MATHEMATICS

FOR TTC

TUTOR'S GUIDE

YEAR

3

OPTIONS: LE

FOREWORD

Dear Tutor,

Rwanda Basic Education Board is honoured to present the tutor's guide for Year three Mathematics in the option of Language Education (LE). This book serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of the mathematics content. The Rwandan educational philosophy is to ensure that student-teachers achieve full potential at every level of education which will prepare them to be well integrated in society and exploit employment opportunities.

Specifically, TTC curriculum was reviewed to train quality teachers who will confidently and efficiently implement the Competence Based Curriculum in pre-primary and primary education. The rationale of the changes is to ensure that TTC leavers are qualified for job opportunities and further studies in Higher Education in different programs under education career advancement.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate their learning process. Many factors influence what they learn, how well they learn and the competences they acquire. Those factors include the relevance of the specific content, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials.

The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market have necessitated the shift to a competence-based curriculum. After a successful shift from knowledge to a competence-based curriculum in general education, TTC textbooks also were elaborated to align them to the new curriculum.

The book provides active teaching and learning techniques that engage student teachers to develop competences. In view of this, your role is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize group discussions for students considering the importance of social constructivism suggesting that learning occurs more effectively when the students works collaboratively with more knowledgeable and experienced people.
- Engage students through active learning methods such as inquiry methods, group discussions, research, investigative activities and group and individual work activities.
- Provide supervised opportunities for students to develop different competences by giving tasks which enhance critical thinking, problem solving, research,

creativity and innovation, communication and cooperation.

- Support and facilitate the learning process by valuing students' contributions in the class activities.
- Guide students towards the harmonization of their findings.
- Encourage individual, peer and group evaluation of the work done in the classroom and use appropriate competence-based assessment approaches and methods.

To facilitate you in your teaching activities, the content of this book is self explanatory so that you can easily use it. It is divided in 3 parts:

The part I explain the structure of this book and give you the methodological guidance;

The part II gives a sample lesson plan;

The part III details the teaching guidance for each concept given in the student book.

Even though this Tutor's guide contains the guidance on solutions for all activities given in the student-teacher's book, you are requested to work through each question before judging student's findings.

I wish to sincerely express my appreciation to the people who contributed towards the development of this book, particularly, REB staff, UR-CE Lecturers, TTC Tutors, Teachers from general education and experts from Local and international Organizations for their technical support. A word of gratitude goes also to the Head Teachers and TTCs principals who availed their staff for various activities.

Dr MBARUSHIMANA Nelson

Director General, REB

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I owe gratitude to different universities and schools in Rwanda that allowed their staff to work with REB in the in-house textbooks production initiative.

I wish to extend my sincere gratitude to lecturers and teachers whose efforts during writing exercise of this tutor`s guide was very much valuable.

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Joan MURUNGI,

Head of Curriculum, Teaching and Learning Resources Department

Contents

PART I. GENERAL INTRODUCTION

1.1 The structure of the guide

The tutor's guide of Mathematics is composed of three parts:

The Part I concerns **general introduction** that discusses methodological guidance on how best to teach and learn Mathematics, developing competences in teaching and learning, addressing cross-cutting issues in teaching and learning and Guidance on assessment.

Part II presents a **sample lesson plan**. This lesson plan serves to guide the tutor on how to prepare a lesson in Mathematics.

The Part III is about **the structure of a unit** and **the structure of a lesson**. This includes information related to the different components of the unit and these components are the same for all units. This part provides information and guidelines on how to facilitate student teachers while working on learning activities. Moreother, all application activities from the textbook have answers in this part.

1.2 Methodological guidance

1.2.1 Developing competences

Since 2015 Rwanda shifted from a knowledge based to a competency-based curriculum for pre-primary, primary, secondary education and recently the TTC curriculum. This called for changing the way of learning by shifting from teacher centred to a learner-centred approach. Teachers are not only responsible for knowledge transfer but also for fostering student-teachers' learning achievement and creating safe and supportive learning environment. It implies also that student-teachers have to demonstrate what they are able to transfer the acquired knowledge, skills, values and attitude to new situations.

The competence-based curriculum employs an approach of teaching and learning based on discrete skills rather than dwelling on only knowledge or the cognitive domain of learning. It focuses on what student-teacher can do rather than what student-teacher knows. Student-teachers develop competences through subject unit with specific learning objectives broken down into knowledge, skills and attitudes through learning activities.

In addition to the competences related to Mathematics, student-teachers also develop generic competences which should promote the development of the higher order thinking skills and professional skills in Mathematics teaching. Generic competences are developed throughout all units of Mathematics as follows:

Generic competences	**Ways of developing generic competences**
Critical thinking	All activities that require student-teachers to calculate, convert, interpret, analyse, compare and contrast, etc have a common factor of developing critical thinking into student-teachers
Creativity and innovation	All activities that require student-teachers to plot a graph of a given algebraic data, to organize and interpret statistical data collected and to apply skills in solving problems of economics have a common character of developing creativity into student-teachers
Research and problem solving	All activities that require student-teachers to make a research and apply their knowledge to solve problems from the real-life situation have a character of developing research and problem solving into student-teachers.
Communication	During Mathematics class, all activities that require student-teachers to discuss either in groups or in the whole class, present findings, debate …have a common character of developing communication skills into student-teachers.
Co-operation, interpersonal relations and life skills	All activities that require student-teachers to work in pairs or in groups have character of developing cooperation and life skills among student-teachers.
Lifelong learning	All activities that are connected with research have a common character of developing into student-teachers a curiosity of applying the knowledge learnt in a range of situations. The purpose of such kind of activities is for enabling student-teachers to become life-long student-teachers who can adapt to the fast-changing world and the uncertain future by taking initiative to update knowledge and skills with minimum external support.
Professional skills	Specific instructional activities and procedures that a teacher may use in the class room to facilitate, directly or indirectly, students to be engaged in learning activities. These include a range of teaching skills: the skill of questioning, reinforcement, probing, explaining, stimulus variation, introducing a lesson; illustrating with examples, using blackboard, silence and nonverbal cues, using audio – visual aids, recognizing attending behaviour and the skill of achieving closure.

The generic competences help student-teachers deepen their understanding of Mathematics and apply their knowledge in a range of situations. As students develop generic competences, they also acquire the set of skills that employers look for in their employees, and so the generic competences prepare students for the world of work.

1.2.2 Addressing cross cutting issues

Among the changes brought by the competence-based curriculum is the integration of cross cutting issues as an integral part of the teaching learning process-as they relate to and must be considered within all subjects to be appropriately addressed. The eight cross cutting issues identified in the national curriculum framework are: *Comprehensive Sexuality Education, Environment and Sustainability, Financial Education, Genocide studies, Gender, Inclusive Education, Peace and Values Education, and Standardization Culture.*

Some cross-cutting issues may seem specific to particular learning areas/subjects but the teacher need to address all of them whenever an opportunity arises. In addition, student-teachers should always be given an opportunity during the learning process to address these cross-cutting issues both within and out of the classroom.

Below are examples of how crosscutting issues can be addressed:

Cross-Cutting Issue	Ways of addressing cross-cutting issues
Comprehensive Sexuality Education: The primary goal of introducing Comprehensive Sexuality Education program in schools is to equip children, adolescents, and young people with knowledge, skills and values in an age appropriate and culturally gender sensitive manner so as to enable them to make responsible choices about their sexual and social relationships, explain and clarify feelings, values and attitudes, and promote and sustain risk reducing behaviour.	Using different charts and their interpretation, Mathematics tutor should lead students to discuss the following situations: "Alcohol abuse and unwanted pregnancies" and advise student teachers on how they can instil student-teachers to fight those abuses. Some examples can be given when learning statistics, powers, logarithms and their properties.

Environment and Sustainability: Integration of Environment, Climate Change and Sustainability in the curriculum focuses on and advocates for the need to balance economic growth, society well-being and ecological systems. Student-teachers need basic knowledge from the natural sciences, social sciences, and humanities to understand to interpret principles of sustainability.	Using Real life models or students' experience, Mathematics Tutor should lead student teachers to illustrate the situation of "population growth" and discuss its effects on the environment and sustainability.
Financial Education: The integration of Financial Education into the curriculum is aimed at a comprehensive Financial Education program as a precondition for achieving financial inclusion targets and improving the financial capability of Rwandans so that they can make appropriate financial decisions that best fit the circumstances of one's life.	Through different examples and calculations on interest rate problems, total revenue and total cost, Mathematics Tutor can lead student teachers to discuss how to make appropriate financial decisions.
Gender: At school, gender will be understood as family complementarities, gender roles and responsibilities, the need for gender equality and equity, gender stereotypes, gender sensitivity, etc.	Mathematics Tutor should address gender as cross-cutting issue through assigning leading roles in the management of groups to both girls and boys and providing equal opportunity in the lesson participation and avoid any gender stereotype in the whole teaching and learning process.
Inclusive Education: Inclusion is based on the right of all student-teachers to a quality and equitable education that meets their basic learning needs and understands the diversity of backgrounds and abilities as a learning opportunity.	Firstly, Mathematics Tutors need to identify/recognize students with special needs. Then by using adapted teaching and learning resources while conducting a lesson and setting appropriate tasks to the level of students, they can cater for students with special education needs. They must create opportunity where student teachers can discuss how to cater for student-teachers with special educational needs.

Peace and Values Education: Peace and Values Education (PVE) is defined as education that promotes social cohesion, positive values, including pluralism and personal responsibility, empathy, critical thinking and action in order to build a more peaceful society.	Through a given lesson, a tutor should: Set a learning objective which is addressing positive attitudes and values, Encourage students to develop the culture of tolerance during discussion and to be able to instil it in colleagues and cohabitants; Encourage students to respect ideas for others.
Standardization Culture: Standardization Culture in Rwanda will be promoted through formal education and plays a vital role in terms of health improvement, economic growth, industrialization, trade and general welfare of the people through the effective implementation of Standardization, Quality Assurance, Metrology and Testing.	With different word problems related to the effective implementation of Standardization, Quality Assurance, Metrology and Testing, students can be motivated to be aware of health improvement, economic growth, industrialization, trade and general welfare of the people.

1.2.3 Guidance on how to help students with special education needs in classroom

In the classroom, students learn in different way depending to their learning pace, needs or any other special problem they might have. However, the teacher has the responsibility to know how to adopt his/her methodologies and approaches in order to meet the learning need of each student in the classroom. Also teachers need to understand that student with special needs, have to be taught differently or need some accommodations to enhance the learning environment. This will be done depending on the subject and the nature of the lesson.

In order to create a well-rounded learning atmosphere, teachers need to:

- Remember that student-teachers learn in different ways so they have to offer a variety of activities (e.g. role-play, music and singing, word games and quizzes, and outdoor activities);
- Maintain an organized classroom and limits distraction. This will help student-teachers with special needs to stay on track during lesson and follow instruction easily;
- Vary the pace of teaching to meet the needs of each child. Some student-teachers process information and learn more slowly than others;

- Break down instructions into smaller, manageable tasks. Student-teachers with special needs often have difficulty understanding long-winded or several instructions at once. It is better to use simple, concrete sentences in order to facilitate them understand what you are asking.
- Use clear consistent language to explain the meaning (and demonstrate or show pictures) if you introduce new words or concepts;
- Make full use of facial expressions, gestures and body language;
- Pair a student-teacher who has a disability with a friend. Let them do things together and learn from each other. Make sure the friend is not over protective and does not do everything for the one with disability. Both student-teachers will benefit from this strategy;
- Use multi-sensory strategies. As all student-teachers learn in different ways, it is important to make every lesson as multi-sensory as possible. Student-teachers with learning disabilities might have difficulty in one area, while they might excel in another. For example, use both visual and auditory cues.

Below are general strategies related to each main category of disabilities and how to deal with every situation that may arise in the classroom. However, the list is not exhaustive because each child is unique with different needs and that should be handled differently.

Strategy to help student-teachers with intellectual impairment:

- Use simple words and sentences when giving instructions;
- Use real objects that student-teachers can feel and handle. Rather than just working abstractly with pen and paper;
- Break a task down into small steps or learning objectives. The student-teacher should start with an activity that she/he can do already before moving on to something that is more difficult;
- Gradually give the student-teacher less help;
- Let the student-teacher with disability work in the same group with those without disability.

Strategy to help student-teachers with visual impairment:

- Help student-teachers to use other senses (hearing, touch, smell and taste) and carry out activities that will promote their learning and development;
- Use simple, clear and consistent language;
- Use tactile objects to help explain a concept;
- If the student-teacher has some sight, ask him/her what he/she can see;
- Make sure the student-teacher has a group of friends who are helpful and who allow him/her to be as independent as possible;
- Plan activities so that student-teachers work in pairs or groups whenever possible;

Strategy to help student-teachers with hearing disabilities or communication difficulties

- Always get the student-teacher's attention before you begin to speak;
- Encourage the student-teacher to look at your face;
- Use gestures, body language and facial expressions;
- Use pictures and objects as much as possible.
- Keep background noise to a minimum.

Strategies to help student-teachers with physical disabilities or mobility difficulties:

- Adapt activities so that student-teachers, who use wheelchairs or other mobility aids, can participate.
- Ask parents/caregivers to assist with adapting furniture e.g. the height of a table may need to be changed to make it easier for a student-teacher to reach it or fit their legs or wheelchair under;
- Encourage peer support when needed;
- Get advice from parents or a health professional about assistive devices if the student-teacher has one.

Adaptation of assessment strategies:

At the end of each unit, the tutor is advised to provide additional activities to help students achieve the key unit competence. These assessment activities are for remedial, consolidation and extension designed to cater for the needs of all categories of students; slow, average and gifted students respectively. Therefore, the tutor is expected to do assessment that fits individual student.

Remedial activities	After evaluation, slow students are provided with lower order thinking activities related to the concepts learnt to facilitate them in their learning. These activities can also be given to assist deepening knowledge acquired through the learning activities for slow students.
Consolidation activities	After introduction of any concept, a range number of activities can be provided to all students to enhance/ reinforce learning.

Extended activities	After evaluation, gifted and talented students can be provided with high order thinking activities related to the concepts learnt to make them think deeply and critically. These activities can be assigned to gifted and talented students to keep them working while other students are getting up to required level of knowledge through the learning activity.

1.2.4. Guidance on assessment

Assessment is an integral part of teaching and learning process. The main purpose of assessment is for improvement of learning outcomes. Assessment for learning/ Continuous/ formative assessment intends to improve students' learning and tutor's teaching whereas assessment of learning/summative assessment intends to improve the entire school's performance and education system in general.

Continuous/ formative assessment

It is an on-going process that arises during the teaching and learning process. It includes lesson evaluation and end of sub unit assessment. This formative assessment should play a big role in teaching and learning process. The teacher should encourage individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods.

Formative assessment is used to:

- Determine the extent to which learning objectives are being achieved and competences are being acquired and to identify which students need remedial interventions, reinforcement as well as extended activities. The application activities are developed in the student-teacher book and they are designed to be given as remedial, reinforcement, end lesson assessment, homework or assignment
- Motivate students to learn and succeed by encouraging students to read, or learn more, revise, etc.
- Check effectiveness of teaching methods in terms of variety, appropriateness, relevance, or need for new approaches and strategies. Mathematics tutors need to consider various aspects of the instructional process including appropriate language levels, meaningful examples, suitable methods and teaching aids/ materials, etc.
- Help students to take control of their own learning.

In teaching Mathematics, formative or continuous assessment should compare performance against instructional objectives. Formative assessment should measure the student's ability with respect to a criterion or standard. For this reason, it is used to determine what students can do, rather than how much they know.

Summative assessment

The assessment can serve as summative and informative depending to its purpose. The end unit assessment will be considered summative when it is done at end of unit and want to start a new one.

It will be formative assessment, when it is done in order to give information on the progress of student-teachers and from there decide what adjustments need to be done.

The assessment done at the end of the term, end of year, is considered as summative assessment so that the tutor, school and parents are informed of the achievement of educational objective and think of improvement strategies. There is also end of level/ cycle assessment in form of national examinations.

When carrying out assessment?

Assessment should be clearly visible in lesson, unit, term and yearly plans.

Before learning (diagnostic): At the beginning of a new unit or a section of work; assessment can be organized to find out what students already know / can do, and to check whether the students are at the same level.

During learning (formative/continuous): When students appear to be having difficulty with some of the work, by using on-going assessment (continuous). The assessment aims at giving students support and feedback.

After learning (summative): At the end of a section of work or a learning unit, the Mathematics Tutor has to assess after the learning. This is also known as Assessment of Learning to establish and record overall progress of students towards full achievement. Summative assessment in Rwandan schools mainly takes the form of written tests at the end of a learning unit or end of the month, and examinations at the end of a term, school year or cycle.

Instruments used in assessment.

- **Observation:** This is where the Mathematics tutor gathers information by watching students interacting, conversing, working, playing, etc. A tutor can use observations to collect data on behaviours that are difficult to assess by other methods such as attitudes, values, and generic competences and intellectual skills. It is very important because it is used before the lesson begins and throughout the lesson since the tutor has to continue observing each and every activity.

- **Questioning**

a) Oral questioning: a process which requires a student to respond verbally to questions

b) Class activities/ exercises: tasks that are given during the learning/ teaching process

c) Short and informal questions usually asked during a lesson

d) Homework and assignments: tasks assigned to students by their tutors to be completed outside of class.

Homework assignments, portfolio, project work, interview, debate, science fair, Mathematics projects and Mathematics competitions are also the different forms/ instruments of assessment.

1.2.5. Teaching methods and techniques that promote active learning

The different learning styles for students can be catered for, if the tutor uses active learning whereby student-teachers are really engaged in the learning process.

The main teaching methods used in mathematics are the following:

- **Dogmatic method** (the tutor tells the student-teachers what to do, What to observe, How to attempt, How to conclude)
- **Inductive-deductive method:** Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples.
- **Analytic-synthetic method:** Analytic method proceeds from unknown to known, 'Analysis' means 'breaking up' of the problem in hand so that it ultimately gets connected with something obvious or already known. Synthetic method is the opposite of the analytic method. Here one proceeds from known to unknown.
- **Skills Laboratory method:** Laboratory method is based on the maxim "learning by doing." It is a procedure for stimulating the activities of the students and to encourage them to make discoveries through practical activities.
- **Problem solving method, Project method and Seminar Method.**

The following are some active techniques to be used in Mathematics:

- Group work
- Research
- Probing questions
- Practical activities (drawing, plotting, interpreting graphs)
- Modelling

- Brainstorming
- Quiz Technique
- Discussion Technique
- Scenario building Technique

What is Active learning?

Active learning is a pedagogical approach that engages student-teachers in doing things and thinking about the things they are doing. Student-teachers play the key role in the active learning process. They are not empty vessels to fill but people with ideas, capacity and skills to build on for effective learning. Thus, in active learning, student-teachers are encouraged to bring their own experience and knowledge into the learning process.

The role of the tutor in active learning	The role of student-teachers in active learning
– The tutor engages student-teachers through active learning methods such as inquiry methods, group discussions, research, investigative activities, group and individual work activities. – He/she encourages individual, peer and group evaluation of the work done in the classroom and uses appropriate competence-based assessment approaches and methods. – He provides supervised opportunities for student-teachers to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation. – The tutor supports and facilitates the learning process by valuing student-teachers' contributions in the class activities.	– A student-teacher engaged in active learning: – Communicates and shares relevant information with peers through presentations, discussions, group work and other student-teacher-centred activities (role play, case studies, project work, research and investigation); – Actively participates and takes responsibility for his/her own learning; – Develops knowledge and skills in active ways; – Carries out research/investigation by consulting print/online documents and resourceful people, and presents their findings; – Ensures the effective contribution of each group member in assigned tasks through clear explanation and arguments, critical thinking, responsibility and confidence in public speaking – Draws conclusions based on the findings from the learning activities.

Main steps for a lesson in active learning approach

All the principles and characteristics of the active learning process highlighted above are reflected in steps of a lesson as displayed below. Generally, the lesson is divided into three main parts whereby each one is divided into smaller steps to make sure that student-teachers are involved in the learning process. Below are those main part and their small steps:

1. Introduction

Introduction is a part where the tutor makes connection between the current and previous lesson through appropriate technique. The tutor opens short discussions to encourage student-teachers to think about the previous learning experience and connect it with the current instructional objective. The teacher reviews the prior knowledge, skills and attitudes which have a link with the new concepts to create good foundation and logical sequencings.

2. Development of the new lesson

The development of a lesson that introduces a new concept will go through the following small steps: discovery activities, presentation of student-teachers' findings, exploitation, synthesis/summary and exercises/application activities.

- **Discovery activity**

Step 1

- The tutor discusses convincingly with student-teachers to take responsibility of their learning
- He/she distributes the task/activity and gives instructions related to the tasks (working in groups, pairs, or individual to instigate collaborative learning, to discover knowledge to be learned)

Step 2

- The tutor let student-teachers work collaboratively on the task;
- During this period the tutor refrains to intervene directly on the knowledge;
- He/she then monitors how the student-teachers are progressing towards the knowledge to be learned and boosts those who are still behind (but without communicating to them the knowledge).

- **Presentation of student-teachers' findings/productions**
  - In this episode, the teacher invites representatives of groups to present their productions/findings.
  - After three/four or an acceptable number of presentations, the teacher decides to engage the class into exploitation of student-teachers' productions.

- **Exploitation of student-teacher's findings/ productions**
  - The tutor asks student-teachers to evaluate the productions: which ones are correct, incomplete or false
  - Then the tutor judges the logic of the student-teachers' products, corrects

those which are false, completes those which are incomplete, and confirms those which are correct.

- **Institutionalization or harmonization (summary/conclusion/ and examples)**
  - The tutor summarizes the learned knowledge and gives examples which illustrate the learned content.
- **Application activities**
  - Exercises of applying processes and products/objects related to learned unit/ sub-unit
  - Exercises in real life contexts
  - The tutor guides student-teachers to make the connection of what they learnt to real life situations. At this level, the role of teacher is to monitor the fixation of process and product/object being learned.

3. Assessment

In this step the teacher asks some questions to assess achievement of instructional objective. During assessment activity, student-teachers work individually on the task/ activity. The teacher avoids intervening directly. In fact, results from this assessment inform the teacher on next steps for the whole class and individuals. In some cases, the teacher can end with a homework/ assignment. Doing this will allow student-teachers to relay their understanding on the concepts covered that day. Teacher leads them not to wait until the last minute for doing the homework as this often results in an incomplete homework set and/or an incomplete understanding of the concept.

PART II: SAMPLE LESSON PLAN

School Name:….   **Tutor's name**: …………………..

Term	Date	Subject	Class	Unit Nº	Lesson Nº	Duration	Class size
III	…./…./2019	Mathematics	Y3LE	2	10 of 14	40 min	….....

Type of special educational needs to be catered for in this lesson and number of student-teachers in each category	3 slow student-teachers and 2 low vision student-teachers
Unit title	Elementary probability
Key unit competency:	Use counting techniques and concepts of probability to determine the probability of possible outcomes of events occurring under equally likely assumptions.
Title of the lesson	Determine the classical and empirical probability of events.
Instructional Objective	Using coins, dice or deck of cards, student- teachers will be able to determine the classical or empirical probability of an event correctly.
Plan for this Class (location: in / outside)	The lesson is held in classroom , the class is organized into groups ,3 slow student-teachers are scattered in different groups ,and 2 low vision student-teachers seat on the front desks near the blackboard in order to see and participate fully in all activities
Learning Materials (for ALL student-teachers)	Manila papers, calculators, coins, dice, deck of cards
References	-TTC syllabus of Mathematics of Languages education. -Student-teachers' Mathematics textbook and Teacher's guide

Timing for each step	Description of teaching and learning activity		Generic competences and cross cutting issues to be addressed + a short explanation
	– Firstly, student – Teachers will be put into small groups then work on the activity 2.3.1 which is in the student book under the guidance of the tutor – Secondly, student-teachers will present their work, and the tutor will harmonize the findings in order to deduce the formula of classical probability. – Finally, the student-teachers will work on other examples the application activity 2.3.1 given in their mathematics book.		
	Tutor activities	**Student-teacher activities**	
Introduction: 5 minutes	– To ask oral questions on the previous lessons related to counting techniques – To form groups and give written exercises to the student-teachers – To guide and correct student-teachers	– To answer oral questions asked by the tutor – To correct exercises by sharing ideas in groups then present results. – To follow the guidance of the tutor.	– Communication skills developed as student- teachers respond to oral questions asked by the tutor, while sharing and when presenting the results. – Gender addressed when both girls and boys work together in the same group – Cooperation developed when student-teachers correct exercises in groups. – Problem solving and critical thinking are developed when correcting given exercises

Development of the lesson **Discovery activity:** **10 minutes**	– To organize student-teachers into groups and ask them to work out activity 2.3.1 which is in the student mathematics book – To move around in class for facilitating student-teachers where necessary and give more clarification on how to determine the classical or empirical probability of an event	– Tofollowinstructions of the tutor and work out the activity 2.3.1 in the student-book. – To analyze and discuss the given task under the guidance of the tutor – etermine the classical probability of an event	– Cooperation and communication skills through discussions – Peace and values education; Cooperation, mutual respect, tolerance through discussions with people with different views and respect one's views during discussions. – Gender addressed when both girls and boys work together in the same group – Problem solving and critical thinking are developed when correcting given exercises

Presentation of student-teacher's findings and exploitation: **15minutes**	– To invite one member from each group with different working steps to present their work where they must explain the working steps; – To encourage student-teachers to follow attentively and to guide them to answer questions. – To ask student-teachers to amend the presentation and to evaluate their work – To harmonize presentations of student-teachers – To help student-teachers to determine and to give the formula of classical probability	To present findings by explainingtheworking steps **Expected answers** (Refer to solution of activity 2.3.1, in Teacher's guide) – To follow the presentation of their neighbors – To ask questions for more clarification. – To evaluate their own findings and findings of others bout the classical probability – To determine the classical probability of an event and to give the formula $P(A)=\frac{Number\ of\ outcomes\ in\ A}{Total\ number\ of\ outcomes\ in\ the\ sample\ space}$ $=\frac{n(A)}{n(\Omega)}$ – To work out through the exercises and examples prepared in their books related to classical probability. – To answer questions asked by the tutor and discover basic rules of probability.	Cooperation and communication/ attentive listening during presentations and group discussions Problem solving and critical thinking are developed when correcting given exercises Cooperation and communication/ attentive listening during presentations and group discussions

	– To guide student teachers to work through various examples which are in their books. – To ask questions that help student teachers to discover basic rules of probability	**Basic rules of probability** **1. The probability cannot be negative or greater than 1** Suppose that an experiment has only a finite number of equally likely outcomes. If A is an event, then $0 \leq P(A) \leq 1$. **2. The probability of a certain event** If the event A is certain to occur, $A = \Omega$, and $P(A) = 1$ and $P(\Omega) = 1$ Probability of impossible event The event that cannot occur is an impossible event $A = \varnothing$, and if $A = \varnothing$ then $P(A) = 0$. **The sum of the probabilities of all the outcomes in the sample space is 1.**	

Conclusion/ **Summary** **5 minutes** - Assessment **5 minutes**	To help student-teachers to construct a short summary on the formula of classical probability and basic formula of classical probability To request student-teachers to write down the summary in their notebooks To open a discussion on the application of classical probability in the real life. Invite student-teachers to work individually application activity 2.3.1	To give the formula of classical probability basic formula of classical probability To write down the main points in their notebooks To give examples of the use of classical probability in the real life. To work out the application activity 2.3.1 and finally make a correction on the chalk board. **Expected answers** (Refer to solution in TG)	Lifelong learning developed as student- teachers continue to do research on where they can apply probability in real life. Critical thinking and problem solving developed as student-teachers use the formula to find out the classical probability
Observation on lesson delivery	To be completed after receiving the feed-back from the student-teachers (what did the student-teachers liked, what challenged them,)		

UNIT 1

BIVARIATE STATISTICS

1.1 Key unit competence

Extend understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines

1.2 Prerequisite

Student teachers will easily learn this unit, if they have a good background on descriptive statistics (Unit 3, Year 1) and (Unit 13, Senior 3 Statistics: Bivariate data).

1.3 Cross-cutting issues to be addressed

a) Inclusive education

(promote education for all while teaching)

b) Peace and value Education

(respect others' view and thoughts during class discussions)

c) Gender

(provide equal opportunity to boys and girls in the lesson)

d) Financial education

(discuss how to make appropriate financial decisions through examples and exercises)

1.4 Guidance on introductory activity 1

- Invite student-teachers to work in groups and give them instructions on how they can do the introductory activity 1.0 found in unit 1 of the student's book;
- Guide students to read and analyse the questions insisting on the analysis of statistical data with two variables (x, y) and how they can interpret the bivariate data using correlation.

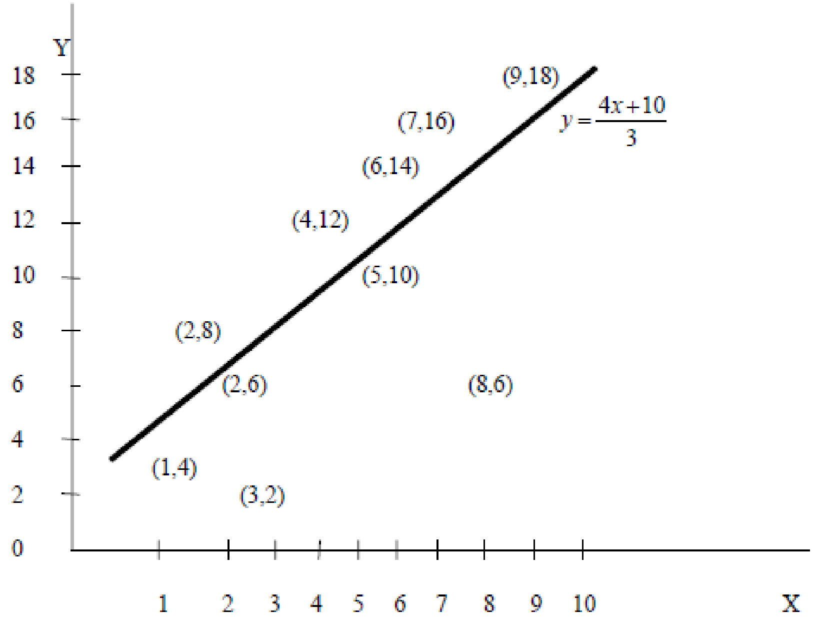
- Move around in the classroom to get aware of struggling groups and provide assistance where necessary;
- Invite some group members to present groups' findings, then try to harmonize their answers;
- Basing on student-teachers' experience, prior knowledge and abilities shown in answering the questions for this activity, open a discussion and use different questions to facilitate them to give their predictions and ensure that you arouse their curiosity on what is going to be leant in this unit.

Answer for introductory activity 1:

a) Scatter diagram:

Plotting the 9 sample points

(1,4), (2,8), (3,4,), (4,12), (5,10), (6,14), (7,16), (8,6), (9,18)



b) The equation of the line is given by $y = mx + c$,. The slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$ .For example; let us take $(2,6)$ as the first point on the line and (5, 10) as another point on the line. The slope $m = \frac{10-6}{5-2} = \frac{4}{3}$ the equation of the line is $y = mx + c = \frac{4}{3}x + c$. For $x = 2, y = 6$ then $6 = \frac{4}{3} \times 2 + c \Leftrightarrow c = \frac{10}{3}$

The equation of the line joining the two points is $y = \frac{4}{3}x + \frac{10}{3}$.

c) Apart from the points (3,2) and (8,6) which are far from the line , other points are close to the line . This means that there is a relation between the variation of the number of cows and the number of domestic ducks.

1.5. List of lessons/sub-headings

Numbering	**Lesson title**	**Learning objectives**	**Number of periods**
0	Introductory activity	To arouse the curiosity of student-teachers on the content of unit 4.	1
1	Bivariate data ,scatter diagram and types of correlation	To define bivariate statistics data and plot the scatter diagram and differentiate types of correlation.	3
2	Covariance	Explain and determine the covariance for bivariate statistics data	3
3	Coefficient of correlation	To define and calculate the Coefficient of correlation for Bivariate statistics data. Appreciate the importance of using the Coefficient of correlation to interpret data and to infer conclusions	4
4	Regression lines	To define ,establish the equation and plot the regression lines Appreciate the importance of using regression line to interpret data.	4
5	Interpretation of statistical data	Analyse, interpretation and predict bivariate statistical data from various areas (Business, Geography, Demography …)	4
6	Interpretation of collected data	To collect, organize and interpret data from the real life situations.	4

	End unit assessment		1
	Total		24

Lesson 1: Bivariate data, scatter diagram and types of correlation

a) Learning objectives

To define bivariate statistics data, plot the scatter diagram and differentiate types of correlation

b) Teaching resources

Student's book and other Reference books to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils...

c) Prerequisites/Revision/Introduction

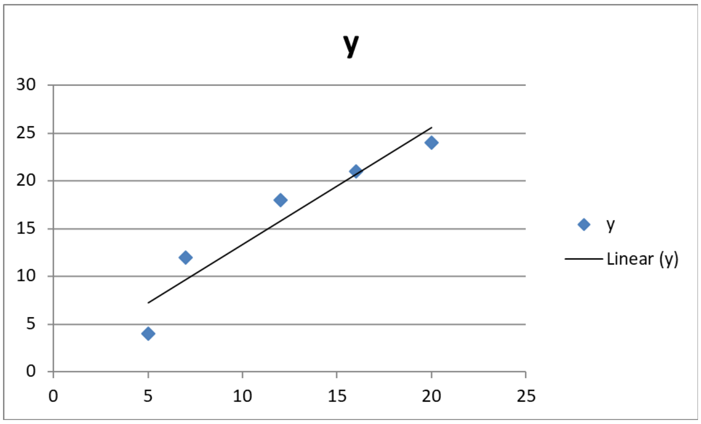
Students will learn better in this lesson if they refer to descriptive statistics (Unit 3Year1 LE) and to (Unit 13 Senior 3 on statistics)

d) Learning activities:

- Invite student-teachers to work in groups and do the activity1.1 found in their Mathematics Student books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work;
- Invite one member from each group with different working steps to present their work explaining the working steps;
- As a tutor, harmonize the findings from presentation and guide them to decide whether they can guess if there is a relationship between two variables;
- Use different probing questions that guide them to explore the content and examples given in the student's book and lead them to discover how to define bivariate statistics data, to plot the scatter diagram and to distinguish different types of correlation between two variables.
- After this step, guide students to do the application activity 1.1 and evaluate whether lesson objectives are achieved.

Answer for activity 1.1

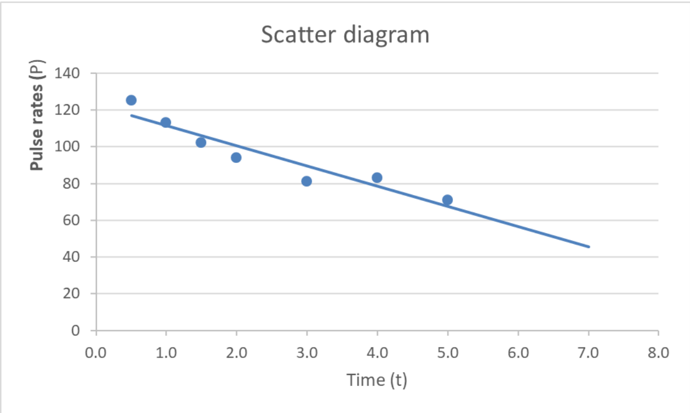
a)



As time is increasing the chemical reaction is also increasing.

Answer for Application activity 1.1

a)



b) From the scatter diagram above, we see that P tends to decrease as t increases. Meaning that

Norman's pulse decreases as time increases.

Lesson 2: Covariance

a) Learning objectives

Explain and determine the covariance for bivariate statistics data

b) Teaching resources

Student-teacher’s book and other Reference books to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils...

c) Prerequisites/Revision/Introduction

Studentswilllearnbetterinthislessoniftheyhaveagoodbackgroundondescriptive statistics (Unit 3Year1 LE) and to (Unit 13 Senior 3 on statistics) and on lesson 1 of this Unit.

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 1.2 found in their Mathematics Student books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a tutor, harmonize the findings from presentation;
- Use different probing questions that guide them to explore the content and examples given in the student’s book and lead them to discover how to determine and explain the covariance for bivariate statistics data.
- After this step, guide students to do the application activity 1.2 and evaluate whether lesson objectives are achieved.

Answer for activity 1.2

x_i	y_i	$x_i - \overline{x}$	$y_i - \overline{y}$	$(x_i - \overline{x})(y_i - \overline{y})$
3	6	-1.3	-2.6	3.38
5	9	0.7	0.4	0.28
7	12	2.7	3.4	9.18
3	10	-1.3	1.4	-1.82
2	7	-2.3	-1.6	3.68
6	8	1.7	-0.6	-1.02
$\sum_{i=1}^{6} x_i = 26$	$\sum_{i=1}^{6} y_i = 52$			$\sum_{i=1}^{6} (x_i - \overline{x})(y_i - \overline{y}) = 13.68$
$\overline{x} = 4.3$	$\overline{y} = 8.6$			

If you divide the expression by total frequency you get **variance in x**

If you divide the expression by total frequency you get **covariance of x and y**

Answer for application activity 1.2

Let M stands for scores of students in Mathematics and P stands for scores of students in Physics.

M	P	$M_i - \overline{M}$	$P_i - \overline{P}$	$(M_i - \overline{M})(P_i - \overline{P})$
2	1	-4	-4	16
3	3	-3	-2	6
4	2	-2	-3	6
4	4	-2	-1	2
5	4	-1	-1	1
6	4	0	-1	0
6	6	0	1	0
7	4	1	-1	-1
7	6	1	1	1
8	7	2	2	4
10	9	4	4	16
10	10	4	5	20
$\overline{M} = \frac{72}{12} = 6$	$\overline{P} = \frac{60}{12} = 5$			$\sum_{i=1}^{12}(M_i - \overline{M})(P_i - \overline{P}) = 71$

$$Cov(M,P) = \frac{1}{n}\sum_{i=1}^{12}(M_i - \overline{M})(P_i - \overline{P}) = \frac{1}{12}\times(71) = 5.91$$

2)

Y X	100	50	25	Total
14	1	1	0	2
18	2	3	0	5
22	0	1	2	3
Total	3	5	2	10

Then,

$$\overline{x} = \frac{1}{10}(100\times 3 + 50\times 5 + 25\times 2) = 60$$

$$\overline{y} = \frac{1}{10}(14\times 2 + 18\times 5 + 22\times 3) = 18.4$$

$$Cov(x,y)=\frac{1}{10}\begin{pmatrix}100\times14\times1+100\times2\times18+100\times0\times22+\\50\times14\times1+50\times18\times3+50\times22\times1+\\25\times14\times0+25\times18\times0+25\times25\times2\end{pmatrix}-60\times18.4=-44$$

Lesson 3: Coefficient of correlation

a) Learning objectives

To define and calculate the Coefficient of correlation for bivariate statistics data.

b) Teaching resources

Student-teacher's book and other Reference books to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils...

c) Prerequisites/Revision/Introduction

Students will learn better in this lesson if they have a good background on lesson 1 & lesson 2 of this Unit. and Unit 13 Senior 3 on descriptive statistics

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 1.3 found in their Mathematics Student books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a tutor, harmonize the findings from presentation;
- Use different probing questions that guide them to explore the content and examples given in the student's book and lead them to discover how to define and calculate the Coefficient of correlation for bivariate statistics data.
- Guide students to do the application activity 1.3 and evaluate whether lesson objectives have been achieved.

Answers for activity 1.3

We know that:

$$\sigma_x=\sqrt{\frac{\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2}{n}},\quad \sigma_y=\sqrt{\frac{\sum_{i=1}^{n}\left(y_i-\overline{y}\right)^2}{n}}$$

$\sigma_x=1.8,\ \sigma_y=1.97$

$$\operatorname{cov}(x,y)=\frac{1}{n}\sum_{i=1}^{6} f_i x_i y_i - \overline{x}\ \overline{y} =$$

$$\operatorname{cov}(x,y)=\frac{41}{18}$$

The ratio $\frac{\operatorname{cov}(x,y)}{\sigma_x\sigma_y}=0.64$

Answers for application activity 1.3

1) $r=0.94$. As the correlation coefficient is very close to one, the correlation is very strong.

2) $r=-0.26$. As the correlation coefficient is very close to zero, the correlation is **negative** and **very weak.**

3) $\sigma=0.14$. There is a week positive correlation between the English and Mathematics rankings.

Lesson 4: Regression lines

a) Learning objectives

To define, establish the equation and plot the regression lines

b) Teaching resources

Student-teacher's book and other Reference books to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils...

c) Prerequisites/Revision/Introduction

Students will learn better in this lesson if they have a good background on lesson 1, lesson 2 and lesson 3 of this Unit and Unit 13 Senior 3 on descriptive statistics.

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 1.4 found in their Mathematics Student books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work.
- Invite one member from each group with different working steps to present

their work where they must explain the working steps;

- As a tutor, harmonize the findings from presentations;
- Use different probing questions and guide them to explore the content and examples given in the student's book then lead them to discover how to define the regression line, establish equations and appreciate the importance of using regression lines to interpret data.
- Guide students to do the application activity 1.4 and evaluate whether lesson objectives have been achieved.

Answers for activity 1.4

x_i	y_i	x_i^2	y_i^2	$x_i y_i$
5	4	25	16	20
7	12	49	144	84
12	18	144	324	216
16	21	256	441	336
20	24	400	576	480
$\sum_{i=1}^{5} x_i = 60$	$\sum_{i=1}^{5} y_i = 79$	$\sum_{i=1}^{5} x_i^2 = 874$	$\sum_{i=1}^{5} y_i^2 = 1501$	$\sum_{i=1}^{5} x_i y_i = 1136$

$$\overline{X} = \frac{1}{n}\sum_{i=-1}^{5} x_i = \frac{60}{5} = 12$$

$$\overline{Y} = \frac{1}{n}\sum_{i=-1}^{5} y_i = \frac{79}{5} = 15.8$$

a) Variance of x= $S_{xx} = \frac{1}{n}\sum_{i=1}^{5} x_i^2 - \overline{X}^2 = \frac{874}{5} - 144 = 30.8$

b) Variance of y= $S_{yy} = \frac{1}{n}\sum_{i=1}^{5} y_i^2 - \overline{Y}^2 = \frac{1501}{5} - 249.64 = 50.56$

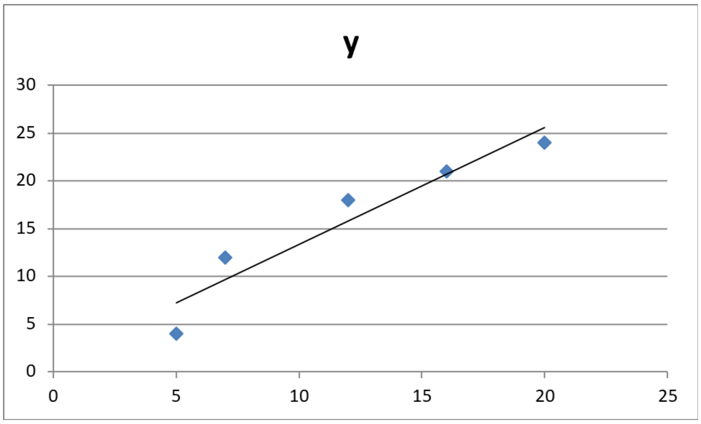
c) Covariance of x and y= $S_{xy} = \frac{1}{n}\sum_{i=1}^{5} x_i y_i - \overline{X}\,\overline{Y} = \frac{1136}{5} - 12\times 15.8 = 37.6$

d) The value a given by $\frac{S_{xy}}{S_{xx}} = \frac{37.6}{30.8} = 1.22$

e) The value b given by $b = \overline{Y} - a\overline{X} = 15.8 - 1.22 \times 12 = 1.15$

f) The equation of the line $y = ax + b = 1.22x + 1.15$

g)



h) The points are closed to the line $y = 1.22x + 1.15$

Application activity 1.4

1. a) $y = 0.19x - 8.098$ b) $y = 4.06$

2. $x = -5.6y + 163.3, \quad y = -0.06x + 21.8$

Lesson 5: Interpretation of statistical data

a) Learning objectives

Analyse, interpretation and predict bivariate statistical data from various areas (Business, Geography, Demography …)

b) Teaching resources

Student-teacher's book and other Reference books to facilitate research, Mathematical set, calculator, manila paper, markers, pens, pencils...

c) Prerequisites/Revision/Introduction

Students will learn better in this lesson if they have a good background on lesson 1, lesson 2 , lesson 3, lesson 4 of this Unit and Unit 13 Senior 3 on descriptive statistics.

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 1.5 found in their Mathematics Student books;

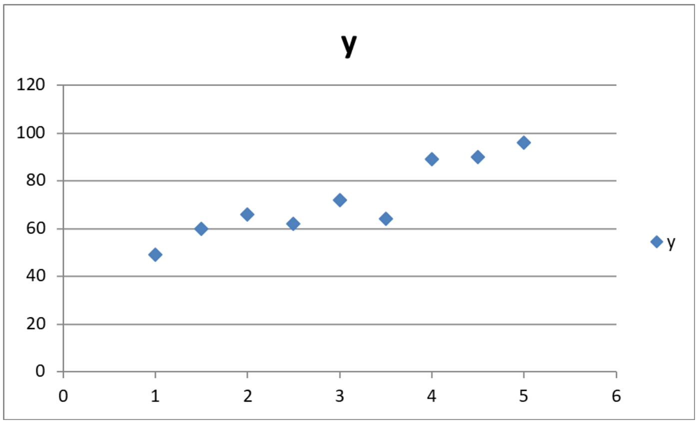
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a tutor, harmonize the findings from presentation;.
- Use different probing questions and guide them to explore the content and examples given in the student's book and lead them to discover how to define the regression line , establish its equation and appreciate the importance of using regression line to interpret data.
- Guide students to do the application activity 1.5 and evaluate whether lesson objectives were achieved.

Answer to activity 1.5 (see the content summary in the student book)

Bivariate analysis is a statistical method that helps people study relationships (correlation) between data sets. Many questions and problems in business, marketing, and social science are analysed and solved using bivariate data sets.

Answers for application activity 1.5

a) Scatter diagram



b) Sample F was damaged.

$$\bar{x}=\frac{\sum x}{n}=\frac{23.5}{8}=2.9375 \text{ and } \bar{y}=\frac{\sum y}{n}=\frac{584}{8}=73$$

c) To calculate r: $s_{xy}=\frac{1}{n}\sum xy-\bar{x}\,\bar{y}=\frac{1}{8}\times 1883-2.9375\times 73=20.9375$

$$s_{xx}=\frac{1}{n}\sum x^2-\left(\bar{x}\right)^2=\frac{1}{8}\times 83.75-(2.9375)^2=1.839...$$

$$s_{yy}=\frac{1}{n}\sum y^2-\left(\bar{y}\right)^2=\frac{1}{8}\times 44622-(73)^2=248.75$$

$$r=\frac{s_{xy}}{s_x s_y}=\frac{20.9375}{\sqrt{1.839...}\sqrt{248.75}}=0.9787...$$, Then, $r=0.98\,(2\,s.f.)$

d) Yes it is sensible to conclude that x *and* y are related. Since $r=0.98\,(2\,s.f.)$ is very closed to 1, it would appear to indicate a very strong position linear correlation.

e) For the regression line $y=a+bx, a=\bar{y}-b\bar{x}$ and

$b=\frac{s_{xy}}{s_{xx}}=\frac{20.9375}{1.839...}=11.38...$; $b=\frac{s_{xy}}{s_{xx}}=\frac{20.9375}{1.839...}=11.38...$ Then,

$$a=\bar{y}-b\bar{x}=73-11.38...\times 2.9375=39.57...$$

$$y=39.6+11.4x\,(3\,s.f.)$$

f) When $x=3.5, y=38.57...+11.38...\times 3.5=79\,(2\,s.f.)$ The constant index would have been 79.

g) No, it would not be sensible to use the regression equation when $x=0$, since this is outside the range of data. Extrapolating outside the data is unreliable.

Note:

The lesson 6 of collecting and interpreting data is a practical activity where student teachers will independently collect data and bring them in class; then use them in analysis and interpretation. As a Tutor;

- Make sure that the data collection is not made between students themselves in class.
- Guide students in data analysis after collection.

1.6 Unit summary

1. **Bivariate** or **double series** includes technique of analysing data in two variables.

2. If each point (x, y) of the data is plotted in an x, y *coordinate* plane, we say that we have the scatter plot or Scatter diagram

3. If $x-coordonates$ increases as $y-coordonates$ increases also; we say that x and y have a positive correlation. When y tends to decrease as x increases,

then x and y have a negative correlation.

4. The **covariance of variables x and y** is a measure of how these two variables change together.
5. If covariance is zero the variables are said to be **uncorrelated,** means that there is no linear relationship between them. Then, $\operatorname{cov}(x,y)=\frac{1}{n}\sum_{i=1}^{k} f_i\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)$
6. The **Pearson's coefficient of correlation** (or **Product moment coefficient of correlation** or simply **coefficient of correlation**), denoted by *r*, is a measure of the strength of linear relationship between two variables.

The coefficient of correlation between two variables *x* and *y* is given by

$$r=\frac{\operatorname{cov}(x,y)}{\sigma_x\sigma_y}$$

Where, $\operatorname{cov}(x,y)$ is covariance of *x* and *y*

σ_x is the standard deviation for *x*

σ_y is the standard deviation for *y*

7. We use the regression line to **predict** a value of *y* for any given value of *x* and vice versa. The "best" line would make the best predictions: the observed *y*-values should stray as little as possible from the line. This straight line is the regression line from which we can adjust its algebraic expressions and it is written as $y=ax+b$, where a *is* the gradient and c is the y - intercept.

The regression line *y* on *x* is written as $y=\frac{\operatorname{cov}(x,y)}{\sigma_x^2}x+\left(\bar{y}-\frac{\operatorname{cov}(x,y)}{\sigma_x^2}\bar{x}\right)$

We may write $L_{y/x}\equiv y-\bar{y}=\frac{\operatorname{cov}(x,y)}{\sigma_x^2}\left(x-\bar{x}\right)$

1.7 Additional information for the tutor

Here the tutor has to emphasize on how to analyze, interpret and predict bi-variate statistical data using regression line and coefficient of correlation from various areas (Business, Geography, Demography …)

1.8 End Unit assessment

$$y-\overline{y}=\frac{\operatorname{cov}(x,y)}{\sigma_x^2}\left(x-\overline{x}\right)$$

1) $y-39.5=\dfrac{\operatorname{cov}(x,y)}{282.24}\left(x-475\right)$

$$y=0.61x-250.6$$

$$x-\overline{x}=\frac{\operatorname{cov}(x,y)}{\sigma_y^2}\left(y-\overline{y}\right)$$

$$x-475=\frac{\operatorname{cov}(x,y)}{116.64}\left(x-39.5\right)$$

$$x=1.478y+416.63$$

$$y-\overline{y}=\frac{\operatorname{cov}(x,y)}{\sigma_x^2}\left(x-\overline{x}\right)$$

2) $y-39.5=\dfrac{\operatorname{cov}(x,y)}{282.24}\left(x-475\right)$

$$y=0.61x-250.6$$

$$x-\bar{x}=\frac{\operatorname{cov}(x,y)}{\sigma_y^2}\left(y-\bar{y}\right)$$

$$x-475=\frac{\operatorname{cov}(x,y)}{116.64}\left(x-39.5\right)$$

1.9 Additional activities

$$x=1.478y+416.63$$

1.9.1. Remedial activitiexs

1. For a given set of data it is known that $\bar{x}=10\, and\, \bar{y}=4$.The gradient of the regression line y on x is 0.6. Find the equation of this regression line and estimate y when $x=12$

Solution:

The equation of the regression line is $y=a+bx, where\, b=0.6$; Then $y=a+0.6x$

The regression line goes through $\left(\bar{x},\bar{y}\right)$, so $\bar{y}=a+0.6\bar{x}$

$4=a+0.6\times 10\Rightarrow a=-2$; Thus, the equation of the regression line is $y=-2+0.6.x$; When $x=12,\ y=-2+0.6\times 12=5.2$.

1.9.2 Consolidation activities

1. Find the regression line of x on y if the line goes through $\left(1,4\right)$ and has gradient 2.

Solution

Equation of regression line x on y is $x=c+dy$; rearranging

$$dy=x-c\Leftrightarrow y=\frac{1}{d}x-\frac{c}{d}$$

Gradient $=\frac{1}{d}$, then $2=\frac{1}{d}\Leftrightarrow d=0.5$; So $x=c+0.5y$

You are given that $\left(1,4\right)$ lies on the line $1=c+0.5\times 4\Leftrightarrow c=-1$.

The equation of regression line x on y is $x=-1+0.5y$

1.9.3 Extended activities

1. A student found the following data for the female life expectancy, x years, and the Gross Domestic Production (GDP) per head, $\$y$, in six countries in South Asia in 1988.

Country	X	y
Afghanistan	42	143
Bangladesh	50	179
Bhutan	47	197
India	58	335
Pakistan	57	384
Sri Lanka	73	423

$$\left[n=6, \sum x=327, \sum y=1661, \sum x^2=18415, \sum y^2=529909, \sum xy=96412\right]$$

It is required to estimate the value of x for Nepal, where the value of y was 160. (i) Find the equation of a suitable line of regression. Simplify your answer as far as possible, giving the constant s correct to three significant figures. (ii) Use your equation to obtain the required estimate.

Use your equation to estimate the value of x for North Korea, where the value of y was 858. Comment on Your answer.

Solution

Neither variable has been controlled in the given data and since you are required to estimate the life expectancy, x years, When the Gross Domestic Product per head, $y is $160, it is sensible to use the regression line of x on y.

The least squares regression line of x on y. Has the equation $x=c+dy$

Where $c=\bar{x}-d\bar{y}$ and $d=\frac{s_{xy}}{s_{yy}}$. $\bar{x}=\frac{\sum x}{n}=\frac{327}{6}$ and $\bar{y}=\frac{\sum y}{n}=\frac{1661}{6}=$'Then

$$s_{xy}=\frac{1}{n}\sum xy-\bar{x}\bar{y}=\frac{1}{6}\times 96412-\frac{327}{6}\times\frac{1661}{6}=981.25$$

$$S_y=\frac{1}{n}\sum y^2-\left(\bar{y}\right)^2=\frac{1}{6}\times 529909-\left(\frac{1661}{6}\right)^2=11681.4\!\!\!7$$

$$d=\frac{s_{xy}}{s_{yy}}=\frac{981.25}{11681.47}=0.084000...$$

$$c=\bar{x}-d\bar{y}\Leftrightarrow c=\frac{327}{6}-0.08400...\times\frac{1661}{6}=31.24...$$

The equation of regression line of x on y is $x=31.2+0.084y(3s.f.)$

UNIT 2

ELEMENTARY PROBABILITY

(ii) When $y=160, x=31.2+0.0840\times160=45\,(2s.f.)$

The estimated value of the life expectancy in Nepal is 45 years.

(b)From the equation, when $y=858$,$y=160, x=31.2+0.0840\times858=103\,(3s.f.$

This would give the life expectancy in North Korea as 103 years, which is clearly not sensible. The value of $y=858$ is a long way outside the range of the data, and should not be used to estimate a value of x.

2.1 Key unit competence

Use counting techniques and concepts of probability to determine the probability of possible outcomes of events occurring under equally likely assumptions.

2.2 Prerequisite

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11.

2.3 Cross-cutting issues to be addressed

a) Inclusive education

(promote education for all while teaching)

b) Peace and value Education

(respect others' view and thoughts during class discussions)

c) Gender

(provide equal opportunity to boys and girls in the lesson)

d) Financial education

(discuss how to make appropriate financial decisions through examples and exercises)

2.4 Guidance on introductory activity 2

- Form groups of students and invite student-teachers to work on questions for introductory activity 2 found in student's book unit 2;
- Guide student-teachers to read and analyse the problem related to different cases of the gender that 3 children can have: they have to write all those cases on a sheet of paper;
- Guide student-teachers to find out many possibilities as they can;
- Invite students with different working steps to present their findings to the whole class discussion;
- As a tutor, harmonize the findings from presentation
- Basing on their experience, prior knowledge and abilities shown in answering the questions for this activity , use different questions to guide student-teachers to give their predictions and ensure that you arouse their curiosity on what is going to be leant in this unit.

2.5 List of lessons

	Lesson title	Learning objectives	Number of periods
0	Introductory activity	To arouse the curiosity of student teachers on the content of unit 2.	1
1	Concept of probability	Define the terms: probability, experiment, sample space and event, complementary events, mutually exclusive events.	4
2	Simple counting techniques	Determine all the possible outcomes using simple counting techniques **(Venn diagram, tree diagrams, and contingency table).**	1
3	Basic sum principle of counting (Mutually exclusive situations).	Perform operations on mutually exclusive events to determine the number of outcomes using Basic sum principle of counting	1
4	Arrangements of n unlike objects in a row.	Perform operations on arrangements of n unlike objects in a row.	1
5	Arrangements of indistinguishable objects (Permutations with repetitions).	Perform operations on the arrangement of n alike objects in line to determine the number of outcomes	1

6	Circular arrangements	Determine the number of outcomes on circular arrangements	1
7	**Distinguishable permutations** (Permutations of r unlike objects selected from n distinct objects).	Use distinguishable arrangements to find the number of outcomes	1
8	Permutations of r objects selected from mixture of n alike and unlike objects	Use permutations of r objects selected from mixture of n alike and unlike objects to determine the number of outcomes	1
9	Combinations	Use combinations to determine the number of outcomes	2
10	Probability of an event	Determine the classical and empirical probability of events.	2
11	Probability of mutually exclusive or incompatible events	Calculate the probability of mutually exclusive events. Use addition rule formula	2
12	Probability of independent events and multiplication rule	Calculate the probability of independent events. Use multiplication rule formula	2
13	Examples of Events in real life and determination of related probability	Calculate the probability of an event using rules and formulas Use real life tasks (games, number of trials,…) to determine the probability of well described events.	2
	End unit assessment		2
	Total		24

Answers for introductory activity 2

Three children who can be born can have the following gender:If F=female or Girl and M= male or Boy,

$\Omega = \{GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB\}$; There are 8 possibilities.

Therefore, there is one case under which the woman can have a girl at the first and the second delivery then a boy at the last delivery. This is the event $V = \{GGB\}$ which means that she has one chance among 8 possible cases.

Lesson 1: Concepts of probability (Sample space and Events)

a) Learning objective

Define the terms: probability, experiment, sample space and event, complementary events, mutually exclusive events.

b) Teaching resources

Playing cards, graph papers, manila papers, calculators, coin, dice.

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11.

d) Learning activities

- Form small groups of student-teachers and give them instructions on how to work on the activity 2.1;
- Walk around groups then ask probing questions leading them to find the total number of cards and the number of specified cards;
- Invite groups to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers highlighting that there are a number of chance of choosing a card.
- Use different probing questions and guide them to explore examples given in the student's book and lead them to explain the main concepts of probability accompanied with examples: events and their types, outcome, sample space, etc.
- Guide students to do the application activity 2.1 and evaluate whether lesson objectives are achieved.

Answers for activity 2.1

1. a) There are 52 cards that can be chosen;

   b) There are 4 kings that can be chosen in 52 cards;

   c) There is 1 ace of hearts that can be chosen.

2. Other examples of event:

Students can give many examples. As a tutor, verify if they are correct events. Example: selecting a black card, selecting a diamond, etc.

Answers for application activity 2.1

Two dice are thrown simultaneously, on has $\{1,2,3,4,5,6\}$ and the second $\{1,2,3,4,5,6\}$. The sample space made by the sum of points is noted $\Omega = \{2,3,4,5,6,7,8,9,10,11,12\}$.

Lesson 2: Simple counting techniques (Venn diagram, tree diagrams, contingency table)

a) Learning objectives

Determine all the possible outcomes using simple counting techniques (Venn diagram, tree diagrams, and contingency table).

b) Teaching resources:

Graph papers, manila papers, calculators, coins, dice.

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11.

d) Learning activities

- Form small groups of student-teachers and give them instructions on how to work on the activity 2.2.1
- Walk around each group and ask probing questions leading them to determine the total number of roads from A to C via B
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers highlighting that there is a technique of finding the total number of outcomes for a given random experiment;
- Use different probing questions and guide them to explore examples given in the student's book and lead them to determine total number of outcomes for a given random experiment using: Venn diagram, tree diagram or a table.
- Guide them to discover that if a sequence of n events in which the first one has n_1 possibilities, the second with n_2 possibilities the third with n_3 possibilities, and so forth until n_k, the total number of possibilities of the sequence will be $= n_1.n_2.n_3...n_k$
- Guide students to do the application activity 2.2.1 and evaluate whether lesson objectives are achieved.

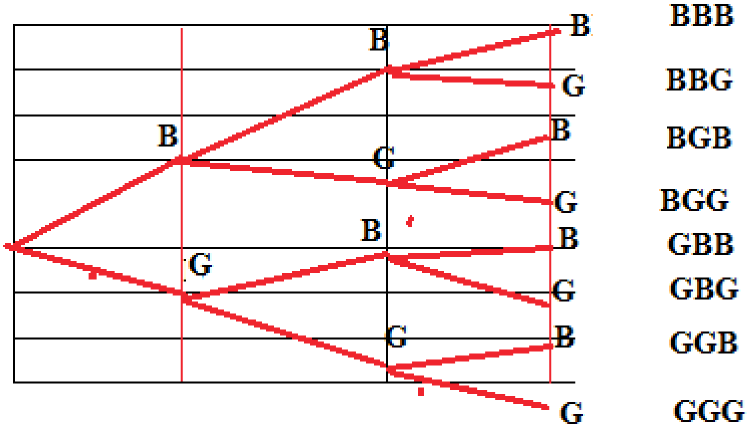
Answers for activity 2.2.1

To find all possible roads, students can use allows to join points or a try and fail method.

$\Omega = \{AB_1C_1, AB_1C_2, AB_1C_3, AB_2C_1, AB_2C_2, AB_2C_3\}$ so they are 6.

Answers for application activity 2.2.1

1) Using the tree diagram, one can find:



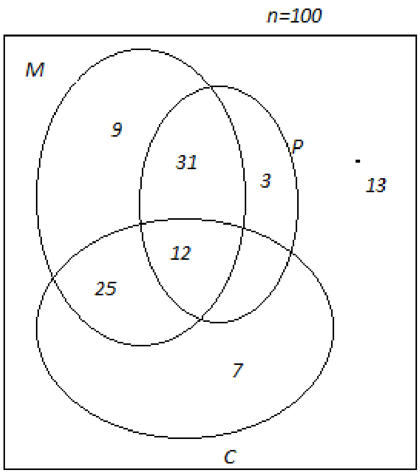
$S = \{\text{BBB}, \text{BBG}, \text{BGB}, \text{BGG}, \text{GBB}, \text{GBG}, \text{GGB}, \text{GGG}\}$

2) The coin can land either head up or tails up.

	1	**2**	**3**	**4**	**5**	**6**
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T4	T4	T5	T6

There are 2.6 = 12 possibilities.

3)



The number of student-teachers among the 100 who were not studying any one of the three sciences is 13

The number of student-teachers among the 100 who were studying both Physics and Mathematics but not Chemistry is 31

The number of ways $=20\times19\times18\times17=116280$.

Lesson 3: Basic sum principle of counting (Mutually exclusive situations)

a) Learning objectives:

Perform operations on mutually exclusive events to determine the number of outcomes using Basic sum principle of counting

b) Teaching resources:

Manila papers, calculators, coins, dice, deck of cards

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11.

d) Learning activities

Invite students to discuss in pairs the activity 2.2.2

Walk around each pair and ask probing questions leading them to determine the total number of outcomes given that the number of outcomes for event one and the outcomes for event 2 are known;

Invite two neighbouring pairs to work together, exchange ideas and improve their work;

Visit each new formed group and identify groups with different working steps;

Invite representatives from groups with different working steps to present their work for a whole class discussion;

As a tutor, harmonize their answers and guide students discover that If Experiment 1 has **m** possible outcomes and if experiment 2 has **n** possible outcomes, then an experiment which might be experiment 1 or experiment 2, called **experiment 1 or 2**, has $m+n$ possible outcomes;

After this step, guide students to do the application activity 2.2.2 and evaluate whether lesson objectives are achieved.

Answers for activity 2.2.2

1. Answers will vary from group to another. But help them to conclude that one has chances of picking either the soup or the juice but not all together. One is allowed two chances.
2. We need the numbers that end in 3: the last digit can be chosen in one way, as it must be a 3, the first digit can then be chosen in 3 ways, the second in 2 ways and the third in 1 way. Thus, there are $1\times3\times2\times1=6$ numbers that end in a 3.

Similarly, there are $1\times3\times2\times1=6$ numbers that end in a 4.

The number that ends in a 3 cannot also end in a 4, so these are mutually exclusive situations.

Thus, there are $6+6=12$ numbers end either in a 3 or in a 4.

Alternatively, this can be solved as follows:

The last digit can be chosen in 2 ways (3 or 4); the first digit can be chosen in 3 ways, the second in 2 ways and the third in 1 way, i.e. , $2\times3\times2\times1=12$ numbers end either in a 3 or in a 4.

Answer for application activity 2.2.2

1. There are $2\times2\,numbers=4\,numbers$
2. $13+13=26\,ways$.

Lesson 4: Arrangements of n unlike objects in a row

a) Learning objective:

Perform operations on arrangements of n unlike objects in a row to determine the number of outcomes.

b) Teaching resources:

Manila papers, cards with letters, calculators, coins, dice, a bench, etc.

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3

d) Learning activities

- Form small groups of student-teachers and give them instructions on how to work on the activity 2.2.3: give each group the letter cards to be used and ask them to make all possible arrangements and permutations of those letters (for example: letter R, E and B);
- Walk around each group and ask probing questions leading them to determine the total number of ways starting by the number of ways to chose the first letter, the second letter and the third letter;
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers highlighting that the arrangement of letters is the same as ways of sitting different people on the same bench and that a permutation is an arrangement of n objects in a specific order.
- Use different probing questions and guide them to explore examples given in the student's book and lead them to discover the formula which gives the number of different permutations of *n* different objects (unlike objects) in a row.
- Guide them to discover that this number corresponds to $n!$ (read *n* factorial) and explore the related properties.
- After this step, guide students to do the application activity 2.2.3 and evaluate whether lesson objectives achieved.

Answers for activity 2.2.3

Possible arrangements for three letters R, E and B are $\{REB, \mathrm{R}\,BE, ERB, EBR, BER, BRE\}$;

The possible arrangement for these three letters is 6. This can be found by: $3! = 3.2.1 = 6$

Answers for application activity 2.2.3

1. a) $\frac{5!}{2!} = \frac{5\times4\times3\times2\times1}{2\times1} = 60$; b)

$$\frac{10!}{6!7!} = \frac{10\times9\times8\times7!}{6\times5\times4\times3\times2\times1\times7!} = \frac{5\times2\times3\times3\times4\times2}{3\times2\times5\times4\times3\times2\times1} = 1$$

2. Four different English books, five different Biology books and ten other books are to be arranged on a bookshelf:

a) (4+5+10)!= 19!= 1.216451004×10^{17};

b) Since the 3 biology books have to be together, consider these bound together as one book, there are now $(16+1)! = 17!$ books to be arranged and these can be calculated using a calculator and find $2.134124569 \times 10^{15}$.

Lesson 5: Arrangements of indistinguishable objects (Permutations with repetitions)

a) Learning objective:

Perform operations on the arrangement of n alike objects in line to determine the number of outcomes.

b) Teaching resources:

Bench, shelves of books, manila papers, calculators, coins, dice.

c) Prerequisites/Revision/Introduction:

Students will perform well in this lesson if they learnt well the content of the previous lesson in this unit;

d) Learning activities

- Form groups of student-teachers and give them instructions on how to work on the activity 2.2.4: give each group the letter cards to be used and ask them to make all possible permutations of those letters in which some letters are the same (for example: letter of the word BOOM, the two O are not separable);
- Walk around each group and ask probing questions leading them to determine the total number of permutations considering that it is not possible to distinguish the two letters "O";
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers highlighting that the number of permutations of will reduce: the total number of permutations 4! Will be divided by the number of permutations of identical letters which is 2! and find $\frac{4!}{2!}$.
- Use different probing questions and guide students to explore examples given in the student's book and lead them to discover the formula which gives the number of different permutations of n **indistinguishable** objects with n_1 alike, n_2 alike, …, which is $\frac{n!}{n_1!n_2!\ldots}$.
- After this step, guide students to do the application activity 2.2.4 and evaluate whether lesson objectives were achieved.

Answers for activity 2.2.4

Let us take numbers from 1,2,3,and 4.

1) $\Omega=\begin{Bmatrix}1234,1243,1324,1342,1423,1432\\2134,2143,2314,2341,2413,2431\\3124,3142,3214,3241,3412,3421\\4123,4132,4213,4231,4312,4321\end{Bmatrix}$ the possible number is 4!=24;

2) Students will try to make possible arrangements but some of them will be the same.

The number of all possible arrangements when writing once the identical arrangement is $\frac{4!}{2!}$.

Answers for application activity 2.2.4

1. a) Arrangements that can be made from the letters of the word ENGLISH are 7!=5,040;

   b) Arrangements that can be made from the letters of the word MATHEMATICS are

$\frac{11!}{(2!)(2!)(2!)}$ because there are 2M, 2A and 2T which are indistinguishable.

2. Alphabet in English=26!= 4.032914611×10^{26};

3. $\Omega=\frac{9!}{4!3!2!}=1,260$.

Lesson 6: Circular arrangements

a) Learning objectives:

Determine the number of outcomes on circular arrangements

b) Teaching resources:

Manila papers, calculators, coins, dice.

c) Prerequisites/Revision/Introduction:

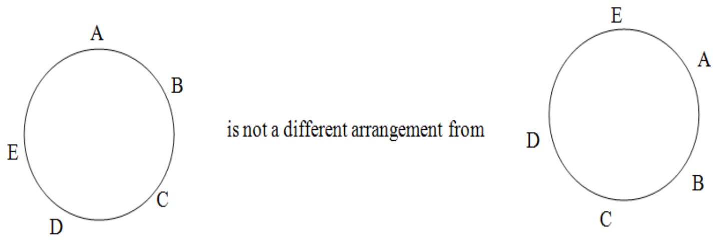
Student-teachers will perform well in this lesson if they learnt well the content of previous lesson and the content on intervals on closed line in P5 unit 7

d) Learning activities

- Form groups of student-teachers and give them instructions on how to work on the activity 2.2.: each group may have a circular table and objects to be arranged on that table;
- Walk around each group and ask probing questions leading them to determine the total number of arrangements when one item is fixed and the remaining items arranged around it;
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers and guide them to discover that the number of arrangements of *n* unlike things in a circle will therefore be $(n-1)!$. Guide students to note that where clockwise and anticlockwise arrangements are not considered to be different, this reduces to $\frac{1}{2}(n-1)!$.
- After this step, guide students to do the application activity 2.2.5 and evaluate whether lesson objectives are achieved.

Answers for activity 2.2.5

As one notebook will be fixed, for example A, must be: $(n-1)! = (5-1)! = 24$



Answers for application activity 2.2.5

1. Five men will seat on a circular table in $(5-1)!\text{ways} = 24\text{ ways}$.
2. Eleven different books will be placed on a circular table in $(11-1)!\text{ways} = 3,268,800\text{ ways}$.

Lesson 7: Distinguishable permutations (Permutations of r unlike objects selected from n distinct objects)

a) Learning objectives:

Use distinguishable permutations to find the number of outcomes

b) Teaching resources:

Manila papers, calculators, coins, dice, deck of cards

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11 and has very well covered the content of the previous lessons of this unit.

d) Learning activities

- Form groups of student-teachers and give them instructions on how to work on the activity 2.2.6: give each group the letter cards to be used and ask them to make all possible ways of selecting 3 letters from 7 given letters of the word *PRODUCT*.
- Walk around each group and ask probing questions leading them to determine the total number of ways of selecting the first letter, the second letter and the third letter;
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers highlighting that the number of ways for selecting 3 letters from 7 is the number of permutations of 3 objects taken from 7 written as ${}^{7}P_{3}$. Guide them to enhance that the first letter can be selected in 7 ways, the second can be selected in 6 ways and the third can be selected in 5 ways;
- Ask all students to guess how they can write the product $7\times6\times5$ using the factorial notation which lead them to guess $7.6.5=\frac{7.6..5.4.3.2.1}{4.3.2.1}=\frac{7!}{(7-3)!}$
- Use different probing questions and guide students to explore examples given in the student's book and lead them to discover the formula which gives the number of different permutations (ways) of *r* unlike objects selected from *n* different objects given by ${}^{n}P_{r}=\frac{n!}{(n-r)!}$ which can also

be written as $P(n,r)=\dfrac{n!}{(n-r)!}$

- After this step, guide students to do the application activity 2.2.6 and evaluate whether lesson objectives are achieved.

Answers for activity 2.2.6

Students will work in different ways; as a tutor, verify whether all possible ways are given:

Selection: PRO: permutations: PRO,POR, OPR, ORP, RPO, ROP

Selection: ROD: permutations: ROD, RDO, ORD, ODR, DRO, DOR

Selection: ODU: permutations: ..., ..., ..., ..., ..., ...

Selection: DUC: Permutations: ..., ..., ..., ..., ..., ...

Selection: UCT: Permutations: ..., ..., ..., ..., ..., ...

.... : .....: ....

There are 35 lines with 6 permutations which gives the number $7.6.5=210$ permutations.

Answers for application activity 2.2.6

1. Number of permutations with 4 letters chosen from letters of the word ENGLISH: ${}^{7}P_{4}=840$
2. Number of permutations with 2 letters chosen from letters of the word PACIFIC: 13
3. Number of permutations with 5 letters chosen from letters A, B, C, D, E, F, and G is ${}^{7}P_{5}$.
4. Number of permutations with 10 letters chosen from English alphabet is ${}^{26}P_{10}$

Lesson 8: Permutations of r objects selected from the mixture of n alike and unlike objects

a) Learning objectives:

Use permutations of r objects selected from the mixture of n alike and unlike objects to determine the number of outcomes

b) Teaching resources:

Manila papers, calculators, coins, dice, deck of cards .

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11 and has very well covered the content of the previous lessons of this unit.

d) Learning activities

- Form groups of student-teachers and give them instructions on how to work on the activity 2.2.7
- Walk around each group and ask probing questions leading them to determine the total number of arrangements when there is selection from the mixture of n alike and unlike objects. Give enough support to students to overcome challenges on this lesson.
- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers and guide them to discover that the number of arrangements is given by determining all possible mutually exclusive events from the given experiment that may occur ;then apply basic sum principle.
- After this step, guide students to do the application activity 2.2.7and evaluate whether lesson objectives are achieved.

Answers for activity 2.2.7

Student-teachers will work in different ways; as a tutor, verify whether all possible ways are given:

Selection: BOO; permutations: BOO, OBO, OOB

Selection: OOM; permutations: OOM, OMO, MOO

Selection: BOM; permutations: BOM, BMO, OBM, OMB, MBO, MOB

There are $3+3+6=12$ permutations.

Alternatively:

Possible selections:

- A selection containing two "O" and one other letter for example "OOB"
- A selection for which all three letters are distinct (B,O,M)

Permutations formed from three letters containing two O and another letter (M or B)		
O	**O**	**M**
O	**M**	**O**
M	**O**	**O**
O	**O**	**B**
O	**B**	**O**
B	**O**	**O**

Permutations formed from three distinct letters (B,M, O)		
B	**M**	**O**
B	**O**	**M**
M	**B**	**O**
M	**O**	**B**
O	**B**	**M**
O	**M**	**B**

There are 12 permutations of 3 letters selected from the letters of the word "**BOOM**" in which 6 contain two O's and one other letter and 6 in which all 3 letters are different.

Or simply

Number of permutations formed from 2 o's +one letter $= 3 \times 2 = 6$

Number of permutations formed from all three different letters (BOM) $= 3! = 6$

Total permutations $= 6 + 6 = 12$

Answers for application activity 2.2.7

1. There are 7 letters including two A's and two N's. To find the total number of different arrangements we consider the possible arrangements as three mutually exclusive situations.

   - The number of arrangements in which all 2 letters are different is ${}^{5}P_{2} = 20$;
   - The number of arrangements containing two A's is 1;
   - The number of arrangements containing two N's is 1;

Therefore, the number of permutations with 2 letters chosen from letters of the word RWANDAN is $20+1+1=22$

2. There are 8 letters including two M's. To find the total number of different arrangements we consider the possible arrangements as two mutually exclusive situations.

   - The number of arrangements in which all 3 letters are different is ${}^{7}P_{3} = 210$
   - The number of arrangements containing two M's and one other letter; the other letter can be one of six letters (E, A, N, U, E or L) and can appear in any of the three positions (before the two M'S, between the two M's, or after the two M's)i.e. $3\times 6=18$.

Thus the total number of arrangements of 3 letters chosen from the word EMMANUEL is $210+18=228$.

Lesson 9: Combinations

a) Learning objectives:

Use combinations to determine the number of outcomes

b) Teaching resources:

Manila papers, calculators, coins, dice, deck of cards.

c) Prerequisites/Revision/Introduction:

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11 and has very well covered the content of the previous lessons of this unit.

d) Learning activities

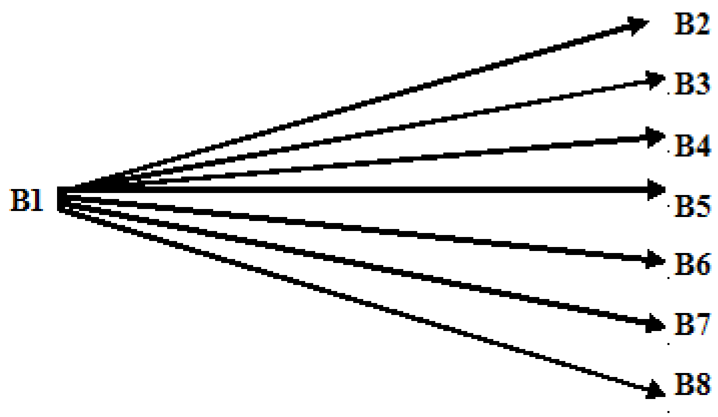
- Form groups of student-teachers and give them instructions on how to work on the activity 2.2.7;
- Walkaroundeachgroupandaskprobingquestionsleadingthemtodetermine

the total number of groups each containing 2 mathematics books from 8 mathematics books;

- Invite groups with different working steps to present their findings to the whole class for discussion;
- As a tutor, harmonize their answers highlighting that in this case, the order in which books are placed is not important (the group of B1B2 is the same as the group B2B1) which is contrary to the permutation of *r* unlike objects selected from *n* different objects where the order in which those objects are placed is important.
- Lead students to see that in this case, we must divide by the 2! (Or generally by the arrangement r!) as the order is not important; we get $\frac{7.8}{2}=\frac{8!}{(8-2)!2!}$.
- Use different probing questions and guide students to explore examples given in the student's book and lead them to discover the formula which gives the number of different groups of *r* items that could be formed from a set of *n* distinct objects with the order of selections being ignored is ${}^{n}C_{r}=\frac{n!}{(n-r)!r!}$
- After this step, guide students to do the application activity 2.2.7 and evaluate whether lesson objectives were achieved.

Answers for activity 2.2.8

The book B1 can be participating in 7 different groups as follows:



Idem, every book Bi can participate in 7 different groups. This means that we have $8.7=56$ groups. However, as for example the group B2B3 and the group B3B2 make a same group, we have to divide by 2.

Which gives $\frac{7.8}{2}$ groups.

By the use of factorial notation we have: $\frac{7.8}{2}=\frac{8!}{(8-2)!2!}=28$

Application activity 2.2.8

1. Four men can be selected from 10 men, i.e ${}^{10}C_4=\frac{10!}{(10-4)!4!}$ ways

Two women can be selected from 12women, i.e ${}^{12}C_2=\frac{12!}{(12-2)!2!}$ ways

By the basic product principle of counting, there are $\left({}^{10}C_4\right)\left({}^{12}C_2\right)$ ways of selecting the committee.

2. In the same ways, groups containing 4 Mathematics books and 5 Physics books is formed from 9 Mathematics books and 10 Physics books in ${}^{9}C_4\times{}^{10}C_5$ ways. $=126\times 252=31752$

Lesson 10: Probability of an event

a) Learning objectives

Determine the classical or empirical probability of events.

b) Teaching resources

Manila papers, calculators, coins, dice, deck of cards .

c) Prerequisites/Revision/Introduction

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11 and has very well covered the content of the previous lessons of this unit.

d) Learning activities

- Let students work in groups and do the activity 2.3.1;
- Go around each group and ask probing questions to guide students to work towards the correct answer;
- Request student-teachers to present their findings in a whole class discussion;
- As a tutor, harmonize answers for students and highlight how to determine the probability of an event using the classical probability

$$P(A)=\frac{Number\ of\ outcomes\ in\ \mathrm{E}}{Total\ number\ of\ outcomes\ in\ the\ sample\ space}=\frac{n(\mathrm{E})}{n(\Omega)}$$

- Use different probing questions and guide students to explore examples given in the student's book and lead them to establish and use properties of probability, determine probability of different events: certain event, impossible event, probability of complementary event, mutually exclusive or incompatible or events.
- After this step, guide students to do the application activity 2.3.1 and evaluate whether lesson objectives are achieved.

Answers for activity 2.3.1

a) There are 26 black cards in an ordinary deck of 52 cards.

b) $P(\mathrm{A})=\frac{n}{number\ of\ all cards}=\frac{26}{52}=0.5$

c) $P(A)=\frac{Number\ of\ outcomes\ in\ \mathrm{E}}{Total\ number\ of\ outcomes\ in\ the\ sample\ space}=\frac{n(\mathrm{E})}{n(\Omega)}$

Answers for application activity 2.3.1

a) $P(\mathrm{A})=\frac{56}{127}$

b) Less than 6 days means 3,4 and 5 days;
$P(\text{less than 6 days})=\frac{15}{127}+\frac{32}{127}+\frac{56}{127}=\frac{103}{127}$

c) At most 4 days means 3 or 4 days; $P(\mathrm{A})=\frac{47}{127}$

d) At least 5 days means 5, 6, or 7 days; $P(\mathrm{A})=\frac{80}{127}$.

Lesson 11: Probability of mutually exclusive or incompatible events

a) Learning objectives

To determine the probability for mutually exclusive events.

b) Teaching resources

Manila papers, calculators, coins, dice, deck of cards .

c) Prerequisites/Revision/Introduction

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11 and has very well covered the content of the previous lessons of this unit.

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 2.3.2 found in their Mathematics Student books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a tutor, harmonize the findings from presentation on the probability for mutually or non mutually exclusive events;
- Use different probing questions and guide them to explore the content and examples given in the student's book and lead them to discover how to determine the probability for mutually exclusive events
- After this step, guide students to do the application activity 2.3.2 and evaluate whether lesson objectives are achieved.

Answers for activity 2.3.2

a) $\textit{the probability } P(A)=\frac{13}{52}=\frac{1}{4}$

b) $\textit{the probability } P(B)=\frac{13}{52}=\frac{1}{4}$

c) $\textit{the probability P to draw a c}\text{lub}\textit{ or a diamond } =\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

d) The probability P is given by the sum of the P (A) and the P (B).

Application activity 2.3.2

a) The P(Prime number) $=\frac{3}{6}=\frac{1}{2}$

b) The P(Odd number) $=\frac{3}{6}=\frac{1}{2}$

c) Even or prime $=\frac{1}{2}+\frac{1}{2}=1$

d) P(less than 4 or multiple of 5) $=\frac{1}{3}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$

e) P(greater than 2 or less than 4) $=\frac{4}{6}+\frac{3}{6}-\frac{1}{6}=\frac{6}{6}=1$

Lesson 12: Probability of independent events and multiplication rule

a) Learning objectives

To define Independent events and determine the probability for independent events

b) Teaching resources

Manila papers, calculators, coins, dice, deck of cards .

c) Prerequisites/Revision/Introduction

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11 and has very well covered the content of the previous lessons of this unit.

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 2.3.3 found in their Mathematics Student books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a tutor, harmonize the findings from presentation on Independent events and determine the probability for independent events ;
- Use different probing questions and guide them to explore the content and examples given in the student's book and lead them to discover how to define Independent events and determine the probability for independent events.
- After this step, guide students to do the application activity 2.3.3 and evaluate whether lesson objectives are achieved.

Answers for activity 2.3.3

The occurrence of event *B* does not affected by occurrence of event *A* because after the first trial the pen is replaced in the box. It means that the sample space does not change.

Answers for application activity 2.3.3

1. $P(A \cup B) = \frac{1}{5} + \frac{1}{3} = \frac{8}{15}$

2. $P(A \cup B) = \frac{1}{3} + x = \frac{7}{10} \Rightarrow x = \frac{11}{30}$

3. a) $\frac{3}{8}$; b) $\frac{5}{8}$; c) $\frac{1}{32}$

Lesson 13: Dependent events

a) Learning objectives

To define Dependent events and determine the probability for dependent events.

b) Teaching resources

Manila papers, calculators, coins, dice, deck of cards.

c) Prerequisites/Revision/Introduction

Student-teachers will perform well in this unit if they make a short revision on the sets S1 unit 1, on the Tree and Venn diagram in S2 unit 11, on problem sets in S3 and on the elementary probability learnt in S1 unit 9 and S2 unit 11 and has very well covered the content of the previous lessons of this unit.

d) Learning activities:

- Invite student-teachers to work in groups and do the activity 2.3.4 found in their Mathematics Student books;
- Move around in the class for facilitating students where necessary and give more clarification on eventual challenges they may face during their work; Verify and identify groups with different working steps;
- Invite one member from each group with different working steps to present their work where they must explain the working steps;
- As a tutor, harmonize the findings from presentation on dependent events;
- Use different probing questions and guide them to explore the content and examples given in the student's book and lead them to discover how to define

Dependent events and determine the probability for dependent events .

- After this step, guide students to do the application activity 2.3.4 and evaluate whether lesson objectives are achieved.

Answers for activity 2.3.4

a) Sample space for the first drawing is $\Omega = 52$, But for the second drawing the sample space is $\Omega = 51$.

b) The outcomes of the first draw has affected the outcome of the second i.e. the outcome of the second dependent on the first drawing.

Answers for application activity 2.3.4

1. Let *A* be the event: "the number is a 4", then $A = \{4\}$

B be the event: "the number is greater than 2", then $B = \{3,4,5,6\}$ and

$$P(B) = \frac{4}{6} = \frac{2}{3}$$

But $A \cap B = \{4\}$ and $P(A \cap B) = \frac{1}{6}$

Therefore,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{\frac{1}{6}}{\frac{2}{3}}$$

$$P(A|B) = \frac{1}{6} \times \frac{3}{2}$$

$$= \frac{1}{4}$$

Lesson 14: Examples of Events in real life and determination of related probability

a) Learning objectives:

Use real life tasks (games, number of trials,) to determine the probability of well described events applying rules and formulas.

b) Teaching resources:

Manila papers, calculators, coins, dice, etc.

c) Prerequisites/Revision/Introduction:

Students will perform well in this lessons if they try to relate the content for all previous lessons for this units to real life problems involving events of chance.

d) Learning activities

- Invite students to work in small groups, discuss on the activity 2.4 and answer to related questions;
- Invite group representatives to present the findings in a whole class discussions;
- Tutor harmonizes answers for students on activity 2.4 and guide students to brainstorm their worries about the betting without a good prediction of probability for winning.
- Guide students to discuss other application of probability in real life and take decisions on eventual risks in betting and other probability related games.
- Use different probing questions and guide students to explore examples given in the student's book and lead them to realize that probability is applicable in the real life.
- After this step, guide students to do the application activity 2.4 and evaluate whether lesson objectives were achieved.

Answers for activity 7.5

1. (a) Let A stands for APR and R stands for RAYON SPORTS; $\Omega = \{RRR, RRA, RAR, RAA, ARR, ARA, AAR, AAA\}$.

Matayo said that APR will gain the first Match only and Rayon Sport will gain the second and the third, it means that $= E = \{ARR\}$ and $P(E) = \frac{1}{8}$

Manasseh said that APR will gain at least two matches, it means that:

$$F = \{RAA, ARA, AAR, AAA\} \text{ and } \quad P(F) = \frac{4}{8} = \frac{1}{2}$$

From these results, we see that Manasseh has more chances of winning that money than Matayo.

b) Normally betting is a game of chance, it is not good to bet much money without a good and clear prediction of the probability for winning. When you bet without such clear prediction, you are wasting your money. We can advise the young people not to spend much money in such games which do not have clear rules which can help the player to predict the probability of winning.

2. Students may come up with different applications of probability in real life, analyse them and organize a session for feedback in which they can discuss their strengths and weaknesses.

Answers for application activity 7.5

1. Let E: a student own a car, $P(\mathrm{E}) = 0.65$, F: a student owns a computer; $P(\mathrm{F}) = 0.82$

We have $P(\mathrm{E} \cap \mathrm{F}) = 0.55$

Question: what is the probability that a given student owns neither a car nor a computer?

i.e $1 - P(\mathrm{E} \cup \mathrm{F}) = ?$

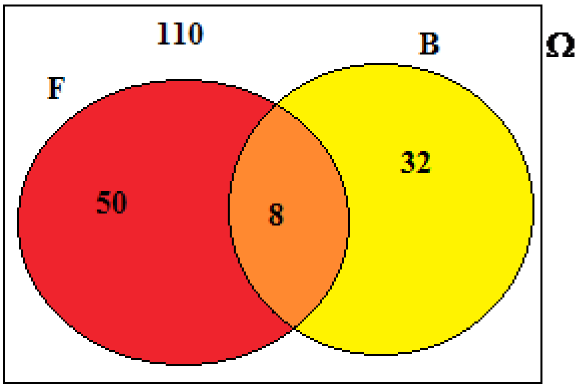
We have:

$$\begin{aligned} P(\mathrm{E} \cup \mathrm{F}) &= P(E) + P(E) - P(E \cup F) \\ &= 0.65 + 0.82 - 0.55 \\ &= 0.92 \end{aligned}$$

Therefore, $1 - P(\mathrm{E} \cup \mathrm{F}) = 1 - 0.92 = 0.08$

2. Using a Venn diagram, one can represent the number of students:

Let Ω be the sample space formed by all students, F the set of students who play Football and B the set of students who play Basket Ball. The number of students can be given in the following sets.



$n(\Omega) = 200$, $n(\mathrm{F}) = 58$, $n(\mathrm{B}) = 40$, and $n(\mathrm{F} \cap \mathrm{B}) = 8$.

The number of students who plays Foot Ball or basket Ball is $n(\mathrm{F} \cup \mathrm{B}) = 90$

The number of student who play neither sport is

$$n(\mathrm{F} \cup \mathrm{B})' = n(\Omega) - n(F \cup B)$$
$$= 200 - 90 = 110$$

2.6 Summary of the unit

Sample space

The totality of all possible outcome (or sample points) of a random experiment constitutes the sample space which is denoted by Ω .

Complementary events

If *E* is an event, then E' is the event which occurs when *E* does not occur. Events *E* and E' are said to be **complementary events**

Mutually exclusive Events

When $A \cap B = \varnothing$, the two events A and B are said to be **mutually exclusive**. This means that they cannot occur at the same time, they do not have outcomes in common.

Counting techniques

- Use of Venn diagram,
- Use of tree diagrams,
- Use of a table,
- The number of different permutations of *n* different objects (unlike objects) in a row is

$$n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 2 \times 1$$

- *The number of different permutations of n indistinguishable objects with* n_1 *alike,* n_2 *alike, …, is* $\dfrac{n!}{n_1!n_2!\ldots}$.
- The number of different permutations (ways) of r unlike objects selected from n different objects is $^nP_r = \dfrac{n!}{(n-r)!}$ or we can use the denotation $P_r^n = \dfrac{n!}{(n-r)!}$

or $P(n,r)=\frac{n!}{(n-r)!}$

Circular arrangements

The number of arrangements of n unlike things in a circle will therefore be $(n-1)!$. In those cases where clockwise and anticlockwise arrangements are not considered to be different, this reduces to $\frac{1}{2}(n-1)!$.

Mutually exclusive situations Mutually exclusive situations

If Experiment 1 has **m** *possible outcomes,*

and if experiment 2 has **n** *possible outcomes,*

then an experiment which might be experiment 1 or experiment 2,

called **experiment 1 or 2** *has* **(m+2)** *possible outcomes*

Combination

The number of different groups of r items that could be formed from a set of n distinct objects with the order of selections being ignored is

$$^{n}C_{r}=\frac{n!}{(n-r)!r!}.$$

Probability of an event

- The probability of an event $A\subset\Omega$, is a real number obtained by applying to *A* the function *P* defined by

$$P(A)=\frac{Number\ of\ outcome\mathrm{s\ in}\ A}{Total\ \mathrm{nu}\,mber\ of\ outcome\mathrm{s\ in}\ the\ sample\ space}=\frac{n(A)}{n(\Omega)}$$

- When *E* and E' are complementary events, $P(E)=1-P(E')$.
- When two events A and B are not mutually exclusive, $A\cap B=\phi$ the probability that A or B occurs is given by:

$$P(A\cup B)=\mathrm{P(A)+P(B)-P}(A\cap B)$$

Note

We have to think big before taking decision regarding our engagement in the games of chance. Such games are for example: betting on card games, slot machine (ikiryabarezi), lotteries, and weather forecasting. In such games, predictions are based on probability and hypotheses are tested by using probability.

Independent events

Two events *A* and *B* are **independent events** if the fact that *A* occurs does not affect the probability of *B* occurring.

When two events are independent, the probability of both occurring is

$$\text{P(FA and B)} = P(A).P(B)$$

Example:

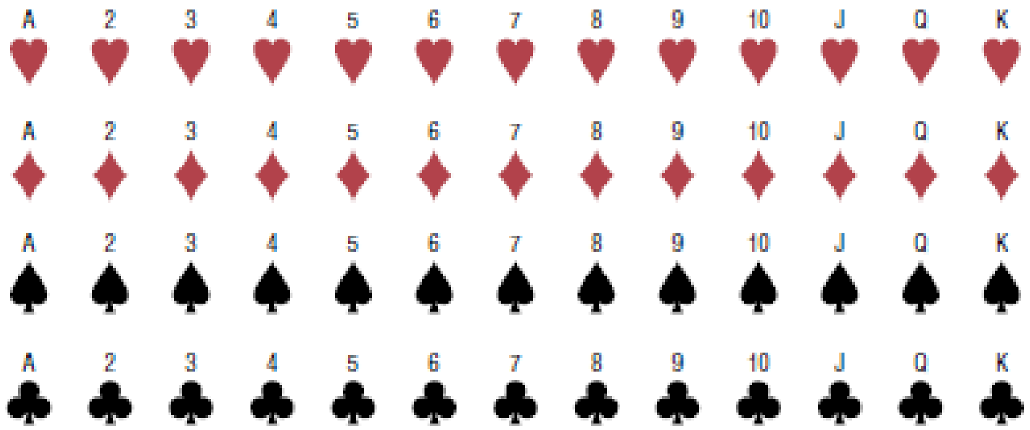
A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Solution

P(Head and 4)= P(head).P(4)= $\frac{1}{2}.\frac{1}{4}=\frac{1}{8}$

2.7 Additional Information for teachers

2.7.1 Components of an ordinary deck of cards:



2.7.2 Law of Large Numbers

When a coin is tossed one time, it is common knowledge that the probability of getting a head is $\frac{1}{2}$. But what happens when the coin is tossed 50 times? Will it come up heads 25 times? Not all the time. You should expect about 25 heads if the coin is fair. But due to chance variation, 25 heads will not occur most of the time.

If the empirical probability of getting a head is computed by using a small number of trials, it is usually not exactly $\frac{1}{2}$. However, as the number of trials increases, the empirical probability of getting a head will approach the theoretical probability of $\frac{1}{2}$, if in fact the coin is fair (i.e., balanced). This phenomenon is an example of the law of large numbers.

2.8 End unit assessment

This question is related to a Lottery that can be played at school to show that it is not good to be blindly engaged in some games of chance as the participant can lose his/her money.

An urn contains 20 lottery tickets numbered from 1 to 20.

To buy a ticket, each one is selected at random and replaced before the next selection. The organizer of the lottery decided to pay 1000Frw to the one who will select a number divisible by 4 and 3 at the same time. He will pay also 500Frw to the one who will select a number which is divisible by 5 and 2 at the same time.

Solution

1.

a) The game is not fair because the organizer decided him/her self the number of money to be collected;

b) The organizer will receive 200Frw x 20 = 4000Frw

c) Let the following events: *A: selecting a number divisible by 4;*

B: selecting the number divisible by 3;

C: selecting the number divisible by 5, and

D: Selecting the number divisible by 2.

E= selecting a number divisible by 3 and 4 $=\{12\}$, P(E)= $\frac{1}{20}$. This means that only one participant will win and get 1000F.

d) F= Selecting a number divisible by 2 and 5=$\{10;20\}$; P(F) = $\frac{2}{20}$. This means that only two participants will win and each one will get 500Frw .

e) The organizer will pay: $1000Frw + 2(500Frw) = 2000Frw$.

Therefore, the organizer will make money.

This money equals to $4000Frw - 2000Frw = 2000Frw$

2. The parents of your friend Anne Marie gave her 200Frw for buying two pens, however, she wants to participate in the lottery to get more money before buying pens. What can you advise her?

Answers

Answers from students will vary, however, guide them to conclude that this is a game of chance where there is a high probability of loosing. It is clear that the organizer planned to get money without investing anything. It is clear that only 3 participants will win. Therefore, there is a high probability for Anne Marie to lose the money which and miss a pen.

2.9 Additional activities

2.9.1 Remedial activity

1. A card is drawn at random from an ordinary pack of 52 playing cards. Find the probability that the card drawn is a club or a diamond.

Solution

There are 13 clubs, then $P(\text{club}) = \frac{13}{52}$

There are 13 diamonds, then $P(\text{diamond}) = \frac{13}{52}$

Since a card cannot be both a club and a diamond, $P(\text{club} \cap \text{diamond}) = 0$

Therefore, P(a club or a diamond) = P(club) + P(diamond)

$\frac{13}{52} + \frac{13}{52} = \quad \frac{13}{52} + \frac{13}{52} = \quad \frac{26}{52} = \quad \frac{1}{2}$

2.9.2 Consolidation activity

1. In a group of 20 adults, 4 out of the 7 women and 2 out of the 13 men wear glasses. What is the probability that a person chosen at random from the group is a woman or someone who wears glasses?

Solution

Let *A* be the event: "the person chosen is a woman".

B be the event: "the person chosen wears glasses".

Now, there are 7 women, then $P(A) = \frac{7}{20}$

There are 6 persons who wear glasses, then $P(B)=\frac{6}{20}$

There are 4 women who wear glasses, then $P(A\cap B)=\frac{4}{20}$

The probability that a person chosen at random from the group is a woman or someone who wears glasses is given by $P(A\ or\ B)$ which is

$$
\begin{aligned}
P(A\cup B) &= P(A)+P(B)-P(A\cap B)\\
&= \frac{7}{20}+\frac{6}{20}-\frac{4}{20}\\
&= \frac{9}{20}
\end{aligned}
$$

On the other hand:

There are 7 women and 6 persons who wear glasses but also 4 women who wear glasses. Then

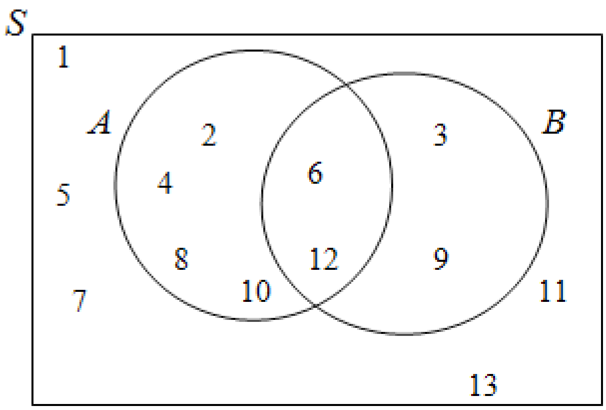
$A\cup B=9$ and $P(A\cup B)=\frac{9}{20}$.

2.9.3 Extended activity

1. An integer is chosen at random from the set $S=\{x: x\in\mathbb{Z}^{+}, x<14\}$. Let A be the event of choosing a multiple of 2 and let B be the event of choosing a multiple of 3.

Find $P(A\cup B)$, $P(A\cap B)$ and $P(A-B)$.

Solution



From the diagram, $\#S = 13$

$$A \cup B = \{2,3,4,6,8,9,10,12\} \Rightarrow \#(A \cup B) = 8, \text{ thus } P(A \cup B) = \frac{8}{13}$$

$$A \cap B = \{6,12\} \Rightarrow \#(A \cap B) = 2, \text{ thus } P(A \cap B) = \frac{2}{13}$$

$$A - B = \{2,4,8,10\} \Rightarrow \#(A - B) = 4, \text{ thus } P(A - B) = \frac{4}{13}$$

2. Suppose, for example, that a researcher in RAB asked 50 staff members how they go home.

The results can be categorized in a frequency distribution as shown in the table below.

Method	**Frequency**
drive	20
Fly	6
Bus	24

Determine:

a) The probability of selecting a person who goes home by driving;
b) Probability of selecting a person who goes home in an air plane;
c) The probability of selecting a person who goes home in a bus.
d) The sum of the probability.

3. In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a) A person has type O blood.
b) A person has type A or type B blood.
c) A person has neither type A nor type O blood.
d) A person does not have type AB blood.

Solution

Type	**Frequency**
A	22
B	5
AB	2
O	21
Total	50

They are mutually exclusive.

a) $P(O)=\frac{f}{n}=\frac{21}{50}$

b) $P(\text{A or B})=\frac{22}{50}+\frac{5}{50}=\frac{27}{50}$

c) Neither A nor O means that a person has either type B or type AB blood.)

$P(\text{neither A nor O})=\frac{5}{50}+\frac{2}{50}=\frac{7}{50}$.

d) Find the probability of not AB by subtracting the probability of type AB from 1

$P(\text{not AB})=1-P(AB)=1-\frac{2}{50}=\frac{48}{50}=\frac{24}{25}$

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