

MATHEMATICS SYLLABUS FOR TTCs

OPTIONS: EARLY CHILDHOOD AND LOWER PRIMARY (ECLPE)

YEAR 1, 2 & 3

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FOREWORD

Rwanda Education Board is honoured to avail the Mathematics Syllabus for Teacher Training Colleges (TTCs) in the ECLPE option. This document serves as official guide to teaching and learning of Mathematics in TTCs, Pre-primary and Primary education.

The Rwandan education philosophy is to ensure that young people at every level of education achieve their full potential in terms of relevant knowledge, skills and appropriate attitudes that prepare them to be well integrated in society and exploit employment opportunities.

The ambition to develop a knowledge-based society and the growth of regional and global competition in the labour market, has necessitated the shift from knowledge to competence based curriculum in TTCs. The TTC curriculum was revised to align it to the Competence-Based Curriculum for basic education to prepare teachers who are competent and confident to implement CBC in pre-primary and primary education. The rationale of the changes is to ensure that TTC leavers are qualified for job opportunities and further studies in Higher Education in different programs under education career advancement.

I wish to sincerely express my appreciation to the people who contributed towards the development of this syllabus, particularly, Consultants, REB staff, UR-CE Lecturers, TTC Tutors, Teachers from general education and experts from Local and international Organizations for their technical support. A word of gratitude goes to the Head Teachers and TTCs principals who availed their staff for various revision activities.

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Dr. NDAYAMBAJE Irénée

Director General REB.

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Joan MURUNGI,

Head of Curriculum Teaching and Learning Resources Department/REB

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GENERAL INTRODUCTION

1.1. Rwanda education sector objectives

The Education Sector objectives are the reference point for the inclusion of education issues into other Rwandan policy documents. These objectives are aligned with those recommended in the Eastern African Curriculum Framework proposals. The Government of Rwanda through law number 36/2018 of 29th June, 2018, determining the organization of education, revised the objectives of the sector. They are to:

1. Provide Rwandans with adequate skills at all levels of general, professional as well as technical and vocational education;
2. Offer quality courses and education at all levels;
3. Promote science, technology and research in order to equip many Rwandans with capacity to speed up national development;
4. Promote the culture of peace, tolerance, justice, respect for human rights, solidarity, democracy and that of avoiding any form of discrimination or favoritism;
5. Provide each Rwandan with an integrated education based on ethical values, science and social welfare and directed towards building a nation to ensure its sustainable development.
6. Instil into Rwandans the love of a job well done, the value of hard work, punctuality and promotion of competence.
7. Train Rwandans to have freedom of thought, be innovative, have abilities to acquire and be analytical towards other people's opinions and to communicate his or her own ideas, to be patriotic and encourage him or her to be updated on the situation prevailing elsewhere;
8. Eliminate all grounds and obstacles that hinder the development of girls and women education as well as of any other groups that need special attention.

These objectives and associated strategies are the backbone for developments in education including the curriculum and assessment policy and the curriculum framework.

1.2. Level competences of pre-primary and primary teacher education in the republic of Rwanda

As stated earlier, Pre-primary and Primary Teacher Education is under the responsibility of Rwanda Education Board. The following are the competences of Teacher Education. By the time a student teacher is exiting the college after three years he or she should:

- be a qualified teacher who can compete not only locally but regionally and internationally;
- have professional ethics and develop an inquiring mind for innovative education;
- be prepared adequately for efficiency in education, administration, management, evaluation and measurement;
- be competent, reliable, honest and responsible;
- be equipped with potentials that enable him/her to explore the learners' abilities and interests;
- be able to develop the child's ability in critical thinking, free expression and ideas.

1.3. Background to curriculum review

The Mathematics syllabus is developed for TTC student-teachers in the option of Early Childhood and Lower Primary Education where Mathematics is a core subject.

The motive of reviewing the syllabus was to ensure that the syllabus is responsive to the needs of the student-teacher and to shift from objective and knowledge-based learning to competence-based learning. Another reason was to align the draft TTC curriculum to the CBC in general education. Emphasis in the review was put more on skills and competences as well as the coherence within the existing content by drawing on the previous syllabus and benchmarking with syllabi elsewhere with best practices.

The Mathematics syllabus guides the interaction between the tutor and student -teachers in the learning processes and highlights the competences a student teacher should acquire during and at the end of each unit of learning. Student- teachers will have the opportunity to apply Mathematics in different contexts, and see its importance in daily life. Tutors help the student- teachers appreciate the relevance and benefits for studying Mathematics from pre/primary to tertiary levels.

The learning of student teacher is influenced by many factors such as curriculum relevancy, necessary and sufficient pedagogical approach by tutors, assessment strategies and sufficient instructional materials. With review of the Mathematics syllabus, these factors have been aligned with the competence-based curriculum for general education. This will lead to having qualified and competent teachers who are ready to implement the competence based curriculum for pre/primary education and perform well in higher education or any fields that require some knowledge of mathematics.

This implies equipping student teachers with relevant knowledge, skills, attitude and values necessary to make them competitive on local, regional and global job market. This revised syllabus will allow future teachers to contribute to the development of equity and quality education at pre/primary levels and then it will enable student teachers to go for further studies.

1.4. Rationale of teaching and learning mathematics

Mathematics subject in the options of Early Childhood and Lower Primary Education (ECLPE) is designed to provide mathematical concepts that make important tools in supporting other subjects such as natural sciences, engineering, medicine, finance and social sciences.

The syllabus concentrates on mathematical concepts that are useful for a student-teacher who intends to: teach in Lower Primary school, collaborate with others to solve real life problems in knowledge based economy society and undertake further studies at higher education. Topics were chosen to help the student-teacher perform well every Mathematics lessons of secondary education and find that mathematical knowledge is an essential tool for learning other subjects such as natural sciences, engineering, medicine, finance and social sciences. The content of this syllabus will help student teacher to enhance critical thinking and problem solving skills and enable him/her to be systematic, creative and self-confident. Student -teachers will be equipped with knowledge, skills, values and attitudes necessary to succeed in an era of rapid technological growth and socio-economic development as well as in effective teaching of Mathematics in Primary Education or pursuing tertiary education.

1.4.1. Mathematics and society

Mathematics plays an important role in society through abstraction and logic, counting, manipulation, calculation, measurement, cutting, classifying, systematic study of shapes and motion. It is also used in natural sciences, engineering, medicine, finance and social sciences. The applied Mathematics like statistics and probability play an important role in game theory, in the national census process, in scientific research, etc. In addition, some crosscutting issues such as financial awareness are incorporated into some of the Mathematics units to improve social and economic welfare of Rwandan society.

Mathematics is key to the Rwandan education ambition of developing a knowledge-based and technology-led economy since it provides to student teachers all required knowledge and skills to be used in different learning areas. Therefore, Mathematics is an important tool in supporting other subjects. This TTC Mathematics syllabus is intended to address gaps in the current Math syllabus which lacks adequate and appropriate knowledge, skills, attitude and values.

1.4.2. Mathematics and student teachers

Student teachers need enough basic mathematical competences to be effective members of Rwandan society including the ability to estimate, analyze, interpret statistics, assess probabilities, and read the commonly used mathematical representations and graphs.

Therefore, Mathematics equips student teachers with knowledge, skills and attitudes necessary to enable them to succeed in an era of rapid technological growth and socio-economic development. Mastery of basic Mathematical ideas and calculations makes student teachers being confident in problem solving. It enables the student teachers to be systematic, creative and self-confident in using mathematical language and techniques to reason; think critically; develop imagination, initiative and flexibility of mind. In this regard, learning of Mathematics needs to include practical problem-solving activities with opportunities for student teachers to plan their own investigations in order to develop their mathematical competences and confidence.

As new technologies have had a dramatic impact on all aspects of life, wherever possible in Mathematics, student- teachers should gain experience of a range of ICT equipment and applications.

1.5. Professional standards and competences

1.5. 1 Competences

Competence is defined as the ability to perform a particular task successfully resulting from having gained an appropriate combination of knowledge, skills, attitudes and values. The present syllabus gives the opportunity to student teachers to develop different broad Mathematics competences as well as the generic competences.

Broad subject competences are highlighted and broken into key competences for each year, these are further broken into key unit competences which are finally split into learning objectives knowledge, skills, attitudes and values) in every learning unit.

Taking into account the rationale behind the overall TTC curriculum review as well as the parameters and constraints of the local context, student- teachers will be equipped with professional standards and generic competences.

For student teachers, professional standards are acquired through generic competences and professional practices that are emphasized and reflected in the learning process. The Mathematics tutors will ensure that student -teachers are exposed to tasks that help them acquire these competences.

1.5.2 Teaching professional standards

These refer to the characteristics that all tutors globally should have. All tutors in Rwanda should have the six characteristics listed below:

- **The teacher has knowledge of CBC and how to implement it.**

The teacher has understanding of CBC and how it works. He/she has knowledge of basic and generic competences and is able to integrate the cross cutting issues within and across subject area.

- **The teacher as an educator**

The teacher enhances and stimulates cognitive, social-emotional, physical and moral development of the children. She/he therefore has a thorough understanding of the child's background, interests, motivations and problems and can adjust her/his actions and the learning environment to the different needs of the students. A competent educator is a role model, showing desired behavior and values. She/he guides and coaches her/his students to become social, self-confident, independent, responsible, open-minded and innovative people and act like a role model. In order to be educator, the student must be supported in developing cooperation, inter personal and life skills.

- **The teacher as a subject expert**

Mathematics teacher develops critical thinking skills, problem solving skills and creativity. She/he uses teaching/learning Mathematics methods and techniques that are appropriate to Mathematics in lower primary education; she/he applies Mathematics content, plan lessons integrating play-based strategies in teaching/learning. The teacher in primary education has a thorough Mathematics knowledge that enables him/her to develop the teaching/learning materials, lesson planning, to deliver the lesson and connect with her/his daily life activities in primary education using correctly the language of instruction. The teacher as Mathematics expert in upper primary education stimulates English in the teaching/learning process by considering the transition from Kinyarwanda to English as medium of Instruction in upper primary.

The teacher as subject expert knows and uses appropriate Mathematics methods/techniques to assess student teachers and give constructive feedback to the whole class. The teacher is able to link the content with other subjects and connect it with real life situations. This enables a student-teacher to acquire critical and problem solving skills as well as the mathematical concepts and skills enabling him/her to pursue tertiary education easily.

- **The teacher as a communicator**

The professional teacher displays a good example in his/her way of expressing him/herself, stimulates and enhances positive and clear communication between him/herself and the students, between the students, college community and the wider society. A good communicator is open-minded and respects diversity within and around the college. This requires teacher to communicate in official languages.

- **The teacher as a guide and an organizer**

The professional teacher facilitates the holistic development of all students, taking into account the differences between them. She/he ensures that the learning environment (class, playground, etc.) is well maintained and conducive for learner's learning outcomes. This requires a student to be equipped with management skills.

- **The teacher as an innovator, researcher and reflective practitioner**

The professional teacher looks at ways to improve her/his teaching, and the wellbeing and results of her/his learners. She/he is a reflective practitioner and knows how to perform small-scale reflective action.

The acquisition of such skills will require students to update knowledge and skills with minimum external support and to cope with evolution of knowledge advances for personal fulfillment in areas that need improvement and development, thus becoming a lifelong learner.

1.5.3. Broad mathematics competences

At the end of the three years the student teacher should be able to:

- Work in systematic way to develop clear, logical, coherent and creative reasoning and imagination;
- Express clearly, comprehensibly, correctly and precisely in verbal and/or in written form, all the reasons and calculations leading to the required result whenever finding a solution to any given exercise;
- Apply mathematical concepts, principles and processes in real life situations/ experiences or problem solving as well as applying them to other subjects.
- Use ICT tools (such as calculators and mathematical software) in teaching and learning Mathematics in sense of research, curiosity and creativity to explore Mathematical concepts and facts;

- Describe, explain, present, analyze, interpret, draw logical conclusions, and make predictions and decisions on scientific data in solving real life word problems.
- Use the acquired Mathematical knowledge and skills to teach at pre/primary Education or pursue further studies;

1.5.4. Mathematics and developing competences

The national policy documents based on national aspirations identify some ‘basic competences’ alongside the ‘generic competences’ that will develop higher order thinking skills and help student teacher learn subject content and promote application of acquired knowledge and skills.

Through observations, constructions, using symbols, applying and generalizing mathematical ideas, and presentation of information during the learning process, the student teacher will not only develop deductive and inductive skills but also acquire cooperation and communication, critical thinking and problem solving skills. This will be realized when student-teachers make presentations leading to inferences and conclusions at the end of learning unit. This will be achieved through student teachers’ group work and cooperative learning which in turn will promote interpersonal relations and teamwork.

The acquired knowledge in learning Mathematics should develop a responsible citizen who adapts to scientific reasoning and attitudes and develops confidence in reasoning independently. The student-teacher should show concern of individual attitudes, environmental protection and comply with the scientific method of reasoning. The scientific method should be applied with the necessary rigor, intellectual honesty to promote critical thinking while systematically pursuing the line of thought.

PEDAGOGICAL APPROACH

The change to a competence-based curriculum is about transforming learning, ensuring that learning is deep, enjoyable and habit-forming.

2.1. Role of the student teacher

In the competence-based syllabus, the student teacher is the principal actor of his/her education. She/ He is not an empty bottle to fill. Taking into account the initial capacities and abilities of the student teacher, the syllabus suggests under each unit, some activities of the student teacher and they all reflect active participation of the student teacher in the learning process.

The teaching and learning processes will be tailored towards creating a student's friendly environment basing on the capabilities, needs, experience and interests.

The following are some of the roles or the expectations from the student teachers:

- Student teachers construct the knowledge either individually or in groups in an active way. From the learning theory, student teachers move in their understanding from concrete through pictorial to abstract. Therefore, the opportunities should be given to student teachers to manipulate concrete objects and to use models.
- Student teachers are encouraged to use hand-held calculator. This stimulates Mathematics as it is really used, both on job and in scientific applications. Frequent use of calculators can enhance student teachers' understanding and mastering of arithmetic.
- Student teachers work on one competence at a time to form concrete units with specific learning objectives (knowledge, skills and attitude).
- Student teachers will be encouraged to do research and present their findings through group work activities.
- A student teacher is cooperative: student teachers work in heterogeneous groups to increase tolerance and understanding.
- Student teachers are responsible for their own participation and ensure the effectiveness of their work.

2.2. Role of the tutor

In the competence-based syllabus, the tutor is a facilitator, organizer, advisor, a conflict solver, ...

The specific duties of the teacher/tutor in a competence-based approach are the following, Tutor is:

- A facilitator, his/her role is to provide opportunities for student teachers to meet problems that interest and challenge them and that, with appropriate effort, they can solve. This requires an elaborated preparation to plan the activities, the place they will be carried, the required assistance;
- An organizer: his/her role is to organize the student- teachers in the classroom or outside and engage them through participatory and interactive methods through the learning processes as individuals, in pairs or in groups. To ensure that the learning is personalized, active and participative, co-operative the teacher/tutor must identify the needs of the student teachers, the nature of the learning to be done, and the means to shape learning experiences accordingly;
- An advisor: he/she provides counseling and guidance for student-teachers in need. He/she comforts and encourages student teachers by valuing their contributions in the class activities;
- A conflict-solver: most of the activities are performed in groups. The members of a group may have problems such as attribution of tasks; they should find useful and constructive the intervention of the teacher as a unifying element.
- Asked for help only when the whole group agrees to ask a question
- Ethical and preaches by examples by being impartial, by being a role-model, by caring for individual needs, especially for slow student teachers and those with physical impairments, through a special assistance by providing remedial activities or reinforcement activities. One should notice that this list is not exhaustive.

2.3. Special needs education and inclusive approach

All Rwandans have the right to access education regardless of their different needs. The underpinnings of this provision would naturally hold that all citizens benefit from the same menu of educational programs. The possibility of this assumption is the focus of special needs education. The critical issue is that we have persons/ student teachers who are totally different in their ways of living and learning as opposed to the majority. The difference can either be emotional, physical, sensory and intellectual learning challenged.

These student teachers equally have the right to benefit from the free and compulsory basic education in the nearby ordinary/mainstream colleges. Therefore, the college's role is to enroll them and also set strategies to provide relevant education to them. The tutor therefore is requested to consider each student teacher's needs during teaching and learning process. Assessment strategies and conditions should also be standardized to the needs of these student teachers. Detailed guidance for each category of student teachers with special education needs is provided for in the guidance for teachers. The Mathematics tutor is advised to work closely with the tutor of special need education to provide appropriate support to any identified student -teacher's needs.

2.4. Skills lab pedagogy in mathematics

Mathematics is a practical subject in which student teachers can learn and explore various mathematical concepts , verify different mathematical facts and theories using a variety of practical activities and material. The use of mathematics laboratory helps to integrate theory and practical work in mathematics teaching and learning where students develop competences and apply acquired skills. Skills lab ensures teachers are accountable to completing all the learning activities and projects as outlined in the syllabus.

The basic characteristics of Skills Lab are:

- Students work in small groups (manageable teams)
- Assessment takes place through portfolio activities
- Students talk more than teachers (10% tutor talk time and the rest is for the students)
- Students receive constructive feedback on their work (Tutor gives quality feedback on student presentations).
- Skills lab consists of three components: build, practice and present.

Skills Lab in Mathematics makes a moment in which the student is active, discusses and argues her/his own choices, builds meanings, learns to collect data and to compare them with the models. It is important for valorising the experience and knowledge of the students, the discovery and the collaborative learning.

This learning must also be done through practice, discussion, exploration, development of thinking about the practice and way for building knowledge.

Practical activities to be done will help to reduce the abstract nature of the Mathematics subject and draw the students to participate in the classroom where the vague theories and imaginary objects of mathematics take real shape which facilitates students to understand and perform better.

Role of the tutor is to prepare practical activities, to set groups of learners and assign them their roles and responsibilities. The tutor has to explore students' works, to analyse how and why the same final answer or product can be the result of different process or constructions followed by students, to mark and provide feedback and remediation to students.

The tutor has also to guess strategies used by students when doing a given activity. This is the analysis of possible solutions for a problem given to students considering that students can present only the starting and the final steps. The tutor has to guess the difficulties and misconception that students may face when they apply such a procedure in such a situation.

Role of the student teacher is to read carefully instructions related to the activity, apply the mathematics process in doing practical activity as provided. This process includes: reasoning, communication and connection, application and modeling, production of mathematics teaching and learning materials, apply thinking skills and heuristics.

Mathematical reasoning: Analyse mathematical situations and construct logical arguments. It is the habit of mind to be developed through application of mathematics in different contexts.

Communication: use mathematics language to express mathematical ideas and arguments precisely, concisely and logically.

Connections: see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

Applications and modeling: connect mathematics that have been learnt to the real life situations, enhance understanding of key mathematical concepts and methods, as well as developing competences. Students will have opportunity to apply problem solving and reasoning skills to tackle a variety of problems: open-ended and real world problems. They will use mathematical models to represent and solve problems.

Thinking skills: Skills used in thinking process: classifying, comparing, analyzing parts and whole, identifying patterns and relationships, induction, deduction, generalizing, and spatial visualization.

Heuristics: rules of thumb of what students can do to tackle a problem when the solution to the problem is not obvious such as using representation (drawing diagrams, tabulating), making a guess (trial and error, guess and check, making a supposition), walking through the process (acting it out, working backwards), and changing the problem (simplifying the problem or considering special cases).

Production of teaching and learning materials: Mathematics student teachers can be given practical activities of organizing or producing teaching materials by the use of local material available in their environment.

Therefore, when doing practical activities, students build the knowledge and understanding, practice acquired skills and present the findings from their researches.

The following are tasks for the students: group building, basic communication, get to know the community and learners, exploration and needs assessment, coaching and guidance on managing challenging situations, investigation, asking, finding problems and to work collaboratively with peers for solving them, self-development on observation and inquiry, using scientific method and scientific research, develop a sense of taking responsibility for one's own learning, represent and build models of relationships among objects and events, communicate and discuss, argue correctly, understand views and arguments of the others, perceive and explore fascinating relationships and structures found in nature and in the creations of the human being.

In this regard, skills lab activities are the time when students combine the competencies acquired to make learning more practical. Thus, in every unit, this syllabus suggests practical and learning activities that the teacher may adapt and give to students to ensure practical application of the competencies acquired.

ASSESSMENT APPROACH

Assessment is the process of evaluating the teaching and learning processes through collecting and interpreting evidence of individual student teacher's progress in learning and to make a judgment about a student teacher's achievements measured against defined standards. Assessment is an integral part of the teaching learning process. In the new competence-based curriculum assessment must also be competence-based; whereby a student teacher is given a complex situation related to his/her everyday life and asked to try to overcome the situation by applying what he/she learned.

3.1. Types of assessments

There are two major types of assessment namely formative and summative assessments. Any form of assessment should reflect the three domains of learning, which are Cognitive, Psychomotor and Affective.

- Knowledge and understanding: Does the student- teacher demonstrate an understanding of the Mathematics concept? Has the student- teacher mastered the Mathematics concepts? Indicators: correctness of answers, coherence of ideas, logical reasoning, use correctly mathematical symbols and concepts, etc.
- Practical skills: How does the student-teacher perform on aptitude and practical tests? Indicators: accuracy, using appropriate methods, quality product, speed and efficiency, coherence.
- Attitude and values: How does the student-teacher respond to a task or a situation? What is the student-teacher's behavior? How the student-teacher persists on solving a given problem?

3.1.1. Formative assessment

Formative assessment helps to check the efficiency of the process of learning. It is done within the teaching/learning process.

Continuous assessment involves formal and informal methods used by schools to check whether learning is taking place. When a tutor is planning his/her lesson, he/she should establish criteria for performance and behavior changes at the beginning of a lesson. Then, at the end of every unit, the tutor should ensure that all the student teachers have mastered the stated key unit competences basing on the criteria stated, before going to the next unit. The tutor will assess how well each student teacher masters both the subject and the generic competences described in the syllabus as well as the professional practices. From this, the tutor will gain a picture of the all-round progress of the student teacher. The tutor will use one or a combination of the following techniques: observation, pen and paper, and oral questioning.

3.1.2. Summative assessment

When assessment is used to record a judgment of a competence development or performance of the student teacher, it serves a summative purpose. Summative assessment gives a picture of a student teacher's competence or progress at any specific moment. The main purpose of summative assessment is to evaluate whether competences have been achieved and to use the results for ranking or grading of student teachers, for deciding on progression, for selection into the next level of education and for certification. This assessment should have an integrative aspect whereby a student must be able to show mastery of all competences. It can be internal College based assessment or external assessment in the form of national examinations. College based summative assessment should take place once at the end of each term and once at the end of the year. College summative assessment average scores for the subjects which are examined at the national level will be weighted and included in the final national examinations grade. For the subjects that are not examined externally, the grade should be part of the continuous assessment reflected in the student's transcript.

3.2. Record keeping

This is gathering facts and evidence from assessment instruments and using them to judge the student's performance by assigning an indicator against the set criteria or standard. Whatever assessment procedures used generate data in the form of scores which will be carefully be recorded and stored in a portfolio. The latter is used in deciding remedial actions, alternative instructional strategy and as well as feed back to the student teacher. The records also are important to parents to check the learning progress and to advice accordingly. Finally, the records are very essential to the final assessment of professional practice of the student teacher at the end of the college.

This portfolio is a folder (or binder or even a digital collection) containing the student teacher's work as well as the student teacher's evaluation of the strengths and weaknesses of the work. Portfolios reflect not only work produced (such as papers and assignments), but also it is a record of the activities undertaken over time as part of student learning. Besides, it will serve as a verification tool for each student teacher that he/she attended the whole learning before she/he undergoes the summative assessment for the subject.

3.3. Item writing in summative assessment

Before developing a question paper, a plan or specification of what is to be tested or examined must be elaborated to show the units or topics to be tested on, the number of questions in each level of revised Bloom's taxonomy and the marks allocation for each question. In a competence-based curriculum, questions from higher levels of Bloom's taxonomy should be given more weight than those from knowledge and comprehension level.

Before developing a question paper, the item writer must ensure that the test or examination questions are tailored towards competence based assessment by doing the following:

- Identify topic areas to be tested on from the subject syllabus.
- Outline subject-matter content to be considered as the basis for the test.
- Identify learning outcomes to be measured by the test.
- Prepare a table of specifications.
- Ensure that the verbs used in the formulation of questions do not require memorization or recall answers only but also testing broad, subject and generic competences as stated in the syllabus.

RESOURCES

4.1. Materials needed for implementation

The following list shows the main materials/equipment needed in the learning and teaching process:

- Materials to encourage group work activities and presentations: Computers (Desktops & laptops) and projectors; Manila papers and markers, textbooks and handouts
- Materials for drawing & measuring geometrical figures/shapes and graphs: Geometric instruments, ICT tools such as Geogebra,
- Mal Math, Math lab, Microsoft student ENCARTA, ...
- Materials for enhancing research skills: Textbooks and internet (the list of the textbooks to consult is given in the reference at the end of the syllabus and those books can be found in printed or digital copies).
- Materials to encourage the development of Mathematical models: scientific calculators, Geogebra Software, Math type, Matlab, different math mobile applications etc

The technology used in teaching and learning of Mathematics has to be regarded as tools to enhance the teaching and learning process and not to replace tutors.

4.2. Human resource

The effective implementation of this curriculum needs a joint collaboration of educators at all levels. Given the material requirements, tutors are expected to accomplish their noble role as stated above. The staff in charge of education at District and sector level should ensure overall support to TTCs for a successful implementation. On the other hand, TTC principles and TTC deputy principals are required to make a close follow-up and assess the teaching and learning of this subject due to their profiles in the schools. These combined efforts will ensure bright future careers and lives for student teachers as well as the contemporary development of the country.

In a special way, the tutor of Mathematics at TTC level should have a firm understanding of mathematical concepts and pedagogical content of teaching Mathematics at primary and secondary levels. He/she should be qualified in Mathematics and have a firm ethical conduct. The tutor should possess the qualities of a good facilitator, organizer, problem solver, listener and adviser. He/she is required to have basic skills and competence of guidance and counseling because students may come to him or her for advice.

SYLLABUS UNITS

5.1 Mathematics syllabus unit for year 1

5.1.1. Key competences at the end of year 1

After completion of year one Early Childhood and Lower Primary Education, the Mathematics syllabus will enable the student-teacher to:

1. Classify sets of numbers into naturals, integers, rational and irrationals basing on their properties and solve related problems.
2. Solve problems that involve the sets of numbers using Venn diagrams and set operations
3. Apply ratios and proportions properties to solve related problems.
4. Perform operations, on polynomials and solve related problems.
5. Use Mathematical logic as a tool of reasoning and argumentation in daily situations.
6. Solve algebraically or graphically problems involving linear and quadratic equations or inequalities.
7. Solve problems related to powers, indices, radical and common logarithms.
8. Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to standard deviation.

5.1.2 Syllabus units for year one

SUBJECT: MATHEMATICS			YEAR ONE	Option : ECLPE
Topic Area: ARITHMETIC			Sub Topic: SETS	
UNIT 1: SET OF NUMBERS			No. of periods:6	
Key unit competence: To be able to Classify numbers into naturals, integers, rational and irrationals.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
<p>-Identify sets of numbers (natural, integers, rational and real) and relationship between them.</p> <p>-Illustrate different sets of numbers on a number line.</p> <p>-Show that irrational numbers cannot be expressed exactly as decimal</p>	<p>-Carry out mathematical operations on set of numbers</p> <p>-Work systematically to determine the operation properties of sets numbers.</p> <p>- Determine the hierarchy of sets on numbers and explain its relationship with operations.</p>	<p>-Appreciate that rational numbers can be represented exactly as a fraction or a decimal which may terminate or occur.</p> <p>-Appreciate that the number line is incomplete without the irrational numbers which cannot be written exactly as a decimal</p>	<p>➤ Classification of numbers</p> <ul style="list-style-type: none"> ❖ Natural numbers <ul style="list-style-type: none"> • Definition • Sub sets of natural numbers (Counting numbers, even numbers, odd numbers, prime numbers, ...) • Operations and properties on natural numbers ❖ Integers <ul style="list-style-type: none"> • Definition • Sub sets of integers (positive integers and negative integers) • Operations and properties on integers ❖ Rational <ul style="list-style-type: none"> • Definitions (fractions, terminating decimal, repeating decimal,) • Operations and properties on rational numbers ❖ Irrational <ul style="list-style-type: none"> • Definition • Operations and properties on natural numbers ❖ Real numbers <ul style="list-style-type: none"> • Definition • Operations and properties on real numbers <p>➤ Presentation of sets of numbers on number line</p>	<p>In groups, different numbers written on manila papers, flash cards, student-teachers are given the tasks to classify them into natural, rational, irrational and integers.</p> <p>In groups, student teachers are asked to do word problems related to set of numbers and present their findings.</p>
Assessment criteria: Identify the sets and use operations to explore properties on sets of numbers and use correctly number line to compare different numbers.				
Link to other subjects: Integrated science, General Science and Mathematics, EST, Mathematics for Primary, life sciences.				
Resources: Text books, manila papers, calculators, papers.				

SUBJECT: MATHEMATICS			YEAR ONE	OPTION: ECLPE
Topic Area: ARITHMETIC			Sub Topic: SETS	
Unit 2: SET THEORY				No. of periods: 12
Key unit competence: Solve problems that involve Sets operations, using Venn diagram.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
-Express mathematical problem set using a Venn diagram. -Represent A mathematical problem using a Venn diagram.	-Use Venn diagram to represent a mathematical problem set -Interpret, model, and solve a mathematical problem set.	-Develop clear, logical and coherent thinking skills in solving real life problem involving sets. -Appreciate the importance of representing and solving a mathematical problem set using Venn diagrams.	<ul style="list-style-type: none"> ➤ Analysis and interpretation of a problem using set language (intersection, union...) ➤ Representation of a problem using Venn diagram ➤ Modelling and solving a problem. 	<p>-In groups, student- teachers observe information given in the Venn diagram analyzes, interpret using set language and solve related problems.</p> <p>-In groups, student-teachers are given learning situation involving set theory, discuss and represent it using Venn diagram, form an equation and solve related questions.</p>
Assessment criteria: Can solve problem sets.				
Link to other subjects: Integrated science , Social studies.				
Resources: Calculators, manila papers, flash cards,				

Subject: MATHEMATICS			YEAR ONE	OPTION: ECLPE
Topic Area: ARITHMETIC			Sub Topic:	
UNIT 3: PROBLEMS ON RATIOS AND PROPORTIONS.				No. of periods: 12
Key unit competence: Apply ratios, proportions and multiplier proportion change to solve real life related problems.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
-Define ratio and proportion. - Express ratios in their simplest form. -Identify a direct and indirect proportion -Differentiate direct from indirect proportion.	-Compare quantities using proportions -Share quantities in a given proportion or ratio. -Apply ratio and unequal share to solve given problems. -Solve real life problems involving direct and indirect proportions using tables and graphs. - Interpret ratio and proportions in practical contexts.	-Appreciate the importance of multiplication when working with ratio and proportion	<ul style="list-style-type: none"> ➤ Equal and unequal share ➤ Ratio, Direct and indirect proportion. ➤ Calculation of proportional change using multiplier. ➤ Compound proportional change or continued proportions. ➤ Problems involving direct and indirect proportions 	- In groups, student teachers solve problems involving direct and indirect proportion, ratios and share adjust recipe amounts for different numbers of people - In pairs, match different representations of ratios and proportions including simplest form then presentation. - In groups, interpret and explain the ratio and proportion in maps and scale drawings/models - In pairs, solve problems in practical contexts involving direct and indirect proportion using tables of values and graphs
Assessment criteria: Solve perfectly problem involving ratio and proportion in a variety of contexts.				
Link to other subjects: Subjects that require proportional reasoning such as integrated sciences, computers science, economics, personal finances.				
Resources: Calculators and digital materials such as computers, software, interactive multi-media content.				

SUBJECT: MATHEMATICS		YEAR ONE		OPTION: ECLPE	
Topic Area: ALGEBRA			Sub Topic: MATHEMATICAL LOGIC		
UNIT 4: PROPOSITIONAL AND PREDICATE LOGIC				No. of periods:15	
Key unit competence: Use Mathematical logic as a tool of reason and argumentation in deal situation.					
Learning objectives			Content	Teaching/ Learning Activities	
Knowledge and understanding	Skills	Attitudes and values			
<ul style="list-style-type: none"> - Distinguish between statement and proposition. - Convert into logical formula composite propositions and vice versa. - Draw the truth table of a composite proposition. 	<ul style="list-style-type: none"> - Use mathematical logic to infer conclusion from given proposition. - Evaluate claims, issues and arguments, and identify mistakes in reasoning and prove the validity or invalidity of arguments in ordinary discourse. - Show that a given logic statement is tautology or a contradiction 	<ul style="list-style-type: none"> -Judge situation accurately and act with equality. - Observe situations and make appropriate decisions. - Appreciate and act with thoughtfulness: grasp and demonstrate carefulness. - Develop and show mutual respect. 	<ul style="list-style-type: none"> ➤ Logical statement or proposition ➤ Propositional logic ❖ Logical connectives ❖ Composite proposition ❖ Truth tables ❖ Tautologies ❖ Contradictions ➤ Quantifier ❖ Existential ❖ Universal 	<ul style="list-style-type: none"> - In groups student teacher are given scenarios to be discussed and deduce if given statement is a proposition or not. - In pairs, student teachers perform research in advance in the library/ computer lab about propositional logic, predicates, quantifier, and method of proof and present their findings to the whole class. 	
Assessment criteria: Use correctly logical connectives, quantifiers in logical statement and draw truth table of a simple or composite proposition.					
Link to other subjects: Integrated Science, FOE, Social studies.					
Resources: Text book, hound out, Manila papers, ICT tools.					

SUBJECT: MATHEMATICS		YEAR ONE	Option : ECLPE
Topic Area: ALGEBRA		Sub Topic: POLYNOMIALS	
UNIT 5: OPERATION ON POLYNOMIALS			No. of periods: 15
Key unit competence: Perform operations, on polynomials and solve related problems.			
Learning objectives		Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values	
<ul style="list-style-type: none"> -Define polynomial -Classify polynomial by degree and number of terms. -Recognize operation properties on polynomials -Give common factor of algebraic expressions. 	<ul style="list-style-type: none"> -Perform operations on polynomials -Expand algebraic expression by removing brackets and collecting like terms. -Apply operation properties to carry out given operation of polynomials -Factorize a given algebraic expression using appropriate methods 	<ul style="list-style-type: none"> -Appreciate the role of numerical value of polynomial and algebraic identities in simplifying mathematical expressions. -Develop critical thinking and reasoning -Ability to classify and follow orders to perform a given task. 	<ul style="list-style-type: none"> ➤ Defining degree of polynomials and comparing polynomials. ➤ Operations on polynomials. ➤ Factorizing and expanding polynomials.
<p>Using hand-out, manila papers, flash cards; In groups' student teachers</p> <ul style="list-style-type: none"> -Classify polynomials according to their degree or to the number of terms -Discuss and perform operations on polynomials - Expand and factorize given mathematical expressions and make presentation to the whole class. 			
Assessment criteria: Perform operations on polynomials by using properties of sets of numbers and solve problems related to polynomials.			
Link to other subjects: Integrated science, General Science and Mathematics, EST, Mathematics for Primary.			
Resources: Text books, manila papers calculators, papers.			

SUBJECT: MATHEMATICS			YEAR ONE	OPTION: ECLPE
Topic Area: ALGEBRA			Sub Topic: POLYNOMIALS	
UNIT 6: LINEAR AND QUADRATIC EQUATION/ INEQUALITIES				No. of periods: 15
Key unit competence: Solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
-List and clarify the steps in modeling a problem by linear or quadratic equations and inequalities.	<ul style="list-style-type: none"> - Solve equations and inequalities in one unknown. - Solve parametric equations and inequalities in one unknown. - Solve simultaneous equations in two unknowns Use equations in solving mathematical problems involving supply and demand, linear motions,	-Appreciate, value and care for situations involving to quadratic equations and quadratic inequalities in daily life situation	<ul style="list-style-type: none"> ➤ Defining and solving equations and inequalities ❖ Linear equations ❖ Linear inequalities ❖ Quadratic equations ❖ Quadratic inequalities ➤ Solving word problems involving linear or quadratic equations (problems involving supply and demand, linear motions, ...) ➤ Solving equations reducible to quadratic equation ➤ Solve and discuss parametric equation and inequalities 	In groups, student teachers: <ul style="list-style-type: none"> - Discuss and use factorization or any other method to solve given quadratic equations and inequalities. - Model given mathematic problems using quadratic equations and solve them. - Solve given equations of high degree reducible to quadratic by using Factorization and Synthetic division - Solve the given parametric equations and inequalities hence discuss them according to the solutions.
Assessment criteria: Can perform operations, factorize polynomials and solve related problems.				
Link to other subjects: Subjects where polynomials are important such as integrated science.				
Resources: textbook, papers, calculators.				

SUBJECT: MATHEMATICS			YEAR ONE	OPTION: ECLPE
Topic Area: ALGEBRA			Sub Topic: POLYNOMIALS	
UNIT 7: PROBLEMS ON POWERS, INDICES, RADICALS AND COMMON LOGARITHMS			No. of periods: 18	
Key unit competence: Solve problems related to powers, indices, radicals and common logarithms.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
<p>-Define a power, an exponent, a radical, a logarithm, the absolute value of a real number</p> <p>-Match a number and the set to which it belongs.</p>	<p>- Illustrate each property of a power, an Exponent, a radical, a logarithm, the absolute value of a real number.</p> <p>-simplify indices in radicals</p> <p>- Use logarithm and exponentials to model simple problems about growth, decay, compound interest, magnitude of an earthquake.</p> <p>-Transform a logarithmic Expression to equivalent power or radical form and vice versa.</p>	<p>Appreciate the importance and the use of properties of operations on powers, indices, radical s and common logarithms</p> <p>- Show curiosity for the study of operations on powers, indices ,radical s and common logarithms</p>	<p>➤ Powers and radicals</p> <p>❖ Definition of power, indices and radicals</p> <p>❖ Properties of indices and radicals</p> <p>❖ Operations on indices and radicals</p> <p>➤ Decimal logarithm</p> <p>❖ Definition</p> <p>❖ Properties</p> <p>❖ operations</p>	<p>- In pairs, student teachers think themselves two numbers or more having different powers but the same base, multiply and divide them. Then draw conclusion.</p> <p>- Individually, student teacher simplifies powers by rationalizing the denominators</p> <p>- In groups, express each of the given powers as the square root of a single number, discuss and reduce power to the simplest possible power form.</p> <p>- Individually, student teacher find the square roots of given numbers by using Square roots methods and calculators.</p> <p>- Tutor facilitate student teachers to use calculator or Microsoft Excel to find images of given real numbers, discuss their existence in the set of real numbers, dress a table of values for given numbers.</p>
Assessment criteria: Be able to Solve problems related to powers, indices, radical and common logarithms.				
Link to other subjects: Integrated Science, Entrepreneurship, Social studies.				
Resources: Manila papers, logarithmic tables, Geometrical instruments, calculators.				

SUBJECT: MATHEMATICS			YEAR ONE	OPTION: ECLPE
Topic Area: STATISTICS			Sub Topic: DESCRIPTIVE STATISTICS.	
UNIT 8: PARAMETERS OF CENTRAL TENDENCIES AND DISPERSION				No. of periods: 15
Key unit competence: Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to the standard deviation.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		Skills
<p>-Define the variance, standard deviation and the coefficient of variation.</p> <p>-Interpret critically data and infer conclusions.</p>	<p>-Calculate and interpret Measures of central tendency</p> <p>- Determine the measures of dispersion of a given statistical series.</p> <p>- Apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data.</p> <p>- Express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.</p>	<p>- Appreciate the importance of measures of dispersion in the interpretation of data.</p> <p>- Show concern on how to use the standard deviation as measure of variability of data.</p>	<ul style="list-style-type: none"> ➤ Collection and presentation of grouped and ungrouped data. ➤ Central tendencies (mean, median, mode) ➤ Graphical representation of grouped and ungrouped data ➤ Measure of dispersion (range, variance, Standard Deviation and coefficient of variation). ➤ Application in daily life. 	<p>- In pairs, student teachers are encouraged to collect, analyze their own data from real life.</p> <p>-In groups, student teacher will be given a task and be asked to:</p> <ul style="list-style-type: none"> . Discuss about the measures of dispersion, interpret them and represent their findings. . Perform research on given problems arising from various situations in daily life, investigate them to include the standard deviation, and represent their findings.
Assessment criteria: Determine, analysis and interpret data arising from questions or problems related to daily life based on parameters of central tendency or dispersion.				
Link to other subjects: Integrated sciences, social studies ,creative performance				
Resources: geometrical instruments, calculators.				

5.2 Mathematics syllabus unit for year 2

5.2.1. Key competences at the end of year 2

After completion of year two Early Childhood and Lower Primary Education, the Mathematics syllabus will enable the student-teacher to:

1. Extend the concept of arithmetic and geometric progression to sequences and series.
2. Locate points and determine equations of a line and a circle in 2D.
3. Apply trigonometric concepts in solving simple problems on triangles.
4. Apply the concepts and definitions of functions to determine the domain of polynomial, rational irrational, trigonometric, functions

5.2.2 Syllabus units for year two

SUBJECT: MATHEMATICS		YEAR TWO		Option : ECLPE	
Topic Area: ARITHMETIC		Sub Topic: SEQUENCES			
UNIT 1: SEQUENCE				No. of periods: 30	
Key unit competence: Extend the concept of arithmetic and geometric progression to sequences and apply them to solve related problems					
Learning objectives			Content	Teaching/ Learning Activities	
Knowledge and understanding	Skills	Attitudes and values			
<ul style="list-style-type: none"> - Define and understand arithmetic progressions and their properties. - Determine the value of “n”, given the sum of the first “n” terms of arithmetic progressions. - Show how to apply formulas to determine the “nth” term and the sum of the first “n” terms of arithmetic progressions. - Extend the concepts of arithmetic progression to harmonic sequences and their properties. 	<ul style="list-style-type: none"> Use basic concepts and formulas of sequences to find the value “n”, given the sum of the first “n” terms of arithmetic progressions. - Use basic concepts and formulas of sequences to find the value “n”, given the sum of the first “n” terms of arithmetic progressions or the “nth” term and the sum of the first “n” terms of geometric progressions. 	<ul style="list-style-type: none"> -Appreciate the relationship between the sequences and other subjects to understand occurring situations. -Appreciate the use of sequence in daily life. 	<ul style="list-style-type: none"> ➤ Generalities on sequences ➤ Arithmetic and harmonic sequences ➤ Geometric sequences. ➤ Applications 	<ul style="list-style-type: none"> -In groups student teachers are given tasks of folding a piece of paper to make them understand the meaning of geometric sequences, and think what should be the last term to the infinity -In groups, student teachers investigate and discuss how the bank rates increase or decrease unexpectedly year by years. -In groups student-teachers will solve problems involving sequence. 	
Assessment criteria: Students-teachers are able to apply concepts of sequences to solve problems involving arithmetic and geometric sequences.					
Link to other subjects: Social studies, entrepreneurship and Integrated Science					
Resources: Graph papers, digital technology including calculators, manila paper, flash cards.					

Subject: MATHEMATICS			YEAR TWO	OPTION: ECLPE
Topic Area: GEOMETRY			Sub Topic: EUCLIDIAN PLANE GEOMETRY.	
UNIT 2: POINTS, STRAIGHT LINES AND CIRCLE IN 2D.				No. of periods: 21
Key unit competence: Determine an equation of a line and a circle				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
-Define the coordinate of a point in 2D. - Define a straight line knowing its: <ul style="list-style-type: none"> • 2 points. • Direction vector • Gradient. 	- Calculate the distance between two points in 2D and the mid-Point of a segment in 2D. - Determine equations of a straight line (vector equation, parametric equation, Cartesian equation). - Apply knowledge to find the center, radius, and diameter to find out the equation of a circle.	- Appreciate that a point is a fixed position in a plane. - Be accurate in representations and calculations. - Manifest a team spirit and think critically in problem solving related to the position of straight lines in 2D.	<ul style="list-style-type: none"> ➤ Location of a point in 2D ➤ Mid-point and Distance between two points in 2D ➤ Determination of equation of a straight line (vector equation, parametric equation, Cartesian equation). 	-Student teachers represent on graphs some points, straight lines and circles in 2D. - In pairs, Student teachers represent on graph paper some chosen points, lines and/or circles and determine their parametric or Cartesian equations.
Assessment criteria: Determine the equation of lines, straight lines and circles in the 2D.				
Link to other subjects: Integrated Science, Entrepreneurship, Social studies.				
Resources: Graph papers, manila papers, digital technology including calculators, mathematical set (ruler, T-square, protractor)				

SUBJECT: MATHEMATICS			YEAR TWO	Option : ECLPE
Topic Area: TRIGONOMETRY			Sub Topic: TRIGONOMETRIC CIRCLE, RATIOS AND IDENTITIES	
UNIT 3 : APPLICATION OF TRIGONOMETRIC CONCEPTS IN SOLVING PROBLEMS				No. of periods: 30
Key unit competence: Apply trigonometric concepts in solving problems on triangles and real life situation.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
<ul style="list-style-type: none"> - Convert radians to degree and vice versa. - Define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle <ul style="list-style-type: none"> - Know trigonometric ratios of special angles ($30^\circ, 45^\circ, 60^\circ$) - Differentiate between complementary angles, supplementary angles and co-terminal angles 	<ul style="list-style-type: none"> - Use trigonometry, including the sine and cosine rules, to solve problems involving triangles 	<ul style="list-style-type: none"> - Appreciate the relationship between the trigonometric values for different angles - Appreciate the use of trigonometry in daily life situation 	<ul style="list-style-type: none"> ➤ Trigonometric concepts: <ul style="list-style-type: none"> ❖ Angles measurement <ul style="list-style-type: none"> • Radian • Degree ❖ Units conversion ❖ Unit circle ❖ Trigonometric ratios <ul style="list-style-type: none"> • Definition • Trigonometric ratio of special angles ❖ Trigonometric identities <ul style="list-style-type: none"> ➤ Reduction to functions of positive acute angles ➤ Triangles and Applications (Bearing, Air Navigation, Inclined plane). 	<p>In groups student teachers:</p> <ul style="list-style-type: none"> - Using protractor and compass, construct special angles ($30^\circ, 45^\circ, 60^\circ$) on a unit circle, then determine the cosine and sine of each angle and deduce that each value is less than 1 - Use unit circle to establish relationship between trigonometric ratios and angles in different quadrants (complementary angles, supplementary angles, opposite angles,...) - Make a research on trigonometric identities and present findings to the whole class. - Make a research on the applications of trigonometry and present findings to the whole class.
Assessment criteria: Solve correctly a triangle based on trigonometric ratios and identities.				
Link to other subjects: Integrated sciences, social studies.				
Resources: Graph paper, ruler, digital technology including calculators, manila papers.				

SUBJECT: MATHEMATICS			YEAR TWO	OPTION: ECLPE
Topic Area: CALCULUS			Sub Topic: LIMITS AND DIFFERENTIATION OF REAL FUNCTIONS	
UNIT 4: POLYNOMIAL FUNCTIONS				No. of periods: 27
Key unit competence: Use concepts and definitions of functions to determine the domain of polynomial functions, solve related problems and represent them graphically in simple cases (plotting linear and quadratic functions)				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
-Define a function - Find whether a function is even, odd, or neither.	- Determine the domain and range of a function. - Plot linear and quadratic functions - Interpret graphs of functions (linear and quadratic) related to practical context and make conclusions - Analyze, model and solve problems involving linear or quadratic functions and interpret the results.	- Appreciate the importance of linear and quadratic functions in daily life. - Show concern of using graphs of linear and quadratic functions in solving mathematical problems	<ul style="list-style-type: none"> ➤ Definition of a polynomial function ➤ Domain and range of a polynomial function ➤ Parity(even , odd) of a polynomial function ➤ Plotting linear and quadratic functions ➤ Solve problem related to linear and quadratic functions. 	-In groups, using manila papers , student teachers perform the tasks below and make presentation to the whole class: . Use the given coordinates to draw graphically linear and quadratic functions, . Model and interpret the problems related to linear and quadratic functions.
Assessment criteria: Plot and interpret graphs of linear and quadratic functions in relation with practical context and make conclusions				
Link to other subjects: Integrated science; Entrepreneurship.				
Resources: Graph paper, ruler, digital technology including calculators, manila papers.				

5.3 Mathematics syllabus unit for year 3

5.3.1. Key competences at the end of year 3

After completion of year three Early Childhood and Lower Primary Education, the Mathematics syllabus will enable the student-teacher to:

1. Determine domain of definition and evaluate limit of a numeric function and deduce related asymptote.
2. Differentiate a real function and apply derivatives to sketch the graphs and solve problems.
3. Solve problem involving the system of linear equations using matrices.
4. Extend the use of concepts and definitions of functions to determine the domain and sketch the graphs of logarithmic and exponential functions.

5.3.2 Syllabus units for year three

SUBJECT: MATHEMATICS			YEAR THREE	Option : ECLPE
Topic Area: CALCULUS			Sub Topic: LIMITS AND DIFFERENTIATION OF REAL FUNCTIONS	
UNIT 1: Polynomial, rational and irrational functions.				No. of periods: 28
Key unit competence: Evaluate correctly domain of definition and limit of a function and determine asymptote to related function.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
<ul style="list-style-type: none"> -Define the domain and range of polynomial functions - State the properties of even and odd functions -Identify properties of polynomial, rational and irrational functions. 	<ul style="list-style-type: none"> - Determine compositions of functions. - Determine the domain and range of a function. - Model and solve problems related to functions in various practical contexts by applying different properties of functions. - Calculate limits of certain elementary functions. - Solve problems involving continuity. 	<ul style="list-style-type: none"> -Appreciate the importance of functions in daily life. - Show concern for the importance, use and determination of limits of functions in daily life. - Appreciate the use of asymptotes in daily life. 	<ul style="list-style-type: none"> ➤ Generalities on numerical functions ❖ Properties ❖ Operations ➤ Domain of definition ❖ Existence condition ❖ Domain and range of a function. ➤ Symmetry of a function ❖ Even function ❖ Odd function 	<ul style="list-style-type: none"> - In groups, student teachers study algebraically and graphically polynomial functions which will help them to find the domain and range of functions and present their work -Guided by the tutor, student teachers will perform different activities such as inscribing polygons in circle and students will discover that the circle is the limiting position of an inscribed polygon when the number of sides approaches to infinity; - Referring to examples of real life such as” the bridge in the road” students will be helped to understand the concept of “continuity” and” discontinuity”. -Guided by the tutor, student teachers will deduce the limit notation, give properties of limits and evaluate limits of functions;

<p>- Define the concept of limit for real-valued functions of one real variable.</p> <p>-State the properties of limits of functions</p> <p>- Explain how to determine the asymptotes of the given function.</p>	<p>- Apply informal methods to explore the concept of a limit including one sided limits.</p> <p>- Use the concepts of limits to determine the asymptotes to the rational and polynomial functions.</p>		<p>➤ Limits and continuity</p> <ul style="list-style-type: none"> ❖ Introduction to limits and continuity ❖ Properties of limits ❖ Operations on limits <ul style="list-style-type: none"> • Finite limits • Infinite limits • Limit at infinity • Indeterminate cases $\left(\frac{\infty}{\infty}, \frac{0}{0}, \infty - \infty, 0 \cdot \infty \right)$ <ul style="list-style-type: none"> ❖ Continuity <ul style="list-style-type: none"> • Continuity of a function at a point or on interval • Properties on continuity • Point of discontinuity ➤ Asymptotes <ul style="list-style-type: none"> ❖ Definition and interpretation ❖ Types and equations of asymptotes 	<p>- In groups of two students learners guided by the tutor will describe and remove indeterminate cases using limits.</p> <p>-Guided by the tutor, student teachers in groups will discover the types of asymptotes and establish the equations of asymptotes</p>
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Assessment criteria: Determine correctly the domain of definition for rational functions and irrational by establishing existence condition and deduce related asymptotes equations from calculation of limits.

Link to other subjects: Integrated Science, Entrepreneurship, Social studies, creative performance.

Resources: Manila papers, graph paper, ruler, markers, digital technology.

SUBJECT: MATHEMATICS			YEAR THREE	Option : ECLPE
Topic Area: CALCULUS			Sub Topic: LIMITS AND DIFFERENTIATION OF REAL FUNCTIONS	
UNIT 2 : DIFFERENTIATION				No. of periods: 20
Key unit competence: Differentiate a real function and apply derivatives to sketch the graphs of the given functions.				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
<ul style="list-style-type: none"> - Define the derivative of a function - Identify techniques of differentiation to be used depending on the type of differentiable function 	<ul style="list-style-type: none"> -Use properties of derivatives to differentiate polynomial, rational and irrational functions. - Use first principles to determine the gradient of the tangent line to a curve at a point. - Apply the concepts and techniques of differentiation to model, analyze and solve rates or optimization problems in different situations. 	<ul style="list-style-type: none"> -Appreciate the use of gradient as a measure of rate of change. - Appreciate the importance of use of differentiation in kinematics (velocity, acceleration). - Show concern on derivatives to help in the understanding of optimization problems. 	<ul style="list-style-type: none"> ➤ Derivative concepts <ul style="list-style-type: none"> ❖ Definition and graphical interpretation ❖ Rules of differentiation ❖ Derivative of elementary function ➤ Use of differentiation for a given function <ul style="list-style-type: none"> ❖ Variation of a function (increasing, decreasing interval or turning points) ❖ Concavity (inflexion points) ➤ Curves sketching <ul style="list-style-type: none"> ❖ Table of variation ❖ Additional points ➤ Application of differentiation (Hospital's rules, tangent and Normal line equation, rates of change problems). 	<ul style="list-style-type: none"> -Tutor will introduce the derivative using a real example like a car travelling at a constant speed, using distance and time, notice that the distance from the starting point increases at a constant rate of that speed each hour. From this example, guided by the tutor, student teachers will deduce the definition of derivative at a point; - Guided by the tutor student teachers in groups will identify the rules of differentiation for basic functions ; -Guided by the tutor, student teachers in groups will apply Hospital rules to remove indeterminate forms; they will also solve problems related to derivatives.
Assessment criteria: Determine, analyze and interpret derivative of a real function and apply it to sketch an accurate curve of a given function or to solve problems related to optimization.				
Link to other subjects: Integrated Science, Entrepreneurship, Social studies, creative performance.				
Resources: Manila papers, graph paper, ruler, markers, digital technology.				

SUBJECT: NUMERACY AND MATHEMATICS		YEAR THREE		Option : ECLPE	
TOPIC AREA: LINEAR ALGEBRA		SUB TOPIC: MATRICES AND SYSTEM OF LINEAR EQUATIONS.			
UNIT 3: Matrices and determinants of order 2 and order 3				No. of periods: 15	
Key unit competence: Solve problems involving the system of linear equations using matrices					
Learning objectives			Content	Teaching/ Learning Activities	
Knowledge and understanding	Skills	Attitudes and values			
<ul style="list-style-type: none"> - Define the order of a matrix. - Explain that a given square matrix of order 2 or order 3 is invertible or not. - Explain using determinant whether a matrix of order 2 or 3 is invertible or not. - Reorganize data into matrices 	<ul style="list-style-type: none"> - Perform operations on matrices of order 2 or 3. - Calculate the determinants of matrices of order 2 and 3. - Determine the inverse of a matrix of order 2 or 3. - Use various methods such as Cramer, inverse matrix, and comparison to solve a system of two/three linear equations in two / three unknowns. -Discuss with respect to a parameter the solutions of a system of three linear equations in three unknown 	<ul style="list-style-type: none"> -Appreciate the importance of matrices of order 2/ 3 and their determinants in organizing data and solving a system of two/ three linear equations in two / three unknowns. 	<ul style="list-style-type: none"> Definition and Size of matrix <ul style="list-style-type: none"> ➤ Operations on matrix ❖ Equality of two matrices ❖ Addition and subtraction ❖ Multiplication ❖ Determinant of 2×2 or 3×3 matrices ❖ Inverse of matrix of order 2×2 or 3×3 <ul style="list-style-type: none"> ➤ System of linear equations ❖ Solve using Cramer method ❖ Solve using inverse matrix 	<ul style="list-style-type: none"> -Guided by the tutor, student teachers use examples like marks of student in different subjects to arrange them in table in order to understand the concept of matrix then deduce the types of matrices; -Guided by the tutor, student teacher in their groups will perform operations on matrices; -Guided by the tutor, student teachers will compute the determinant of matrices using different methods. -Student teachers, in groups discuss whether a matrix of order 2 or 3 is invertible or not and present their findings. 	
Assessment criteria: Solve correctly the problem involving the system of linear equations using matrices					
Link to other subjects: Integrated Science					
Resources: Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators					

SUBJECT: NUMERACY AND MATHEMATICS			YEAR THREE	Option : ECLPE
Topic Area: CALCULUS			Sub Topic: LIMITS AND DIFFERENTIATION OF REAL FUNCTIONS.	
UNIT 4: LOGARITHM AND EXPONENTIAL FUNCTION				No. of periods:18
Key unit competence: Extend the use of concepts and definitions of functions to determine the domain and sketch the graphs of natural logarithmic and exponential function				
Learning objectives			Content	Teaching/ Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
<ul style="list-style-type: none"> - Define logarithm or exponential function using properties of logarithms in any base. - State and demonstrate properties of logarithms and exponentials for any base. - Carry out operations using the change of base of logarithms. 	<ul style="list-style-type: none"> -Use the properties of logarithms to solve logarithmic and exponential equations. - Convert the logarithm to exponential form for any base - Apply logarithms or exponential to solve rates problems, mortgage problems, population growth problems. 	<ul style="list-style-type: none"> -Appreciate the use of logarithmic equations to model and solve problem involving logarithms such radioactive-decay problems, carbon dating problems, problems about alcohol and risk of car accident. 	<ul style="list-style-type: none"> ➤ Introducing natural logarithm function (<i>with base e</i>) ❖ Domain and range of logarithmic functions ➤ Properties and operations on natural logarithmic function ➤ Logarithmic equations ❖ Domain of validity ❖ Solutions of logarithmic equation ➤ Algebraic and graphical determination of limit of $f(x) = \ln x$ ❖ Calculation of limit of logarithmic functions (<i>with base e</i>) ❖ Determination of asymptotes related to logarithmic functions (<i>with base e</i>) 	<ul style="list-style-type: none"> -Student teachers use scientific calculators to evaluate logarithms and exponentials of real numbers; they conclude about the domain (the allowed input values) and the range (the set of possible outputs). - Student teachers may use software, such as Geogebra, to graph logarithmic and exponential functions and report to class their findings about the general trend of the graphs. - From definition of derivative, find differentiation formulas of logarithmic and exponential functions. - Discuss in groups the applications of logarithms and exponentials in real life.

			<ul style="list-style-type: none"> ➤ Differentiation of natural logarithmic functions ❖ Definition and calculation ❖ Increasing and/or decreasing interval and turning points ❖ Concavity and inflection points ➤ Graphical representation ❖ Table of variation ❖ Additional points ❖ Curve plotting ➤ Modeling and solving problems involving natural logarithms ➤ Introducing exponential function ❖ Domain and range of exponential functions (<i>with base e</i>) ➤ Properties and operations on exponential function ➤ Exponential equations ❖ Domain of validity ❖ Solutions of logarithmic equation ➤ Algebraic and graphical determination of limit of $f(x) = e^x$ ❖ Calculation of limit of exponential functions (<i>with base e</i>) ❖ Determination of asymptotes related to exponential functions (<i>with base e</i>) 	
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			<ul style="list-style-type: none"> ➤ Graphical representation ❖ Table of variation ❖ Additional points ➤ Modeling and solving problems exponents ➤ Modeling and solving problems involving logarithms or exponents for any base. 	
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Assessment criteria: Determine correctly the domain of definition of logarithmic and exponential functions based on existence conditions, sketch the corresponding graphs and solve problems involving logarithms or exponentials related to real life.

Link to other subjects: Social Studies, Entrepreneurship, SET, ICT, Integrated Science

Resources: Geometric instruments (ruler, T-square, compass), graph paper, digital technology including calculators, manila paper, ICT tools.

REFERENCE

1. A. J. Sadler, D. W. S. Thorning: *Understanding Pure Mathematics*, Oxford University Press 1987.
2. Arthur Adam, Freddy Goossens and Francis Lousberg (1991). *Mathematisons 65*, DeBoeck, 3rd edition.
3. Balakrishnan, V.K . (1995). Schaum's Outline, Combinatorics.
4. Bird, J. (2003). *Engineering Mathematics* (4th Edition).
5. Bronson, R. (2003). Schaum's Outline, Differential equation.
6. Buxkemper, A. (2001). *Mathematics for elementary education: Preliminary edition*. New York: McGraw-Hill
7. Clements, D.H.& Sarama, J.(2009). *Learning and Teaching Early Math: The learning trajectories approach*. London: Routledge.
8. CrashwshaW,J.,& Chambers,J.(2001).Advanced Level Statistics with worked examples (Fourth Edition). Nelson Thornes Ltd:UK
9. Curriculum Planning and Development Division (2006). Secondary Mathematics Syllabuses. Ministry of Education: Singapore
10. David Rayner, *Higher GCSE Mathematics*, Oxford University Press 2000
11. Frank Ebos, Dennis Hamaguchi, Barbara Morrison & John Klassen (1990), *Mathematics Principles & Process*, Nelson Canada A Division of International Thomson Limited .
12. George B. Thomas, Maurice D. Weir & Joel R. Hass (2010), *Thomas' Calculus Twelfth Edition*, Pearson Education.
13. H.K. DASS... *Engineering Mathematics*. New Delhi, S.CHAND&COMPANY LTD, thirteenth revised edition 2007.
14. Hoffman, L.D.; Bradley, G.L.& Rosen, K.H.(2005). *Applied Calculus*. New York: McGraw-Hill.
15. J CRAWSHAW, J CHAMBERS (1984).*A concise course in A-Level statistics with worked examples*, Stanley Thornes (Publishers) LTD.
16. John, A.; Robert, G.; & Sorgenfrey (1993). *Trigonometry with application*. Boston: Houghton Mifflin Company.

17. Ministry of Education (2015). *Upper Primary level Mathematics Curriculum (P4, P5, P6)*.
18. National Curriculum Development Centre (2008). *Mathematics teaching syllabus: Uganda Certificate of Education Senior1-4. Pupils' Books-Grade 1-3*. Nairobi: The Jomo Kenyata Foundation.
19. Rwanda Education Board (2015). *Mathematics syllabus for advanced level S4 – S6*. Kigali
20. Sanchez , A.V, & Ruiz, M.P. (2008). *Competence-Based Learning : A proposal for the assessment of Generic Competences*(Eds).
21. Shampiyona, A. (2005). *Mathématiques 6*. Imprimerie de Kigali: Kigali
22. Spiegel,M. R.; Schiller, J. & Srinivasan, A. (2009). *Schaum's Outline, Probability and Statistics: Third Edition. Teacher's Books-Grade 1-3*. Nairobi: The Jomo Kenyata Foundation.
23. Uworwabayeho, A. , Ndung'u, R. , Sebashumba Kamali, & Rwamanywa, N. (2010) *Primary Mathematics*.
24. Uworwabayeho, A. , Ndung'u, R. , Sebashumba Kamali, & Rwamanywa, N. (2010) *Primary Mathematics*.
25. Van de Walle, J& Lovin, H.L. (2016). *Teaching Student-centered mathematics: Grades K-3*. Volume 1. New York: Pearson Education.
26. Ngezahayo, E. P. (2016). *Advanced Mathematics for Rwanda Secondary Schools. Learners' Book Senior Four*. Kigali: Fountain.
27. Ngezahayo, E. P. (2017). *Advanced Mathematics for Rwanda Secondary Schools. Learners' Book Senior Five*. Kigali: Fountain.
28. Ngezahayo, E. P. (2018). *Advanced Mathematics for Rwanda Secondary Schools. Learners' Book Senior Six*. Kigali: Fountain.
29. Peter Smythe: *Mathematics HL & SL with HL options, Revised Edition*, Mathematics Publishing Pty. Limited, 2005.
30. Igbokwe DI (2000): Dominant factors and Error Types inhibiting the understanding of Mathematics. 41st Annual Conference Proceedings of STAN, 242-249.

Annexes

7.1 Annex 1: TTC subjects and time allocation

No	Subject	Number of Periods								Number of Syllabi
		ECLPE		SSE		SME		LE		
		Y1	Y2&3	Y1	Y2&3	Y1	Y2&3	Y1	Y2&3	
1	Foundations of Education	6	6	6	5	6	5	6	5	1
2	English	5	5	5	4	5	4	7	7	3
3	Kinyarwanda	5	5	2	2	2	2	5	6	3
4	Mathematics	3	3	3	3	6	6	2	2	3
5	Integrated Science	4	4	1	1	11	0	1	1	3
6	Physics	0	0	0	0	0	5	0	0	1
7	Biology	0	0	0	0	0	5	0	0	1
8	Chemistry	0	0	0	0	0	5	0	0	1
9	Social Studies	4	4	11	0	2	2	2	2	3
10	History	0	0	0	5	0	0	0	0	1
11	Geography	0	0	0	5	0	0	0	0	1
12	Economics	0	0	0	5	0	0	0	0	1
13	Creative Performance (Music and Fine Arts)	4	4	4	4	2	2	4	4	2
14	Physical Education	1	1	2	1	1	1	1	1	1
15	Entrepreneurship	2	2	2	2	2	2	2	2	1
16	ICT	2	2	2	2	2	2	2	2	1

17	TMP	7	7	4	4	6	4	4	4	11
18	SNE	2	2	2	2	2	2	2	2	1
19	Religious Education	1	1	4	3	1	1	1	1	2
20	French	4	4	2	2	2	2	7	7	3
21	Kiswahili	1	1	1	1	1	1	5	5	2
22	Co-Curricular	1	1	1	1	1	1	1	1	0
23	Individual Study	8	8	8	8	8	8	8	8	0
24	School Attachment	Year 3 (First term)								
	TOTAL	60	60	60	60	60	60	60	60	46

7.2 Annex 2: overview of Mathematics for ECLPE

TOPIC AREA	SUB TOPIC AREA	COMPETENCES		
		YEAR 1	YEAR 2	YEAR 3
I.ARITHMETIC	1.1.SETS	<ul style="list-style-type: none"> ▪ Classify numbers into naturals, integers, rational and irrationals. ▪ Solve problems that involve Set operations using Venn diagram. 		
	1.2.RATIOS AND PROPORTIONS	Apply ratios, proportions and multiplier proportion change to solve real life related problems.		
	1.3.SEQUENCES (PROGRESSIONS)		<ul style="list-style-type: none"> ▪ Extend the concept of arithmetic and geometric progression to sequences and series. ▪ Apply concepts of sequences and series to solve related problems. 	
II.ALGEBRA	2.1. ELEMENT OF LOGIC	Use Mathematical logic as a tool of reason and argumentation in daily life situation.		
	2.2. POLYNOMIALS	Perform operations on polynomials and solve related problems.		

	2.3 EQUATIONS AND INEQUALITIES	<ul style="list-style-type: none"> ▪ Solve algebraically or graphically linear and quadratic equations or inequalities. ▪ Solve and discuss parametric equations of second degree. 		
	2.4. POWERS ,INDICES, RADICALS AND LOGARITHMS	Solve problems related to powers, indices, radical and common logarithms.		
III.GEOMETRY	3.1. EUCLIDIAN PLANE GEOMETRY.		Determine an equation of a line and a circle.	
IV.TRIGONOMETRY	4.1. TRIGONOMETRIC CIRCLE, RATIOS AND IDENTITIES	Apply trigonometric concepts in solving problems on triangles and real life situation.		
V.STATISTICS	5.1. DESCRIPTIVE STATISTICS.	Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to the standard deviation.	Extend understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines.	

VI.CALCULUS	6.1 LIMITS; DIFFERENTIATION OF REAL FUNCTIONS	Use concepts and definitions of functions to determine the domain of polynomial functions and represent them graphically in simple cases(plotting linear and quadratic functions)	<ul style="list-style-type: none"> ▪ Apply the concepts and definitions of functions to determine the domain of rational and irrational functions. ▪ Evaluate correctly limit of function and deduce asymptotes of a real function. ▪ Differentiate a real function and apply derivatives to sketch the graphs. 	<ul style="list-style-type: none"> ▪ Evaluate correctly domain of definition and limit of a function and determine asymptote to related function. ▪ Differentiate a real function and apply derivatives to sketch the graphs of the given functions. ▪ Extend the use of concepts and definitions of functions to determine the domain and sketch the graphs of natural logarithmic and exponential function
VII.LINEAR ALGEBRA	7.1. MATRICES AND SYSTEM OF LINEAR EQUATIONS.			Solve problems involving the system of linear equations using matrices(Cramer's methods, Gaussian elimination methods and inverse matrix)