SUBSIDIARY MATHEMATICS BOOK

FOR LKK, HLP & HGL

SENIOR 4

STUDENT'S BOOK

Experimental version

Kigali, 2022

FOREWORD

Dear Student,

Rwanda Basic Education Board (REB) is honoured to present Senior 4 Mathematics book for the students in the Literature in English, French, Kinyarwanda and Kiswahili (LFK); History, Geography and Literature in English History, Geography and Literature in English (HGL) and History, Literature in English and Psychology (HLP) combinations which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of the Mathematics. The Rwandan educational philosophy is to ensure that you achieve full potential at every level of education which will prepare you to be well integrated in society and exploit employment opportunities.

The government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate your learning process. Many factors influence what you learn, how well you learn and the competences you acquire. Those factors include the relevance of the specific content, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials available. In this book, we paid special attention to the activities that facilitate the learning process in which you can develop your ideas and make new discoveries during concrete activities carried out individually or in groups.

In competence-based curriculum, learning is considered as a process of active building and developing knowledge and meanings by the learner where concepts are mainly introduced by an activity, situation or scenario that helps the learner to construct knowledge, develop skills and acquire positive attitudes and values.

For efficiency use of this textbook, your role is to:

- Work on given activities which lead to the development of skills;
- Share relevant information with other learners through presentations, discussions, group work and other active learning techniques such as role play, case studies, investigation and research in the library, on internet or outside;
- Participate and take responsibility for your own learning;
- Draw conclusions based on the findings from the learning activities.

To facilitate you in doing activities, the content of this book is self-explanatory so that you can easily use it yourself, acquire and assess your competences. The book is made of units as presented in the syllabus. Each unit has the following structure: the unit title and key unit competence are given and they are followed by the introductory activity before the development of mathematical concepts that are connected to real world problems more especially to production, finance and economics.

The development of each concept has the following points:

- Learning activity which is a well set and simple activity to be done by students in order to generate the concept to be learnt;
- Main elements of the content to be emphasized;
- Worked examples; and

- Application activities to be done by the user to consolidate competences or to assess the achievement of objectives.

Even though the book has some worked examples, you will succeed on the application activities depending on your ways of reading, questioning, thinking and handling calculations problems not by searching for similar-looking worked out examples.

Furthermore, to succeed in Mathematics, you are asked to keep trying; sometimes you will find concepts that need to be worked at before you completely understand. The only way to really grasp such a concept is to think about it and work related problems found in other reference books.

I wish to sincerely express my appreciation to the people who contributed towards the development of this book, particularly, REB staff and secondary school teachers for their technical support. A word of gratitude goes to Secondary Schools Head Teachers who availed their staff for various activities.

Any comment or contribution for the improvement of this textbook for the next edition is welcome.

Dr. MBARUSHIMANA Nelson
Director General, REB.

ACKNOWLEDGEMENT

I wish to express my appreciation to the people who played a major role in the development of this teacher`s guide for Mathematics in in the Literature in English, French, Kinyarwanda and Kiswahili (LFK); History, Geography and Literature in English History, Geography and Literature in English (HGL) and History, Literature in English and Psychology (HLP) combinations. It would not have been successful without active participation of different education stakeholders.

I owe gratitude to different schools in Rwanda that allowed their staff to work with REB in the in-house textbooks production initiative.

I wish to extend my sincere gratitude to teachers whose efforts during writing exercise of this teacher`s guide was very much valuable.

Finally, my word of gratitude goes to the Rwanda Basic Education Board staffs who were involved in the whole process of in-house textbook writing.

MURUNGI Joan

Head of Curriculum, Teaching and Learning Resources Department/REB

Contents

UNIT 1: ARITHMETICS

Key Unit competence: Use arithmetic operations to solve simple real life problems.

1.0 Introductory activity 1

The simple interest earned on an investment is $I = prt$ where I is the interest earned, p is the principal, r is the interest rate and t is the time in years. Assume that 50,000Frw is invested at annual interest rate of 8% and that the interest is added to the principal at the end of each year.

a) Discuss the amount of interest that will be earned each year for 5 years.

b) How can you find the total amount of money earned at the end of these 5 years? Classify and explain all Mathematics operations that can be used to find that money.

1.1 Fractions and related problems

Activity 1.1

1. Sam had 120 teddy bears in his toy store. He sold $\frac{2}{3}$ of them at 12 Rwandan francs each. How much did he receive?
2. Simplify the following fractions and explain the method used to simplify.

   a. $\frac{8x^2y^3}{2x^3y}$ b. $\frac{2x^2+5x^3}{2x^2+4x^3}$
3. Considering that the denominator is different from zero, explain how to work out $\frac{1}{x+1}-\frac{1}{2x+2}$
4. Do you some times use fractions in your life? Explain your answer.

CONTENT SUMMARY

Consider the expressions $\frac{x}{4}+\frac{3y}{x}, \frac{5}{x+4}$ in each of these the numerator or the denominator or both contain a variable or variables. These are examples of algebraic fractions. Since the letter used in these fractions stand for real numbers, we deal with algebraic fractions in the same way as we do with fractions in arithmetic. Fractions can be simplified and operations on fractions such as addition, subtraction, multiplication and division applied in simple arithmetic for solving related problems.

Adding fractions:

To add fractions there is a simple rule $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$ where $(b \neq 0)$

Example:

$$\frac{x}{2}+\frac{y}{5}=\frac{5x+2y}{(2)(5)}=\frac{5x+2y}{10}$$

Subtracting fractions

Subtracting fractions is very similar to addition of fractions, except that the sign change

Example: Considering that the denominator is different from zero,

$$\frac{x+2}{x}-\frac{x}{x-2}=\frac{(x+2)(x-2)-(x)(x)}{x(x-2)}=\frac{-4}{x^2-2x} \text{ with } x \neq 0 \textit{ and } \quad x \neq 2$$

Multiplying fractions

Multiplying fractions is the easiest one of all, just multiply the numerators together, and the denominators together

$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$, where $(b \neq 0, d \neq 0)$

Example: Considering that the denominator is different from zero,

$$\frac{3x}{x-2}\times\frac{x}{3}=\frac{(3x)(x)}{3(x-2)}=\frac{3x^2}{3(x-2)}=\frac{x^2}{x-2} \qquad (x\neq 2)$$

Dividing fractions

To divide fractions, first flip the fraction we want to divide by, then use the same method as for multiplication: $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$ where $(b\neq 0, c\neq 0, d\neq 0)$.

Example: Considering that the denominator is different from zero, we have:

$$\frac{3y^2}{x+1}\div\frac{y}{2}=\frac{3y^2}{x+1}\times\frac{2}{y}=\frac{(3y^2)(2)}{(x+1)(y)}=\frac{6y^2}{(x+1)(y)}=\frac{6y}{x+1} \qquad (x\neq -1)$$

As fraction is a symbol indicating the division of integers. For example: $\frac{15}{9},\frac{3}{8}$ are fractions and are called Common Fraction. The dividend (upper number) is called the numerator $N(x)$ and the divisor (lower number) is called the denominator $D(x)$ from the operations of fractions, different fractions can be found by taking Low Common Multiple (L.C.M.) and then add all the fractions.

For **example:** $\frac{1}{x-1}+\frac{2}{x+2}=\frac{3x}{(x-1)(x+2)}$ (for $x\neq 1$ and $x\neq -2$);

We split up a single fraction into a number of fractions whose denominators are the factors of denominator of that fraction. These fractions are called **Partial fractions t**o express a single rational fraction into the sum of two or more single rational fractions is called **Partial fraction resolution**.

Example: Considering that the denominator is different from zero,

$$\frac{2x+x^2-1}{x(x^2-1)}=\frac{1}{x}+\frac{1}{x-1}-\frac{1}{x+1}$$

$\frac{2x+x^2-1}{x(x^2-1)}$ is the resultant fraction and $\frac{1}{x}$, $\frac{1}{x-1}$ and $\frac{1}{x+1}$ are its partial fractions.

Rational fraction

We know that $\frac{p}{q}$, $q \neq 0$ is called a rational number. Similarly the quotient of two polynomials $\frac{N(x)}{D(x)}, D(\text{x}) \neq 0$ with no common factors. There are two types of rational fractions such **proper or improper fractions.**

Proper fraction: A rational fraction $\frac{N(x)}{D(x)}$ is called a proper fraction if the **degree of numerator is less than the degree of Denominator** $D(x)$**.**

Example: Considering that the denominator is different from zero,

$\frac{6x+27}{3x^3-9x}$ is a proper fraction.

Improper Fraction:

A rational fraction $\frac{N(x)}{D(x)}$ is called improper fraction if the **degree of numerator is greater than or equal to the degree of Denominator** $D(x)$**.**

Example: $\frac{6x^3-5x^2-3x-10}{x^2+1}$

An algebraic fraction exists only if the denominator is not equal to zero. The values of the variable that make the denominator zero is called a restriction on the variable(s). An algebraic fraction can have more than one restriction.

Examples:

1) Identify the restriction on the variable in the fraction $\frac{3xy}{(x+3)(x-2)}$

Solution

In the fraction $\frac{3xy}{(x+3)(x-2)}$, $(x+3)$ $(x-2)$ is the denominator.

As the denominator must be different from zero, we have $x+3 \neq 0$ or $x-2 \neq 0$.

Therefore, $x \neq -3$ or $x \neq 2$.

If $x = -3$, $(x+3)(x-2) = 0$ and if $x = 2$, $(x+3)(x-2) = 0$

In the fraction $\frac{3xy}{(x+3)(x-2)}$, the restrictions are $x \neq -3$ and $x \neq 2$.

2) Simplify $\frac{3x^2y}{4a^2} \div \frac{9xy}{5a}$

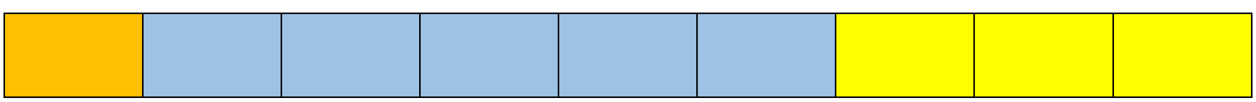
Solution: $\frac{3x^2y}{4a^2} \times \frac{5a}{9xy} = \frac{(3x^2y)(5a)}{(4a^2)(9xy)} = \frac{(x)(5)}{(4a)(3)} = \frac{5x}{12a}$ where $a \neq 0$, $x \neq 0$ and $y \neq 0$.

Problems related to fractions

Examples

1) One ninth of the shirts sold at peter's shop are stripped. $5/8$ of the remainder are printed. The rest of the shirts are plain colour shirts. If peter's shop has 81 plain colour shirts, how many more printed shirts than plain colour shirts does the shop have?

Solution



Striped printed plain

3 units equals to 81

1 unit $= 81 \div 3 = 27$

Printed shirts have 2 parts more than plain shirts.

2 units $= 27 \times 2 = 54$

Peter's shop has 54 more printed colour shirts than plain shirts.

2) Oscar sold 2 glasses of milk for every 5 sodas he sold. If he sold 10 glasses of milk, how many sodas did he sell?

Solution

Let x be the number of Sodas that Oscar will sell.

Set up a proportion of milk with soda as $\frac{milk}{soda}$, $\frac{2}{5}=\frac{10}{x} \Rightarrow 2x=50$, $x=\frac{50}{2}=25$

He will sell 25 sodas.

Application Activity 1.1

1. A proper fraction is such that its numerator and denominator have a difference of 2. If one is added to the denominator and three subtracted from the numerator, the fraction becomes $\frac{2}{3}$. Find the fraction and explain your colleague how to do it.

2. What is a partial fraction? Express $\frac{x^2+1}{x^3+4x^2+3x}$ in partial fractions.

1.2 Decimals and related problems

Activity 1.2

Refer to the meaning of decimals and fractions learnt in previous years and

1) Calculate 50:100 and write it in the form of

a) a fraction b) a decimal number.

2) Express $\frac{1}{3}$ *and* $\frac{22}{7}$ in the form of decimal numbers . Explain the relationship between the set of fractions and the set of decimal numbers.

3) Given that the set D is a set of the limited decimal numbers, discuss the following:

(a) D is a subset of $\mathbb{Q}$; (b) $D \subset \mathbb{Q}$, (c) $\mathbb{Z} \subset D$, (d) $\mathbb{N} \subset \mathbb{Z} \subset D \subset \mathbb{Q} \subset \mathbb{R}$.

4) Do you some times use decimal numbers in your life? Explain your answer.

CONTENT SUMMARY

Decimals are just another way of expressing fractions

$0.1=\frac{1}{10}$, $0.01=\frac{1}{100}$, $0.001=\frac{1}{1000}$.

Thus 0.234 is equivalent to 234/1,000. Most of the time you will be able to perform operations involving decimals by applying what was learnt in previous years or by using a calculator.

In mathematics a decimal format is often required for a value that is usually specified as a fraction in everyday usage. For example, $\frac{62}{100}=0.62$.

Because some fractions cannot be expressed exactly in decimals, one may need to 'round off' an answer for convenience. In many of the economic problems (of various books) there is not much point in taking answers beyond two decimal places. Where this is done then we denote '(to 2 dp)' is normally put after the answer.

For **example**: 1/7 as a decimal is 0.14 (to 2 dp).

Example:

1) $1.345+0.00041=1.34541$
2) $2.463\times 38=93.954$
3) $360.54\div 0.04=9,013.5$

Application activity 1.2

Evaluate the following: 1) $1.345+0.00041+0.20023=$
2) $93.954\div 2.4=$

1.3 Percentages and related problems

Activity 1.3

Refer to the meaning of decimals and percentage learnt in previous years and

1) Calculate 60:100 and write it in the form of a) a fraction, b) a percentage and c) a decimal number.

2) Express $\frac{1}{3}$ *and* $\frac{22}{7}$ in the form of decimal numbers . Is it possible to express this number in the form of percentage? Is the pecentage obtained an exact number?

3) 24 students in a class took English test. If 18 students passed the test, what percentage of those who did not pass?

4) Are percentages used by bank managers? Explain your answer.

CONTENT SUMMARY

Percentage is defined as the proportion, rate or ratio expressed with a denominator of 100. For **example :** $\frac{3}{100}, \frac{25}{100}$ **etc**

Fraction can be expressed in the form of percentage as $\frac{1}{4} \times 100 = 25\%$.

Decimal format is often required for a value that is usually specified as a percentage in everyday usage. For example, interest rates are usually specified as percentages. A percentage format is really just another way of specifying a decimal fraction,

$62\% = \frac{62}{100} = 0.62$ and so, percentages can easily be converted into decimal fractions by dividing by 100. Because some fractions cannot be expressed exactly in decimals, one may need to 'round off' an answer for convenience. In many of the economic problems (of various books) there is not much point in taking answers beyond *two decimal places (2dp)*. Where this is done then we denote 'to 2 dp' normally put after the answer.

For example, 1/7 as a percentage is 14.29% (to 2 dp).

In solving word problems involving percentage, 3 steps can help you:

1. Make sure you understand the question

2. Sort out the information to make a basic percent problem

3. Apply the operations to find out what asked.

Example:

A town council imposes different taxes on different fixed assets as follows: Commercial property 25% per year, Residential property 15% per year, Industrial property 20% per year.
An investor owns a residential building on a plot all valued at 80 000 000 Frw an industrial plot worth 75 000 000 Frw and a commercial premises worth 12 500 000 Frw. How much tax does the investor pay annually?

Solution:

Commercial: $\frac{25}{100}\times 12,500,000Frw = 3,125,000Frw$

Residential: $\frac{15}{100}\times 80,000,000Frw = 12,000,000Frw$.

Industrial: $\frac{20}{100}\times 75,000,000Frw = 15,000,000Frw$

Total tax the investor can pay annually:

$3,125,000Frw + 12,000,000Frw + 15,000,000Frw = 30,125,000Frw$.

Application activity 1.3

1. In the middle of the first term, the school organize the test and the tutor of mathematics prepared 20 questions for both section A and B. peter get 80% correct. How many questions did peter missed?
2. Student earned a grade of 80% on mathematics test that has 20 questions. How many did the student answered correctly? And what percentage of that not answered correctly?
3. John took a mathematics test and got 35 correct answers and 10 incorrect answers. What was the percentage of correct answers?
4. As a future teacher, is it necessary for a student teacher to know how to determine the percentage? Explain it with supportive examples.

1.4 Ratios and related problems

Activity 1.4

1. Suppose that two brothers from your village received 7 000 Frw from their child who lives in the city. The condition to share this amount of money is that for every 2 Frw that the young brother gets, the other one gets 3 Frw. The two brothers have come to you for help after they disagreed on how to share such money.

   (i) In what ratio would you share the money between them?

   (ii) Tell your partner how you would share the money and how much each would get.

2. Read carefully the following word problem and express the given data into fraction or ratio.

   (a) John and Lucy partnered to save money for x time and later buy a taxi. For every 800 Frw that John saved, Lucy saved 120 Frw. In what fraction or ratio were their contributions? What other simplest fraction or ratio is same as this?

   b) Jane and David sold milk to a vendor in the morning. Jane sold 4 500 ml while David sold 7.5 litres. In what fraction or ratio are their milk sales?

CONTENT SUMMARY

The mathematical term '**ratio**' defines the relationship between two numbers of the same kind. The relationship between these numbers is expressed in the form "**a to b**" or more commonly in the form $a : b$

A ratio is used to represent how much of one object or value there is in relation to another object or value. The two numbers in a ratio can only be compared when they have the same unit.

Example:

If there are 10 apples and 5 oranges in a bowl, then the ratio of apples to oranges would be 10 to 5 or 10: 5. This is equivalent to 2:1. In contrast, the ratio of oranges to apples would be 1:2.

Ratios occur in many situations such as in business where people compare profit to loss, in sports where compare wins to losses etc.

A comparison of two or more numbers is a **ratio.** You can write a 2-term ratio in fractional form when the second term is non-zero. Thus $a:b=\frac{a}{b}$, *where* $b \neq 0$ and we read *a is to b*.

To solve problems about ratios you need to know the following:

- The order of the terms of ratio is very important. If the order of the terms is changed, then the meaning of the ratio also changes.
- A ratio is in the lowest terms or simplest form if the terms of the ratio have no integral common factor. For example to write $4:8:12$ in lowest terms, you divide each term by the greatest common factor of the terms.

  Therefore, $4:8:12=1:2:3$ *it means that* $\frac{4}{4}:\frac{8}{4}:\frac{12}{4}$
- When you equate two ratios you are writing a **proportion.** A proportion is a relationship between four numbers or quantities in which the ratio of the first pair equals the ratio of the second pair, and is written as $a:b \ = \ c:d$ and is read as "**a is to b while c is to d**"

  Therefore $a:b=c:d$ *and* $4:8:12=1:2:3$ are proportions
- The proportion $a:b=c:d$ is written as $\frac{a}{b}=\frac{c}{d}$ where *a and d* are known as **extremes** of the proportion. While *b and c* are known as **means**
- The product of the extremes equals the product of the means.

  If $\frac{a}{b}=\frac{c}{d}$, $b \neq 0$ *and* $d \neq 0$, $a \times d = b \times c$

Experimental version

Note :

To share a quantity into two parts in the ratio $a:b$, the quantity is split into $a+b$ equal parts. The required parts become $\frac{a}{a+b}$ and $\frac{b}{a+b}$ of the quantity

Example

Share 38 400 Frw between Linda and Jean in the ratio 5:7 respectively.

Solution

38 400 Frw is to be shared in the ratio 5:7. It is split into 12 equal parts i.e 5 + 7 = 12 equal parts.

The amount of money Linda receives is $\frac{5}{12} \times 38\ 400\ Frw = 16\ 000\ Frw$

The amount of money Jean receives is $\frac{7}{12} \times 38\ 400\ Frw = 22\ 400\ Frw$

Example:

The working capital of "X" Ltd has deteriorated in recent years and now stands as under current assets of 980 000Frw to the current liabilities of 700 000Frw. Compute the current ratio.

Solution: The current ratio is given by $\frac{\text{current assets}}{\text{current liabilities}} = \frac{980\ 000 Frw}{700\ 000 Frw} = 1:4$.

Hence the current ratio is $1:4$.

Application activity for Lesson 1.4

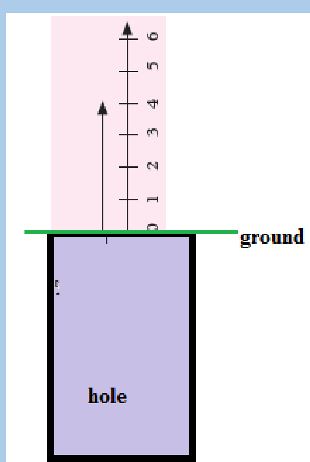
1. John weighs 56.7 kilograms. If he is going to reduce his weight in the ratio 7 : 6, find his new weight.
2. Ingabire, Mugenzi and Gahima have jointly invested in buying and selling of shares in the Rwanda stock exchange market. In one sale as they invested different amount of money, they realised a gain of 1 080 000 Frw and intend to uniquely share it in the ratio $2:3:4$ respectively. How much did Mugenzi get?
3. The average age of three boys is 25 years and their ages are in the proportion 3 : 5 : 7. Find the age of the youngest boy.

1.5 Negative numbers and related problems

Activity 1.5

1. The temperature of a juice in the bottle was $20^0 C$. They put this juice in the fridge so that its temperature decreases by $30^0 C$. What is the temperature of this juice? What can you advice the child who wishes to drink that juice?

2. Suppose that you have a long ruler fixed from the hole and graduated such that the point 0 corresponds to the ground level as illustrated on the following figure.



What is the coordinate of the point position for an insect which is at 3 units below the ground level in a hole?

CONTENT SUMMARY

There are numerous instances where one comes across negative quantities, such as temperatures below zero or bank overdrafts. There are instances, however, where it is not usually possible to have negative quantities. For example, a firm's production level cannot be negative.

From the above activity, you have learnt that, you can need to use **negative or a positive numbers.**

For example, when measuring temperature, the value of the temperatures of the body or surrounding can be negative or positive. The normal body temperature is about $+37^0 C$ and the temperature of the freezing mercury is about $-39^0 C$.

Example 1 :

Calculate $\frac{24}{-5} \div \frac{-32}{-10}$

Solution:

$$\frac{24}{-5} \div \frac{-32}{-10} = \frac{24}{-5} \times \frac{-10}{-32} = \frac{3}{1} \times \frac{2}{-4} = \frac{6}{-4} = -\frac{3}{2}$$

Example 2:

Eight students have an overdraft (scholarship advance) of 21,000Frw for each. What is their total bank balance?

Solution:

The total balance in the bank is $8 \times (-21{,}000) = -168{,}000$Frw. The sign negative means that students have the credit to be paid.

Application activity 1.5

Question1: Answer to the following questions

a) $\frac{(-10)\times(-5)\times(-6)}{(-3)\times(-2)}=$ b) $\frac{(-30)\times(+2)\times(-10)}{(-50\times(+2)}=$

c) Where negative numbers are applied in the real life? Do you think that computers of bank managers deal with negative numbers when operating loans for clients?

Question2:

Rebecca made a cup of tea with a temperature of 90^0C. She left it to cool, but forgot about it for 20 minutes, which meant that its temperature dropped by 74^0C. She decided to reheat her tea in the microwave which increased its temperature by 58^0C, and then she drank it. How hot was her tea at the point of drinking?

1.6 Concept and properties of absolute value

1.6.1 Meaning of absolute value

Activity 1.6.1

1) Draw a number line and state the number of units found between:

a) 0 and -8 b) 0 and 8

c) 0 and $\frac{1}{2}$ d) 4 and 17

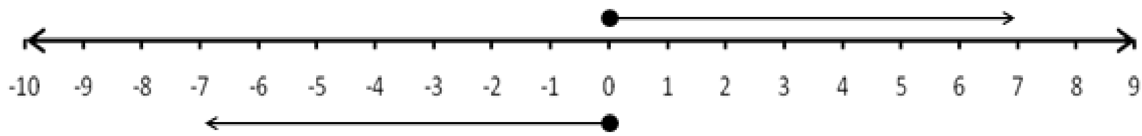
2) Do you think that a distance can be expressed by a negative number?

CONTENT SUMMARY

Absolute value of a number is the distance of that number from the origin (zero point) on a number line. The symbol $|\ |$ is used to denote the absolute value.

Example:

7 is at 7 units from zero, thus the absolute value of 7 is 7 or $|7| = 7$. Also -7 is at 7 units from zero, thus the absolute value of -7 is 7 or $|-7| = 7$. So $|-7| = |7| = 7$ since -7 and 7 are on equal distance from zero on number line.

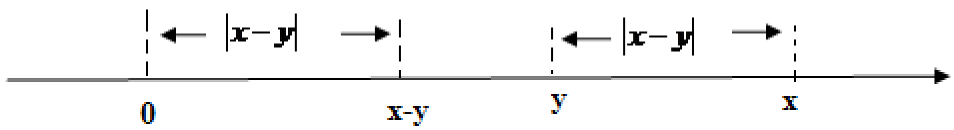


Note:

- The absolute value of zero is zero
- The absolute value of a non-zero real number is a positive real number.
- Given that $|x| = k$ where k is a positive real number or zero, then $x = -k$ *or* $x = k$.
- $|x| = \begin{cases} x, & if \ x \geq 0 \\ -x, & if \ x < 0 \end{cases}$
- **Geometrically**, $|x|$ represents the nonnegative distance from x to 0 on the real line.

  More generally, $|x - y|$ represents the nonnegative distance between the point x and y on the real line, since this distance is the same as that from the point $x - y$ to 0.(See the following figure)

  $|x - y| = \begin{cases} x - y, if \ x \geq y \\ -(x - y) = y - x, if \ x \leq y. \end{cases}$



Example*:*

Find x in the following:

a) $|x| = 5$ b) $|x| + 5 = 1$ c) $|x - 4| = 10$

Solution

a) $|x|=5,\ x=-5\ \ or\ \ x=5$

b) $|x|+5=1$

$\Leftrightarrow |x|=1-5 \Rightarrow |x|=-4$; This is impossible in the set $\mathbb{R}$ of real numbers.

There is no value of x since the absolute value of x must be a positive real number.

c) $|x-4|=10$

$x-4=-10\ \ or\ \ x-4=10$

$x=-10+4\ \ or\ \ x=10+4$

$x=-6\ \ or\ \ x=14$

Example:

Simplify

a) $-|40-12|$ b) $|4(-3)-(2)(5)|$ c) $|-4(-2)|$

Solution

a) $-|40-12|=-|28|=-28$

b) $|4(-3)-(2)(5)|=|-12-10|=|-22|=22$

c) $|-4(-2)|=|8|=8$

1.6.2 Properties of the Absolute Value

Activity 1.6.2

Evaluate and compare the following:

1) $|3|$ and $|-3|$ 2) $|3\times 5|$ and $|3|\times|5|$ 3) $|(-8)+5|$ and $|-8|+|5|$

From activity 1.6.2, you deduce the following properties:

1. Opposite numbers have equal absolute value.

$|a| = |-a|$

Example:

$|5| = |-5| = 5$

2. The absolute value of a product is equal to the product of the absolute values of the factors.

$|ab| = |a||b|$

Example:

$|4(-6)| = |4||-6|$

$|4(-6)| = |-24| = 24$

$|4||-6| = 4 \times 6 = 24$

3. The absolute value of a sum is less than or equal to the sum of the absolute values of the ends.

$|a+b| \le |a| + |b|$

Example:

$|-3+2| \le |-3| + |2|$

$|-1| \le 3+2$

$1 \le 5$

Application activity 1.6

1. Find the value(s) of x: a) $|x|=6$ b) $|x+1|=0$

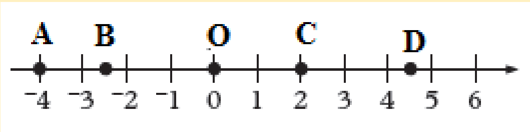
c) $|x-3|-4=2$ d) $|2x+1|=4$ e) $|x-3|+3=5$

2. Simplify

1) $|-5|$ (2) $|-4||-5|$ (3) $|-7|+|4|$ (4) $-|4\times 6|$ (5) $-|-6+8|$

2. Let a and b be the coordinates of points A and B, respectively, on a coordinate line. The distance between A and B, denoted by d(A,B) is defined by $d(A,B)=|b-a|$.

a) Refer to the figure below and determine the distance d(A,B), d(C,B) and d(O,A).



b) Dr. Makoma went from Kabgayi to Gitarama City on foot at the constant speed of 100m/min. If he used 60 min to go and 60 min to come back, explain to your colleague the distance covered by Dr. Makoma.

1.7 Problems involving absolute value

Activity 1.7

Using absolute value, solve the following equations

a) $|4x-7|=3$ b) $|2-4x|=1$ c) $|1+3u|=-6u$

d) $|12z+4|=|4z-6|$

CONTENT SUMMARY

Solving problems involving absolute value, use the definition of absolute value and then solve. After applying the definition to given equation, you will have two equations to solve.

In fact, when solving absolute value equations, you will usually get two solutions. That is important to keep in mind!

Remember that

- If x is positive, $|x| = x$
- If x is negative, $|x| = -x$

Example:

Solve for x when $|x - 4| = 7$

Before, we apply the definition, let's make a useful substitution

Let $y = x - 4$, so $|x - 4| = 7$ becomes $|y| = 7$. You must understand this step.

Now, let's apply the definition to $|y| = 7$. Again, you will have two equations to solve

Once again, when solving absolute value equations, you will usually get two solutions.

If y is positive, $|y| = y$, so the first equation to solve is $y = 7$. You have to substitute $x - 4$ for y

After substitution, becomes $x - 4 = 7$

$x - 4 + 4 = 7 + 4$ or $x = 11$

If y is negative, $| y | = -y$, so the second equation to solve is $-y = 7$ or $y = -7$.
You have to substitute $x - 4$ for y

You get
$x - -4 = -7$ or $x + -4 + 4 = -7 + 4$ or $x = -3$

The solutions are $x = -3$ and $x = 11$.

Example 2:
Solve for x when $| 3x + 3 | = 15$
Before, we apply the definition, let's make a useful substitution.
Let $y = 3x + 3$, so $| 3x + 3 | = 15$ becomes $| y | = 15$.
Now, let's apply the definition to $| y | = 15$.

Lastly, when solving absolute value equation, you will usually get two solutions. If y is positive, $| y | = y$, so the first equation to solve is $y = 15$. You have to substitute $3x + 3$ for y
After substitution, $y = 7$ becomes $3x + 3 = 15$
$3x + 3 = 15$ or $3x + 3 - 3 = 15 - 3$
Or $3x = 12$ or $x = 12/3$ or $x = 4$.
If y is negative, $| y | = -y$, so the second equation to solve is $-y = 15$ or $y = -15$.
After substitution, becomes $3x + 3 = -15$ or $3x = -18$ or $x = -6$
The solutions are $x = 4$ and $x = -6$

Application activity 1. 7

1. Solve the following equations

   a) $|2x-3|=7$ b) $|4-5x|=16$ c) $|x-1|=-3$

2. You have money in your wallet, but you don't know the exact amount. When a friend asks you to borrow him/her 5000frw, you say that if you do, from the amount you have, you remain with or miss1500frw. Use an absolute value equation to find least and biggest amount of money in your pocket?

1.8 Concept and properties of Powers

Activity 1.8

Knowing that $4=2\times2$ *or* 2^2, examine each expression and rewrite it in power form.

a) $5\times5\times5\times5\times5\times5\times5$ b) $3\times3\times3\times3\times t\times t\times s\times s\times s\times s\times s$ c) $(x+2)(x+2)(x+2)$

CONTENT SUMMARY

We call n^{th} power of a real number a that we note a^n, the product of n factors of a. that is

$$a^n=\underbrace{a\cdot a\cdot a...a}_{n\ factors}\quad \begin{cases} n \text{ is an exponent} \\ a \text{ is the base}\end{cases}$$

Example 1

$$2^4=\underbrace{2\cdot2\cdot2\cdot2}_{4\ factors}=16 \qquad 3^3=\underbrace{3\cdot3\cdot3}_{3\ factors}=27$$

Notice

- $a^1=a$
- $a^0=1, a\neq0$
- If $a=0, a^0$ is not defined

Properties of powers

Let $a,b\in\mathbb{R}$ and $m,n\in\mathbb{R}$

a) $a^m\cdot a^n=a^{m+n}$

In fact, $a^m\cdot a^n=\underbrace{a\cdot a\cdot a\cdots a}_{m\ factors}\times\underbrace{a\cdot a\cdot a\cdots a}_{n\ factors}=\underbrace{a\cdot a\cdot a\cdots a}_{m+n\ factors}=a^{m+n}$

b) $(a^m)^n=a^{mn}$

In fact, $\left(a^m\right)^n = \underbrace{\underbrace{a \cdot a \cdot a \cdots a}_{m\ factors} \times \underbrace{a \cdot a \cdot a \cdots a}_{m\ factors} \cdots \underbrace{a \cdot a \cdot a \cdots a}_{m\ factors}}_{n\ factors} = a^{mn}$

c) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

In fact,

$$\left(\frac{a}{b}\right)^m = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} \cdots \frac{a}{b}}_{m\ factors} = \frac{\underbrace{a \cdot a \cdot a \cdots a}_{m\ factors}}{\underbrace{b \cdot b \cdot b \cdots b}_{m\ factors}} = \frac{a^m}{b^m}$$

d) $\dfrac{1}{b^m} = b^{-m}$

In fact,

$$\frac{1}{b^m} = \frac{1}{b^m}^m = \left(\frac{1}{b}\right)^m = \left(b^{-1}\right)^m = b^{-m}$$

e) $\dfrac{a^m}{a^n} = a^{m-n}$

In fact,

$$\frac{a^m}{a^n} = a^m \frac{1}{a^n} = a^m a^{-n} = a^{m-n}$$

f) $\left(ab\right)^m = a^m b^m$

In fact,

$$\left(ab\right)^m = \underbrace{ab \cdot ab \cdots ab}_{m\ factors} = \underbrace{a \cdot a \cdots a}_{m\ factors} \times \underbrace{b \cdot b \cdots b}_{m\ factors} = a^m b^m$$

These properties help us to simplify some powers.

There is no general way to simplify the sum of powers, even when the powers have the same base. For instance, $2^5 + 2^3 = 32 + 8 = 40$, and 40 is not an integer power of 2. But some products or ratios of powers can be simplified using repeated multiplication models of a^n.

Example:

a) $2^4 \cdot 2^3 \cdot 4 = 2^4 \cdot 2^3 \cdot 2^2 = 2^9 = 512$

b) $a^4 \cdot b^3 \cdot a^5 \cdot b^8 = a^4 \cdot a^5 \cdot b^3 \cdot b^8 = a^9 \cdot b^{11}$

$a^9 \cdot b^{11}$ cannot be simplified further because the bases are different.

c) $\frac{y^9}{y^2} = y^{9-2} = y^7$

Application activity 1.8

1) Simplify

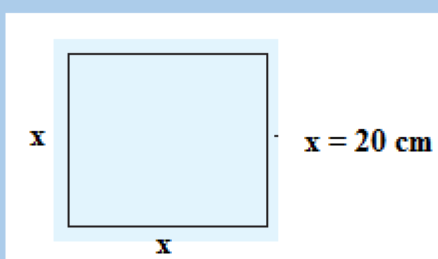
a) x^3x^2 (b) $\left(xy^3\right)^2 + 4x^2y^6$ (c) $\frac{6xy^2}{3xy}$ (d) $\frac{ab}{a^3} - \frac{a^3b^2}{a^5b}$ (e) $\frac{yx}{4xy}$

2) Referring to your real life experience, where powers are used?

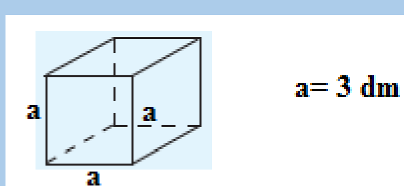
1.9 Powers and related problems

Activity 1.9

1. How can you find the area of the following paper in the form of a square?



2. Given a cube of the following form:



Determine the volume of this cube.

Considering the case in which P is invested in a bank for *n* interest periods per year at the rate r expressed as a decimal. The accumulated amount of money A after the time t (number of years P is invested) is given by:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

where *n* is the number of interest periods per year, r is the interest rate expressed as decimal, A is the amount after t years.

Note:

1) When the interest rate is compounded per year, $A = P(1+r)^{n}$ where r is expressed as a decimal for example $r = 9\% = 0.09$.

2) When the interest rate is compounded monthly, $A = P\left(1+\frac{r}{12}\right)^{12t}$ where t is the number of years and r expressed as a decimal.

This is called the compound interest formula. It is conveniently used in solving problems of compound interest especially those involving long periods of investments or payment.

In this method, the accrued compound interest is obtained by subtracting the original principal from the final amount.

Thus, Compound interest I = Accumulated amount (A) – Principal (P)

Note that the principal and the interest earned increased after each interest period.

We can also deduce that $I = A - P$

Example:

Application activity 1.6

1) Simplify

a) x^3x^2 (b) $\left(xy^3\right)^2+4x^2y^6$ (c) $\frac{6xy^2}{3xy}$ (d) $\frac{ab}{a^3}-\frac{a^3b^2}{a^5b}$ (e) $\frac{yx}{4xy}$

2) Referring to your real life experience, where powers are used?

1.10 Concept and properties of radicals

Activity 1.10

1) Evaluate the following powers using a calculator

a) $\sqrt{81}$ b) $(216)^{\frac{1}{3}}$ c) $(-27)^{\frac{1}{3}}$ d) $\sqrt[4]{16}$

2) Using examples, explain whether there is a difference between the square and the square root of a number. How do you calculate a square root of a given number?

CONTENT SUMMARY

A radical, or root, is the mathematical opposite of an exponent, in the same sense that addition is the opposite of subtraction. The smallest radical is the square root, represented with the symbol $\sqrt{\ }$. The next radical is the cube root, represented by the symbol $\sqrt[3]{\ }$. The small number in front of the radical is its index.

The n^{th} root of a real number is $\frac{1}{n}$ power of that real number. It is noted by $\sqrt[n]{b}$,

$b\in\mathbb{R}, n\in\mathbb{N}\setminus\{1\}$. $\forall a,b\in\mathbb{R}, \sqrt[n]{b}=a \Leftrightarrow b^{\frac{1}{n}}=a \Leftrightarrow b=a^n$

$$\begin{cases} n \text{ is called the index} \\ b \text{ is called the base or radicand} \\ \sqrt[n]{\ } \text{ is called the radical sign} \end{cases}$$

Example

a) $\sqrt[3]{27}=(27)^{\frac{1}{3}}=\left(3^{3}\right)^{\frac{1}{3}}=3^{3\times\frac{1}{3}}=3$

b) $\sqrt[4]{16}=(16)^{\frac{1}{4}}=\left(2^{4}\right)^{\frac{1}{4}}=2$

If $n=2$, we say square root and is written as $\sqrt{b}$. Here b must be a positive real number or zero.

If $n=3$, we say cube root noted $\sqrt[3]{b}$. Here b can be any real number.

If $n=4$, we say 4th root noted $\sqrt[4]{b}$. Here b must be a positive real number or zero.

Generally, for index n, we say n^{th} root noted $\sqrt[n]{b}$. Here if n is even, b must be a positive real number or zero and if n is odd b can be any real number.

Example:

$\sqrt{-9}$ is not defined in $\mathbb{R}$ the index in radical is even but $\sqrt[3]{-27}=(-27)^{\frac{1}{3}}=\left[(-3)^{3}\right]^{\frac{1}{3}}=-3$

Properties of radicals

$\forall n\in\mathbb{N}\setminus\{1\}, m\in\mathbb{R}$

a) $\sqrt[n]{a^{m}}=a^{\frac{m}{n}}$

In fact, $\sqrt[n]{a^{m}}=\left(a^{m}\right)^{\frac{1}{n}}=a^{m\times\frac{1}{n}}=a^{\frac{m}{n}}$

b) $\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$

In fact, $\sqrt[n]{ab}=(ab)^{\frac{1}{n}}=a^{\frac{1}{n}}b^{\frac{1}{n}}=\sqrt[n]{a}\sqrt[n]{b}$

c) $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

In fact, $\sqrt[n]{\frac{a}{b}}=\left(\frac{a}{b}\right)^{\frac{1}{n}}=\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

d) $\sqrt[n]{\sqrt[m]{a}}=\sqrt[nm]{a}=a^{\frac{1}{nm}}$

In fact,

$$\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{m}\times\frac{1}{m}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$$

Example:

Simplify

a) $\sqrt{46656}$ b) $\sqrt[3]{\sqrt{64}}$ c) $\sqrt[3]{ab}\times\sqrt[3]{a^2b^2}$ d) $\sqrt{\frac{36}{81}}$

Solution

a) $\sqrt{46656} = \sqrt{6^6} = 6^3 = 216$

b) $\sqrt[3]{\sqrt{64}} = \sqrt[6]{64} = \sqrt[6]{2^6} = 2$

c) $\sqrt[3]{ab}\times\sqrt[3]{a^2b^2} = \sqrt[3]{a^3b^3} = \sqrt[3]{(ab)^3} = ab$

d) $\sqrt{\frac{36}{81}} = \frac{\sqrt{36}}{\sqrt{81}} = \frac{6}{9} = \frac{2}{3}$

Addition and subtraction

When adding or subtracting the radicals we may need to simplify if we have similar radicals. Similar radicals are the radicals with the same indices and same bases.

Example:

$$\sqrt{2}+\sqrt{8} = \sqrt{2}+\sqrt{2\times 4} = \sqrt{2}+\sqrt{2}\times\sqrt{4} = \sqrt{2}+2\sqrt{2} = 3\sqrt{2}$$

$$\sqrt{3}-\sqrt{27} = \sqrt{3}-\sqrt{3\times 9} = \sqrt{3}-\sqrt{3}\times\sqrt{9} = \sqrt{3}-3\sqrt{5} = -2\sqrt{3}$$

Application activity 1.10

1. Simplify the following expression

a) $\sqrt{a}\times\sqrt{ab^3}\times\sqrt{bc^2}$ b) $\sqrt[3]{abc}\times\sqrt[3]{a^2b^2c^2}$ c) $\sqrt[3]{\frac{8}{27}}$

d) $\sqrt{\sqrt[4]{x^8}}$ e) $\sqrt{\frac{x^3y^4}{4x}}$ f) $\sqrt{18}+\sqrt{2}$

g) $\sqrt{12}-3\sqrt{3}$ h) $\sqrt{2}\times\sqrt{3}$ i) $\frac{\sqrt{6}}{\sqrt{2}}$

2. Compute the following radicals

a) $\sqrt{20}+\sqrt{5}$ b) $4\sqrt{3}-\sqrt{12}$ c) $5\sqrt{7}-\sqrt{28}$

d) $\sqrt{18}\times\sqrt{8}$ e) $\sqrt{45}+\sqrt{80}+\sqrt{180}$ f) $\sqrt{108}-\sqrt{48}$

1.11 Rationalizing radicals

Activity 1.11

Make the denominator of each of the following rational

1. $\frac{1}{\sqrt{2}}$ 2. $\frac{2-\sqrt{3}}{2\sqrt{5}}$ 3. $\frac{2}{1-\sqrt{6}}$ 4. $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}+\sqrt{5}}$

CONTENT SUMMARY

Rationalizing is to convert a fraction with an irrational denominator to a fraction with rational denominator. To do this,

1) If the denominator involves radicals we multiply the numerator and denominator by the conjugate of the denominator when this is made by two terms.

Some examples: the conjugate of $a+\sqrt{b}$ is $a-\sqrt{b}$ and the conjugate of $a-\sqrt{b}$ is $a+\sqrt{b}$.

2) When the denominator is made by one term, we multiply the numerator and denominator by the same radical. For example, use $\sqrt{a}$ is if the denominator is $\sqrt{a}$; you can use $\sqrt{b}$ when the denominator is $a\sqrt{b}$.

Remember that $(a+b)(a-b)=a^2-b^2$

Example:

a) $\dfrac{1}{1+\sqrt{2}}=\dfrac{1-\sqrt{2}}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)}=\dfrac{1-\sqrt{2}}{1-2}=\dfrac{1-\sqrt{2}}{-1}=\sqrt{2}-1$

b) $\dfrac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}=\dfrac{\sqrt{2}\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}=\dfrac{\sqrt{10}+\sqrt{6}}{5-3}=\dfrac{\sqrt{10}+\sqrt{6}}{2}$

c) $\dfrac{\sqrt{3}+\sqrt{7}}{4\sqrt{2}}=\dfrac{\left(\sqrt{3}+\sqrt{7}\right)\sqrt{2}}{\left(4\sqrt{2}\right)\sqrt{2}}=\dfrac{\sqrt{6}+\sqrt{14}}{8}$

Application activity 1.11

1. Rationalize the denominator

a) $\dfrac{2\sqrt{2}}{4+3\sqrt{3}}$ b). $\dfrac{a-\sqrt{b}}{\sqrt{d}}$ c) $\dfrac{3\sqrt{3}+2\sqrt{2}}{1+2\sqrt{2}}$

2. Find the value of a and b if

$$\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$$

1.12 Problems involving radicals

Activity 1.12

Juan is going to Nene's house to do a school project. Instead of walking two perpendicular streets to his classmates' house, Juan will take a short-cut diagonal path through the city plaza. Juan is 15 meters away from Nene's street. The distance from the intersection of the two streets to Nene's house is 8 meters.

1) How will you illustrate the problem?
2) How far will Juan travel along the shortcut?
3) How many meters will he save by taking the short cut rather than walking along the sidewalks.
4) If one of the distances increases or decreases, what might happen to the distance of the short cut? Justify your answer.
5) What Mathematical concepts did you use?

CONTENT SUMMARY

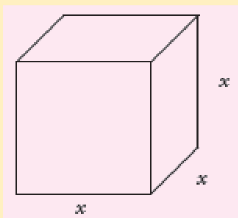
Basic word problems involving radical equations are word problems where the radical equation is given along with some of the known information that we can plug into the equation before we simplify/solve for what we are being asked to find.

Since the equation is already given in a basic word problem, the tricky part of solving this type of problem is knowing how to solve an equation involving a radical.

Application activity 1.12

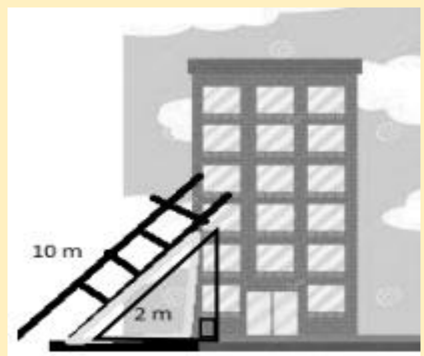
1. On a clear day, the distance d (in meters) that can be seen from the top of a tall building of height h (in meter) can be approximated by $d = 1.2\sqrt{h}$. Approximate the distance that can be seen from Kigali Tower which is 30m tall.

2. A cube has a total surface area of 96 square cm. Find the volume of that cube.



3.

To save a woman from a burning building, the firemen placed a 10- meter ladder against the window. The foot of the ladder was 2 meters from the wall as illustrated on the right figure. Approximately, how far up the wall is the ladder touching the wall?



4) Discuss orally how to determine the square of a square root of a number. Did you ever need to use a square root in your real life experience? Explain the answer.

1.13 Concept and properties of decimal logarithms

Activity 1.13

1) What is the real number at which 10 must be raised to obtain:

a) 1 b) 10 c) 100 d) 1000 e) 10000 f) 100000

2) Explain your classmate how you can find the number x if $x^3 = 64$.

CONTENT SUMMARY

The **decimal logarithm** of a positive real number *x* is defined to be a real number *y* for which 10 must be raised to obtain *x*. We write $\forall x > 0$, $y = \log x$ as $x = 10^y$.

$\log x$ is the same as $\log_{10} x$ and is defined for all positive real numbers only. 10 is the base of this logarithm. In general notation we do not write this base for decimal logarithm.

In the notation $y = \log x$, *x* is said to be the **antilogarithm of** *y*.

Example 1:

log (100) = ?

We are required to find the power to which 10 must be raised to obtain 100

So log(100) = 2 as $100 = 10^2$

$y = \log x$ means $10^y = x$

Be careful! $\log 2x + 1 \neq \log(2x+1)$

$$\log 2x + 1 = (\log 2x) + 1$$

Properties

$\forall a, b \in]0, +\infty[$

a) $\log a = \log b \Leftrightarrow a = b$ This is known as one to one property

Experimental version

b) $\log ab = \log a + \log b$

c) $\log \frac{1}{b} = -\log b$

d) $\log \frac{a}{b} = \log a - \log b$

e) $\log a^n = n \log a$

f) $\log \sqrt{a} = \log a^{\frac{1}{2}} = \frac{1}{2} \log a$

g) $\log \sqrt[m]{a^n} = \log a^{\frac{n}{m}} = \frac{n}{m} \log a$

h) $\operatorname{colog} x = \log\left(\frac{1}{x}\right) = -\log x$

Example 2: Calculate in function of $\log a, \log b \, and \log c$

a) $\log a^2 b^2$

b) $\log \frac{ab}{c}$

c) $\log \frac{ab}{\sqrt{c}}$

Solution

a) $\log a^2 b^2 = \log (ab)^2$

$$= 2 \log ab$$
$$= 2(\log a + \log b)$$

b) $\log \frac{ab}{c} = \log ab - \log c$

$$= \log a + \log b - \log c$$

c) $\log \frac{ab}{\sqrt{c}} = \log ab - \log \sqrt{c}$

$$= \log a + \log b - \log (c)^{\frac{1}{2}}$$
$$= \log a + \log b - \frac{1}{2} \log c$$

Example 3: Given that $\log 2 = 0.30$, $\log 3 = 0.48$ and $\log 5 = 0.7$. Calculate

a) $\log 6$ b) $\log 0.9$

Solution

a) $\log 6 = \log(2\times 3)$

$$= \log 2 + \log 3$$
$$= 0.30 + 0.48$$
$$= 0.78$$

b) $\log 0.9 = \log \frac{9}{10}$

$$= \log 9 - \log 10$$
$$= \log 3^2 - \log(2\times 5)$$
$$= 2\log 3 - \log 2 - \log 5$$
$$= 2(0.48) - 0.30 - 0.7$$
$$= -0.04$$

Co-logarithm

Co-logarithm, sometimes shortened to **colog**, of a number is the logarithm of the reciprocal of that number, equal to the negative of the logarithm of the number itself,

$$\operatorname{colog} x = \log\left(\frac{1}{x}\right) = -\log x$$

Example 4:

$$\operatorname{colog}\ 200 = -\log\ 200 = -2.3010$$

Change of base formula

If u $(u > 0)$ and if a and b are positive real numbers different from 1, $\log_b u = \frac{\log_a u}{\log_a b}$

This means that if you have a logarithm in any other base, you can convert it in the decimal logarithm in the following way where $a = 10$:

$$\log_b u = \frac{\log_{10} u}{\log_{10} b} = \frac{\log u}{\log b}.$$

This is for example: $\log_2 5 = \dfrac{\log_{10} 5}{\log_{10} 2} \approx 2.322$

There is another special logarithm called natural logarithm which has the base a number $e \approx 2.71828$. This logarithm is written as : $\log_e x = \ln x$.

Application activity 1.13

1. Without using calculator, compare the numbers *a* and *b*.
   a) $a = 3\log 2$ and $b = \log 7$
   b) $a = \log 2 + \log 40$ and $b = 4\log 2 + \log 5$
   c) $a = 2\log 2$ and $b = \log 16 - \log 3$
2. Given that $\log 2 = 0.30$, $\log 3 = 0.48$ and $\log 5 = 0.69$. Calculate
   a) $\log 150$ b) $\log \dfrac{9}{2}$ c) $\log 0.2 + \log 10$
3. Find co-logarithm of
   a) 100 b) 42 c)15

1.14 Simple equations involving decimal logarithms

Activity 1.14

Using appropriate properties solve the following logarithmic equations

$\log x = 3$ 2) $\log 2x = 4$ 3) $\log x = \log(4x - 9)$

CONTENT SUMMARY

Logarithmic equations can be solved using the laws of logarithms. These laws allow us to rewrite logarithms and form more convenient expressions.

The goal is to reduce to the logarithmic equation until you get a single logarithm on each side or a single logarithm on one side. Based on this, we can distinguish two types of logarithmic equations. We have to recognize these two types to facilitate solving the equations.

Types of logarithmic equations

Generally, after applying the laws of logarithms to reduce the equation, we can end up with one of two types of logarithmic equations:

- The first type looks like this:

$$\log_b P = \log_b Q \quad \rightarrow \quad P = Q$$

In cases where we end up with only one logarithm on each side of the equation, we can eliminate the logarithms if they have the same base and we can form an equation with the arguments. For example, in the expression above, the arguments are the algebraic expressions represented by ***P*** and ***Q***.

- The second type looks like this:

$$\log_b P = Q \quad \rightarrow \quad P = b^Q$$

In cases where we end up with a single logarithm on only one side of the equation, we can write the logarithm as an exponential expression and solve it that way.

Example 1: Solve the following equation

$$\log(2x+2)+\log 2 = \log(x+1)+\log 3$$

Solution:

In this case, we have a sum of logarithms on each side of the equation. Therefore, we are going to use the law of the product on both sides to get:

$$\log(2x+2)2 = \log(x+1)3$$

We can expand the multiplication on both sides to get:

$$\log(4x+4) = \log(3x+3)$$

Now, we eliminate the logarithms and form an equation with the arguments:

$$4x+4=3x+3$$

The linear equation can be easily solved:

$$4x+4=3x+3 \Leftrightarrow 4x-3x=3-4 \Leftrightarrow x=-1.$$

Example 2: What is the value of x in $\log(x+3)-\log 2 = \log(x-1)-\log 7$

Solution:

In this case, we have log subtractions on both sides of the equation, so we can apply the law of the logarithm quotient.

Therefore, applying this law to both sides, we have:

$$\log(x+3)-\log 2=\log(x-1)-\log 7 \Leftrightarrow \log\left(\frac{x+3}{2}\right)=\log\left(\frac{x-1}{7}\right)$$

Expressions within logarithms can no longer be simplified. However, we can eliminate the logarithms since they both have the same base:

$$\frac{x+3}{2}=\frac{x-1}{7}$$

We can cross multiply to simplify:

$$7(x+3)=2(x-1)$$

We multiply using the distributive property:

$$7x+21=2x-2$$

We solve the linear equation:

$$7x-2x=-2-21$$

$$5x=-23$$

$$x=-\frac{23}{5}$$

Example 3:

What is the value of x in $\log(4x+60)=2$?

Solution:

In this equation, we have a one-sided logarithm. This is an equation of the second case mentioned above:

$$\log p=q \Rightarrow p=10^q$$

We can solve this equation by writing it in exponential form. Therefore, we remove the logarithm from the left side and write its argument. On the right-hand side, 2 is the exponent and 10 is the base (the base of the logarithm):

$$\log(4x+60)=2 \Rightarrow 4x+60=10^2$$

We apply the exponent and solve the linear equation:

$$4x+60=100 \Leftrightarrow 4x=40 \Leftrightarrow x=10$$

Example 4:

Find the value of x in $\log 3x-2=\log\left(2x-5\right)$

Solution:

We have to move the logarithms to one side of the equation and the constant terms to the other side:

$$\log 3x-2=\log\left(2x-5\right)\Leftrightarrow \log 3x-\log\left(2x-5\right)=2$$

Now, we simplify the left part using the quotient law:

$$\log 3x-\log\left(2x-5\right)=2\Leftrightarrow \log\left(\frac{3x}{2x-5}\right)=2$$

To solve, we have to write the equation in its exponential form. The argument remains in the same place and we eliminate the logarithm. We raise the 2 (the base of the logarithm) to the exponent 2:

$$\log\left(\frac{3x}{2x-5}\right)=2\Leftrightarrow \frac{3x}{2x-5}=10^2$$

$$\Leftrightarrow \frac{3x}{2x-5}=100$$

$$\Leftrightarrow 3x=200x-500$$

$$\Leftrightarrow -197x=-500$$

$$\Leftrightarrow x=\frac{500}{197}$$

Application activity for Lesson 14

Solve the following logarithmic equations

1) $\log x=5$ 2) $\log 3x=-3$ 3) $\log 3x=\log(x+2)$

4) $\log x+\log 2=2$ 5) $3=\log x-\log 3$ 6) $\log 2x=\log 3x-\log x$

1.15 Simple interest

Activity 1.15

Make a research from library or internet to find the meaning of principal and simple interest in investment and discover how to calculate a simple interest from a given principal after a given time. .

Suppose you invest 10,000Frw at 100% in a bank at simple interest.

a) How much money will you have after first year?
b) If you keep all your 10,000Frw with its interest in the same bank, how much money will you have after two years?
c) How much money will you have after three years of SIMPLE INTEREST?

CONTENT SUMMARY

Simple interest is the amount charged when one borrows money or loan from a financial institution which accrue yearly.

This interest is a fixed percentage charged on money/loan that is not yet paid.

This interest is calculated based on the original principal or loan and is paid at regular intervals

From, above example, we see that when the principal (P), rate in percentage (R) and time in year (T) are given, then simple interest (I) for the given period is given by

$$I = P.\frac{R}{100}.T = \frac{PRT}{100}$$

The total amount (A) paid back by the borrower or the financial institution on the expiry of the interest period is the sum of the principal and the interest earned (I).

Thus, $A = P + I$

Example 1:

Find the simple interest earned from $3400Frw$ borrowed for 3years at the rate of 10%p.a.

Solution:

$$Interst, I = \frac{PRT}{100} = \frac{3400 \times 10 \times 3Frw}{100} = 1020Frw$$

Example 2:

Gatete borrowed 32 000Frw from a lending institution to start a business. If the institution charged interest at a rate of 8%p.a., calculate the simple interest and the total amount she eventually paid back after 4years.

Solution:

$$Interest, I = \frac{PRT}{100} = \frac{32000 \times 8 \times 4 Frw}{100} = 10240 Frw$$

$$Amount, A = (32000 + 10240) Frw = 42\,240 Frw.$$

Application activity

1. A 2-year loan of $500 is made with 4% simple interest. Find the interest earned.
2. You put 100,000Frw into your savings account which pays you by simple interest at 7% per annum and you leave it in the account for 5 years. Calculate your interest from your investment after five years. How much money will you have after five years?
3. In how many years will a sum of 4,000Frw yield a simple interest of 1,440 Frw at 12% per annum?

1.16 Compound interest and related problems

Activity 1.16

Make a research from the library or internet for finding how to differentiate compound interest from simple interest and then answer the following problem.

Suppose you invest 10,000Frw at 100% in a bank

a) How much money will you have after first year?
b) If you keep all your 10,000Frw with its interest in the same bank, how much money will you have after two years?
c) How much money will you have after three years of COMPOUND INTEREST?

The accumulated amount A after the time t (number of years P is invested) is given by: $A=P\left(1+\frac{r}{n}\right)^{nt}$ where n is the number of interest periods per year, r is the interest rate expressed as decimal, A is the amount after t years.

A general formula for calculating the interest rate can be derived. Starting with the familiar final sum formula

Compound interest = Accumulated amount (A) – Principal (P);

Compound interest is $I=A-P=P\left(1+\frac{r}{n}\right)^{nt}-P=P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)$ where A is accumulated amount and P principal.

Example 1:

A trader deposited $63000Frw$ in a fixed deposit account with a local bank which attracted an interest of 8% per annual compound interest. Find:

(a) the total amount after 4 years;

(b) compound interest.

Solution:

$P=63000Frw; n=1,\ t=4;$

$r=8\%\, p.a.$

$$A=P\left(1+\frac{r}{100}\right)^{t}$$

(a) $A=63000\left(1+\frac{8}{100}\right)^{4}$

$$=63000(1.08)^{4}=63000\times 1.36048896$$
$$=85710.80Frw$$

(b) $I=(85710.80-63000)Frw$

$=22710.80Frw.$

Example 2:

Find the compound interest earned on $15000 Frw$ invested for 3years, at 20%p.a. compounded quarterly.

Solution:

Here, each year has 4 interest period (quarterly) i.e in 3years, there are 12 interest period $(3\times 4=12)$.

The rate, $r\%=20\div 4=5\% p.a.$

$$A=15000\left(1+\frac{5}{100}\right)^{12}$$

$$A=15000(1.05)^{12}=26937.84 Frw$$

$$I=26937.84-15000=11937.84 Frw$$

Example 3:

How long will it take10,000Frw to double it on account earning 2% compounded quarterly?

Solution

For this problem, we'll use the compound Interest formula,

$F=P(1+i)^{n}$ where F is the final value, P the initial value of investment.

Since we want to know how long it will take, let t represent the time in years. The number of compounding periods is four times the time or $n=4t$. The original amount is $P=10,000$ and the future value is the double or F = 20,000. The interest rate per period is $i=\frac{0.02}{4}=0.005$

When these values are substituted into the compound interest formula, we get the exponential equation

$$20000=10000(1.005)^{4t}.$$

To solve this equation for t, isolate the exponential factor by dividing both sides by 10,000 to give

$$2=(1.005)^{4t}$$

Convert this exponential form to logarithm form and divide by 4

$$t=\frac{\frac{\log 2}{\log 1.005}}{4}=\frac{\log 2}{4\log 1.005} \text{ or } t=34.7$$

It will take 34.7 years, to double $10,000$ frw on account earning 2%.

Note:

It is interesting to note that the starting amount is irrelevant when doubling. If we started with P dollars and wanted to accumulate 2P at the same interest's rate and compounding periods, we would need to solve

$2P = P(1.005)^{4t}$ **.**

This reduces to the same equation as above $2=(1.005)^{4t}$ when both sides are divided by P. This means it takes about 37.4 years to double any amount of money at an interest rate of 2% compounded quarterly.

Considering the case in which P is invested in a bank for *n* interest periods per year at the rate r expressed as a decimal. The accumulated amount A after the time t (number of years P is invested) is given by:

$$A=P\left(1+\frac{r}{n}\right)^{nt}$$

where *n* is the number of interest periods per year, r is the interest rate expressed as decimal, A is the amount after t years.

Application activity 1.16

1. You put 100,000Frw into your savings account which pays you by compound interest at 7% per annum and you leave it in the account for 5 years. Calculate your interest from your investment after five years.
2. If you deposit $6,500\,Frw$ into an account paying 8% annual interest compounded monthly, how much money will be in the account after 7 years?
3. If you deposit $4,000\,Frw$ into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 5 years?
4. How much money would you need to deposit today at 9% annual interest compounded monthly to have $12,000\,Frw$ in the account after 6 years?
5. If you deposit $8000 into an account paying 7% annual interest compounded quarterly, how long until there is $12,400 in the account?

1.17 Final investment

Activity 1.17

1. A business takes out a simple interest loan of $10,000 at a rate of 7.5%. What is the total amount the business will repay if the loan is for 8 years?
2. Kwizera takes out a simple interest loan of P frw at rate r.
   a) Find the total amount Kwizera will repay if the loan is for n years and establish the formula giving final value F for simple interest.
   b) Determine the formula giving final value F for compound interest at rate r of principal P after t years.

CONTENT SUMMARY

Consider an investment at compound interest where:

P is the initial sum invested; A is the final value of the investment, r is the interest rate per time period (as a decimal fraction) and n is the number of time periods.

The value of the investment at the end of each year will be $1+r$ times the sum invested at the start of the year.

Thus, for any investment,

The value after one year $= P(1+r)$

Value after 2 years $= P(1+r)(1+\mathrm{r}) = \mathrm{P}(1+\mathrm{r})^2$

Value after 3 years $= P(1+r)(1+\mathrm{r})(1+r) = \mathrm{P}(1+r)^3 \quad etc$

We can see that each value is multiplied by $(1+r)$ to the power of number of years that the sum is invested. Thus, after n years the initial sum A is multiplied by $(1+r)^n$.

The formula for the initial value A. Thus, after an investment of P money, for n time periods at interest rate r is therefore $A = P(1+r)^n$.

Example 1:

1) If $600 is invested at compound interest of 8% per year, determine the final sum after 3 years.

   Solution:

$P = \$600; \quad n = 3; \quad r = 8\% = 0.08$.

Thus the final sum will be

$A = P(1+r)^n = 600(1.08)^3 = \$755.83$

Example 2:

If $4,000 is invested for 10 years at an interest rate of 11% per annum what will be the final value of the investment?

Solution:

P = $4,000, n = 10   r = 11% = 0.11

$11{,}253.68 \quad F = P\left(1+i\right)^n = 4{,}000\left(1+0.11\right)^{10} = 11{,}253.68$

The final value of the investment be $\$11{,}253.68$.

Experimental version

So far we have only calculated F for given values of P, i and n. However, if the values of any three of the variables in this equation are given then one can usually calculate the fourth.

Initial amount

A formula to calculate P, when values for F, i and n are given, can be derived as follows.
Since the final sum formula is

$$F = P(1+r)^n,$$

then, dividing through by $(1+r)^n$ (we get the initial sum formula

$$P = \frac{F}{(1+r)^n} \quad or\ P = F(1+r)^{-n}$$

Example 3:

How much money needs to be invested now in order to accumulate a final sum of $12,000 in 4 years' time at an annual rate of interest of 10%?

Solution:

Using the formula derived above, the initial amount is

$$P = F(1+r)^{-n} = 12,000(1.1)^{-4} = \$8,196.16$$

What we have actually done in the above example is to find the sum of money that is equivalent to $12,000 in 4 years' time if interest rates are 10%. An investor would therefore be indifferent between (a) $8,196.16 now and (b) $12,000 in 4 years' time. The $8,196.16 is therefore known as the 'present value' (PV) of the $12,000 in 4 years' time. We shall come back to this concept in the next few sections when methods of appraising different types of investment project are explained.

Time period

Calculating the time period is rather more tricky than the calculation of the initial amount.
From the final sum formula.

$A = P(1+r)^n$, then $\dfrac{A}{P} = (1+r)^n$

If the values of A, P and *r* are given and one is trying to find *n* this means that one has to work out to what power (1 + r) has to be raised to equal $\dfrac{A}{P}$. One way of doing this is via logarithms.

Example 4:

For how many years must $1,000 be invested at 10% in order to accumulate $1,600?

Solution

P= $1,000, *A* = $1,600, r = 10% = 0.1

Substituting these values into the formula

$\dfrac{A}{P} = (1+r)^n$ then, $\dfrac{1600}{1000} = (1+0.1)^n$

We get $1.6 = (1.1)^n$

Since to find the *n*th power of a number its logarithm must be multiplied by *n*. Finding logs, this means that our equation becomes

$\log 1.6 = n \log(1.1)$

And $n = \dfrac{\log(1.6)}{\log(1.1)} = 4.93$. Given that 4,93 years is approximately 5years,

If investments must be made for whole years then the answer is 5 years.

This answer can be checked using the final sum formula

$F = \mathrm{P}(1+r)^n = 1000(1.1)^5 = 1{,}610.51 \approx 1600$

If the $1,000 is invested for a full 5 years then it accumulates to just over $1,600, which checks out with the answer above.

A general formula to solve for *n* can be derived as follows from the final sum formula:

$A = P(1+r)^n$, $\dfrac{A}{P} = (1+r)^n$ and $n = \dfrac{\log(A/P)}{\log(1+r)}$.

An alternative approach is to use the iterative method and plot different values on a spreadsheet. To find the value of n for which $1.6 = (1.1)^n$.

This entails setting up a formula to calculate the function $y = (1.1)^n$ and then computing it for different values of n until the answer 1.6 is reached. Although some students who find it difficult to use logarithms will prefer to use a spreadsheet, logarithms are used in the other examples in this section. Logarithms are needed to analyze other concepts related to investment and so you really need to understand how to use them.

Example 5:

How many years will \$2,000 invested at 5% take to accumulate to \$3,000?

Solution:

P = 2,000; A = 3,000; r= 5% = 0.05

Using these given values in the time period formula derived above gives

$$n = \frac{\log(A/P)}{\log(1+r)} = \frac{\log 1.5}{\log 1.05} = 8.34$$

This money will need to be invested in 8.34 years.

Example 6:

How long will any sum of money take to double its value if it is invested at 12.5%?

Solution

Let the initial sum be A. Therefore the final sum is

$A = 2P$ and $r = 12.5\% = 0.125$

Substituting these value for A and r into the final sum formula $A = P(1+r)^n$, we find

$2A = A(1.125)^n$

Or $2 = (1.125)^n$ which gives $n = \dfrac{\log 2}{\log 1.125} = 5.9$

For any sum of money, it takes 5.9 years to double its value if it is invested at 12.5%.

Interest rates

A method of calculating the interest rate on an investment is explained in the following example.

Example 7:

If \$4,000 invested for 10 years is projected to accumulate to \$6,000, what interest rate is used to derive this forecast?

Solution

$P = 4,000$ A = 6,000 and $n = 10$

Substituting these values into the final sum formula

$A = P(1+r)^n$ gives $6000 = 4000(1+r)^{10}$

$1.5 = (1+r)^{10}$

$1+r = \sqrt[10]{1.5}$

$r = 4.14\%$

Application activity 1.17

1. A town has a population of 20,000. The population increases by 10% per year. What will be the population after 2 years?
2. You have 90,000Frw to deposit. Bank A offers 12 percent per year compounded monthly, while Bank B offers 12 percent but will only compound annually. How much will your investment be worth in 10 years at each bank?

1.18 Arc of elasticity for demand

Make a research in the library or on internet and categorize problems of Economics and Finance that are easily solved with the use of arithmetic.

Focus on the following: Elasticity of demand, Arc of elasticity for demand.

CONTENT SUMMARY

Price elasticity of demand is a measure of the responsiveness of demand to changes in price. It is usually defined as

$$e = (-1)\frac{\%change\, in\, quantity\, demand}{\%change\, in\, price}$$

The (-1) in this definition ensures a positive value for elasticity as either the change in price or the change in quantity will be negative. When there are relatively large changes in price and quantity it is best to use the concept of 'arc elasticity' to measure elasticity along a section of a demand schedule.

This takes the changes in quantity and price as percentages of the averages of their values before and after the change.

Thus arc elasticity is usually defined as

$$arc\, e = (-1)\frac{\dfrac{change\, in\, quantity}{0.5(1st\, quantity + 2nd\, quantity)}.100}{\dfrac{change\, in\, price}{0.5(1st\, price + 2nd\, price)}.100}$$

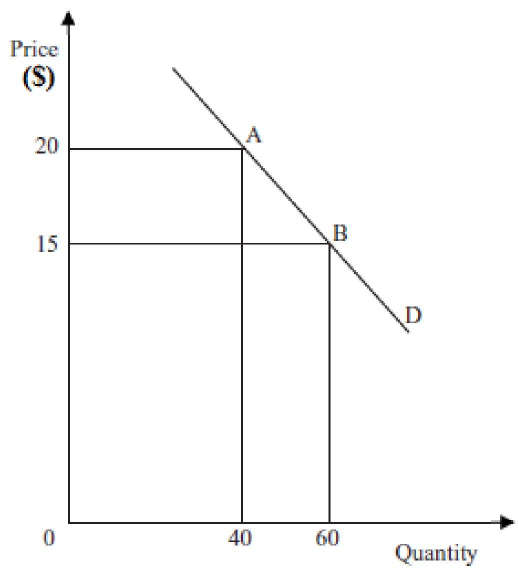
Although a positive price change usually corresponds to a negative quantity change, and vice versa, it is easier to treat the changes in both price and quantity as positive quantities. This allows the (-1) to be dropped from the formula. The 0.5 and the 100 will always cancel top and bottom in arc elasticity calculations.

Thus we are left with $$arc\,e = (-1)\frac{\dfrac{change\,in\,quantity}{0.5(1st\,quantity + 2nd\,quantity)}}{\dfrac{change\,in\,price}{0.5(1st\,price + 2nd\,price)}}$$

as the formula actually used for calculating price arc elasticity of demand

Example:

Calculate the arc elasticity of demand between points A and B on the demand schedule shown In the following figure:



Solution:

Between points A and B price falls by 5 from 20 to 15 and quantity rises by 20 from 40 to 60. Using the formula defined above:

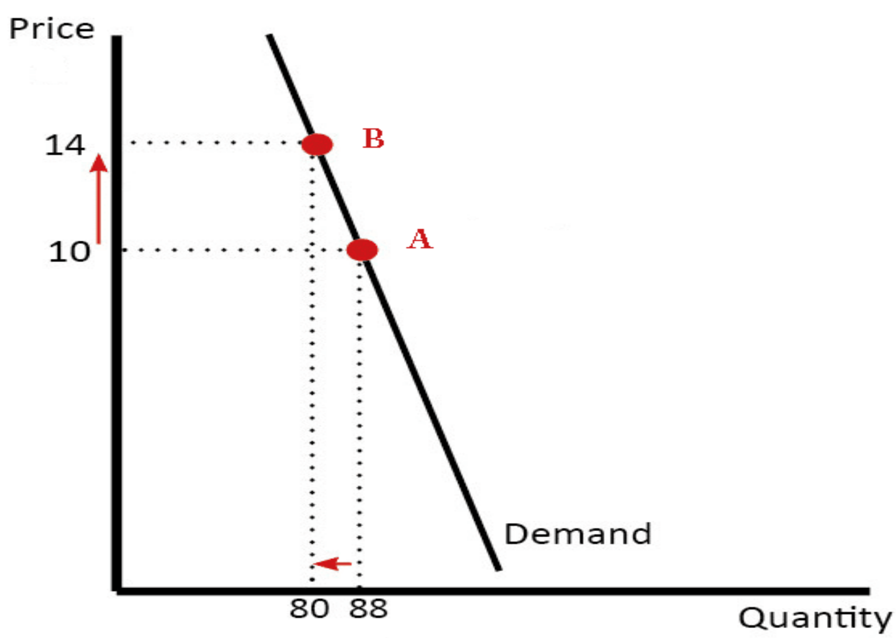
$$arce = \frac{\dfrac{20}{40+60}}{\dfrac{5}{20+15}} = \frac{7}{5}$$

Application activity 1.18

1. There are 100 people who need to get home on Christmas day and the average cost per ride is 2000 Frw. At this rate, all people are willing to pay that cost to get home. Transport agencies raise the average cost to 3000 per ride (a price increase of 50%). However, most people don't live too far from home, so 75 of them decide to walk. This leaves only 25 people who take buses (a 75% decrease in demand). Calculate price elasticity of demand. Interpret your result.

2. Once again, there are 100 people who need to get home on Christmas day. The average cost per ride is 2000 Frw. At this rate, all people are willing to pay this cost. Assume that transport agencies raise the average cost to 3000 Frw per ride (a price increase of 50%). 25 of them end up calling friends or family instead, leaving only 75 people to join transport agencies (a decrease in demand of 25%). Calculate price elasticity of demand, and interpret your result.

3. Calculate arc elasticity of demand between points A and B on the demand schedule shown in the following figure



1.10 End unit assessment

1. Why is it necessary for a student teacher to study arithmetic? Explain your answer on one page and be ready to defend your arguments in a classroom discussion;

2. The price of a house was 2 000 000Frw in the year 2000. At the end of each year the price has increased by 6%.

a) Find the price of the house after one year

b) Find the price of that house after 3 years

c) Find the price that such a house should have in this year.

UNIT 2: EQUATIONS AND INEQUALITIES

Key Unit competence: Apply equations and inequalities to solve daily life problems.

2.0 Introductory activity

1. By the use of library and computer lab, do the research and explain the linear equation.
2. If x is the number of pens for a learner, the teacher decides to give him/her two more pens. What is the number of pens will have a learner with one pen?

a) Complete the following table called table of value to indicate the number $y = f(x) = x + 2$ of pens for a learner who had x pens for $x \geq 0$.

x	-2	-1	0	1	2	3	4
$y = f(x) = x + 2$			2				
(x,y)			(0,2)				

b) Use the coordinates of points obtained in the table and complete them in the Cartesian plan.

c) Join all points obtained. What is the form of the graph obtained?

d) Suppose that instead of writing $f(x) = x + 2$ you write the equation $y = x + 2$. Is this equation a linear equation or a quadratic equation? What is the type of the inequality "$x + 2 \geq 0$"?

3. Find out an example of problem from the real life situation that can be solved by the use of linear equation in one unknown.

2.1 Linear equations in one unknown

Activity 2.1

1) Assume that in a competitive market, the supply schedule is $p = 60 + 0.4q$ where p is a price function and q is the quantity supplied. Is the price increasing or decreasing.

a) Find the price for q = 600 units.

b) What is the value of p for $q = 0$. What does it mean for an industry which is producing a certain good and has just fixed a price p_0 at the beginning ($q = 0$). Can you justify this price p_0.

2) Solve in set of real numbers the following equations:

a) $3 - x = 3$

b) $x = 16 - x$

CONTENT SUMMARY

An equation is a mathematical statement expressing the equality of two quantities or expressions. Equations are used in every field that uses real numbers. A linear equation is an equation of a straight line.

Consider the statement $x - 1 = 0$, this statement is equation as there is two quantities to be equal and is true for the value $x = 1$. The value $x = 1$ is called the solution of the statement $x - 1 = 0$ the number 1 is called the root of the equation. Thus, to find a solution to the given equation is to find the value that satisfies that equation.

To do this, rearrange the given equation such that variables will be in the same side and constants in the other side and then find the value of the variable.

Example:

Solve in set of real numbers the following equations

a) $x + 2 = 10$ b) $3 + x = 18 - 2x$ c) $x = 36 - 3x$

Solutions:

a) $x+2=10$

$x=10-2$

$x=8$

$S=\{8\}$

b) $3+x=18-2x$

$x+2x=18-3$

$3x=15$

$x=\frac{15}{3}$

$x=5$

$S=\{5\}$

c) $x=36-3x$

$x+3x=36$

$4x=36$

$x=\frac{36}{4}$

$x=9$

$S=\{9\}$

Application activity 2.1

Solve in set of real numbers:

a. $4x + 5 = 20 + x$

b. $x-31=50-8x$

c. $3+x=9-2x$

d. $\frac{2x+5}{x-6}=4$

2.2 Real life problems involving linear equations

Activity 2.2

A school expects to enrol students into the next academic year. The school leaders expect four more than twice the number of girls to enrol into the school than boys. The school buildings have the capacity of enrolling 316 students. How many of each gender would the school have to enrol to meet this requirement?

Problems which are expressed in words are known as problems or applied problems. A word or applied problem involving unknown number or quantity can be translated into linear equation consisting of one unknown number or quantity. The equation is formed by using conditions of the problem. By solving the resulting equation, the unknown quantity can be found.

In solving problem by using linear equation in one unknown the following steps can be used:

i. Read the statement of the word problems
ii. Represent the unknown quantity by a variable
iii. Use conditions given in the problem to form an equation in the unknown variable
iv. Verify if the value of the unknown variable satisfies the conditions of the problem.

Examples

1) The sum of two numbers is 80. The greater number exceeds the smaller number by twice the smaller number. Find the numbers.

Solution

Let the smaller number be x

Therefore the greater number be $80 - x$

According to the problem,

$(80 - x) - x = 2x$

$80 - x - x = 2x$

$80 - 2x = 2x$

$80 = 2x + 2x$

$80 = 4x$

$4x = 80$

$x = \frac{80}{4}$

$x = 20$

Now substitute the value of $x = 20$ in $80 - x$ we get $80 - 20 = 60$

Therefore, the smaller number is 20 and the greater number is 60

2) A boat covers a certain distance downstream in 2 hours and it covers the same distance upstream in 3 hours. If the speed of the stream is $2km/_{hr}$. Find the speed of the boat.

Solution

Let the speed of the boat be $x km/_{hr}$

Speed of the stream=$2km/hr$

Speed of the boat downstream $= (x+2)\, km/hr$

Speed of the boat upstream $= (x-2)\, km/hr$

Distance covered in both the cases in same.

$2(x+2) = 3(x-2)$

$2x + 4 = 3x - 6$

$2x - 3x = -6 - 4$

$-x = -10$

$x = 10$

Therefore, the speed of the boat is $10km/hr$.

Product equation

The equation in the form $(ax+b)(cx+d) = 0$ is product equation since the product of factors is null (zero) either one of them is zero. To solve this we proceed as follows:

$(ax+b)(cx+d) = 0$

$ax + b = 0 \ or\ cx + d = 0$

$x = \frac{-b}{a} \qquad or\ \ x = \frac{-d}{c}$

Example

Solve the equation $(2x+4)(x-1) = 0$

Solution: $\quad 2x + 4 = 0 \ or\ \ x - 1 = 0$

$x = -2 \qquad or\ \ x = 1$

Experimental version

Fractional equation of the first degree

The general form $\frac{ax+b}{cx+d}=0$ to find solution of this equation we need to have the condition of existence $cx+d\neq 0$ and $\frac{ax+b}{cx+d}=0 \Rightarrow ax+b=0$

We solve $ax+b$ and we take the value(s) which verify the condition of existence

Example

Solve $\frac{2x-6}{x+1}=0$

Solution: The existence condition is $x+1\neq 0$, $x\neq -1$

$2x-6=0$

$2x=6$

$x=3$

$S=\{3\}$

Application Activity 2.2

1) Mr. Peter wants to fence his garden of rectangle form where the length of that rectangle is twice its breadth. If the perimeter is 72 metres, help Peter to find the length and breadth of the rectangle.

2) The sum of two numbers is 25. One of the numbers exceeds the other by 9. Explain how you can determine these numbers.

3) Find the number whose one fifth is less than the one fourth by 3.

2.3 Meaning of inequalities

Activity 2.3

1. Find the value(s) of x such that the following statements are true

   a. $x < 5$ b. $x > 0$ c. $-4 < x < 12$

2. Use library and computer lab to do the following:

a) Discuss the difference between linear equations and linear inequalities

b) Find out example of linear inequalities and try to solve them.

c) Examples on how linear inequalities are applied when solving real word problems.

CONTENT SUMMARY

Just as we use the symbol = to represent "**is equal to**", we also use the symbols < and > to represent "**is less than**" and "is greater than", respectively.

If a and b are real numbers, then various statements of inequality can be made:

- $a \neq b$ says that a is not equal to b
- $a < b$ says that a is less than b
- $a > b$ says that a is greater than b
- $a \leq b$ means that a is less than or equal to b
- $a \geq b$ means that a is greater than or equal to b

The table below shows the inequality symbols:

Symbol	**Meaning**
$\neq$	Not equal
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to

Inequalities are used to compare two values and determine the range or ranges of values that satisfy the conditions of a given variable.

Note that:

- When the same real number is added or subtracted from each side of inequality the direction of inequality is not **changed**.

  It means that for all real numbers a, b and c,

  i) $a > b$ if and only if $a + c > b + c$

  ii) $a > b$ if and only if $a - c > b - c$

- The direction of the inequality is **not changed** if both sides are multiplied or divided by the same **positive real number.**

  It means that for all real numbers a, b and c, with $c > 0$

  $a > b$ if and only if $ac > bc$

- The direction of the inequality is **reversed** if both sides are multiplied or divided by the **same negative real** number.

  It means that for all real numbers a, b and c, with $c < 0$

  $a > b$ if and only if $ac < bc$

Example

The statement $x + 3 = 10$ is true only when $x = 7$. If x is replaced by 5, we have a statement $5 + 3 = 10$ which is **false**. To be true we may say that 5 + 3 is less than 10 or in symbol $5 + 3 < 10$. If x is replaced by 8, the statement 8+3=10 is also **false**. In those two cases we no longer have equality but inequality.

Suppose that we have the inequality $x + 3 < 10$, in this case we have an inequality with one unknown. Here the real value of x satisfies this inequality is not unique. For example, 1 is a solution but 3 is also a solution. In general, all real numbers less than 7 are solutions. In this case we will have many solutions combined in an interval.

Now, the solution set of $x+3<10$ is an open interval containing all real numbers less than 7 whereby 7 is excluded. How?

We solve this inequality as follow

$$\begin{aligned}&x+3<10\\ \Leftrightarrow\ &x<10-3\\ \Leftrightarrow\ &x<7\end{aligned}$$

And then $S=]-\infty,7[$

Application activity 2.3

Solve the linear inequality in one variable:

a) $7x+3<5x+9$

b) $3x-5\leq 3-x$

c) $4x+5<6x+9$

2.4 Concept and properties of intervals

Activity 2.4

Make research in the library or computer lab to do the following:

Suppose a and b be two real numbers such that $a<b$, how to denote

a) the open interval from a to b?

b) the closed interval from a to b?

c) The half-open interval?

CONTENT SUMMARY

A subset of real line is called an interval if it contains at least two numbers and also contains all real numbers between any two of its elements. For example, the set of real numbers x such that $x > 6$ is an interval, but the set of real numbers y such that $y \neq 0$ is not an interval.

If a and b are real numbers and $a < b$, we often refer to:

(i) The open interval from a to b, denoted by (a, b) or $]a,b[$, consisting of all real numbers x satisfying $a < x < b$

(ii) The closed interval from a to b, denoted by $[a, b]$, consisting of all real numbers x satisfying $a \leq x \leq b$

(iii) The half-open interval $[a, b[$, consisting of all real numbers x satisfying the inequalities $a \leq x < b$

(iv) Half-open interval $]a, b]$, consisting of all real numbers x satisfying the inequalities $a < x \leq b$

Examples

Solve the following inequalities and express the solution in terms of intervals.

a) $2x - 1 > x + 3$ b) $-\frac{x}{3} \geq 2x - 1$ c) $2(x+5) > 2x-8$ d) $2x+5 \leq 2x+4$

Solution:

$$
\begin{aligned}
a)\quad & 3x-1 > 4x+3 \\
& 3x > 4x+3+1 \\
& 3x-4x > 4 \\
& -x > 4 \\
& x < -4
\end{aligned}
$$

The solution set is the interval $]-\infty, -4[$

Experimental version

b) $-\frac{2x}{3} \geq -6x+3$

$-2x > 3(-6x+3)$

$-2x > -18x+9$

$-2x+18x > 9$

$16x > 9$

$x > \frac{9}{16}$

The solution set is the interval $\left]\frac{9}{16}, +\infty\right[$

c) $3(x+5) > 2x+3$

$3x+15 > 2x+3$

$3x > 2x+3-15$

$3x-2x > -12$

$x > -12$

$x > -12$

The solution set is the interval $]-12, +\infty[$

d) $2x+5 \leq 2x+4$

$2x-2x \leq 4-5$

$0x \leq -1$

Since any real number times zero is zero and zero is not less or equal to -1 then the solution set is the empty set; $S = \varnothing$.

Application activity 2.4

1) Complete the following table by converting the given inequalities to interval notation.

Inequality	Interval notation
$x \leq 3$	
$x < 5$	
$x \geq 2$	

2) Solve the following inequalities and express the solution in terms of intervals.

a) $2x - 1 > x + 3$

b) $-\frac{x}{3} \geq 2x - 1$

c) $2(x+5) > 2x - 8$

2.5 Problems involving intervals

Activity 2.5

1. For a person to be elected as the president, he/she should be a minimum of 35 years old. Give an interval representing this information.
2. For a school to participate in an Olympiad exam, the number of students from the school be a minimum of 10. Represent this information using an interval notation.

CONTENT SUMMARY

Introducing intervals, which are bounded sets of numbers and are very useful when describing domain and range. We can use interval notation to show that a value falls between two endpoints. For example, the learners need to score above 65 marks and up to 100 marks. The interval for this information will be written as $65 < x \leq 100$ *or* $]65,100]$. The intervals can also be used to determine the amount of time between two given points in time. For example, "The time interval between three o'clock and four o'clock is one hour.

Example:

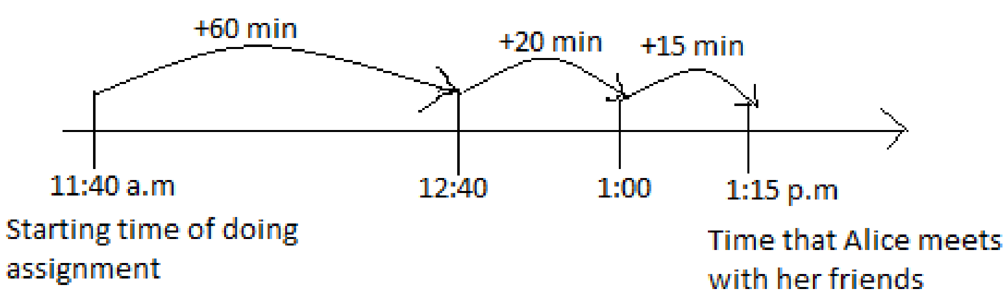
Alice wants to meet her friends downtown. Before leaving home, she does assignment for 60 minutes and eats lunch for 20 minutes. The walk downtown takes 15 minutes. Alice starts her assignment at 10:45 a.m. At what time does she meet her friends?

Solution:

We need to find what time Alice meet her friends downtown.

The information given is: the time Alice starts her assignment before leaving home, the time used for eating lunch, the time it takes to do assignment, and the time it takes to walk downtown.

We can start with the time Alice starts her assignment before leaving home and draw a number line to find the time Alice meets her friends.



From the number line we see that Alice meets with her friends at 1:15 p.m

Application activity 2.5

1. Peter needs at least 1300 calories a day but the calorie intake should not exceed 1700 calories. Represent the possible amount of calories she could eat using interval notation.
2. Karake gets out of the school at 2:45 P.M. It takes him 10 minutes to walk home. The he spends 10 minutes eating a snack. He spends 8 minutes putting on his football uniform. It takes 20 minutes for Karake's father to drive him to football ground for doing practice.

a) By using number line, determine the time for which Karake will arrive at football ground.

b) How do you know that the answer is reasonable?

2.6 Inequalities involving products

Activity 2.6

Explain the method you can use to solve the following inequalities:

1. $(x+1)(x-1)<0$
2. $(x-3)(-2x+4)(x+1)\geq 0$

CONTENT SUMMARY

Suppose that we need to solve the inequality of the form $(ax+b)(cx+d)<0$. For this inequality we need the set of all real numbers that make the left hand side to be negative. Suppose also that we need to solve the inequality of the form $(ax+b)(cx+d)>0$. For this inequality we need the set of all real numbers that make the left hand side to be positive.

We follow the following steps:

a) First we solve for $(ax+b)(cx+d)=0$
b) We construct the table called **sign table**, find the sign of each factor and then the sign of the product.
c) Write the interval considering the given inequality sign.

Example

Solve in set of real numbers the following inequalities

$a)$ $(x+2)(x-5)<0$

$b)$ $(-2x+2)(x-5)(3x-9)\geq 0$

Solution

$a)$ $(x+2)(x-5)<0$

Start by solving $(x+2)(x-5)=0$

$$x+2=0 \quad or \quad x-5=0$$
$$x=-2 \quad or \quad x=5$$

Then, we find the sign table.

x	$-\infty$		-2		5		$+\infty$
$x+2$		-	0	+		+	
$x-5$		-		-	0	+	
$(x+2)(x-5)$		+	0	-	0	+	

Since the inequality is $(x+2)(x-5)<0$ we will take the interval where the product is negative. Thus, $S=]-2,5[$

$b)\quad (-2x+2)(x-5)(3x-9)\geq 0$

$-2x+2=0 \Rightarrow x=1, \quad x-5=0 \Rightarrow x=5, \quad 3x-9=0 \Rightarrow x=3$

Then, we find the sign table.

x	$-\infty$		1		3		5		$+\infty$
$-2x+4$		-	0	+		+		+	
$x-5$		-		-		-	0	+	
$3x-9$		-		-	0	+		+	
$(-2x+2)(x-5)(3x-9)$		-	0	+	0	-	0	+	

Since the inequality is $(-2x+2)(x-5)(3x-9)\geq 0$ we will take the interval where the product is positive. Thus, $S=[1,3]\cup[5,+\infty[$

Application activity 2.6

Solve in set of real numbers the following inequalities

1. $(3x+7)(x-2)<0$
2. $(2x+7)(3x-2)\geq 0$

2.7 Inequalities involving quotients

Activity 2.7

Explain the method you can use to solve the following inequality:

1) $\frac{2x-3}{x}<0$

2) $\frac{x+2}{x-1}\leq 0$

CONTENT SUMMARY

Suppose that we need to solve the inequality of the form $\frac{ax+b}{cx+d}<0$. For this inequality we need the set of all real numbers that make the left hand side to be negative. Suppose also that we need to solve the inequality of the form $\frac{ax+b}{cx+d}>0$. For this inequality we need the set of all real numbers that make the left hand side to be positive.

We follow the following steps:

a) The first thing we need to do is get a zero on one side of the given inequality. That means we solve for $ax+b=0$ and $cx+d=0$

b) We construct the table called **sign table**, find the sign of each factor and then the sign of the quotient.

   For the quotient the value that makes the denominator to be zero is always excluded in the solution. For that value we use the symbol || in the row of quotient sign.

c) Write the interval considering the given inequality sign.

Example

Solve in set of real numbers the following inequalities

1) $\frac{4-x}{x+3}\geq 0$

2) $\frac{2x+3}{4-x}<0$

Solution

1) $\dfrac{4-x}{x+3} \geq 0$

$4-x=0 \Rightarrow x=4$

$x+3=0 \Rightarrow x=-3$

Then, we find the sign table

x	$-\infty$		-3		4		$+\infty$
$4-x$		-		-	0	+	
$x+3$		-	0	+	+	+	
$\dfrac{4-x}{x+3}$		+	||	-	0	+	

$S =]-\infty, -3[\cup [4, +\infty[$

2) $\dfrac{2x+3}{4-x} < 0$

$2x+3=0 \Rightarrow x=-\dfrac{3}{2}$

$4-x=0 \Rightarrow x=4$

Then, we find the sign table.

x	$-\infty$		$-\dfrac{3}{2}$		4		$+\infty$
$2x+3$		-	0	+		+	
$4-x$		+		+	0	-	
$\dfrac{2x+3}{4-x}$		-	0	+	||	-	

$$S = \left]-\infty, -\frac{3}{2}\right[\cup \left]4, +\infty\right[$$

Application activity 2.7

Solve in set of real numbers the following inequalities

1) $\frac{x+4}{2x-1} \geq 0$

2) $\frac{x-2}{x+3} < 0$

3) $\frac{x+2}{x+4} \leq 0$

2.8 Real life problems involving linear inequalities

Activity 2.8

Sam and Alex play in the same team at their school. Last Saturday their team played with another team from other school in the same district, Alex scored 3 more goals than Sam. But together they scored less than 9 goals.

What is the set of the possible number of goals Alex scored?

Inequalities can be used to model a number of real life situations. When converting such word problems into inequalities, begin by identifying how the quantities are relate to each other, and then pick the inequality symbol that is appropriate for that situation. When solving these problems, the solution will be a range of possibilities. Absolute value inequalities can be used to model situations where margin of error is a concern.

Examples

1) The width of a rectangle is 20 meters. What must the length be if the perimeter is at least 180 meters?

Solution:

Let x be length of rectangle

$perimeter = 2length + 2width$

$2x + 2(20) \geq 180$

$2x \geq 180 - 40$

$x \geq 70$

The length must be at least 70 meters.

2) John has 1 260 000 Rwandan Francs in an account with his bank. If he deposits 30 000 Rwanda Francs each week into the account, how many weeks will he need to have more than 1 820 000 Rwandan Francs on his account?

Solution:

Let *x* be the number of weeks

We have total amount of deposits to be made plus the current balance is greater than the total amount wanted.

That is $30000x + 1260000 > 1820000$

$30000x > 1820000 - 1260000$

$30000x > 560000$

$$x > \frac{560000}{30000} \approx 19$$

Thus, John need at least 19 weeks to have more than 1 820 000 Rwandan Francs on his account.

Application activity 2.8

1) John enters a race where he has to cycle and run. He cycles a distance of 25 km, and then runs for 20 km. His average running speed is half of his average cycling speed. Joe completes the race in less than 2½ hours, what can we say about his average speeds?

2) Explain your colleague whether or not a solution set for an inequality can have one element.

2.9 Solving algebraically Simultaneous linear equations in two unknowns (Solving by equating two same variables)

Activity 2.9

In each of the following systems find the value of one variable from one equation and equalize it with the same value of another variable from second equation. Calculate the values of those variables.

1. $\begin{cases} 3x - 2y = -6 \\ x + y = -2 \end{cases}$ 2. $\begin{cases} x + 2y = 10 \\ -3x + 2y = 12 \end{cases}$ 3. $\begin{cases} x + 4y = 8 \\ y - x = 2 \end{cases}$

CONTENT SUMMARY

To find the value of unknown from simultaneous equation by equating the same variable in terms of another, we do the following steps:

i) Find out the value of one variable in first equation

ii) Find out the value of that variable in the second equation

iii) Equate the same values obtained

iv) Solve the equation obtained to find out the unknown variables.

Example

1) Algebraically, solve the simultaneous linear equation by equating the same variables.

$$\begin{cases} 4x + 5y = 2 \\ x + 2y = -1 \end{cases}$$

Solution:

$$\begin{cases} 4x + 5y = 2 \\ x + 2y = -1 \end{cases}$$

From equation (1) $4x + 5y = 2 \Rightarrow x = \frac{2-5y}{4}$, from equation (2) $x + 2y = -1 \Rightarrow x = -1 - 2y$

Equalize the values of x from equation (1) and (2)

$$\frac{2-5y}{4} = -1 - 2y$$

$$2 - 5y = 4(-1 - 2y)$$

$$2 - 5y = -4 - 8y$$

$$-5y + 8y = -4 - 2$$

$$y = -2$$

$x = \frac{2-5y}{4}$, $x = \frac{2-5(-2)}{4} = \frac{12}{4} = 3$ then, $S = \{(3, -2)\}$

2) Solve algebraically the following system by equating two same variables

$$\begin{cases} x + y = 1 \\ 2x + 3y = 2 \end{cases}$$

Solution

From equation $x + y = 1 \Rightarrow y = 1 - x$

From equation $2x + 3y = 2 \Rightarrow y = \frac{2-2x}{3}$

By equating those values of y, $1 - x = \frac{2-2x}{3}$

$3(1 - x) = 2 - 2x$

$3 - 3x = 2 - 2x$

$-3x + 2x = 2 - 3$

$x = 1 \quad , \quad y = 1 - x \Rightarrow y = 1 - 1 = 0$

$s = \{(1,0)\}$

3) Solve the following simultaneous equation

$\begin{cases} x + 4y = 8 \\ y - x = 2 \end{cases}$

Solution

From equation (1), $x + 4y = 8 \Rightarrow x = 8 - 4y$

From equation (2), $y - x = 2 \Rightarrow x = -2 + y$

By equating values from (1) and (2)

$8 - 4y = -2 + y$

$-4y - y = -2 - 8$

$-5y = -10$

$y = 2\,, \quad x = -2 + 2 = 0$ Then, $s = \{(0,2)\}$

Application activity 2.9

Solve the simultaneous linear equations by equating two same variables

1. $\begin{cases} x - y = 3 \\ 2x - 2y = 6 \end{cases}$
2. $\begin{cases} x + y = 2 \\ 2x + 2y = 4 \end{cases}$
3. $\begin{cases} 3x - 5y = 10 \\ 2x + y = 12 \end{cases}$
4. $\begin{cases} x - 4y = 1 \\ x - y = 2 \end{cases}$
5. $\begin{cases} -x + y = 0 \\ x + 2y = 3 \end{cases}$
6. $\begin{cases} 5y + 3x = 2 \\ 10x + 6y = 0 \end{cases}$

2.10 Solving algebraically Simultaneous linear equations in two unknowns (solving by row operations or elimination method)

Activity 2.10

For each of the following, find two numbers to be multiplied to the equations such that one variable will be eliminated when making the addition or subtraction of the two equations.

$$\begin{cases} -2x+5y=-7 \\ 7x-3y=-19 \end{cases}$$

CONTENT SUMMARY

To eliminate one of the variables from either of equations to obtain an equation in just one unknown, make one pair of coefficients of the same variable in both equations negatives of one another by multiplying both sides of an equation by the same number. Upon adding the equations, that unknown will be eliminated.

Example

1) Solve the system of equations using elimination method.

$$\begin{cases} x+y=1 \\ 2x+3y=2 \end{cases}$$

Solution

$$\begin{cases} x+y=1 \\ 2x+3y=2 \end{cases}\left|\begin{array}{l} -2 \\ \ \ 1 \end{array}\right. \Leftrightarrow \begin{cases} -2x-2y=-2 \\ \ \ 2x+3y=2 \end{cases}$$

$$\overline{\qquad\qquad\qquad y=0}$$

$x+y=1 \Leftrightarrow x=1-y=1$

$S=\{(1,0)\}$

2) Solve the system of equation by using elimination method

$$\begin{cases} -2x + 5y = -7 \\ 7x - 3y = -19 \end{cases}$$

Solution

$$\begin{cases} -2x + 5y = -7 \\ 7x - 3y = -19 \end{cases} \Leftrightarrow \begin{array}{l} 7(-2x + 5y) = 7(-7) \\ 2(7x - 3y) = 2(-19) \end{array}$$

$$\begin{array}{r} -14x + 35y = -49 \\ 14x - 6y = -38 \end{array} \Leftrightarrow \begin{array}{r} -14x + 35y = -49 \\ 14x - 6y = -38 \\ \hline 29y = -87 \end{array}$$

$$29y = -87 \Leftrightarrow y = -3$$

$$x = \frac{-7 - 5y}{-2} = \frac{-7 - 5(-3)}{-2} = -4$$

Then, $s = \{(-4, -3)\}$

Application activity 2.10

1) Solve the following system of equation by using elimination method

a) $\begin{cases} 3x - 4y = 1 \\ x - 3y = 2 \end{cases}$ b) $\begin{cases} x - 4y = 1 \\ x - y = 2 \end{cases}$

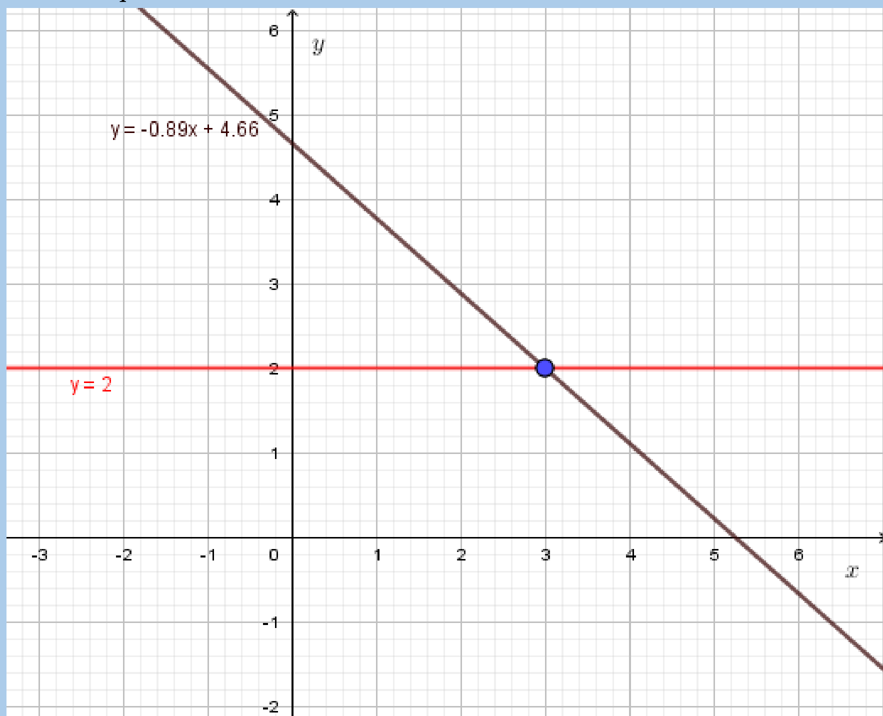
c) $\begin{cases} 3x - 2y = -6 \\ x + y = -2 \end{cases}$ d) $\begin{cases} 2x + 3y = 8 \\ x - y = 2 \end{cases}$

2) Use your own words to explain how to solve algebraically simultaneous linear equations.

2.11 Solving graphically simultaneous linear equations in two unknowns

Activity 2.11

1) Discuss how you can find the coordinate of the point intercept of two lines whose equations are known.



2) Given the system of equations

$$\begin{cases} 3x - 2y = -6 \\ x + y = -2 \end{cases}$$

a. For each equation from the system, choose any two values of x and use them to find values of y . this gives you two points in the form

b. Plot the obtained points in XY plane and join these points to obtain the lines.

c. What is the point of intersection for two lines?

d. Write the obtained point as solution of the system.

CONTENT SUMMARY

One way to solve a system of linear equations is by graphing. The intersection of the graphs represents the point at which the equations have the same x-value and the same y-value. Thus, this ordered pair represents the solution common to both equations. This ordered pair is called the solution to the system of equations.

The following steps can be applied in solving system of linear equation graphically:

1. Find at least two points for each equation.
2. Plot the obtained points in XY plane and join these points to obtain the lines. Two points for each equation give one line.
3. The point of intersection for two lines is the solution for the given system

Examples

1) Solve the following system by graphical method

$$\begin{cases} x+y=4 \\ x-y=2 \end{cases}$$

Solution

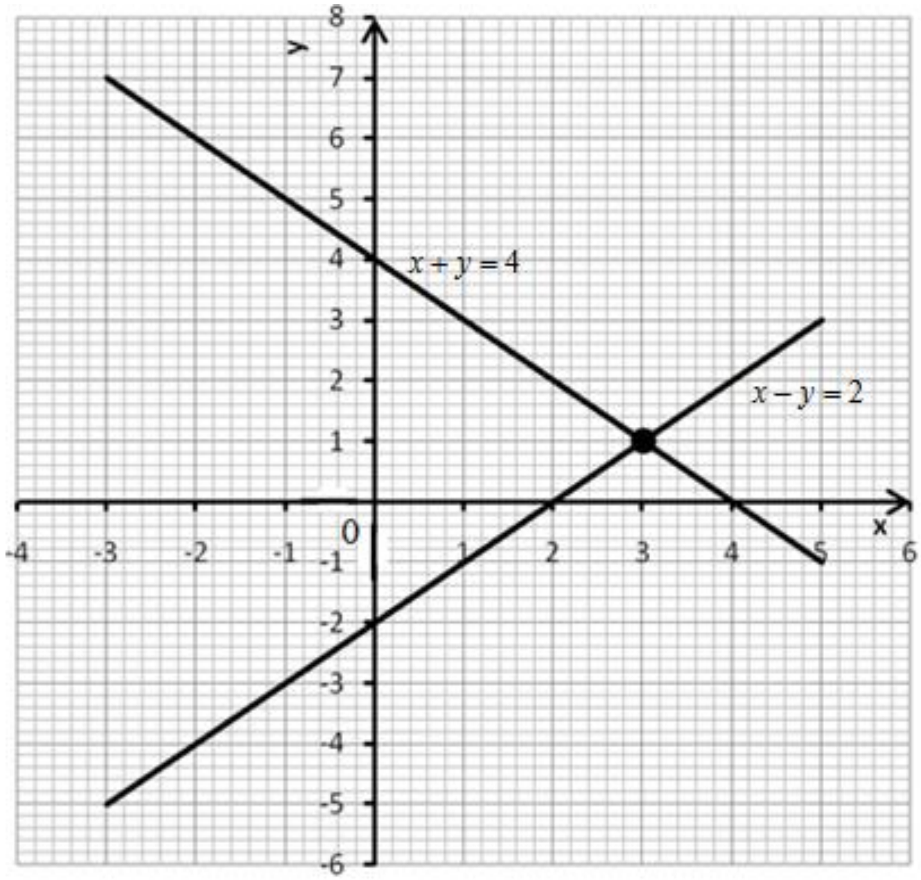
For $x+y=4$

x	-3	5
y	7	-1

For $x-y=2$

x	-3	5
y	-5	3

Graph



The two lines intersect at point $(3,1)$. Therefore the solution set is $S=\{(3,1)\}$.

2) Solve graphically the following system of linear equations

Solve the following equations graphically

$$\begin{cases} x+y=2 \\ 2y=4-2x \end{cases}$$

Solution

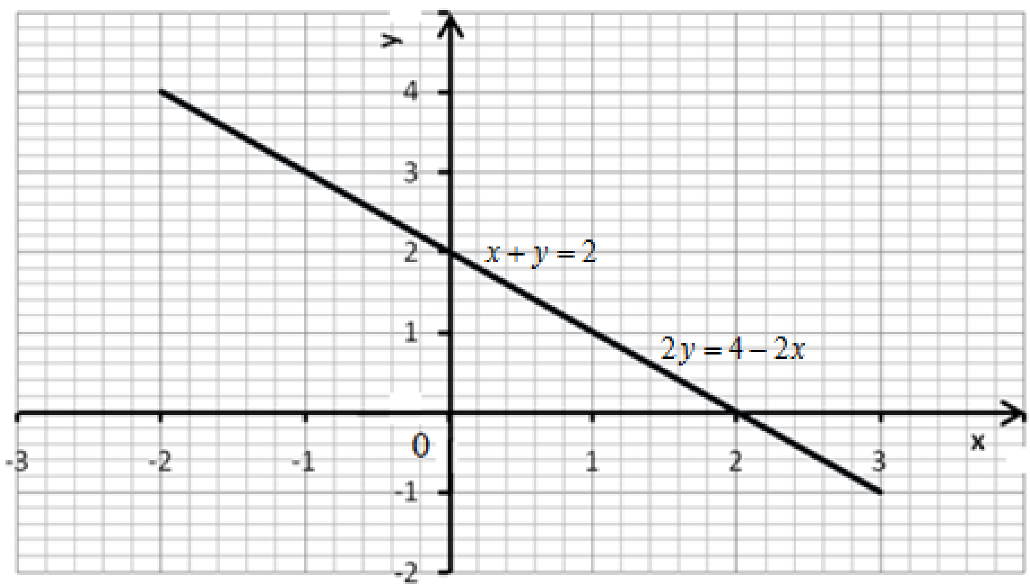
For $x+y=2$

x	-2	3
y	4	-1

For $2y = 4 - 2x$

x	-2	3
y	4	-1

Graph



We see that the two lines coincide as a single line. In such case there is infinite number of solutions.

Application activity 2.11

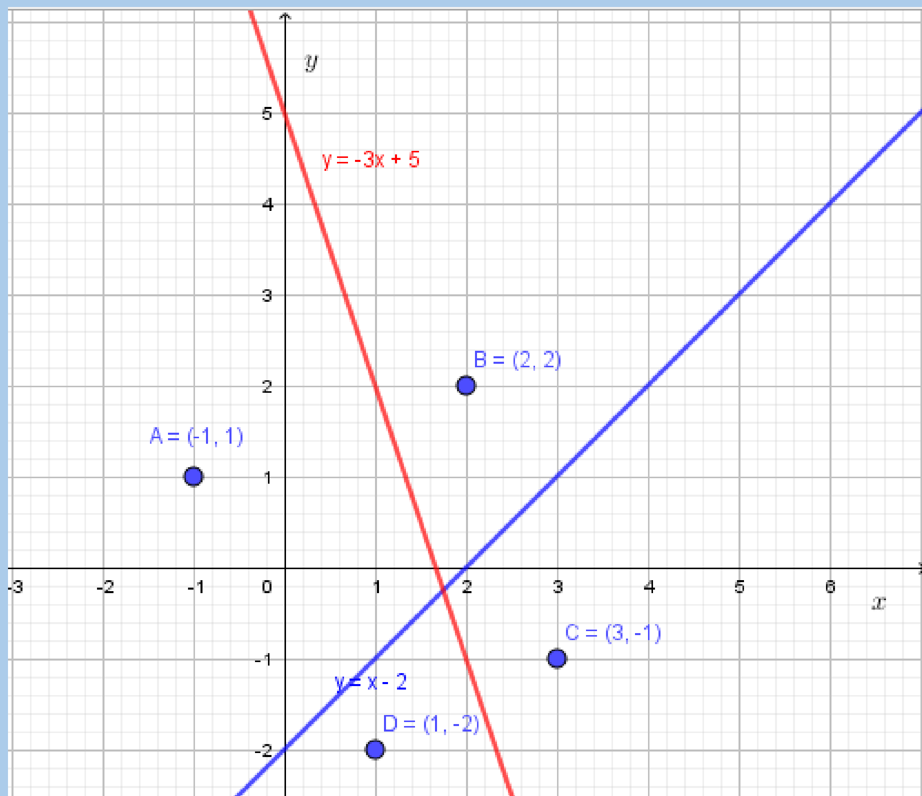
Solve the graphically the following system of linear equations

1. $\begin{cases} 4y + x = 8 \\ -x + y = 2 \end{cases}$ 2. $\begin{cases} 3x + y = 1 \\ x - y = -1 \end{cases}$ 3. $\begin{cases} 2x + y = 3 \\ 6x + 3y = 9 \end{cases}$

2.12 Solving algebraically and graphically simultaneous linear inequalities in two unknowns

Activity 2.12

1) The following graph illustrates two lines and their equations



For each point A, B, C, and D, replace its coordinate in the two inequalities to verify which one satisfies the following system:

$$\begin{cases} y < x - 2 \\ y > -3x + 5 \end{cases}$$

CONTENT SUMMARY

A system of inequalities consits of a set of two inequalities with the same variables. The inequalities define the conditions that are to be considered simultaneously.

Each inequality in the set contains infinetely many ordered pair solutions defined by a region in rectangular coordinate plane. When considering two of these inequalities together, the intersection of these sets will define the set of simultaneous ordered pair solutions.

Linear inequalities with two unknowns are solved to find a range of values of the two unknowns which make the inequalities true at the same time. The solution is represented graphically by a region.

In finding solution, first, graph the "equals" line, then shade in the correct area.

The following steps used to find the solution of simultaneous inequalities graphically:

1. Rearrange the equation so "y" is on the left and everything else on the right.
2. Plot the y line (make it a solid line for $y \leq$ or $y \geq$ and a dashed line for $y <$ or $y >$
3. Shade above the line for a greater than $y >$ or $y \geq$ or below the line for a less than $y <$ or $y \leq$
4. The intersection will define the set of simultaneous ordered pair solutions.

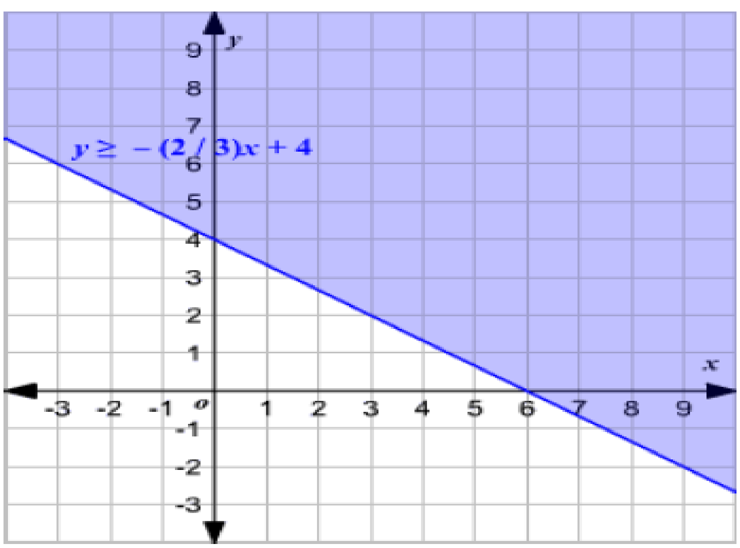
Example

Solve the system of inequalities by graphing:

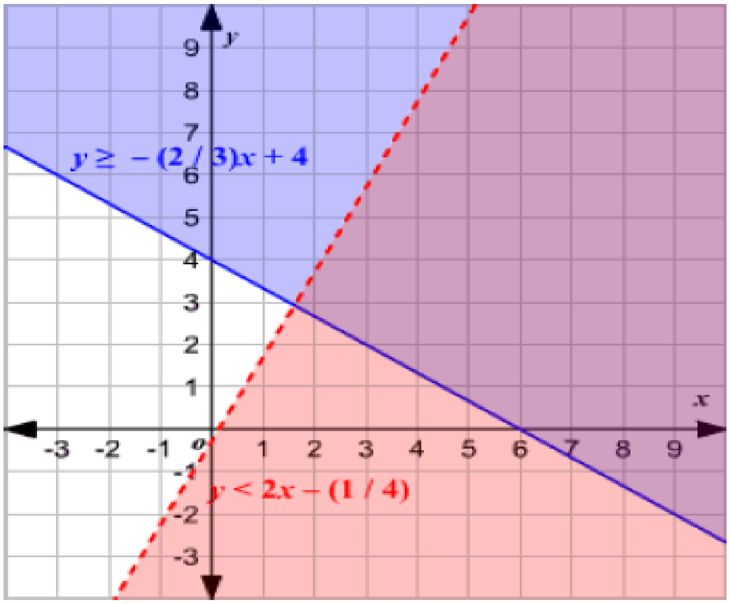
$$\begin{cases} 2x + 3y \geq 12 \\ 8x - 4y > 1 \\ x < 4 \end{cases}$$

Solution

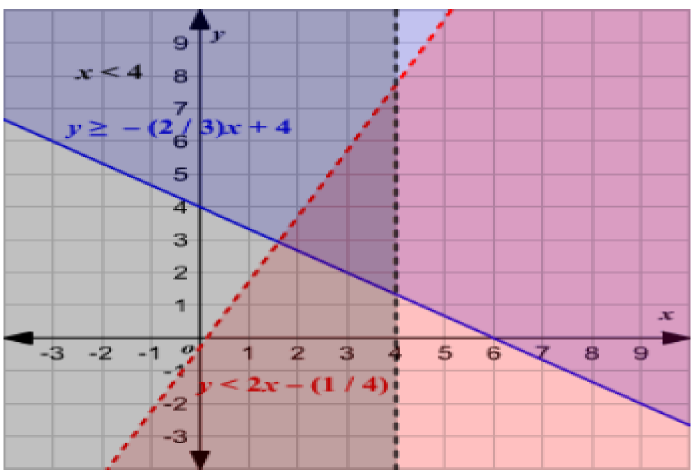
From Each inequality from the system we have $2x + 3y \geq 12$ which implies $y \geq -\frac{2}{3}x + 4$, the related equation to this is $y = -\frac{2}{3}x + 4$ since the inequality is $\geq$, not a strict one, the border line is solid. Graph the line $y = -\frac{2}{3}x + 4$



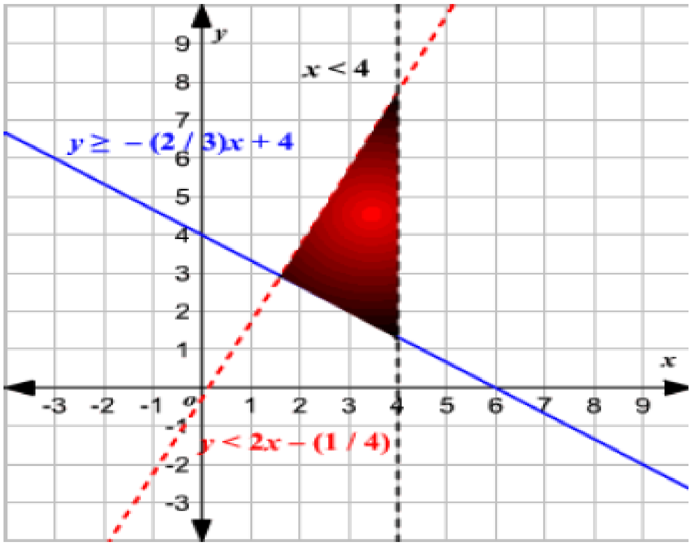
Similarly, draw a dashed line of related equation of the second inequality $y < 2x - \frac{1}{4}$ which has a strict inequality.



Draw the dashed vertical line $x = 4$ which is the related to the equation of the third inequality.



The solution of the system of inequalities is the intersection region of the solutions of the three inequalities as it is done in the following figure.



Application activity 2.12

1) Algebraically and graphically, solve the following simultaneous inequalities

a) $\begin{cases} y - 2x \leq 1 \\ x + y \leq 10 \\ x \geq 0 \end{cases}$ b) $\begin{cases} -3x + 2y > 6 \\ 6x - 4y > 8 \end{cases}$ c) $\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + y \leq 4 \end{cases}$

2) Use your own words to explain how to solve graphically the simultaneous inequalities.

2.13 Solving quadratic equations by the use of factorization

Activity 2.13

1) The sum of the square of a number and 15 is the same as eight times the number.
   a) Write the equation in the form $ax^2+bx+c=0$
   b) Hence, factorize the equation and find all such numbers.
2) John threw a rock off a bridge into the river. The distance from the rock to the river is modeled by the equation $y=t^2-7t-30$, where y is the height in meter and t is the time in seconds. Find how long it took the rock to hit the surface of the water.
   HINT: $-7t=-10t+3t$

CONTENT SUMMARY

Equations which are written in the form of $ax^2+bx+c=0$ $(a\neq 0)$ are called quadratic equations. To find solution of this equation the two main ways can be used in solving such equation

Use of factorization or finding square roots

Grouping terms or decomposition can be used to factorize the quadratic equations and later help us to find the solution of equation. By having the product of ac and the sum of those two integers which gives b, it helps you to decompose into a product of factors.

Example

1) Solve in $\mathbb{R}$: $6y^2+5y-25=0$

Solution: To obtain a common factor, terms that have a common factor are grouped together, and the common factor of each group is divided as follows

$6y^2+5y-25=0$

Guess two integers whose sum is 5 and product is -150

$6y^2 + 15y - 10y - 25 = 0$

$(6y^2 + 15y) - (10y + 25) = 0$

$3y(2y+5) - 5(2y+5) = 0$

$(2y + 5)(3y - 5) = 0$

$(2y + 5) = 0$ or $(3y - 5) = 0$

$y = \frac{-5}{2}$ or $y = \frac{5}{3}$

2) Solve the equation $x^2 - 4y - 45 = 0$

Solution: $x^2 - 4x - 45 = 0$

$(x - 9)(x + 5) = 0$

$x = 9 \quad or \;\; x = -5$

Then, the solution set is $S = \{-5, 9\}$.

3) Find the solution set of $(3x - 1)(2x + 3) = -5$

Solution: we expand the product as the equation is different from zero

$6x^2 + 7x - 3 = -5$

$6x^2 + 7x - 3 + 5 = 0$

$(3x + 2)(2x + 1) = 0$

$3x + 2 = 0 \qquad or \qquad 2x + 1 = 0$

$x = -\frac{2}{3}$ or $x = -\frac{1}{2}$ Thus, the solution set is $\left\{-\frac{2}{3}, -\frac{1}{2}\right\}$.

Experimental version

Application activity 2.13

a) Use factorization and discriminant to solve the following equations

1) $3x^2 = 10 - x$ 2) $x^2 - 3x = -11$

3) $x^2 - 12x + 11 = 0$ 4) $x^2 + 16 = 8x$

b) The area of a rectangular garden is 30 meter square. If the length is 7 meter longer than the width, find the dimensions of the rectangle.

c) Does a quadratic equation have more than one solution? Explain your answer.

2.14 Concept of discriminant

Activity 14

Make research in the library or computer lab and by means of examples prepare a short presentation on the following:

a) What is the discriminant of the quadratic equation of the form $ax^2 + bx + c = 0$?

b) How to find the discriminant of a quadratic equation?

c) Identify the different discriminant rules.

CONTENT SUMMARY

Equations which are written in the form of $ax^2 + bx + c = 0$ $(a \neq 0)$ are called quadratic equations.

- The discriminant of a quadratic equation of the form $ax^2 + bx + c = 0$ is $\Delta = b^2 - 4ac$
- To find the discriminant given the quadratic equation $ax^2 + bx + c = 0$, simply record the values of a, b, and c and then substitute them into the discriminant formula:

$\Delta = b^2 - 4ac$. This will give the value of the discriminant. It tells if the solutions to the quadratic equation are real number or not as well as how many solutions to the equation exist.

➢ Here are the rules for the discriminant: Suppose that $f(x)=0$ is a quadratic equation, then:

- If $\Delta = b^2 - 4ac > 0$, there are two real roots
- If $\Delta = b^2 - 4ac = 0$, there is one double real root
- If $\Delta = b^2 - 4ac < 0$, there are no real roots

Example:

Equation	Discriminant	Nature of solutions
$4x^2 - 7x - 1 = 0$	$\Delta = b^2 - 4ac = (-7)^2 - 4(4)(-1) = 49 + 16 = 65$	Two real roots
$4x^2 + 12x + 9 = 0$	$\Delta = b^2 - 4ac = (12)^2 - 4(4)(9) = 144 - 144 = 0$	One double real root
$5x^2 + 2x + 1 = 0$	$\Delta = b^2 - 4ac = (2)^2 - 4(5)(1) = 4 - 20 = -16$	There are no real roots

Application activity 2.14

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation:

a) $9x^2 - 12x + 4 = 0$

b) $2x^2 + 16x + 33 = 0$

c) $2x^2 - 5x + 1 = 0$

2.15 Solving quadratic equations by the use of discriminant

Activity 2.15

1.For what value of k will the equation $x^2+2x+k=0$ have one double roots. Find that root.

2. Use discriminant to solve in $\mathbb{R}$ the following equation: $6x^2+5x-25=0$

CONTENT SUMMARY

The quadratic equations which are written in the form of $ax^2+bx+c=0$ $(a\neq 0)$ can be solved by the use of discriminant.

In quadratic equation: $ax^2+bx+c=0$, Let $\Delta=b^2-4ac$ be discriminant,

- ✓ The two values of unknown x are generated as follows:

$$x_1=\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

- ✓ Note that, when the $\Delta=0$ two equal roots are generated as $x_1=x_2=\frac{-b}{2a}$.
- ✓ When $\Delta<0$ the equation has no root in the set of real numbers.

These two values of x can help us to write down factor form of a quadratic expression.

Examples

1) Solve in $\mathbb{R}$: $6x^2+5x-25=0$

Solution: $\boldsymbol{a=6}$, $\boldsymbol{b=5}$, $\boldsymbol{c=-25}$

$\Delta=b^2-4ac$, $\Delta=\boldsymbol{b^2-4ac}$

$\Delta = (5)^2 - 4(6)(-25) = 625$,

$\sqrt{\Delta} = \sqrt{625} = 25$

$x_1 = \frac{-5+25}{12} = \frac{20}{12} = \frac{5}{3}$ $\qquad$ $x_2 = \frac{-5-25}{12} = \frac{-30}{12} = \frac{-5}{2}$

Then, the solution set is $S = \left\{-\frac{5}{2}, \frac{5}{3}\right\}$.

2) For what value of k will the equation: $4x^2 - 4x + k = 0$ have one double roots? Find that root.

Solution:

For one double root $\Delta = 0$**.**

$\Delta = b^2 - 4ac$

$\Delta = (-4)^2 - 4 \times 4 \times \mathrm{k}$

$\Delta = 16 - 16\mathrm{k}$

$16 - 16\mathrm{k} = 0 \Rightarrow -16k = -16 \Rightarrow k = 1$

Thus, the value of k is 1.

That root is $x = \frac{-b}{2a} = \frac{-(-4)}{2 \times 4} = \frac{4}{8} = \frac{1}{2}$. Then, the solution set is $S = \left\{\frac{1}{2}\right\}$.

Application activity 2.15

a) Use discriminant to solve the following equations

1) $3x^2 = 10 - x$ 2) $x^2 - 3x = -11$

3) $x^2 - 12x + 11 = 0$ 4) $x^2 + 16 = 8x$

b) The area of a rectangular garden is 30 meter square. If the length is 7 meter longer than the width, find the dimensions of the rectangle.

c) Does a quadratic equation have more than one solution? Explain your answer

2.16 Applications of linear equations: Problems about supply and demand (equilibrium price), finance

Activity 2.16

Assume that a firm can sell as many units of its product as it can manufacture in a month at 180 Rwandan francs each. It has to pay out 2400 Rwandan francs fixed costs plus a marginal cost of 140 Rwanda francs for each unit produced. How much does it need to produce to break even (where total revenue equals to total cost)?

CONTENT SUMMARY

When only two or single variables and equations are involved, a simultaneous equation system can be related to familiar graphical solutions, such as supply and demand analysis.

For **example,** assume that in a competitive market the demand schedule is given by

$p = 420 - 0.2q$ *a*nd the supply schedule is given by $p = 60 + 0.4q$,

If this market is in equilibrium, the equilibrium price and quantity will be where the demand and supply schedules intersect. This requires you to solve the system formed by the two simultaneous equations. Its solution will correspond to a point which is on both the demand schedule and the supply schedule. Therefore, the equilibrium values of p and q will be such that both equations (1) and (2) hold.

Example

1) In a competitive market the demand schedule is given by $p = 420 - 0.2q$ *a*nd the supply schedule is given by $p = 60 + 0.4q$, solve for p and q the simultaneous equation and determine the point at which the market is in equilibrium.

Solution:

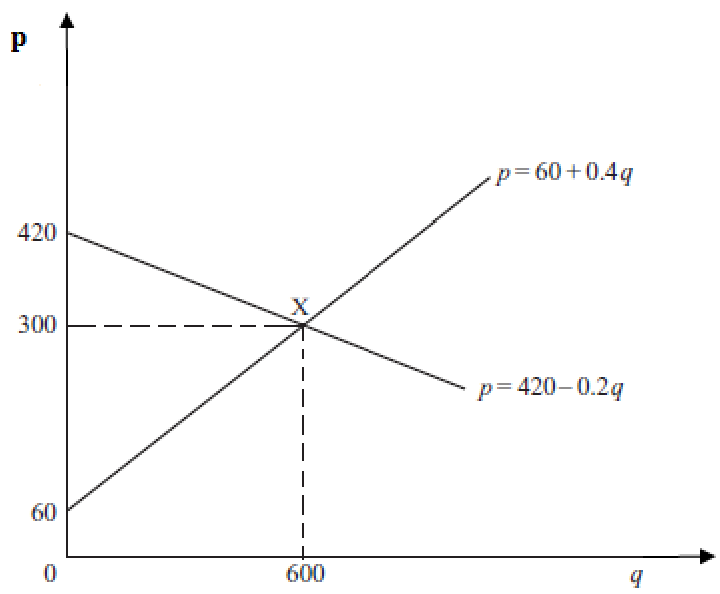
Let us solve the system

$$\begin{cases} p = 420 - 0.2q \\ P = 60 + 0.4q \end{cases}$$

Equalizing the value of p, we find $420 - 0.2q = 60 + 0.4q$.

Which gives $q = 600$. Replacing this value in the given two equations, we find $p = 300$. The market is in equilibrium at the point $q = 600$.

These two functional relationships are plotted in the figure and both hold at the intersection point $X(600, 300)$.



Experimental version

2) Calculate the equilibrium values of p and q in a competitive market where the demand schedule is $p = 200q^{-1}$ and the supply is $p = 30 + 2q$

Solution

In equilibrium, demand price equals supply price. Therefore

$200q^{-1} = 30 + 2q$

Multiplying through by q, $200 = 30q + 2q^2$

$0 = 30q + 2q^2 - 200$

$0 = (2q - 10)(q + 20)$

Therefore $2q - 10 = 0$ and $q + 20 = 0$

$q = 5$ or $q = -20$

We can ignore the second solution as negative quantities cannot exist.

Thus, the equilibrium quantity is 5.

Substituting this value into the supply function gives equilibrium price

$p = 30 + 2 \times 5 = 40$

3) A firm makes two goods A and B which require two inputs K and L. One unit of A requires 6 units of K plus 3 units of L and one unit of B requires 4 units of K plus 5 units of L. The firm has 420 units of K and 300 units of L at its disposal. How much of A and B should it produce if it wishes to exhaust its supplies of K and L totally?

Solution:

This question requires you to use the economic information given to set up a mathematical problem in a format that can be used to derive the desired solution.

The total requirements of input K are 6 for every unit of A and 4 for each unit of B, which

Can be written as $K = 6A + 4B$

Similarly, the total requirements of input L can be specified as $L = 3A + 5B$

As we know that $K = 420$ and $L = 300$ because all resources are used up

Then, $420 = 6A + 4B$ and $300 = 3A + 5B$

Solve the system of linear equations to find values of A and B

$$\begin{cases} 420 = 6A + 4B \\ 300 = 3A + 5B \end{cases}$$

From first equation $420 = 6A + 4B \Longrightarrow A = \frac{420-4B}{6}$

From second equation $300 = 3A + 5B \Longrightarrow A = \frac{300-5B}{3}$

$$\frac{420-4B}{6} = \frac{300-5B}{3}$$

$$1260 - 12B = 1800 - 30B$$

$$-12B + 30B = 1800 - 1260$$

$$18B = 540$$

$B = 30$ or $A = \frac{420-4(30)}{6} = 50$

The firm should therefore produce 50 units of A and 30 units of B.

3) Two student teachers were driving at constant speeds to the same direction during their holidays. The first travelling at 40 km/hr left Kigali at 8:30 a.m. The second travelling at 60 km/hr followed him after 1 hour.

a) What is the distance between the two divers one hour after the departure of the first driver?

b) If t is the same time used by the two drivers after the departure of the second driver and y the entire distance covered,

i) Establish the functions $y = f(t)$ of the first;

ii)) Establish the functions $y = g(t)$ of the second driver

iii) When did the second bus over take the first bus?

iv) Illustrate the two functions on the Cartesian plan from $t = 0$ to $t = 5hrs$ and show the point where the second met the first driver.

Solution:

The speed of the first driver is 40 km/hour. $v_1 = 40km/hr$

The second driver has a speed of 60km/hr and left after the departure of the first. $v_2 = 60km/hr$.

a) Distance covered by the first driver is $y_0 = v_1.t = (40km/hr).1hr = 40km$

b) i) The distance covered by the first driver is the initial distance plus the distance covered after the departure of the second driver.

This is $y = y_0 + v_1.t = 40 + 40.t$.

Then $y = f(t) = 40 + 40.t$

ii) The distance y covered by the second driver is $y = v_2.t = 60.t$.

Therefore, $y = g(t) = 60.t$

iii) After the departure of the second driver, the two drivers will be using the same time t and when this second driver will meet the first, the two will be covered the same distance

$d = y = g(t) = f(t)$

This means: $40 + 40.t = 60.t$

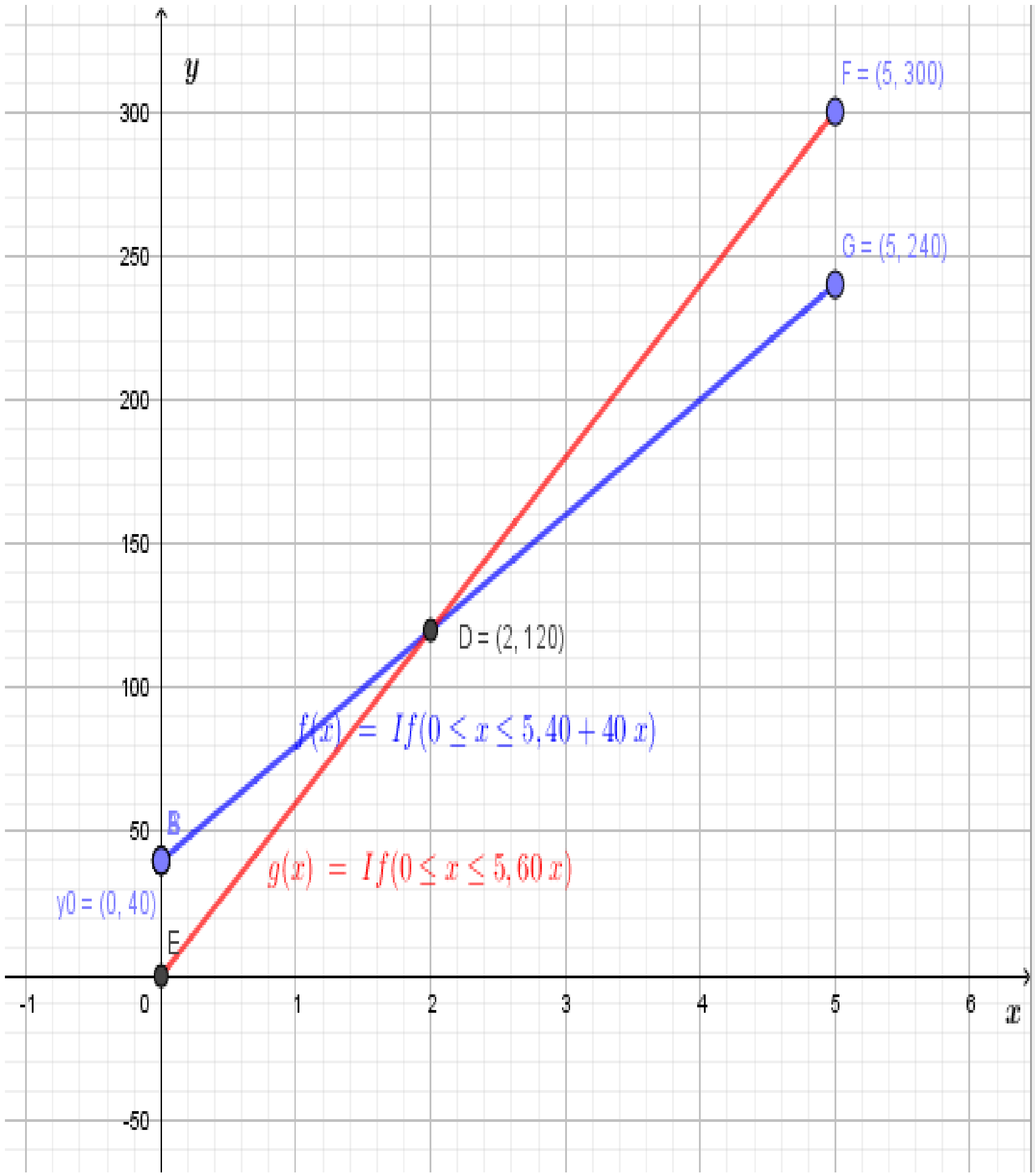
$$\begin{aligned}
&40 + 40.t = 60.t \\
&60t - 40t = 40 \\
&20t = 40 \\
&t = 2
\end{aligned}$$

The second driver will meet the first after 2 hours. It will be at 11h30 because

9h30+2hs =11h30.

iv) The table of values can help to draw the graphs of the function f and g.

T	**0**	**1**	**2**	**3**	**4**	**5**
$y = f(t) = 40 + 40.t$ **(in km)**	40	80	120	160	200	240
$y = g(t) = 60.t$ **(in km)**	0	60	120	180	240	300



Experimental version

Application activity 2.16

Read carefully the following word problems and solve the related equations

1. Assume that in a competitive market the demand schedule is $p = 420 - 0.2q$ and the supply schedule is $p = 60 + 0.4q$ ($p = price$, $q = quantity$). If the market is in equilibrium, then the equilibrium price and quantity will be where the demand and supply schedules intersect. As this will correspond to a point which is on both the demand schedule and the supply schedule the equilibrium values of p and q will be such that both equations hold. Find the equilibrium quantity and the equilibrium price by solving the following $420 - 0.2q = 60 + 0.4q$

2. Today a house-worker has 3000 Frw. His boss pays him/her 400Frw per day and he saves all the money received.

   a) What is the money the house-worker will have after 2 days, 5 days and after 10 days ?

   b) Discuss the function $y = f(t)$ for the money saved by the house-worker where t represents the time in days.

   c) Illustrate the function $y = f(t)$ in the Cartesian plan on a manila paper;

   d) Discuss the money the house-worker will have after 2 months.

   e) Is it possible for a house-worker to save the money? Explain your answer.

3. The national income in the basic Keynesian macroeconomic model is given by $Y = C + I$.

If C is given by $40 + 0.5Y$ and $I = 200$. Calculate the national income in the basic Keynesian macroeconomic model.

2.17 End unit assessment

1. Solve by factorization the quadratic equation $2x^2 - 6x - 20 = 0$
2. Solve the inequalities $-4x - 3 > -2x - 11$
3. Algebraically and graphically, solve the simultaneous inequalities $\begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + y \leq 4 \end{cases}$
4. The length of a rectangular garden is $5cm$ more than its width and the area is $50cm^2$. Calculate the length and width of the garden.
5. A ball is thrown upwards from a rooftop, 80m above the ground. It will reach a maximum vertical height and then fall back to the ground. The height of the ball from the ground at time t is h, which is given by, $h = -16t^2 + 64t + 80$.

a. What is the height reached by the ball after 1 second?

b. What is the maximum height reached by the ball?

c. How long will it take before hitting the ground?

6. Two cyclists move away from a town along two perpendicular paths at $20km/hr$ and $40km/hr$ respectively. The second cyclist starts the journey an hour later than the first one. Find the time taken for them to be $100km$ apart.

UNIT 3: DESCRIPTIVE STATISTICS

Key Unit competence: Analyse and interpret statistical data from daily life situations

3.0 Introductory activity

1. At the market a fruit-seller has the following daily sales Rwandan francs for five consecutive days: 1000Frw, 1200Frw, 125Frw, 1000Frw, and 1300Frw. Help her to determine the money she could get if the sales are equally distributed per day to get the same total amount of money.

2. During the welcome test of Mathematics for the first term 10 student-teachers of year one language education scored the following marks out of 10: 3, 5,6,3,8,7,8,4,8 and 6.

   a) What is the mean mark of the class?

   b) Chose the mark that was obtained by many students.

   c) Compare the differences between the mean of the group and the mark for every student teacher.

3.1 Definition and type of data

Activity 3.1

Carry out research on statistics to determine the meanings of statistics and types of data. Use your findings to select qualitative and quantitative data from this list: Male, female, tall, age, 20 sticks, 45 student-teachers, and 20 meters, 4 piece of chalks.

CONTENT SUMMARY

Statistics is the branch of mathematics that **deals with** data collection, data organization, summarization, analysis and draws conclusions from data.

The use of graphs, charts, and tables and the calculation of various statistical measures to organize and summarize information is called descriptive statistics. Descriptive statistics helps to reduce our information to a manageable size and put it into focus.

Every day, we come across a wide variety of information in form of facts, numerical figures or table groups. A **variable** is a characteristic or attribute that can assume different values. **Data** are the values (measurements or observations) that the variables can assume.

Variables whose values are determined by chance are called **random variables.** A collection of data values forms a **data set.** Each value in the data set is called a **data value** or a **datum.**

For example, information related to profit/ loss of the school, attendance of students and tutors, used materials, school expenditure in term or year, etc. These facts or figure which is numerical or otherwise, collected with a definite purpose is called data. This is the word derived from Latin word Datum which means pieces of information.

Qualitative variable

The qualitative variables are variables that cannot be expressed using a number. A qualitative data is determined when the description of the characteristic of interest results is a non-numerical value. A qualitative variable may be classified into two or more categories. Data obtained by observing values of a qualitative variable are referred to as **qualitative data**.

Examples

Qualitative variable	**Possible categories of data**
Marital status	Single, married, divorced
Gender	Male, Female
Pain level	None, moderated, severe
Colour	Red, black, green, yellow

The possible categories for qualitative variables are often coded for the purpose of performing computerized statistical analysis so that we have qualitative data.

Quantitative variable

Quantitative variables are variables that are expressed in numerical terms, counted or compared on a scale. A quantitative data is determined when the description of the characteristic of interest results in numerical value. When measurement is required to describe the characteristic of interest or when it is necessary to perform a count to describe the characteristic, a quantitative variable is defined.

Discrete data is a quantitative data whose values are countable. Discrete data usually result from counting.

Continuous data is a quantitative data that can assume any numerical value over an interval or over several intervals. Continuous data usually results from making a measurement of some type. Data obtained by observing values of a quantitative variable are referred to as **quantitative data**. Quantitative data obtained from a discrete variable are also referred to as discrete data and quantitative data obtained from a continuous variable are called continuous data.

Examples

Discrete variable	**Possible values of data**
The number of defective needles in boxes of 100 diabetic syringes	0,1,2,……………..100
The number of individuals in groups of 30 with a type **A** personality	0,1,2,3,……………30
Number of student-teachers in classroom	0,1,2,………………45

Continuous variable	**Possible values of data**
The household income for households with incomes less than or equal to 200,000 Rwandan francs	All the real numbers between A and 200,000 where A is the smallest household income in the population
Height	All values for the length in length measurements.

Data can be used in different ways. The body of knowledge called statistics is sometimes divided into two main areas, depending on how data are used. The two areas are descriptive statistics and inferential statistics.

Descriptive statistics consists of the collection, organization, summarization, and presentation of data.

Inferential statistics consists of generalizing from samples to populations, performing estimations and hypothesis tests, determining relationships among variables, and making predictions.

A **population** consists of all subjects (human or otherwise) that are being studied.

A **sample** is a group of subjects selected from a population.

Application Activity 3.1

1. Select qualitative and quantitative data from the list below:

   Product rating, basketball team classification, number of student-teachers in the classroom, weight, age, number of rooms in a house, number of tutors in school.
2. The table below gives the fasting blood sugar reading for five patients at a small medical clinic. What is the variable? Are they continuous? Compare this data and give your observations.

patient	Fasting blood sugar level
1^{st} Patient	125
2^{nd} patient	175
3^{rd} patient	160
4^{th} patient	110

3.2 Presentation of data using frequency distribution table

Activity 3.2

At the beginning of the school year, a Mathematics test was administered in Senior four to 50 student-teachers to test their level of understanding. Their results out of 20 were recorded as follow:

16	10	11	11	17	12	13	16	15	15
10	9	9	10	11	9	17	10	16	8
18	10	17	9	11	10	14	12	9	8
17	18	15	13	10	10	18	9	10	18
9	10	10	11	11	9	16	11	11	9

Construct a frequency distribution table to help the teacher and the school administration to easily recognize the level of understanding for the Senior four learners.

CONTENT SUMMARY

After the collection of data, the researcher needs to organize and present them in order to help those who will benefit from the research and lead him or her to the conclusion. When the data are in original form, they are called **raw data** and are listed next.

Raw data

After the collection of data, one can present them in the raw form presentation.

Example

The following are numbers of notebooks for 54 student-teachers.

3 4 5 6 6 7 3 2 5 4 5 7 4 3 2 3 6 5

8 5 6 4 2 6 5 3 4 7 9 8 6 7 4 5 4 6

3 4 3 6 7 8 2 7 6 5 4 6 4 7 8 9 9 5

Since little information can be obtained from looking at raw data, the researcher organizes the data into what is called a *frequency distribution.* A frequency distribution consists of the number of times each value appears in the raw data and sometimes the corresponding percentage vis-à-vis the total number in the sample.

1. Frequency distribution

Data can be presented in various forms depending on the type of data collected. A **frequency distribution** is the organization of raw data in the table form by the use of frequencies.

Example

The following data represent marks obtained by 12 student-teachers out of 20 in mathematics test of a certain TTC.

13	10	15	17	17	18
17	17	11	10	17	10

The set of outcomes is displayed in a frequency table, as illustrated below:

<table>
<tr><th>Marks</th><th>Tallies</th><th>Frequencies (fi)</th></tr>
<tr><td>10</td><td>||</td><td>2</td></tr>
<tr><td>11</td><td>|</td><td>1</td></tr>
<tr><td>13</td><td>|</td><td>1</td></tr>
<tr><td>15</td><td>||</td><td>2</td></tr>
<tr><td>17</td><td><s>||||</s></td><td>5</td></tr>
<tr><td>18</td><td>|</td><td>1</td></tr>
<tr><td colspan="2">Total</td><td>12</td></tr>
</table>

The total frequency is the number of items in the population. In the survey about the marks of student-teachers above, the total frequency is the number of all student-teachers. We find this by adding up the numbers from the third column of the table:
$2+1+1+2+5+1$ which is equal to 12.

Grouped data

When the number of data is too large, a simple distribution is not appropriate. In this case we come up with a grouped frequency distribution. A grouped frequency distribution organizes data into **groups or classes.**

Definitions related to grouped frequency distribution

a) **Class limits:** The class limits are the lower and upper values of the class
b) **Lower class limit:** Lower class limit represents the smallest data value that can be included in the class.
c) **Upper class limit:** Upper class limit represents the largest data value that can be included in the class.
d) Class mark or class midpoint:

$$class\,mid\ po\mathrm{int} = \frac{Lower\ class + upper\ class}{2}$$

Example:

The following data represent the marks obtained by 40 students in Mathematics test. Organize the data in the frequency table; grouping the values into classes, stating from 41-50:
54 83 67 71 80 65 70 73 45 60 72 82 79 78 65 54 67 64 54 76 45 63 49 52
60 70 81 67 45 58 v69 53 65 43 55 68 49 61 75 52.

Solution:

Classes	Class midpoint x	Frequency f
41-50	45.5	6

51-60	55.5	10
61-70	65.5	13
71-80	75.5	8
81-90	85.5	3

The classes: 41-50, 51-60, 71-80, 81-90

Lower class limits: 41, 51, 61, 71, and 81

Upper class limits: 50, 60, 70, 80, and 90

$$Class\ midpoint\ for\ the\ first\ class = \frac{41+50}{2} = 45.5$$

Class boundaries

Class boundaries are the midpoints between the **upper class limit** of a class and **the lower class limit of the next class.**

Therefore each class has a lower and an upper class boundary.

Example

Classes	**Class midpoint**	**Frequency**
$[5{,}10[$	7.5	2
$[10{,}15[$	12.5	6
$[15{,}20[$	17.5	4
$[2{,}25[$	22.5	3
$[25{,}30[$	27.5	4
$[30{,}35[$	32.5	1

For$[10{,}15[The\ lower\ class\ boundary\ is\ 10, The\ upper\ class\ boundary]$ is 15

$$Class\ width = 15 - 10 = 5$$

We have **Categorical Frequency Distributions and Grouped Frequency Distributions**

Experimental version

The **categorical frequency distribution** is used for data that can be placed in specific categories, such as nominal- or ordinal-level data.

Example:

Twenty-five army inductees were given a blood test to determine their blood type.

The data set is:

A B B AB O O O B AB B B B O

A O A O O O AB AB A O B A

Class	Tally	Frequency	Percent
A	𝍸	5	20
B	𝍸𝍷𝍷	7	28
AB	𝍸𝍷𝍷𝍷𝍷	9	36
O	𝍷𝍷𝍷𝍷	4	16
Total		25	100

Grouped Frequency Distributions

When the range of the data is large, the data must be grouped into classes that are more than one unit in width, in what is called a **grouped frequency distribution.** For example, a distribution of the number of hours that boat batteries lasted is the following

Class limits	Class boundaries	Tally	Frequency
24–30	23.5–30.5	///	3
31–37	30.5–37.5	/	1
38–44	37.5–44.5	~~////~~	5
45–51	44.5–51.5	~~////~~ ////	9
52–58	51.5–58.5	~~////~~ /	6
59–65	58.5–65.5	/	1
			25

The procedure for constructing a grouped frequency distribution, i.e., when the classes contain more than one data value

These data represent the record high temperatures in degrees Fahrenheit (*F) for each of the 50 states. Construct a grouped frequency distribution for the data using 7 classes.

112	100	127	120	134	118	105	110	109	112
110	118	117	116	118	122	114	114	105	109
107	112	114	115	118	117	118	122	106	110
116	108	110	121	113	120	119	111	104	111
120	113	120	117	105	110	118	112	114	114

Source: *The World Almanac and Book of Facts.*

1) Determine the classes

- Find the highest value and lowest value: H =134 and L=100.
- Find the range: R = highest value-lowest value H -L, so R=134 -100 = 34

- Select the number of classes desired (usually between 5 and 20). In this case, 7 is arbitrarily chosen.

- Find the class width by dividing the range by the number of classes

$$width = \frac{R}{Number\ of\ classes} = \frac{34}{7} = 4.9$$

- Select a starting point for the lowest class limit. This can be the smallest data value or any convenient number less than the smallest data value. In this case, 100 is used. Add the width to the lowest score taken as the starting point to get the lower limit of the next class. Keep adding until there are 7 classes, as shown, 100, 105, 110, etc.

- Subtract one unit from the lower limit of the second class to get the upper limit of the first class. Then add the width to each upper limit to get all the upper limits. 105-1=104.

The first class is 100–104, the second class is 105–109, etc.

- Find the class boundaries by subtracting 0.5 from each lower class limit and adding 0.5 to each upper class limit:

99.5–104.5, 104.5–109.5, etc.

2) Tally the data

3) Find the numerical frequencies from the tallies.

The completed frequency distribution is

Class limits	Class boundaries	Tally	Frequency
100–104	99.5–104.5	//	2
105–109	104.5–109.5	~~////~~ ///	8
110–114	109.5–114.5	~~////~~ ~~////~~ ~~////~~ ///	18
115–119	114.5–119.5	~~////~~ ~~////~~ ///	13
120–124	119.5–124.5	~~////~~ //	7
125–129	124.5–129.5	/	1
130–134	129.5–134.5	/	1
			$n = \Sigma f = 50$

2. Cumulative frequency

The cumulative frequency corresponding to a particular value is the sum of all frequencies up to the last value including the first value. Cumulative frequency can also defined as the sum of all previous frequencies up to the current point.

Example:

The set of data below shows marks obtained by student-teachers in Mathematics. Draw a cumulative table for the data.

11	15	18	15	10	16	11	10	17
13	17	11	17	16	17	15	13	16

Solution:

The cumulative frequency at a certain point is found by adding the frequency at the present point to the cumulative frequency of the previous point.

Marks	**Frequencies** (fi)	**Cumulative frequencies** (cufi)
10	2	2
11	3	$2+3=5$
13	2	$5+2=7$
15	3	$7+3=10$
16	3	$10+3=13$
17	4	$13+4=17$
18	1	$17+1=18$

3. Relative frequency and percentage

The relative frequency is obtained by dividing the frequency for every data by the sum of all the frequencies. The percentage for a category is obtained by multiplying the relative frequency for that category by 100.

Example:

Marks	**Frequencies** (fi)	**Relative frequency**	**percentage**
10	2	$2/18=0.111$	11.1
11	3	$3/18=0.167$	16.7
13	2	$2/18=0.111$	11.1
15	3	$3/18=0.167$	16.7

16	3	$3/18 = 0.167$	16.7
17	4	$4/18 = 0.222$	22.2
18	1	$1/18 = 0.55$	5.5

Application activity 3.2

1. Suppose that a teacher conducted a test for learners and the marks out of 10 were as follows: 3 3 3 5 6 4 6 7 8 3 8 8 8 10 9 10 9 10 8 10 6
Present this data using frequency distribution table.
2. In a quiz, the marks obtained by 20 learners out of 30 are given as follows:
12, 15, 15, 29, 30, 21, 30, 30, 15, 17, 19, 15, 20, 20, 16, 21, 23, 24, 23, 21
Present this data in the form of a frequency distribution.
3. The mass of 50 tomatoes (measured to the nearest g) were measured and recorded in the table below.

86	101	114	118	87	92	93	116	105
102	92	93	101	111	96	117	100	106
118	101	107	96	101	102	104	92	99
107	98	105	113	100	103	108	92	109
95	100	103	110	113	99	106	116	101
105	86	88	108	92				

Construct a frequency distribution table, using equal class intervals of width 5g and taking the lower class boundary of the first interval as 84.5g.

3.3 Presentation of data using bar chart

Activity 3.3

At the beginning of the school year student-teachers came to school with all requirements materials and others don't attend the school on first day. The table below shows the number of student-teachers who attended the school in 5 classrooms on first day.

Number of student-teachers	4	5	6	7	8
Classroom	A	B	C	D	E

Present the above data on bar chart and discuss what you can say when interpreting the chart.

CONTENT SUMMARY

A bar chart or bar graph is a chart or graph that presents numerical data with rectangular bars with heights or lengths proportional to the values that they represent. The bars can be plotted vertically or horizontally. A vertical bar chart is sometimes called a line graph.

To draw the bar chart, always choose a suitable scale for the vertical axis.

Example

The table below shows number of learners per class in a school in Rwanda.

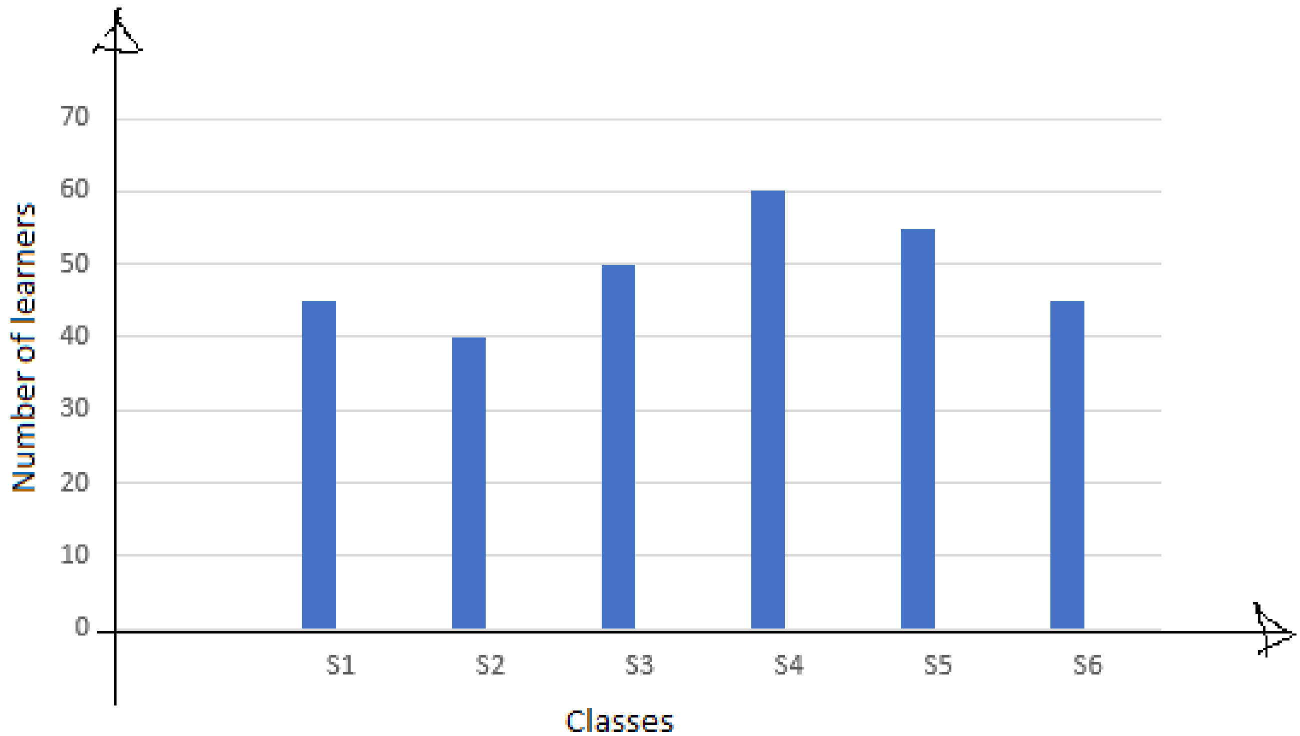
Class	S1	S2	S3	S4	S5	S6
Number of learners	45	40	50	60	55	45

a) Represent the data in a bar chart

b) How many learners are in the whole school?

Solution

a) A bar graph showing number of learners per class in a school



b) The number of learners that are in the whole school $= 45 + 40 + 50 + 60 + 55 + 45 = 295$

The school has 295 learners.

Application activity 3.3

An insurance company determines vehicle insurance premiums based on known risk factors. If a person is considered a higher risk, their premiums will be higher. One potential factor is the color of your car. The insurance company believes that people with some color cars are more likely to get in accidents. To research this, they examine police reports for recent total loss collisions. The data is summarized in the frequency table below:

Color	Frequency
Blue	25
Green	52
Red	41
White	36
Black	39
Grey	23

a) From the frequency table above, identify the highest frequency and the lowest frequency.
b) Present the car data on bar chart indicating frequency against vehicle color involved in total loss collision.

3.4 Presentation of data using pie chart

Activity 3.4

1. Conduct a research in the library or on the internet about the meaning of pie chart and explain steps to be done when statistical data are presented using pie chart.
2. One teacher of any secondary school wants to check the level of how his/her learners like different subjects taught in Language Education option. The survey is done on 60 student-teachers. Here is the number of participants of the survey.

Subject	**Number of students**
English	12
Mathematics	24
French	6
Entrepreneurship	10
Kinyarwanda	8

Present these data on a pie chart. Using that chart, explain the data.

CONTENT SUMMARY

A pie chart, sometimes called a circle chart, is a way of summarizing a set of data in circular graph. This type of chart is a circle that is divided into sections or wedges according to the percentage of frequencies in each category of the distribution. Each part is represented in degrees.

To present data using pie chart, the following steps are respected:

Step 1: Write all the data into a table and add up all the values to get a total.

Step 2: To find the values in the form of a percentage divide each value by the total and multiply by 100. That means that each frequency must also be converted to a percentage by using the formula

$$\% = \frac{f}{n}.100\%$$

Step 3: To find how many degrees for each pie sector we need, we take a full circle of 360° and use the formula:

$$Angle\ for\ sector\ S = \frac{Frequency\ of\ S \times 360^0}{Total\ frequency}$$

Since there are 360^0 in a circle, the frequency for each class must be converted into a proportional part of the circle. This conversion is done by using the formula:

$$Degrees = \frac{f}{n}.360^0$$

where f is frequency for each class and n is the sum of the frequencies. Hence, the following conversions are obtained. The degrees should sum to 360.

Step 4: Once all the degrees for creating a pie chart are calculated, draw a circle (pie chart) using the calculated measurements with the help of a protractor, and label each section with the name and percentages or degrees.

Example.

1) In the summer, a survey was conducted among 400 people about their favourite beverages: 2% like cold-drinks, 6% like Iced-tea, 12% like Cold-coffee, 24% like Coffee and 56% like Tea.

- **a)** How many people like tea?
- **b)** How many more people like coffee than cold coffee?
- **c)** What is the total central angle for iced tea and cold-drinks?
- d) Draw a pie chart to represent the provided information.

Solution:

a) Total number of people = 400

Number of people like tea $= 400 \times \frac{56}{100} = 224$

b) Number of people like coffee $= 400 \times \frac{24}{100} = 96$

Number of people like cold-coffee $= 400 \times 12 = 48$

Number of people like coffee more than cold-coffee $= 96 - 48 = 48$

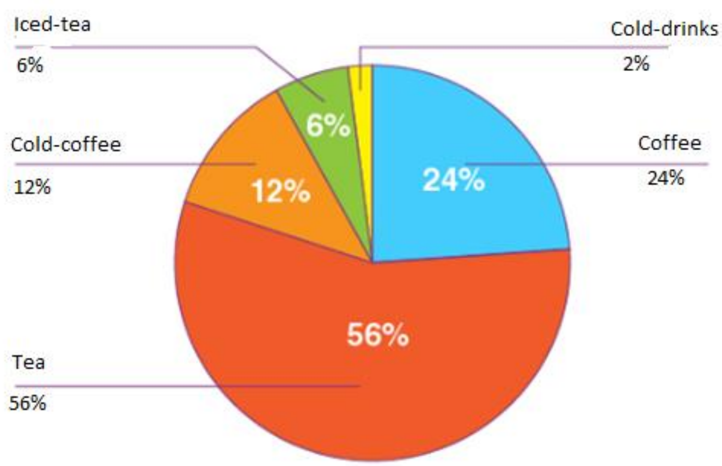
c) Number of people like iced-tea $= 400 \times \frac{6}{100} = 24$

Number of people like cold-drinks $= 400 \times \frac{2}{100} = 8$

Central angle for iced-tea $= \frac{24}{400} \times 360^0 = 21.6^0$

Central angle for cold-drinks $= \frac{8}{400} \times 360^0 = 7.2^0$ = 8/400 × 360^o = 7.2^o

Total central angle $= 21.6^0 + 7.2^0 = 28.8^0$ = 21.6^o + 7.2^o = 28.8^o.



2) Construct a pie graph showing the blood types of the army inductees described above. The frequency distribution is repeated here.

Class	Frequency	Percent
A	5	20
B	7	28
O	9	36
AB	4	16
	25	100

Solution:

Step 1: Find the number of degrees for each class, using the formula

$Degrees = \frac{f}{n} \times 360^0$.

For each class then the following results are obtained:

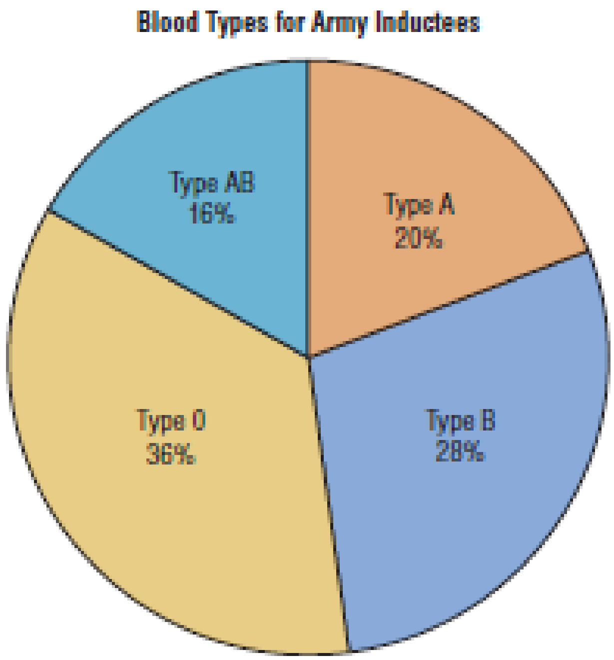
$A: \ \frac{5}{25}.360^0 = 72^0$; $\quad$ $\text{B}: \ \frac{7}{25}.360^0 = 100.8^0$;

$\text{O}: \ \frac{9}{25}.360^0 = 129.6^0$; $\quad$ $AB: \ \frac{4}{25}.360^0 = 57.6^0$

Step 2: Find the percentages.

A: $\frac{5}{25} \times 100\% = 20\%$ **B:** $\frac{7}{25} \times 100\% = 28\%$ **O:** $\frac{9}{25} \times 100\% = 36\%$ **AB:** $\frac{4}{25} \times 100\% = 16\%$

Step 3: Using a protractor, graph each section and write its name and corresponding percentage, as shown in the Figure



Application activity 3.4

A person spends his time on different activities daily (in hours):

Activity	Office work	exercise	Travelling	Watching shows	Sleeping	Miscellaneous
Number of hours	9	1	2	3	7	2

a) Find the central angle and percentage for each activity.

b) Draw a pie chart for this information

c) Use the pie chart to comment on these findings.

3.5 Presentation of data using histogram and polygon

Activity 3.5

The mass of 50 tomatoes (measured to the nearest g) were measured and recorded in the table below.

Class boundaries (in g)	84.5 – 89.5	89.5 – 94.5	94.5 – 99.5	99.5 – 104.5	104.5 – 109.5	109.5 – 114.5	114.5 – 119.5
Frequency	4	7	6	13	10	5	5
Cumulative frequency	4	11	17	30	40	45	50

a) Determine cumulative frequency

b) Try to draw a histogram

CONTENT SUMMARY

After you have organized the data into a frequency distribution, you can present them in graphical form. The purpose of graphs in statistics is to convey the data to the viewers in pictorial form. It is easier for most people to comprehend the meaning of data presented graphically than data presented numerically in tables or frequency distributions.

The three most commonly used graphs in research are

1. The histogram.

2. The frequency polygon.

3. The cumulative frequency graph or ogive (pronounced o-jive).

a) The Histogram

The **histogram** is a graph that displays the data by using contiguous vertical bars (unless the frequency of a class is 0) of various heights to represent the frequencies of the classes.

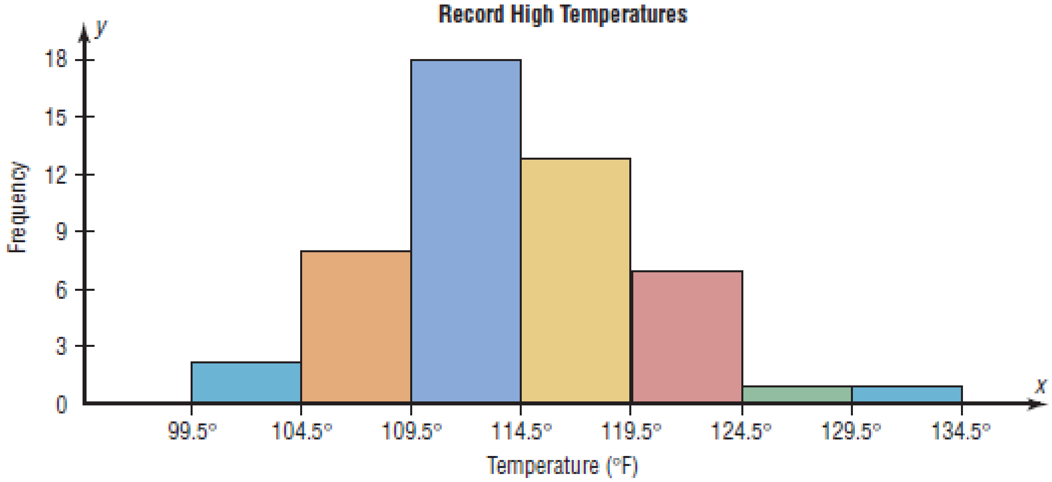
Example: Construct a histogram to represent the data shown for the record high temperatures for each of the 50 states.

Class boundaries	Frequency
99.5–104.5	2
104.5–109.5	8
109.5–114.5	18
114.5–119.5	13
119.5–124.5	7
124.5–129.5	1
129.5–134.5	1

Step 1: Draw and label the *x* and *y* axes. The *x* axis is always the horizontal axis, and the *y* axis is always the vertical axis.

Step 2: Represent the frequency on the *y* axis and the class boundaries on the *x* axis.

Step 3: Using the frequencies as the heights, draw vertical bars for each class. See Figure below



b) The Frequency Polygon

The **frequency polygon** is a graph that displays the data by using lines that connect points plotted for the frequencies at the midpoints of the classes. The frequencies are represented by the heights of the points.

Experimental version

Example:

Using the frequency distribution given in Example 2–4, construct a frequency polygon

Step 1: Find the midpoints of each class. Recall that midpoints are found by adding the upper and lower boundaries and dividing by 2:

$\frac{99.5+104.5}{2}=102$, $\frac{104.5+109.5}{2}=107$ and so on.

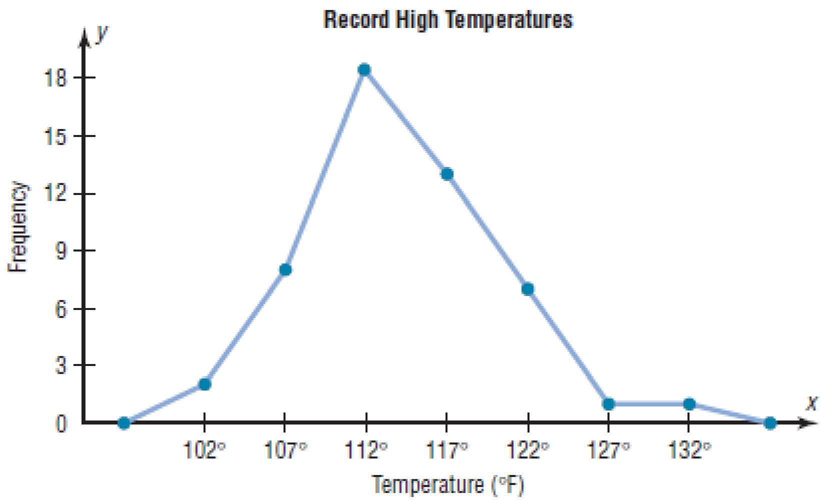
The midpoints are:

Class boundaries	Midpoints	Frequency
99.5–104.5	102	2
104.5–109.5	107	8
109.5–114.5	112	18
114.5–119.5	117	13
119.5–124.5	122	7
124.5–129.5	127	1
129.5–134.5	132	1

Step 2: Draw the *x* and *y* axes. Label the *x* axis with the midpoint of each class, and then use a suitable scale on the *y* axis for the frequencies.

Step 3: Using the midpoints for the *x* values and the frequencies as the *y* values, plot the points.

Step 4: Connect adjacent points with line segments. Draw a line back to the *x* axis at the beginning and end of the graph, at the same distance that the previous and next midpoints would be located, as shown in Figure 2–3.



The frequency polygon and the histogram are two different ways to represent the same data set. The choice of which one to use is left to the discretion of the researcher.

c) The Ogive

The **ogive** is a graph that represents the cumulative frequencies for the classes in a frequency distribution.

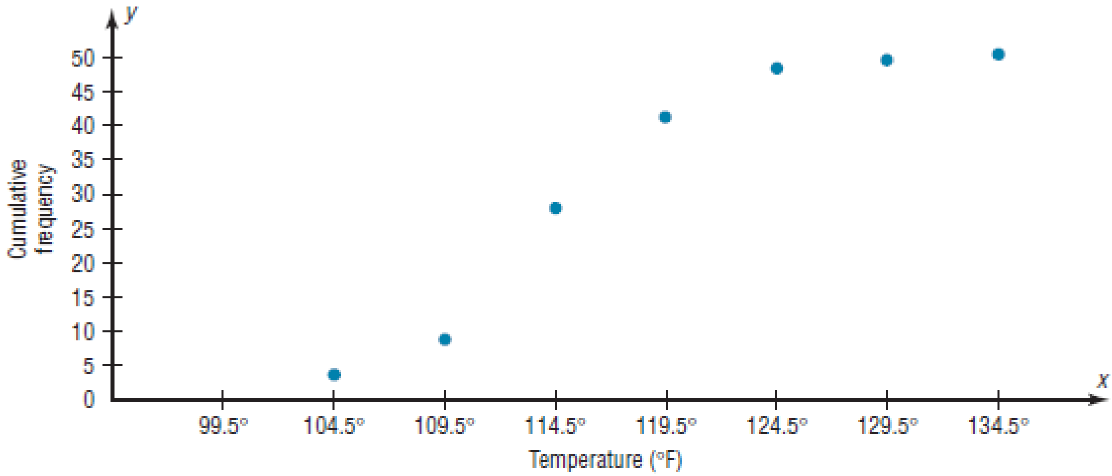
Step 1: Find the cumulative frequency for each class.

	Cumulative frequency
Less than 99.5	0
Less than 104.5	2
Less than 109.5	10
Less than 114.5	28
Less than 119.5	41
Less than 124.5	48
Less than 129.5	49
Less than 134.5	50

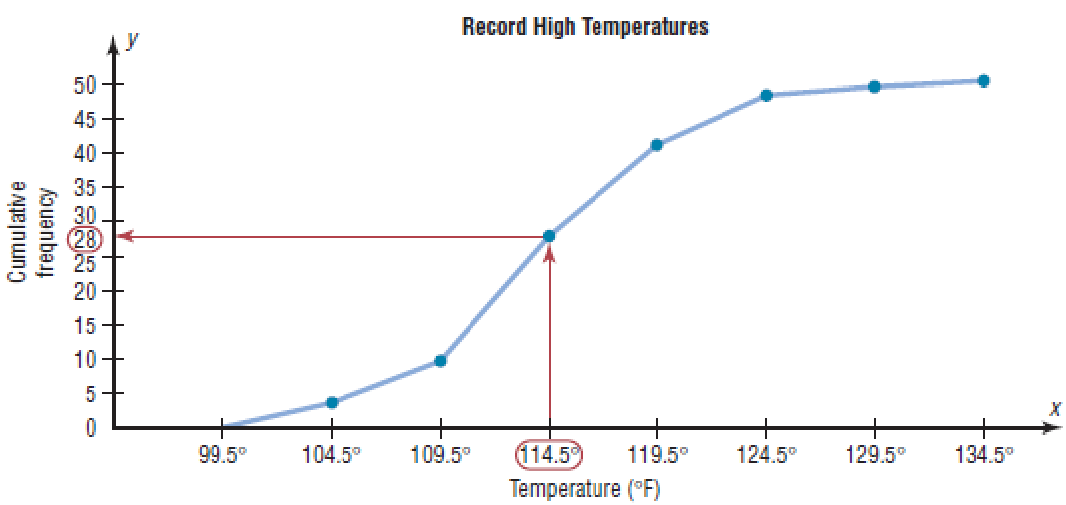
Step 2: Draw the *x* and *y* axes. Label the *x* axis with the class boundaries. Use an appropriate scale for the *y* axis to represent the cumulative frequencies.
(Depending on the numbers in the cumulative frequency columns, scales such as 0, 1, 2, 3, . . . , or 5, 10, 15, 20, . . . , or 1000, 2000, 3000, . . . can be used.

Do *not* label the *y* axis with the numbers in the cumulative frequency column.) In this example, a scale of 0, 5, 10, 15, . . . will be used.

Step 3: Plot the cumulative frequency at each upper class boundary, as shown in Figure below. Upper boundaries are used since the cumulative frequencies represent the number of data values accumulated up to the upper boundary of each class.



Step 4: Starting with the first upper class boundary, 104.5, connect adjacent points with line segments, as shown in the figure. Then extend the graph to the first lower class boundary, 99.5, on the *x* axis.



Experimental version

Cumulative frequency graphs are used to visually represent how many values are below a certain upper class boundary. For example, to find out how many record high temperatures are less than 114.5_F, locate 114.5_F on the *x* axis, draw a vertical line up until it intersects the graph, and then draw a horizontal line at that point to the *y* axis. The *y* axis value is 28, as shown in the figure.

d) Relative Frequency Graphs

The histogram, the frequency polygon, and the ogive shown previously were constructed by using frequencies in terms of the raw data. These distributions can be converted to distributions using *proportions* instead of raw data as frequencies. These types of graphs are called **relative frequency graphs.**

Example:

Construct a histogram, frequency polygon, and ogive using relative frequencies for the distribution (shown here) of the kilometers that 20 randomly selected runners ran during a given week.

Class boundaries	Frequency
5.5–10.5	1
10.5–15.5	2
15.5–20.5	3
20.5–25.5	5
25.5–30.5	4
30.5–35.5	3
35.5–40.5	2
	20

Solution:

Step 1: Convert each frequency to a proportion or relative frequency by dividing the frequency for each class by the total number of observations.

For class 5.5–10.5, the relative frequency is $\frac{1}{20} = 0.05$; for class 10.5–15.5, the relative frequency is $\frac{2}{20} = 0.10$; for class 15.5–20.5, the relative frequency is $\frac{3}{20} = 0.15$; and so on.

Place these values in the column labeled Relative frequency.

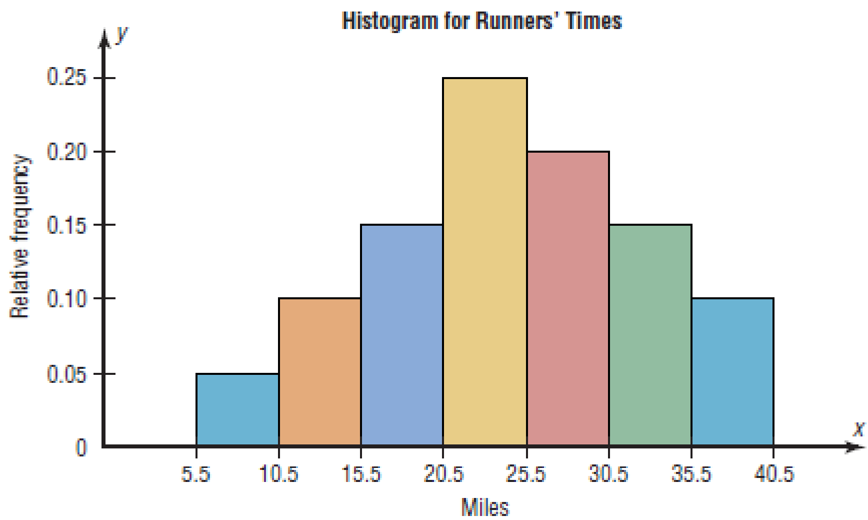
Class boundaries	Midpoints	Relative frequency
5.5–10.5	8	0.05
10.5–15.5	13	0.10
15.5–20.5	18	0.15
20.5–25.5	23	0.25
25.5–30.5	28	0.20
30.5–35.5	33	0.15
35.5–40.5	38	0.10
		1.00

Step 2: Find the cumulative relative frequencies. To do this, add the frequency in each class to the total frequency of the preceding class. In this case, 0 +0.05 = 0.05, 0.05 + 0.10 = 0.15, 0.15 +0.15 =0.30, 0.30 + 0.25 = 0.55, etc. Place these values in the column labeled Cumulative relative frequency.

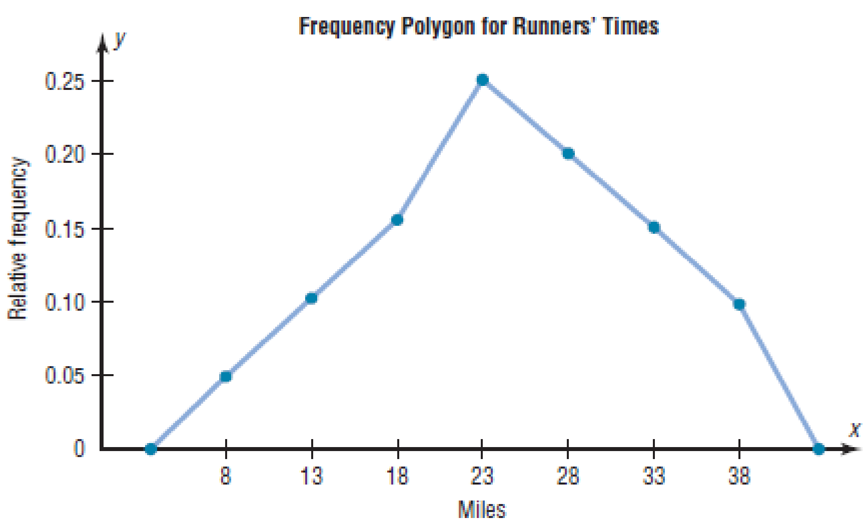
An alternative method would be to find the cumulative frequencies and then convert each one to a relative frequency.

	Cumulative frequency	Cumulative relative frequency
Less than 5.5	0	0.00
Less than 10.5	1	0.05
Less than 15.5	3	0.15
Less than 20.5	6	0.30
Less than 25.5	11	0.55
Less than 30.5	15	0.75
Less than 35.5	18	0.90
Less than 40.5	20	1.00

Step 3: Draw each graph as shown in Figure 2–7. For the histogram and ogive, use the class boundaries along the *x* axis. For the frequency polygon, use the midpoints on the *x* axis. The scale on the *y* axis uses proportions.

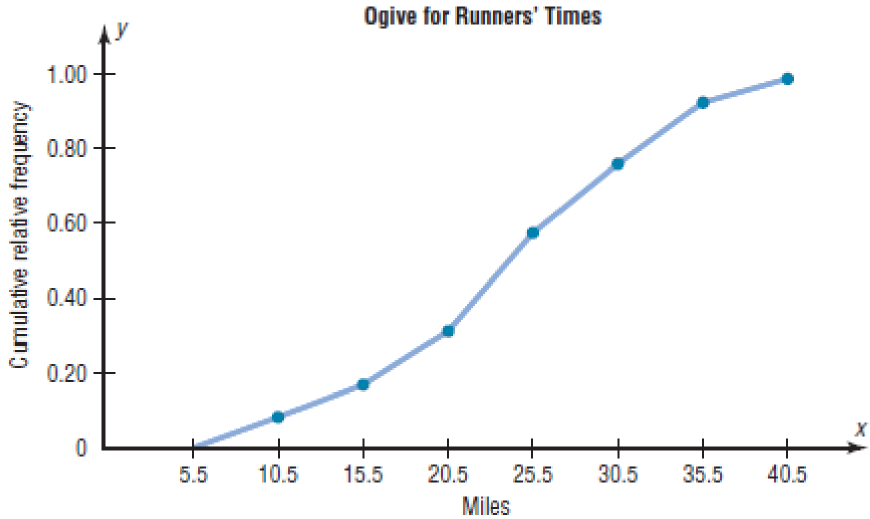


(a) Histogram



(b) Frequency polygon

Experimental version



(c) Ogive

Application activity 3.5

1. The cumulative frequency distribution table below shows distances (in km) covered by 20 runners during the week of competition.

Class boundaries	**Frequency**	**Cumulative frequency**
5.5-10.5	1	1
10.5-15.5	2	3
15.5-20.5	3	6
20`5-25.5	5	11
25.5-30.5	4	15
30.5-35.5	3	18
35.5-40.5	2	20

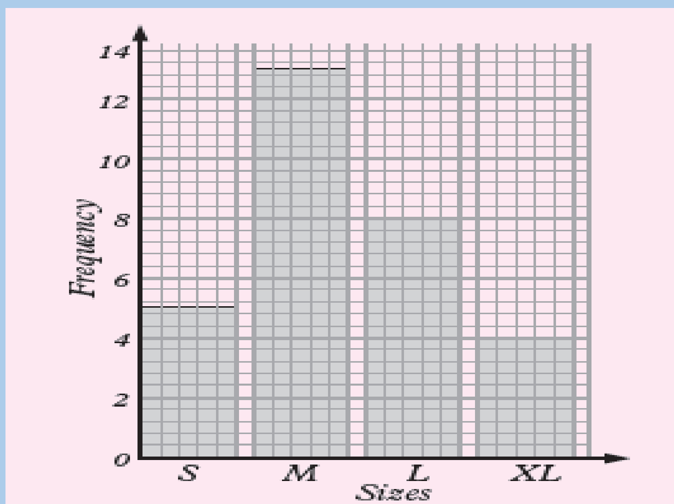
a) Construct a histogram

b) Construct the frequency polygon

3.6 Graph interpretation

Activity 3.6

The graph below shows the sizes of sweaters worn by 30 year 1 students in a certain school. Observe it and interpret it by answering the questions below:



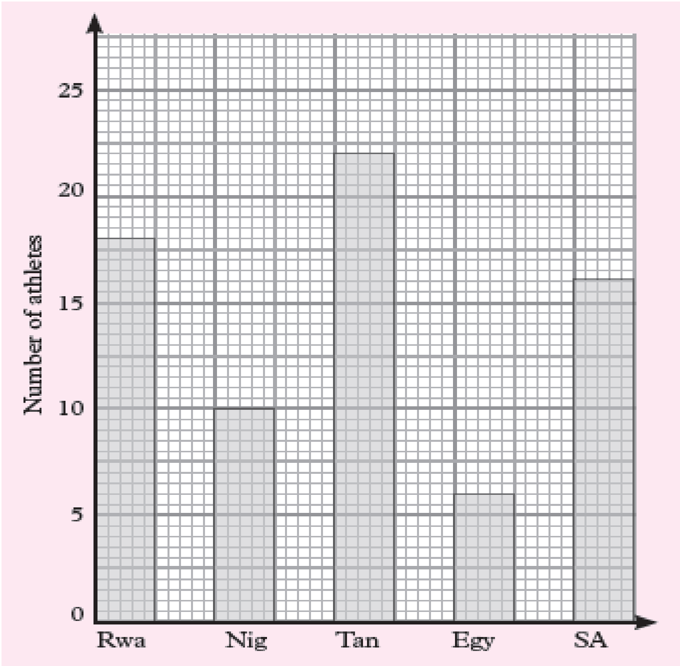
a. How many students are with small size?
b. How many students with medium size, large size and extra large size are there?

CONTENT SUMMARY

Once data has been collected, they may be presented or displayed in various ways including graphs. Such displays make it easier to interpret and compare the data.

Examples

1) The bar graph shows the number of athletes who represented five African countries in an international championship.



a) What was the total number of athletes representing the five countries?

b) What was the smallest number of athletes representing one country?

c) What was the most number of athletes representing a country?

d) Represent the information on the graph on a frequency table.

Solution:

We read the data on the graph:

a) Total number of athletes are: 18 + 10 + 22 + 6 + 16 = 72 athletes

b) 6 athletes

c) 22 athletes

d) Representation of the given information on the graph on a frequency table.

Country	**Number of athletes**
Rwanda	18
Nigeria	10
Tanzania	22
Egypt	6
South Africa	16
Total	**72**

2) Use a scale vertical scale 2cm: 10 students and Horizontal scale 2cm: 10 represented on histogram below to answers the questions that follows

a. estimate the mode
b. Calculate the range

Solution:

a) To estimate the mode graphically, we identify the bar that represents the highest frequency. The mass with the highest frequency is 60 kg. It represents the mode.

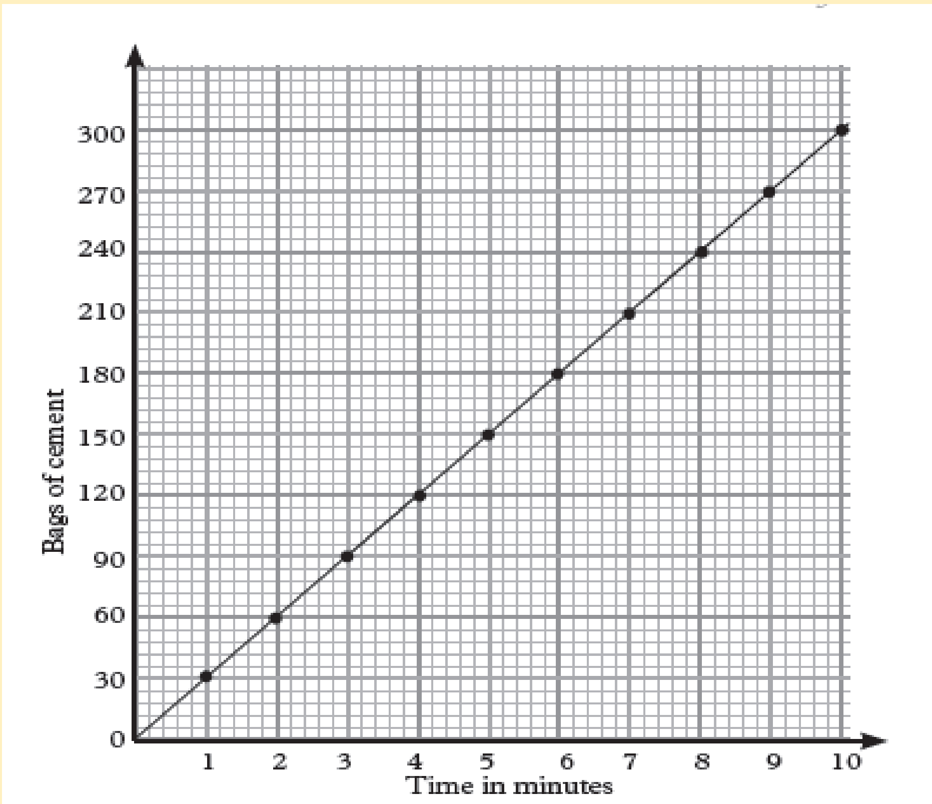
b) The highest mass = 67 kg and the lowest mass = 57 kg

Then, The range=highest mass-lowest mass= $67kg - 57kg = 10kg$

Experimental version

Application activity 3.3

The line graph below shows bags of cement produced by CIMERWA industry cement factory in a minute.



a) Find how many bags of cement will be produced in: 8 minutes, 3 minutes12 seconds, 5 minutes and 7 minutes.

b) Calculate how long it will take to produce: 78 bags of cement.

c) Draw a frequency table to show the number of bags produced and the time taken.

3.7 Measures of central tendencies for ungrouped data: mode and mean

Activity 3.4

Conduct a research in the library or on the internet and explain measures of central tendency, their types and provide examples.

Insist on explaining how to determine the Mean, Mode and their role when interpreting statistic data.

CONTENT SUMMARY

Measures of central tendency were studies in S1 and S2.

1. The mean

The *mean,* also known as the *arithmetic average,* is found by adding the values of the data and dividing by the total number of values.

The following steps are used to calculate the mean:

Step 1: Add the numbers

Step 2: Count how many numbers there are in the data set

Step 3: Find the mean by dividing the sum of the data values by the number of data values.

Suppose that a fruit seller earned the flowing money from Monday to Friday respectively: 300, 200, 600, 500, and 400 Rwandan francs. The mean of this money explains the same daily amount of money that she should earn to totalize the same amount in 5 days.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{300 + 200 + 600 + 500 + 400}{5} = 400$$

Or $\bar{x} = \frac{1}{n}\sum xfi$

This is called the mean and is equivalent to sharing out all data evenly.

2. The mode:

The mode is the number that appears the most often from the set of data. It represents the value which appears more frequently in the data. No calculation is needed to find the mode. You may have to sort the data, and then you count to find the value that appears more frequently.

The following steps are used to calculate the mode:

Step 1: First sort the data, writing the values in order from smallest to largest.

Step 2: Identify the mode, which is the value that appears the most often

Examples:

1) Calculate the mean and mode of the following set of numbers: 3, 4, 4, 6, 8, 5, 4, and 8

Solution: $\bar{x} = \frac{1}{n}\sum xfi$

x	fi	xfi
3	1	3
4	3	12
5	1	5
6	1	6
8	2	16
	$\sum fi = n = 8$	$\sum xfi = 42$

The mean is given by $\bar{x} = \frac{1}{n}\sum xfi$, $\bar{x} = \frac{42}{8} = 5.25$

Arrange data first 3, 4, 4, 4, 5, 6, 8,8. Total observation is 8,   Mode is 4.

2) Considering the following set data: 4; 6; 7; 3; 4; 8; 4; 2; 9.

Find the mean value and mode of the data set.

Solution

To find the mean:

Step 1: Add the numbers.

Sum of the data values = 4 + 6 + 7 + 3 + 4 + 8 + 4 + 2 + 9 = 47

Step 2: Count how many numbers there are in the data set.

Number of data values = 9.

Step 3: Find the mean by dividing the sum of the data values by the number of data values.

$$\textit{The mean} = \frac{\text{the sum of the data values}}{\text{the number of data values}} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{47}{9} = 5.2$$

The mean is 5.2.

Arrange the data first: 2, 3, 4, 4, 4, 6, 7, 8, 9. The mode is 4

Application activity 3.7

1) A group of learners from language education were asked how many books they had read in previous year, the results are shown in the frequency table below. Calculate the mean and mode of the number books read.

Number of books	0	1	2	3	4	5	6	7	8
Frequency (number of student teachers)	5	5	6	9	11	7	4	2	1

2) During oral presentation of internship report for year three student-teachers, the first 10 student-teachers scored the following marks out of 10:

8, 7, 9, 10, 8, 9, 8, 6, 7 and 10

Calculate the mean and the mode of the group.

3.8 Measures of central tendencies for ungrouped data: median

Activity 3.8

Conduct a research in the library or on the internet and explain measures of central tendency, their types and provide examples.

Insist on explaining how to determine the Median and its role when interpreting statistic data.

CONTENT SUMMARY

The median:

If the data is well arranged in an order from the smallest to the largest, the median is the middle number or the central number of the range.

When total observation ($\sum fi = n$) is odd the median is given by $\left(\frac{n+1}{2}\right)^{th}$ number which is located on this position. On the other side when n is even, $\frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right]$, then the median is a half of the sum of number located on those two positions.

Examples

1) Calculate the median of the following numbers: 4, 5, 7,2, 1

Solution:

Arrange data from lowest to highest number as 1, 2, 4, 5, 7, the position of the median $=\left(\frac{n+1}{2}\right)^{th}$ element ,

the position of the median $Me = \left(\frac{5+1}{2}\right)^{th} = 3^{rd}$ $position, Then\ Me = 4$

2) Calculate the median of the following numbers: 4, 5, 7, 2, 1 and 8

Solution:

Arrange numbers in ascending order: 1, 2, 4, 5, 7, 8. Total observation $(n)=6$ since the total observation is even then, the position of the median is $\left[\left(\frac{n}{2}\right)+\left(\frac{n}{2}+1\right)\right]^{th}$ element and $Me=\frac{1}{2}\left(\left[\left(\frac{n}{2}\right)+\left(\frac{n}{2}+1\right)\right]^{th}\text{ element}\right)$

Then, the position of the median is $\left[\left(\frac{6}{2}\right)+\left(\frac{6}{2}+1\right)\right]^{th}$ element **which is the** $(3+4)^{th}$ **and** $Me=\frac{4+5}{2}=4.5$**.**

Application activity 3.8

A secondary school has two cricket teams: a junior and a senior team. The junior team consists of 17 players (including reserves) and the senior team consists of 16 players (including reserves). The mass of each team member is given below. Use the data to answer the questions that follow:

Junior team masses (kg) $= \{56, 60, 67, 45, 51, 53, 64, 49, 56, 48, 42, 51, 64, 52, 64, 49, 50\}$

Senior team masses (kg) $= \{88, 81, 53, 62, 83, 68, 70, 62, 91, 78, 64, 74, 73, 54, 62, 62\}$

1) What is the mean mass of the senior team?
2) Arrange the masses of the senior team in ascending order.
3) Determine the mode of the senior team.
4) Determine the median of the senior team.
5) Calculate the mean of the masses of the junior team correct to one decimal digit.
6) Arrange the masses of the junior team in ascending order.
7) Calculate the mode of the masses of the junior team.
8) Calculate the median of the masses of the junior team.
9) Look at the answers you found for the junior and senior teams. Which measure do you think gives the best measure of the real ‘average’ of each data set?

Lesson 9: Measure of central tendency for grouped data (mode)

Activity 3.9

Carry out a research from reference books or from internet to discover how to determine the mode for grouped data and then answer the following question.

Suppose we have the following frequency distribution that shows the number of points scored per game by 60 basketball players:

Points Scored	Frequency
1-10	8
11-20	25
21-30	14
31-40	9
41-50	4

Determmine the mode of the above statistical data.

The mode for grouped data is found from the class with the largest frequency which is called **modal class** and it is determined using the following formula:

$$Mode = L + w\left(\frac{f_m - f_1}{(2f_m - f_1 - f_2)}\right)$$

Where:

- L : the lower limit of the modal class
- f_m : the modal frequency
- w : modal class width
- f_1 : the frequency of the immediate class below the modal class
- f_2 : the frequency of the immediate class above the modal class

Example:

Find the modal class for the frequency distribution of kilometers that 20 runners ran in one week.

Class	Frequency
5.5–10.5	1
10.5–15.5	2
15.5–20.5	3
20.5–25.5	5 ← Modal class
25.5–30.5	4
30.5–35.5	3
35.5–40.5	2

The modal class is 20.5–25.5, since it has the largest frequency. Sometimes the midpoint of the class is used rather than the boundaries; hence, the mode could also be given as 23 km per week. ??????????????????

Application activity 3.9

Suppose we have the following frequency distribution that shows the exam scored receive by 40 students in a certain class:

Exam Score	Frequency
51-60	4
61-70	8
71-80	15
81-90	8
91-100	5

Determine the modal of the above frequency distribution.

Lesson 10: Measure of central tendency for grouped data (mean)

Activity 3.10

1) Conduct a research in the library or on the internet and explain measures of central tendency for grouped data and provide examples.

Insist on explaining how to determine the Mean, Mode, Median and their role when interpreting statistic data.

2) The data below represent the number of kilometers (distance) covered by a sample of 20 runners during one week. Using the frequency distribution, find the mean.

A Class	B Frequency f	C Midpoint X_m	D $f \cdot X_m$
5.5–10.5	1		
10.5–15.5	2		
15.5–20.5	3		
20.5–25.5	5		
25.5–30.5	4		
30.5–35.5	3		
35.5–40.5	2		
	$n = 20$		

What does this mean represent considering the class in which it is located in the data?

CONTENT SUMMARY

The process of finding the mean is the same as the one applied in the ungrouped data with the exeption that the midpoints x_m of each class in grouped data plays the role of x_i used in ungrouped data.

$$\bar{X} = \frac{\sum f.X_m}{n}$$

Weighted mean

It is a mean of a data set in which not all values are equally represented. Find the weighted mean of a variable X by multiplying each value by its corresponding weight and dividing the sum of the products by the sum of the weights.

$$\bar{X} = \frac{w_1 X_1 + w_2 X_2 + w_3 X_3 + \ldots \mathrm{w}_n X_n}{w_1 + w_2 + w_3 + \ldots . + w_n} = \frac{\sum wX}{\sum w}$$

Where $w_1, w_2, \ldots w_n$ are the weights and $X_1, \mathrm{X}_2, \ldots \mathrm{X}_n$ are the values.

Example

A student received an A in English Composition I (3 credits), a C in Introduction to Psychology (3 credits), a B in Biology I (4 credits), and a D in Physical Education (2 credits). Assuming A=4 grade points, B=3 grade points, C =2 grade points, D =1 grade point, and F=0 grade points, find the student's grade point average.

Solution

Course	**Credits** (w)	**Grade** (x)
English Composition	**3**	**A (4 points)**
Introduction	**3**	**C (2 points)**
Biology I	**4**	**B (3 points)**
Physical Education	**2**	**D (1 points)**

$$\bar{X} = \frac{\sum wX}{\sum w} = \frac{3\times 4 + 3\times 2 + 4\times 3 + 2\times 1}{3+3+4+2} = \frac{32}{12} = 2.7$$

Application activity 3.5

The data below shows the marks scored by a group of students in a mathematics out of 100: 72; 63; 51; 25; 31; 49; 51; 27; 46; 42; 25; 39; 38; 39; 55; 38; 35; 64;67;37.

Use a grouped data of 5 intervals and determine the mean mark.

Lesson 11: Measure of central tendency for grouped data (median)

Activity 3.11

Carry out a research from reference books or from internet to discover how to determine the mode for grouped data and then answer the following question.

Suppose we have the following frequency distribution that shows the number of points scored per game by 60 basketball players:

Points Scored	Frequency
1-10	8
11-20	25
21-30	14
31-40	9
41-50	4

Determmine the median of the above statistical data.

CONTENT SUMMARY

In the case of continuous frequency of distribution, we first locate the median by cumulating the frequency until $(\frac{n}{2})^{th}$ point is reached. Finally, the median is determined within this class by using formula. The procedures thus involve the following steps:

1. Compute cumulative frequencies
2. Locate the median class in cumulative frequency column where the size of $\left(\frac{n}{2}\right)^{th}$ item falls.
3. Obtain the median value by applying the formula mentioned below

The median represents the value that lies directly in the middle of a dataset, when all of the values are arranged from smallest to largest.

Median of Grouped Data $= L + \frac{w}{f}\left(\frac{N}{2} - C_{<}\right)$ where:

L : Lower limit of median class

w : Width of median class

N : Total Frequency

$C_<$: Cumulative frequency up to median class

f : Frequency of median class median class.

Example

The following table shows the weekly consumption of electricity of 56 families

Weekly consumption	0-10	10-20	20-30	30-40	40-50
Number of families	16	12	18	6	4

Calculate the median weekly consumption.

Solution

Weekly consumption	**Frequency**	**Cumulative frequency**
0-10	16	16
10-20	12	28
20-30	18	46
30-40	6	52
40-50	4	56
Sum	56	

Median is the value of $\left(\frac{n}{2}\right)^{th}$ position $=\left(\frac{56}{2}\right)^{th} = 28^{th}$ position , which lies in the class 10-20. Thus 10-20 is the median class

$$Median = L + \frac{w}{f}\left(\frac{N}{2} - C_<\right)$$

$$Median = 10 + \frac{20-10}{12}\left(\frac{56}{2} - 16\right)$$

$$Median = 10 + 10 = 20$$

Hence the median is 20 units.

Question

1. Calculate the median for the following distribution

Class interval	130-140	140-150	150-160	160-170	170-180	180-190	190-200
Frequency	5	9	17	28	24	10	7

Application activity 3.11

Suppose we have the following frequency distribution that shows the exam scored receive by 40 students in a certain class:

Exam Score	Frequency
51-60	4
61-70	8
71-80	15
81-90	8
91-100	5

Determine the median of the above frequency distribution.

Lesson 12: Measures of dispersion (Quartiles)

Carry out a research from reference books or from internet to discover how to determine the quartiles for grouped data and then answer the following question.

The following table gives the amount of time (in minutes) spent on the internet each evening by a group of 56 students. Compute the cumulative frequency for the following frequency distribution.

Time spent on internet	9.5 – 12.5	12.5 – 15.5	15.5 – 18.5	18.5 – 21.5	21.5 – 24.5
Number of students	3	12	15	24	2

Find quartiles for the given grouped data.

CONTENT SUMMARY

The word dispersion has a technical meaning in statistics. The average measures the center of the data. It is one aspect of observations. Another feature of the observations is how the observations are spread about the center. The observation may be close to the center or they may be spread away from the center. If the observation are close to the center (usually the arithmetic mean or median), we say that dispersion, scatter or variation is small. If the observations are spread away from the center, we say that dispersion is large.

The study of dispersion is very important in statistical data. If in a certain factory there is consistence in the wages of workers, the workers will be satisfied. But if some workers have high wages and some have low wages, there will be unrest among the low paid workers and they might go on strikes and arrange demonstrations. If in a certain country some people are very poor and some are very high rich, we say there is economic disparity. It means that dispersion is large.

The extent or degree in which data tend to spread around an average is also called the dispersion or variation. Measures of dispersion help us in studying the extent to which observations are scattered around the average or central value. Such measures are helpful in comparing two or more sets of data with regard to their variability.

Quartiles are values that split up a dataset into four equal parts.

The middle term, between the median and first term is known as the first or Lower Quartile and is written as Q_1 . Similarly, the value of mid-term that lies between the last term and the median is known as the third or upper quartile and is denoted as Q_3. Second Quartile is the median and is written as Q_2.

You can use the following formula to calculate quartiles for grouped data:

$$Q_i = L + \frac{w}{f}\left(\frac{iN}{4} - C_<\right), i = 1,2,3$$

where:

L is lower limit of ith quartile class

w is width of ith quartile class

N is total frequency

$C_<$ is cumulative frequency of the class previous to the ith quartile class

f is frequency of the interval that contains the ith quartile.

Application activity 3.12

1. The Scores of students in a Math test is given in the table below:

Class Interval	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$	$60-70$
Frequency	6	8	12	10	5	4

Find quartiles for the given grouped data.

2. Suppose we have the following frequency distribution:

Class interval	$0.5-5.5$	$5.5-10.5$	$10.5-15.5$	$15.5-20.5$	$20.5-25.5$	$25.5-30.5$	$30.5-35.5$	$35.5-40.5$
Frequency	6	19	13	20	12	11	6	5

Calculate the value of the third quartile (Q_3) of this distribution.

Lesson 13: Measures of dispersion (Variance)

Before starting the third tern, tutor calculated the mean mark of five student-teachers got in second term in Mathematics and he/she obtained that the mean mark is $\overline{x} = 16.875$. Use this mean to complete the table below:

x	f	$x-\overline{x}$	$\left(x-\overline{x}\right)^2$	$f\left(x-\overline{x}\right)^2$
12	4			
13	2			
15	1			
19	4			
21	5			
	$\sum f =$			$\sum f\left(x-\overline{x}\right)^2 =$

Explain the expression $\sum f\left(x-\overline{x}\right)^2$ in your own words.

CONTENT SUMMARY

Variance measures how far a set of numbers is spread out. A variance of zero indicates that all the values are identical. Variance is always non-negative: a small variance indicates that the data points tend to be very close to the mean and hence to each other, while a high variance indicates that the data points are very spread out around the mean and from each other.

The variance is denoted and defined by

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2}{n}$$

Developing this formula we have

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i^2 - 2x_i\overline{x} + \left(\overline{x}\right)^2\right)}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \frac{1}{n}2\overline{x}\sum_{i=1}^{n} x_i + \frac{1}{n}\left(\overline{x}\right)^2\sum_{i=1}^{n}1$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - 2\overline{x}\overline{x} + \left(\overline{x}\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\overline{x}\right)^2$$

Thus, the variance is also defined by

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\overline{x}\right)^2$$

Recall that the mean of the set of *n* values $x_1, x_2, x_3, ..., x_n$ is denoted and defined by

$$\overline{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \sum_{i=0}^{n}\frac{x_i}{n} = \frac{1}{n}\sum_{i=0}^{n} x_i$$

Example

Calculate the variance of the following distribution: 9, 3, 8, 8, 9, 8, 9, 18

Solution

$$\overline{x} = \frac{9+3+8+8+9+8+9+18}{8} = 9$$

$$\sigma^2 = \frac{(9-9)^2 + (3-9)^2 + (8-9)^2 + (8-9)^2 + (9-9)^2 + (8-9)^2 + (9-9)^2 + (18-9)^2}{8} = 15$$

Application activity 3. 13

1. The heights (in meters) of six children are 1.42, 1.35, 1.37, 1.50, 1.38 and 1.30.

Calculate the mean height and the variance of the heights.

2. The number of customers served lunch in a restaurant over a period of 60 days is as follows:

Number of customers served lunch	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79
Number of days in the 60-day period	6	12	16	14	8	4

Find the mean and variance of the number of customers served lunch using this grouped data.

3.14 Standard deviation

Activity 3.14

Conduct a research in the library or on the internet and explain standard deviation as measure of dispersion.

Insist on explaining how Standard deviation is calculated and why it is used in real life problems for grouped data and for ungrouped data.

CONTENT SUMMARY

The standard deviation has the same dimension as the data, and hence is comparable to deviations from the mean. We define the **standard deviation** to be the square root of the variance.

Thus, the standard deviation is denoted and defined by:

a) For ungrouped data:

$$\sigma=\sqrt{\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}} \text{ or } \sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2-\left(\bar{x}\right)^2}$$

b) For grouped data

$$\sigma=\sqrt{\frac{\sum\left\{f_i\left(x_i-\bar{x}\right)^2\right\}}{\sum f_i}}\text{, where } \sigma^2=\frac{\sum\left\{f_i\left(x_i-\bar{x}\right)^2\right\}}{\sum f_i} \text{ is the variance.}$$

The following results follow directly from the definitions of mean and standard deviation:

- When all the data values are multiplied by a constant *a*, the new mean and new standard deviation are equal to *a* times the original mean and standard deviation. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $a\left(\bar{x}\right)$ and the standard deviation is $a\sigma$.
- When a constant value, *b*, is added to all data values, then the new mean is increased by *b*. However standard deviation does not change. That is, the mean of $ax_1, ax_2, ax_3, ..., ax_n$ is $\bar{x}+b$ and the standard deviation is σ.

Example 1:

The six runners in a 200 meter race clocked times (in seconds) of 24.2, 23.7, 25.0, 23.7, 24.0, 24.6.

Find the mean and standard deviation of these times.

Solution

$$\bar{x}=\frac{24.2+23.7+25.0+23.7+24.0+24.6}{6}=24.2 \text{ seconds}$$

$$\sigma=\sqrt{\frac{(24.2-24.2)^2+(23.7-24.2)^2+(25.0-24.2)^2+(23.7-24.2)^2+(24.0-24.2)^2+(24.6-24.2)^2}{6}}$$

$$=0.473 \text{ seconds}$$

The method which uses the formula for the standard deviation is not necessarily the most efficient.

Consider the following: $\frac{\sum\left(x-\bar{x}\right)^2}{n}=\frac{1}{n}\sum x^2-\left(\bar{x}\right)^2$

Example 2:

The heights (in meters) of six children are 1.42, 1.35, 1.37, 1.50, 1.38 and 1.30. Calculate the mean height and the standard deviation of the heights.

Solution:

$$\text{Mean}=\frac{1}{6}(1.42+1.35+1.37+1.50+1.38+1.30)=1.39 \quad m$$

$$\text{Variance}=\frac{1}{6}\left(1.42^2+1.35^2+1.37^2+1.50^2+1.38^2+1.30^2\right)-1.39^2=0.00386 \ m^2$$

$$\text{Standard deviation}=\sqrt{0.00386 \ m^2}=0.0621 \ m$$

Application activity 3.14

1. In the following statistical series, calculate the standard deviation of the following set of data

56,54,55,59,58,57,55

2. In the classroom of language education the first ten student-teachers scored the following marks out of 10 in a quiz of French

5, 6, 5, 2, 4, 7, 8, 9, 7, 5.

a. Calculate the mean, median and the modal mark

b. Calculate the quartiles and inter-quartile range

C. Calculate the variance and the standard deviation

Lesson 15: Measures of dispersion (Coefficient of variation)

Activity 3.15

Two plants C and D of a factory show the following results about the number of workers and the wages paid to them.

	C	D
No. of workers	5000	6000
Average monthly wages	\$2500	\$2500
Standard deviation	9	10

Using coefficient of variation, find in which plant, C or D there is greater variability in individual wages.

In which plant would you prefer to invest in?

CONTENT SUMMARY

The coefficient of variation measures variability in relation to the mean (or average) and is used to compare the relative dispersion in one type of data with the relative dispersion in another type of data. It allows us to compare the dispersions of two different distributions if their means are positive. The greater dispersion corresponds to the value of the coefficient of greater variation.
By determining the coefficient of variation of different securities, an investor identifies the risk-to-reward ratio of each security and develops an investment decision. Generally, an investor seeks a security with a lower coefficient (of variation) because it provides the most optimal risk-to-reward ratio with low volatility but high returns. However, the low coefficient is not favorable when the average expected return is below zero.

Formula for Coefficient of Variation

The coefficient of variation is a calculation built on other calculations: the standard deviation and the mean as follows:

$$Cv = \frac{\sigma}{\bar{x}} \times 100$$

Where:

- σ is the standard deviation
- $\bar{x}$ Is the mean.

In the context of finance, we can re-write the above formula in the following way:

$$Coefficient\ of\ Variation = \frac{\text{Volatility}}{\text{Expected Return}} \times 100\%$$

Example:

One data series A has a mean of 140 and standard deviation 28.28. The second data series B has a mean of 150 and standard deviation 24. Which of the two has a greater dispersion?

Solution:

$$CV_1 = \frac{28.28}{140} \times 100 = 20.2\%$$

$$CV_2 = \frac{24}{150} \times 100 = 16\%$$

From the given two data series (A and B), data series A has a greater dispersion.

Application activity 3.15

Fred wants to find a new investment for his portfolio. He is looking for a safe investment that provides stable returns. He considers the following options for investment:

- Stocks: Fred was offered stock of ABC Corp. It is a mature company with strong operational and financial performance. The volatility of the stock is 10%, and the expected return is 14%.
- ETFs: Another option is an Exchange-Traded Fund (ETF) which tracks the performance of the S&P 500 index. The ETF offers an expected return of 13% with a volatility of 7%.
- Bonds: Bonds with excellent credit ratings offer an expected return of 3% with 2% volatility.

In order to select the most suitable investment opportunity, Fred decided to calculate the coefficient of variation of each option. In which option does Fred want to invest?

.

Lesson 16: Practical activity in statistics

Activity 3.16

Student teachers were interested in getting information on the number of hours patients spent in the hospital in a certain week. The table below summarizes the data they recorded where the frequency indicates the number of patients.

Class boundaries(of hours)	**Frequency (number of patients)**
7.5-12.5	3
12.5-17.5	5
17.5-22.5	15
22.5-27.5	5
27.5-32.5	2

Determine and interpret the average, the modal class, the median, the variance and the standard deviation related to the number of hours a sick person spends in that hospital.

What is the advice you can provide to the manager of that hospital if he/she has a few number of beds?

CONTENT SUMMARY

With observation, data are collected through direct observation. Information could also be collected using an existing table that shows types of data to collect in a certain period of time, this method of collecting data is reliable and accurate.

Once data has been collected, they may be presented or displayed in various ways. Such displays make it easier to interpret and compare the data.

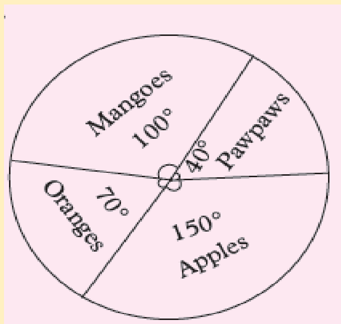
After the collection of data there is need of interpreting them and here there are some tips:

- Collect your data and make it as clean as possible.
- Choose the type of analysis to perform: qualitative or quantitative
- Analyze the data through various statistical methods such as mean, mode, standard deviation or Frequency distribution tables
- Reflect on your own thinking and reasoning and analyze your data and then interpret them referring to the reality.

During interpretation, avoid subjective bias, false information and inaccurate decisions.

Application activity 3.16

1) After selling fruits in a market, Martha had a total of 144 fruits remaining. The pie chart below shows each type of fruit that remained.



a) Find the total cost of mangoes and paw-paws remaining if a mango sells at 30 Frw and a pawpaw at 160Frw
b) Which type of fruit remained the most?
c) What was the median number of fruit that remained?
d) Draw a frequency table to display the information on the pie chart.

2) Work in groups and use tailor's meters to correct data on the height for 20 people.

a) Organize the corrected data with a frequency distribution

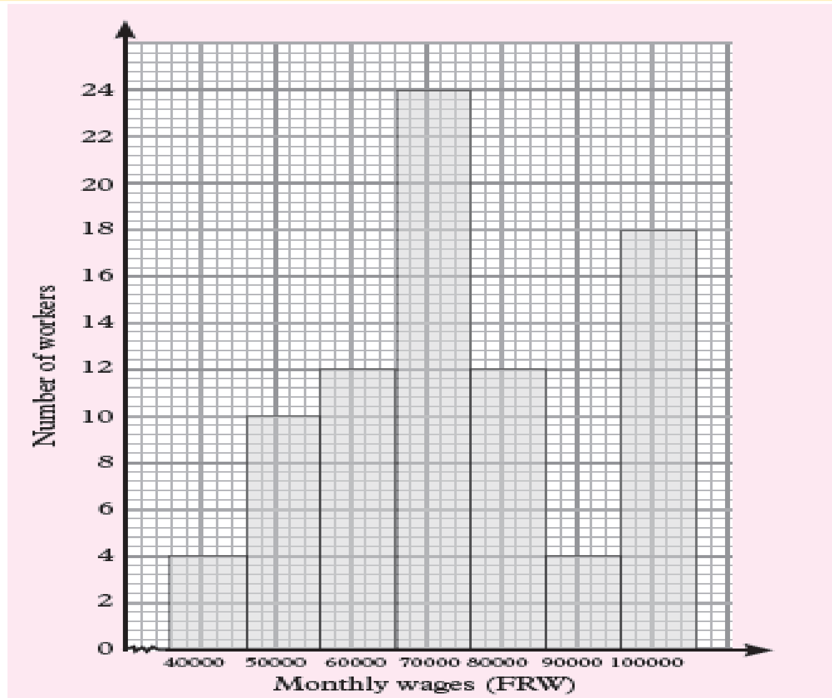
b) Determine and interpret the mean, the mode and the median for the data.

c) Determine and interpret the range, the quartiles and the standard deviation.

d) Organize data into a grouped data distribution of 10 groups and determine: the modal class and the standard deviation. Compare this standard deviation with the one found in (c).

3.8 End unit assessment

1. Use the graph below to answer the questions that follow



   a. Estimate the mode.
   b. State the range of the distribution.
   c. Draw a frequency distribution table from the graph
   d. Draw the frequency polygon represented by the histogram
2. In test of mathematics, 10 student-teachers got the following marks:

   6, 7, 8, 5, 7, 6, 6, 9, 4, 8

   a. Calculate the mean, mode, quartiles and inter-quartile range

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