REPUBLIC OF RWANDA

MINISTRY OF EDUCATION

Rwanda Education Board

CURRICULUM FOR SUSTAINABLE DEVELOPMENT

SUBSIDIARY MATHEMATICS SYLLABUS S4-S6

DIGNITY AND NATIONAL IDENTITY
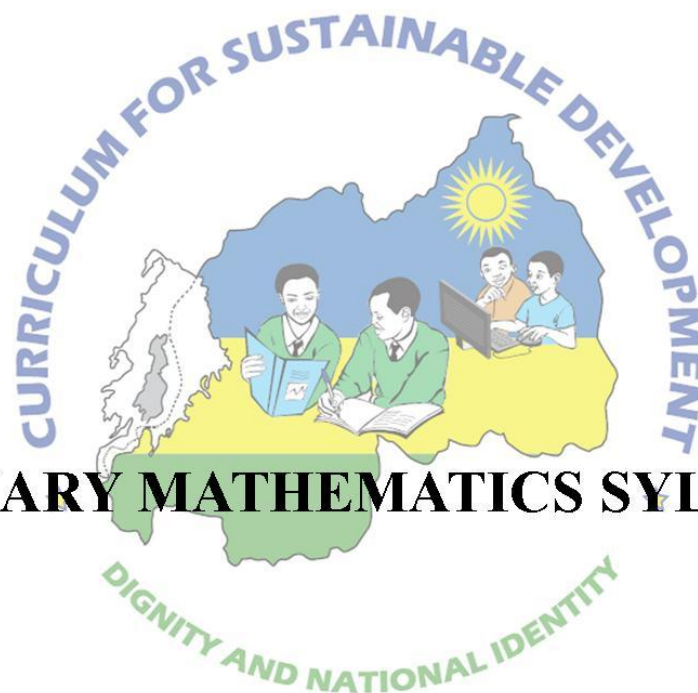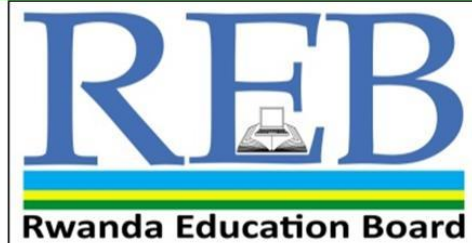
Kigali, 2015

SUBSIDIARY MATHEMATICS SYLLABUS FOR ADVANCED LEVEL S4 – S6

Kigali, 2015

FOREWORD

The Rwanda Education Board is honored to avail Syllabuses which serve as official documents and guide to competence-based teaching and learning in order to ensure consistency and coherence in the delivery of quality education across all levels of general education in Rwandan schools.

The Rwandan education philosophy is to ensure that young people at every level of education achieve their full potential in terms of relevant knowledge, skills and appropriate attitudes that prepare them to be well integrated in society and exploit employment opportunities.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of aligning the syllabus, teaching and learning and assessment approaches in order to ensure that the system is producing the kind of citizens the country needs. Many factors influence what children are taught, how well they learn and the COMPETENCES they acquire, among them the relevance of the syllabus, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials available. The ambition to develop a knowledge-based society and the growth of regional and global competition in the jobs market has necessitated the shift to a competence-based syllabus. With the help of the teachers, whose role is central to the success of the syllabus, learners will gain appropriate skills and be able to apply what they have learned in real life situations. Hence they will make a difference not only to their own lives but also to the success of the nation.

I wish to sincerely extend my appreciation to the people who contributed towards the development of this document, particularly REB and its staff who organized the whole process from its inception. Special appreciation goes to the development partners who supported the exercise throughout. Any comment of contribution would be welcome for the improvement of this syllabus.

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ACKNOWLEDGEMENT

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1. INTRODUCTION

1.1. Background to curriculum review

The Subsidiary Mathematics syllabus is developed to help leaners in combinations where Mathemtics is not core subject and draws on the previous syllabus for Pysics-Chemistry and Biology combianation.

The motive of reviewing the syllabus was to ensure that the syllabus is responsive to the needs of the learner and to shift from objective and knowledge-based learning to competence-based learning. Emphasis in the review is put more on skills and COMPETENCES and the coherence within the existing content by benchmarking with syllabi elsewhere with best practices.

The new Mathematics syllabus guides the interaction between the teacher and the learners in the learning processes and highlights the COMPETENCES a learner should acquire during and at the end of each unit of learning.

Learners will have the opportunity to apply Mathematics in different contexts, and see its importance in daily life. Teachers help the learners appreciate the relevance and benefits for studying this subject in advanced level.

This syllabus is prepared to be taught in three periods a week in the following combinations:

Biology - Chemistry - Geography **(BCG),** History – Economics - Geography (**HEG**), History - Economics - Literature in English (**HEL**), Literature in English – Economics - Geography (**LEG**), Physics - Chemistry - Biology **(PCB)**.

1.2. Rationale of teaching and learning mathematics

1.2.1. Mathematics and society

Mathematics plays an important role in society through abstraction and logic, counting, calculation, measurement, systematic study of shapes and motion. It is also used in natural sciences, engineering, medicine, finance and social sciences. The applied

mathematics like statistics and probability play an important role in game theory, in the national census process, in scientific research,etc. In addition, some cross-cutting issues such as financial awareness are incorporated into some of the Mathematics units to improve social and economic welfare of Rwandan society.

Mathematics is key to the Rwandan education ambition of developing a knowledge-based and technology-led economy since it provide to learners all required knowledge and skills to be used in different learning areas. Therefore, Mathematics is an important subject as it supports other subjects.This new curriculum will address gaps in the current Rwanda Education system which lacks of appropriate skills and attitudes provided by the current education system.

1.2.2. Mathematics and learners

Learners need enough basic mathematical COMPETENCES to be effective members of Rwandan society including the ability to estimate, analyse, interpret statistics, assess probabilities, and read the commonly used mathematical representations and graphs.

Therefore, Mathematics equips learners with knowledge, skills and attitudes necessary to enable them to succeed in an era of rapid technological growth and socio-economic development. Mastery of basic Mathematical ideas and calculations makes learners being confident in problem-solving. It enables the learners to be systematic, creative and self confident in using mathematical language and techniques to reason; think critically; develop imagination, initiative and flexibility of mind. In this regard, learning of Matheamtics needs to include practical problem-solving activities with opportunities for students to plan their own investigations in order to develop their mathematical competence and confidence.

As new technologies have had a dramatic impact on all aspects of life, wherever possible in Mathematics, learners should gain experience of a range of ICT equipment and applications.

1.2.3. Competences

Competence is defined as the ability ability to perform a particular task successfully, resulting from having gained an appropriate combination of knowledge, skills and attitudes.

The Mathematics syllabus gives the opportunity to learners to develop different COMPETENCES, including the generic COMPETENCES.

Basic COMPETENCES are addressed in the stated broad subject competences and in objectives highlighted year on year basis and in each of units of learning. The generic COMPETENCES, basic competences that must be emphasized and reflected in the learning process are briefly described below and teachers will ensure that learners are exposed to tasks that help the learners acquire the skills.

Generic Competences and Values

- **Critical and problem solving skills:** Learners use different techniques to solve mathematical problems related to real life situations.They are engaged in mathematical thinking, they construct, symbolize, apply and generalize mathematical ideas.
  The acquisition of such skills will help learners to think imaginatively and broadly to evaluate and find solutions to problems encountered in all situations.

- **Creativity and innovation** : The acquisition of such skills will help learners to take initiatives and use imagination beyond knowledge provided to generate new ideas and construct new concepts. Learners improve these skills through Mathematics contest, Mathematics competitions,...

- **Research:** This will help learners to find answers to questions basing on existing information and concepts and to explain phenomena basing on findings from information gathered.

- **Communication:** Learners communicate effectively their findings through explanations, construction of arguments and drawing relevant conclusions.

  Teachers, irrespective of not being teachers of language, will ensure the proper use of the language of instruction by learners which will help them to communicate clearly and confidently and convey ideas effectively through speaking and writing and using the correct language structure and relevant vocabulary.

- **Cooperation, inter personal management and life skills**: Learners are engaged in cooperative learning groups to promote higher achievement than do competitive and individual work.

  This will help them to cooperate with others as a team in whatever task assigned and to practice positive ethical moral values and respect for the rights, feelings and views of others. Perform practical activities related to environmental conservation and protection. Advocating for personal, family and community health, hygiene and nutrition and Responding creatively to the variety of challenges encountered in life.

- **Lifelong learning**: The acquisition of such skills will help learners to update knowledge and skills with minimum external support and to cope with evolution of knowledge advances for personal fulfillment in areas that need improvement and development

Broad Mathematics Competences

During and at the end of learning process, the learner can:

1. Develop clear, logical, creative and coherent thinking.
2. Master basic mathematical concepts and to usethem correctly in daily life problem solving;
3. Express clearly, comprehensibly, correctlyandpreciselyin verbal and/orin written form all the reasons and calculations leading to the required result whenever findingasolution to any givenexercise;
4. Master the presented mathematical models and to identify their applications in the learner's environment.

5. Arouse learner's mathematical interest and research curiosity in theories and their applications.

6. Use the acquired mathematical concepts and skills to follow easily higher studies (Colleges,Higher Institutions and Universities);

7. Use acquired mathematical skills to develop work spirit, team work, self-confidenceand timemanagement without supervision;

8. UseICT tools to explore Mathematics(examples: calculators,computers, mathematical software,…).

9. Demonstrate a sense of research, curiosity and creativity in their areas of study.

Mathematics and Developing Competences

The national policy documents based on national aspirations identify some 'basic COMPETENCES' alongside the 'Generic COMPETENCES'' that will develop higher order thinking skills and help student learn subject content and promote application of acquired knowledge and skills.

Through observations, constructions, using symbols, applying and generalizing mathematical ideas, and presentation of information during the learning process, the learner will not only develop deductive and inductive skills but also acquire cooperation and communication, critical thinking and problem solving skills. This will be realized when learners make presentations leading to inferences and conclusions at the end of learning unit. This will be achieved through learner group work and cooperative learning which in turn will promote interpersonal relations and teamwork.

The acquired knowledge in learning Mathematics should develop a responsible citizen who adapts to scientific reasoning and attitudes and develops confidence in reasoning independently. The learner should show concern of individual attitudes, environmental protection and comply with the scientific method of reasoning. The scientific method should be applied with the necessary rigor, intellectual honesty to promote critical thinking while systematically pursuing the line of thought.

The selection of types of learning activities must focus on what the learners are able to demonstrate such COMPETENCES throughout and at the end of the learning process.

2. PEDAGOGICAL APPROACH

The change to a competence-based curriculum is about transforming learning, ensuring that learning is deep, enjoyable and habit-forming.

2.1. Role of the learner

In the competence-based syllabus, the learner is the principal actor of his/her education. He/she is not an empty bottle to fill. Taking into account the initial capacities and abilities of the learner, the syllabus lists under each unit, the activities of the learner and they all reflect appropriate engagement of the learner in the learning process

The teaching- learning processes will be tailored towards creating a learner friendly environment basing on the capabilities, needs, experience and interests. Therefore, the following are some of the roles or the expectations from the learners:

- Learners construct the knowledge either individually or in groups in an active way. From the learning theory, learners move in their understanding from concrete through pictorial to abstract. Therefore, the opportunities should be given to learners to manipulate concrete objects and to use models.
- Learners are encouraged to use hand-held calculator. This stimulates mathematics as it is really used, both on job and in scientific applications. Frequent use of calculators can enhance learners' understanding and mastering of arithmetic.
- Learners work on one competence at a time in form of concrete units with specific learning objectives broken down into knowledge, skills and attitude.
- Learners will be encouraged to do research and present their findings through group work activities.

- A learner is cooperative: learners work in heterogeneous groups to increase tolerance and understanding.
- Learners are responsible for their own participation and ensure the effectivness of their work.
- Help is sought from within the group and the teacher is asked for help only when the whole group agrees to ask a question
- The learners who learn at a faster pace do not do the task alone and then the others merely sign off on it.
- Participants ensure the effective contribution of each member, through clear explanation and argumentation to improve the English literacy and to develop sense of responsibility and to increase the self-confidence, the public speech ability, etc.

2.2. Role of the teacher

In the competence-based syllabus, the teacher is a facilitator, organiser, advisor, a conflict solver, ...

The specific duties of the teacher in a competence-based approach are the following:

- He/she is a facilitator, his/her role is to provide opportunities for learners to meet problems that interest and challenge them and that, with appropriate effort, they can solve. This requires an elaborated preparation to plan the activities, the place they will be carried, the required assistance.
- He/she is an organizer: his/herrole is to organize the learners   in the classroom or outside and engage them through participatory and interactive methods through the learning processes as individuals, in pairs or in groups. To ensure that the learning is personalized, active and participative , co-operative theteacher must identify the needs of the learners, the nature of the learning to be done, and the means to shape learning experiences accordingly
- He/she is an advisor: he/she provides counseling and guidance for learners in need. He/she comforts and encourages learners by valuing their contributions in the class activities.

- He/she is a conflict-solver: most of the activities competence-based are performed in groups. The members of a group may have problems such as attribution of tasks; they should find useful and constructive the intervention of the teacher as a unifying element.

- He/she is ethical and preaches by examples by being impartial, by being a role-model, by caring for individual needs, especially for slow learners and learners with physical impairments, through a special assistance by providing remedial activities or reinforncement activities. One should notice that this list is not exhaustive.

2.3. Special needs education and inclusive approach

All Rwandans have the right to access education regardless of their different needs. The underpinnings of this provision would naturally hold that all citizens benefit from the same menu of educational programs. The possibility of this assumption is the focus of special needs education. The critical issue is that we have persons/ learners who are totally different in their ways of living and learning as opposed to the majority. The difference can either be emotional, physical, sensory and intellectual learning challenged traditionally known as mental retardation.

These learners equally have the right to benefit from the free and compulsory basic education in the nearby ordinary/mainstream schools. Therefore, the schools' role is to enrol them and also set strategies to provide relevant education to them. The teacher therefore is requested to consider each learner's needs during teaching and learning process. Assessment strategies and conditions should also be standardised to the needs of these learners. Detailed guidance for each category of learners with special education needs is provided for in the guidance for teachers.

3. ASSESSMENT APPROACH

Assessment is the process of evaluating the teaching and learning processes through collecting and interpreting evidence of individual learner's progress in learning and to make a judgment about a learner's achievements measured against defined standards. Assessment is an integral part of the teaching learning processes. In the new competence-based curriculum assessment must also be competence-based; whereby a learner is given a complex situation related to his/her everyday life and asked to try to overcome the situation by applying what he/she learned.

Assessment will be organized at the following levels: School-based assessment, District examinations, National assessment (LARS) and National examinations.

3.1. Types of Assessment

3.1.1 Formative Assessment

Formative assessment helps to check the efficiency of the process of learning. It is done within the teaching/learning process. Continuous assessment involves formal and informal methods used by schools to check whether learning is taking place. When a teacher is planning his/her lesson, he/she should establish criteria for performance and behavior changes at the beginning of a unit. Then at the end of every unit, the teacher should ensure that all the learners have mastered the stated key unit COMPETENCES basing on the criteria stated, before going to the next unit. The teacher will assess how well each learner masters both the subject and the generic COMPETENCES described in the syllabus and from this, the teacher will gain a picture of the all-round progress of the learner. The teacher will use one or a combination of the following: (a) observation (b) pen and paper (c) oral questioning.

3.1.2 Summative assessment

When assessment is used to record a judgment of a competence or performance of the learner, it serves a summative purpose. Summative assessment gives a picture of a learner's competence or progress at any specific moment. The main

purpose of summative assessment is to evaluate whether learning objectives have been achieved and to use the results for the ranking or grading of learners, for deciding on progression, for selection into the next level of education and for certification. This assessment should have an integrative aspect whereby a student must be able to show mastery of all COMPETENCES.

It can be internal school based assessment or external assessment in the form of national examinations. School based summative assessment should take place once at the end of each term and once at the end of the year. School summative assessment average scores for each subject will be weighted and included in the final national examinations grade. School based assessment average grade will contribute a certain percentage as teachers gain more experience and confidence in assessment techniques and in the third year of the implementation of the new curriculum it will initially contribute 10% of the final grade, but will be progressively increased. Districts will be supported to continue their initiative to organize a common test per class for all the schools to evaluate the performance and the achievement level of learners in individual schools. External summative assessment will be done at the end of P6, S3 and S6.

3.2. Record keeping

This is gathering facts and evidence from assessment instruments and using them to judge the student's performance by assigning an indicator against the set criteria or standard. Whatever assessment procedures used shall generate data in the form of scores which will be carefully be recorded and stored in a portfolio because they will contribute for remedial actions, for alternative instructional strategy and feed back to the learner and to the parents to check the learning progress and to advice accordingly or to the final assessment of the students.

This portfolio is a folder (or binder or even a digital collection) containing the student's work as well as the student's evaluation of the strengths and weaknesses of the work. Portfolios reflect not only work produced (such as papers and assignments), but also it is a record of the activities undertaken over time as part of student learning. Besides, it will serve as

a verification tool for each learner that he/she attended the whole learning before he/she undergoes the summative assessment for the subject.

3.3. Item writing in summative assessment

Before developing a question paper, a plan or specification of what is to be tested or examined must be elaborated to show the units or topics to be tested on, the number of questions in each level of Bloom's taxonomy and the marks allocation for each question. In a competence based curriculum, questions from higher levels of Bloom's taxonomy should be given more weight than those from knowledge and comprehension level.

Before developing a question paper, the item writer must ensure that the test or examination questions are tailored towards competence based assessment by doing the following:

- Identify topic areas to be tested on from the subject syllabus.
- Outline subject-matter content to be considered as the basis for the test.
- Identify learning outcomes to be measured by the test.
- Prepare a table of specifications.
- Ensure that the verbs used in the formulation of questions do not require memorization or recall answers only but testing broad COMPETENCES as stated in the syllabus.

Structure and format of the examination

There will be one paper in Mathematics at the end of Primary 6. The paper will be composed by two sections, where the first section will be composed with short answer items or items with short calculations which include the questions testing for knowledge and understanding, investigation of patterns, quick calculations and applications of Mathematics in real life

situations. The second section will be composed with long answer items or answers with constructions, more calculations, investigation of patterns and generalization, analysis, interpratation and drawing conclusions. The items for the second section will emphasize on the mastering of Mathematics facts, the understanding of Mathematics concepts and its applications in real life situations. In this section, the assessment will find out not only what skills and facts have been mastered, but also how well learners understand the process of solving a mathematical problem and whether they can link the application of what they have learned to the context or to the real life situation. The Time required for the paper is three hours (3hrs).

The following topic areas have to be assessed: Trigonometry; algebra; analysis; linear algebra; geometry; statistics and probability.Topic areas with more weight will have more emphasis in the second section where learners should have the right to choose to answer 3 items out of 5.

3.4. Reporting to Parents

The wider range of learning in the new curriculum means that it is necessary to think again about how to share learners' progress with parents. A single mark is not sufficient to convey the different expectations of learning which are in the learning objectives. The most helpful reporting is to share what students are doing well and where they need to improve.

4. RESOURCES

4.1. Materials needed for implementation

The following list shows the main materials/equipments needed in the learning and teaching process:

- Materials to encourage group work activities and presentations: Computers (Desk tops&lab tops) and projectors; Manila papers and markers

- Materials for drawing & measuring geometrical figures/shapes and graphs: Geometric instruments, ICT tools such as geogebra, Microsoft student ENCARTA, ...

- Materials for enhancing research skills: Textbooks and internet (the list of the textbooks to consult is given in the reference at the end of the syllabus and those books can be found in printed or digital copies).

- Materials to encourage the development of Mathematical models: scientific calculators, Math type, Matlab, etc

The technology used in teaching and learning of Mathematics has to be regarded as tools to enhance the teaching and learning process and not to replace teachers.

4.2 Human Resource

The effective implementation of this curriculum needs a joint collaboration of educators at all levels. Given the material requirements, teachers are expected to accomplish their noble role as stated above. On the other hand school head teachers and directors of studies are required to make a follow-up and assess the teaching and learning of this subject due to their profiles in the schools. These combined efforts will ensure bright future careers and lives for learners as well as the contemporary development of the country.

In a special way, the teacher of Mathematics at ordinary level should have a firm understanding of mathematical concepts at the leavel he / she teaches. He/she should be qualified in Mathematics and have a firm ethical conduct. The teacher should possess the qualities of a good facilitator, organizer, problem solver, listener and adviser. He/she is required to have basic skills and competence of guidance and counseling because students may come to him or her for advice.

Skills required for the Teacher of Religious Education

The teacher of Mathematics should have the following skills, values and qualities:

- Engage learners in variety of learning activities
- Use multiple teaching and assessment methods
- Adjust instruction to the level of the learners
- Have creativity and innovation the teaching and learning process
- Be a good communicator and organizer
- Be a guide/ facilitator and a counsellor
- Manifest passion and impartial love for children in the teaching and learning process
- Make useful link of Mathematics with other Subjects and real life situations
- Have a good master of the Mathematics Content
- Have good classroom management skills

5. SYLLABUS UNITS

5.1. Presentation of the structure of the Syllabus Units

Subsidiary Mathematics is developed to be taught and learnt in advanced level of secondary education, i.e. in S4, S5 and S6 respectively. It means that subsidiary Mathematics syllabus is developed for combinations where Mathematics is not core subject. At every grade, the syllabus is structured in Topic Areas, sub-topic Areas where applicable and then further broken down into Units to promote the uniformity, effectivness and efficiency of teaching and learning Mathematics. The units have the following elements:

1. Unit is aligned with the Number of Lessons.
2. Each Unit has a Key Unit Competence whose achievement is pursued by all teaching and learning activities undertaken by both the teacher and the learners.
3. Each Unit Key Competence is broken into three types of Learning Objectives as follows:

a. *Type I:* Learning Objectives relating to Knowledge and Understanding (*Type I* Learning Objectives are also known as Lower Order Thinking Skills or LOTS)

b. *–Type II a*nd *Type III:* These Learning Objectives relate to acquisition of skills, Attitudes and Values (*Type II* and *Type III* Learning Objectives are also known as Higher Order Thinking Skills or HOTS) – These Learning Objectives are actually considered to be the ones targeted by the present reviewed curriculum.

4. Each Unit has a Content which indicates the scope of coverage of what to be tought and learnt in line with stated learning objectives

5. Each Unit suggests a non exhaustive list of Learning Activities that are expected to engage learners in an interactive learning process as much as possible (learner-centered and participatory approach).

6. Finally, each Unit is linked to Other Subjects, its Assessment Criteria and the Materials (or Resources) that are expected to be used in teaching and learning process.

The Mathematics syllabus for ordinary level has got 7 Topic Areas: Trigonometry, Algebra, Analysis, Linear algebra, Geometry, Statistics and Probability and these topic areas are found in each of the three grades of the advanced level which are S4, S5 and S6. As for units, they are 10 in S4, 8 in S5 and 4 in S6

5.2. Secondary four

5.2.1. Key Competences at the end of Secondary Four

After completion ofsecondary 4, the mathematics syallabus will help the learnerto:

1. Use the trigonometric concepts and formulas in solving problem related to trigonometry;
2. Think critically and analyze daily life situations efficiently using mathematical logic concepts and infer conclusion.
3. Model and solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities.
4. Represent graphically simple numerical functions.
5. Determine algebraic representations of lines, straight lines and circles in the 2D

6. Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.

7. Use matrices and determinants of order 2 to solve systems of linear equations and to define transformations of 2D

8. Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.

9. Use counting techniques and concepts of probability to determine the probability of possible outcomes of events occurring under equally likely assumptions

10. Evaluate correctly limits of functionsand apply them to solve related problems

11. Use differentiation to solve and interpret rates and optimization problems in various contexts

5.2.2. Mathematics units for Secondary Four

Topic Area: TRIGONOMETRY			Sub-topic Area: TRIGONOMETRIC CIRCLE AND IDENTITIES	
S4- MATHEMATICS		**Unit 1 :** Fundamentals of trigonometry		**No. of lessons: 15**
Key unit Competence: Use the trigonometric concepts and formulas to solve related problems in Physics, Air navigation, Water navigation, bearings, Surveying, ...				
Learning Objectives			**Contents**	**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Define sine, cosine, and tangent (cosecant, secant and cotangent) of any angle – know special values ($30°$, $45°$, $60°$) ❖ Convert radians to degree and vice versa. ❖ Differentiate between	❖ Represent graphically sine, cosine and tangent, functions and, together with the unit circle, ❖ Use trigonometry, including the sine and cosine rules,	❖ Appreciate the relationship between the trigonometric values for different angles	❖ Trigonometric concepts: • Angle and its measurements • Unit circle • Trigonometric ratios • Trigonometric	❖ Mental task – imagine a point on the edge of a wheel – as the wheel turns how high is the point above the centre? – sketch the graph ❖ Practical – on graph paper draw circle radius 10cm and measure half chord length and distance from centre to chord for angles (say multiples of $15°$) – plot the graphs – use calculator to determine which is sine and cosine. What is the radius of the calculator's circle? – unit circle

complementary angles, supplementary angles and co-terminal angles	to solve problems involving triangles		identities ❖ Triangles and Applications: • Bearing • Air • Navigation • Inclined plane…	❖ Use of dynamic geometry and graph plotting to illustrate relationship e.g. geogebra ❖ In groups use unit circle and graphs to determine the relationship between trigonometric functions of any angle ❖ Group investigation -What angle subtends an arc length equal to the radius? – define a radian, make a table of equivalences ❖ Derive trigonometric identities, sine and cosine rules ❖ Apply trigonometry to practical problems involving triangles and angles.

Links to other subjects: *Physics (optics, wave, electricity), Geography, Architecture, Engineering,...*

Assessment criteria: *Use the trigonometric concepts and formulas to solve related problems in Physics, Air navigation, Water navigation, bearings, Surveying,*

Materials:*Geometric instruments (ruler,T-square,compass), graph papers, digital technology including calculators*

Topic Area: ALGEBRA				Sub-topic Area: NUMBERS AND OPERATIONS	
S4 - MATHEMATICS		Unit 2:SET OF REAL NUMBERS		No. of lessons: 12	

Key Unit Competence: Think critically to understand and perform operations on the set of real numbers

Learning Objectives			Content	Learning Activities
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Match a number and the set to which it belongs ❖ Define a power, an exponential ,a radical, a logarithm, the absolute value of a real number	❖ Classify numbers into naturals, integers, rational and irrationals ❖ Determine the restrictions on the variables in rational and irrational expressions ❖ Illustrate each property of a power, an exponential ,a radical, a logarithm, the absolute value of a real number ❖ Use logarithm and exponentials to model simple problems about growth, decay, compound interest, magnitude of an earthquake... ❖ Transform a logarithmic expression to equivalent power or radical form and vice versa ❖ Rewrite an expression containing "absolute value" using order relation	❖ Appreciate the importance and the use of properties of operations on real numbers ❖ Show curiosity for the study of operations on real numbers	❖ Absolute value and its properties ❖ Powers and radicals ❖ Decimal logarithms and properties.	❖ Group investigation – Make research in advance in the library about Sets of numbers (natural numbers, integers, rational numbers and irrational numbers ❖ Mental task What is the main facts about sets R of real numbers Apply operations on set of real numbers to illustrate relation to arithmetic

Links to other subjects: *Physics, e.g. converting temperature from degree Celsius to degree Fahrenheit, converting seconds to minutes and vice versa Entrepreneurship and in Economics Organisation and computation of data from sales ,Chemistry: e.g. The decay process Biology: e.g. growth of bacteria ,Geography: e.g. magnitude of an earthquake*

Assessment criteria: *Think critically to understand and perform operations on the set of real numbers*

Materials: *Graph papers, manila papers, digital technology including calculators,...*

Topic Area: ALGEBRA			Sub-topic Area: EQUATIONS AND INEQUALITIES	
S4 - MATHEMATICS		**Unit 3:** Linear, Quadratic equations and inequalities		**No. of lessons: 12**
Key unit Competence: Model and solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities.				

Learning Objectives			Content	Learning Activities
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ List and clarify the steps in modeling a problem by linear equations and inequalities	❖ Solve graphically and algebraically linear equations and inequalities to model a given situation	❖ Appreciate the importance of linear equations and inequalities in a given situation ❖ Listen colleagues arguments in solving linear equations and inequalities	❖ Equations and inequalities in one unknown ❖ Simultaneous equations in two unknowns ... ❖ Applications: •Economics (Problems about supply and demand analysis, ...) •Physics (Linear motions, Electric circuits, projectile motions, ...) •Chemistry (Balancing equations,...) •Masonry (Arched shape ...)	❖ Group investigation - discuss in groups the importance and necessity of linear equations and inequalities and how it takes place in the trade ❖ Practical - solve linear equations and simultaneous equations on a graph paper
Links to other subjects: *Physics (kinematics), Chemistry, Economics...*				
Assessment criteria: *Model and solve algebraically or graphically daily life problems using linear, quadratic equations or inequalities.*				
Materials: *Geometric instruments (ruler-square ....), Digital technology including calculator,...*				

Topic Area: ANALYSIS				Sub-topic Area: FUNCTIONS	
S4 - MATHEMATICS			**Unit 4:** Polynomial, Rational and Irrational functions		**No. of lessons: 9**
Key unit Competence: Use concepts and definitions of Polynomial, Rational and Irrational functions to determine the domain of Polynomial, Rational and Irrational functions and represent them graphically in simple cases…					
Learning Objectives			**Content**		**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**			
❖ Identify a function as a rule and recognize rules that are not functions ❖ Determine the domain and range of a function ❖ Construct composition of functions ❖ Find the even and odd parts of a function ❖ Demonstrate an understanding of operations on, and compositions of, functions.	❖ Perform operations on functions ❖ Apply different properties of functions to model and solve related problems in various practical contexts. ❖ Analyse, model and solve problems involving linear or quadratic functions and interpret the results.	❖ Increase self-confidence and determination to appreciate and explain the importance of functions and how they are related or how are helpful to different event occurring in different domains. ❖ Show concern on patience, mutual respect and tolerance	❖ Generalities on numerical functions: • Definitions • Domain and range • Parity of a function (odd or even) • Graphical representation of linear and quadratic functions and their use and interpretation in Economics, Physics…		❖ Study algebraically and graphically polynomial functions. ❖ Practical: discuss in groups patiently in mutual respect and tolerance, different operations on factorizations ❖ Model or interpret the problems related to polynomial functions
Links to other subjects: : *Physics (eg: Use a quadratic function to model the fall of a ball,…), Economics (Use of polynomials to represent the cost of producing* **"x"** *units of a commodity, or marginal cost,), Chemistry (use polynomial to express the rate of reaction in chemistry)*					
Assessment criteria: *Use concepts and definitions of Polynomial, Rational and Irrational functions to determine the domain of Polynomial, Rational and Irrational functions and represent them graphically in simple cases…*					
Materials: *Pair of compasses, Graph Papers, ruler, Digital technology (including calculators,…)*					

Topic Area: ANALYSIS			Sub-topic Area: LIMITS, DIFFERENTIATION AND INTEGRATION	
S4 - MATHEMATICS		**Unit 5:** Limits of polynomial, rational and irrational functions		No. of lessons: 9

Key unit Competence: Evaluate correctly limits of functionsand apply them to solve related problems

Learning Objectives			Content	Learning Activities
Knowledge and understanding	Skills	Attitudes and values		
❖ Define the concept of limit for real-valued functions of one real variable ❖ Evaluate the limit of a function and extend this concept to determine the asymptotes of the given function.	❖ Calculate limits of certain elementary functions ❖ Develop introductory calculus reasoning. ❖ Solve problems involving continuity. ❖ Apply informal methods to explore the concept of a limit including one sided limits. ❖ Use the concepts of limits to calculate the asymptotes to the rational and polynomial functions	❖ Show concern on the importance, the use and determination of limit of functions ❖ Appreciate the use of intermediate-value theorem	❖ Concepts of limits: • Neighborhood of a real number • Limit of a variable • Definition and graphical interpretation of limit of a function • One-sided limits • Squeeze theorem • Limits of functions at infinity. • Operations on limits ❖ Indeterminate cases: $\dfrac{\infty}{\infty}, \dfrac{0}{0}, \infty-\infty, 0.\infty$ ❖ Applications: • Continuity of a function at a point or on interval I • Asymptotes	❖ Learners discuss in group to evaluate the limit of a function at a point both algebraically and graphically, extend this understanding to determine the asymptotes. ❖ Learners represent on graph papers limits of some chosen functions and draw the possible asymptotes

Links to other subjects: *Physics (Calculation of velocity, acceleration using concepts of limits)*

Assessment criteria: *Evaluate correctly limits of functionsand apply them to solve related problems*

Materials: *Manila papers, Graph Papers, ruler, markers ,Digital technology,…*

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Topic Area: ANALYSIS			Sub-topic Area: LIMITS. DIFFERENTIATION AND INTEGRATION	
S4 - MATHEMATICS	**Unit 6:** Differentiation of polynomials, rational and irrational functions and their applications			**No. of lessons:9**
Key unit Competence: Use the gradient of a straight line as a measure of rate of change and apply this to line tangent and normal of curves in various contexts and use these concepts of differentiation to solve and interpret related rates and optimization problems in various contexts				

Learning Objectives			Content	Learning Activities
Knowledge and understanding	**Skills**	**Attitudes and values**		

❖ Evaluate derivatives of functions using the definition of derivative.
 	
❖ Define and evaluate from first principles the gradient at a point.
 	
❖ Distinguish between techniques of differentiation to use in an appropriate context.
	❖ Perform operations on derivative of rational functions and simple trigonometric functions (sine, cosine, tangent and cotangent).
	❖ Use first principles to determine the gradient of a straight line at a point.
	❖ Use the derivative to find the equation of a line tangent or normal to a curve at a given point.
	❖ Apply the concepts of and techniques of differentiation to model, analyse and solve rates or optimisation problems in different situation.
		❖ Appreciate the use of gradient as a measure of rate of change (economics)
		❖ Appreciate the importance and use of differentiation in Kinematics (velocity, acceleration)
		❖ Show concern on derivatives to help in the understanding of situation (Entrepreneurship ,production in a factory)
			❖ Concepts of derivative of a function:
			  • Definition
			  • High order derivatives
			❖ Rules of differentiation
			❖ Applications of differentiation :
			  • Geometric interpretation of derivatives:
			    - Equation of the tangent to a curve
			    - Equation of normal to a curve
			  • Hospital's theorem
			  • Variations of functions
			  • Optimization problems
			  • Related rates problems
				❖ Group investigation Determine the gradient of different functions at a point using definition of derivatives, from first principles, chain rule, and interpret the results.
				❖ Practical - represent on graph papers the gradient of a straight line and interpret it geometrically in various practical problems.
				❖ In group, learner use different techniques of differentiation to model, to analyze and solve rates or optimization problems.
				❖ In group, learner determine rate of change from practical various problems and interpret the results

Links to other subjects: *Physics, Economics (in Optimization problems , related rates problems, ...)*				
Assessment criteria: *Use the gradient of a straight line as a measure of rate of change and apply this to line tangent and normal of curves in various contexts and use these concepts of differentiation to solve and interpret related rates and optimization problems in various contexts*				
Materials: *Manila paper, graph paper, digital technology including calculators ...*				

Topic Area: LINEAR ALGEBRA				Sub-topic Area: VECTORS
S4 - MATHEMATICS		Unit 7: Vector Space of real numbers		No. of lessons: 6
Key unit Competence: Use concepts of vectors in 2D to solve related problems such as distance, angles,…				
Learning Objectives			**Content**	**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Define the scalar product of two vectors ❖ Give examples of scalar product ❖ Determine the magnitude of vector and angle between two vectors	❖ Calculate the scalar product of two vectors ❖ Analyse a vector in term of size. ❖ Determine the angle between two vectors ❖ Use concepts of vectors in 2D to solve related problems in 2D such as distance, …	❖ Apply and transfer the skills of dot product, magnitude to other area of knowledge	❖ Euclidian Vector space $\mathbb{R}^2$ • Dot product and properties • Modulus or Magnitude of vectors • Angle between two vectors	❖ Group investigation : Learners discuss about the scalar product of two vectors , - Determine the magnitude of vector and measure the angle between two vectors
Links to other subjects: *Physics (Dynamics), Geography,…*				
Assessment criteria: *Use concepts of vectors in 2D to solve related problems such as distance, angles,…*				
Materials: *Manila papers, Graph papers, Geometric instruments : rulers , T-square ,Protectors, Computers …*				

Topic Area: LINEAR ALGEBRA			Sub-topic Area: LINEAR TRANSFORMATION IN 2D	
S4 - MATHEMATICS		**Unit 8:** Matrices of and determinants of order 2		**No. of lessons: 9**

Key Unit Competence: Use matrices and determinants of order 2 to solve other related problems such as organisation of data in a shopping, in Cryptography, in Physics (problems about quantum or circuits), …

Learning Objectives			Content	Learning Activities
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Define the order of a matrix ❖ Define a linear transformation in 2D by a matrix ❖ Define operations on matrices of order 2 ❖ Show that a square matrix of order 2 is invertible or not	❖ Reorganise data into matrices ❖ Perform operations on matrices of order 2 ❖ Determine the inverse of a matrix of order 2 ❖ Use matrices to solve problems such as organisation of data in a shopping, in Cryptography, in Physics (problems about quantum or circuits), …	❖ Appreciate the importance and the use of matrices in organising data ❖ Show curiosity for the study of matrices of order 2 and their applications in different domains	❖ Operations on matrices: • Equality of matrices • Addition • Multiplication by a scalar • Multiplication of matrices • Transpose of a matrix • Inverse of a square matrix ❖ Determinant of a matrix of order 2 • Definition • Applications of determinants	In group: ❖ Learners should be given a task to reorganize given data into matrices be asked to perform different operations on matrices by calculating their determinant, ❖ Learners in group discuss about to show how a matrix of order 2 is invertible ❖ Learners should make research about the importance and use of matrices for example in Physics, Economics, Entrepreneurship, …, and report the findings

Links to other subjects:
Physics (problems about quantum or circuits), Entrepreneurship and in Economics(Organisation of data from sales)

Assessment criteria: Use matrices and determinants of order 2 to solve other related problems such as organisation of data in a shopping, in Cryptography, in Physics (problems about quantum or circuits), …

Materials: Geometric instruments (ruler, T-square, pair of compasses), graph papers, digital technology including calculators,…

Topic Area: STATISTICS AND PROBABILITY				Sub-topic Area: DESCRIPTIVE STATISTICS	
S4 - MATHEMATICS			Unit 9: Measures of dispersion		No. of lessons: 6
Key Topic Competence: Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.					
Learning Objectives				Content	Learning Activities
Knowledge and understanding	Skills	Attitudes and values			
❖ Define the variance, standard deviation and the coefficient of variation ❖ Analyse and interpret critically data and infer conclusion.	❖ Determine the measures of dispersion of a given statistical series. ❖ Apply and explain the standard deviation as the more convenient measure of the variability in the interpretation of data ❖ Express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.	❖ Appreciate the importance of measures of dispersion in the interpretation of data ❖ Show concern on how to use the standard deviation as measure of variability of data.		❖ Variance ❖ Standard deviation (including combined set of data) ❖ Coefficient of variation ❖ Application: • Problems to include measure of dispersion and explain the standard deviation as the more convenient measure of the variability in the interpretation of data • Problems to include measure of dispersion and express the coefficient of variation as a measure of the spread of a set of data as a proportion of its mean.	In group, learners will be given a task and be asked to: ❖ Discuss about the measures of dispersion, interpret them and represent their findings. ❖ Represent data on graph papers, interpret them and infer conclusion. Make a research on given problems arising from various situation in daily life, investigate them to include the standard deviation, nad represent their findings.
Links to other subjects: Physics, Biology, Chemistry, Geography, Finance, Economics,...					
Assessment criteria: *Extend understanding, analysis and interpretation of data arising from problems and questions in daily life to include the standard deviation.*					
Materials: *Manila papers, Graph Papers, ruler, digital technology including calculators ...*					

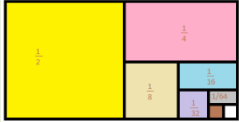| Topic Area: STATISTICS AND PROBABILITY			Sub-topic Area: COMBINATORIAL ANALYSIS AND PROBABILITY	
S4 - MATHEMATICS			**Unit 10:** Elementary probability	No. of lessons: 21
Key unit Competence: Use combinations and permutations to determine probabilities of occurrence of an event				
Learning Objectives			**Content**	**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**		

❖ Define the combinatorial analysis
❖ Define factorial and recognize that for n different items there are different combinations
❖ Construct Pascal's triangle
❖ Distinguish between permutations and combinations
❖ Define notion, axioms of probability and explain probability as a measure of chance
❖ Distinguish between mutually exclusive and non-exclusive events and compute their probabilities
❖ Compute expectations and determine the probability of events arising from an experiment with a number of possible outcomes
	❖ Determine the number of permutations and combinations of "n" items, "r" taken at a time.
	❖ Use counting techniques to solve related problems.
	❖ Use and apply elementary properties of probability to calculate the number possible outcomes of occurring event under equally likely assumptions
	❖ Determine and explain expectations from an experiment with possible outcomes
		❖ Appreciate the importance and the use of counting techniques
		❖ Appreciate the use of probability as a measure of chance
		❖ Show concern on patience, mutual respect, tolerance and curiosity of the possible outcomes of occurring event under equally likely assumptions
			❖ Counting techniques:
			  • Venn diagram
			  • Tree diagrams
			  • Contingency table
			  • Multiplication principles
			❖ Arrangement and Permutations:
			  • Arrangements with or without repetition
			  • Permutations with or without repetition
			❖ Combinations:
			  • Definitions and properties
			  • Pascal's triangles
			  • Binomial expansion
			❖ Concepts of probability:
			  • Random experiment
			  • Sample space
			  • Event
			  • Definition of probability of an event under equally likely assumptions
			❖ Properties and formulas
				❖ Mental task:
				Imagine you are a photographer sitting a group in a row for pictures. You need to determine how many different ways you can seat the group. Learners find out.
				
				❖ Learners solve in in group questions about counting techniques for example " In how many different ways could a committee of 5 people be chosen from a class of 30 students? "
				❖ Learners form their proper words using letters of "MISSISSIPI", without a prior instructions, then give feedback..
				❖ Learners are given a task to sit 3 men and 4 women at random in a row. In groups, they discuss about the probability that either all the men or all the women end up sitting together, and then give feedback.

Links to other subjects: English, Physics, Biology, Chemistry, Geography, Finance, Economics, Medical sciences…				
Assessment criteria: Use combinations and permutations to determine probabilities of occurrence of an event				
Materials: Manila papers, Graph Papers, ruler, digital components including calculators …				

5.3. Secondary Five

5.3.1 Key Competences at the end of Secondary Five

After completion of secondary 5, the mathematics syallabus will help the learnerto:

1. Extend the usethetrigonometric concepts and transformation formulas to solve problems involving trigonometric equations, inequalities and or trigonometric identities

2. Use arithmetic, geometric and harmonic sequences, including convergence to understand and solve problems arising in various context.

3. Solve equations involving logarithms or exponentials and apply them to model and solve related problems.

4. Studyand to representgraphicallyanumerical function.

5. Apply theorems of limits and formulas to solve problems involving differentiation including optimization, …

6. Extend the use of matrices and determinants to order 3 to sove problems in various contexts

7. Use algebraic representations of lines, spheres and planes in 3D space and solve related problems.

8. Extend the understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines

9. Solve problems using Bayes theorem and data to make decisions about likelihood and risk.

5.3.2 Mathematics units for Secondary Five

Topic Area: TRIGONOMETRY			Sub-topic Area: TRIGONOMETRIC FUNCTIONS AND EQUATIONS		
S5 - MATHEMATICS			**Unit 1:** Trigonometric functions and equations		No. of lessons: 15
Key unit Competence: Solve trigonometric equations and related problems using trigonometric functions and equations					
Learning Objectives			**Contents**		**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**			
❖ Show how to use apply transformation formula to simplify the trigonometric expressions ❖ Extend the concepts of trigonometric ratios and their properties to trigonometric equations ❖ Analyze and discuss the solution of trigonometric inequalities	❖ Apply the transformation formulas to simply trigonometric expressions ❖ Use trigonometric functions and equations to model and solve problems involving trigonometry concepts.	❖ Appreciate the relationship between trigonometry and other subjects. ❖ Show concern on patience, mutual respect, tolerance and curiosity in the solving and discussion about problems involving trigonometric functions and equations.	❖ Transformation formulas: • Addition and subtraction formulas • Double-angle and half-angle formulas • Sum, Difference and Product Formulas ❖ Trigonometric equations		❖ In groups, learners discuss on how to simplify trigonometric expressions using transformation formulas – solve problems involving trigonometric equations
Links to other subjects: *Physics, Complex numbers, …*					
Assessment criteria: *Apply trigonometry functions, transformations formulas and equations to solve problems related to trigonometry.*					
Materials: *Geometric instruments (ruler, T-square, compass), graph papers, calculators,…*					

Topic Area: ALGEBRA				Sub-topic Area: NUMBER PATTERNS
S5 - MATHEMATICS		**Unit 2:** Sequences		**No. of lessons: 12**
Key unit Competence: Understand, manipulate and use arithmetic, geometric sequences				

Learning Objectives			**Contents**	**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Define a sequence and understand arithmetic progressions and their properties ❖ Determine the value of "n", given the sum ofthe first "n"terms of arithmetic progressions. ❖ Show how to apply formulas to determine the "nth" term and the sum of the first "n"terms of arithmetic progressions ❖ Define and explaingeometric progressions and their properties ❖ Determine the value of "n", given the sum ofthe first "n"terms of geometric progressions ❖ Show how to apply formulas to determine specific terms,the "nth" term and the sum of the first "n"terms of geometric progressions.	❖ Use basic concepts and formulas of sequences to find the value "n", given the sum ofthe first "n"terms of arithmetic progressions - the "nth" term and the sum of the first "n"terms of arithmetic progressions ❖ Use basic concepts and formulas of sequences to find the value "n", given the sum of the first "n"terms of arithmetic progressions - the "nth" term and the sum of the first "n"terms of geometric progressions ❖ Apply the concepts of sequences to solve problems involving arithmetic and geometric sequences.	❖ Appreciate the relationship between the sequences and other subjects to understand occurring situations (in Economics: Value of annuity, future value of money ...) ❖ Show concern on, patience, mutual respect, tolerance and curiosity to discuss about sequences and their applications.	❖ Arithmetic sequences ❖ Geometric sequences ❖ Applications: • Problems including population growth • Problems including compound and simple interests • Half-life and Decay problems in Radioactivity • Bacteria growth problems in Biology ...	• Group led approach: Learners can be given a task of folding a piece of paper to make them understand the meaning of geometric sequences , and think what should be the last term to the infinity $$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \frac{1}{2^n}$$ - Group investigation : If the bankrates increase or decrease unexpectedly, learners discuss or investigate how in the next n- years: - they come out ahead - the deal stays fair

Links to other subjects: *Demography in Population growth Problems, Economics in Compound and simple interests, Chemistry in Half-life and Decay...*
Assessment criteria: *Apply concepts of sequences to solve problems involving arithmetic and geometric sequences*

Topic Area: ALGEBRA			Sub-topic Area: EQUATIONS AND INEQUALITIES	
S5 - MATHEMATICS		**Unit 3:** Logarithmic and exponential equations		No. of lessons:15
Key unit Competence: Solve equations involving logarithms or exponentials and apply them to model and solve related problems.				
Learning Objectives			**Contents**	**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Define logarithm or exponential equations using properties of logarithms in any base ❖ State and demonstrate properties of logarithms and exponentials ❖ Carry out operations using the change of base of logarithms	❖ Explain the properties of logarithms in any base ❖ Solve logarithmic and exponential equations ❖ Convert the logarithm to exponential form ❖ Apply logarithms or exponential to solve rates problems, mortgage problems, population growth problems	❖ Appreciate the use of logarithmic equations to model and solve problem involving logarithms such radioactive-decay problems, Carbon dating problems, problems about alcohol and risk of car accident, etc. ❖ Show concern on patience, mutual respect and tolerance in solving problems involving logarithmic or exponential equations	❖ Logarithmic equations, including natural logarithms. ❖ Exponential equations ❖ Application: • Interest rates problems • Mortgage problems • Population growth problems • Radioactive decay problems • Earthquake problems • Carbon dating problems • Problems about alcohol and risk of car accident	In group or individually, learners: • Once they have the shape of a logarithmic graph, they can shift it vertically or horizontally, stretch it, shrink it, reflect it, check answers with it, and the most important is to interpret the graph. • Given for example a growth or decay situation, learners after investigating the situation, they write an exponential function and evaluate it for a given input.
Links to other subjects: Demography (Population growth Problems), Economics (Interest rates problems, annuity value of money), etc.				
Assessment criteria: Apply concepts of logarithmic and exponential equations to solve problems involving logarithms or exponentials.				
Materials: Geometric instruments (ruler, T-square, compass), graph papers, digital technology including calculators, Manila paper…				

Topic Area: ANALYSIS			Sub-topic Area: LIMITS, DIFFERENTIATION AND INTEGRATION	
S5 - MATHEMATICS		**Unit 4:** Trigonometric and inverse trigonometric functions		**No. of lessons: 12**
Key unit Competence: Apply theorems of limits and formulas of derivatives to solve problems including trigonometric functions, optimization, motion,				

Learning Objectives			Contents	Learning Activities
Knowledge and understanding	**Skills**	**Attitudes and values**		

❖ Extend the concepts of function, domain, range, period,inverse function, limits to trigonometric functions.
 	
❖ Extend the concepts of limits or / and differentiation to model and solve problems involving trigonometric or inverse trigonometric functions.
	❖ Apply concepts and definition of limits, to calculate the limits of trigonometric functions and remove their indeterminate forms – Calculate also their high derivatives.
	❖ Derive techniques of differentiation to model and solve problems related to trigonometry.
	❖ Apply technique of differentiation to solve problems involving trigonometric functions such as optimization, motion or trigonometric functions,...
		❖ Appreciate that questions of optimization, motion, … involving trigonometric functions can be solved using concepts of limits or / and techniques of derivatives.	❖ Trigonometric functions
			
			• Generalities:
			
			  - Definitions
			  - Domain and range of a function
			  - Parity of a function (odd or even)
			  - Periodic functions
			• Limits, including indeterminate cases $\dfrac{0}{0}$, $0.\infty$
			
			Differentiation of trigonometric functions – Extend this to high derivatives
				• Learners in groups plot the graphs of trigonometric functions e.g $y = \sin x$ or $y = a \sin bx$ and investigate it, they discuss about its period, they find its domain of definition and range. – Generalise this activities to other trigonometric functions. – Calculate high derivatives of these trigonometric functions.
				• Derive techniques of differentiation to differentiate trigonometric functions and apply them to solve related practical problems such as: optimization, motion, Etc.

Links to other subjects: optimization, motion …	
Assessment criteria: Apply theorems of limits and formulas of derivatives to solve problems including trigonometric functions, optimization, motion,	
Materials: Geometric instruments (ruler, T-square, compass), graph papers, digital technology including calculators	

Topic Area: LINEAR ALGEBRA					Sub-topic Area: VECTORS IN 3D
S5 - MATHEMATICS			**Unit 5:** Vector space of real numbers		**No. of lessons: 9**
Key unit Competence: Apply vectors of $\mathbb{R}^3$ to solve problems related to angles using the scalar product in $\mathbb{R}^3$ and use the vector product to solve also mensuration problems in $\mathbb{R}^3$					
Learning Objectives			Contents		Learning Activities
Knowledge and understanding	Skills	Attitudes and values			
❖ Define the dot product and the cross product of two vectors in a three-dimensional vector space and list their properties. ❖ Define the magnitude of a three-dimensional vector and list its properties ❖ Distinguish between the dot product and the cross product.	❖ Determine the dot product and the vector product of two vectors in a three-dimensional space and use them to solve practical related problems. ❖ Explain geometrically the dot product and the cross product	❖ Appreciate the usefulness of vectors of $\mathbb{R}^3$ in the description of quantities such as force, velocity ...	❖ Euclidian Vector space $\mathbb{R}^3$ • Dot product and properties • Modulus or Magnitude of vectors • Angle between two vectors • Vector product and properties		Learners perform specific tasks in group, patiently, in mutual respect and tolerance such as • To draw a three-dimensional coordinate system and plot some chosen points and represent the corresponding vectors • Choose some learners to simulate points and vectors in three-dimensional space and ask the audience to describe vectors and related operations • Study vectors in three-dimensional coordinate system to describe quantities such as force, velocity, acceleration ...
Links to other subjects: Physics (force, velocity, acceleration), ...					
Assessment criteria: Apply vectors of $\mathbb{R}^3$ to solve problems related to angles using the scalar product in $\mathbb{R}^3$ and use the vector product to solve also mensuration problems in $\mathbb{R}^3$					
Materials: Geometric instruments (ruler, T-square, compass), graph papers, digital technology including calculators					

Topic Area: **LINEAR ALGEBRA**			Sub-topic Area: **LINEAR TRANSFORMATION IN 3D**	
S5 - MATHEMATICS		**Unit 6:** Matrices and determinants of order 3		**No. of lessons: 18**
Key unit Competence: Apply matrix and determinant of order 3 to solve related problems.				
Learning Objectives			**Contents**	**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Define operations on matrices of order 3 ❖ Illustrate the properties of determinants of matrices of order3. ❖ Show that a square matrix of order 3 is invertible or not ❖ Discuss with respect to a parameter the solutions of a system of three linear equations in three unknowns	❖ Perform operations on matrices of order 3 ❖ Calculate the determinants of matrices of order 3 ❖ Explain using determinant whether a matrix of order 3 is invertible or not ❖ Determine the inverse of a matrix of order 3 ❖ Reorganise data into matrices ❖ Apply matrices to solve related problems (e.g in physics.) ❖ Use Cramer's rule to solve a system of three linear equations in three unknowns ❖ Apply properties of determinants to solve problems related to matrices of order 3.	❖ Appreciate the importance of matrices of order 3 and their determinants in organising data and solving related problems.	❖ Operations on matrices: • Equality of matrices • Addition • Multiplication by a scalar • Multiplication of matrices • Transpose of a matrix • Inverse of a square matrix ❖ Determinant of a matrix of order 3 • Definition ❖ Applications of determinants	❖ Learners discuss in group patiently, in mutual respect and tolerance, how to organize data into matrices of order 3 and apply these concepts to solve related problems. ❖ Learners discuss in group, with respect to a parameter, the solutions of a system of three linear equations in three unknowns.
Links to other subjects: *Physics (Expressing force, velocity, acceleration,…), Engineering,...*				
Assessment criteria: *Apply matrix and determinant of order 3 to solve related problems.*				
Materials: *graph papers, digital technology including calculators*				

Topic Area: STATISTICS AND PROBABILITY			Sub-topic Area: DESCRIPTIVE STATISTICS	
S5 - MATHEMATICS		**Unit 7:** Bivariate statistics	**No. of lessons: 12**	
Key unit Competence: Extend understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines				
Learning Objectives			**Contents**	**Learning Activities**
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ Define the covariance, coefficient of correlation and regression lines. ❖ Analyse, interpret data critically then infer conclusion.	❖ Determine the coefficient of correlation, covariance and regression lines of bivariate data of dispersion of a given statistical series. ❖ Apply and explain the coefficient of correlation and standard deviation as the more convenient measure of the variability in the interpretation of data.	❖ Appreciate the importance of regression lines and coefficient of correlation analyse, interpret data to infer conclusion - Predict event e.g after analysing the population growth of a given country, we can make a decision about the future generation.	❖ Covariance ❖ Correlation coefficient of bivariate data ❖ Regression lines ❖ Applications: ❖ Data analysis, interpretation and prediction problems in various areas (Biology, Business, Engineering, Geography, Demography ...)	❖ Learners discuss in groups, about, the correlation between class results and rank in school for example. They investigate them, they analysethe relationship between them, and check how the coefficient of correlation reflects the amount of variability that is shared between them and what they have in common. They finally infer conclusion. ❖ Learners plot visually data on scatter diagram or scatter plot to represent a correlation between two variable. – Analyse the graph, infer conclusion using coefficient of correlation to make predictions about the variables studied. E.g
Links to other subjects: *Geography (spatial statistics research, Air pollution in different year...), Biology (Bio-statistics, ...), Chemistry, Demography (Population growth,...),...*				
Assessment criteria: *Extend understanding, analysis and interpretation of bivariate data to correlation coefficients and regression lines*				
Materials: *Geometric instruments (ruler, T-square, compass), graph papers, digital technology including calculators*				

Topic Area: STATISTICS AND PROBABILITY			Sub-topic Area: PROBABILITY	
S5 - MATHEMATICS		Unit 8:Conditional probability and Bayes theorem	No. of lessons: 15	
Key unit Competence: Solve problems using Bayes theorem and use data to make decisions about likelihood and risk				
Learning Objectives			Contents	Learning Activities
Knowledge and understanding	Skills	Attitudes and values	Contents	Learning Activities
❖ Extend the concept of probability to explain it as a measure of chance. ❖ Compute the probability of an event B occurring when event A has already taken place. ❖ Interpret data to make decision about likelihood and risk.	❖ Apply theorem of probability to calculate the number possible outcomes of occurring independent events under equally likely assumptions. ❖ Determine and explain results from an experiment with possible outcomes ❖ Apply Bayes theorem to calculate the number of possible outcomes of occurring independent events under equally likely assumptions.	❖ Appreciate the use of probability theorem as measure of chance. ❖ Show concern on patience, mutual respect, tolerance and curiosity about the possible outcomes of event B occurring when event A has already taken place. ❖ Appreciate the use of Bayes theorem to determine the probability of event B occurring when event A has already taken place.	❖ Conditional probability: • Probability of event B occurring when event A has already taken place. • Basic formulae and properties and of conditional Probability • Independent events • Probability tree diagram ❖ Bayes theorem and its applications	❖ Learners discuss in groups patiently in mutual respect and tolerance, about number of possible outcomes of event B occurring when even A has already taken place. ❖ In a given task, learners use Bayes theorem to determine the probability of event B occurring when event A has already taken place.
Links to other subjects: *Geography, Biology, Chemistry, Demography …*				
Assessment criteria: *Solve problems using Bayes theorem and use data to make decisions about likelihood and risk*				
Materials: *Manila papers, ,markers, digital technology including calculators*				

5.4 Secondary Six

5.4.1 Key competences at the end of Secondary Six

After completion of secondary 6, the mathematics syallabus will help the learner to:

1. Extend understanding of sets of numbers to complex numbers
2. Solve polynomial equations in the set of complex numbers and solve related problems in physics, ...
3. Extend the use of concepts and definitions of functions to determine the domain of logarithmic and exponential functions.
4. Use integration as the inverse of differentiation and as the limit of a sum and apply them to finding area and volumes to solve various practical problems.
5. Use differential equations to solve related problems that arise in a variety of practical contexts
6. Relate the sum and the intersection of subspaces of a vector space by the dimension formula

5.4.2 Mathematics Units for Secondary Six

Topic Area: ALGEBRA			Sub-topic Area: NUMBERS AND OPERATIONS	
S6 - MATHEMATICS			Unit 1: COMPLEX NUMBERS	No. of lessons: 27

Key unit competence: Perform operations on complex numbers in different forms and use them to solve related problems in Physics (voltage and current in alternating current), Computer Science(fractals), Trigonometry (Euler's formula to transform trigonometric expressions), …

Learning Objectives			Content	Learning Activities
Knowledge and understanding	**Skills**	**Attitudes and values**		

❖ Identify the real part and the imaginary part of a complex number
 ❖ Convert a complex number from one form to another
 ❖ Represent a complex number on Argand diagram
 State De Moivre's formula and Euler's formula
	❖ Apply the properties of complex numbers to perform operations on complex numbers in algebraic form, in polar form or in exponential form
	 ❖ Find the modulus and the square roots of a complex number
	 ❖ Solve in the set of complex numbers a linear or quadratic equation
	 ❖ Apply De Moivre's formula and Euler's formula to transform trigonometric expressions
		❖ Appreciate the importance of complex numbers to solve related problems such as in Physics (voltage and current in alternating current), Computer Science(fractals), Trigonometry (Euler's formula to transform trigonometric expressions), …	❖ Algebraic form of Complex numbers
			 • Definition and properties of " i "
			 • Operations:
			  - Addition, subtraction, multiplication, powers, Conjugate and division
			 • Modulus of a complex number
			 • Square roots in the set $\Box$ of complex numbers
			 • Equations in the set $\Box$ of complex numbers
			 ❖ Geometric representation of complex numbers
			 ❖ Polar form of complex numbers
			 • Definition
			 • Modulus and argument of a complex number
			 • Operations
			 • De Moivre's formula
			 ❖ Exponential forms of complex numbers:
			  - Definition and operations
			  - Euler's formula of complex numbers
				❖ Mental work; Use definition of the multiplication of complex numbers to determine the complex number whose square is -1 and draw conclusion about the properties of "i"
				 ❖ Learners derive properties of operations on complex numbers in trigonometric form and apply complex numbers to transform trigonometric formulas
				 ❖ Use internet to determine the generation of fractals by complex numbers and print the different shapes to present in class

Links to other subjects: Physics (alternating current), Computer science(fractals) ,…

Assessment criteria: *Perform operations on complex numbers in different forms and use them to solve related problems in Physics (voltage and current in alternating current), Computer Science(fractals), Trigonometry (Euler's formula to transform trigonometric expressions), …*

Materials: *Geometric instruments (ruler, T-square, compass), IT equipments, ...*

Topic Area: **ANALYSIS**	Sub-topic Area: **LIMITS, DIFFERENTIATION and INTEGRATION**

S6 - MATHEMATICS	Unit 2: LOGARITHMIC AND EXPONENTIAL FUNCTIONS	No. of lessons: 24

Key unit competence: Extend the concepts of functions to investigate fully logarithmic and exponential functions and use them to model and solve problems about interest rates,population growth or decay,magnitude of earthquake,etc·

Learning Objectives			Content	Learning Activities
Knowledge and understanding	**Skills**	**Attitudes and values**		
❖ State the restrictions on the base and the variable in a logarithmic function ❖ Extend the concept of functions to investigate fully logarithmic and exponential functions ❖ Perform operations on logarithmic and exponential functions in any base ❖ Recall the differentiation formulas for logarithmic and exponential functions	❖ Transform a logarithm from a base to another ❖ Find the domain and the range of a logarithmic or an exponential function ❖ Calculate limits of logarithmic and exponential functions ❖ Determine possible asymptotes of a logarithmic or an exponential function ❖ Determine the derivative of a logarithmic or an exponential function ❖ Solve related problems involving logarithms	❖ Show concern on the importance of logarithmic and exponential functions in solving problems such as carbon dating in Chemistry, ... ❖ Develop patience, dedication and commitment in solving problems about logarithmic and exponential functions	❖ Logarithmic functions • Domain of definition • Limits of logarithmic functions and their applications to continuity and asymptotes • Differentiation and its applications ❖ Exponential functions • Domain of definition • Limits of logarithmic functions and their applications to continuity and asymptotes • Differentiation and its applications ❖ Applications of logarithmic and exponential functions: • Interest rates problems • Mortgage problems • Population growth problems • Radioactive decay problems • Earthquake problems • Carbon dating problems • Problems about alcohol and risk of car accident.	❖ Learners use scientific calculators to evaluate logarithms and exponentials of real numbers; they conclude about the domain (the allowed input values) and the range(the set of possible outputs) ❖ Learners may use software ,such as Geogebra, to graph logarithmic and exponential functions and to report to class their findings about the general trend of the graphs ❖ Derive formulas about differentiation of logarithmic and exponential functions ❖ Discuss in groups the applications of logarithms and exponentials in real life and report the results.

Links to other subjects:*English, Physics (Newton's law of cooling), Economics(Compounded interest), Biology(population growth), Chemistry(carbon dating)*

Assessment criteria: *Learner is able to* Extend the concepts of functions to investigate fully logarithmic and exponential functions and use them to model and

solve problems about interest rates,population growth or decay,magnitude of earthquake,etc	
Materials:Graph Papers, ruler, digital technologies including calculators ...	

Topic Area: ANALYSIS			Sub-topic Area: LIMITS, DIFFERENTIATION and INTEGRATION	
S6 - MATHEMATICS		**Unit 3:** INTEGRATION		No. of lessons: 30
Key Unit Competence:				
Use integration as the inverse of differentiation and as the limit of a sum and apply them to find area of a plane shapes.				
Learning Objectives				
Knowledge and understanding	**Skills**	**Attitudes and values**	**Content**	**Learning Activities**

❖ Define the differential of a function
❖ Interpret geometrically the differential of a function
❖ List the differentiation formulas
❖ State and clarify the relationship between derivative and antiderivative of a function
❖ Illustrate the use of basic integration formulas
❖ Extend the concepts of indefinite integrals to definite integrals.
	❖ Use differentials to approximate a function and to calculate the percentage error in an estimation
	❖ Calculate integrals. Using appropriate techniques
	❖ Use properties of integrals to simplify the calculation of integrals
	❖ Calculate a limit of a sum to infinity as a definite integral
	❖ Apply definite integrals to calculate the area, volume, arc length
	❖ Analyze the convergence of an improper integral
	❖ Use integrals to solve problems in Physics (work,..), Economics (marginal and total cost),etc.
		❖ Show concern on the importance of integral calculus in solving problems from daily life .
		❖ Appreciate various techniques of integration and show patience, commitment and tolerance in the evaluation of integrals
			❖ Indefinite integrals
			 • Antiderivatives
			 • Definition and properties
			 • Techniques of integration:
			  - Basic Integration Formulas
			  - Integration by change of variables
			  - Integration by Parts
			❖ Definite integrals
			 • Definition
			 • Properties
			 • Techniques of integration
			 • Applications of definite integrals
			  - Calculation of area of a plane surface
				❖ Graphical approach: Learners shade the area between a curvethe x-axis and two vertical .Considering consecutive subintervals, calculate the areas of corresponding rectangles ,then introduce the concept of integral as sum to infinity, when the width tends to zero

Links to other subjects:English, ...				
Assessment criteria: Learner is able to Use integration as the inverse of differentiation and as the limit of a sum and apply them to find area of a plane shapes.				
Materials: Manila papers, Graph Papers, ruler, markers ...				

Topic Area: ANALYSIS			Sub-topic Area: LIMITS, DIFFERENTIATION and INTEGRATION	
S6 - MATHEMATICS			Unit 4: ORDINARY DIFFERENTIAL EQUATIONS	No. of lessons: 27
Key unit competence: Use ordinary differential equations of first to model and solve related problems tin Physics, Economics, Chemistry, Biology, …				
Learning Objectives				
Knowledge and understanding	Skills	Attitudes and values	Content	Learning Activities
❖ Extend the concepts of differentiation and integration to ordinary differential equations ❖ State the order and the degree of an ordinary differential equation ❖ Express the auxiliary quadratic equation of a homogeneous linear differential equation of second order with constant coefficients ❖ Predict the form of the particular solution of an ordinary linear differential equation of second order	❖ Determine whether an ordinary differential equation of first order is with separable variables, homogeneous or linear ❖ Use appropriate method to solve an ordinary differential equation of first order ❖ Solve an ordinary linear differential equation of first order by "variation of constant" and by "integrating factor" ❖ Solve an ordinary linear differential equation of second order ❖ Use differential equations to model and solve problems in Physics (simple harmonic motion,…),Economics(point elasticity,…),etc.	❖ Appreciate the use of differential equations in solving problems occurring from daily life ❖ Show patience, commitment and dedication when solving a differential equation or modelling a problem using differential equations ❖ When discussing in groups the solution of a differential equation, make sense of other learners' thinking, show tolerance and mutual respect.	❖ Definition and classification ❖ 1st Order differential equations • Differential equations with separable variables • Linear differential equations • Applications ❖ 2nd Order differential equations • Linear equations with constant coefficients: - The right hand side is equal to zero • Applications: - Physics - Economics - Chemistry - Crime investigation - Etc.	❖ Mental task: imagine the motion of a child on a swing. Express the displacement as function of time. Differentiate the function to find the velocity and acceleration, and then express the relation between the function and its derivatives. Report your results. ❖ Use graph plotting to illustrate the general solution of a differential equation ❖ Discuss in groups the solutions of a differential equation with respect to a parameter and present the result to the class, show ability to communicate your thinking and reasoning - Use internet to find the applications of differential equations in sciences and report your findings to the class

Links to other subjects: *Physics (simple harmonic motion), Chemistry (radioactive decay), Economics (point elasticity and demand function),etc…*
Assessment criteria: *Learner is able to Use ordinary differential equations of first to model and solve related problems tin Physics, Economics, Chemistry, Biology, …*
Materials: *Geometric instruments, graph papers, calculators, ICT equipments,…*

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7. APPENDIX: SUBJECTS AND WEEKLY TIME ALOCATION FOR A'LEVEL

Subjects in Secondary 4-6		Number of periods per week (1 period = 40 min.)		
Core subjects		**S4**	**S5**	**S6**
1. Mathematics		7	7	7
2. Physics		7	7	7
3. Computer Science		7	7	7
4. Chemistry		7	7	7
5. Biology		7	7	7
6. Geography		7	7	7
7. History		7	7	7
8. Economics		7	7	7
9. Literature in English		7	7	7
10. Kinyarwanda major		7	7	7
11. Kiswahili major		7	7	7
12. French major		7	7	7
13. Religion major		7	7	7
14. Entrepreneurship		6	6	6
15. General Studies and Communication Skills		3	3	3
16. Subsidiary Mathematics		3	3	3
Electives Subjects	17. English minor	4	4	4
	18. French minor	4	4	4
	19. Kinyarwanda minor	4	4	4
	20. Kiswahili minor	4	4	4
Co-curricular Activities	Religious activities	2	2	2
	Sports/ Clubs	2	2	2
	Computer/library	2	2	2