b) Moscow with -8°C
c) $-2^{\circ}\text{C} - 19^{\circ}\text{C} = -21^{\circ}\text{C}$. Berlin is 21°C less than Boston
- 4)

t(min)	0	1	2	3	4	5
T $^{\circ}\text{C}$	-8°C	-7°C	-6°C	-5°C	-4°C	-3°C

- 5) The new temperature $-7^{\circ}\text{C} - 10^{\circ}\text{C} = -17^{\circ}\text{C}$

1.6.3 Rounding and estimating decimal numbers.

Learning activity 1.3

Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

- Facilitate learners in Group work, in answering the activity 1.3
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply their skills in calculation to accurately make rounding and estimating decimal numbers.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 1.3** and evaluate whether lesson objectives were achieved.

Answers for activity 1.3

1) $2:3=0.6$

$7:11=0.6363$

$1:5=0.2$

$\sqrt{11} = 3.33166$

$\sqrt{16} = 4$

- 2) I can write $\frac{22}{7} = 0.3142$ or 0.314 or 0.31 or 0.3 . It depends on instructions

Answers to Application activity 1.3

1) a) 22.68

b) 34.63

c) 34.58

- d) 56.67
e) 30.00
- 2) a) $30 \times 60 \approx 1800$
b) $360 : 20 \approx 18$
c) $30 \times 4 \approx 120$
d) $36 - 1360 \approx 23$
e) $20 + 60 \approx 80$

1.6.5 Ratios, proportions and rates.

Learning activity 1.5

Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

- Facilitate learners in Group work, in answering the activity 1.5
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply their skills on fractions and equivalent fractions to accurately solve problems related to Ratios, proportions and rates.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 1.5** and evaluate whether lesson objectives were achieved.

Answers for activity 1.5

1) a) Since $\frac{8}{12}$ prefer Maternal and child health

$\frac{4}{12}$ do not prefer it

$$\frac{4}{12} \times 100 = 33.3\%$$

b) $\frac{8}{12} = \frac{2}{3}$

c) $\frac{2}{3} \times 100 = 66.7\%$

2) Total amount to be paid by a patient is 17 500 Frw + 45 000 Frw + 30 000 Frw = 92 500 Frw

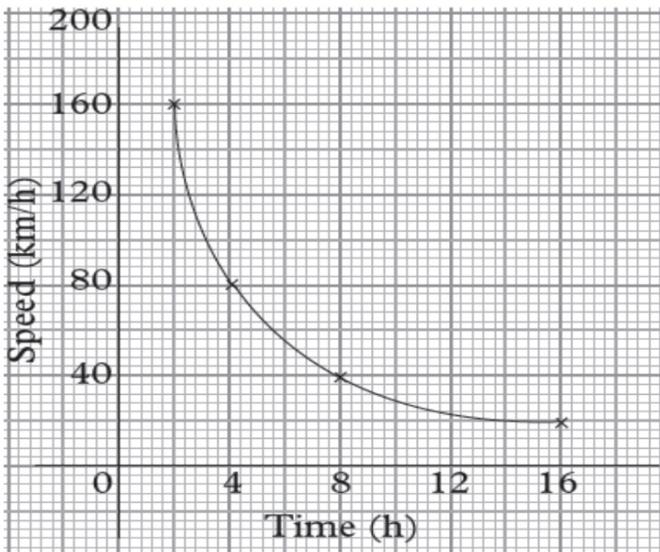
3) $\frac{85}{100} \times 92\,500 \text{ Frw} = 78\,625 \text{ Frw}$, RSSB paid 78 625 Frw

4) $\frac{15}{100} \times 92\,500 \text{ Frw} = 13\,875 \text{ Frw}$, Patient paid 13 875 Frw

5) Graph to be drawn

6) i) We notice that when then the speed is doubled the time is decreasing

ii)



iii) From the above graph you notice that the speed increases in the ratio $1:2$, the time decreases in the ratio $2:1$ and vice versa. Thus the two quantities are said to be **inversely proportional**.

It means that the car is decelerating with acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{20 - 160}{16 - 2} = -10 \text{ kmh}^{-2}$$

$$7) \frac{17 \text{beats}}{15 \text{seconds}} = \frac{17}{15 \times \frac{1}{60 \text{minutes}}} = \frac{17 \times 4 \text{beats}}{1 \text{minute}} = 68 \frac{\text{beats}}{\text{minute}}$$

Answers to Application activity 1.5

8) Simplifying ratio

$$36 \text{males} : 48 \text{females} = 3 \text{males} : 4 \text{females} \text{ (for physiology)}$$

$$64 \text{males} : 80 \text{females} = 4 \text{males} : 5 \text{females} \text{ (for clinical)}$$

Therefore, the clinical has largest ratio of males than physiology.

9) She would take 1litre in $12\frac{1}{2}$ hours

10) a) 2 tablets b) 160mg c) 1.3ml d) 60drops / min e) 7.5mil

1.6.6 Absolute value and its properties

a) Content summary

This section introduces the absolute value of a real number and properties of absolute value.

b) Materials, Methodology, Answers to activities and exercises

Learning activity 1.6.1

Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

Facilitate learners in Group work, then questioning .

Answers for activity 1.6.1



1. 8 units 2. 8 units 3. $\frac{1}{2}$ units 4. 13 units

Answers to application activity 1.6.1

1. -6 or 6 2. -1 3. -3 or 9
4. $\frac{-5}{2}$ or $\frac{3}{2}$ 5. 1 or 5

Learning activity 1.6.2

Materials

Notebook, pens and calculator.

Methodology

Facilitate learners in Group work, then questioning .

Answers for activity 1.6.2

1. 3 and 3 2. 15 and 15 3. 3 and 13

Answers to application 1.6.2

1. 5 2. 20 3. 28 4. -24 5. -2

1.6.7 Powers and radicals

This section introduces powers and radicals in set of real numbers and their properties.

Learning activity 1.7.1

Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

- Facilitate learners in pairs, in answering the activity 1.7.1
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to apply their skills in prime factors to accurately solve problems related to **powers and power properties**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 1.7.1** and evaluate whether lesson objectives were achieved.

Answers for activity 1.7.1

1.

Week	Dollars
One	$2=2$
Two	$2 \times 2 = 2^2 = 4$
Three	$2 \times 2 \times 2 = 2^3 = 8$
Four	$2 \times 2 \times 2 \times 2 = 2^4 = 16$
Five	$2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

2. $2^7 = 128$, $2^{10} = 1024$

3. No, his parent will not agree with his suggestion. Allowance is very increasing at certain time.

Answers to application activity 1.7.1

1. x^5

2. $5x^2y^6$

3. $2y$

4) $10^{-2}kg = \frac{1}{10^2}kg = \frac{1}{100}kg = 0.01kg$

5) $1.67 \times 10^{-27} \times 6 \times 10^3 = 1.002 \times 10^{-25}$

6) a) $10^3 = 10 \times 10 \times 10$

b) $10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$

c) $10^4 = 10 \times 10 \times 10 \times 10$

Learning activity 1.7.2

Materials

Manila paper or exercise books, markers or pens, Instruments of geometry.

Methodology

- Facilitate learners in pairs, in answering the activity 1.7.2
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply their skills in prime factors to accurately solve problems related to powers and power properties.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.

- After this step, guide students to do the application activity 1.7.2 and evaluate whether lesson objectives were achieved.

Answers for activity 1.7.2

1. 9

2. 6

3. -3

4. 2

Answers to application activity 1.7.2

1. ab^2c

2. abc

3. $\frac{2}{3}$

4. x

5. $\frac{xy^2}{2}$

Learning activity 1.7.3

Materials

Note book, pens

Methodology

- Facilitate learners in pairs, in answering the activity 1
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to apply their skills in prime factors to accurately solve problems related to **powers and radicals**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 1.7.3** and evaluate whether lesson objectives were achieved.

Answers for activity 1.7.3

1. $4\sqrt{2}$

2. $-\sqrt{3}$

3. $\sqrt{6}$

4. $\sqrt{3}$

Answers to application activity 1.7.3

1. $3\sqrt{5}$ 2. $2\sqrt{3}$ 3. $3\sqrt{7}$ 4. 12 5. $13\sqrt{5}$ 6. $2\sqrt{3}$

Learning activity 1.7.4

Materials

Note book, pens

Methodology

- Facilitate learners in pairs, in answering the activity 1.7.4
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate them to apply their skills in operations to accurately solve problems related to **powers and radicals**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 1.7.4** and evaluate whether lesson objectives were achieved.

Answers for activity 1.7.4

1. $\frac{\sqrt{2}}{2}$ 2. $\frac{2\sqrt{5}-\sqrt{15}}{10}$
3. $\frac{-2+2\sqrt{6}}{5}$ 4. $\frac{-3-\sqrt{6}+\sqrt{10}+\sqrt{15}}{2}$

Answers to applicatin activity 1.7.4

1. $\frac{5\sqrt{7}}{7}$

2. $1-\sqrt{2}$

3. $\frac{\sqrt{12}-\sqrt{18}-\sqrt{20}+\sqrt{50}}{4}$

4. $\frac{6\sqrt{6}-8\sqrt{2}}{23}$

5. $\frac{a\sqrt{d}-\sqrt{bd}}{d}$

6. $\frac{-3\sqrt{3}+6\sqrt{6}-2\sqrt{2}+8}{7}$

1.6.8 Decimal Logarithm

Learning activity 1.8

Materials

Note book, pens and scientific calculator.

Methodology

- Facilitate learners in pairs, in answering the activity 1.8
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglng students.
- Facilitate them to apply their skills in powers to accurately solve problems related **Decimal logarithm**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 1.8** and evaluate whether lesson objectives were achieved.

Answers for activity 1.8

1. 0 2. 1 3. 2
4. 3 5. 4 6. 5

Answers to application activity 1.8

1. a) $a > b$ b) $a < b$
2. a) 2.17 c) 0.30
3. a) -2 b) -1.62 c) -1.176

Answers to end unit assessment 1

- 1) a) yz b) $2a^3b^6$ c) $2\sqrt{2}$
- 2) a) $\frac{3\sqrt{35} + \sqrt{14}}{14}$ b) $2\sqrt{2} + 2\sqrt{5} + \sqrt{6} + \sqrt{15}$
c) $\frac{-2 - \sqrt{6} + \sqrt{10} + \sqrt{15}}{3}$
- 3) a) 1.08 b) -0.35 c) 0.56
- 4) a) \rightarrow iii b) \rightarrow ii c) \rightarrow i
- 5) 96 6. 64
- 7) 287l 8) $19.3 \text{ gr} / \text{cm}^3$ 9) 45times
- 10) a) 13,041 people tested negative.
b) 459 people tested positive
- 11) Ingabire's share : 240,000Frw
Mugenzi's share : 360,000Frw
Gahima's share : 480,000Frw

12) No, if you take for example , $\frac{3}{5}$ and add 1 to both numerator

and denominator, it shows that $\frac{3+1}{5+1} = \frac{4}{6} \neq \frac{3}{5}$

Unit 2

Fundamentals of Trigonometry

2.1. Key Unity Competency

Use the trigonometric concepts and formulas to solve related problems in Physics, Air navigation, Water navigation, bearings, Surveying and modern medicine.

2.2. Objectives

By the end of this unit, the learners will:

- define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle – know special values.
- convert radians to degree and vice versa.
- use trigonometric identities.
- apply trigonometric formulae in real world problems.

2.3. List of lessons

Week	Lesson number	Content	Number of Periods
5	1	Angle and its measurements	1
	2	Unit circle	1
	3	Definition and identification of Trigonometric ratios of acute angles	1
6	4	Trigonometric ratios of special angles	2
	5	Trigonometric identities	1
7	6	Solving a triangle by cosine law	2
	7	Solving a triangle by Sine law	1

8&9	8	Applications trigonometry: Cardiology, Bearing, Air plane directions, Navigation, Inclined plane.	5
	9	End unit assessment and Remediation	1

2.4. Vocabulary

Trigonometry, radian, grade, sine, cosine, secant, cosecant, tangent and cotangent.

2.5. Guidance on introductory activity 2.0

Students work on the introductory activity to understand the use of trigonometry.

Let them read and do the introductory activity 1 in the Student's book.

Make sure that all students are activating and performing well.

Through class discussions, let students think of different ways of application of trigonometry.

Through different examples, help students to understand the importance of trigonometry by showing their application in real life for example in construction, satellite systems and astronomy, naval and aviation industries, land surveying and in cartography (creation of maps) and so on.

Answer of introductory activity 2.0

Pythagoras theorem is not enough for finding the height of the given cathedral. By using sine rule, the required height can be determined as

$$\frac{\sin 60^\circ}{h} = \frac{\sin 30^\circ}{280} \Leftrightarrow h \sin 30^\circ = 280 \sin 60^\circ \Leftrightarrow \frac{h}{2} = \frac{280\sqrt{3}}{2}$$

$$\Leftrightarrow h = 280\sqrt{3} \approx 484.97m$$

2.6. Contents and activities

Trigonometric concepts

This section introduces the conversion of angles in their units of measurements (degrees, radians and grades). It looks at trigonometric ratios, trigonometric number of an angle and trigonometric identities.

$$\frac{D}{180} = \frac{R}{\pi} = \frac{G}{200}, \text{ where D stands for degree, R for radians, G for}$$

grades and $\pi = 3.14\dots$

In a right triangle

$$\sin \alpha = \frac{\textit{opposite side}}{\textit{hypotenuse}}, \cos \alpha = \frac{\textit{adjacent side}}{\textit{hypotenuse}}, \tan \alpha = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$\csc \alpha = \frac{\textit{hypotenuse}}{\textit{opposite}} = \frac{1}{\sin \alpha}, \sec \alpha = \frac{\textit{hypotenuse}}{\textit{adjacent}} = \frac{1}{\cos \alpha}$$

$$\text{and } \cot \alpha = \frac{\textit{adjacent}}{\textit{opposite}} = \frac{1}{\tan \alpha}$$

2.6.1 Angle and its measurements

Learning activity 2.1

Materials

Manila paper if not sheets of paper or notebook, Markers or pens, Instruments of geometry

Methodology

To form a group of atleast four learners and facilitate them during the activity 2.1

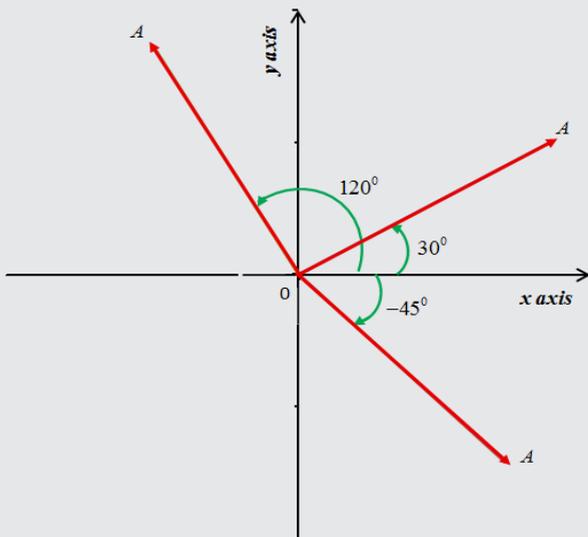
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglng students.
- Facilitate the use of geometric materials to make accurate graphs and measuments.

- Make sure that the notebooks of student-teachers include squared papers/graph papers.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 2.1** and evaluate whether lesson objectives were achieved..

To check if each group is rotating the given vector correctly on the manila paper (or sheet of paper).

Invite one group for representation of its work to other group.

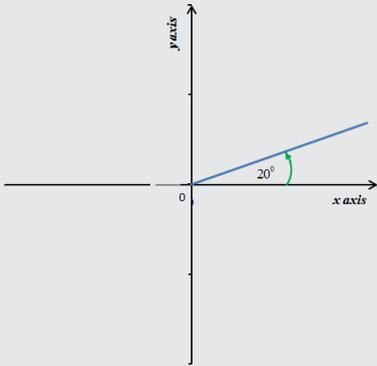
Answers for activity 2.1



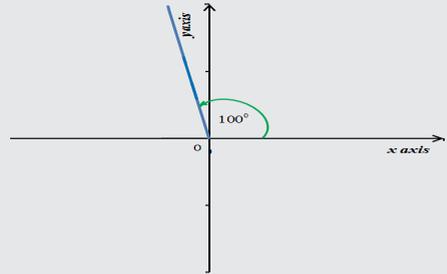
Answers to application activity 2.1

1.

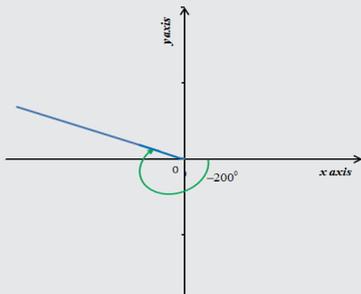
a)



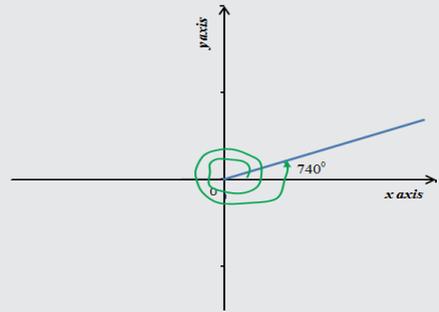
b)



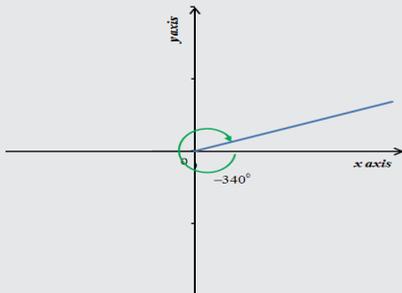
c)



d)



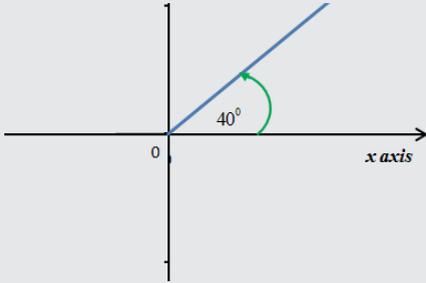
e)



740° and -340° are co terminal to 20°

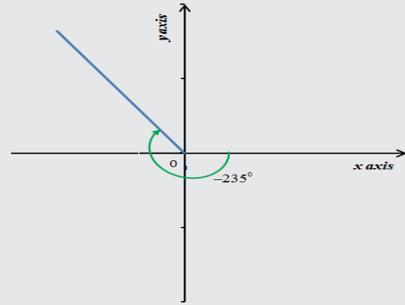
2.

a)



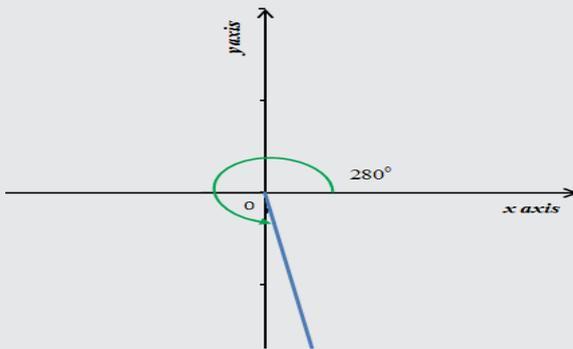
40° is in first quadrant

b)



-235° is in second quadrant

c)



280° is in fourth quadrant

2.6.2 Measures of angles

Learning activity 2.2

Materials

Notebook, pens, scientific calculator

Methodology

This is a pairing activity. Facilitate every learner during the activity 2.2.

Get different answers from learners and emphasize on conversion.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.

- Facilitate them to make accurate conversions.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 2.2** and evaluate whether lesson objectives were achieved.

Answers for activity 2.2

$$1. \frac{80}{3} \qquad 2. \frac{2480}{9} \qquad 3. \pi \qquad 4. \frac{3\pi}{2}$$

Answers to application activity 2.2

$$1. \frac{11}{10} \pi \text{ radians, } 198^\circ \qquad 2. \frac{62\pi}{9} \text{ radians, } \frac{12400}{9} \text{ grades}$$

$$3. 72^\circ \qquad 4. 5^\circ 36' 00''$$

2.6.3 Definition and identification of trigonometric ratios of acute angle

Learning activity 2.3

Materials

Note book, Pens, Instruments of geometry

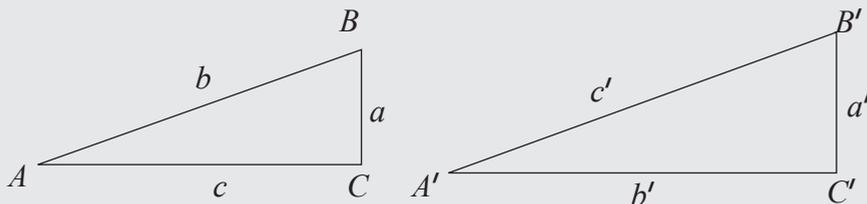
Methodology

Group work and guide learners during the activity 2.3 of drawing a right triangle and show different ratios in that triangle.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.

- Facilitate them to relate sides and angle to make accurate ratios and explore the trigonometric ratios.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 2.3** and evaluate whether lesson objectives were achieved.

Answers for activity 2.3



The longest side is called the hypotenuse.

Angle	$\frac{\textit{opposite side}}{\textit{hypotenuse}}$	$\frac{\textit{adjacent side}}{\textit{hypotenuse}}$	$\frac{\textit{opposite side}}{\textit{adjacent side}}$
A	$\frac{a}{c}$	$\frac{b}{c}$	$\frac{a}{b}$
B	$\frac{b}{c}$	$\frac{a}{c}$	$\frac{b}{a}$
C	$\frac{c}{c} = 1$	$\frac{0}{c} = 0$	$\frac{c}{0}$ does not exist

Angle	$\frac{\textit{opposite side}}{\textit{hypotenuse}}$	$\frac{\textit{adjacent side}}{\textit{hypotenuse}}$	$\frac{\textit{opposite side}}{\textit{adjacent side}}$
A'	$\frac{a'}{c'}$	$\frac{b'}{c'}$	$\frac{a'}{b'}$
B'	$\frac{b'}{c'}$	$\frac{a'}{c'}$	$\frac{b'}{a'}$
C'	$\frac{c'}{c'} = 1$	$\frac{0}{c'} = 0$	$\frac{c'}{0}$ does not exist

From Thales' theorem: $\frac{a}{c} = \frac{a'}{c'}, \frac{b}{c} = \frac{b'}{c'}, \frac{c}{c} = 1 = \frac{c'}{c'}$

For both triangle, the ratio

- Opposite side to the considered angle and hypotenuse is the same
- Adjacent side and hypotenuse is the same
- Opposite side to the considered angle and adjacent side is the same.

Answers to application activity 2.3

$$\csc B = \frac{15}{9} = \frac{5}{3}$$

$$\csc A = \frac{15}{12} = \frac{5}{4}$$

$$\sec B = \frac{15}{12} = \frac{5}{4}$$

$$\sec A = \frac{15}{9} = \frac{5}{3}$$

$$\csc C = 1$$

$$\cot B = \frac{12}{9} = \frac{4}{3}$$

$$\cot A = \frac{9}{12} = \frac{3}{4}$$

sec C does not exist

$$\cot C = 0$$

2.6.4 trigonometric ratios of special angles

Learning activity 2.4

Materials

Note book, Pens and calculator

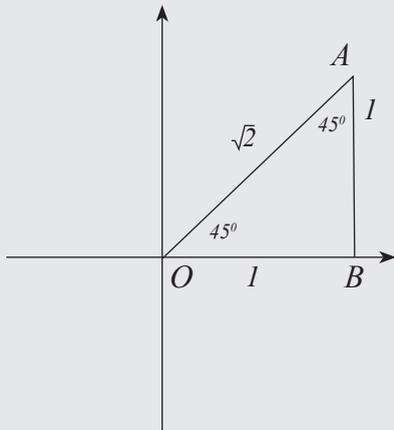
Methodology

Group work and Questioning for facilitating learners during the activity 2.4 and developing brainstorming.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate them to explore accurately the trigonometric ratios on special angles.
- Call students to present the findings and promote gender where possible.

- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application activity 2.4 and evaluate whether lesson objectives were achieved.

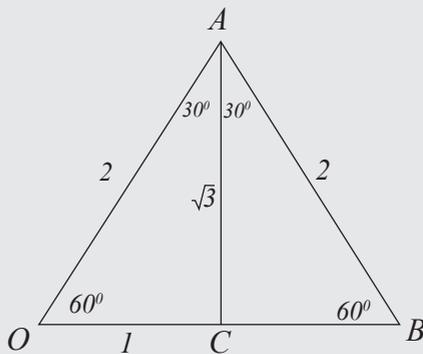
Answers for activity 2.5



From the diagram

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$



From $\triangle OAC$,

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}, \quad \csc 60^\circ = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3},$$

$$\sec 60^\circ = 2,$$

$$\cot 60^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3},$$

From $\triangle OAC$,

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \tan 30^\circ = \frac{\sqrt{3}}{3}, \quad \csc 30^\circ = 2, \quad \sec 30^\circ = \frac{2\sqrt{3}}{3},$$

$$\cot 30^\circ = \sqrt{3}$$

Answers to application activity 2.4

1. 1	2. 1	3. $\frac{\sqrt{3}}{3}$
4. 0	5. does not exist	6. 0
7. does not exist	8. does not exist	9. 0
10. does not exist	11. 0	

2.6.5 CAST Rule

Learning activity 2.5

Materials

Note book, Pens, Instruments of geometry

Methodology

Group work , facilitating learners (HINT: to construct trigonometric circle) for the activity 2.5.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglng students.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book .

After this step, guide students to do the application activity 2.5 and evaluate whether lesson objectives were achieved.

Answers for activity 2.5

	Quadrant			
Value	<i>I</i>	<i>II</i>	<i>III</i>	<i>IV</i>
sin	+	+	-	-
cos	+	-	-	+
tan	+	-	+	-

Answers to application activity 2.5

- a) Second quadrant b) Second quadrant
c) First quadrant d) First quadrant
e) Third quadrant f) Second quadrant

2.6.6 Trigonometric identities

Learning activity 2.6

Materials

Note book, Pens and calculator

Methodology

Group work, brainstorming for the activity 2.6.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragging students.
- Facilitate them to demonstrate and generate accurately the trigonometric identities.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.

- After this step, guide students to do the application activity 2.6 and evaluate whether lesson objectives were achieved.

Answers for activity 2.6

In this triangle,

$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}, \quad \sin \alpha = \frac{x}{r}, \quad \cos \alpha = \frac{y}{r}$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = (\sin \theta)^2 + (\cos \theta)^2 = \sin^2 \theta + \cos^2 \theta$$

and then $\sin^2 \theta + \cos^2 \theta = 1$

$$\sin^2 \alpha + \cos^2 \alpha = \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

Answers to application activity 2.6

1. 1

2. 0

3. $\frac{1 + \tan a}{3 - \tan a}$

2.6.7 Solving a triangle by cosine rule

Triangle and applications

This section introduces the sine and cosine rule and their applications.

The cosine law says that

$$\begin{cases} a^2 = b^2 + c^2 - 2bc \cos A \\ b^2 = a^2 + c^2 - 2ac \cos B \\ c^2 = a^2 + b^2 - 2ab \cos C \end{cases}$$

The sine law is

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Learning activity 2.7

Materials

Manila paper or Note book, markers or Pens

Methodology

Facilitate learners in Group work for the activity 2.7.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to accurately use cosine rule to solve related problems.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book .
- After this step, guide students to do **the application activity 2.7** and evaluate whether lesson objectives were achieved.

Answers activity 2.7

$$1. \cos A = \frac{AX}{b}$$

$$2. b^2 = h^2 + (AX)^2 \Rightarrow h^2 = b^2 - (AX)^2$$

$$3. a^2 = h^2 + (XB)^2 \Rightarrow h^2 = a^2 - (XB)^2$$

$$h^2 = b^2 - (AX)^2 \text{ and } h^2 = a^2 - (XB)^2 \text{ gives } b^2 - (AX)^2 = a^2 - (XB)^2$$

$$4. \text{ But } XB = c - AX, \text{ then}$$

$$b^2 - (AX)^2 = a^2 - (c - AX)^2$$

$$\Leftrightarrow b^2 - (AX)^2 = a^2 - (c^2 - 2cAX + (AX)^2)$$

$$\Leftrightarrow b^2 - (AX)^2 = a^2 - c^2 + 2cAX - (AX)^2$$

$$\Leftrightarrow b^2 + c^2 - 2cAX = a^2$$

But $\cos A = \frac{AX}{b} \Rightarrow AX = b \cos A$. Then

$$\Leftrightarrow b^2 + c^2 - 2cb \cos A = a^2$$

$$\Leftrightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

Answers to application activity 2.7

1. 9.43 cm 2. $c = 21.7 \text{ cm}$ 3. $A = 12.7^\circ, B = 22.3^\circ, c = 14.4 \text{ cm}$

2.6.8 Solving a triangle by sine rule

Learning activity 2.8

Materials

Manila paper or Note book, markers or Pens

Methodology

- Work in Group, then control participation of learners in answering the activity 2.8
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to accurately use sine rule to solve related problems.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book .

After this step, guide students to do **the application activity 2.8** and evaluate whether lesson objectives were achieved.

Answers for activity 2.8

1. $\sin B = \frac{h}{a}$, $\sin A = \frac{h}{b}$.

$$h = a \sin B \text{ and } b \sin A = h, \text{ then } a \sin B = b \sin A \text{ or } \frac{a}{\sin A} = \frac{b}{\sin B}$$

2. $\sin A = \frac{k}{c}$, $\sin C = \frac{k}{a}$.

$$k = c \sin A \text{ and } k = a \sin C, \text{ then } c \sin A = a \sin C \text{ or } \frac{c}{\sin C} = \frac{a}{\sin A}$$

3. Now, $\frac{a}{\sin A} = \frac{b}{\sin B}$ and $\frac{c}{\sin C} = \frac{a}{\sin A}$. This gives $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Answers to application activity 2.8

1. 8.45 cm

2. $A = 56.8^\circ$, $C = 81.2^\circ$ or $A = 123.2^\circ$, $C = 14.8^\circ$

3. 6.18 cm

Answers to end unit assesment 2

1. a)
$$\frac{\sin a(1 + \cos a)}{(1 - \cos a)(1 + \cos a)} = \frac{\sin a(1 + \cos a)}{\sin^2 a}$$
$$= \frac{1 + \cos a}{\sin a}$$

b)
$$\frac{1}{\cos^2 a} + \frac{1}{\sin^2 a} = \frac{\sin^2 a + \cos^2 a}{\cos^2 a \sin^2 a}$$
$$= \frac{1}{\cos^2 a} \frac{1}{\sin^2 a}$$

c)
$$= \sec^2 a \csc^2 a$$

$$\begin{aligned} (\sec^2 a + \tan^2 a)(\sec^2 a - \tan^2 a) &= (\sec^2 a + \tan^2 a) \left(\frac{1}{\cos^2 a} - \frac{\sin^2 a}{\cos^2 a} \right) \\ &= (\sec^2 a + \tan^2 a) \left(\frac{1 - \sin^2 a}{\cos^2 a} \right) \\ &= (\sec^2 a + \tan^2 a) \left(\frac{\cos^2 a}{\cos^2 a} \right) \\ &= \sec^2 a + \tan^2 a \end{aligned}$$

$$d) \sqrt{\frac{(1-\cos a)(1-\cos a)}{(1+\cos a)(1-\cos a)}} = \sqrt{\frac{(1-\cos a)^2}{\sin^2 a}} = \frac{1-\cos a}{\sin a}$$

2. $\tan \theta = 3.18$

3. $\cos \theta = -0.8$; $\tan \theta = -0.75$

4. $\cos 14^\circ = \sqrt{1 - \sin^2 14^\circ}$ or $\cos 14^\circ = \sin(90^\circ - 14^\circ) = \sin 76^\circ$

5. $x \approx 37.7$ 6. $p = 60^\circ$ 7. $h \approx 116.6$ 8. $4.12m$ 9. 7°

10. a) $88.6 km$ b) $179.3 km$ 11. $954 km, 133^\circ$ 12. $76.5 m/s$

13. 81.2189° 14. $117^\circ 39' 26''$ 15. $2^\circ 56'$ 16. 75.3°

Unit 3

LINEAR, QUADRATIC EQUATIONS AND INEQUALITIES

3.1. Key Unity Competency

Model and solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities.

3.2. Objectives

By the end of this unit, the learners will:

- solve equation of the first degree and second degree.
- solve inequality of the first degree and second degree.
- solve a system of linear equations.
- use equations and inequalities to solve word problems.
- apply equations and inequalities in real life problems.

3.3. List of lessons

Week	Lesson number	Content	Number of Periods
10	1	Simple linear equations in one unknown	2
	2	Products / quotients with linear equations.	1
11	3	Simple Linear inequalities in one unknown	2
	4	Products/ quotients form of inequalities.	1
12	5	Quadratic equations in one unknown	3
13	6	Simultaneous equations in two unknowns	1
	7	Solving problems from real life situations involving equations and inequalities	1
	8	End unit assessment and Remediation	1

3.4. Vocabulary

Parameter

3.5. Guidance on introductory activity 3.0

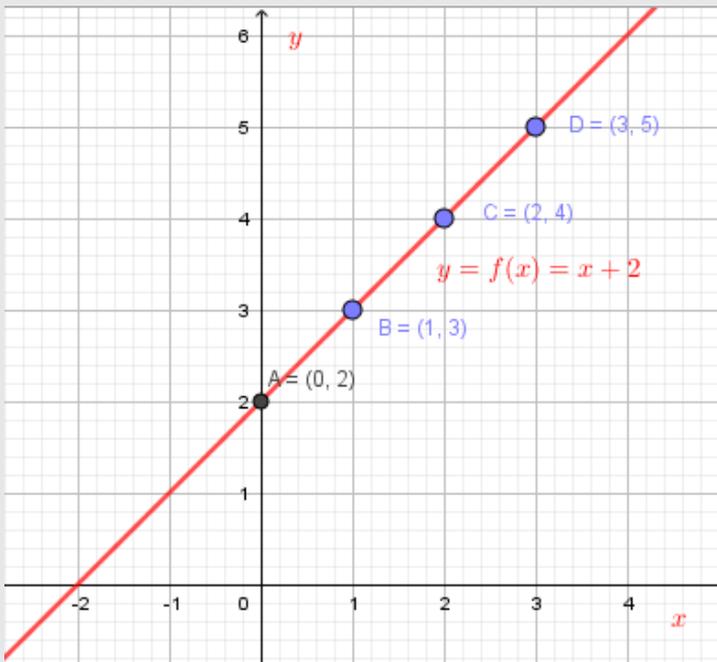
- Invite learners to work in groups where they read and analyse the problem in introductory activity 3.0
- During instruction, tel them that they can use a library or computer lab to search on the definition of linear equation and its application in real life.
- Ask learners to complete the table found in introductory activity by using the information obtained from research.
- Invite all groups to present their findings to the whole class.
- Basing on their experience, results from their own research, prior knowledge and abilities shown in answering the questions for this activity, use different questions to facilitate learners give their predictions and ensure that you arouse their curiosity on what is going to be leant in this unit.

Answer of introductory activity 3.0

1. Refer to the student's book and verify answers for students.
2. If x is the number of pens for a learner, the teacher decides to give him/her two more pens. A learner with one pen will have $(1 + 2)$ pens = 3 pens
3. a) $y = f(x) = x + 2$

X	-2	-1	0	1	2	3	4
$y = f(x) = x + 2$	0	1	2	3	4	5	6
(x,y)	(-2;0)	(-1;1)	(0,2)	(1;3)	(2;4)	(3;5)	(4;6)

b) The graph obtained is the following:



c) The graph obtained is a line.

d) $y = x + 2$ this is a linear equation because its graph is a line.
Identically, $x + 2 \geq 0$ is a linear inequality.

4. Students will give different examples. Verify whether the solution involves the linear equation.

5. a) $y = -16t^2 + 1600$, for $y = 1000$, we have $1000 = -16t^2 + 1600$

Solve this equation to find the time requested. $t = \frac{\sqrt{600}}{4} \approx 6.1$

The jumper is in free fall for about 6 seconds.

b) Table of value:

t	0	1	2	3	4	5	6
y	1600	1616	1664	1744	1856	2000	2176

3.6. Content & activities

3.6.1 Simple linear Equations in One Unknown

Learning activity 3.1

Materials

Note book and pens.

Methodology

- Facilitate learners in pairs, then questioning during the activity 3.1.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply their skills in solving equations to accurately solve problems involving linear equations.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application activity 3.1 and evaluate whether lesson objectives were achieved.

Answers for activity 3.1

- | | | |
|--------|------|-------|
| 1. 4 | 2. 2 | 3. -3 |
| 4. -34 | 5. 3 | 6. 14 |

Answers to application activity 3.1

- | | | | | |
|------|------------------|-------------------|-------|--------|
| 1. 4 | 2. 0 | 3. 5 | 4. 15 | 5. -4 |
| 6. 5 | 7. $\frac{1}{2}$ | 8. $-\frac{7}{3}$ | 9. -1 | 10. 10 |

II) *Uwamahoro's resting heart rate*(r):

$$r + 55 = 123$$

$$r = 68 \text{beats / min}$$

III) *Earth's average distance(d) from the Sun :*

$$d - 42,000,000 = 108,000,000$$

$$d = 150,000,000\text{km}$$

3.6.2 Products / quotients with linear equations.

Learning activity 3.2

Materials

Note book and pens.

Methodology

- This is a pairing activity, Facilitate every learner during the activity 3.2.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply their skills in solving equations to accurately solve problems involving products / quotients with linear equations.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.2** and evaluate whether lesson objectives were achieved

Answers for activity 3.2

1. $AB = 0 \Leftrightarrow A = 0$ or $B = 0$ Either $x + 1 = 0$ or $x - 1 = 0$
2. $AB = 0 \Leftrightarrow A = 0$ or $B = 0$ Either $2x - 3 = 0$ or $x = 0$
3. Cross product: $2(2x - 3) = x$, with $x \neq 0$

Answers to application activity 3.2

1. $S = \{-2, 5\}$
2. $S = \left\{-\frac{9}{2}\right\}$
3. $S = \left\{-8, -\frac{1}{2}\right\}$
4. $S = \left\{\frac{47}{11}\right\}$

3.6.3 Simple Linear inequalities in one unknown

Learning activity 3.3

Materials

Note book and pens

Methodology

- Group work and guide learners during the activity 3.3
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to apply their skills in solving inequalities to accurately solve problems involving linear inequalities in one unknown.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application activity 3.3 and evaluate whether lesson objectives were achieved

Answers for activity 3.3

1. All numbers less than 5
2. All positive numbers
3. All numbers between -4 and 12
4. All numbers less than or equal to 100

Answers to application activity 3.3

- I. 1. $]-\infty, 9[$ 2. $]-\infty, 10[$ 3. $]-\infty, 5]$
4. $]\frac{26}{3}, +\infty[$ 5. $[5, +\infty[$ 6. $]-\infty, 4[$
7. $]-\infty, +\infty[$ or \mathbb{R} 8. $\{ \}$ 9. $]-\infty, -11]$ 10. $]-3, +\infty[$

II) $n < 5$

III) $f \geq 5$

3.6.4 Products/ quotients form of inequalities.

Learning activity 3.4.1

Materials

Note book and pens

Methodology

- Group learners at least in three and facilitate them during the activity 3.4.1.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply their skills in solving inequalities to accurately solve problems involving products/ quotients form of inequalities.
- Call students to present the findings and promote gender where possible.

- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.4.1** and evaluate whether lesson objectives were achieved

Answers for activity 3.4.1

In each case, first construct the sign table. The solution will be given by interval showing negative values for $<$

Answers to application activity 3.4.1

- | | |
|--------------------------------------|------------------------------------|
| 1. $]-\infty, -3[\cup]3, +\infty[$ | 2. $\left[\frac{3}{4}, 1\right]$ |
| 3. $]-\infty, -5[\cup]-1, 2[$ | 4. $\left]-\frac{4}{3}, -1\right[$ |
| 5. $]-\infty, -2[\cup]3, +\infty[$ | |

Learning activity 3.4.2

Materials

Note book, pens and instruments of geometry

Methodology

- Group work and questioning for facilitating learners during the activity 3.4.2 and developing brainstorming.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate them to apply their skills in solving inequalities to accurately solve problems related to inequalities involving absolute value.
- Call students to present the findings and promote gender where possible.

- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.4.2** and evaluate whether lesson objectives were achieved

Answers for application activity 3.4.2

$$1. S = \{x \in \mathbb{R} : |x| > 4\} \quad 2. S = \{x \in \mathbb{R} : |x| < 6\}$$

Answers to application activity 3.4.2

$$1.]-\infty, -2[\cup]3, +\infty[\quad 2.]-\infty, -\frac{9}{2}] \cup \left[\frac{3}{2}, +\infty[\quad 3.]0, 3[$$

Learning activity 3.4.3

Materials

Note book, pens and calculator.

Methodology

- Group work and questioning for facilitating learners during the activity 3.4.3 and developing brainstorming
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply their skills in solving equations and inequalities to accurately solve real life problems.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.

- After this step, guide students to do **the application activity 3.4.3** and evaluate whether lesson objectives were achieved

Answers for activity 3.4.3

1. Let x be the age of the son, then the age of father has $x+30$ years old. Set $x+30-5=4(x-5)$ and then solve.

Here some learners can take x as the age of the father.

2. Let x be the money Betty had originally. Set

$$x - \frac{x}{5} - \frac{1}{2}\left(x - \frac{x}{5}\right) - 7000 = 13000$$

Answers to application activity 3.4.3

1. The two numbers are 8 and 17.
2. The two numbers are 84 and 36.
3. Length of the rectangle is 24 m and breadth of the rectangle is 12 m.
4. Present age of Ron is 6 years and present age of Aaron is 1 year.
5. Sam scored less than 3 goals, which means that Sam could have scored 0, 1 or 2 goals. Alex scored 3 more goals than Sam did, so Alex could have scored 3, 4, or 5 goals.
6. His average speed running is greater than 13 km/h and his average speed cycling is greater than 26 km/h

3.6.5 Simultaneous equations in two unknowns

Learning activity 3.5.1

Materials

Note book, pens and calculator.

Methodology

- In group, facilitate learners for the activity 3.5.1

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply addition or elimination method to accurately solve problems involving **simultaneous equations in two unknowns**
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.5.1** and evaluate whether lesson objectives were achieved

Answers for activity 3.5.1

$$1. \begin{cases} x+y=12 & \times -1 \\ 2x+y=4 & \times 1 \end{cases} \Leftrightarrow \begin{cases} -x-y=-12 \\ 2x+y=4 \end{cases}$$

$$x = -8$$

$$2. \begin{cases} 3x-y=20 & \times 1 \\ -x+2y=4 & \times 3 \end{cases} \Leftrightarrow \begin{cases} 3x-y=20 \\ -3x+6y=12 \end{cases}$$

$$5y = 32$$

$$3. \begin{cases} x-2y=10 & \times 1 \\ 2x+y=14 & \times 2 \end{cases} \Leftrightarrow \begin{cases} x-2y=10 \\ 4x+2y=28 \end{cases}$$

$$5x = 38$$

There are many different possible numbers

Answers to application activity 3.5.1

- | | | |
|---|-----------------|---------------------------------------|
| 1. Infinity solutions | 2. $x = y = 0$ | 3. $x = 3, y = \frac{2}{3}$ |
| 4. $x = -\frac{1}{8}, y = \frac{15}{8}$ | 5. $x = y = -1$ | 6. $x = \frac{7}{3}, y = \frac{1}{3}$ |

Learning activity 3.5.2

Materials

Note book, pens and calculator.

Methodology

- Group work, brainstorming for the activity 3.5.2 then questioning.
- Make sure that everybody is engaged/ involved.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply Substitution method to accurately solve problems involving **simultaneous equations in two unknowns**
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.5.2** and evaluate whether lesson objectives were achieved

Answers for application activity 3.5.2

1. $x = 5 + y$ and $5 + y + 2y = 6 \Leftrightarrow 5 + 3y = 6$
2. $x = 10 - 2y$ and $-3(10 - 2y) + 2y = 12 \Leftrightarrow -30 + 8y = 12$
3. $x = -10 - y$ and $4(-10 - y) + y = 0 \Leftrightarrow -40 - 3y = 0$

Some learners can first find the value of y from any equation

Answers to application activity 3.5.2

- | | | |
|---|---------------------------------------|---------------------------------------|
| 1. Infinity solutions | 2. $x = y = 0$ | 3. $x = \frac{3}{2}, y = \frac{9}{2}$ |
| 4. $x = \frac{13}{4}, y = \frac{15}{4}$ | 5. $x = \frac{3}{5}, y = \frac{2}{5}$ | 6. $x = 1, y = 0$ |

Learning activity 3.5.3

Materials

Note book, pens and calculator.

Methodology

- Group work, brainstorming for the activity 3.5.3 then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply Cramer's rule (determinants method) to accurately solve problems involving **simultaneous equations in two unknowns**
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.5.3** and evaluate whether lesson objectives were achieved

Answers for activity 3.5.3

1. 0

2. 17

3. 25

Answers to application activity 3.5.3

1. $x = 2, y = 0$

2. $x = y = 1$

3. $x = \frac{19}{9}, y = \frac{4}{9}$

4. $x = -\frac{3}{8}, y = \frac{5}{8}$

5. $x = 1, y = -\frac{2}{3}$

6. $x = \frac{70}{13}, y = \frac{16}{13}$

Learning activity 3.5.4

Materials

Note book, pens, instruments of geometry and calculator.

Methodology

- Facilitate learners in Group work for the activity 3.5.4, then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to apply Graphical method to accurately solve problems involving simultaneous equations in two unknowns
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application activity 3.5.4 and evaluate whether lesson objectives were achieved

Answers for activity 3.5.4

1. For $3x + y = 10$

Let $x = 3$, then $y = 1$.

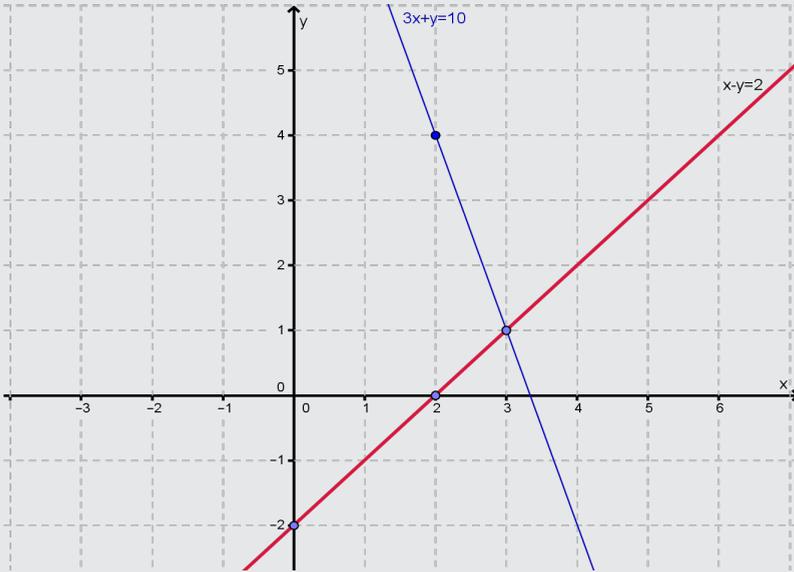
Let $y = 4$, then $x = 2$. We have $(3,1)$ and $(2,4)$

For $x - y = 2$

Let $x = 0$, then $y = -2$.

Let $y = 0$, then $x = 2$. We have $(0,-2)$ and $(2,0)$

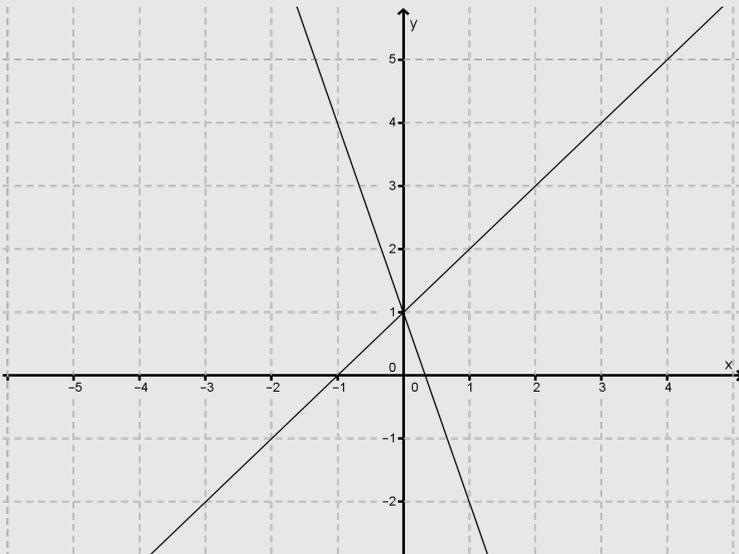
2. Graph



3. The point of intersection is $(3, 1)$

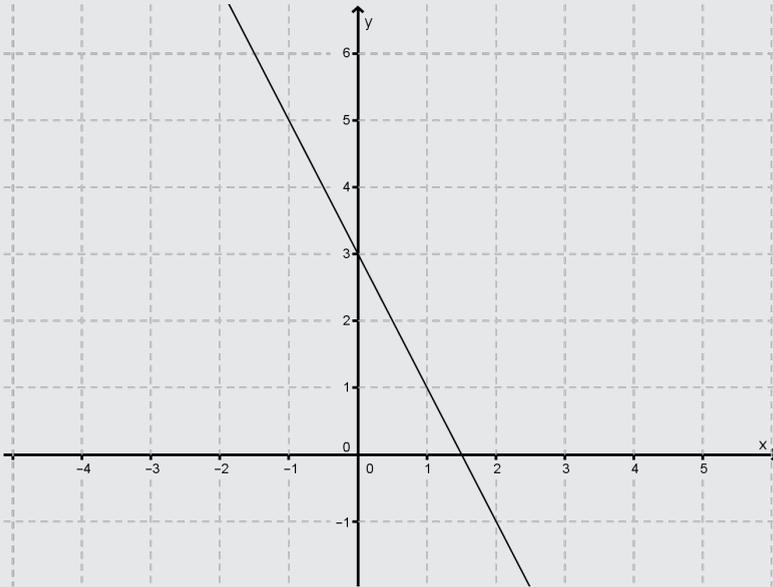
Answers to application activity 3.5.4

1. Graph



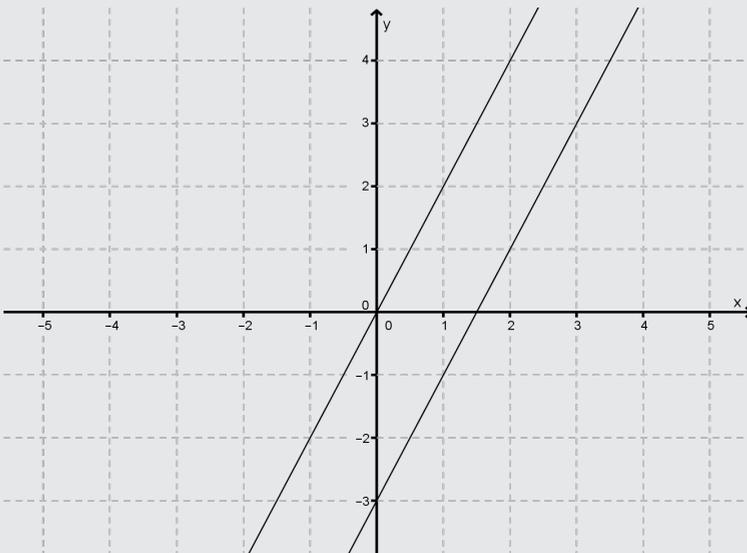
$x = 0, y = 1$

2. Graph



The two lines coincide. There is infinite number of solution

3. Graph



Parallel and distinct lines. No solution.

Learning activity 3.5.5

Materials

Note book, pens and calculator.

Methodology

- Facilitate learners in Group work for the activity 3.5.5, then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to apply skills in solving equations to accurately solve word problems using **simultaneous equations**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.5.5** and evaluate whether lesson objectives were achieved

Answers for activity 3.3.5

1. Let x = cost per cat, y = cost per dog

$$\begin{cases} 164x + 24y = 4240 \\ x = 2y \end{cases}$$

Then solve the system

2. Let x = cost of water slide, y = cost of Ferris wheel

$$\begin{cases} 3x + 3y = 17.70 \\ 2x + 3y = 15.55 \end{cases}$$

Then solve the system

Answers to application activity 3.5.5

1. There are 5 multiple choice questions.
There are 15 T/F questions.
2. The small pitcher holds 2 cups of water.
The large pitcher holds 4 cups of water.
3. Number of students in van is 8 and number of students in bus is 22 students
4. 34
5. The speed of the boat in still water is 12 miles/hour and the speed of the current is 9 miles/hour.

3.6.6 Quadratic equations in one unknown

Learning activity 3.6.1

Materials

Exercise books, pens and calculator.

Methodology

- Group work, facilitating learners for the activity 3.6.1 then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to apply **factorizing and square root** to accurately solve **Quadratic equations**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.6.1**

and evaluate whether lesson objectives were achieved

Answers for activity 3.6.1

$$y = -16x^2 + 1600$$

$$1000 = -16x^2 + 1600$$

$$-16x^2 = -600$$

$$x^2 = 37.5 \Rightarrow x = \pm 6.1$$

The jumper is in free fall for about 6.1s

Answers to application activity 3.6.1

1. $\{-4, 2\}$ 2. $\{-1, 3\}$ 3. $\{-2, -1\}$ 4. $\left\{-\frac{\sqrt{77}}{6}, \frac{\sqrt{77}}{6}\right\}$

Learning activity 3.6.2

Materials

Exercise books, pens and calculator.

Methodology

- Group work, facilitating learners for the activity 3.6.2 then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate them to apply completing **the square method** to accurately solve **Quadratic equations**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.

- After this step, guide students to do **the application activity 3.6.2** and evaluate whether lesson objectives were achieved

Answers for activity 3.6.2

$$\text{a) } y = ax^2 + bx + c, a \neq 0$$

$$y = ax^2 + bx + c, a \neq 0$$

$$\Leftrightarrow y = (ax^2 + bx) + c$$

$$\Leftrightarrow y = a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} \right] + c$$

$$\Leftrightarrow y = a \left(x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a} + c$$

$$\Leftrightarrow y = a \left(x + \frac{b}{2a} \right)^2 + \left(c - \frac{b^2}{4a} \right)$$

Hence $y = a \left(x + \frac{b}{2a} \right)^2 + \left(c - \frac{b^2}{4a} \right)$ if $a \neq 0$ as required

$$\text{b) From a) } 2x^2 - 7x - 4 = 0$$

$$\Leftrightarrow 2 \left(x^2 + \frac{7}{2} \right) - 4 = 0 \Leftrightarrow 2 \left(x + \frac{7}{4} \right)^2 + \left(-4 - \frac{7^2}{8} \right) = 0$$

$$\Leftrightarrow 2 \left(x + \frac{7}{4} \right)^2 = \left(-4 - \frac{7^2}{8} \right) = 0$$

Answers to application activity 3.6.2

1. -8; 5

2. 4; 9

3. $-1\frac{1}{2}$; 2

4. -3; $1\frac{1}{3}$

Learning activity 3.6.3

Materials

Exercise books, Pens, Scientific calculator.

Methodology

- Group work, facilitating learners for **the activity 3.6.3** then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate them to apply **equations by the formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to accurately solve **Quadratic equations**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.6.3** and evaluate whether lesson objectives were achieved

Answers for activity 3.6.3

1. 2 and 2
2. 2 and 3
3. -3 and -4
4. 1 and $\frac{1}{2}$
5. -7 and 5

Answers to application activity 3.6.3

1. $S = \{1, 11\}$
2. $S = \{-7, 5\}$
3. $S = \emptyset$
4. $S = \left\{ \frac{1}{3}, 2 \right\}$
5. $S = \{-11, 11\}$

Learning activity 3.6.4

Materials

Exercise books, Pens, Scientific calculator.

Methodology

- Group work, facilitating learners for the activity 3.6.4 then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate them to accurately express **quadratic expression in Factor form.**
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.6.4** and evaluate whether lesson objectives were achieved

Answers for activity 3.6.4

1. $x^2 + 3x - 4$

2. $3x^2 - 21x + 30$

3. $x^2 + 3x + 2$

4. $6x^2 - 66x + 144$

5. $x^2 - 4x - 12$

In each case the original form is the factor form

Answers to application activity 3.6.4

1. $(x-8)(x-2)$

2. No factor form

3. $(2x-1)(3x-1)$

4. $(x-1)(x+5)$

5. $(x+2)(4x-1)$

Learning activity 3.6.5

Materials

Exercise books, Pens

Methodology

- In group, facilitate learners for the activity 3.6.5 then questioning.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to accurately express/reduce biquadratic equations into quadratic equations and solve them.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.6.5** and evaluate whether lesson objectives were achieved

Answers for activity 3.6.5

1. $u^2 - 2u + 2 = 0$
2. $6u^2 + 5u + 1 = 0$
3. $u^2 - 13u + 36 = 0$

Answers to application activity 3.6.5

1. $S = \{-3, -2, 2, 3\}$
2. $S = \{1, \sqrt[3]{6}\}$
3. $S = \{-3, -1, 1, 3\}$
4. $S = \{-6, -5, 5, 6\}$

Learning activity 3.6.6

Materials

Exercise books, Pens, Scientific calculator.

Methodology

- Group work, facilitating learners for the activity 3.6.6 .
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to explore how the radical like of $\sqrt{A \pm \sqrt{B}}$ can be

transformed to $\sqrt{x} \pm \sqrt{y}$ and solve it to get the values of x and y .

- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 3.6.6** and evaluate whether lesson objectives were achieved

Answers for activity 3.6.6

$$\left(\sqrt{4+\sqrt{12}}\right)^2 = \left(\sqrt{x} + \sqrt{y}\right)^2$$

$$\Leftrightarrow 4 + \sqrt{12} = x + 2\sqrt{xy} + y$$

$$\Leftrightarrow 4 + \sqrt{12} = x + y + \sqrt{4xy}$$

$$\begin{cases} x + y = 4 \\ 4xy = 12 \end{cases} \Rightarrow \begin{cases} x + y = 4 \\ xy = 3 \end{cases}$$

We need two numbers such that their sum is 4 and their product is 3

$$\Rightarrow x = 3, y = 1 \text{ or } x = 1, y = 3$$

Answers to application activity 3.6.6

1. $\sqrt{5} - 1$

2. $1 + \sqrt{2}$

3. $\sqrt{2} + \sqrt{3}$

Learning activity 3.6.7

Materials

Exercise books, Pens, Scientific calculator.

Methodology

- In group of at least four, facilitate learners for the activity 3.6.7.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate them to explore how to solve the Irrational equations (the equation involving radical sign. We will see the case the radical sign is a square root.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application **activity 3.6.7** and evaluate whether lesson objectives were achieved

Answers for activity 3.6.7

$$1. \sqrt{x+8} = x+2 \Leftrightarrow x+8 = (x+2)^2$$

$$2. \quad x+8 = x^2 + 4x + 4$$

$$x^2 + 3x - 4 = 0$$

Either $x = 1$ or $x = -4$

3. We test these two values to the given equation. We see that -4 is a false solution and it must be deleted. The only solution is 1.

Answers to application activity 3.6.7

$$1. \quad S = \{162\}$$

$$2. \quad S = \emptyset$$

$$3. \quad S = \emptyset$$

Learning activity 3.6.8

Group work, facilitating learners for the activity 3.6.8 then questioning.

Materials

Exercise books, Pens, Scientific calculator.

Methodology

In group work, teacher facilitates learners in developing their critical

thinking

Answers for activity 3.6.8

$$1. \quad ax^2 + bx + c + \frac{b}{x} + \frac{a}{x^2} = 0 \quad 2. \quad a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0$$
$$3. \quad ay^2 + by + c - 2a = 0$$

Answers to application activity 3.6.8

$$1. \quad s =]-\infty, 5 - 3\sqrt{5}[\cup]5 + 3\sqrt{5}, +\infty[$$
$$2. \quad s = \left[\frac{1}{3}, \frac{1}{2}\right] \quad 3. \quad s = \mathbb{R} \quad 4. \quad s = \emptyset \quad 5. \quad s = [8, 9]$$

3.6.7 Solving problems from real life situations involving equations and inequalities

Learning activity 3.7

Group work, facilitating learners for the activity 3.7 then questioning.

Materials

Note book, pens.

Methodology

Facilitate learners for their research

Answers for activity 3.7

1. Linear equations can be used in daily life in many different ways like:
 - In economics, supply and demand analysis
 - Linear motion
 - Balancing equation.
2. Quadratic equations are used in daily life like:
 - Calculating areas
 - Figuring out a profit
 - In athletics

- Finding speeds and so on
- There are many different answers.

Answers to application activity 3.7

1) Let T represents the temperature

$$T + 25^{\circ}C = 100^{\circ}C$$

$$T = 75^{\circ}C$$

2)

$$a) \begin{cases} T = 99 - 3.5m \\ T = 0 + 2.5m \end{cases}$$

$$\therefore T = 41.25^{\circ}C, \quad m = 16.5 \text{ min}$$

b) After 16.5 min, the temperature of either piece will be $41.25^{\circ}C$.

3)

$$a) P \leq \frac{1}{2}(20) + 110$$

$P \leq 120$, this is a minimum normal systolic pressure.

$$P = \frac{1}{2}(50) + 110 = 135$$

Answers to End unit assessment

1. a) -19 b) 0 or -9 c) 9 d) 1
e) 90 f) 1 g) 4 h) 3

2. a) $]-\infty, -1[$ b) $]1, \infty[$ c) $]-2, 0[$

d. $]-\infty, -\frac{5}{3}]$ e) $]-\infty, -1[\cup]1, \infty[$ f) $]-\infty, -4[\cup]-1, \infty[$

g) $]-\infty, -9[\cup]-1, 2[$

3. a) 7, 10 b) $\frac{-45 \pm \sqrt{2569}}{8}$ c) $5 \pm 2\sqrt{6}$

d) $]-\infty, 2] \cup [5, \infty[$ e) $\left[\frac{1}{3}, \frac{1}{2}\right]$ f) \mathbb{R}
 g) \emptyset h) \emptyset i) \mathbb{R}

j) $]-\infty, -1[\cup [2, 3]$ k) $-\infty, 2[\cup]3, +\infty[$

4. a) 30 b) $\frac{26}{3}$ c) -2 d. 1

e) 3 f) 0 or 2 g) $-\frac{13}{8}$

5. a) $\pm \frac{\sqrt{2}}{2}$ b) 25 c) \emptyset d) $-\frac{1}{8}$ or 125

6. a) $\sqrt{2}-1$ b) $\sqrt{4}+\sqrt{3}$ c) $2\sqrt{2}-\sqrt{7}$

d) $2(\sqrt{2}+1)$ e) $\sqrt{6}-\sqrt{3}$

7. 18, 59

8. \$4, \$7

9. The number is 40 . The two parts are 15 and 25.

10. Robert: 10 years, his father: 40 years.

11. 25 and 30.

12. 39° and 51°

13. Each chair: \$125, each table: \$165.

14. The velocity is between 5 m/s and 15 m/s between 1 second and 3 seconds after it is thrown.

15. The width must be between 1 m and 7 m (inclusive) and the length is 8m width

16. 12 metres by 16 metres

17. -33 and -34

18. 1.5 metres

19. 9.75 metres on a side

Unit 4

POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS

4.1. Key Unity Competency

Use concepts and definitions of Polynomial, Rational and Irrational functions to determine the domain of Polynomial, Rational and Irrational functions and represent them graphically.

4.2. Objectives

By the end of this unit, the learners will:

- demonstrate an understanding of operations on polynomials, rational and irrational functions, and find the composite of two functions.
- identify a function as a rule and recognize rules that are not functions.
- determine the domain and range of a function.
- construct composition of functions.
- find whether a function is even, odd, or neither.

4.3. List of lessons

Week	Lesson number	Content	Number of Periods
14	1	<ul style="list-style-type: none">- Definition.- Types of functions (Polynomial, rational, irrational functions)- Injective, surjective and bijective functions,- Existence conditions for a given function.	3

15	2	Domain of definition and range of a numerical function (Polynomial functions, rational functions, irrational functions).	2
	3	Factorization and expansion of polynomials	1
16	4	Parity of a function	1
	5	Graphical representation of polynomial, rational and irrational functions, their use and interpretation in Economics or Physics	2
17	6	Application of polynomial functions in Physics, chemistry and medicine.	2
	7	End unit Assessment and Remediation	1

4.4. Vocabulary

Domain of definition, odd function, even function.

4.5. Guidance on introductory activity 4.0

Help the learners to take into their mind the general idea of the whole unit: definition, classification (polynomial, rational and irrational functions), operations (composite, inverse, etc), qualities (such as being odd, even, etc.), some specific sets (such as domain, range, etc.).

You can proceed as follows:

- Give clear instructions for learners to form small groups and to work on the introductory activity;
- As they are discussing, circulate around to note the relevancy of the discussion and to provide guidance where necessary;
- Facilitate working, especially the stragglers
- Ensure that the learners have understood what the unit will be about and they are eager to learn; you can observe this through a clear and concise presentation of a group chosen randomly and the degree of attention other students are paying to the presentation;

- Sustain the curiosity of the learners by a proper management of your class;
- A prior knowledge of software such as geogebra and malmath would be very useful for you and for your students;

Answer of introductory activity 4

- The use (what something is made for);
 - A quantity whose value depends on the value of another quantity
 - A quantity whose value depends on the value of another quantity
- Independent variable: x
– Dependent variable: y
 - Independent variable: r
– Dependent variable: A
 - Independent variable: A
– Dependent variable: S
- Irrational function
 - Rational function
 - Irrational function
 - Polynomial function
 - Rational function
- Domain: $Dom f =]-\infty; 1[\cup]1; +\infty[$

Range: $Im f =]-\infty; 0[\cup]0; +\infty[$: The function $y = \frac{4x-4}{(x-1)^2}$ is equivalent to $y = \frac{4}{x+1}$; the only value that y cannot assume is 0;

ii. Domain: $Dom f =]-\infty; +\infty[$: as any expression, r can assume any value;

Range: $Im f = [0; +\infty[$: the expression $A = \pi r^2 \geq 0$, for any value of r

iii. As A can always take positive values or can be zero, then $Dom f = [0; +\infty[$ and the range is $Im f = [0; +\infty[$

4.6. Contents and activities

4.6.1 Generalities on numerical functions

Learning activity 4.1

Materials

Exercise books and pens.

Methodology

- Facilitate learners in Group work, then questioning during the activity 4.1.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to explore generalities on numerical functions.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application activity 4.1 and evaluate whether lesson objectives were achieved

Answers for activity 4.1

$1 \rightarrow 3$ $2 \rightarrow 4$ $3 \rightarrow 2$
 $4 \rightarrow 5$ $5 \rightarrow 5$

Answers to application activity 4.1

1. The first two relations
2. $Domain = \{a, b, d, e\}$, $Codomain = \{1, 2, 3, 4, 5, 6, 7\}$, $Range = \{1, 2, 3, 4\}$
3. a) 8 b) 0 c) $2d + 4$ d) $a = -4$
4. We have been made one to many by colonialists, NDI
UMUNYARWANDA is making us many to one

4.6.2 Classification of functions.

Learning activity 4.2

Materials

Exercise books, pens

Methodology

- Facilitate learners in pairs during the activity 4.2, then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to explore the types of numerical functions.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 4.2** and evaluate whether lesson objectives were achieved

Answers for activity 4.2

1. Polynomial function
2. Rational function
3. Irrational function

4.6.3 Domain of definition and range of a numerical function

(Polynomial functions, rational functions, irrational functions).

Learning activity 4.3.1

Materials

Note book, pens

Methodology

- Facilitate learners in pairs during the activity 4.3.1, then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate them to discuss on the domain of definition and range of a numerical function (Polynomial functions, rational functions).
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the application activity 4.3.1 and evaluate whether lesson objectives were achieved

Answers for activity 4.3.1

1. No value

2. 0

3. 1

Answers to application activity 4.3.1

1. $Domf = \mathbb{R}$

2. $Domf = \mathbb{R}$

3. $Domf = \mathbb{R} \setminus \{5\}$

4. $Domf = \mathbb{R} \setminus \{3,5\}$

5. $Domf = \mathbb{R}$

Learning activity 4.3.2

Materials

Exercise books, pens and calculator.

Methodology

- Facilitate learners in group during the activity 4.3.2, then questioning
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.

- Facilitate them to discuss on the **domain of definition and range of a numerical function (irrational functions)**.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 4.3.2** and evaluate whether lesson objectives were achieved

Answers for activity 4.3.2

1. $]-\infty, -\frac{1}{2}[$ 2. None 3. $[-1, 2[$

Answers to application activity 4.3.2

1. $[2, +\infty[$ 2. $]-\infty, -6] \cup [1, +\infty[$
 3. $\mathbb{R} \setminus \{-4\}$ 4. $]-\infty, -5[\cup]5, +\infty[$ 5. $]-4, +\infty[$

4.6.4 Operations on functions

Learning activity 4.4

Materials

Exercise books, pens

Methodology

- Facilitate learners in Group work, during the activity 4.4 then questioning and developing brainstorming.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.

- Facilitate them to apply operations on functions.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 4.4** and evaluate whether lesson objectives were achieved

Answers for activity 4.4

$$1. \frac{2(x^2 - 1)}{2x - 3}$$

$$2. \frac{-2(x^2 - x - 2)}{2x - 3}$$

$$3. \frac{x^2 + 2x + 1}{2x - 3}$$

$$4. \frac{1}{2x - 3}$$

Answers to application activity 4.4

$$1. 2x^3 + 8x - 5$$

$$2. 6x^5 - 13x^4 + 28x^3 - 30x^2 + 25x - 12$$

$$3. \left[-\frac{3}{2}, +\infty \right[$$

$$4.]-4, +\infty[$$

$$5. 2x^3 + 4x^2 - 9x - 6$$

4.6.5 Parity of a function

Learning activity 4.5

Materials

Exercise books, pens.

Methodology

- Facilitate learners in pairs, during the activity 4.5 then questioning and developing brainstorming.
- Make sure that everybody is engaged/ involved.

- Facilitate working, especially straggling students.
- Facilitate them to study the parity of a function.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 4.5** and evaluate whether lesson objectives were achieved

Answer for activity 4.5

1. $f(-x) = x^2 - 2x + 3, -f(x) = -x^2 - 2x - 3, f(-x) \neq -f(x), f(-x) \neq f(x)$
2. $f(-x) = \sqrt[3]{-x^3 - x} = -\sqrt[3]{x^3 + x}, -f(x) = -\sqrt[3]{x^3 + x}, f(-x) = -f(x)$
3. $f(-x) = \frac{x^2 - 3}{x^2 + 1}, -f(x) = \frac{-x^2 + 3}{-x^2 + 1}, f(-x) = f(x)$

Answers to application activity 4.5

1. Neither odd nor even
2. Neither odd nor even
3. Odd
4. Even
5. Neither odd nor even

4.6.6 Factorisation of polynomials

Learning activity 4.6

- Organize the student-teachers into small groups and ask them to do the **Activity 4.6** from student's book and lead them to extend the factorization of mathematics expressions to the factorization of polynomials;
- Move around to ensure all students participate actively in their groups.

- Call upon groups with different working steps to present their findings.
- Harmonize their findings, insisting on different methods of factoring polynomials;
- Use different probing questions to guide students to explore examples and the content related to different methods of factorising, as a teacher you can refer to other reference books;
- Guide students to perform individually application **activity 4.6** to assess their competences.

Answers for activity 4.6

Factorization is writing an expression as the product of its prime factors. In factorization the operation is used depending on the polynomial to be factorized.

a) $2(a+b)$; b) $3r(1+2r)$; c) $xy(1+a)$; d) $3xy(3x+5y)$

Answers to application activity 4.6

a) $2(ab+2c)$; b) $-3b(b+3)$; c) $3x(x^2+2x-3)$;

4.6.7 Expansion of polynomials

Learning activity 4.7

- Organize the student-teachers into small groups and ask them to do **the Activity 4.7** from student's book and lead them to extend the expansion of mathematics expressions to the expansion of polynomials;
- Move around to ensure all students participate actively in their groups.
- Call upon groups with different working steps to present their findings.

- Harmonize their findings, insisting on different properties applied when expanding polynomials;
- Use different probing questions to guide students to explore examples and the content related to the expansion of polynomials and the determination of the value of a polynomial when the values of unknown variables were given;
- Guide students to perform individually **application activity 4.7** to assess their competences.

Answers for activity 4.7

$$(i) (x+4)(x-2) = x^2 + 2x - 8$$

$$(ii) (x+4)(x+4) = x^2 + 8x + 16$$

$$(iii) (x-1)(x-1) = x^2 - 2x + 1$$

a) We see that all the final polynomials both have 3 terms for each.

b) **All of the 3 polynomials are perfect square trinomials** and the highest exponent for the variable x in all expressions is 2, while the lowest exponent for the variable x in all expressions is 1.

Note: If a trinomial is a perfect square,

- The first term must be a perfect square.
- The last term must be a perfect square.
- The middle term must be twice the product of numbers that were squared to give the first and last terms.

Answers to application activity 4.7

$$1. i) x^2 + x + 3x + 3 \Rightarrow x^2 + 4x + 3;$$

$$ii) 6x + 3 - 2x^2 - x \Rightarrow -2x^2 + 5x + 3;$$

$$iii) 8a^2 - 12a - 6a + 9 \Rightarrow 8a^2 - 18a + 9;$$

$$\text{iv) } 4b^2 + 24b - b - 6 \Rightarrow 4b^2 + 23b - 6;$$

$$\text{v) } 12y - 3y^2 + 24 - 6y \Rightarrow -3y^2 + 6y + 24$$

$$\text{2. i) } \begin{cases} x^2 = x \times x \\ 16 = 4 \times 4 \Rightarrow (x+4)(x+4); \\ 8x = 2(4 \times x) \end{cases} \quad \text{ii) } \begin{cases} x^2 = x \times x \\ 36 = 6 \times 6 \Rightarrow (x+6)(x+6) \\ 12x = 2(6 \times x) \end{cases}$$

4.6.8 Graph of linear and quadratic functions

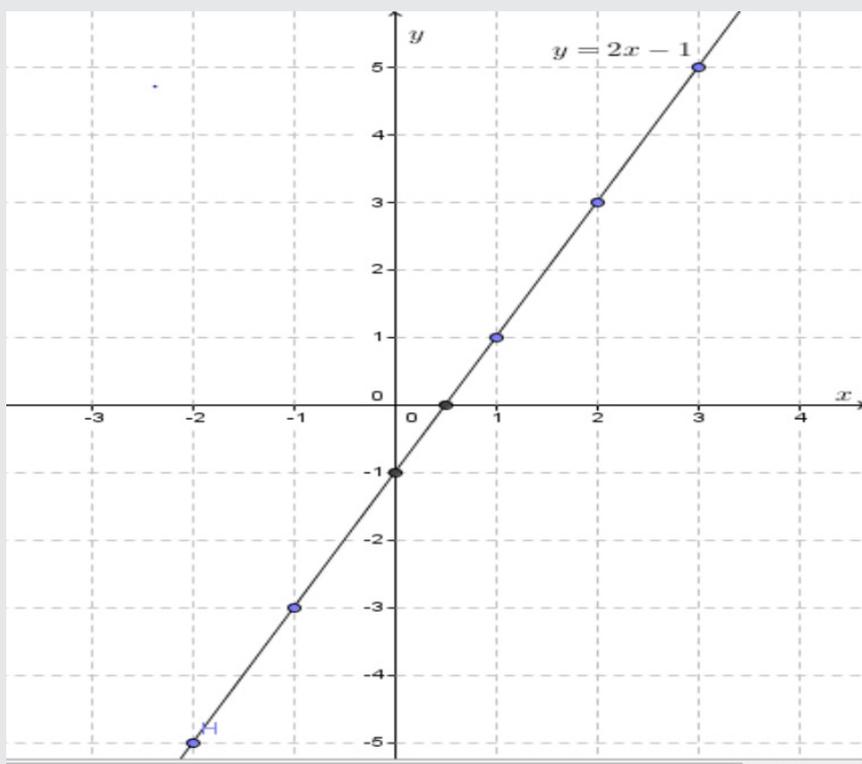
Learning activity 4.8

- Ask students in pairs to read and discuss on the activity 4.8
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate the use of geometric materials to make accurate graphs.
- Make sure that the notebooks of student-teachers include squared papers/graph papers.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book and lead students to verify the parity of different functions.
- After this step, guide students to do the **application activity 4.8** and evaluate whether lesson objectives were achieved.

Answers for activity 4.8

$$y = 2x - 1 \text{ for } -3 \leq x \leq 3$$

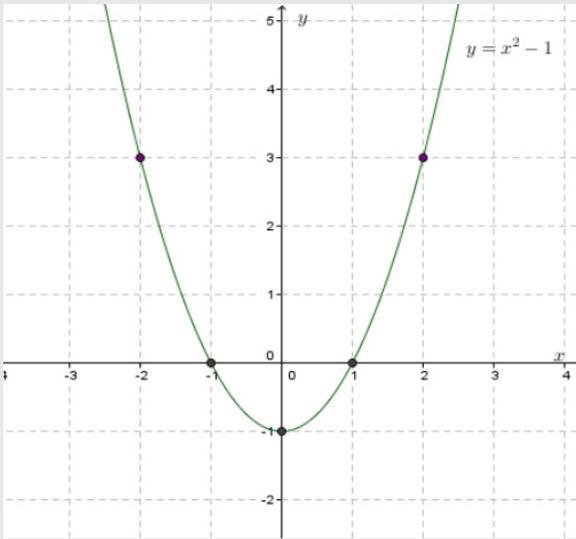
x	-3	-2	-1	0	1	2	3
$y = 2x - 1$	-7	-5	-3	-1	1	3	5



This is linear function

$$y = x^2 - 1 \text{ for } -3 \leq x \leq 3$$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 1$	8	3	0	-1	0	3	8

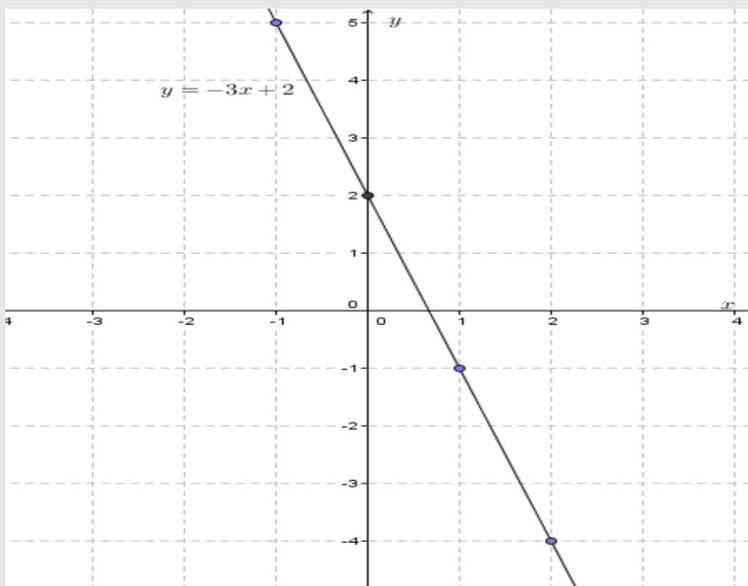


This is quadratic function

Answers to application activity 4.8

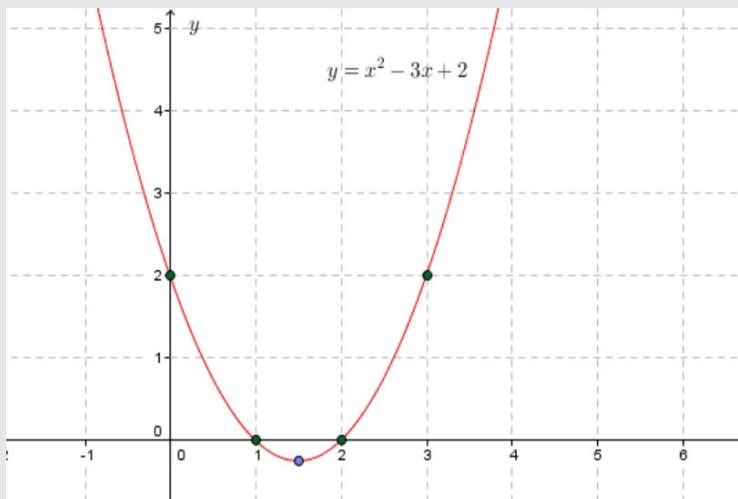
1. a) $y = -3x + 2$

x	-1	0	1	2
$y = -3x + 2$	5	-2	-1	-4

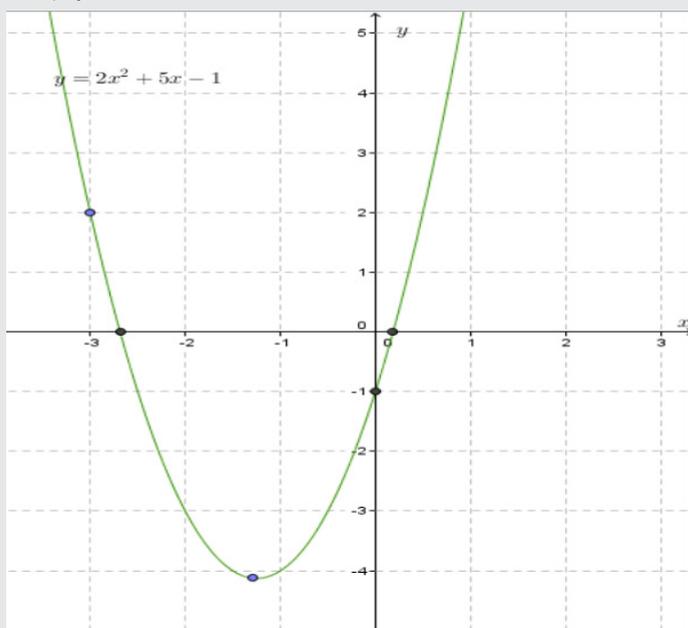


b) $y = x^2 - 3x + 2$

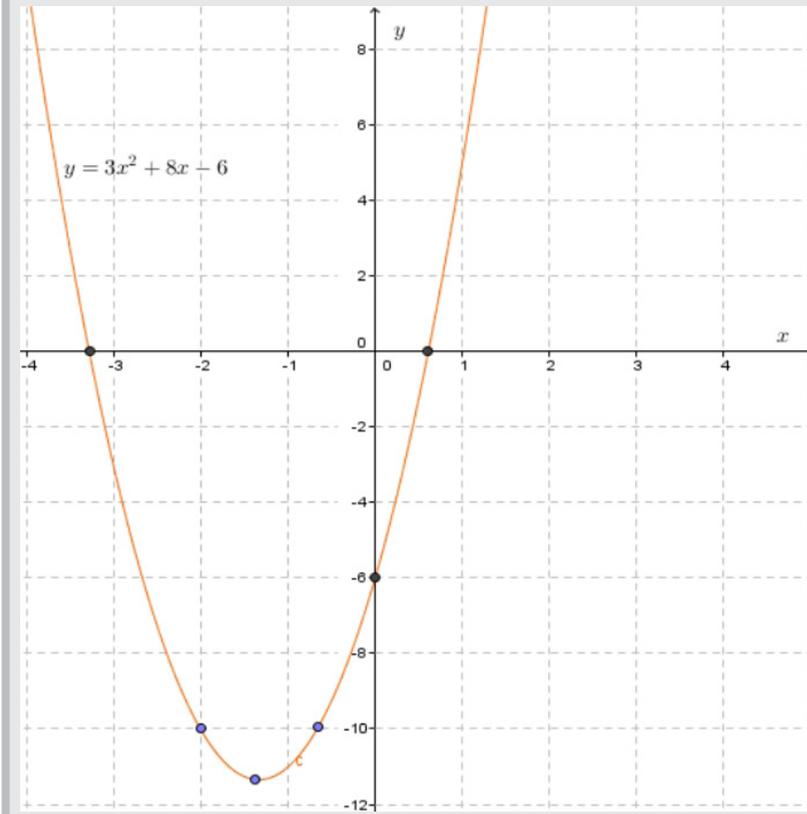
x	-1	0	1	2	3
$y = x^2 - 3x + 2$	6	2	0	0	2



2. a) $y = 2x^2 + 5x - 1$



b) $y = 3x^2 + 8x - 6$



4.6.9 Application of polynomial functions in Physics, chemistry and medicine.

Learning activity 4.9

Materials

Exercise book and pens.

Methodology

- Organize and facilitate the students into small groups and ask them to do the Activity 4.9 from student's book
- Move around to ensure all students participate actively in their groups.

- Call upon groups with different working steps to present their findings.
- Harmonize their findings, insisting on different properties applied when expanding polynomials;
- Use different probing questions to guide students to explore examples and the content related to the application of polynomials and the determination of the value of a polynomial when the values of unknown variables were given;
- Guide students to perform individually application activity 4.9 to assess their competences.

Answers for activity 4.9

Polynomials and functions are used in different ways in daily life:

- Polynomials are used to graph curves.
- Business people also use polynomials to model markets.
- Functions are important in calculating medicine, building structures

Answers to application activity 4.9

1) graph

2)

$$T = 10 \left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10} \right)$$

a) Initial temperature($t=0$ h) of the food is 75 degree Celsius

b) Temperature of the food after 3hour is 43.87 degree Celsius

Answers to end unit assesment

- a and d
- a) (iii) b) (i) c) (iii) d) (ii)
- a) 14 b) 50 c) 2 d) 11 e) $3t^2 + 2$
- a) 21 b) $-\frac{5}{3}$ c) $\frac{\sqrt[3]{5}+2}{\sqrt[3]{5}}$ d) $\frac{\pi+1}{\pi-1}$ e) $\frac{a}{a-2}$
- a) does not exist b) -8 c) $\frac{1}{3}$ d) $\frac{10}{31}$
e) does'nt exist
- 3 7. 1 or $-\frac{3}{2}$
- $a=1, b=-1, g(-4)=17$
- a) $\{y: 0 \leq y \leq 7\}$ one to one b) $\{y: 0 \leq y \leq 9\}$ many to one
c) $\{y: 0 < y \leq 1\}$ one to one d) $\{y \in \mathbb{R}: y \neq 0\}$ one to one
- (a) $]-\infty, 3[\cup]3, +\infty[$ b) $]-\infty, -\frac{7}{5}[\cup]-\frac{7}{5}, +\infty[$
c) $]-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty[$ d) $]-\infty, -2] \cup [1, +\infty[$ e) $]-\infty, +\infty[$
f) $]-\infty, +\infty[$ g) $]-\infty, -3[$ h) $[5, +\infty[$
i) $]-\infty, +\infty[$ j) $]-\infty, -3] \cup [2, +\infty[$
- a) $]-\infty, +\infty[$ b) $]-\infty, -5[\cup]-5, 3[\cup]3, +\infty[$
c) $]-\infty, \frac{1}{2}]$ d) $[-3, 4]$ e) $[\frac{1}{3}, +\infty[$
f) $[-2, -1] \cup]1, +\infty[$ g) $]-2, 1] \cup]4, +\infty[$
- 6 13. -3
- a) $x^3 + 2x^2 - x - 2$ b) $x^3 + 4x^2 - 3x - 2$ c) $x^5 + 2x^4 - 5x^3 + 2x$
- a) odd b) neither c) odd
d) odd e) even f) odd
g) even

16. a) $(f \circ g)(x) = 216, (g \circ f)(x) = 6$

b) $(f \circ g)(x) = 1024x^{10} + 3584x^8 + 4992x^6 + 34,$

$(g \circ f)(x) = 4x^{10} - 8x^9 + 4x^8 - 24x^5 + 24x^4 + 39$

c) $(f \circ g)(x) = x^{12} + 4x^9 + 4x^6 - 5,$

$(g \circ f)(x) = x^{12} - 6x^{10} + 40x^6 - 96x^2 - 63$

17. a) $\frac{-x-2}{9}$

b) $\frac{5x-2}{x+1}$

c) $\frac{2x-9}{3x-1}$

Unit 5

LIMITS OF POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS

5.1. Key Unity Competency

Evaluate correctly limits of functions and apply them to solve related

5.2. Objectives

By the end of this unit, the learners will:

- calculate limits of certain elementary functions.
- apply informal methods to explore the concept of a limit including one sided limits.
- solve problems involving continuity.
- use the concepts of limits to determine the asymptotes to the rational and polynomial functions.

5.3. List of lessons

Week	Lesson number	Content	Number of Periods
18	1	Neighborhood of a real number, limit of a variable, definition and graphical interpretation of limit of a function	1
	2	One-sided limits	1
	3	Infinite limits and Limits at infinity	1
19	4	The Squeeze theorem and Operations on limits	1
	5	Indeterminate cases	2

20	6	Applications of limits: Continuity of a function , Asymptotes.	2
	7	End unit Assessment and Remediation	1

5.4. Vocabulary

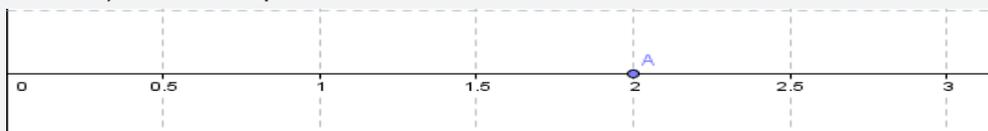
Limit, continuity, asymptote.

5.5. Guidance on introductory activity 5.0

- Invite learners to work in group and do the activity 8 found in their Mathematics books;
- Move around in the class for facilitating where necessary and give more clarification on eventual challenges they may face during their work;
- As they are discussing, concentrate on slow learners for further explanation and provide assistance to groups in need
- Invite one member from each group to present their work;
- A teacher, harmonize the findings from presentation of learners and guide them to explore the content and examples given in the student's book where they will be able to differentiate the neighbourhood of a real number and the value of a function at a given point.
- After the lesson, guide learners to do the application activity 8 and evaluate whether lesson objectives were achieved.

Answer of introductory activity 5.0

a) Yes, it is possible



b) Open intervals: $]1.9; 2.1[$ and $]1.99; 2.01[$

c) When x approaches 2, $f(x)$ approaches 4
 $2.00001 \approx 2$ and $3.999985000 \approx 4$
 $1.99999 \approx 2$ and $4.000015000 \approx 4$

5.6. Contents an activities

5.6.1. Concepts of limits

Learning activity 5.1.1

Materials

Exercise books, pens.

Methodology

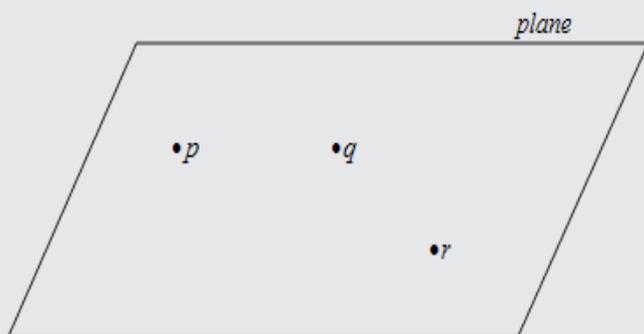
- Facilitate learners in Group work, then questioning during the activity 5.1.1
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate the students to understand the Concepts of limits through discussions.
- Call students to present the findings and promote gender where possible.
- Help them to harmonize the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 5.1.1** and evaluate whether lesson objectives were achieved.

Answers for activity 5.1.1

Lesotho is completely surrounded by South Africa. Swaziland is surrounded by South Africa and Mozambique.

Answers to application activity 5.1.1

1. San Marino, a state surrounded by Italy.
Vatican City, a state forming part of Rome, thereby surrounded by Italy.
2. $(-6, 0)$, $(-7, -1)$, $(-6, -4)$. There are many possible answers (all open intervals containing -5)
3. No. Because no small disk on the circle around any point of the circle
4. The following plane is a neighborhood of points p , q and r .



Learning activity 5.1.2

Materials

Exercise books, pens.

Methodology

- Facilitate learners in pairs during the activity 5.1.2, then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate the students to explore the limits of numerical functions through different probing questions, the content and examples given in the student's book.
- Call students to present the findings and promote gender where possible.

- After this step, guide students to do the **application activity 5.1.2** and evaluate whether lesson objectives were achieved.

Answers for activity 5.1.2

1. $\frac{3}{4}$ 2. 2 3. 98

Answers to application activity 5.1.2

1. 2 2. -8 3. 1 4. -8 5. 12

5.6.2 One-sided limits

Learning activity 5.2

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in pairs during the activity 5.2, then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate the students to explore one sided limits of numerical functions through content and examples given in the student's book.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do the application activity 5.1.2 and evaluate whether lesson objectives were achieved.

Answers for activity 5.2

1. 1.8 2. 1.9 3. 1.99 4. 2.15
5. 2.03 6. 2.003

Answers to application activity 5.2

1. 7 2. 1 3. 0 4. Does not exist

5.6.3 Infinite limits and Limits at infinity

Learninf activity 5.3.1

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in group during the activity 5.3.1, then questioning.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglng students.
- Facilitate the students to explore Infinite limits and Limits at infinity through content and examples given in the student's book.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do the **application activity 5.3.1** and evaluate whether lesson objectives were achieved.

Answers for activity 5.3.1

- | | | |
|------------------|--------------|------------------|
| 1. a) -65.6 | b) -99 | c) -199 |
| d) 201 | e) 101 | e) 67.6 |
| 2. a) $+\infty$ | b) $-\infty$ | c) indeterminate |
| d) $-\infty$ | e) $-\infty$ | f) $+\infty$ |
| g) indeterminate | | |

Answers to application activity 5.3.1

- | | | |
|--|------|-------------|
| 1. $\frac{1}{2}$ | 2. 0 | 3. ∞ |
| 4. $\lim_{x \rightarrow -4^-} \frac{x+1}{x+4} = +\infty$, $\lim_{x \rightarrow -4^+} \frac{x+1}{x+4} = -\infty$ | | |
| 5. $\lim_{x \rightarrow 3^-} \frac{x^2+2x+1}{x-3} = -\infty$, $\lim_{x \rightarrow 3^+} \frac{x^2+2x+1}{x-3} = +\infty$ | | |

5.6.4. The Squeeze theorem and Operations on limits

Learning activity 5.4

Materials

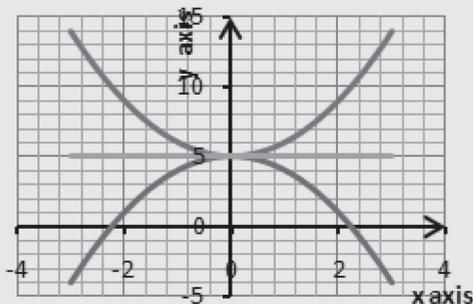
Exercise books, pens, instruments of geometry and calculator

Methodology

- Facilitate learners in Group work, during the activity 5.4 then questioning and developing brainstorming.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglng students.
- Facilitate the students to apply the squeeze theorem and operations on limits, let them explore the content and examples given in the student's book.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do **the application activity 5.4** and evaluate whether lesson objectives were achieved.

Answers for activity 5.4

1.



The curve of $h(x) = 5$ lies between other two curves and the three curves meet at the same point $(0, 5)$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} h(x) = 5$$

2. a) -3, -3

b) 0, -1, -1

c) -2, 5, $-\frac{2}{5}$

d) 1, 6, 6

e) 289, 289

A constant can be moved through a limit sign,

Limit of sum is the sum of limits,

Limit of quotient is the quotient of limits, provided that the denominator is not zero,

Limit of a power is the power of limit.

Answers to application activity 5.4

1. 0

2. a) 3

b) -5832

3. a) $\infty - \infty$ is indeterminate form not zero

$$\text{b) } \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0^+} \frac{x-1}{x^2} = \frac{0-1}{(0^+)^2} = \frac{-1}{0^+} = -\infty$$

x	$-\infty$	0	1	$+\infty$
$x-1$		-	0	+
x^2	+	0	+	
$\frac{x-1}{x^2}$	-		- 0	+

Hence, $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{x^2} \right) = -\infty$

5.6.5. Indeterminate cases

Learning activity 5.5.1

Materials

Exercise books, pens.

Methodology

- Facilitate learners in group of at least four students, during the activity 5.5.1
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate the students to discover strategies used when the limits are indeterminate cases, let them explore the content and examples given in the student's book.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do **the application activity 5.5.1** and evaluate whether lesson objectives were achieved.

Answers for activity 5.5.1

a) $x-1$ b) $x-2$

Answers to application activity 5.5.1

1. $+\infty$ 2. $-\infty$ 3. 20

Learning activity 5.5.2

Materials

Exercise books, pens

Methodology

- Facilitate learners in Group work, during the activity 5.5.2
- Make sure that everybody is engaged/ involved.

- Facilitate working, especially stragglers students.
- Facilitate the students to discover strategies used when the limits are indeterminate forms in irrational functions, let them explore the content and examples given in the student's book.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do the **application activity 5.5.2** and evaluate whether lesson objectives were achieved.

Answers for activity 5.5.2

a) $\sqrt{x^2-2}-3$ b) $\sqrt{x-2}+1$

Answers to application activity 5.5.2

1. $\lim_{x \rightarrow 4^-} \frac{\sqrt{x^2-6}-10}{x-4} = +\infty, \lim_{x \rightarrow 4^+} \frac{\sqrt{x^2-6}-10}{x-4} = -\infty$

2. -1

5.6.6. Applications of limits

Learning activity 5.6.1

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in Group, during the activity 5.6.1.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate the students to discover different areas where the limits are applied, let them explore the content and examples of continuity of a function at a point or on interval I, continuity at the left and continuity at the right of a point, Continuity at an endpoint

and Continuity on an interval given in the student's book and make research for more examples.

- Call students to present the findings and promote gender where possible.
- After this step, guide students to do **the application activity 5.6.1** and evaluate whether lesson objectives were achieved.

Answers for activity 5.6.1

a) 4

b) 4

$f(2)$ and $\lim_{x \rightarrow 2} f(x)$ exist and are equal.

Answers to application activity 5.6.1

1. The function is not continuous at $x = -3$ and $x = 5$
2. $k = 6$
3. $a = -1, b = 1$

Learning activity 5.6.2

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in groups, during the activity 5.6.2
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate the students to discover different areas where the limits are applied, let them explore the content and examples of Apparent (or removable) discontinuity and continuous extensions, discontinuity of the first kind or jump discontinuity, and discontinuity of the second kind, in the student's book and make research for more examples.

After this step, guide students to do the **application activity 5.6.3** and evaluate whether lesson objectives were achieved.

Answers for activity 5.6.3

1. $a = -1, b = 1$ 2. $a = -3, b = -2$ 3. $a = -3, b = -2$

Answers may vary.

Answers to application activity 5.6.3

Consider the given interval and apply theorem

Learning activity 5.6.4

Materials

Exercise books, pens.

Methodology

- Facilitate learners in Group work, during the activity 5.6.4 then questioning and developing brainstorming.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Facilitate the students to discover different types of asymptotes of the functions, and evaluate the limits at the boundaries of the domain.
- Let them explore the content and examples in the student's book for more clarification.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do the **application activity 5.6.4** and evaluate whether lesson objectives were achieved.

Answers for activity 5.6.4

As x increases or decreases the curve comes closer and closer to the line B. As x approaches 3 from the right or from the left, the curve comes closer and closer to the line A.

Answers to application activity 5.6.4

1. Horizontal asymptote: $y = 1$, vertical asymptotes: $x = -1$, $x = 0$
2. Horizontal asymptote: $y = 0$
3. Vertical asymptote: $x = \frac{9}{4}$, oblique asymptote: $y = \frac{1}{4}x + \frac{21}{16}$
4. No asymptote
5. Horizontal asymptote: $y = 3$, vertical asymptotes: $x = -2$, $x = 2$

Learning activity 5.6.5

Methodology

- Facilitate learners in Group work, during the activity 5.6.5 then questioning and developing brainstorming.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers students.
- Facilitate the students to discover different types of asymptotes of the functions, and evaluate the limits at the boundaries of the domain.
- Call students to present the findings and promote gender where possible.
- Let them explore the content and examples in the student's book for more clarification.
- Motivate students to search and provide more examples on the application of limits in daily life especially in medicine.

- After this step, guide students to do the **application activity 5.6.5** and evaluate whether lesson objectives were achieved.

Answers for activity 5.6.5

a) The model is $C = 0.5x + 5000$

b) The average cost is given by $\bar{C} = 0.5 + \frac{5000}{x}$

$$c) C(100) = 0.5 + \frac{5000}{100} = 50.5 \text{ \$}$$

$$C(1000) = 0.5 + \frac{5000}{1000} = 5.5 \text{ \$}$$

$$C(10000) = 0.5 + \frac{5000}{10000} = 1 \text{ \$}$$

$$d) \lim_{x \rightarrow \infty} 0.5 + \frac{5000}{x} = 0.5 \text{ \$}$$

e) The more quantities are sold, the average cost price decreases. But there is reserved price for the seller such that if he/she goes below that he/she can lose his/her money. So the reserved price is 0.5\$.

In the real life situation if you buy more quantities they reduce the price for you.

Answers to application activity 5.6.5

1.

$$a) C(25\%) = \frac{528(25)}{100 - 25} = \$176$$

$$C(50\%) = \frac{528(50)}{100 - 50} = \$352$$

$$C(75\%) = \$528$$

b) $\lim_{x \rightarrow 100^-} \left(\frac{528p}{100-p} \right) = +\infty$, this means that there are no more expenses of money when 100% of illegal drug is stopped. The country is safe at that time!

The tutor goes from this to inform to student-teacher that drugs destroy their brain.

2.

a. $\bar{C} = 0.5 + \frac{500}{x}$

b. $\bar{C}(250) = 0.5 + \frac{500}{250} = 2.5$ \$

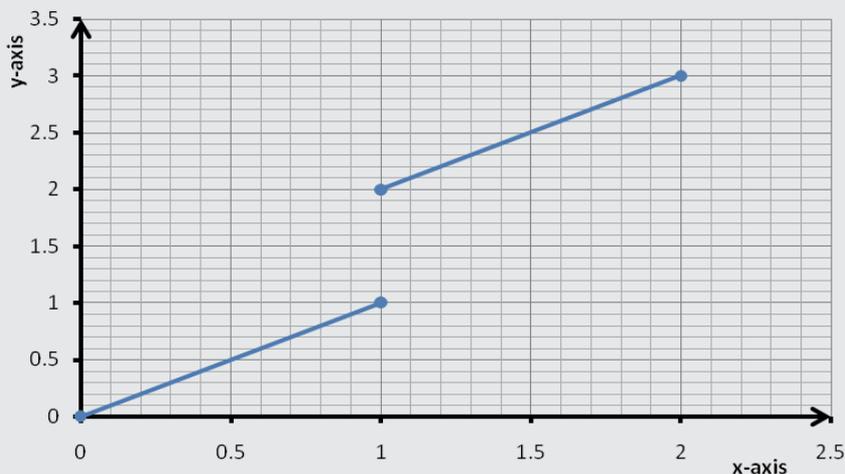
c. $\bar{C}(1250) = 0.5 + \frac{500}{1250} = 0.9$ \$

d. $\lim_{x \rightarrow \infty} \left(0.5 + \frac{500}{x} \right) = 0.5$ \$

The more quantities are sold, the average cost price decreases gradually. But there is a reserved price for the seller such that if he/she goes below that he/she can lose his/her money. So the reserved price is 0.5\$.

Answers to end unit assessment 5

1. $-\frac{1}{2}$
2. 2
3. 2
4. $\frac{1}{2}$
5. 8
6. 1
7. 4
8. 2
9. Does not exist
10. -6
11. $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$. The given function is continuous at $x = 2$
12. 0
13. $x = 1$



14. $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$. The given function is continuous at $x = 0$

 $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$. The given function is continuous at $x = 1$
15. Elimination discontinuity at $x = 4$
16. Discontinuity of the second kind at $x = -2$
17. Discontinuity of the first kind at $x = 3$
18. Vertical asymptote $x = a$
19. No asymptotes

20. Vertical asymptote $x=1$, oblique asymptotes $y=-x-\frac{1}{2}$ and $y=x+\frac{1}{2}$
21. Vertical asymptote $x=0$
22. Vertical asymptote $x=2$, oblique asymptote $y=2x+2$
23. Vertical asymptote $x=-1$, oblique asymptote $y=x+1$
24. Vertical asymptote $x=2$, oblique asymptote $y=x+\frac{1}{2}$
25. Vertical asymptote $x=\frac{5}{3}$, horizontal asymptote $y=\frac{2}{3}$
26. No asymptotes
27. Vertical asymptote $x=0$, oblique asymptote $y=ax$
28. Vertical asymptotes $x=1$ and $x=-1$, horizontal asymptote $y=2$
29. Vertical asymptote $x=a$
30. Oblique asymptote $y=\frac{x}{2}-\frac{1}{2}$
31. Vertical asymptote $x=\frac{1}{2}$
32. Horizontal asymptotes: $y=-1$ and $y=1$
33. Horizontal asymptotes: $y=0$ and $y=-1$, vertical asymptote $x=0$

Unit 6

DIFFERENTIATION OF POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS

6.1. Key Unity Competency

Use differentiation to solve and interpret problems in various contexts.

6.2. Objectives

By the end of this unit, the learners will:

- use properties of derivatives to differentiate polynomial, rational and irrational functions.
- use first principles to determine the gradient of the tangent line to a curve at a point.
- apply the concepts of and techniques of differentiation to model, analyze and solve rates or optimization problems in different situations.

6.3. Vocabulary

Week	Lesson number	Content	Number of Periods
21	1	Concepts of derivative of a function: Definition and Differentiation from first order to high order derivatives	1
	2	Rules of differentiation	2
22&23	3	Geometric interpretation of derivatives: Equation of the tangent to a curve, Equation of normal to a curve	1
	4	Hospital's theorem	1
	5	Variations of a function	2

	6	Optimization problems and Rates of change problems	1
	7	End unit Assessment and Remediation	1

6.4. Vocabulary

Differentiation, derivative.

6.5. Guidance on introductory activity 6.0

- Give clear instructions for learners to form small groups and to work on the introductory activity;
- The learners have encountered gradients before. Ask them what they recall about the concept of gradients
- As they are discussing, circulate around to note the relevancy of the discussion and to provide guidance where necessary;
- Facilitate working, especially the stragglers
- Ensure that the learners have understood what the unit will be about and they are eager to learn; you can observe this through a clear and concise presentation of a group chosen randomly and the degree of attention other students are paying to the presentation;
- Sustain the curiosity of the learners by a proper management of your class;
- A prior knowledge of software such as geogebra and malmath would be very useful for you and for your learners.

Answer of introductory activity 6.0

1.

a) we have $x_0 = 1$ and $h = \Delta x = 1$

$$\text{The slope is given by } m_p = \frac{\Delta y}{\Delta x} = \frac{f(x_0 + h) - f(x_0)}{(x_0 + h) - x_0} = \frac{4 - 2}{2 - 1} = 2$$

$$\begin{aligned}
 \text{b) } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[(x+h)^2 + 1] - (x^2 + 1)}{h} \\
 &= \frac{x^2 + 2hx + 1 - x^2 - 1}{h} \\
 &= \frac{2hx}{h} = 2x \\
 f'(x) &= 2x \\
 \text{for } x_0 = 1 &\Rightarrow f'(x_0) = f'(1) = 2
 \end{aligned}$$

The slope $m_p = f'(x_0) = 2$

2. Possible answers:

The derivative of a function $f(x)$ with respect to x is denoted

by $f'(x)$ or $\frac{d}{dx} f(x)$ and defined as $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ provided that the limit exists.

Or the derivative of a function $y = f(x)$ of a variable x is a measure of the rate at which the value y of the function changes with respect to the change of the variable x . It is called the derivative of $f(x)$ with respect to x .

Materials

Instrument of geometry, scientific calculator.

6.6. Contents and activities

6.6.1. Concepts of derivative of a function

Learning activity 6.1.1

Materials

Exercise books, pens.

Methodology

- Facilitate learners in Group work, then questioning during the activity 6.1.1
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate the students to discover the definition of derivative of a function.
- Let them explore the content and examples in the student's book for more clarification.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do the **application activity 6.1.1** and evaluate whether lesson objectives were achieved..

Answers for activity 6.1.1

$$1. \frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

2. Q will coincide with P

$$3. m_{\tan} = \lim_{x \rightarrow x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$4. m_{\tan} = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Answers to application activity 6.1.1

1. 1

2. 12

3. $8x - 1$

4. $8x + 3$

5. 0

Learning activity 6.1.2

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in pairs during the activity 6.1.2, then questioning.

- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Facilitate the students to find Right-hand and left-hand derivatives of a function.
- Let them explore the content and examples in the student's book for more clarification.
- Call students to present the findings and promote gender where possible.
- After this step, guide students to do the **application activity 6.1.2** and evaluate whether lesson objectives were achieved.

Answers for activity 6.1.2

1. 4, 0

2. 3, 1

Answers to application activity 6.1.2

1. Not differentiable at 1

2. Not differentiable at 2

3. Not differentiable at 0

4. Not differentiable at 4

5. 0

6.6.2. Rules of differentiation

Learning activity 6.2.1

Materials

Exercise books, pens.

Methodology

- Invite students to work in groups and do the activity 6.2.1 in their Mathematics books, to discover the rules that they can use for derivative of constant function and Powers.
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face

during their work; Verify and identify groups with different working steps;

- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a tutor, harmonize the findings from presentation.
- After this step, through examples, guide students to do the **application activity 6.2.1** and evaluate whether lesson objectives were achieved.

Answers for activity 6.2.1

$$\begin{aligned}
 1. \quad f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 2. \quad f'(x_0) &= \lim_{x \rightarrow x_0} \frac{x^n - x_0^n}{x - x_0} = \lim_{x \rightarrow x_0} \frac{(x^{n-1} + x^{n-2}x_0 + \dots + x_0^{n-1})(x - x_0)}{x - x_0} \\
 &= \lim_{x \rightarrow x_0} (x^{n-1} + x^{n-2}x_0 + \dots + x_0^{n-1}) \lim_{x \rightarrow x_0} \frac{(x - x_0)}{x - x_0} \\
 &= (x_0^{n-1} + x_0^{n-2}x_0 + \dots + x_0^{n-1})x' \\
 &= (x_0^{n-1} + x_0^{n-1} + \dots + x_0^{n-1})x' \\
 &= nx_0^{n-1}x' = nx_0^{n-1} \quad (1) \text{ as we have } n \text{ terms}
 \end{aligned}$$

Answers to application activity 6.2.1

1. 0

2. $3x^2 + 6x + 3$

3. $\frac{4x+1}{2\sqrt{2x^2+x-2}}$

Learning activity 6.2.2

Materials

Exercise books, pens.

Methodology

- Facilitate students to work in pairs and do the activity 6.2.2 in their Mathematics books, to discover the rules that they can use for derivative of multiplication by a scalar and product of two functions.
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide students to do the **application activity 6.2.2** and evaluate whether lesson objectives were achieved.

Answers for activity 6.2.2

1.

$$\begin{aligned}(cf)'(x_0) &= \lim_{x \rightarrow x_0} \frac{(cf)(x) - (cf)(x_0)}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{cf(x) - cf(x_0)}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{c[f(x) - f(x_0)]}{x - x_0} \\ &= c \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \\ &= cf'(x_0) \\ (cf)' &= cf'\end{aligned}$$

2.

$$\begin{aligned}(f \cdot g)'(x_0) &= \lim_{x \rightarrow x_0} \frac{(f \cdot g)(x) - (f \cdot g)(x_0)}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{f(x)g(x) - f(x_0)g(x_0)}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{f(x)g(x) - f(x_0)g(x) + f(x_0)g(x) - f(x_0)g(x_0)}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{(f(x) - f(x_0))g(x) + f(x_0)(g(x) - g(x_0))}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{(f(x) - f(x_0))g(x)}{x - x_0} + \lim_{x \rightarrow x_0} \frac{f(x_0)(g(x) - g(x_0))}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{(f(x) - f(x_0))}{x - x_0} \lim_{x \rightarrow x_0} g(x) + \lim_{x \rightarrow x_0} f(x_0) \lim_{x \rightarrow x_0} \frac{(g(x) - g(x_0))}{x - x_0} \\ &= f'(x_0)g(x_0) + f(x_0)g'(x_0) \\ &= (f'g + fg')(x_0)\end{aligned}$$

$$(f \cdot g)' = f'g + fg'$$

Answers to application activity 6.2.2

1. $3x^2 - 4x + 6$ 2. $8x - 17$ 3. $15x^2 - 20x$ 4. 6

Learning activity 6.2.3

Materials

Exercise books, pens.

Methodology

- Engage students to work in pairs and do the activity 6.2.3 in their Mathematics books, to discover the rules that they can use for derivative of Sum or difference of functions.
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;

- Be aware of those who have disabilities.
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide students to do the **application activity 6.2.3** and evaluate whether lesson objectives were achieved.

Answers for activity 6.2.3

$$\begin{aligned}
 (f \pm g)'(x_0) &= \lim_{x \rightarrow x_0} \frac{(f \pm g)(x) - (f \pm g)(x_0)}{x - x_0} \\
 &= \lim_{x \rightarrow x_0} \frac{f(x) \pm g(x) - f(x_0) \mp g(x_0)}{x - x_0} \\
 &= \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \pm \lim_{x \rightarrow x_0} \frac{g(x) - g(x_0)}{x - x_0} \\
 &= f'(x_0) \pm g'(x_0) \\
 (f \pm g)' &= f' \pm g'
 \end{aligned}$$

Answers to application activity 6.2.3

1. $-8x + 7$ 2. $750x^5 - 105x^4$ 3. $96x^3 - 6x^2$

Learning activity 6.2.4

Materials

Exercise books, pens.

Methodology

- Engage students to work in pairs and do the activity 6.2.4 in their Mathematics books, to discover the rules that they can use for derivative of reciprocal function and quotient.
- Move around in the class for facilitating students where necessary

and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;

- Be aware of those who have disabilities.
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide students to do the **application activity 6.2.4** and evaluate whether lesson objectives were achieved.

Answers for activity 6.2.4

$$\begin{aligned}
 1. \quad \left(\frac{1}{f}\right)'(x_0) &= \lim_{x \rightarrow x_0} \frac{\left(\frac{1}{f}\right)(x) - \left(\frac{1}{f}\right)(x_0)}{x - x_0} \\
 &= \lim_{x \rightarrow x_0} \frac{\frac{1}{f(x)} - \frac{1}{f(x_0)}}{x - x_0} \\
 &= \lim_{x \rightarrow x_0} \frac{f(x_0) - f(x)}{f(x)f(x_0)(x - x_0)} \\
 &= \lim_{x \rightarrow x_0} \frac{f(x_0) - f(x)}{(x - x_0)} \lim_{x \rightarrow x_0} \frac{1}{f(x)f(x_0)} \\
 &= -\lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{(x - x_0)} \lim_{x \rightarrow x_0} \frac{1}{f(x)f(x_0)} \\
 &= -f'(x_0) \frac{1}{f(x_0)f(x_0)} \\
 &= \frac{-f'(x_0)}{[f(x_0)]^2}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \left(\frac{1}{f}\right)' &= \frac{-f'}{f^2} \\
 \left(\frac{f}{g}\right)' &= \left(f \frac{1}{g}\right)' \\
 &= f' \frac{1}{g} + f \left(\frac{1}{g}\right)' \\
 &= \frac{f'}{g} + f \left(-\frac{g'}{g^2}\right) \\
 &= \frac{f'}{g} - \frac{fg'}{g^2} \\
 &= \frac{f'g - fg'}{g^2} \\
 \left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2}
 \end{aligned}$$

Answers to application activity 6.2.4

$$\begin{aligned}
 1. \quad & \frac{18x^5 + 12x^3 + 6}{2x^2 + 4x + 1} - \frac{(3x^6 + 3x^4 + 6x - 9)(4x + 4)}{(2x^2 + 4x + 1)^2} \\
 2. \quad & \frac{-3x^2 - 4x}{(x^3 + 2x^2 + 6)^2}
 \end{aligned}$$

Learning activity 6.2.5

Materials

Exercise books, pens and calculator

Methodology

- Call students to work group and do the activity 6.2.5 in their Mathematics books, to discover the rules that they can use for derivative of a composite function: Chain rule.
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face

during their work; Verify and identify groups with different working steps;

- Be aware of those who have disabilities.
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide students to do the **application activity 6.2.5** and evaluate whether lesson objectives were achieved.

Answers for activity 6.2.5

1. $x^2 + 5x - 3$

2. $2x + 5$

3. $2x + 3$

4. $2x + 5$

5. 1

6. $2x + 5$

Results in 2. and 6. are the same.

Answers to application activity 6.2.5

1. 2

2. x

3. $4x^3 + 12x$

4. 0

5. 0

Learning activity 6.2.6

Materials

Exercise books, pens and calculator

Methodology

- Facilitate students to work group and do the activity 6.2.6 in their Mathematics books, to apply successive derivatives.
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Be aware of those who have disabilities.

- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide students to do the **application activity 6.2.6** and evaluate whether lesson objectives were achieved

Answers for activity 6.2.6

1. $6x^5 + 5x^4 + 9x^2 - 4x + 1$

2. $30x^4 + 20x^3 + 18x - 4$

3. $120x^3 + 60x^2 + 18$

4. $360x^2 + 120x$

5. $720x + 120$

Answers to application activity 6.2.6

1. $3360x^3$

2. 0

3. $\frac{-6}{(x-2)^2}$

4. 0

6.6.3 applications of differentiation

Learning activity 6.3

Materials

Exercise books, pens, calculator and instrument of geometry

Methodology

- Facilitate students to work group and do the activity 6.3 in their Mathematics books, to apply derivatives rules.
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Be aware of those who have disabilities.
- Invite one member from each group with different working steps

to present their work where they must explain the working steps;

- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide and motivate students to search for more applications of derivatives in daily life especially in medicine and evaluate whether lesson objectives were achieved

Answers for activity 6.3

1. Solve $-x^2 + 3x = 3x$. You obtain $x = 0$ and also $y = 0$
Some learners can draw the two function in Cartesian plane to see the intersection.
2. 3
3. They are the same.

Answers to End Unit Assessment 6

1. a) $4x^3$ b) $12x^2$ c) $16x$ d) $2x - 4$
2. a) $2x + y = -6$ b) $y = -\frac{4}{3}$
3. a) 44 or 4 b) $\frac{84}{3}$
4. a) 5, 0 b) $6x - 6, 6$
- c) $6x^2 - 10x + 4, 12x - 10$ d) $3x^2 + \frac{2}{x^2}, 6x - \frac{4}{x^3}$
5. a) $(19.6 - 9.8t)ms^{-1}, -9.8ms^{-2}$ b) 2s c) 19.6m
d) 0.586s, 3.41s

6. $a = -3, b = -12$

7. a) 3, -9 b) $\frac{32}{27}, 0$ c) $\frac{32}{27}, -\frac{49}{27}$

8. $8\frac{ft^2}{ft}$ 9. $16\pi\frac{m^3}{m}$

10. a) Increases on $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}}, +\infty\right)$, decreases on $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$

b) Increases on $(-2, 0)$ and $(2, +\infty)$, decreases on $(-\infty, -2)$ and $(0, 2)$

c) Increases on $(-\infty, 3)$ and $(5, +\infty)$, decreases on $(3, 5)$

11. $\frac{1-y}{2+x}$

12. $\frac{2x+y}{3y^2-x}$

13. $\frac{2-2xy^3}{3x^2y^2+1}$

14. $-\frac{3x^2+2xy}{x^2+4y}$

15.
$$\begin{cases} y' = -14(3-2x)^6 \\ y'' = 168(3-2x)^5 \\ y''' = -1680(3-2x)^4 \end{cases}$$

16.
$$\begin{cases} y' = -12(x-1)^{-3} \\ y'' = 36(x-1)^{-4} \\ y''' = -144(x-1)^{-5} \end{cases}$$

17.
$$\begin{cases} y' = \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{3}x^{-\frac{4}{3}} \\ y'' = -\frac{2}{9}x^{-\frac{5}{3}} - \frac{4}{9}x^{-\frac{7}{3}} \\ y''' = \frac{10}{27}x^{-\frac{8}{3}} + \frac{28}{27}x^{-\frac{10}{3}} \end{cases}$$

18.
$$\begin{cases} y' = \frac{5}{2}x^{\frac{3}{2}} + \frac{3}{2}x^{-\frac{1}{2}} \\ y'' = \frac{15}{4}x^{\frac{1}{2}} - \frac{3}{4}x^{-\frac{3}{2}} \\ y''' = \frac{15}{8}x^{-\frac{1}{2}} + \frac{9}{8}x^{-\frac{5}{2}} \end{cases}$$

Unit 7

Vector Space of Real numbers

7.1 Key Unity Competency

Use concepts of vectors in 2D to solve related problems such as distance, angles,....

7.2. Objectives

By the end of this unit, the learners will:

- find the norm of a vector.
- calculate the scalar product of two vectors.
- calculate the angle between two vectors.
- apply and transfer the skills of vectors to other area of knowledge.

7.3. List of lessons

Week	Lesson number	Content	Number of Periods
24	1	Introduction to vector in the Euclidian Vector space	1
	2	Operation of vectors: sum and difference of 2 vectors	1
	3	Dot product and properties	1
25	4	Modulus or Magnitude of Vectors	1
	5	Angle between two vectors	1
	6	End unit Assessment and Remediation	1

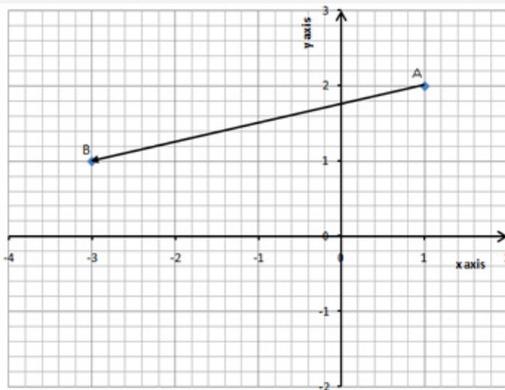
7.4. Vocabulary

Linear combination, dependent, independent, dimension

7.5. Guidance on introductory activity 7.0

- Form groups of students and invite learners to work on the introductory activity found in student's book
- Move around each group in order to give the support where it is necessary
- Guide the learners to present their findings and help them to harmonize their findings basing on their experience, prior knowledge and abilities shown in answering the questions for this activity.
- Open a discussion with the students on how the vectors are drawn in the plane. This will lead to the introduction of the vector space of real number.

Answer of introductory activity 7.0



7.6. Contents and activities

7.6.1. Concept of vector and operations in 2D

Learning activity 7.1

Materials

Exercise books, pens and calculator

Methodology

- Facilitate students to in pairs and do the activity 7.1 in their Mathematics books, to use properties of operations to explore vectors in 2D.
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Be aware of those who have disabilities.
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide students to do **application activity 7.1** and evaluate whether lesson objectives were achieved

Answers for activity 7.1

a) To draw

b) i) to draw

$$\text{ii) } \vec{v} + \vec{w} = (1, 2) + (2, 4) = (3, 6)$$

$$\vec{w} + \vec{v} = (2, 4) + (1, 2) = (3, 6)$$

$$\therefore \vec{v} + \vec{w} = \vec{w} + \vec{v}$$

Answers to application activity 7.1

1) to draw

$$2) 2\vec{v} - 3\vec{w} = 2(2, 3) - 3(3, -4) = (-5, -6)$$

3) to draw

7.6.2 Angle between two vectors

Learning activity 7.2

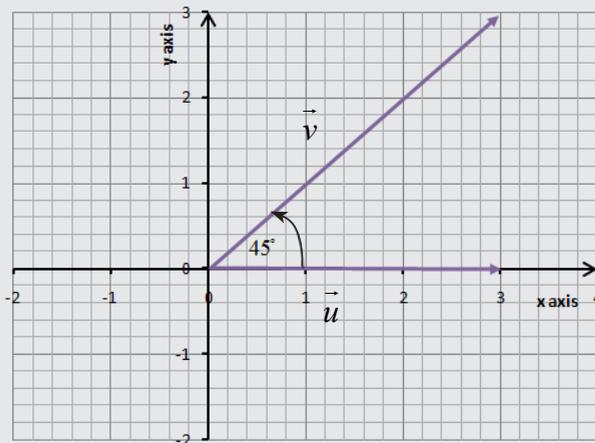
Materials

Exercise books, pens and instrument of geometry

Methodology

- Engage students in group to do the activity 7.2 in their Mathematics books, to calculate the angle between vectors,
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Be aware of those who have disabilities.
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a teacher, harmonize the findings from presentation.
- After this step, through examples, guide students to do **application activity 7.2** and evaluate whether lesson objectives were achieved.

Answers for activity 7.2



The vector drawn in a. is the adjacent side with length 3 and the vector drawn in b. is the hypotenuse. The other side is the opposite side with length 3.

$$\text{Then } \frac{\textit{adjacent side}}{\textit{hypotenuse}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

$$\text{Also } \frac{\textit{opposite side}}{\textit{hypotenuse}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

Answers to application activity 7.2

1. a) 53.84° b) 40.43°
2. a) 3 b) $-\frac{4}{3}$ c) $\frac{-26\sqrt{3} + 48}{3}$

Answers to End Unit Assessment 7

1. a) $\sqrt{153}$ b) $\sqrt{97}$ c) $2\sqrt{34}$
2. a) 29 b) -20 c) 4
3. $\frac{13\sqrt{290}}{290}$, 40.2° , 0.70 rad
4. a) 16.3 degrees b) 36.9 degrees
5. a) 6 b) $-\frac{3}{2}$ c) -9 or 1

Unit 8

MATRICES AND DETERMINANTS OF ORDER TWO

8.1. Key Unity Competency

Use matrices and determinants of order 2 to solve problems involving the system of 2 linear equations with 2 unknowns.

8.2. Objectives

By the end of this unit, the learners will:

- define matrices.
- perform operations on matrices of order 2.
- determine the inverse of a matrix of order 2.

8.3. List of lessons

Week		Content	Number of Periods
26	1	Definition and example of matrices	1
	2	Addition and subtraction of matrices and Transpose of a matrix	2
27	3	Multiplication of matrices	1
	4	Determinant of a matrix of order 2	1
	5	Inverse of a square matrix	1
28	6	Applications of matrices in solving daily life problems (physics, buying and selling, medicine).	2
	7	End unit Assessment and Remediation	1

8.4. Vocabulary

Transpose

8.5. Guidance on introductory activity 8.0

- Form groups of students and invite learners to work on the introductory activity found in student's book
- Move around each group in order to give the support where it is necessary
- Guide the learners to present their findings and help them to harmonize their findings basing on their experience, prior knowledge and abilities shown in answering the questions for this activity.
- Open a discussion with the students on how the numbers were presented before solving the problems. This will lead to the introduction of the matrix concept.

Answer of introductory activity 8.0

a)

Cocks	Rabbits	Prices
5	4	35,000
3	6	30,000

These numbers can also be presented as follows: $\begin{pmatrix} 5 & 4 \\ 3 & 6 \end{pmatrix}$
and $\begin{pmatrix} 35000 \\ 30000 \end{pmatrix}$ respectively.

b) Let x be the cost of one cock and y be the cost of one rabbit,

$$\text{then, } \begin{cases} 5x + 4y = 35,000 \\ 3x + 6y = 30,000 \end{cases}$$

$$\begin{cases} 5x + 4y = 35,000 \times (3) \\ 3x + 6y = 30,000 \times (-5) \end{cases} \Rightarrow \begin{cases} 15x + 12y = 105,000 \\ -15x - 30y = -150,000 \end{cases}$$

$$\Rightarrow -18y = -45,000 \Rightarrow y = 2,500$$

If we replace y in the first equation we obtain

$$5x + 4(2500) = 35,000 \Rightarrow 5x = 25,000 \Rightarrow x = 5,000$$

Thus the cost of 1 cock is 5,000Frw and the cost of one rabbit is 2,500Frw.

8.6 Content and activities

8.6.1. Square Matrices of order two

Learning activity 8.1

Materials

Exercise books, pens

Methodology

- Facilitate learners individually to do the activity 8.1, arranging data in matrix form,
- Guide cooperation and communication when sharing the results in pairs,
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Call students to present the findings and promote gender where possible.
- Monitor harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 8.1** and evaluate whether lesson objectives were achieved.

Answers for activity 8.1

$$\begin{array}{l} \text{cell phones} \\ \text{computers} \end{array} \left| \begin{pmatrix} 20 & 45 \\ 31 & 23 \end{pmatrix} \right.$$

Answers to application activity 8.1

There are many answers. Some of them

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ -20 & 13 \end{pmatrix}, \begin{pmatrix} 1 & -3 \\ 8 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 6 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} -11 & 3 \\ 37 & 18 \end{pmatrix}$$

8.6.2 Operations on matrices

Learning activity 8.2

Materials

Exercise books, pens and calculator

Methodology

- Monitor learners in pairs during the activity 8.2.
- Guide cooperation and communication when sharing ideas,
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 8.2** and evaluate whether lesson objectives were achieved.

Answers for activity 8.2

1. $\begin{pmatrix} 34 & 34 \\ 15 & 22 \end{pmatrix}$

2. $\begin{pmatrix} 19 & -2 \\ 9 & 16 \end{pmatrix}$

3. $\begin{pmatrix} 13 & 6 \\ 4 & 10 \end{pmatrix}, \begin{pmatrix} 7 & 3 \\ 10 & 4 \end{pmatrix}$

Answers to application activity 8.2

1. a) $\begin{pmatrix} 5 & 0 \\ 3 & 2 \end{pmatrix}$

b) $\begin{pmatrix} -9 & -12 \\ 1 & 2 \end{pmatrix}$

c) $\begin{pmatrix} -12 & -6 \\ 8 & 4 \end{pmatrix}$

d) $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$

e) $\begin{pmatrix} 294 & 147 \\ 98 & 49 \end{pmatrix}$

2. $x = 7, y = -1, z = \frac{-3}{2}$

a) $A = \begin{pmatrix} 8 & -3 \\ 1 & 1 \end{pmatrix}$

b) $A^t = \begin{pmatrix} 8 & 1 \\ -3 & 1 \end{pmatrix}$

8.6.3 Determinants and inverse of matrices

Learning activity 8.3

Materials

Exercise books, pens and calculator

Methodology

- Engage learners in groups during the activity 8.3.
- Monitor cooperation and communication when sharing ideas,
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the

content and examples given in the student's book.

- After this step, guide students to do the **application activity 8.3** and evaluate whether lesson objectives were achieved.

Answers for activity 8.3

1. 1 2. 0 3. 18 4. 114

Answers to application activity 8.3

1. 2 2. 0 3. 0 4. 2

8.6.4 Matrix inverse

Learning activity 8.4

Materials

Exercise books, pens and calculator

Methodology

- Engage learners in pairs during the activity 8.4
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Lead the students to find the inverse of matrix.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 8.4** and evaluate whether lesson objectives were achieved.

Answers for activity 8.4

$$A \cdot A' = \begin{pmatrix} 10 & 2 \\ 6 & 3 \end{pmatrix} \cdot \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} = \begin{pmatrix} 10x_1 + 2x_3 & 10x_2 + 2x_4 \\ 6x_1 + 3x_3 & 6x_2 + 3x_4 \end{pmatrix}$$

$$x_1 = \frac{3}{18}, x_2 = \frac{2}{18}, x_3 = -\frac{6}{18}, x_4 = \frac{10}{18}$$

$$\frac{1}{18} \begin{pmatrix} 3 & -2 \\ -6 & 10 \end{pmatrix}$$

Answers to application activity 8.4

$$1. \quad A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

2. No inverse

3. No inverse

$$4. \quad (A^t)^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

5. No inverse

8.6.5 Applications of matrices in solving daily life problems

Learning activity 8.5

Materials

Exercise books, pens

Methodology

- Facilitate learners in pairs during the activity 8.5.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Lead the students to find the inverse of matrix.
- Call students to present the findings and promote gender where

possible.

- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- Motivate students to search and provide more examples on the application of matrices in daily life especially in medicine.
- After this step, guide students to do the **application activity 8.4** and evaluate whether lesson objectives were achieved.

Answers for activity 8.5

$$1. \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} \quad 2. \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 19 \\ 10 \end{pmatrix}$$

Answers to application activity 8.5

1. $x = 3, y = 4$
2. Infinity solution
3. $x = 1, y = 1$
4. $x = 2, y = 0$

Answers to End Unit Assessment 8

1. a) $x = -4, y = 11$ b) $x = 2, y = -1$ c) $x = 5, y = 19$
2. 4, -11 3. $-\frac{1}{2}$ or 3
4. a) $x = 3, y = -2$ b) $x = 1, y = -\frac{2}{3}$ c) $x = \frac{3}{2}, y = \frac{1}{2}$

Unit 9

Measures of Dispersion

9.1 Key Unity Competency

Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.

9.2. Objectives

By the end of this unit, the learners will:

- determine the measures of dispersion of a given statistical series.
- apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.
- express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.

9.3. List of lessons

Week	Lesson number	Content	Number of Periods
29	1	Recall on bivariate data and examples	1
	2	Variance	1
	3	Standard deviation (including combined set of data) and The Coefficient of variation	1
30	4	Application of measures of dispersion in nursing and medicine	2
	5	End unit assessment and Remediation	1

9.4. Vocabulary

Variance, Standard deviation, Coefficient of variation

9.5. Guidance on introductory activity 9.0

- In small groups or pairs, let learners read and do the introductory activity in the students' book.
- Facilitate learners to think on different ways of getting solutions.
- Through question-answer, facilitate learners to understand how statistics is important/used in everyday life.
- After a given time invite learners to present their findings and harmonize them.
- From presentations, the teacher decides to engage the class into discussions that help to the introduction of the unit.

Answer of introductory activity 9.0

1. The table below shows the types and the number of sold fruits in one week.

Type of fruit	A (Banana)	B (Orange)	C (Pineapple)	D (Avocado)	E (Mango)	F (apple)
Number of fruits sold	1100	962	1080	1200	884	900

- The highest number of fruits sold is 1200 (Avocados)
- The least number of fruits sold is 884 (mangoes)
- The total number of fruits sold during the week is 6126 fruits
- The average number of fruits sold per day is $\frac{6126}{6} = 1021$

2.

a) The mean mark of the class is

$$\frac{3+5+6+3+8+7+8+4+8+6}{10} = \frac{58}{10} = 5.8 .$$

b) The mark that was obtained by many students is 8

c) Comparing the mean mark of the class and the mark for every student-teacher, one can find that 4 students have the marks (3 , 4 and 5) below the mean, 2 students scored the mark near the mean while 4 students have scored higher marks than the mean. Mathematics tutor should prepare remedial activities for students whom their marks are below and near the mean

9.6 Content & activities

Variance measures how far a set of numbers is spread out. A variance of zero indicates that all the values are identical. Variance is always non-negative: a small variance indicates that the data points tend to be very close to the mean and hence to each other, while a high variance indicates that the data points are very spread out around the mean and from each other.

9.6.1. Variance

Learning activity 9.1

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in Group work, then questioning during the activity9.1
- Make sure that everybody is engaged/ involved.

- Facilitate working, especially stragglers.
- Lead the students to calculate the **Variance of given data**.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 9.1** and evaluate whether lesson objectives were achieved.

Answers for activity 9.1

x	f	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
12	4	-4.875	23.76563	95.0625
13	2	-3.875	15.01563	30.03125
15	1	-1.875	3.515625	3.515625
19	4	2.125	4.515625	18.0625
21	5	4.125	17.01563	85.07813
	16			231.75

Answers to application activity 9.1

1. $\frac{81}{25}$ 2. 6 3. $\frac{308}{81}$ 4. $\frac{11}{25}$ 5. $\frac{38}{9}$

9.6.2. Standard deviation

Learning activity 9.2

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in pairs during the activity 9.2.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Lead the students to find the **Standard deviation of given data**.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 9.2** and evaluate whether lesson objectives were achieved.

Answers for activity 9.2

x	f	x^2	fx	fx^2
3	2	9	6	18
4	3	16	12	48
5	5	25	25	125
7	1	49	7	49
9	6	81	54	486
	17		104	726

Answers to application activity 9.2

1. $\frac{3\sqrt{2}}{2}$

2. $\frac{2\sqrt{1224499}}{7}$

3. $\frac{4\sqrt{10}}{7}$

4. $\sqrt{2}$

5. $\frac{2\sqrt{34}}{7}$

9.6.3. Coefficient of variation

Learning activity 9.3

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in pairs during the activity 9.3.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Lead the students to find Coefficient of variation of data.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 9.3** and evaluate whether lesson objectives were achieved.

Answers for activity 9.3

x	f	x^2	fx	fx^2
10	10	100	100	1000
14	2	196	28	392
16	14	256	224	3584
18	8	324	144	2592
20	6	400	120	2400
	40		616	9968

Unit 10

ELEMENTARY PROBABILITY

10.1 Key Unity Competency

Use combinations and permutations to determine probabilities of occurrence of an event.

10.2. Objectives

By the end of this unit, the learners will:

- determine the sample space of an experiment.
- explain different concepts relating to events.
- find probability of different events.
- determine probability of an event with an equiprobable sample space.

10.3. List of lessons

Week	Lesson number	Content	Number of Periods
31-32	1	Counting techniques: Venn diagram, Tree diagrams, Contingency table, and Multiplication principles.	4
	2	Arrangements	2
33	3	Permutations	2
	4	Combinations: Definitions and properties	1
34	5	Pascal's triangles and Binomial expansion	1
	6	Concepts of probability: Event, Random experiment, Sample space	2

35	7	Probability of an event A under equally likely assumptions: Definition and formula	2
	8	Rules or properties of probability	1
36	9	Determination of probability for different events occurring under equally likely assumptions	1
	10	Examples of events in medicine and determination of related probability	1
	11	End unit assessment and Remediation	1

10.4. Vocabulary

Permutation, arrangement, factorial, combination, probability, sample space and event.

10.5. Guidance for the introductory activity 10.0

- In small groups or pairs, let learners read and do the introductory activity in the students' book.
- Form small groups of learners and give them instructions on how to work on the given introductory activity;
- Walk around to each group and ask probing questions leading them to determine the total number of roads from A to C via B;
- Ask each group to share their answers with neighbouring group and ask them to support each other for improvement;
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a facilitator, harmonize their answers highlighting that there is a technique of finding the total number of outcomes for a given random experiment;

- Use different probing questions and guide them to explore examples given in the student's book and lead them to determine total number of outcomes for a given random experiment using: Venn diagram, tree diagram or a table.
- Guide them to discover that if a sequence of n events in which the first one has n_1 possibilities, the second with n_2 possibilities the third with n_3 possibilities, and so forth until n_k , the total number of possibilities of the sequence will be $= n_1 \cdot n_2 \cdot n_3 \dots n_k$
- From presentations, the teacher decides to engage the class into discussions that help to the introduction of the unit.

Answer of introductory activity 10.0

1. Many answers may be found by students
2. To find all possible roads, students can use allows to join points or a try and fail method.

$$\Omega = \{AB_1C_1, AB_1C_2, AB_1C_3, AB_2C_1, AB_2C_2, AB_2C_3\} \text{ so they are 6.}$$

3. a) There are 25 black cards in an ordinary deck of 52 cards.

$$\text{b) } P(A) = \frac{n}{\text{number of all cards}} = \frac{26}{52} = 0.5$$

$$\text{c) } P(A) = \frac{\text{Number of outcomes in } E}{\text{Total number of outcomes in the sample space}} = \frac{n(E)}{n(\Omega)}$$

10.6. Contents and activities

10.6.1. Permutations and arrangements

This section introduces the concepts of permutations and arrangements. It looks at addition and multiplication principles.

Learning activity 10.1.2

Materials

Exercise book and pens

Methodology

- Facilitate learners in Group work, then questioning during the activity 10.1.2
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Lead the students to discover how to calculate the number of permutations without having to list them all (Permutations of n objects)
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 10.1.2** and evaluate whether lesson objectives were achieved.

Answers for activity 10.1.2

- All possible different words from three letters A, B and C (not necessarily sensible):
ABC, ACB, BAC, BCA, CAB,CBA. Number of possible arrangements for three letters A, B and C is 6

- All possible different arrangements:
 ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CBAD, CBDA, CABD, CADB, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA. Number of possible arrangements for four letters A, B, C and D is 24

Answers to application activity 10.1.2

1. a) 19! b) 17!3!
 2. a) 60 b) 1

Learning activity 10.1.3

Materials

Exercise book, pens

Methodology

- Facilitate learners in Group work, then questioning during the activity 10.1.3
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Lead the students to find permutations of indistinguishable objects.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 10.1.3** and evaluate whether lesson objectives were achieved.

Answers for activity 10.1.3

- All possible different arrangements:

BOOM, BOMO, BMOO, OBOM, OMOB, OMBO, OBMO, OOMB, OOBM, MOOB, MOBO, MBOO. Number of possible arrangements is 12

- All possible different arrangements:

CLASS, CLSAS, CLSSA, CSLSA, CSSLA, CSSAL, CALSS, CASLS, CASSL, CSLAS, CSASL, CSALS, LCASS, LCSSA, LCSAS, LACSS, LASCs, LASSC, LSCAS, LSACS, LSASC, LSSAC, LSSCA, LSCSA, ACLSS, ACSLS, ACSSL, ALCSS, ALSCS, ALSSC, ASSLC, ASSCL, ASCLS, ASLCS, ASLSC, ASCSL, SCLAS, SCALS, SCSAL, SCSLA, SCLSA, SCASL, SLCAS, SLCSA, SLACS, SLASC, SLSAC, SLSCA, SACLs, SACSL, SALCS, SALSC, SASLC, SASCL, SSACL, SSALC, SSLAC, SSLCA, SSCLA, SSCAL.

Number of possible arrangements is 60

Answers to application activity 10.1.3

1. 5040

2. $26!$

3. 1260

Learning activity 10.1.4

Materials

Exercise book, pens

Methodology

- Facilitate learners in Group, then questioning during the activity 10.1.4
- Make sure that everybody is engaged/ involved.

- Facilitate working, especially stragglers.
- Lead the students to find Circular arrangements.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 10.1.4** and evaluate whether lesson objectives were achieved.

Answers for activity 10.1.4

- There are 24 different ways.

Answers to application activity 10.1.4

1. 24 2. 10!

Learning activity 10.1.5

Materials

Exercise book, pens

Methodology

- Facilitate learners in Group work, then questioning during the activity 10.1.5
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Lead the students to explore mutually exclusive situation.
- Call students to present the findings and promote gender where possible.

- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 10.1.5** and evaluate whether lesson objectives were achieved.

Answers for activity 10.1.5

- You must pick one or the other but not both

Answers to application activity 10.1.5

1. $(3+15+60+180+360+360)$ or 978
2. 4
3. 336

Learning activity 10.1.6

Materials

Exercise book, pens

Methodology

- Facilitate learners in Group, then questioning during the activity 10.1.6
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Lead the students to explore distinguishable permutations.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.

- After this step, guide students to do the **application activity 10.1.6** and evaluate whether lesson objectives were achieved

Answers for activity 10.1.6

- Selection: NOW, permutations:
NOW, NWO, ONW, OWN, WNO, WON
- Selection: KOW, permutations:
KOW, KWO, OKW, OWK, WKO, WOK
- Selection: KNW, permutations:
KNW, KWN, NKW, NWK, WKN, WNK
- Selection: KNO, permutations:
KNO, KON, NKO, NOK, OKN, ONK
In total, there are 24 possible permutations.

Answers to application activity 10.1.6

1. ${}^7P_4 = 840$ 2. 20 3. 7P_5 4. ${}^{26}P_{10}$

10.6.2 Combinations

This section looks at combinations of r unlike objects selected from n different objects. It also looks at binomial theorem and Pascal's triangle.

Learning activity 10.2.1

Materials

Exercise book, pens

Methodology

- Facilitate learners in pairs during the activity 10.2.1
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.

- Lead the students to explore and apply basic formula for combinations.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 10.2.1** and evaluate whether lesson objectives were achieved

Answers for activity 10.2.1

- There are 18 groups

Answers to application activity 10.2.1

1. ${}^{10}C_4 \times {}^{12}C_2$

2. ${}^9C_4 \times {}^{10}C_5$

Learning activity 10.2.2

Materials

Exercise book and pens

Methodology

- Facilitate learners in pairs during the activity 10.2.2.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Lead the students to explore binomial expansion.
- Call students to present the findings and promote gender where possible.

- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do **the application activity 10.2.2** and evaluate whether lesson objectives were achieved

Answers for activity 10.2.2

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Power	Coefficient of powers of a and b					Binomial expression
0	1					$(a+b)^0$
1	1	1				$(a+b)^1$
2	1	2	1			$(a+b)^2$
3	1	3	3	1		$(a+b)^3$
4	1	4	6	4	1	$(a+b)^4$

Answers to application activity 10.2.2

1. 240

2. 0

3. 9039811410

4. $x^7 + 28x^6 + 336x^5 + 2240x^4 + 8960x^3 + 21504x^2 + 28672x + 16384$

5. $8x^3 - 36x^2 + 54x - 27$

10.6.3 Concepts of probability

Probability is the chance that something will happen-how likely it is

that some event will happen. A random experiment is an experiment that, atleast theoretically, may be repeated as often as we want and whose outcome cannot be predicted, the roll of a die. Each time an experiment is repeated, an elementary outcome is obtained. The set of all elementary outcomes of a random experiment is called the sample space, which is denoted by Ω . Sample space may be discrete or continuous. An event is a set of elementary outcomes. That is, it is a subset of the sample space.

Learning activity 10.3

Materials

Exercise books and pens

Methodology

- Facilitate learners in pairs during the activity 10.3.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglng students.
- Lead the students to explore Concepts of probability.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 10.3** and evaluate whether lesson objectives were achieved

Answers for activity 10.3

1. a) 52, b) 4, c) 1 2. 1

Answers to application activity 10.3

1. c 2. d 3. d
4. a) Exhaustive: $X \cup Y \cup Z = \Omega$
b) Exhaustive: $X \cup Y \cup Z = \Omega$
c) Exhaustive: $X \cup Y \cup Z = \Omega$

10.6.4 Properties and formula

The probability of an event $A \subset \Omega$, is a real number obtained by applying to A the function P defined by

$$P(A) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{\#A}{\#\Omega}$$

Addition probability law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Learning activity 10.4.1

Materials

Exercise books and pens

Methodology

- Facilitate learners in pairs during the activity 10.4.1.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Lead the students to explore and apply properties and formula.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.

- After this step, guide students to do the **application activity 10.4.1** and evaluate whether lesson objectives were achieved.

Answers for activity 10.4.1

- a) 11 b) $4, \frac{4}{11}$ c) $7, \frac{7}{11}$
- d) (i) Empty set
- (ii) $\{O, A, I, I, P, R, B, B, L, T, Y\}$
- (iii) $\{P, R, B, B, L, T, Y\}$
- (iv) $\{O, A, I, I\}$

Answers to application activity 10.4.1

1. a) $\frac{2}{11}$ b) $\frac{2}{11}$
2. a) $\frac{13}{19}$ b) $\frac{3}{19}$ c) $\frac{3}{19}$

Learning activity 10.4.2

Materials

Exercise books, pens and calculator

Methodology

- Facilitate learners in pairs during the activity 10.4.2.
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Lead the students to explore and apply Sum law.
- Call students to present the findings and promote gender where possible.

- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 10.4.2** and evaluate whether lesson objectives were achieved

Answers for activity 10.4.2

a) $\frac{2}{3}$ b) $\frac{2}{3}$

The two results are the same.

Answers to application activity 10.4.2

1. $\frac{5}{6}$ 2. $\frac{1}{2}$

Learning activity 10.4.3

Materials

Exercise books and pens

Methodology

- Facilitate learners in pairs during the activity 10.4.3
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially stragglers.
- Lead the students to explore probability for mutually exclusive events.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.

- After this step, guide students to do the **application activity 10.4.3** and evaluate whether lesson objectives were achieved

Answers for activity 10.4.3

a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) 0 d) 1 e) 1

Answers to application activity 10.4.3

1. $\frac{8}{15}$ 2. $\frac{11}{30}$

Learning activity 10.4.4

Materials

Exercise books and pens

Methodology

- Facilitate learners in pairs during the activity 10.4.4
- Make sure that everybody is engaged/ involved.
- Facilitate working, especially straggling students.
- Lead the students to explore probability for exhaustive events.
- Call students to present the findings and promote gender where possible.
- Facilitate harmonization of the answers.
- Use different probing questions and guide them to explore the content and examples given in the student's book.
- After this step, guide students to do the **application activity 10.4.4** and evaluate whether lesson objectives were achieved

Answers for activity 10.4..4

a) 1 b) 1

The two results are the same

Application activity 10.4.4

a) $\frac{3}{8}$ b) $\frac{5}{8}$ c) $\frac{1}{32}$

End unit assessment 9

- a) 144 b) 72
- 216, 72, 108 3. 18 4. 10!
- 8! 6. 24
- a) 210 b) $\frac{1}{110}$ c) 600 d) 81
- a) $n-1$ b) $(n+2)(n+1)$ c) $n(n^2+3n+1)$
- 5040 10. 168
- a. 240 b) 600
- a. 81 b) 256
- 36 14. 2522520
- 151 a) 73 b) 78
c) 13 d) 138
- a) 462 b) 56 c) 20
- 1260
- a) 5 b) 85 c) 365
- a) 126 b) 280
- 480, 172800, 462, 425

21. a) $27 + 27x + 9x^2 + x^3$

b) $125 + 150x + 60x^2 + 8x^3$

c) $16 + 32x + 24x^2 + 8x^3 + x^4$

d) $16 - 32x + 24x^2 - 8x^3 + x^4$

e) $32y^5 + 80y^4x + 80y^3x^2 + 40y^2x^3 + 10yx^4 + x^5$

f) $32x^5 - 240x^4y + 720x^3y^2 - 1080x^2y^3 + 810xy^4 - 243y^5$

g) $x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}$

h) $x^5 - 10x^3 + 40x - \frac{80}{x} + \frac{80}{x^3} - \frac{32}{x^5}$

22. $1 + 12x + 66x^2 + 220x^3$

23. $64x^5 + 160x^{-1} + 20x^{-7}$

24. 0, 1 (trivial) and 6

25. 2

26. 30.43168

27. $a^{10} - 30a^9x + 405a^8x^2 - 3240a^7x^3$

28. $1 + 10x + \frac{95}{2}x^2 + \frac{285}{2}x^3 + \frac{4845}{16}x^4$

29. $16, \frac{1}{8}$

30. a) ${}^{10}C_5 \times 3^5$

b) ${}^{12}C_8 \times 4^{10}$

c) ${}^6C_4 \times 3^2 \times 2^4$

d) $2^4(2 \times {}^{10}C_5 + {}^{10}C_6)$

31. a) 1.0937

b) 0.9860837

c) 0.9044

d) 973.9

32. a) 0.97980

b) 10.1980

c) 2.0199

d) 1.01943

e) 2.05828

33. $1 - \frac{3}{2}x + \frac{15}{8}x^2 - \frac{51}{16}x^3, |x| < \frac{1}{2}$

34. 2, -9, 29, -82

35. a) ${}^8C_3 \times 5^5 \times 3^3$

b) ${}^{-7}C_3 \times 7^4 \times 2^3$

36. a) 560

b) -590625

c) -720

d) -448

e) 1966080

f) $-\frac{7}{144}$

37. $\pm \frac{2}{3}$

38. ${}^{12}C_8 \times 4^4 \times x^8$

39. -20

40. $\frac{3}{2}, \frac{1}{2}, \frac{1970}{1393}$ 41. All of above 42. Landing on red

43. Choose a letter at random from the word SCHOOL

44. Choosing a yellow jelly bean

45. Landing on a number less than 8

46. $\frac{9}{20}$

47. $\frac{10}{21}$

48. $\frac{1}{3}$

49. $\frac{21}{46}$

50. $\frac{1}{221}$

51. $\frac{3}{13}$

52. a) 0.5

b) 0.14

c) 0.07

d) 0.43

53. a) $\frac{1}{6}$

b) $\frac{5}{126}$

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