Mathematics

Senior 3

Teacher's Guide

FOREWORD

Dear teacher,

Rwanda Basic Education Board is honoured to present Senior Three Mathematics Teacher`s Guide which serves as a guide to competence-based teaching and learning to ensure consistency and coherence in the learning of the Mathematics subject. The Rwandan educational philosophy is to ensure that learners achieve full potential at every level of education which will prepare them to be well integrated in society and exploit employment opportunities.

In line with efforts to improve the quality of education, the government of Rwanda emphasizes the importance of aligning teaching and learning materials with the syllabus to facilitate their learning process. Many factors influence what they learn, how well they learn and the competences they acquire. Those factors include the relevance of the specific content, the quality of teachers' pedagogical approaches, the assessment strategies and the instructional materials available. We paid special attention to the activities that facilitate the learning process in which learners can develop ideas and make new discoveries during concrete activities carried out individually or with peers. With the help of the teachers, learners will gain appropriate skills and be able to apply what they have learnt in real life situations. Hence, they will be able to develop certain values and attitudes allowing them to make a difference not only to their own life but also to the nation.

This is in contrast to traditional learning theories which view learning mainly as a process of acquiring knowledge from the more knowledgeable who is mostly the teacher. In competence-based curriculum, learning is considered as a process of active building and developing of knowledge and understanding, skills and values and attitude by the learner where concepts are mainly introduced by an activity, situation or scenario that helps the learner to construct knowledge, develop skills and acquire positive attitudes and values.

In addition, such active learning engages learners in doing things and thinking about the things they are doing and they are encouraged to bring their own real experiences and knowledge into the learning processes. In view of this, your role is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize group discussions for learners considering the importance of social constructivism suggesting that learning occurs more effectively when the learner works collaboratively with more knowledgeable and experienced people.
- Engage learners through active learning methods such as inquiry methods, group discussions, research, investigative activities and group and individual work activities.

- Provide supervised opportunities for learners to develop different competences by giving tasks which enhance critical thinking, problem solving, research, creativity and innovation, communication and cooperation.
- Support and facilitate the learning process by valuing learners' contributions in the class activities.
- Guide learners towards the harmonization of their findings.
- Encourage individual, peer and group evaluation of the work done in the classroom and use appropriate competence-based assessment approaches and methods.

To facilitate you in your teaching activities, the content of this teacher`s guide is self-explanatory so that you can easily use it. Even though this teacher`s guide contains the answers for all activities given in the learner's book, you are requested to work through each question and activity before judging learner's findings.

I wish to sincerely extend my appreciation to REB staff who organized the editing process of this Teacher`s Guide. Special gratitude also goes to lecturers, teachers, illustrators and designers who supported the exercise throughout. Any comment or contribution would be welcome to the improvement of this textbook for the next edition.

Dr. MBARUSHIMANA Nelson

Director General, REB

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Joan Murungi,

Head of CTLRD

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PART 1: INTRODUCTION

1.1: Basic Information

1.1.1 Organisation of the book

This teacher's guide is organised into two main sections.

Part 1 is the section of general **introduction** detailing information on competence based curriculum and pedagogical issues.

The main elements of Part 1 are:

- *Basic information*- It gives a brief overview of the organisation of the syllabus and background to the new curriculum which details the types of competences and their acquisition, crosscutting issues to be addressed during learning, special needs education and inclusivity.
- *Pedagogical approaches*- It highlights the teacher's and learner's roles for effective teaching/learning of mathematics, teaching/learning resources, grouping learners for learning and teaching methods
- *Assessment and evaluation methods*- it gives an overview of types of assessment, record keeping and how to report the learners performance to parents.
- *Content map:* It gives a brief highlight in tabular form how each unit has addressed the various aspects required in the Senior 3 Competence-based Mathematics Curriculum.
- *Preparation for teaching and the teaching process*- It highlights importance of planning for teaching, guides the teacher on the actual process of planning, and provides a sample lesson plan.

Part 2 provides a **unit-to -unit guide** to the teacher on how to facilitate learners to acquire the knowledge, skills and attitudes envisaged in each unit . This part is therefore structured into units

The main elements of of each unit guide are:

- *Unit heading* – This gives the unit title as stated in the syllabus.
- *Key Unit Competence:* This gives the broad competence which the learner will achieve once he/she has met all the learning objectives stipulated in the unit.
- *Learning Objectives:* This section outlines three categories of syllabus-specified learning objectives that a learner is expected to achieve through his/interaction with the concepts and activities planned for the unit. These categories are.
  - *Knowledge and understanding:* Though, knowledge and understanding fall at the lower levels of the Bloom's taxonomy they enhance they enhance learning at the higher levels.
  - *Skills:* It is through the skills that students apply their learning and engage in higher order thinking. These skills relate to the upper levels of Bloom's taxonomy and they lead to deep rather than surface learning.
  - *Attitudes and values:* Truly engaging with the learning requires appropriate attitudes and values that relate to the unit.
- *Prerequisite to the unit:* This section outlines key knowledge, skills attitudes and values that learners

need to have acquired earlier that will facilitate easier acquisition of the new knowledge, skills attitudes and values envisaged in this unit. It also guides the teacher on how to find out that the learners posses them before they start learning the concepts in this unit, and how to help learners incase they do not posses them.

- ***Assessment Criteria:*** This is meant to evaluate whether learners have succeeded in achieving the Key Unit Competence(s) intended. It helps the teacher in assessing whether the unit objectives have been met.
- ***Crosscutting issues to be addressed in the unit:*** The section outlines the specific crosscutting issues that will be addresses by infusion as the learners do the activities and interacts with concepts planed for the unit This is meant to make the teacher conscious on and be on the look out for suitable opportunities through out the teaching/learning process in the entire unit to address the cited crosscutting issues. This issues will be discussed in detail later in this book
- ***Generic competences to be addressed in the unit:*** The section outlines the specific generic competences to be acquired by the learners by doing the activities and interacting with concepts planed for the unit This competences will be discussed in detail later in this book
- ***Vocabulary/keywords:*** This is a list of new words that the learners will come across and interact with in the unit. The teacher is encouraged to help the learners to understand their meanings and usage in in order to help them build their language and vocabulary.

Guidance on the problem statement: This section guides the teacher on how to facilitate the leaners to do the introductory activity, known as the problem statement, given at the start of the unit in the Learners book. The purpose of the problem statement activity is to challenge and motivate learners to be eager and to actively participate in the activities planned for the unit with a drive to get the answers to the problem outlined in the activity.

- ***Attention to special needs:*** The section guides the teacher on how to handle learners with special needs as they do the learning activities organised in the unit.
- ***List of lessons:*** This is a list in tabular form of the structuring of the unit into lessons, each with a lesson title and the number of periods allocated to each lesson. Note that the total number of periods must be equal to those allocated to that unit in the syllabus.
- ***Lesson development:*** This section provides guidance to the teacher on how to facilitate learning in each lesson. The guidance for each lesson is structured as follows:
  - ***Lesson title***
  - ***Specific objectives*** of the lesson
  - ***Teaching and learning aids.***
  - ***Learning activities:*** Under each activity the teacher is guided on how to ***conduct*** the activity and facility the ***synthesis*** of the knowledge being acquired. The ***answers to the activities*** are also provided here.

– *Assessment of the lesson*: Here, the teacher is guided on how to assess whether learners have acquired the knowledge, skills and attitudes after going through the lesson.

- ***Summary of the unit:*** This section guides the teacher on how to summarise the main concepts learnt in the unit. It also guides the teacher on how to help the learners determine the solution to the problem statement activity provided at the start of the unit, since by now they have acquired enough knowledge and skills to enable them solve the problem
- ***End of unit assessment:*** This section guides the teacher on how to assess whether learners have acquired the knowledge, skills and attitudes after going through all the lesson in the unit.
- ***Additional information for the teacher:*** This section gives the teacher more information than what the syllabus recommends for purposes of enriching his/her knowledge.
- ***Remedial activities:*** This is a description of the activities that the teacher should give to the slow learners in order to help them master the concepts.
- ***Extended activities***: This is a description of the activities that the teacher should give to the fast learners in order to help advance their mastery the concepts.
- ***Answers to all exercises.*** This section provides answers to all exercises in the unit, extended and remedial activities in the teacher's guide.

1.1.2 Background Information on the new curriculum

The aim of a competence-based curriculum is to develop in the learners competences that will enable them interact with the environment in more practical ways.

It clearly defines the **knowledge, skills** and **attitudes** that the learner should acquire by doing the specified learning activities.

(a) Types of competences and their acquisition

Comptences are statements of the characteristics that students should demonstrate, which indicate they are prepared and have the ability to perform independently in professional practice. The two types of comptences envisaged in this curriculum are **basic** and **generic** competences.

(i) Basic competences

Basic competences are addressed in the stated broad subject competences and in objectives highlighted in each of units of learning.

They include:

Literacy

This competency will be acquired by S3 mathematics students as they:

- Read and interpret learning activities, and questions in exercises.
- Write down in English their observation and answers to guiding questions given in the activities.
- Listen and communicate their ideas in group and class discussions.

Numeracy

This competency will be acquired by S3 mathematics students as they:

- Compute and manipulate numbers, mathematical symbols, quantities, shapes and figures to accomplish a task involving calculations, measurements, and estimations in units in Units, 3, 4 and 7.
- Use numerical patterns and relationships to solve problems related to everyday activities like commercial arithmetic and financial management in Unit 2.
- Interpret basic statistical data using tables, diagrams, charts, and graphs in Unit 13

ICT and digital competences

This competency will be acquired by S3 mathematics students as they:

- Assess, retrieve and exchange information via internet or cell phones.
- Use information and communication technologies to enhance learning and teaching.

Citizenship and national identity

This competency will be acquired by S3 mathematics students as they:

- Perform activities outlined in the book, that are based on Rwanda environment and setting.
- As they do activities, interpret information, mathematical facts and data derived from Rwanda sources in the problems provided in the book. Good examples of units with abundance of for example in Unit 7 and 13 where there is a lot of commercial arithmetic data on Rwanda

Entrepreneurship and business development

This competency will be acquired by S3 mathematics students as they do activities, interact with and interpret commercial arithmetic data provided in various settings especially in Unit 7.

(ii) Generic competences

Generic comptences that S3 mathematics will acquire in the process of doing activities and learning the concepts planned for S3 mathematics include:

Critical thinking and problem solving skills:

The students will acquire this competence as they use their minds to evaluate mathematical situations and come up with solutions to the problems. Unit 1 on sets is particularly suitable in developing this competence.

Creativity and innovation:

The students will acquire this competence as they use imagination beyond knowledge provided in classroom to generate new ideas and construct new concepts.

Research skills:

The students will acquire this competence as they interrogate sources of information including the Internet, reference books and resource persons to gather information that will help solve real life mathematical problems.

Communication in official languages

The students will acquire this competence as they read and interpret learning instructions, activities, and questions in exercises, and communicate their ideas to others in group and class discussions

Cooperation, inter-personal management and life skills

The students will acquire this competence as they work in groups and cooperate in teams to do the tasks given in learning activities. They learn to respect the opinions of others and the complementary roles played by people through cooperation.

Lifelong learning:

The students will acquire this competence as they continuously learn to learn i.e continuously discover new ways of getting information through research. This is why they should be exposed to more research activities.

(b) Cross-cutting issues to be addressed during learning

These emerging issues give an indication of some national priorities and hence need to be incorporated in the learning process. The eight cross-cutting issues are:

Peace and Values Education

Peace is critical for a society to flourish and for every individual to focus on personal achievement and his or her contribution to the success of the nation.

The teacher of S3 Mathematics should be in the fore front to educate his/her students on the need for peace and values, for example by encouraging group work in the learners activities and showing the learners ways of solving interpersonal problems that occasionally arise during interactions and discussions.

Financial Education

Financial education provides the tools for sound financial management practices on earnings, spending, saving, borrowing and investing. Unit 7 in S3 mathematics provides the teacher with many opportunities to educate his/her students on financial matters.

Standardization Culture

Standardisation Culture develops learners' understanding of the importance of standards as a pillar of economic development. The S3 Mathematics teacher should use the opportunities provided by the various activities and problem setting that involve use of objects with many options to sensitize the students on the importance of acquiring and using standard items.

Environment and sustainability

A well conserved environment is obviously key to our health and survival. It is therefore important for the S3 mathematics teacher to make use of the opportunities that arise in the process of teaching and learning mathematics through activities to sensitise learners on the importance of conserving the environment. One way is by ensuring that the learners always dispose off the waste materials at the end of an activity in ways that do not pollute the environment.

Gender education

There is a strong moral imperative to accord every individual their basic human rights. Gender inequality results in women and girls being treated less favourably than men.

The S3 mathematics teacher should make use of opportunities and situations that arise in the classroom to educate students on the need for equal treatment of both genders. A good example is ensuring that discussion groups have a balanced number of boys and girls.

Comprehensive sexuality education (HIV/AIDS, STI, Family planning, Gender equality and reproductive health)

Few young people receive adequate preparations for their sexual lives. This leaves them potentially vulnerable to coercion, abuse and exploitation, unwanted pregnancies and sexually transmitted infections (STIs) including HIV/AIDS.

The S3 mathematics teacher should use the opportunities provided in the students book by the questions set on sexuality and HIV scenario to educate learners on these aspects.

Inclusive Education

Inclusive education refers to ensuring all learners regardless of gender or ability/inability are engaged in education to help them realise their potential. To achieve this, the S3 mathematics teacher should plan the teaching/learning resources and activities in ways that give all learners a chance to participate in the learning.

(c) Special needs education and inclusivity

All Rwandans have the right to access education regardless of their different needs. This provision allows all citizens to benefit from the same menu of educational programs. This necessitates the focus on special needs education. The critical issue ro consider is that we have persons/ learners who are very different in their ways of living and learning as opposed to the majority. The difference can either be emotional, physical, sensory and intellectual learning challenges traditionally known as mental retardation. These learners equally have the right to benefit from the free and compulsory basic education. Therefore, the schools' role is to enrol them and set strategies to provide relevant education to them. The teacher therefore is required to consider each learner's needs during teaching and learning process. Assessment strategies and conditions should also be tailored to the needs of these learners. Also, the teacher should include learners with special educational needs in classroom activities as much as possible.

The special needs in learners fall in any of the following common categories:

- Physical difficulties
- Visual difficulties
- Hearing difficulties
- Mental difficulties
- Genocide traumatized learners

The teacher should identify such cases and help facilitate the affected learners in learning. For example, learners with visual and hearing difficulties should sit near the teacher's table for easy supervision and assistance. The following are some suggestions on how to support special needs children in your class.

(i) Learners with physical impairment

These are learners, whose some of the body parts are not able to function normally due to physical problems. For example, some learners have partial or total incapacitation in the use of some limbs or hands. In such cases, the learners will need assistance during activities that involve movement. This could for example be during a nature walk and other activities that learners have to stand. The teacher should organize for the learner's ease of movement . The learner should also be given time to catch up with the others.

In case the hands are affected, the learners should be given more time to finish their work. In both cases, the learners should not be pressurized to do things that can cause injury or ridicule.

(ii) Learners with visual impairment

These learners have problems with their eyesight. The may be longsighted, short sighted or have some eye sicknesses. They should sit at a location where they are able to see the chalkboard without straining

The material to be observed should be brought to appropriate location where these learners can be able to see. The magnifying glasses can be used where necessary. The teacher should use large diagrams, charts and labels. In some cases, the learners can be allowed to touch and feel whatever they are looking at.

The teacher should read aloud most of the things he/she writes on the chalkboard. Other learners can also assist by reading aloud. The lighting system in the classroom can also be improved.

(iii) Learners with hearing impairment

The affected part in this case is the ear. The learner can have **hearing aids**. The teacher should use as many visual aids as possible. They should also project their voice and always talk while facing the learners. Use of gestures and signs while talking helps the learner figure out what the teacher is saying as well.

(iv) Learners with speech challenges

One of the most speech challenges is stammering. Such learners speak with many difficulties. The teacher should be patient with them and encourage such learners to express themselves in their own way. Such learners should be given more written exercises.

() Learners with mental challenges

The teacher should identify the nature and level of the mental difficulty with such learners. Such learners should then be given special assistance and attention at individual levels. They can be given special tests or assessments.

In general, all the learners with difficulties should be well facilitated. This encourages and motivates them. The teacher and the rest of the class should never ridicule learners with any of the difficulties. Note that generally, the people with any kind of disability can be very sensitive to any kind of negative comments or criticism.

Remind them that ***'Disability is not inability'.***

Treat them fairly but not with undue favours.

1.2: Effective teaching and learning of S3 Mathematics

1.2 .1 Teacher's and learner's roles for effective teaching/ learning of S3 mathematics

(a)Teacher's role and basic skills

The teacher is the most important resources for an effective mathematics lesson

Some of the key roles of the S3 mathematics teacher include:

- Organising the classroom to create a suitable learning environment.
- Preparing appropriate materials for learning activities.
- Encouraging and accepting student autonomy and initiative.
- Using raw data and primary sources, along with manipulative, interactive, and physical materials.
- Using cognitive terminology such as classify, analyse, predict, and create when framing tasks.
- Allowing student responses to drive lessons, shift instructional strategies.
- Familiarizing themselves with students' understandings of concepts before sharing their own understandings of those concepts.
- Encouraging students to engage in dialogue, both with the teacher and one another.
- Engaging students in experiences that pose contradictions to their initial hypotheses and then encouraging discussion.
- Providing time for students to construct relationships and create metaphors.
- Engaging students in variety of learning activities.
- Using a variety of teaching and assessment methods.
- Adjusting instructions to the level of the learner.
- Nurturing students' natural curiosity.
- Motivating learners to make them ready for learning.
- Coordinate learners' activities so that the desired objectives can be achieved.
- Assessing learners' activities and suggest solutions to their problems.
- Assist learners to consolidate their activities by summarising the key points learnt.

Some of the **key skills** that the S3 mathematics teacher should have include:

- Creativity and innovation.
- Makes connections/relations with other subjects.
- A high level of knowledge of the content.
- Effective disciplining skills manage adequately the classroom
- Good communicator.
- Guidance and counselling.

(a) Learner's role in learning Mathematics

Learning takes place only when the learner acquires the intended knowledge, skills and attitudes. As such, learning is a highly personal and individual process. Thus, a learner must be actively engaged in the learning exercise.

For active participation in learning, the learner should:

- Raise questions about what is observed.
- Suggest solutions to the problems observed.
- Take part in planning investigations with appropriate controls to answer specific questions.
- Carry out investigations to search for answers with the help of materials in search of patterns and relationships while looking for solutions to problems.
- Working collaboratively with others, communicating their own ideas and considering others' ideas.
- Expressing themselves using appropriate mathematical terms and representations in writing and talk.
- Engaging in lively public discussions in defence of their work and explanations.
- Applying their learning in real-life contexts.
- Reflecting critically on the processes and outcomes of their inquiries.

1.2.2: Teaching/learning resources

These refer to things that the teacher requires during the teaching process. They include:

- The classroom.
- Textbooks.
- Wall charts and wall maps.
- Materials and apparatus.
- Various tools and equipment.
- Mathematics models.
- Resource persons.
- Social facilities such as health centres, other learning institutions, community organisations etc.
- Enterprises such as agricultural farms, industries, among others.

(a) Classroom as a learning environment

A classroom generally refers to the place where learning takes place. Learners learn from everything that happens around them, such as the things that they hear, see, touch, taste, smell and play with.

Classroom organization

It is important for the teacher to make the classroom an attractive and stimulating environment. This can be done by:

- Carefully arranging the furniture in the classroom in an organised way. to allow free movement of learners and the teacher.
- Putting up learning and teaching aids on the walls. Examples are wall charts, pictures and photographs.
- Displaying teaching models.
- Providing objects for play for example toys.
- Having a display corner in the classroom where learners display their work.
- Setting a corner for storing materials so as not to obstruct learners or distract them.
- Spreading out the learners evenly so that they do not interfere with one another's activities.
- Setting up the materials for the series of lessons or activities going on for a number of days or weeks in a location where they do not interfere with other daily activities

- Organizing the sitting arrangement such that learners face the lighted areas of the room.
- Choosing the most appropriate location for the teacher and the chalkboard such that they are visible to all learners and the teacher has a good view of all learners in the class.

(b) Apparatus and materials

For learners to study mathematics through the activity method, a number of materials and apparatus are required. The important role played by materials in learning has been felt for centuries. This is noted for instance in the old Chinese proverb that says:

- *What I hear I forget*
- *When I see I remember*
- *When I do I understand*

Since Mathematics is largely a practical subject, materials help the teacher to convey his/ her points, information or develop skills, simply and clearly, and to achieve desired results much faster.

Most of the materials that a teacher requires for Mathematical activities and calculations can be collected from the local environment.

Many others can be improvised while some have to be purchased. Whether collected, improvised or purchased, there are certain materials that are valuable to have around almost all the time. These include:

(i) Mathematical Kit

A Mathematical kit is a special box containing materials, apparatus, and equipment necessary to conduct any mathematical operations and the performance of specific tasks. The content of the mathematical kit depends on the curriculum requirements per level. Most Mathematical kits are commercially available and target particular levels of learners. However, the teacher is encouraged to come up with a kit based on the specific unit and syllabus requirements.

Some of the materials within a mathematical kit includes:

- Dice
- Playing cards
- Blackboard; - ruler, Set square, Divider, Compass
- Meter rule
- Calculator
- Number cards etc.

(ii) Mathematical set

It is important for every learner to have a mathematical set containing at least; protractor, compass, set squares, rulers, divider, pencil, sharpener and eraser. The learner needs these materials especially during geometrical constructions.

(iii) Models

A model refers to a three-dimensional representation of an object and is usually much smaller than the object. Several models are available commercially in shops. Examples of mathematics models include 3 - D model of cubes, cuboids, pyraminds, globe among others. These can be purchased by schools for use during mathematics activities.

(iv) Resource persons

A resource person refers to anybody

with better knowledge on a given topic area. Examples include health practitioners such as doctors, nurses and laboratory technologists, agricultural extension officers, environmental specialists among others. Depending on the topic under discussion, the teacher can organize to invite a resource person in that area to talk to learners about the topic. The learners should be encouraged to ask as many questions as possible to help clarify areas where they have problems.

(v) Improvisation

If each learner is to have a chance of experimenting, cheap resources must be made available. Complicated apparatus may not always be available in most schools. Such sophisticated equipment made by commercial manufacturers are usually expensive and majority of schools cannot afford them. The teacher is therefore advised to improvise using locally available materials as much as possible.

(vi) Scheduling learning activities and venues

The collection of mathematical data in handling topics like probability and statistics are done at some particular specific weather condition and not at other times. For example, when collecting data on different makes of vehicles that pass through a particular route, the weather and other physical/social conditions must be considered for accuracy and to avoid bias. The teacher should therefore think ahead while making the scheme of work so that the prevailing weather pattern and the most appropriate timing are considered..

1.2.3 Grouping learners for learning

Most of the Mathematical activities are carried out in groups and therefore the teacher should place 2 or 3 desks against each other and then have a group of learners sitting around those desks.

In certain activities, the teacher may wish to carry out a demonstration. In this case, the learners should be sitting or standing in a semicircle, or arranged around an empty shape of letter "U" such that each learner can see what the teacher is doing clearly and without obstruction or pushing. If the learners are involved in individual work, each learner can work on the floor or on the desk or a portion of the desk if they are sharing. In this case, they need not face each other.

Grouping learners for learning has increasingly become popular in recent years. In fact, the shift from knowledge-based to competence curriculum will make grouping the norm in the teaching process. Grouping learners can be informed by one or all of the following:

a. Similar ability grouping.
b. Mixed ability grouping.
c. Similar interests grouping.
d. Needs grouping.
e. Friendship grouping.
f. Sex grouping.

Grouping learners in a mathematics class has several advantages that include:

a. The individual learner's progress and needs can easily be observed.
b. The teacher-learner relationship is enhanced.

c. A teacher can easily attend to the needs and problems of a small group.
d. Materials that were inadequate for individual work can now be easily shared.
e. Learners can learn from one another.
f. Cooperation among learners can easily be developed.
g. Many learners accept correction from the teacher more readily and without feeling humiliated when they are in a small group rather than the whole class.
h. Learners' creativity, responsibility and leadership skills can easily be developed.
i. Learners can work at their own pace.

The type of "grouping" that a teacher may choose depends on:

a. The topic or task to be tackled.
b. The materials available.
c. Ability of learners in the class (fast, average, slow).

However, the teacher must be flexible enough to adjust or change his/her type of grouping to cope with new situations.

There is no fixed number of learners that a group must have. This again will be dictated by such factors as the task to be done, the materials, characteristics of learners in your class, size and the space available. However, groups should on average have between four to five learners. You can also resort to pair work depending on the nature of the content being taught at the time.

There is no one method or approach to teaching that is appropriate to all lessons. A teacher should, therefore, choose wisely the method to use or a combination of methods depending on the nature of the topic or subtopic at hand.

1.2.4: Teaching methods

There is a variety of possible methods in which a teacher can help the learners to learn. These include :

(a) Direct exposition
(b) Discovery or practical activity
(c) Group, class or pair discussion
(d) Project method
(e) Educational visit/ field trips
(f) Teacher demonstration
(g) Experimentation/ Research

The particular technique that a teacher may choose to use is influenced by several factors such as:

- The particular group of learners in the class.
- The skills, attitudes and knowledge to be learned.
- Learning and teaching aids available. The local environment.
- The teacher's personal preference.
- The prevailing weather.
- The requirements of mathematics syllabus.

(a) Direct exposition

This is the traditional way of teaching whereby the teacher explains something while the learners listen. After the teacher has finished, the learners may ask questions. However,in competence-based curriculum, this technique should be used very minimally.

(b) Guided Discovery

In this technique, the teacher encourages learners to find out answers to problems by themselves. The teacher does this by:

- Giving learners specific tasks to do.

- Giving learners materials to work with.
- Asking structured or guided questions that lead learners to the desired outcome.

Sometimes learners are given a problem to solve and then left to work in an open-ended manner until they find out for themselves.

This is the most preferred method of teaching in the implementation of competency based curriculum.

(c) Group/class discussion or pair work

In this technique, the teacher and learners interact through question and answer sessions most of the time. The teacher carefully selects his/her questions so that learners are prompted to think and express their ideas freely, but along a desired line of thought. The method leads learners from the known to unknown in a logical sequence; and works well with small groups. The method boosts confidence in learners and improve interpersonal and communication skills.

The main disadvantage of this method is that some learners maybe shy or afraid to air their opinions freely in front of the teacher or their peers. It may give them more confident learners a chance to dominate the others.

(d) Project method

In this approach, the teacher organizes and guides a group of learners or the whole class to undertake a comprehensive study of something in real life over a period of time such as a week or several weeks.

Learners using the project method of studying encounter real life problems which cannot be realistically brought into a normal classroom situation. A project captures learners' enthusiasm, stimulates their initiative and encourages independent enquiry. The teacher, using the project method, must ensure that the learners understand the problem to be solved and then provides them with the necessary materials and guidance to enable them carry out the study.

The main disadvantage of this method is that if a project is not closely supervised, learners easily get distracted and therefore lose track of the main objective of their study. Studying by the project method does not work well with learners who have little or no initiative.

(e) Educational visits and trips/ nature walks

This is a lesson conducted outside the school compound during which a teacher and the learners visit a place relevant to their topic of study. An educational visit/nature walk enables learners to view their surroundings with a broader outlook that cannot be acquired in a classroom setting. It also allows them to learn practically through first-hand experience. In all "educational visit/ nature walk lessons", learners are likely to be highly motivated and the teacher should exploit this in ensuring effective learning. However, educational visits are time consuming and require a lot of prior preparation for them to succeed. They can also be expensive to undertake especially when learners have to travel far from the school.

(f) Demonstration lessons

In a demonstration, the teacher shows the learners an experiment, an activity or a procedure to be followed when investigating or explaining a particular problem. The learners gather around the teacher where each learner can observe what the teacher is doing. It is necessary to involve the learners in a demonstration, for example by:

- Asking a few learners to assist you in setting up the activity.
- Requesting them to make observations.
- Asking them questions as you progress with the demonstration.

This will help to prevent the demonstration from becoming too teacher-centred.

When is a demonstration necessary?

A teacher may have to use a demonstration, for example when:

- The experiment/procedure is too advanced for learners to perform.
- The experiment/ procedure is dangerous.
- The apparatus and materials involved are delicate for learners to handle.
- Apparatus and equipment are too few.

1.3: List of lessons

Term 1 (60 periods)

UNIT 1: PROBLEMS ON SETS (6 periods)		
Week	**Content**	**Number of Periods**
1	Mathematical problem on sets, analysis and interpretation of a problem using set language: union, intersection and complement of sets	2
	Representation of a problem using Venn diagrams involving two sets	1
	• Representation of a problem using Venn diagrams involving Venn diagrams involving three sets	2
	• End unit assessment and Remediation	1

UNIT 2: NUMBER BASES (12 periods)		
Week	**Content**	**Number of Periods**
2	Definition and examples of different number bases and numerals, the abacus and place value and number	1
	Converting a number from base ten to any other base like base 2, 3, or 5 and vice versa.	2
	Converting a number from one base to another (e.g. base 2 to base 3).	1
	• Addition and subtraction exercises on number bases	2
3	• Multiplication and division exercises on number bases	2
	• Special bases 2 and 12	2
	• Solving equations involving number bases	1
	• End unit assessment and Remediation	1

UNIT 3: ALGEBRAIC FRACTIONS (24 periods)		
Week	**Content**	**Number of Periods**
4	Defining, identifying algebraic fractions and restriction on variables or conditions of existence of an algebraic fraction	3
	Simplification of fractions whose denominator /or numerator is a binomial or a quadratic expression.	3
5-6	Addition/subtraction of algebraic fractions with linear Denominator	4
	Multiplication and simplification of two algebraic fractions.	4
	Division and simplification of two algebraic fractions	4
7	Solving rational equations with linear denominators	4
	End unit assessment	1
	Remediation	1

UNIT 4: Simultaneous Linear Equations and Inequalities (18 periods)		
Week	**Content**	**Number of Periods**
8	Introductory activity on equations and inequalities	1
	Graphical solution of simultaneous linear equations in two unknowns	3
	Forming and solving simultaneous linear equations from word problems	2
9-10	Graphical representation of linear inequalities in one unknown	3
	Simultaneous linear inequalities in one unknown	2
	Definition and examples of simultaneous linear inequalities in two unknowns	1
	Solving simultaneous linear inequalities in two unknowns	3
	Forming linear inequalities from graphs of inequalities	2
	End unit assessment and Remediation	1
11	**Exams**	

TERM 2: 72 periods

UNIT 5: QUADRATIC EQUATIONS (24 periods)		
Week	**Content**	**Number of Periods**
1	• Definitions and examples of quadratic equations	2
	• Factorizing Quadratic equations	2
	• Solving Quadratic Equations by factorization method	3
2	• Solving Quadratic equations by graphical method	2
	• Solving Quadratic equations by use of Perfect Squares	2
	• Solving quadratic equations by formula method	2
3	• Completing squares and solving quadratic equations	3
	• Solving quadratic equations by synthetic division method or Horner's method	3
4	• Solving problems from real life situations involving quadratic equations	2
	• End unit assessment	2
	• Remediation	1

UNIT 6: LINEAR AND QUADRATIC FUNCTIONS (24 periods)		
Week	**Content**	**Number of Periods**
5	• General form of Cartesian equation of a straight line	2
	• Slopes/gradients of a linear function	2
	• Finding equation of a straight line given gradient and one point on the line	2
6	• Equation of a straight line joining two points	2
	• Parallel lines	2
	• Perpendicular lines	2

7	• Introduction to quadratic functions	2
	• Table of values	2
	• Vertex of a quadratic function and axis of symmetry	2
8	• Intercepts, vertices and sketching a quadratic function	3
	• End unit assessment	2
	Remediation	1

UNIT 7: Compound Interest, Reverse Percentage, and Compound Proportional (12 periods instead of 20 periods)		
Week	**Content**	**Number of Periods**
9	Introduction (review of percentages, loan, savings and simple interest)	2
	Reverse percentage	1
	Compound interest – step by step method	3
10	The compound interest formula	2
	The compound proportional change or continued proportions using multipliers.	3
	End unit assessment and Remediation	1

UNIT 8: RIGHT-ANGLED TRIANGLES (12 periods /18 periods)		
Week	**Content**	**Number of Periods**
11	Review of Pythagoras theorem	1
	Median theorem of a right-angled triangle	2
	Altitude (Height) theorems of a right-angled triangle	1
	Leg theorem of a right-angled triangle	2

12	Introduction to trigonometry (**Sine and cosine of an acute angle)**	2
	Finding sine and cosine using calculators	1
	Using sines and cosines to find angles and lengths for sides of right-angled triangles	2
	Assessment	1
13	**Exams**	

TERM 3: 72 periods

UNIT 8: **RIGHT-ANGLED TRIANGLES** (6 Remaining periods/18 periods)		
Week	**Content**	**Number of Periods**
1	Tangent of an acute angle	2
	Using calculators to find tangent of angles	
	Using tangents to solve triangles	2
	Application of trigonometric ratios (sine, cosine and tangent)	1
	End unit assessment and Remediation	1

UNIT 9: CIRCLE THEOREM (18 periods)		
Week	**Content**	**Number of Periods**
2	Elements of a circle and disk: Center, radius, diameter, circumference, area, chord, tangent, secant, sector.	2
	Theorem 1:Angles at the centre and the circumference of a circle	2
	Theorem 2:Angle in a semicircle	1
	Theorem 3: Angles in the same segment	1
3	Theorem 4: Angles in a cyclic quadrilateral	2
	Theorem 5: Tangents to a circle	3
	sixth circle theorem (angle between circle tangent and radius), seventh circle theorem (alternate segment theorem)	2

4	Properties of chords: eighth circle theorem (perpendicular from the centre and bisects the chord).	3
	End unit assessment	1
	Remediation	1

UNIT 10: COLLINEAR POINTS AND ORTHOGONAL VECTORS (6 periods)		
Week	**Content**	**Number of Periods**
5	Collinear points	2
	Orthogonal vectors	1
	Problems about points and vectors in two dimensions	1
	End unit assessment and Remediation	1

UNIT 11: ENLARGEMENT AND SIMILARITY IN 2D (18 instead of 22 Periods)		
Week	**Content**	**Number of Periods**
6	Introduction, definition and properties of similarity	1
	Similar triangles	1
	Similar polygons	2
	Calculating lengths of sides of similar shapes using similarity and Thales theorem	2
7	Introduction, definition and properties of enlargement	1
	Enlargement with positive scale factor	1
	Enlargement with negative scale factor	1
	Locating the centre of enlargement and finding scale factor	2
8	Enlargement in the Cartesian plane	2
	Finding lengths of sides of similar shapes using Thales theorem.	
	Area scale factor and volume scale factor	2
	Composite and inverse enlargements	1
	End unit assessment and Remediation	1

UNIT 12: INVERSE AND COMPOSITE TRANSFORMATIONS IN 2D (12 periods)		
Week	**Content**	**Number of Periods**
9	Introduction to composite transformations in two dimensions	2
	Composite translations in two dimensions	
	Composite reflections in two dimensions	2
	Composite rotations in two dimensions	2
10	Mixed transformations in two dimensions	2
	Inverse transformations in two dimensions.	2
	End unit assessment	1
	Remediation	1

UNIT 13: STATISTICS (BIVARIATE DATA) (12 Periods)		
Week	**Content**	**Number of Periods**
11	Definition and examples bivariate data.	2
	Frequency distribution table of bivariate data.	2
	Review of data representation using graphs.	2
12	Scatter diagrams	2
	Correlation: positive and negative correlations	2
	End unit assessment	1
	Remediation	1
13	**Exams**	

1.4: Planning to teach

The two most important documents in planning to teach are the schemes of work and the lesson plan.

1.4.1 Schemes of work

A scheme of work is a collection of related units and subunits drawn from the syllabus and organized into lessons week by week for every term. It is also a forecast or plan that shows details under these sub-headings:

Date

Refers to the date of the day when the lesson will be taught.

Week

Refers to the week in the term e.g. 1, 2, 3, etc.

Unit title

This specifies the title of the unit from which the lesson is derived.

Lesson title and evaluation

Refers to the lesson being taught in that week e.g. lesson 1, 2, 3 and 4, etc, and the type of evaluation to be carried out.

Learning Objectives

Specifies what learners are expected to achieve at the end of the lesson.

Teaching methods, and techniques and evaluation procedures

Indicates the methods and techniques to used in the teaching/learning process and how evaluate.

Learning resources and references

Resources refers to any materials that will be used by the learner and the teacher for learning and teaching.

References

These are books or other materials that will be consulted or used in the teaching process. Books that learners will use should also be shown here; indicating the actual pages.

Observations

This should be a brief report on the progress of the lesson planned in the scheme of work. Such reports could include 'taught as planned'. 'Not taught due to abrupt visit by County Director of Education.' 'Children did not follow the lesson, it will be repeated on... (Specific date).

1.4.2 Lesson plan

A lesson plan is a detailed outline of how the teacher intends to carry out a specific lesson.

The following Important sub-headings of a Lesson Plan

Administrative details

Date………… Subject……….

Class…………Teacher

Time………… Roll…........

Unit title

The name of the unit as in the syllabus.

Key unit competence

This is/are the competence(s) that the learner is expected to achieve at the end of the unit.

Lesson title

The content area to be taught in the lesson.

Instructional Objectives

These represent what the teacher anticipates learners to achieve by the end of the lesson. Objectives should be clear and specific. They should also be stated in behavioural terms, that is, in a way that the outcome can be seen, displayed or measured. In mathematics, one should distinguish between knowledge, skill and attitude objectives.

Learning/teaching resources

Any materials and apparatus that the learners and the teacher will use during the lesson.

References

Any resources consulted or used by the teacher to prepare the lesson as well as any books that the learners will use during the lesson.

Introduction

This is the start of the lesson. The teacher should motivate the learners by creating learning situations that interest learners e.g. posing a problem, telling an amusing but relevant story or episode, showing an object or picture that arouse their interest. The introduction should link what the learners have already learnt with what they are going to learn.

*Presentation/lesson developmen*t

This should mainly include the activities that learners and the teacher will perform in order to achieve the stated objectives; as well as the questions that learners will answer as they do the various activities.

It is convenient to distinguish between the learners' and teacher's activities under two columns.

Summary/conclusion: (Consolidation)

This is the step in which the lesson activities are tied up or consolidated to emphasise the main points, summarize the lessons or make conclusions. The summary should correspond to the objectives stated for that lesson.

Comments/self-evaluation:

Teacher should write remarks on whether the objectives were achieved or not and what he or she intends to do to improve on the weak points noted during the lesson.

This teacher's book has been written to help you guide learners to learn mathematics in the most enjoyable and captivating manner. You are reminded to always arouse the curiosity of learners as you teach. Some things that you may do before you go for a lesson include:

- Go through the expected learning outcomes – this should help guide the manner of teaching.
- Read through the unit for the lesson in advance to get an overview of the content required.
- Form a mental picture of the teaching situation and the ways in which you will interact with learners when dealing with the suggested activities.
- Collect the materials that will be needed during the lesson in advance
- In some cases. try out the suggested activities / experiments in advance to avoid embarrassments like - the experiment failing to work during the lesson.
- Remember: The suggested teaching activities in this book are just a guide. You may not need to follow them to the letter! Feel free to incorporate other innovative teaching methods that will help in delivering the intended content optimally.

Lesson Plan

<table>
<tr><td>Term: 1</td><td>Date:</td><td>Subject:
MATHEMATICS</td><td>Class:
3N</td><td>Unit
No. 1</td><td>Lesson
No. 1</td><td>Duration:
40 min</td><td>Class Size: 40</td></tr>
<tr><td colspan="4">Type of Special Educational Needs to be catered for in the lesson and number of learners in each category</td><td colspan="4">1 student with hearing problem:</td></tr>
<tr><td>Unit title</td><td colspan="7">Problems on sets</td></tr>
<tr><td>Key Unit Competence</td><td colspan="7">By the end of this unit, learners should be able to solve problems involving sets.</td></tr>
<tr><td>Title of the Lesson</td><td colspan="7">Venn diagrams involving two sets</td></tr>
<tr><td>Instructional Objectives</td><td colspan="7">By the end of this lesson the learner should be able to solve problems involving two sets accurately using Venn diagrams.</td></tr>
<tr><td>Plan for this Class (location): in /outside:</td><td colspan="7">This class will be held indoors</td></tr>
<tr><td>Learning materials</td><td colspan="7">Charts with Venn diagrams drawn on them
Reference books,
Chalk and chalkboard
Pens and pencils
Calculators</td></tr>
<tr><td rowspan="3">Timing for each step</td><td colspan="6">Description of Teaching and Learning Activity</td><td rowspan="3">Generic comptences and cross cutting issues to be addressed</td></tr>
<tr><td colspan="6">Although groups and guidance of teacher, learners will do activities, presentations and take summary of the lesson and do conclusion</td></tr>
<tr><td colspan="3">Teacher Activity</td><td colspan="3">Learner Activity</td></tr>
<tr><td>Introduction
5 min</td><td colspan="3">Organise the learners in groups of three.
Guiding the learners to do Acivity 1.2 Mathematics for Rwanda Schools Student's Book 3 on solving a set problem analytically and then using a Venn diagram.
Allow the learners to present their findings.</td><td colspan="3">Doing Activity 1.2 in Mathematics for Rwanda Schools Student's Book 3 in groups of three on solving a set problem analytically and then using a Venn diagram.
Present their findings through group leaders.</td><td>Critical thinking enhanced by solving problems
Communication skills enhanced through group discussions</td></tr>
</table>

Development of Lesson **30 min**	Guiding the students in a class discussion on how to solve problems on two sets using Venn diagram. Guiding the leaners in a discussion of Examples 1.3, and 1. 4 in Mathematics for Rwanda Schools Student's Book 3 respectively. Highlighting and emphasising key concepts on solving problems on sets using Venn diagrams. Giving students an assessment/ practice task from Exercise 1.2 The teacher should ensure that the one student with hearing challenges (hears only loud voices) sits near the chalkboard to hear the teacher loudly. The teachers should also make effort to speak loudly and always confirm with that students whether he has hears him well, even repeating the speech to him here necessary.	Listening to the teacher Asking and answering questions Noting down the concepts and examples to their notebooks. Solving question 2, 3 and 5 from Exercise 1.2 in Mathematics for Rwanda Schools Student's Book 3 to practice the concepts learned	Listening and speaking skills enhanced through Critical thinking and cooperation enhanced through solving problems The teacher should provide hearing devices ,if available or shift the learners to hearing problems to the front seat or use sign languages .
Conclusion **5 min** **Summary** **Assessment**	Conclude the lesson by using Question and answer method to highlight key concepts, methods and skills learnt in the lesson including how to draw and analyse Venn diagrams. Giving the students assignment	Listen to the teacher Answer questions Taking notes and writing down the assignment in their note books.	Listening and speaking skills of the learners will be enhanced through asking and answering questions Learners with healing problems will helped to fit in class without any learning difficulty
Evaluation	The lesson was successfully covered because all students satisfied the requirements of assessment criteria		

1.5: Assessment and evaluation methods

1.5.1 Definition of assessment

Assessment is the process of evaluating the teaching and learning processes through collecting and interpreting evidence of individual learner's progress in learning and to make a judgment about a learner's achievements measured against defined standards. Assessment is an integral part of the teaching and learning processes. In the new competence-based curriculum assessment must also be competence-based; whereby a learner is given a complex situation related to his/her everyday life and asked to try to overcome the situation by applying what he/she learned.

1.5.2 Types of assessment

The two types of assessment that will be employed in the new curriculum is **formative** and **summative** assessment.

(a) Formative and continuous assessment (assessment for learning)

Formative or continuous assessment involves formal and informal methods used by schools to check whether learning is taking place.

T The teacher will use one or a combination of the following:

- Observation to judge the extend of skills acquisition
- Written tests
- Oral questions
- Project work
- Attitude change – this can be done by asking probing questions and checking body language as learners respond to the questions.

To achieve this, the student's Student's Book provides an assessment exercise immediately following the teaching/learning of each concept to immediately test the learners understanding. It also provides a unit test at the end of each unit to test the learners mastery of the concepts learnt in the entire unit. The Teacher's guide in addition provides remedial and extended activities and exercises for slow and fast learners respectively.

(i) Written tests

Under this, learners are given questions or tasks and are required to respond in writing. Examples of written tests are: short answer type questions, structured type questions, filling blanks, multiple choice questions, true-false questions and matching items.

(ii) Practical work or Activity

In this category, learners are required to perform a task or solve a problem practically. The teacher then assesses the finished work by looking at the materials used, procedures followed, whether it works or not or whether it is finished. He or she then awards marks accordingly.

(iii) Observation

This involves the teacher observing learners as they perform a practical task to assess acquisition of skills and attitude change. The teacher checks ability of the learner to measure, classify, communicate findings, etc. He or she also assesses the learner's curiosity, patience, team and co-operation spirit among others.

(iv) Oral questions or interviews

Asking learners questions which require a verbal response such as naming parts of human body, a system or short explanations of a process such as digestion can also be used to assess a learner's level of competence.

(v) Drawing

This involves asking learners to draw something they have observed or learnt about. They can also collect data and draw graphs and interpret the graph and give conclusions. This helps to assess their skill in communication through recording.

(vi) Project work

In a project, learners undertake a comprehensive study of something in real life over a period of time such as several weeks or even months after which they present a report. In project work, let learners begin from planning stage (come up with a schedule of events), execute the plan, analyse the results and look back (reflect on the challenges encountered during the project and come up with solutions to those challenges (problem-solving skills).

A teacher can use one or several of these assessment methods depending on the subtopic being studied or the purpose for which assessment is required.

When should the teacher assess learning progress?

The teacher should decide whether to assess learners at the end of the lesson or at any other appropriate time when enough content has been covered. T

(b) Summative assessment (assessment of learning)

When assessment is used to record a judgment of a competence or performance of the learner, it serves a summative purpose. Summative assessment gives a picture of a learner's competence or progress at any specific moment. The main purpose of summative assessment is to evaluate whether learning objectives have been achieved and to use the results for the ranking or grading of learners, for deciding on progression, for selection into the next level of education and for certification. This assessment should have an integrative aspect whereby a student must be able to show mastery of all comptences.

It can be internal school based assessment or external assessment in the form of national examinations. School based summative assessment should take place once at the end of each term and once at the end of the year. Districts will be supported to continue their initiative to organize a common test per class for all the schools to evaluate the performance and the achievement level of learners in individual schools. External summative assessment will be done at the end of S3.

Item writing in summative assessment

Before developing a question paper, a plan or specification of what is to be tested or examined must be elaborated to show the units or topics to be tested on, the number of questions in each level of Bloom's taxonomy and the marks allocation for each question. In a competency based curriculum, questions from higher levels of Bloom's taxonomy should be given more weight than those from knowledge and comprehension level.

Before developing a question paper, the item writer must ensure that the test or examination questions are tailored towards competency based assessment by doing the following:

- Identify topic areas to be tested on from the subject syllabus.
- Outline subject matter content to be considered as the basis for the test.
- Identify learning outcomes to be measured by the test.
- Prepare a table of specifications.
- Ensure that the verbs used in the formulation of questions do not require memorization or recall answers only but testing broad comptences as stated in the syllabus.

1.5.3 Record Keeping

This is gathering facts and evidence from assessment instruments and using them to judge the student's performance by assigning an indicator against the set criteria or standard. Whatever assessment procedures used shall generate data in the form of scores which will be carefully be recorded and stored in a portfolio because they will contribute for remedial actions, for alternative instructional strategy and feed back to the learner and to the parents to check the learning progress and to advice accordingly or to the final assessment of the students.

This portfolio is a folder (or binder or even a digital collection) containing the student's work as well as the student's evaluation of the strengths and weaknesses of the work. Portfolios reflect not only work produced (such as papers and assignments), but also it is a record of the activities undertaken over time as part of student learning. Besides, it will serve as a verification tool for each learner that he/she attended the whole learning before he/she undergoes the summative assessment for the subject.

1.5.4 Reporting to parents

The wider range of learning in the new curriculum means that it is necessary to think again about how to share learners' progress with parents. A single mark is not sufficient to convey the different expectations of learning, which are in the learning objectives. The most helpful reporting is to share what students are doing well and where they need to improve.

1 PROBLEMS ON SETS

Key unit competence

By the end of this unit, the learner should be able to solve problems involving sets.

Learning objectives

Knowledge and understanding skills

- Express a mathematical problem set using a Venn diagram.
- Represent a mathematical problem using a Venn diagram.

Skills

- Use Venn diagram to represent a mathematical problem set.
- Interpret, model, and solve a mathematical problem set.

Attitudes and Values

- Develop clear, logical and coherent thinking skills in solving real life problems involving sets.
- Appreciate the importance of representing and solving a mathematical problem set using Venn diagrams.

Pre-requisite to this unit

For the learners to be able to understand the concepts in this unit, there is a need to have acquired the following skills in their earlier work.

- Ability to express a mathematical problem set using a Venn diagram as studied in S1 Unit 1.
- Interpret, model, and solve a mathematical problem set.

Cross-cutting issues to be addressed in this unit

- **Financial Education**: This unit will involve learners working with some financial related problems. Kindly guide the learners how they can manage their finances.
- **Environment and sustainability:** Some activities may involve environment and sustainability. The teacher should guide the learners to understand how they can maintain and sustain a better environment.
- **HIV and AIDS:** Some Exercises have questions set on the HIV and AIDS scenario. The teacher should guide the learners on the dangers of being infected with HIV and how it can be prevented within the youth to save the nation.

- **Inclusive education**: The teacher should provide for learning of all learners including those with special needs. A teacher should use sign language for those with hearing difficulty and those who are dumb.

Generic competences to be addressed in this unit

The generic competences to be addressed in this unit are:

- **Critical thinking**: This competence will be achieved when the teacher involve learners in activities that involve analysing problems on sets using Venn diagrams.
- **Problem solving**: This competence will be achieved when the teacher involve learners in activities and exercises on how to complete and solve problems involving Venn diagrams
- **Communication skills and listening**: This competence will be achieved when learners will be allowed to express themselves while presenting the discussion results to the whole class.

Vocabulary/keywords

In the course of learning the concepts in this unit, the learners will discover the meaning of the following new words:

- Intersection of a set
- Union of a set
- Complement of a set
- A set

Guide the learners to understand the meanings of these words and construct and speak out mathematical statements involving them in order to master their meaning and usage.

Guidance on the problem statement

- In order to motivate the learners to actively participate in learning the concepts, and lesson planned for this unit, organise them in groups of 5 to do the Unit Focus Activity outlined in the Student's Book.
- Encourage them to use the knowledge they have so far acquired on sets in S1 to do the activity.
- Allow the learners to be as creative as possible. It is most likely that different groups will use different symbols to represent the sets.
- Give the groups a chance to present their work to the rest of the class in a class discussion.
- Most likely, most of the groups may not get the correct answer to the activity. To the groups that may get the answer, congratulate them and ask them to explain to the class how to do it.
- Make sure that all the learners with disabilities are able to do catared for in the lesson such that they can also get the content without difficulties.

- After this, let the whole class know that the concepts involved in the activity form the content of the unit, hence by keenly participating in all the lessons for this unit, they will discover how to do this unit focus activity. This will motivate them and make them eager to learn all the concepts in this unit.

Answers to the unit focus activity

Let F represent those who play football.

V represent those who play volley ball

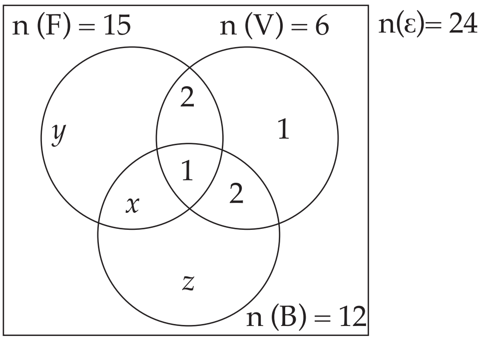
B represent those who play Basket ball

$n(F) = 15$, $n(V) = 6$, $n(B) = 12$, $n(\varepsilon) = 24$,

$n(F \cap V \cap B') = 2$, $n(F \cap B \cap V') = x$, $n(V \cap B \cap F') = 2$,

$n(F \cap V \cap B) = 1$, $n(F)$ only = y, $n(B)$ only = z

Representing the data in the Venn diagram



The number of those who like football and basketball but not volleyball can be calculated as follows:

For basketball,

$x + z + 1 + 2 = 12$

$x + z = 12 - 3$

$x + z = 9$ ... (i)

For football

$x + y + 1 + 2 = 15$

$x + y = 15 - 3$

$x + y = 12$ ... (ii)

For all the games

$x + y + z + 2 + 1 + 2 + 1 = 24$

$x + y + z + 6 = 24$

$x + y + z = 24 - 6$

$x + y + z = 18$ ... (iii)

Solving (i) and (ii) simultaneously by elimination method.

$$-\begin{vmatrix} x + z = 9 \\ \underline{x + y = 12} \end{vmatrix}$$

$$z - y = 9 - 12$$

$$z - y = -3 \Rightarrow y = z + 3 \text{ ... (iv)}$$

Substituting y = z + 3 into equation (iii),

$x + (z + 3) + z = 18$

$x + 2z + 3 = 18$

$x + 2z = 15$ ... (v)

Solving (i) and (v),

$$-\begin{vmatrix} x + 2z = 15 \\ \underline{x + z = 9} \end{vmatrix}$$

$$z = 6$$

Those who like basket ball only are 6.

So, $x + z = 9$

$x + 6 = 9 \Rightarrow x = 3$

Those who like both football and basket ball are 3.

Also, $y = z + 3$

$y = 6 + 3$

$y = 9$

Those who like football only are 9.

Those who like football and basketball but not volleyball are 3.

List of Lessons

Lesson No.	Lesson Title	No. of periods
1	Review of union, intersection and complement of sets	1
2	Venn diagrams involving two sets	2
3	Venn diagrams involving three sets	2
4	Unit Assessment	1

Lesson development

Lesson 1: Review of union, intersection and complement of sets

Lesson objective

By the end of the lesson the learner should be able to define the term sets and establish the process involved in it.

Teaching Aids

- books,
- chalk
- Pens and pencils
- Classroom Chalkboard
- Other writing materials
- Calculators
- Mathematical sets and other geometrical Instruments

Learning Activities

Activity 1.1

- Organise the class into groups of three learners. Prompt then to realise that they need a group leader. Ensure that each group has a secretary to record and report the group's findings.
- Ask the groups to do Activity 1.1.
- Let the groups present their findings in a class discussion through their secretaries, and allow other members of the class to point out omissions or errors in the facts presented.
- Take the learners through the main points of the activity by guiding them where they go wrong.

Synthesis

- Having guided the learners through Activity 1.1 take them through a class discussion. This will help them to remember that:
  - Union of two or more sets is the set formed by combination of elements of two or more sets together.
  - Intersection of two or more sets is the set formed by the common elements between two or more sets.
  - The compliment of a set A is formed by all elements that are not in A.
- Guide the leaners through Examples 1.1 and 1.2.

Assessment of the lesson

- Ask the learners to do Exercise 1.1. Move round to help those who may encounter problems.
- Assess whether the lesson objectives have been met.

Answers to Activity 1.1

1. A set is the collection of same objects.
2. (a) (i)A = {Rwanda, Burundi, Uganda, Tanzania, Kenya, South Sudan}

   (ii) B = {Burundi, Uganda, Tanzania, Democratic Republic of Congo}

   (iii) C = {Uganda, Kenya, Tanzania}

   (b) (i) n (A) = 6 (ii) n(B) = 4 (iii) n(C) = 3

(c) (i) $A \cap B$ = {Burundi, Uganda, Tanzania}

(ii) $A \cap C$ = {Uganda, Kenya, Tanzania}

(iii) $A \cup B \cup C$ = {Rwanda, Burundi, Kenya, Tanzania, Uganda, South Sudan, DR Congo}

(iv) $A \cup B \cap C$ = {Uganda, Kenya, Tanzania}

3. (i) $A' = \{1, 4, 6\}$ (ii) $B' = \{1, 2, 6\}$ (iii) $A' \cap B = \{4\}$

(iv) $A' \cup B' = \{1, 2, 4, 6\}$ (v) $(A \cap B)' = \{1, 2,4,6\}$ (vi) $A' \cap B' = \{1, 6\}$

Lesson 2: Venn diagrams involving two sets

Lesson objectives

By the end of the lesson, the learner should be able to interprete and represent problems involving two sets using Venn diagrams and solve them.

Teaching Aids

- Books,
- Pens and pencils
- Calculators
- Chalks and Class chalkboard
- Other writing materials.
- Mathematical sets and other geometrical instruments

Learning Activities.

Activity 1.2

- Organise the learners in groups of three to do Activity 1.2.
- When the activity is done, listen as different secretaries present their findings.
- Verify their findings and help them to conclude; emphasize the key points and correct possible erroneous conclusions.

Synthesis

- Wind up the conclusions by doing Examples 1.3, 1.4 and 1.5.
- Ensure that all the learners have understood the examples well.

Assessment

- Ask the learners to do Exercise 1.2 and numbers 1 to 5 and move to help those who will encounter problems. For quick learners ask them to continue with numbers 6 to 8.
- Assess whether learning has taken place and the objective of the activity has been achieved.

Answers to Activity 1.2

(a) Let M represent those who bought milk

B represent those who bought bread

$n(M) = 52$, $n(B) = 32$, $n(\varepsilon) = 79$

Let those who bought both milk and bread be represented by x i.e. $n(M \cap B) = x$

$52 - x + 32 - x + x + 15 = 79$

$15 + 84 - x = 79$

$x = 99 - 79$

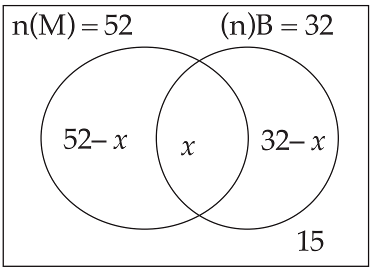
$x = 20$

(i) Those who bought milk and bread are 20.

(ii) Those who bought bread only are $32 - 20 = 12$.

(iii) Those who bought milk only are $52 - 20 = 32$.

(b)



$(\varepsilon) = 79$

$n(M \cup B) = n(M) + n(B) - n(M \cap B)$

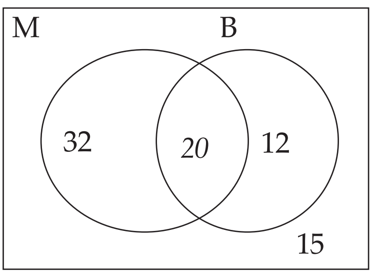
$n(\varepsilon) = n(M \cup B) + n(\overline{M \cup B})$

$52 - x + x + 32 - x + 15 = 79$

$-x + 99 = 79$

$-x = -20$

$x = 20$



$\varepsilon = 79$

(i) Those who bought milk and bread are 20.

(ii) Those who bought bread only are $32 - 20 = 12$.

(iii) Those who bought milk only are $52 - 20 = 32$.

(c) It is easier to use a Venn diagram because the elements in each region can easily be visualized.

Lesson 3: Venn diagrams involving three sets

Lesson objective

By the end of the lesson, the learner should be able to interpret and represent problems involving three sets using Venn diagrams and solve them.

Teaching Aids

- Charts
- Books
- Geometrical sets.
- Chalks and class Chalkboard
- Any other writing materials

Learning Activities

Activity 1.3

- Organise the learners in pairs to do Activity 1.3 given in the Student's Book.
- After the end of the activity, ask the groups to present their results to the class each at a time as the other groups compare with other groups results.
- At the end of the presentations, guide the whole class to identify members who eat only one type of food and those who eat more than one type.
- Note that the data given in this activity was on the types of food eaten. Take this opportunity to sensitize the learners on the dangers of eating too much one type of food.

Synthesis

- Wind up the activity summary and do Examples 1.6, 1.7 and 1.8.
- Make sure learners have understood the activity very well.

Assessment of the lesson

- Ask the learners to do question 1 in Exercise 1.3. Go round checking their work and give more guidance to the slow learners who could be having challenges.
- When you are satisfied with the learner's mastery of the concept, ask them to do questions 2 – 4 of Exercise 1.3.
- Ask the quick learners who may finish in time to do questions 5 and 6. These questions may be a bit challenging.

Answers to Activity 1.3

(a) $n(\varepsilon) = 50$, $n(R) = 15$, $n(S) = 30$, $n(P) = 19$, $n(R \cap S) = 8$, $n(R \cap P) = 12$, $n(S \cap P) = 7$, $n(R \cap S \cap P) = 5$, $n(P) = 19$

(b) i) Let those who don't take any of the foods be represented by x.

$15 - (8 - 5 + 12 - 5 + 5) + 30 - (8 - 5 + 7 - 5 + 5) + 19 - (12 - 5 + 5 + 7 - 5) + 8 - 5 + 12 - 5 + 7 - 5 + 5 + x = 50$

$0 + 20 + 5 + 3 + 7 + 2 + 5 + x = 50$

$x + 42 = 50$

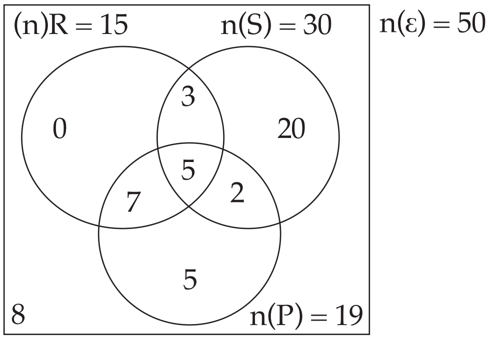
$x = 50 - 42$

$x = 8$

ii) $n(P \cap R \cap S') = n(P \cap R) - n(P \cap R \cap S) = 12 - 5 = 7$

iii) $n(R \cap S \cap P') = n(R \cap S) - n(R \cap P \cap S) = 8 - 5 = 3$

(c)



(i) Those who do not eat any are 8.

(ii) Those who eat both rice and posho are 7.

(iii) those who eat both rice and sweet potatoes are 3.

d) It is not easy to answer without a venn diagram.

Summary of the unit

- Asking different learners to take the different concepts highlighted in the unit summary.
- Ask the class probing questions to help them recall the concepts correctly.

End of unit assessment

- A teacher should ask learners to do unit 1 test to assess whether learners understood the whole unit.

Additional information to the teacher

- A teacher should know the concepts of sets taught in Senior 1 and some concepts of applications of sets in S2 fully in order to teacher this unit properly.

Remedial Activities

- For the slow learners, a teacher should organize remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

Remedial Exercise

1. Last June, there were 15 windy days and 20 rainy days, yet 5 days were neither windy nor rainy. 'How can this be, when June only has 30 days?
2. A travel agent surveyed 100 people to find out how many of them had visited the cities of Kigali and Huye. Thirty-one people had visited Kigali, 26 people had been to Huye, and 12 people had visited both cities. Draw a Venn diagram to find the number of people who had visited:

   (a) Kigali or Huye (b) Huye but not Kigali

   (c) Only one of the two cities
3. Twenty-four people go on holidays. If 15 go swimming, 12 go fishing, and 6 do neither, how many go swimming and fishing? Draw a Venn diagram and fill in the number of people in all four regions.
4. A cafe records how people take their coffee. Altogether 71 take milk and 36 take sugar. 21 take both and 24 take neither.

   (a) Fill in the Venn diagram. (b) How many people were in the survey?
5. A group of 100 people were asked if they owned a pair of Converse shoes and if they owned a pair of Nike trainers. 48 people owned Converse, 14 people owned both types of footwear, and 18 people owned neither.

   (a) Draw a Venn Diagram showing this information.

   (b) How many people own Nike?

   (c) How many people only own Converse?

Extended activities

- The calculations in this unit may not be challenging to the specially gifted learners since they finish doing the task so fast, then get bored and start distracting others. Give learners the questions below.

Extended Exercises

1. A group of 80 people were asked if they liked different fruits. 38 said they liked apples, 42 like bananas, and 14 like cherries. 6 liked all 3 fruits. 20 people like apples and bananas, 10 of the people who liked cherries also liked bananas. 6 people like cherries and apples.

   (a) Draw a Venn diagram showing this information

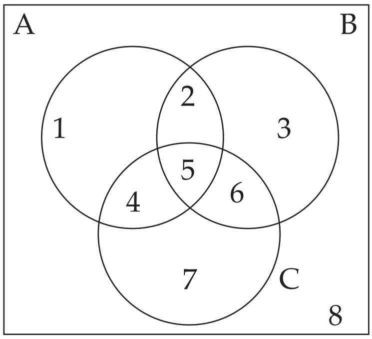
   (b) How many people liked apples and bananas but did not like cherries

   (c) How many people liked none of the fruit?

   (d) How many people liked just one of the fruits?

   (e) How many people liked at least 2 of the fruits?

2. Study the Venn diagram below.



The figure above shows a Venn diagram with 3 sets A, B and C.
How would you describe each region of the diagram using unions $\cup$ intersections $\cap$ of sets A, B, C, A', B' and C'?

1	2	3	4
5	6	7	8

3. A group of 40 people were asked what they ate for lunch. $\frac{3}{4}$ of the group had a sandwich, $\frac{1}{2}$ the group had some fruit, and 12 people had yoghurt. 10 of the people who had yoghurt also had fruit, $\frac{1}{4}$ of the group had all 3 items, $\frac{2}{3}$ of the sandwich eaters also had fruit. 11 of the sandwich eaters also had yoghurt.

(a) Draw a Venn diagram to show this information

(b) How many people had all the three items?

(c) How many people had fruit and yoghurt but no sandwich?

(d) How many people had none of the named items?

(e) How many people had yoghurt only?

(f) How many people had only one item?

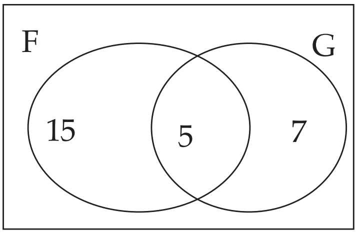
Answers to exercises, unit test, remedial activities, extended activities

Exercise 1.1)

1. (a) 4 (b) 3 (c) 3 (d) 7 (e) 4 (g) 7
   (f) {1, 2, 3, 4, 6, 8, 10} (h) 6
2. { –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 12,14,16,18,20}
3. (i) $(P \cup Q)' = \varnothing$ (ii) $(P \cap Q)' = \{1,3,5,7,9,11,13,15\}$ (iii) $\varepsilon = \{1,2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ (iv) $n(\varepsilon) = 15$
4. (i) {7, 9, 11, 13} (ii) {3, 5, 7, 9, 11, 13} (iii) {3, 5}
   (iv) {3, 5, 7,9, 11,13} (v) {7, 9, 11, 13} (vi) {3, 5, 9, 11, 13}

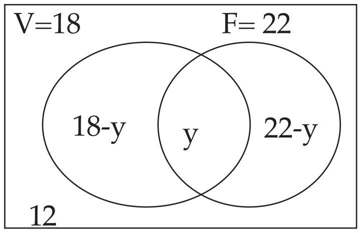
Exercise 1.2

1. (a)



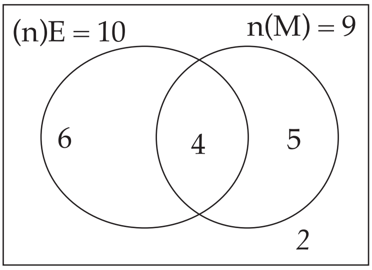
(b) 27

2. (a) $n(\varepsilon) = 30$



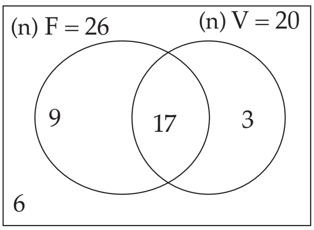
(b) y = 10

3. (a) $n(\varepsilon) = 17$



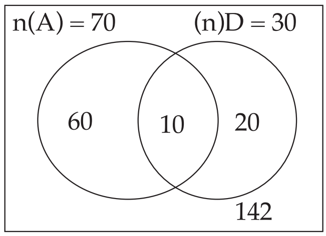
(b) (i) 4 (ii) 11 (iii) 2

4. (a) $n(\varepsilon) = 35$



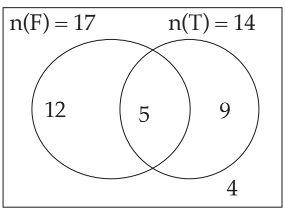
(b) 6

5. (a) $n(\varepsilon) = 232$



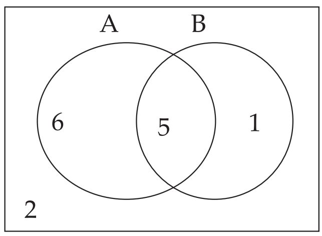
(b) x=10 and n(D) only = 20

6. (a) $n(\varepsilon) = 30$



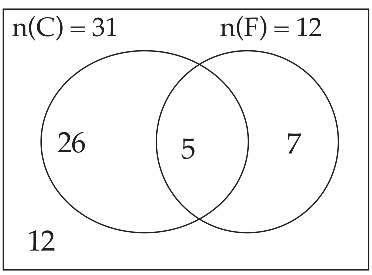
(b) 12 (c) 4

7. (a) $n(\varepsilon) = 14$



(b) (i) 9 (ii) 3

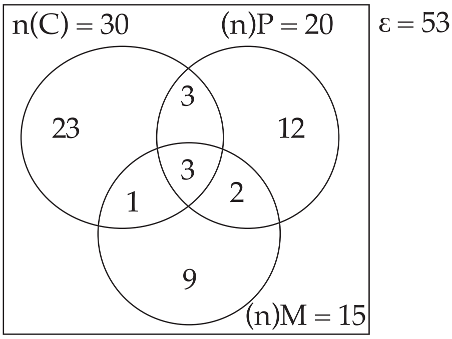
8. (a) $n(\varepsilon) = 50$



(b) (i) 7 (ii) 26 (iii) 12

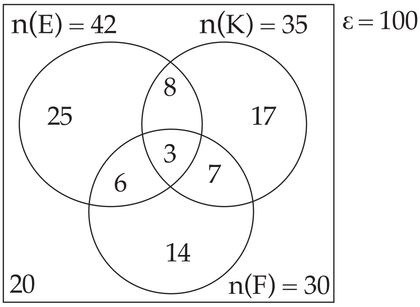
Exercise 1.3

1. (a)



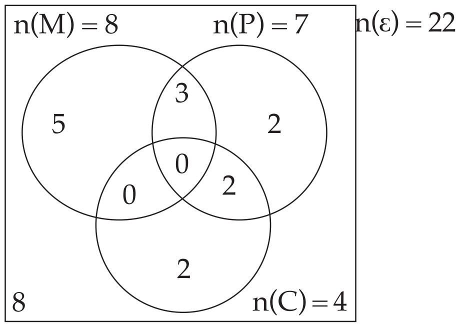
(b) 3 (c) (i) 12 (ii) 15 (iii) 6

2. (a)



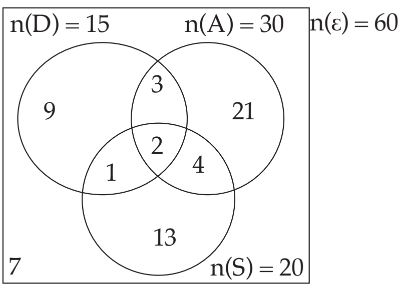
(b) 3 (c) 25 (d) 10

3. (a)



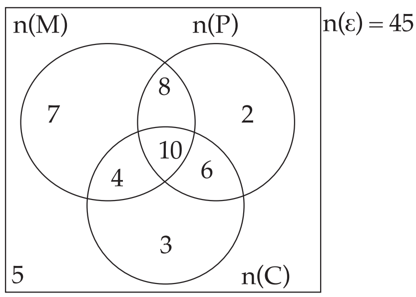
(b) (i) 5 (ii) 2 (iii) 8

4. (a)



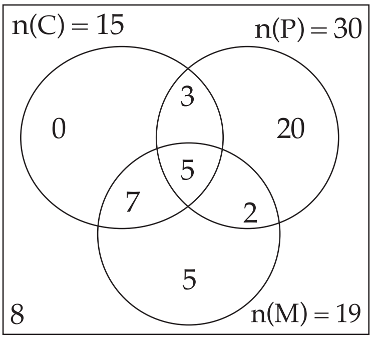
(b) (i) 43 (ii) 10 (iii) 45

5. (a)



(b) (i) 5 (ii) 16

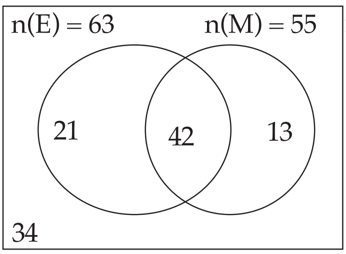
6. (a)



(b) (i) 0 (ii) 3 (iii) 8

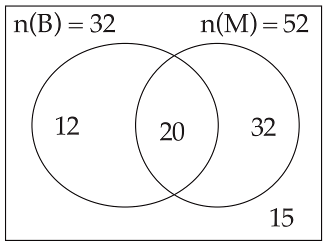
Unit 1 Test

1. (a)



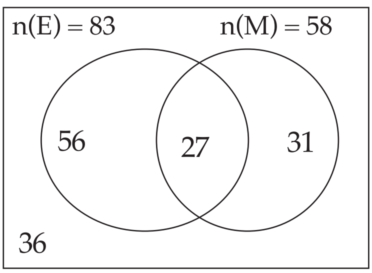
(b) (i) 76 (ii) 42 (iii) 13

2. (a)



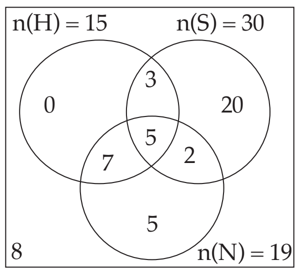
(b) (i) 20 (ii) 12 (iii) 32

3.



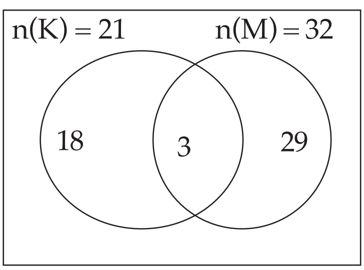
Both careers 27

4.



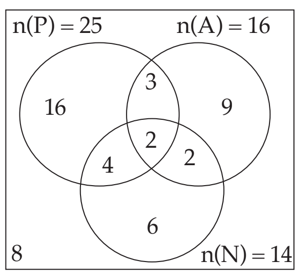
(a) None (b) 3 (c) 8

5. (a)



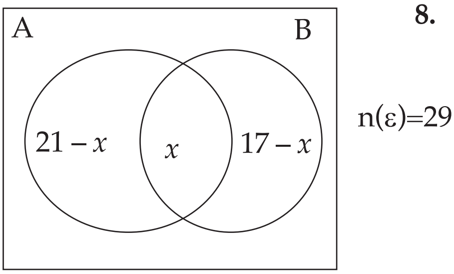
(b) 47

6. (a)



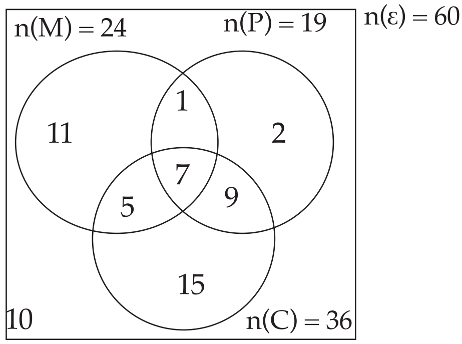
(b) (i) 42 (ii) 31 (iii) 16

7. (a)



(b) 9

8. (a)

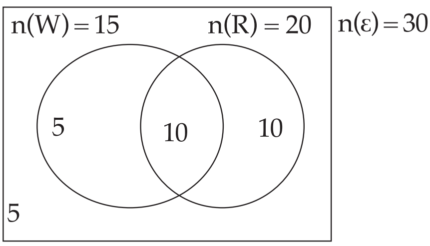


(b) 10 students

(c) 24 students

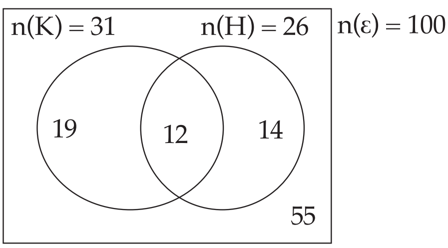
Answers for remedial exercises

1.



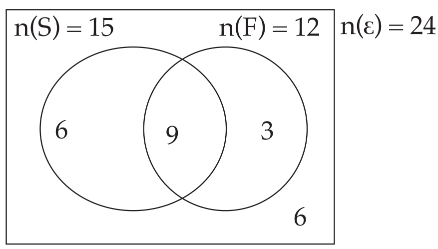
- 5 days were only windy
- 10 days were both windy and rainy
- 10 days were only rainy
- 5 days were neither windy nor rainy

2.



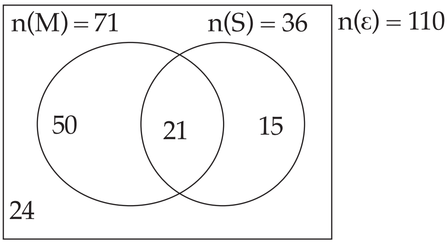
(a) 45 (b) 14 (c) 33

3.



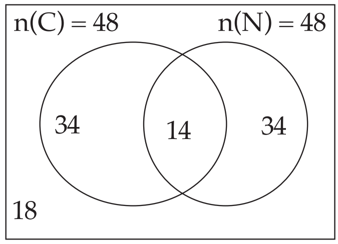
∴ 9 people go swimming and fishing

4. (a)



(b) 110 people were in the survey.

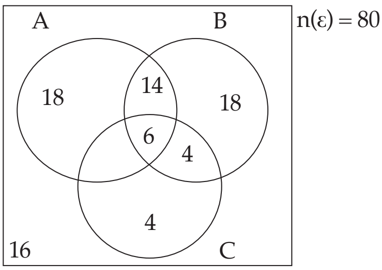
5. (a)



(b) 48 (c) 34

Answers for extended exercises

1. (a)



(b) 14 (c) 16 (d) 40 (e) 24

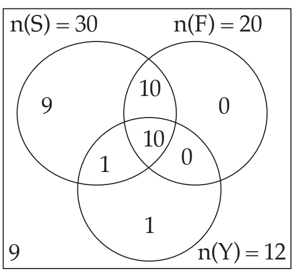
2. 1: $A \cap B' \cap C'$ 2: $A \cap B \cap C'$ 3: $A' \cap B \cap C'$

4: $A \cap C \cap B'$ 5: $A \cap B \cap C$ 6: $B \cap C \cap A'$

7: $A' \cap B' \cap C$ 8: $(A \cup B \cup C)'$

3.

(a)



(b) 10 (c) 0 (d) 9 (e) 1 (f) 10

UNIT
2
NUMBER BASES

Key unit competence

By the end of this unit, the learner should be able to represent numbers in different number bases and solve related problems.

Learning objectives

Knowledge and understanding

- List the digits used in a given base.
- Convert numbers from base ten to any other base and vice versa.

Skills

- Carry out addition, subtraction, multiplication, and division on numbers bases.
- Solve equations involving bases.

Attitudes and values

- Develop clear, logical and coherent thinking while solving problem sets.
- Appreciate the importance of bases in various contexts.

Pre-requisite to this unit

For learners to be able to acquire the knowledge, understanding, skills, attitudes and values desired in this unit, there is need to have acquired the following: -

- Knowledge of decimal system of numbers. This includes number place values.
- Skills on performing operations on numbers in base ten.
- Defining **place value** of all the digits in a given number.

Cross-cutting issues to be addressed in this unit

The specific cross cutting issues to be addressed in this unit are:

Inclusive education

Teacher should provide the means for effective learning for all student's including those with special physical needs. Seek intervention of need be, including acceptance by fellow learners, regardless of their physical status or learning challenges.

Peace and value education

There is need for peace and value education. The teacher to sensitize their learners the need for peace and the consequences of lack of it. Let the learners know the importance of value education in order to contribute positively to the welfare of the society.

Financial education

Teacher to sensitize learners on requirement of accuracy in packaging goods for business, using different groups of items.

Generic competences

The specific generic competences to be addressed in this unit are:

- **Team work:** When the teacher groups learners to work together, the learners learn to **share knowledge and to cooperate.**
- **Communication and listening skills:** are promoted when learners are given a chance to organize and report their findings in a class discussion,
- **Research techniques:** These are developed when you ask learners to find meaning of new words or terminology. Learners are encouraged to use different methods to do so.

Vocabulary/key words

In this unit the learners will discover the meaning of new/keywords such as:

- Number bases
- Place value
- The abacus
- Numeral
- Digit
- Denary, duodecimal

Help the learners to understand the meanings and the mathematical usage of these words.

Guidance on the problem statement

Organise learners to do the following introductory activity in groups. This will help them to have a general idea about the unit they are about to begin.

Materials:

A basic framework with rows or columns of beads i.e. abacus materials, internet, reference books, writting materials, chalks and chalkboard

- Ask learners to do unit focus activity in the Student's Book. Let them count items such as beads, coins etc using different groups. This will help them grasp the idea of using different bases in mathematical operations.
- Use their findings to introduce this unit, relating their counting groups to **number bases**, describing **place value** of a digit in a number using base 10 system.

- Help learners to understand that just as we do in base **ten** operations, the same can be done using any other base.

Answers to unit focus activity

(i) Groups of 6

(ii) Groups of 8

(iii) Symbolically, 3: ~~//~~ (three items make a group)

5 : ~~////~~ (five items)

6 : ~~/////~~ (six items)

7 : ~~//////~~ (seven items)

The strokes against each number, show the number of items in each group.

(iv) Addition, Substruction and Multiplication can be done in groups. For example, if we have groups of 10s, we have 10+10+10=30 ie. three groups of tens.

Attention to special needs

- You should know your learners well so that you are aware of their physical, emotional and psychological needs in order to help them learn effectively.
- Identify for how may have impaired vision, hearing problems etc and address the issue accordingly. Be on the lookout for a learner who suddenly becomes inattentive, dull, tends to dose in class, not doing his/her homework etc may be in need of help. Find out and act accordingly.
- For fast learners, you may give them extra work to keep them busy and practice more so that they do not feel unappreciated. For slow learners organize remedial work/activity to help them understand the concept.

List of lessons

Lesson No	Lesson Title	No. of periods
1	Number bases – and numerals, the abacus and number place value	1
2	Change of base	3
3	Addition and subtraction	2
4	Multiplication and division	2
5	Special bases 2 and 12	1
6	Solving problems	2
7	Unit assessment	1

Lesson Development

Lesson 1: Numbers and numerals

Lesson objective

By the end of the lesson, the learner should be able to distinguish between number, numeral and digit and identify the place value of digits in any number given, in a base.

Introduction

Introduce the lesson by sensitizing the learners about different number systems, the measuring and significance of number place value with reference to the decimal system of numbers. Discuss the idea of counting or packaging items using different groups as a way to introduce number bases.

Teaching Aids

- Access to internet.
- Dictionary: Mathematics/English
- Model of a basic framework of an abacus including beads,.
- Calculator

Learning Activities

Activities 2.1 to 2.3

- Organize learners into group of three to do Activity 2.1 in the Student's Book to find the meaning of number, numeral digit.
- Ask learners (groups) to report their findings in a class discussion. Ensure that all learners are involved in the activity fully. Encourage them to make their own notes as the activity progresses.

Synthesis

- Summarise group findings, using this opportunity to emphasize the key points and facts i.e. the importance of place value, difference between number and numeral the meaning of digit etc.
- Emphasize the significance of place value of numbers as explained in the Student's Book.

Answers to Activity 2.1

- A number is an idea describing a quantity as in counting or in identification. Such an idea can be represented by a mathematical symbol called a numeral.
  For example twenty five is a idea representing a quantity, and 25 is the symbol representing the number.

- Each of the symbol such as 2 and 5 is called a digit. This means that a number may be made up of one or more digits.

- In the Hindu-Arabic system of numbers that we use, there are ten digits namely, 0, 1, 2, 3, 4, 5, 6, 7, 8,9

Generally the terms number and numeral are used interchangeably.

Activity 2.2

Organize the students to do Activity 2.2 in the student's Book on describing an abacus and how it is used to count in base 10.

Let them present their findings to the rest of the class.

Synthesis

With the help of an actual abacus (if available) and Fig. 2.1 in the Student's Book, guide the learners to understand that an abacus is a device that is used to demonstrate counting in various bases. Demonstrate to them how it works then allow them to practice counting using it. Take the learners through the discusion in the Student's Book.

Answers to Activity 2.2

An abacus is a counting/calculating device used in elementary schools. This device consists of balls or beads strung on wires or thin rods set in a frame. Fig 2.1 Student's Book shows a basic such device. This device helps develop place value concept of numbers. It helps young children to develop counting skills as well as identifying place value of numbers.

Activity 2.3

Organise the students to do Activity 2.3 in the Student's Book on finding out the meaning of a number base and giving examples. Let them present their findings to the rest of the class.

Synthesis

With the help of an actual abacus (if available) and Fig. 2.2 and 2.3 in the Student's Book respectively, guide the learners to understand the meaning of number bases. Take the learners through the discussion following immediately after the Activity 2.3.

Guide them through Example 2.1 in the Student's Book, giving them a chance by explaining how to work it out at each step.

Answers to Activity 2.3

A number bases refer to number systems where we count using different groups. For example we normally count in groups of ten i.e. 23 means 2 groups of 10 and 3 ones. Thus, we call this base ten system. Similarly, we could count in groups of 2, 3, 4, 4, …. which could be called base 2, base 3, base 4 etc.

In base 10, a number like 25 means 2 x 10 + 5

If we express 25 in groups of 6 and 1 one unit

This means 25 base 10 can also be expressed as 41 base 6 denoted as

$25_{ten} = 41_{six}$; $81_{ten} = 213_{six}$; $324_{ten} = 1300_{six}$;

Assessment

- Formulate simple questions which the learners can illustrate on the board.
- Encourage the learners to formulate own questions based on the Activities 2.1, 2.2, 2.3 which they then discuss as a class.

Lesson 2: Change of number base (Changing from base 10 to any other base)

Lesson objective

By the end of the lesson, the learner should be able to convert any number in base 10 to any other base, any number to base 10 and any number from one base to another base.

Teaching Aids

- An abacus
- ICT tools i.e. calculator, computer

Introduction to the lesson

Introduce the lesson by requiring each learner to make a sketch of an abacus framework in their books as was used in the place value exercise. Point out and emphasize the fact that in base ten, we use digits from 0 to 9.

Learning Activities

Activities 2.4, 2.5

- Organise learners to work in pairs to do Activity 2.4 in the Student's Book.
- Move round the class as they work so that you can help those who may have a challenge.
- When the activity is done, ask the groups to present their findings to the rest of the class. This presentation should be done in a class discussion. Encourage those who may have questions or comments to give their opinions or ask their questions.

Synthesis

- When the activity is done, take the learners through a class discussion guided by the discussion of the Student's Book to ensure that they all understand the concept and the skills that were meant to be achieved.

- Summarize the learner’s findings, explaining the process of changing from base 10 to base 6 using Example 2.2, and converting from base 10 to base 8 using axample 2.3.
- Take the learners through Examples 2.2 and 2.3 in the Student's Book guiding them on how to convert numbers from base 10 to any base. Later guide them through Example 2.4. after discussing Activity 2.5 in the Student's Book.

Answers to Activity 2.4

$$\begin{array}{c|c|l} 8 & 725 & \\ 8 & 90 & r5 \\ 8 & 11 & r2 \\ 8 & 1 & r3 \end{array} \qquad : 725_{10} = 1325_8$$

Answers to Activity 2.5

(a) 1 has a value of $6^2 = 36$

2 has a value of $2 \times 6 = 12$

5 means 5 ones

(b) $6^2 + 2 \times 6 + 5 = 53$

(c) 53_{10}

Assesesment

Ask the learners to do questions 1,2 and 3 Exercise 2.1, in the Student's Book.

Lesson 3: Changing a number from one base to another

Lesson objective

By the end of the lesson, the leaner should be able to change a number from one base to another.

Introduction to the lesson

Introduce the lesson by reminding the learners how to convert numbers from base 10 to other bases and vice versa.

Learning Activities

- Organise learners in groups to do Activity 2.6 in the Student's Book.
- Move around as the leaners are doing the activities to identify those with difficulties and help them accordingly.
- Let the learners present their findings through class discussions and not down the errors made.

Synthesis

- Summarise the learners presentations by correcting them where they are wrong.
- Pass through Example 2.5 for the learners to understand the concepts in the Student's Book.

Answers to Activity 2.6

a) 47710 = 4 × 7 + 7 × 7

28 + 7

35_{10}

b)
$$\begin{array}{r|r|l}
5 & 34 & \\
5 & 6 & r4 \\
 & 1 & r1
\end{array} \qquad \therefore\ 34_{10} = 114_5$$

c) To convert any number from base x to base y, first convert the number from base x to base 10, then convert it from base 10 to base y by successive division by *y*.

Assessment

- Let the leaners do Exercise 2.3 in the Student's Book.
- Correct the exercise and guide all the learners to understand.

Lesson 4: Operations using different bases

Lesson objective

By the end of the lesson, the leaner should be able to perform the basic operations of addition, subtraction, using different number bases.

Materials

Abacus, calculator, chalk, pens, charts chalkboard.

Introduction to the lesson

Introduce the lesson by reminding learners of the properties of addition and subtraction of numbers in the decimal system. Let them give the properties and you verify them. Remind them that we never use a numeral equal to the base we are working with. Refer the learners back to the notes on bases and numerals earlier in this unit.

Learning Activities

Activity 2.7

- Let the learners work individually to do Activity 2.7 in the Student's Book.
- After the activity is done, ask learners to report on their findings through class discussion.
- Encourage them to illustrate their findings on the board.

Synthesis

- Using a board illustration verify the learners findings. Point out only obvious misunderstanding.
- Take the learners through the discussion following immediately after the activity on how to add numbers in the same base.
- In a class discussion take the class through Examples 2.6 and 2.7 in the Student's Book.

Answers to Activity 2.7

Below is the addition table for base 6.

+	0	1	2	3	4	5	10
0	0	1	2	3	4	5	10
1	1	2	3	4	5	10	11
2	2	3	4	5	10	11	12
3	3	4	5	10	11	12	13
4	4	5	10	11	12	13	14
5	5	10	11	12	13	14	15
10	10	11	12	13	14	15	20

$15 - 10 = 5$; $13 - 5 = 4$; $12 - 3 = 5$

The table is using base 6.

$15_6 - 10_6 = 5_6$

$13_6 - 5_6 = 4_6$

$12_6 - 3_6 = 5_6$

Assessment

Ask learners to do Exercise 2.5 in the Student's Book. Move around the classroom to assist those who are experiencing difficulties.

Lesson 5: Multiplication and Division

Lesson objective

By the end of the lesson, the learner should be able to multiply and divide numbers using different number bases.

Introduction to the lesson

Introduce the lesson by asking learners to make a multiplication table for base 10. Use the table to formulate simple division questions.

Learning Activities

(a) Activity 2.8

Learners should work individually to complete their multiplication table.

After they have completed the tables guide them through the activity inviting the learners to report their findings. Encourage the learners to discuss their findings.

(b) Activity 2.9

- Ask the learners to do Activity 2.9 in the Student's Book.
- Encourage the learners to do this activity individually. To ensure that all actively participate.
- When the activity is done, encourage the learners to present their findings in a class discussion.

  Verify that the learners conclusions are correct and accurate.

Synthesis

- Take the learners through Example 2.8 in the Student's Book.
- Take learners through Example 2.9 in the Student's Book.

Answers to Activity 2.8

(b) Table 2.5 shows the required table of multiplication.

x	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8
2	0	2	4	6	8	11	13	15	17
3	0	3	6	10	13	16	20	23	26
4	0	4	8	13	17	22	26	31	35
5	0	5	11	16	22	27	33	38	44
6	0	6	13	20	26	33	40	46	56
7	0	7	15	23	31	38	46	54	62
8	0	8	17	26	35	44	53	62	71

Table

(a) Base 9

(c) (i) $x = 2$ (ii) $x = 7$ (iii) $x = 6$

(d) answers to this part will vary.

Answers to Activity 2.9

(a) (i) $a = \frac{c}{b}$ (ii) $b = \frac{c}{a}$

(iii) in part (i) divide both sides of the equation by $\underset{\sim}{b}$

in part (ii) divide both sides by $\underset{\sim}{a}$

(b) (i) $2_{six} = \frac{14_{six}}{5_{six}}$ (ii) $5_{six} = \frac{14_{six}}{2_{six}}$

(iii) m (i) divide by 5_{six} and in (ii) divide by 2_{six}

(c)

×	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	10	12	14
3	0	3	10	13	20	23
4	0	4	12	20	24	32
5	0	5	14	23	32	41

From the table, we confirm that $2_{six} \times 5_{six} = 14_{six}$

Similarly, $2_{six} = 14_{six} \div 5_{six}$ and $5_{six} = 14_{six} \div 2_{six}$

(d) Answers to this part will vary with different learners.

Emphasize to the learners that division using bases other than base ten is not as straightforward as multiplication. Let them know that they can also convert both the divisor and the dividend to base ten, carry out the division and convert the answer back to the required base.

Assessment

Ask the learners to do Exercise 2.6 in the Student's Book.

Lesson 6: Special bases: Base 2 (binary) and base 12 (duodecimal)

Lesson objective

By the end of the lesson, the learner should be able to perform operations using special bases 2 and 12.

Introduction

Introduce this lesson by impressing on the learners that working using base 2 or base 12 uses the same principle as any other base. Only in base two there are only two numerals, 0 and 1 while in base 12 there are eleven numerals i.e. 0 to 9 then 10 and 11 and since we cannot use 10 and 11 as they are we have to define a symbol for 10 and eleven. Generally we use a. for 10 and b. for 11. Any other symbol can be used provided they are defined as 10 and eleven respectively.

Learning Activities

Activities 2.10, and 2.11

- Ask the learners to work individually to convert base ten numerals to base two in Activity 2.10 in the Student's Book.
- When they are through with the activity ask them to present their findings in a class discussion. (you need to time the learners so that they do not spend a lot of time on this simple exercise).
- Ask the learners to carryout Activity 2.11 for base twelve numerals in the Student's Book.

Synthesis

- Summarise the outcome of the activity using class findings. Verify their findings and if any learner has a challenge deal with them individually.
- In a class discussion take the learners through Examples 2.10 2.11 and 2.12 on performing operations on the two bases in the Student's Book.

Answers to Activity 2.10

(a) The digits used in base 10 system are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

(b) Now convert base 10 numerals to base 5.

In both base 10 and base 5, the numerals 0 to 4 are the same i.e. $0_{10} = 0_5$ $1_{10} = 1_5$

$2_{10} = 2_5$ $3_{10} = 3_5$ and $4_{10} = 4_5$

$5_{10} = 10_5$

$6_{10} = 11_5$

$7_{10} = 12_5$

$8_{10} = 13_5$

$9_{10} = 14_5$

The required table

Base 10	0	1	2	3	4	5	6	7	8	9
Base 5	0	1	2	3	4	10	11	12	13	14

(a) Answers to Activity 2.11 vary with the individual learners.

(b) The word dozen means a group of twelve. This could be items or persons. Example like a dozen of books, pens, pencils, plate, etc.

Assessment

- Ask them to do Exercise 2.8 in the Student's Book.

Lesson 7: Solving equations involving numbers in other bases

Lesson objective

By the end of the lesson, the learner should be able to solve equations using different number bases.

Introduction to the lesson

Introduce this lesson by asking the learners to suggest how to solve equations in one unknown in decimal system. Let them give you the properties of equations i.e. what can be done and what cannot be done. You can give them a simple equation to be used in this discussion.

Learning Activities

Activity 2.12

- Organize the learners to work in pairs to do Activity 2.12 in the Student's Book.
- When the activity is done, ask them to present their solutions in a class discussion.

Synthesis

- From the class findings, summarise as you verify their answers.
- Now discuss the argument presented immediately after Activity 2.12 in the student's Book.
- In a class discussion take learners through Examples 2.13 and 2.14 in the Student's Book.

Activity 2.12

Working in base 10 system, the solutions are as follows:

(a) (i) $x - 6 = 10$

$x = 16$

(ii) $x - (-3) = -5$

$x + 3 = -5$

$x = -5 - 3$

$x = -8$

(iii) $\frac{1}{4}x + 3 = 5$

$\frac{1}{4}x = 2$

$x = 8$

Now, working in base 6, 6 and 10 in equation (i) must first be converted to base 6 i.e. $6_{10} = 10_6$; $10_{10} = 14_6$

(b) (i) $x - 6 = 10 \Rightarrow x - 10_6 = 14_6$

$x = 14_6 + 10_6$

$= 24_6$

(ii) $x - (-3) = -5$

$x + 3 = -5$

$x = -12_6$

(iii) $\frac{1}{4}x + 3 = 5$

$\frac{1}{4}x = 5 - 3 = 2_6$

$x = 2_6 \times 4_6$

$= 12_6$

Answers to Activity 2.12

(i) $x = 16$ (ii) $x = -8$ (iii) $x = 8$

In base six:

(i) $x = 24_{six}$ (ii) $x = -12_{six}$ (iii) $x = 12_{six}$

Assessment

Ask the learners to do Exercise 2.9 given in the Student's Book.

Summary of the Unit

- Ask the learners to take the whole class through the different concepts highlighted in the unit summary given in the Student's Book. Take the learners through a question and answer session to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit. Let them discuss and solve the unit focus activity stated at the beginning of this unit. Ask them probing questions to gauge their understandings on this topic.

Unit assessment

Ask the learners to do the Unit 2 Test at the end of the unit in the Student's Book. Mark their work to assess their areas of weaknesses and strengths. From the result, give extra remedial exercise to the weaker students while giving the extended exercise to the learners who are comfortable with the rest of the exercise.

Additional information to the teacher

The hindus first developed a system of numerals using a different symbol for each of the numbers. These single numerals which are now written as 1,2,3,4,5,6,7,8,9, are called digits.

In this system of numbers, the position of any digit is very important because it determines the value of a given number. In the Hindus/ Arabs system which we use today, ten was used as a suitable groups for counting, the choice often must have been due to an obvious reasons that most human have ten fingers on our hands. If we hand, say 8 fingers or 6 etc most probably we would be counting in group. 8 or 6. So, number bases simply refers to the size of a group anybody may use for counting. Numbers such as 56_{eight}, 56_{nine} and 56_{ten} represent different systems of numbers i.e. 56_8 means 5 groups of 8 plus 6 ones.

569 means 5 groups of 9 and 6 ones and

56_{10} means 5 groups of 10 and 6 ones.

Remember the importance of position of a digit in a number. It determines the value of that digit. Emphasize the importance of place value of a digit in a number. Remember we can operate on a number using different number bases, provided we remember that no digit equal to a number base can be used. It is also possible to change a number from one base to another. Suppose a number is in base *m*, we can change to another base *n* by first changing to base 10 and then from base 10 to base *n*.

The original computer contained a large number of electric circuits. In this system, no current in a circuit corresponded to the digit 0 while a current flowing corresponded to the digit 1. No other symbol system were used so this is an application of base two systems.

The arithmetic involving the two digits 0 and 1 is called **binary**.

What is the result of adding 1 to 1 in binary? Since the use of digit equal to the base is not allowed, we cannot use 2 in binary.

Thus $1_{two} + 1_{two} = 10_{two.}$

Meaning one group of 2 and no unit.

This binary base is referred to as a special base.

There is another special base called base twelve, known as duodecimal system. Think of an example where units can be counted in groups of 12. Ask the leaners to think of a situation where base 12 is used in packaging for business.

What does the word dozen mean?

Remember, when we use a base greater than 10, the values from 10 and above must be represented by a single symbols that you must create and define. For example, in base 12, the digits are 0,1,2,3,4,5,6,7,8,9,$\underline{10}$,$\underline{11}$. We must create a symbol for 10 and 11 and define it. So, 10 can be represented by a letter A and 11 by letter B. so the digits in base 12 are 0,1,2,3,4,5,6,7,8,9, AB. As in base 10 we can solve equations using different number bases.

Remedial Exercise

1. What does 2 mean in:

   (a) 32_{six}? (b) 25_{six}? (c) 231_{six}?

2. Write down in words the meaning of 147_{eight}
3. (a) Express 415_{eight} as a number in base 10.

   (b) Express 85_{ten} as a number in base six.

4. Evaluate: $332_{six} + 25_{six}$
5. Evaluate: $52_{eight} - 23_{eight}$
6. Multiply: $35_{six} \times 23_{six}$
7. Divide: (a) 32_{six} by 5_{six}

   (b) 24_{six} by 4_{six}

8. Calculate the following in binary.

   (a) 101 + 10101 (b) 1001 – 101

Extended Exercise

1. Convert to the binary system.

   (a) 18_{ten} (b) 65_{seven}

2. (a) Calculate: $1011_{two} \times 101_{two}$ (b) Convert 1000010_{two} to the decimal
3. Convert 245_{twelve} to base 10.
4. Divide the following binary.

   (a) 1000001÷101 (b) 10010110÷110

5. Evaluate given that A=10 and B=11.

   (a) $A7_{twelve} + 5_{twelve}$ (b) Convert 1705_{ten} – to base twelve

6. Invent and describe any necessary symbols to work out.

   (a) $859_{fifteen} + 268_{fifteen}$ (b) $A356_{15} - 65B7_{15}$

7. Solve for x in $33_X - 24_X = 6_{X.}$

Answers

Exercises 2.1 (Student's Book)

1 . a) 200_5 b) 121_5 c) 1411_5

2. a) 101_9 b) 231_9 c) 170_9 d) 405_9

3. a) 10010001_2 b) 40032_6 c) 1100100000_2

d) 1672_8 e) 200222_3 f) $8A8_{12}$ g) 380_{16}

Exercise 2.2 (Student's Book)

1. a) 77 b) 9 c) 348 d) 5 e) 306

   f) 47 g) 110

2. a) Invalid b) Valid c) Invalid d) Invalid

Exercise 2.3 (Student's Book)

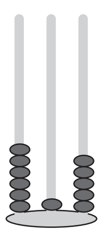
1. a) 211_7 (b) 232_7 (c) 14_7 d) 101_7
2. 140_5
3. 6
4. a) 2031_6 b) 451 c) 551_9 d) 111000011_2 e) 13003_4

Exercise 2.4 (Student's Book)

1. (a) 0, 1, 2, 3, 4, 5 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 30, 31 (base 6 numeral)
   (b) 0, 1, 2, 3, 4, 5, 6 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25 base 7 numerals
   (c) 0, 1, 2, 3, 4, 5, 6, 7 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23 base 8 numerals
2. (a) Ones (b) tens
   (c) hundreds (d) from extreme left: ten thousands, thousands, hundreds.
3. (a) two six-sixes and three ones
   (b) three - four fours and 2 two ones
   (c) one six and five ones
   (d) three eight cubed, two eight eights one eight and five ones.
4. (a) 2031_6 (b) 451 (c) 551_{nine}
5. (a) 106 (b) 121 (c) 135 (d) 612
6. 140_{five}
7. (a) One group of three plus two units.
   (b) two groups of four and one unit
   (c) One group of 5^2 plus 4 groups of five and 2 units
   (d) One group of nine squared eight groups of nine and no units
8. (a)

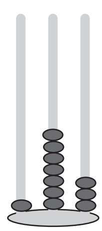
Base 5

(b)



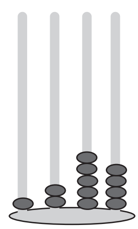
base 7

(c)



base 8

(d)



base 10

9. 343_{five}

Exercise 2.5 (Student's Book)

1. (a) 230_8 (b) 120_8 (c) 741_8
2. (a) 2_{six} (b) 54_{six} (c) 12_{six} (d) 305_{six}
3. (a) 217_{nine} (b) 6_{nine} (c) 8_{nine} (d) 112_{nine}
4. (a) base 8 (b) base 8 (c) base 5 (d) base10
   (e) base 9 (f) base 7
5. (a) base 10 (b) (i) base 9 (ii) base 9 (iii) base 9
   (iv) 177 + 19 should be 177 + 20 = 207_{nine}

Exercise 2.6 (Student's Book)

1.

x	0	1	2	3	4	5	6	7	10
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	10
2	0	2	4	6	10	12	14	16	20
3	0	3	6	11	14	17	22	25	30
4	0	4	10	14	20	24	30	34	0
5	0	5	12	17	24	31	36	43	50
6	0	6	14	22	30	36	44	52	60
7	0	7	16	25	34	43	52	61	70
10	0	10	20	30	40	50	60	70	100

   (a) 6_8 (b) 7_8 (c) 4_8 (d) 4_8
2. (a) 205_{six} (b) 252_{six} (c) 41_{six} (d) 23_6
3. (a) 32_4 (b) 132_4
4. (a) 332_8 (b) 135_6

Exercise 2.7 (Student's Book)

1. (a) 11000_{two} (b) 1000100_{two} (c) 110101_{two} (d) 1000010_{two}
2. (a) 1010_{two} (b) 1010_{two} (c) 1101_{two} (d) 1000110_{two}
3. (a) 1111_{two} (b) 11000011_{two} (c) 111111_{two} (d) 1100010_{two}
4. (a) 1110_{two} Rem 1_{two} (b) 101101_{two} Rem 100_{two} (c) 1100_{two} Rem 111_{two}
   (d) 110111_{two} (e) 10010000_{two} Rem 1
5. 10_{two}, 10111_{two}
6. (a) 10010_{two} (b) 111011_{two} (c) 101111_{two} (d) 11101_{two}
7. 112_4
8. (a) 100110_{two} (b) 10111_{two}

Exercise 2.8 (Student's Book)

1. (a) 115 (b) 131 (c) 118 (d) 139
2. (a) 188_{12} (b) $B101_{12}$
3. (a) 11_{12} (b) $32A_{12}$
4. (a) $93731A_{12}$ (b) 7472_{12}
5. $A6_{12}$
6. $BA1_{12}$
7. 173_{12}; 231_{10}
8. (a) 500_{12} (b) 150_{12} (c) 534_{12} (d) 407_{12} (e) 89_{12}

Exercise 2.9 (Student's Book)

1. (a) $x = 9$ (b) $x = 24$
2. $x = 65$
3. (a) $x = 12$ (b) $x = 12$
4. $x = 8$
5. (a) 6 (b) 6 (c) 6
6. (a) 9 (b) $x = 7$
   (c) 8 (d) 8
7. (a) 885 (b) 1311_9

Unit 2 Test (Student's Book)

1. 4042_{six}
2. (a) 1000001_{two} (b) 111100_{two}; 60_{ten}
3. $2n^2 + 3n + 0n$
4. $n = 7$
5. (a) $101\frac{11}{100_{two}}$
   (b) $21\frac{1}{100_{five}}$
6. Base 7
7. t = 0; r = 1, sum = 100001_{two}
8. (a) 1101_{six} (b) 15243_{six} (c) 12010_{six}
9. (a) 689_{12} (b) 146_{12} (c) 540_{12} (d) $37A_{12}$
   (e) $6B_{12}$ (f) $A1_{12}$ (g) $AA0_{12}$
10. x = 12

Remedial Exercise

1. (a) 2 ones (b) 2sixes (c) 2 six sixes 2. One eight eight four eights and seven

3. (a) 269 (b) 221_6 4. 401_6 5. 27_8 6. 1333_6

7. (a) 4_{six} (b) 4_{six} 8. (a) 11010 (b) 100

Extended Exercise

1. (a) 10010 (b) 101111

2. (a) 110111 (b) 66

3. 341

4 (a) 1101 (b) 11001

5 (a) Ac 2 (c) 1191D

7. x = 7

UNIT 3 ALGEBRAIC FRACTIONS

Key unit competence

By the end of this unit, the learner should be able to perform operations on rational expressions, and use them in different situations.

Learning objectives

Knowledge and understanding

- Define an algebraic fraction.
- State the restriction on the variable in algebraic fraction.
- Recognise the rules applied to addition, subtraction, multiplication, division and simplification of algebraic fractions.

Skills

- Perform operations on algebraic fractions.
- Solve rational equations with linear denominators.
- Simplify algebraic fractions.

Attitudes and values

- Develop clear, logical and coherent thinking while working on algebraic fractions.
- Show patience, mutual respect, tolerance, team spirit, and curiosity in group activities while solving and discussing mathematical situations involving algebraic fractions.

Pre-requisites to this unit

For the learners to acquire the learning objectives envisaged in this unit with ease, they need to have acquired the following: -

- Define a fraction with reference to a numerator and a denominator.
- Finding and LCM of numbers to perform operations on simple fractions.
- Describing the condition under which a fraction does not exist i.e. any number which cannot be used as a denominator in a fraction.

Cross-cutting issues to be addressed

The specific cross cutting issues to be addressed.

- Standardisation of culture: learners should be able to solve and apply fractions accurately to the required standards.

- **Inclusive education:** Learners with physical disabilities should be helped so as to fit in the society of other learners.

Generic competences

The specific generic competences to be addressed in this unit are:

- **Ability to work as a team*:*** This competence will be achieved when you organize learners to work in groups. This will promote the spirit of cooperation, respect of other peoples ideas and opinions and sharing of ideas and knowledge. This will also promote tolerance and patience with each other.
- **Communication skills*:*** Working in groups and reporting findings requires a group leader and a secretary. In this case, learners improve their communication ability during the group discussion and reporting.
- **Problem solving*:*** In doing activities and exercises in the Student's Book, the skills of solving problems are developed and refined.

Vocabulary/key words

In the course of learning the concepts in this unit the learners will discover the meaning of terms such as:

- Restriction on variables.
- Prime algebraic expression.
- Reciprocal of a number.
- Equivalent fractions.
- Rational equations.

Guide the learners to understand the meanings of these phrases/expressions in this unit.

Guidance on the problem statement

To challenge the learners and give them a reason to study this unit. Organise them to do the introductory unit focus activity outlined in the Student's Book.

• The activity involves:

- Identifing value(s) of x for which the fraction is not defined.
- Finding LCM of given groups of numbers.
- Performing operations on given fractions, including divisions by. This activity could be done individually as a take away quiz.

Hence the need to actively partcipate in all the lesson for the unit to acquire such skills.

It is likely that most learners may not get the correct results as it is the first time they are working with algebraic fractions. Appreciate the teachers for their efforts.

- Emphasize to the learners that the concepts summarized in this unit activity points generally to the concepts dealt with in this unit.

Answers to unit focus activity

(a) (i) $\frac{-1}{2}$; $^{-}1$, undefined / does not exist; 1; $\frac{1}{2}$

(ii) Yes, when $x = 2$, the denominator of $\frac{2}{2x-4}$ becomes zero. Therefore the $\frac{2}{0}$ fraction does not exist / undefined.

(b) (i) 12 (ii) $2x(x + 3)$

$1\frac{1}{12}$; $\frac{1}{6}$; $\frac{5x+9}{2x(x+3)}$; $\frac{3}{2x}$

(c) (i) $\frac{1}{4}$; $\frac{1}{9}$; $\frac{9}{16}$

(ii) 1 ; 1 ; 1 ; 1 All answers are equal to 1.

(iii) 1 ; 1 ; 1 ; 1

(iv) $\frac{1}{4} \times \frac{4}{1}$; $\frac{1}{5} \times \frac{5}{1}$; $\frac{2}{3} \times \frac{3}{2}$

d) In general, $\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c}$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ where a, b, c and d are integers.

Attention to special needs

- You should ensure that the needs of all the learners are addressed. These special needs could be physical, psychological or emotional.
- For gifted learners who may be working ahead of their class, provide them with appropriate extra problems to keep them busy.
- For slow learners identify them and address their needs accordingly. Organize remedial work for them.

Lesson No.	Lesson Title	No. of periods
1	Defining and identifying algebraic fractions	2
2	Simplification of monomial fractions	2
3	Simplification of fractions whose denominator/ or numerator is a binomial or a quadratic expression	4
4	Addition/subtraction of algebraic fractions with linear denominator	3
5	Multiplication of fractions	2
6	Division of fractions	4
7	Solving rational equations with linear denominators	3
8	Unit assessment	1

Lesson Development

Lesson 1: Definition and examples of algebraic fractions

Lesson objective

By the end of the lesson, the learner should be able to identify and describe examples of algebraic fractions.

Teaching Aids

Calculators, writting materials eg. pens, chalk, pencils and chalkboard.

Introduction to the lesson

Introduce the lesson by asking the learners to define a fraction and to distinguish between a denominator and a numerator. Ask them to name a number which a denominator cannot take.

Learning Activities

Activity 3.1

- Organise the learners in groups to do Activity 3.1 in the Student's Book on the existence of a fraction.
- Ask the learners to identify algebraic fractions among the given fractions and explain why some are not.
- Using the given fractions, ask the learners to explain the conditions under which each of the fractions is meaningful.

Synthesis

- Through a class discussion on the findings of the activity guide the learners to summarise their results, including the comments given in the Student's Book immediately after the activity.
- Emphasize that in any algebraic fraction, any value of the variable that makes the denominator zero is called a restriction on the variable.
- Through a class discussion take the learners through Example 3.1 in the Student's Book.

Answers to Activity 3.1

1. (i) $\frac{3y}{1-x}$, $\frac{5}{y+4}$, $\frac{x+y}{2}$, $\frac{5x-6}{x+3}$ are algebraic fractions.

2. (i) $x + 3$ can only be equal to zero if $x = -3$

   i.e. $x + 3 = 0$ means

   $x + 3 - 3 = 0 - 3$ subtracting from both sides

   $x = -3$

   (ii) $y + 4$ equal zero only if $y = -4$

   i.e. $y + 4 = 0$ means

$y = -4$ ……… subtracting 4 from both sides.

(iii) $1 - x$ can only be equal to zero if $x = 1$

i.e. $1 - x = 0$ means

$-x = -1$ …………….. subtracting 1 from both sides

$x = 1$ …………….. and multiplying both sides by -1

3. (i) If $x = -3$, $\frac{5(-3)-6}{-3+3} = \frac{-21}{0}$ undefined

But division by 0 is not possible

Therefore $\frac{5x-6}{x+3}$ exists only if $x \neq -3$

(ii) if $y = -4$, then $\frac{5}{y+4} = \frac{5}{-4+4}$

$= \frac{5}{0}$ undefined

Similarly $\frac{5}{y+4}$ exists only if $y \neq -4$

(iii) Similarly, $\frac{3y}{1-x}$ exists only if $x \neq 1$

Note that in this case, x can take any value.

Assessment

Ask the learners to do Exercise 3.1 in the Student's Book.

Lesson 2: Simplification of Fractions

Lesson objective

By the end of the lesson, the learner should be able to identify and use a common factor in a given fraction.

Introduction to the lesson

Introduce this lesson by asking learners to describe a common factor in a given set of numbers or monomial expressions. Ask them to create fractions in which both numerator and denominator have a common factor.

Learning Activities

Activity 3.2

- Organise the learners to individually do Activity 3.2 in the Student's Book.
- Ask them to explain the meaning of **simplest form** of a fraction.

Synthesis

- Having done the Activity 3.2, guide the learners through the class discussion immediately after the activity in the Student's Book.
- Ensure that the learners understand the meaning of **equivalent fraction**.
- Ensure that they understand how to create an equivalent fraction from a given fraction, either by division or by multiplication.
- Discuss with the learners Example 3.2 in the Student's Book.

Activity 3.2 discussion

A fraction is in the simplest form if both numerator and denominator have no common factor.

$\frac{8}{12} = \frac{8}{12} \div \frac{4}{4} = \frac{2}{3}$ (common factor 2)

$\frac{9}{12} = \frac{9}{12} \div \frac{3}{3} = \frac{3}{4}$ (common factor 3)

$\frac{12a}{15a^2} = \frac{4}{5a}$ (the common factor is 3a)

$\frac{8a^2c}{16a^2c^2} = \frac{1}{2c}$ (the common factor is $8a^2c$)

$\frac{12x^2y^4}{3xy^5} = \frac{4x}{y}$ (common factor is $3xy^4$)

Assessment

Ask the learners to do Exercise 3.2 questions 1 to 7 given in the Student's Book.

Lesson 3: Simplification of a fraction with numerator or denominator or both contain a binomial or quadratic expression

Lesson objective

By the end of the lesson, the learner should be able to: factorise binomial and quadratic expression and hence simplify fractions involving such denominators.

Introduction to the lesson

Introduce the lesson by asking the learners to suggest different methods of factorizing algebraic expressions. Use their suggestions to remind them how to factorize various binomial expressions and quadratic expressions.

Learning Activities

Activity 3.3

- Organise the learners to do Activity 3.3 in the Student's Book.
- Ask different groups to demonstrate and explain the procedure used to factorize the given expressions.

Synthesis

- Having done the Activity 3.3, guide the learners through the discussion in the Student's Book after the activity.
- Ensure that the learners are able to identify the difference of two squares as in (a) and (b).
- Remind them of the procedure of factorizing quadratic expressions of the form (i) $x^2 + bx + c$ and (ii) $ax^2 + bx + c$ where a, b and c are integers.
- Through a class discussion take learners through Example 3.3 in the Student's Book.

Answers to Activity3.3

(a) $x^2 - 81$ represents a difference of two squares.

Thus $\sqrt{x^2} = x$ and $\sqrt{81} = 9$.

The factors of a difference of two squares is the product of the sum and the difference of their square roots.

$\therefore\ x^2 - 81 = (x - 9)(x + 9)$

(b) $3x^2 - 3 = 3(x^2 - 1)$ (3 is a common factor of the two terms

$= (3(x - 1)(x + 1)$ since $x^2 - 1$ is a difference of two squares x^2 and 1

(c) $x^2 - 2x - 12$ is a quadratic expression. Possible factors of -12 are -3×4, 3×-4, -1×6, 1×-6, -1×12 or 1×-12

Of these pairs of factors, $3 \times (-4)$ is the suitable one.

$\therefore\ x^2 - x - 12 = x^2 - 4x + 3x - 12$

$x(x - 4) + 3(x - 4)$

$= (x - 4)(x + 3)$

(d) $2x^2 - 9x + 10$ is a quadratic expression in the form $ax^2 + bx + c$ where a = 2, $b = -9$, $c = 10$

$\therefore$ ac = 2 x 10 = 20 possible factors of 20 are

4×5, -5×-4, -2×-10, -1×-20, 1×20

$2x^2 - 9x + 10 = 2x^2 - 4x - 5x + 10$

$= 2x(x - 2) - 5(x - 2)$ factorise by grouping

$= (x - 2)(2x - 5)$

Assessment

Ask learners to do questions 8 and 9 from Exercise 3.2 in the Student's Book.

Lesson 4: Addition and subtraction of algebraic fractions with linear denominators

Lesson objective

By the end of the lesson, the learner should be able to add and subtract fractions with linear denominators.

Introduction to the lesson

Introduce the lesson by emphasizing

- The use of LCM in addition and subtraction of fractions.
- The procedure of LCM of a given set of numbers.
- Addition, subtraction and equivalent fractions.

Learning Activities

Activity 3.4

- Organize learners in pairs to do Activity 3.4 in the Student's Book.
- Ask learners to find the LCM of the given numbers. Ability to find LCM of numbers or algebraic expression is necessary to be able to develop the required skills in this unit.
- Ask the pairs to report their findings to the rest of the class.
- After doing the activity, guide them through the discussion after Activity 3.4 in the Student's Book.

Synthesis

From the above Activity and the class discussion after it, guide the learners to summarise the findings as follows: -

- To add or subtract fractions, the denominator of the fractions must be the same.
- Through a class discussion ensure that learners can find equivalent fractions, find and use LCM in addition and subtraction.
- Guide the learners through Examples 3.4 to 3.6 in the Student's Book.

Activity 3.4 discussion

1) Express each of the numbers 2, 6, 7 as a product of prime factors.

Thus $2 = 2 \times 1$, $6 = 2 \times 3$, $7 = 1 \times 7$

$\therefore$ the LCM of 2, 6 and 7 is $2 \times 3 \times 7 = 42$.
Since the LCM of 2, 6 and 7 is 42.

2) We can express $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{7}$ as fractions with common denominator 42

$$\frac{1}{2} = \frac{1 \times 21}{2 \times 21} = \frac{21}{42}\ ;\ \frac{1}{6} = \frac{1 \times 7}{6 \times 7} = \frac{7}{42}\ ;\ \frac{1}{7} = \frac{1 \times 6}{7 \times 6} = \frac{6}{42}$$

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{7c} = \frac{21}{42} + \frac{7}{42} + \frac{6}{42}$$

$= \frac{21 + 7 + 6}{42}$ (To add fractions with a common denominator add the numerators and express the sum over the common denominator.)

$= \frac{34}{42}$

$= \frac{17}{21}$ (divide both numerator and denominator by the common factor 2)

3) $\frac{1}{2a} + \frac{1}{6b} + \frac{1}{7c} = \frac{21bc + 7ac + 6ab}{42abc}$

Assessment

Ask the learners to do Exercise 3.3 in the Student's Book.

Lesson 5: Multiplication of algebraic fractions

Lesson objective

By the end of the lesson, the learner should be able to multiply fractions.

Introduction to the lesson

- Introduce the lesson by asking learners to identify common factors in given algebraic fractions.
- Ensuring that learners can factorise different expressions i.e. quadratics.
- Emphasizing the need to be able to factorize both numerator and denominator of fractions in order to facilitate multiplication or cancellation.

Learning Activities

Activity 3.5 discussion

- Organise learners in pairs to do Activity 3.5 in the Student's Book.
- Ask the learners to factorise the expressions both numerators and denominators.
- Ask the learners (groups) to write the given multiplication in factor form.
- Ask the learners to identify the factors in the numerator and denominator which can cancel out.
- Let each group demonstrate the working to the activity on the chalk board.

Synthesis

- Through class discussion take learners through the Activity and the argument just after the it.
- In a class discussion take learners through Example 3.7 in the Student's Book.

Activity 3.5 discussion and answers

Simplify: $\frac{2x^3 + 8x^2}{x^2 - 16} \times \frac{x^2 + 5x + 6}{3x^2 - 6x}$

We start by factorising individual expressions i.e.

(a) (i) $2x^3 + 8x^2 = 2x^2 (x + 4)$

(ii) $x^2 - 16 = (x - 4) (x + 4)$

(iii) $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$= x (x + 3) + 2(x + 3)$

$= (x + 3) (x + 2)$

(iv) $3x^2 + 6x = 3x (x + 2)$

(b) (i) $\frac{2x^3 + 8x^2}{x^2 - 16} = \frac{2x^2 (x + 4)}{(x - 4) (x + 4)}$ (Express product in factor form, then cancel by the common factor)

$\frac{2x^3 + 8x^2}{x^2 - 16} = \frac{2x^2}{x - 4}$

(c) $\frac{x^2 + 5x + 6}{x^2 + 6x} = \frac{(x + 3) \cancel{(x + 2)}}{3x \cancel{(x + 2)}} = \frac{x + 3}{3x}$

(d) $\frac{2x^3 + 8x^2}{x^2 - 16} \times \frac{x^2 + 5x + 6}{3x^2 + 6x} = \frac{2x^2}{(x - 4)} \times \frac{(x + 3)}{3x} = \frac{2x (x + 3)}{3 (x - 4)}$

Assessment

Ask the learners to do Exercise 3.4 in the Student's Book.

Lesson 6: Division of Algebraic fractions

Lesson objective

By the end of the lesson, the learner should be able to simplify given algebraic fractions by use of factors.

Introduction to the lesson

- Introduce the lesson by reminding the learner division is actually the reverse or opposite of multiplication.
- Ask the learners to related division of fractions to multiplication.

- Use their responses to convert division of fractions into multiplication in order to point the learners to the concept of reciprocal of a number.

  From our work on operations on integers, we know that division is the reverse or opposite of multiplication. For example we know that;

  $2 \times 3 = 6$ means that $6 \div 3 = 2$ and $6 \times \frac{1}{3} = 2$

  Also, $6 \div 2 = 3$ and $6 \times \frac{1}{2} = 3$

  $\therefore\ 6 \div 3 = 6 \times \frac{1}{3}$ and $6 \div 2 = 6 \times \frac{1}{2}$

  Similarly, $3 \times \frac{2}{3} = \frac{6}{5}$ means that $\frac{6}{5} \div 3 = \frac{2}{5}$ and $\frac{6}{5} \times \frac{1}{3} = \frac{2}{5}$

  $\frac{6}{5} \div \frac{2}{5} = 3$

  and $\frac{6}{5} \times \frac{5}{2} = 3$

  $\therefore\ \frac{6}{5} \div \frac{2}{5} = \frac{6}{5} \times \frac{5}{2}$ and $\frac{6}{5} \div 3 = \frac{6}{5} \times \frac{1}{3}$.

Lesson 7: Division of Algebraic fractions

Lesson objective

By the end of the lesson, the learner should be able to divide algebraic fractions.

Activity 3.6

- Organise learners in groups to do Activity 3.6. in the Student's Book.
- Ask the learners to factorize the expressions both numerators and denominators.
- Evaluate the given products.
- Having done the activity ask the learners to comment on their results.
- Generalise the product of the given pairs.

Synthesis

- In a class discussion take the learners through the argument in the Student's Book immediately after Activity 3.6.
- In a class discussion take leaners through Examples 3.8 to 3.10 in the Student's Book.

Activity 3.6 discussion and answers

(1) Answers will vary depending on the pairs of numbers listed by different groups.
Ensure that class discusses as many of their choices as possible.

Answers to Activity 3.6

1) $(1, 1), (2, \frac{1}{2}), (\frac{1}{4}, 4), (\frac{5}{2}, \frac{2}{5}), (\frac{7}{2}, \frac{2}{7})$

Assessment

Ask the learners to do Exercise 3.5 in the Student's Book.

Lesson 8: Solving rational equations

Lesson objective

By the end of the lesson, the learner should be able to find LCM of algebraic expressions and solve rational equations.

Introduction to the lesson

- Introduce the lesson by asking learners to suggest a method of finding the LCM of algebraic expression.
- Ask them how they can use the LCM in solving equations involving fractions.
- In this section, you will combine their skills of solving linear equations, with their skills of simplifying fractions to solve another type of equations i.e. equations involving algebraic fractions.

Learning Activities

- Organise learners to do Activity 3.7 in the Student's Book.
- Ask them to find LCM of the given groups of numbers.
- Ask the learners to list their answers, compare with those of other members of the class.
- When solving equations involving fractions, ask the learners to explain the use of the LCM.

Synthesis

- Having done the activity, guide the learners through a class discussion to help them revise the concept of LCM.
- To find the LCM, they must be able to factorise the denominators.
- To solve a rational equation, we multiply each term in the equation by the LCM to eliminate the denominator.
- In a class discussion, guide the learners through Examples 3.11 and 3.12 in the Student's Book.

Activity 3.7 discussion and answers

Find the LCM.

(a) 12, 16, 24

$12 = 4 \times 3 = 2^2 \times 3; 16 = 2 \times 8 = 2^4; 24 = 4 \times 6 = 2^3 \times 3$

LCM of 12, 16, 24 $= 2^4 \times 3 = 48$

(b) a, b LCM of a and b is ab

(c) $a - 3,\ 2a^2 - 18$

$(a - 3)$ has no factors

$2a^2 - 18 = 2\ (a^2 - 9) = 2(a - 3)\ (a + 3)$

LCM of $(a - 3)$ and $2a^2 - 18$ is $2(a - 3)\ (a + 3)$

(d) a^2, $(a + 1)$ since $a + 1$ has no factors and a^2 is a repeated factor.

LCM of a^2 and $a + 1$ is $a^2\ (a + 1)$

(e) LCM of b, 6, $3b^2$

b has no factors

$6 = 2 \times 3$

$3b^2 = 3 \times b \times b$

$\text{LCM} = 2 \times 3 \times b^2$

$= 6b^2$

Answers to Activity 3.7

(a) LCM 48 (b) ab (c) $2(a - 3)\ (a + 3)$ (d) $a^2(a + 1)$ (e) $6b^2$

Assessment

Ask the learners to do Exercise 3.6 in the Student's Book.

Summary of the Unit

- Take the whole class through the different concepts highlighted in the unit summary given in the Student's. Take the learners through a question and answer session to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit. Let them discuss and solve the unit focus activity stated at the beginning of this unit. Ask them probing questions to gauge their understanding on this topic.

Unit assessment

Ask the learners to do the Unit 3 Test at the end of the unit Student's Book. Mark their work in order to identify their areas of weaknesses and strengths. From the result, give extra remedial exercise to the weaker students while giving the extended exercise to the learners who are comfortable with the rest of the exercise.

Additional information to the teacher

An importance characteristic of Mathematics is the potential of building on the skills already learned, in order to learn further skills.

When sampling rational expressions with monomial denominators, the skills used in arithmetic are extended to algebra. In arithmetic we deal with numbers. In algebra we deal with algebraic expressions.

Example

Arithmetic $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

Algebra $\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{x+y}{xy}$

Expressions such as $\frac{x+y}{xy}$ and $\frac{x^2+6x+5}{x+1}$ are examples of rational expressions. Since division by zero is not permissible, the denominat of rational must not be equal to zero. When working with rational expression, you must note the restrictions on the variables. For example,

in $\frac{x+y}{xy}$, $\begin{matrix} x \neq y \\ x \neq 0 \end{matrix}$; and in $\frac{x^2+6x+5}{x+1}$, $x+1 \neq 0$ or $x \neq -1$

Before you simplify rational expressions, you must be able to recognize which skills to use.

- Look for common factor in both numerators and denominators.
- Check yours factors to see if any of them can factor again.
- Then cancel with the common factor.

Simplifying by addition and or subtraction.

- Factorize both numerators and denominators.
- Cancel by common factors.
- Express both or difference and sum with a single denominators.
- If need be express the fraction with common denominators which is the LCM of the individual denominators.

These skills of adding, subtracting rational expressions prepare us to solve rational equations.

In the section of solving rational equations, we combine skills, learned this far ie. finding LCM of the denominators, multiplying terms by the LCM to eliminate the denominators, then proceed and solve the resulting, simpler equations.

Remedial exercise

1. Identify the restrictions on the variables of each rational expression.
   (a) $\frac{x+2}{x}$ (b) $\frac{2x}{y-1}$ (c) $\frac{7}{3x+6}$
2. Find the least common multiples of:
   (a) $4x$, $6x$ (b) x, x^2 $2x$ (c) $2x$, $3x$
3. Simplify:
   (a) $\frac{3x}{2}+\frac{5x}{3}+\frac{2x}{6}$ (b) $\frac{2a}{x}+\frac{5a}{xy}-\frac{6a}{x^2}$
4. Simplify: $\frac{6x+1}{3}+\frac{x-2}{4}$ (b) $\frac{8x-2}{9}-\frac{x-6}{6}$
5. List the restrictions in:

(a) $\frac{5}{x} - \frac{4}{x-4}$ (b) $\frac{3}{x+1} - \frac{3}{x}$ (c) $\frac{6}{x-y} - \frac{7}{x+y}$

6. Evaluate: (a) $\frac{-24x^2}{45} \times \frac{-9}{15x}$ (b) $\frac{-3x^2y}{4a^2} \div \frac{-9xy}{-5a}$

7. Write the reciprocal of each of the following:

(a) $\frac{3y}{-x}$ (b) $\frac{-2x^2}{y}$ (c) $2y$ (d) $\frac{-2x}{3}$

8. Divide the following:

(a) $\frac{y}{x} \div \frac{a}{b}$ (b) $\frac{-x}{y} \div \frac{-a}{b}$ (c) $\frac{-6}{y} \div \frac{5}{k}$

Extended Exercises

1. What is your first step in solving the equations?

(a) $\frac{2}{3y} + \frac{3}{5} = \frac{1}{4y}$ (b) Solve the equation

2. Solve the equations.

(a) $\frac{2x-1}{3x} = 1$ (b) $\frac{x-3}{x+3} = 2$

3. Solve for x in:

$\frac{3}{x} + 2 = \frac{2x+3}{x-1}$

4. Simplify:

(a) $\frac{36}{y^2+2y} \times \frac{y+2}{9}$ (b) $\frac{x+1}{x^2-1} \div \frac{x}{x-1}$

5. Simplify:

(a) $\frac{4}{y^2-9} \div \frac{8}{3-y}$ (b) $\frac{x^2-y^2}{x^2+xy} \times \frac{(y+2)^2}{xy} \div \frac{x^2-y^2}{x^2+xy}$

6. Solve and verify by substituting what the restrictions are:

(a) $\frac{2x-1}{3x} = 1$ (b) $\frac{y+1}{y+2} = \frac{y+3}{y-4}$

7. One of the roots of the following equation is 3.

$\frac{x+6}{x} = \frac{x+k}{x-k} - k$. Find the other root.

Answers to exercises

Exercise 3.1

1. (a) $a \neq 0$ (b) $y \neq 2$ (c) -2 (d) $w \neq -3$ (e) $a \neq -3$

(f) $a \neq \frac{1}{2}$ (g) $x \neq 2$ (h) $w \neq 1$ (i) $a \neq 2$ (j) $a \neq -3$

2. (a) $y \neq 4, -8$ (b) $y \neq 3$ (c) $y \neq 3, -2$ (d) $a \neq \frac{2}{3}, 0$

3. (a) $x \neq 5$ (b) $y \neq 2, 1$ (c) $y \neq -3, \frac{7}{2}$ (d) $x \neq \frac{1}{2}, -1$

Exercise 3.2

1. $\frac{3x}{2y}$ 2. $\frac{r}{p}$ 3. $\frac{2}{3x}$

4. $\frac{3}{4x^2}$ 5. $\frac{2x}{3y}$ 6. $\frac{a}{d}$

7. (i) $\frac{a}{d}$

8. (a) (i) $\frac{(x-5)(x+3)}{4(x-5)}$ (ii) $x \neq 5$ (iii) $\frac{x+3}{4}$

   (b) (i) $\frac{2y-7}{(2y-7)(y+3)}$ (ii) $y \neq 3\frac{1}{2}$; -3 (iii) $\frac{1}{y+3}$

   (c) (i) $\frac{(2x-1)(3x+1)}{(2x-1)(2x+1)}$ (ii) $x \neq \frac{1}{2}, \frac{-1}{2}$ (iii) $\frac{3x+1}{2x+1}$

   (d) (i) $\frac{(x-3)(x-2)}{x-3}$ (ii) $x \neq 3$ (iii) $x-2$

   (e) (i) $\frac{(y-6)(y-1)}{y-6}$ (ii) $y \neq 6$ (iii) $y-1$

   (f) (i) $\frac{(x-5)(x-4)}{x-5}$ (ii) $x \neq 5$ (iii) $x-4$

9. (a) (i) $\frac{(x-5y)(x-5y)}{x+5y}$

   (ii) (x, y) such that $x \neq -5y$ or $y \neq \frac{-1}{5}x$

   (iii) $\frac{(x-5y)(x-5y)}{x+5y}$

   (b) (i) $\frac{(2x-y)(3x-y)}{(2x-y)(3x+y)}$ (ii) (x, y) such that $2x \neq y$ and $y \neq -3x$ or $x \neq \frac{-1}{3}y$

   (iii) $\frac{3x-y}{3x+y}$

   (c) (i) $\frac{(x-y)(x+y)}{(x-y)(3x+y)}$ (ii) (x, y) such that $x \neq y$ and $y \neq -3x$ or $x \neq \frac{-1}{3}y$

   (iii) $\frac{x+y}{3x+y}$

   (d) (i) $\frac{(2x-y)(x+y)}{(2x-y)(2x+y)}$ (ii) (x, y) such that $y \neq 2x$ and $y \neq -2x$ or $x \neq \frac{-1}{2}y$

   (iii) $\frac{x+y}{2x+y}$

Exercise 3.3 (Student's Book)

1. (a) $12x$ (b) 6mn (c) xy (d) $2ab$

   (e) x^2y (f) $4b$

2. (a) $\frac{11x}{4}$ (b) $\frac{11}{4a}$ (c) $\frac{19x}{6a}$ (d) $\frac{2nx-3mx}{mn}$

(e) $\frac{6y + 2}{xy}$ (f) $\frac{6x + 8y}{x^2y^2}$

3. (a) $\frac{-5}{2x}$ (b) $\frac{a + 3ax}{2x}$ (c) $\frac{x + 4}{4x}$ (d) $\frac{2x - 11}{6}$

4. (a) $\frac{-5}{2x}$ (b) $\frac{2an - 3bn + 2cm}{6mn}$ (c) $\frac{1}{3a}$

5. (a) $\frac{18x + 13}{12}$ (b) $\frac{13x + 5}{18}$ (c) $\frac{-17x + 25}{12}$

6. (a) $\frac{-x^2 + 5x - 3}{(x - 2)(x - 1)}$ (b) $\frac{33 - 5x}{(x - 3)(x + 3)}$ (c) $\frac{x + 3}{x - 1}$ (d) $\frac{-c}{b(b - c)}$

(e) $\frac{a - 4b}{2a - 4b}$ (f) $\frac{-4}{(a + 2)(a - 2)}$ (g) $\frac{b^2}{b - c}$ (h) $\frac{y^2}{y - z}$

7. (a) $\frac{7b - a}{(a - b)(a + b)}$ (b) $\frac{10 + 5y - 3y^2}{(y - 1)(y + 5)}$ (c) $\frac{-6}{(x + 3)(x - 3)}$

(d) $\frac{x - 5}{(x + 1)(x - 1)}$ (e) $\frac{-y}{x - y}$ (f) $\frac{2cd}{(c - d)(c + d)}$

(g) $\frac{rs}{r + s}$

8. (a) $\frac{yz + y^2 + z^2}{y^2z + yz^2}$ (b) $\frac{b^2 - 2bc - c^2}{b^2 - c^2}$ (c) $\frac{n^2 + 2n - 1}{n(n - 1)(n + 1)}$

9. (a) $r - 3$ (b) $-(y + z)$ (c) $-(y - 5)$

Exercise 3.4

1. (a) $\frac{4}{y}$ (b) $\frac{a + b}{a + 4}$ (c) $\frac{3}{4}$ (d) $\frac{3x^2}{(x + 1)^2}$

(e) $\frac{x - y}{y(x^2 - 2xy - y^2)}$ (f) 1 (g) $\frac{x}{y(x - 1)}$ (h) $\frac{1}{x}$

(i) $\frac{1}{2(x + y)}$

2. (a) $\frac{bm}{an}$ (b) $\frac{2x}{5}$ (c) $\frac{5m}{2}$ (d) -1

(e) $\frac{3a^2}{2b^4}$ (f) $6y$ (g) $\frac{x}{y}$ (h) $-x^2$

(i) $-4y$ (j) $\frac{-4}{3x}$

3. (a) $\frac{3(x + 1)}{x - 3}$ (b) $\frac{x(x - 2)(x + 4)}{(x - 1)(x + 1)}$

Exercise 3.5

1. (a) $\frac{x}{3y}$ (b) $\frac{y}{2x^2}$ (c) $\frac{-1}{2x}$ (d) $\frac{3}{2y}$

(e) $\frac{4}{5y}$ (f) $\frac{-5y}{3x}$

2. (a) $\frac{bx}{ay}$ (b) $\frac{-bx}{ay}$ (c) $\frac{2x}{21}$ (d) $-2y$

3. (a) $\frac{4xy}{5}$ (b) -1 (c) $\frac{-3}{20y}$ (d) $\frac{-3ab}{7}$

4. (a) $(x-3)^2$ (b) -1 (c) $\frac{1}{a-2b}$

5. (a) $\frac{-c}{d}$ (b) $\frac{y+1}{y-1}$

6. (a) $\frac{38}{51}$ (b) $\frac{2x(y+6x)}{9y}$ (c) $\frac{-4a+9b}{6(2a+3b)}$

7. (a) $\frac{38}{21}$ (b) $\frac{4a+9b}{6(2a+3\boldsymbol{b})}$

Exercise 3.6 (Student's Book)

1. (a) $\frac{-25}{36}$; $x \neq 0$ (b) $\frac{-20}{3}$ (c) $\frac{-3}{2}$; $x\neq 0$, $x \neq 1$ (d) $x = 1$ or $x = 2$ Remember that $x \neq 2 \therefore x = 1$

2. (a) 12 (b) 2 ; $x \neq 0$ (c) 3; $x \neq 0$ or -3 (d) $\frac{1}{2}$; $y \neq 2$ or 4

3. (a) $\frac{1}{6}$; $x \neq 0$ (b) 1, $\frac{-21}{2}$; $x \neq \pm 3$ (c) $\frac{8}{3}$ $a \neq 0, 1$
   (d) $x \neq \pm 1$ or 0 (e) $2\frac{2}{15}$; $x \neq 0$ (f) 11; $x \neq -2$

4. (a) $\frac{2}{9}$; $x \neq \pm\frac{1}{3}$ (b) 6 (c) $\frac{10}{13}$, -2

5. (a) $\frac{12}{5}$ (b) -12 (c) 2, 22.908, 1.091 (to 3 d.p)

Unit 3 Test (Student's Book)

1. (a) $\frac{47a}{60}$ (b) $\frac{13x}{12}$ (c) $\frac{x+5}{3}$ (d) $\frac{10x-3}{12}$

2. (a) $\frac{11a}{20}$ (b) $\frac{3x-2y+8}{6}$ (c) $\frac{14xy}{15y} = \frac{14x}{15}$ (d) $\frac{-x-9}{12}$

3. (a) $\frac{4}{y}$ (b) $\frac{5}{6b}$ (c) $\frac{3ab^2c^2d}{2}$ (d) $\frac{6a^2d^3}{bc}$

4. (a) $\frac{3b^2c}{a}$ (b) $\frac{21b^2d}{2ac}$

5. (a) $\frac{9}{4x}$ (b) $\frac{7m}{6x}$ (c) $\frac{x+2}{6}$ (d) $\frac{a+12b}{6}$

6. (a) $m = 3\frac{3}{4}$ (b) $x = -15$ (c) $x = \frac{1}{2}$ (d) $x = 13$

7. (a) $\frac{-8x}{25}$ (b) $\frac{4x^2}{x-4}$ (c) $\frac{4x}{5}$ (d) $\frac{3}{4y}$

8. (a) $\frac{-5b}{4}$ (b) $\frac{-a(a+b)}{a^2+b^2}$ (c) $\frac{x-1}{3x}$ (d) $\frac{-4}{2d+1}$

9. (a) $\frac{4y+3}{6y^2}$ (b) $\frac{x^2-6x+1}{4x^2}$

10. (a) $\frac{x+2}{3}$ (b) $\frac{10x+3}{5}$

11. (a) $\frac{5x}{12a}$ (b) $\frac{22}{51}$

12. (a) $\frac{1}{3x}$ (b) $x \neq 0, \pm 1$

13. (a) Multiply each item by the LCM to eliminate the denominator.
    (b) $x = -1$

14. (a) $x = \frac{-3}{2}$; $x \neq -1$ or $\frac{-4}{3}$ (b) $x = 2$; $x \neq -1$

Remedial Exercise

1. (a) $x \neq 0$ (b) $y \neq 1$ (c) $x \neq -2$

2. (a) $12x$ (b) $2x^2$ (c) $6x$

3. $\frac{7x}{2}$ (b) $\frac{(2axy + 5ax) - 6ay}{x^2y}$

4. (a) $\frac{27x-2}{12}$ (b) $\frac{13x+14}{18}$

5. (a) $x \neq 0, x \neq 4$ (b) $x \neq -1, x \neq 0$ (c) $x \neq y, x \neq -y$

6. (a) $\frac{8x}{25}$ (b) $\frac{-5x}{12a}$

7. (a) $\frac{-x}{3y}$ (b) $\frac{-y}{2x^2}$ (c) $\frac{1}{2y}$ (d) $\frac{-3}{2x}$

8. (a) $\frac{yb}{xa}$ (b) $\frac{xb}{ya}$ (c) $-\frac{6k}{5y}$

Extended Exercise

1. a) Find the LCM of the demonstration.

b) $y = \frac{-25}{36}$

2. (a) x=-1 (b) x = -9

3. $x = \frac{-3}{2}$

4. (a) $\frac{4}{y}$ (b) $\frac{1}{x}$

5. (a) $\frac{-1}{2\,(y+3)}$ (b) $\frac{(y+2)^2}{xy}$

6. $x = -1$, $x \neq 0$ (b) $y = -\frac{5}{4}$

7. $x = -2$

4 SIMULTANEOUS LINEAR EQUATIONS AND INEQUALITIES

Key unit competence

By the end of this unit, the learner should be able to solve problems involving simultaneous linear equations and inequalities.

Learning objectives

Knowledge and understanding

- Define simultaneous linear inequalities in two unknowns.
- Give examples of simultaneous linear inequalities in two unknowns.
- Show solution set of simultaneous linear equations and inequalities in two unknowns given their graphs.

Skills

- Solve graphically simultaneous linear equations and inequalities in two unknowns.
- Interpret graphical solutions of simultaneous linear equations and inequalities in two unknowns.
- Solve word problems leading to simultaneous linear equations.

Attitudes and values

- Appreciate how simultaneous linear equations in two unknowns are important to represent and solve mathematical word problems.
- Develop clear, logical, and coherent thinking while solving simultaneous linear equations and inequalities in two unknowns.
- Show patience, mutual respect, tolerance, team spirit, and curiosity in group activities while solving and discussing mathematical situations involving simultaneous linear equations and inequalities in two unknowns.

Pre-requisites to this unit

For learners to be able to understand the concepts in this unit, there is need to have acquired the following in their earlier work.

- Ability to solve simple simultaneous linear equations by algebraic methods as was learned in S2 unit 3 under the same heading.
- Ability to solve linear inequalities in one unknown as in S2 unit 3.
- Making tables of values of linear relations and using them to draw linear graphs.
- Interpreting and analyzing such graphs.

Cross-cutting issues addressed in this unit

Financial education

This unit involves some business related questions. Sensitize the learners on how they can use graphs to solve business related questions such as sales, profit and loss; cost of production, constraints for manufacturing etc.

Inclusive education

Ensure that all learners have equal opportunities to benefit and participate in the teachers instruction. Embrace all learners regardless of their physical or learning challenges.

Peace and value education

As the learners discuss through the various activities in the unit, the teacher to emphasize the importance of respecting each other's opinon whether they agree with it or not. Emphasize to the learners the need for peace and value education. These are a sure recipe for personal and societal development which ensures the success of the nation. All must endeavor to give a positive contribution to this.

Standardisation

This is where learners have to solve some axamples and exercises and graphs and able to draw the graphs accurately.

Generic comptences

The specific generic comptences to be addressed in this unit are;

- **Problem solving skills**: This competence will be achieved when the learners are involved in doing activities and solving equations and inequalities.
- **Team work**: When learners work in groups they will learn to share information, and tolerate each other, respect each other's opinion. This cultivates humility in the learner.
- **Communication and listening skills**: This will be developed when you require learners to work in groups and therefore the learner will make an effort to listen and to communicate with others.

Vocabulary/keywords

- Interpretation
- Independent and dependent
- Solution set
- Region

Guide the learners to understand the meanings of these words and how to use them appropriately in mathematics.

Ensure that learners identify and relate a region to the appropriate line depending on whether the line is solid (continuous) or is broken, attaching the appropriate inequality symbol to the required region.

Guidance on the problem statement

In order to guide the learners to the general direction of the unit and challenge them to be active during the lessons for this unit, organize learners and ask them to do the introductory unit focus activity in the Student's Book.

Unit Focus Activity

Materials

- Graph papers/books
- Geometrical instruments
- Calculators

(a) Ask learners to do the unit focus activity as follows.

  (i) To draw the graphs of the lines whose equations are given in the Student's Book;

  (ii) Find the coordinates of the point of intersection and describe those values with reference to x and y.

(b) Solve the given equations:

  $-2x + y = -1$ ....(i) $2y + x = 4$ ....(ii)

  Using different algebraic methods as suggested in the Student's Book.

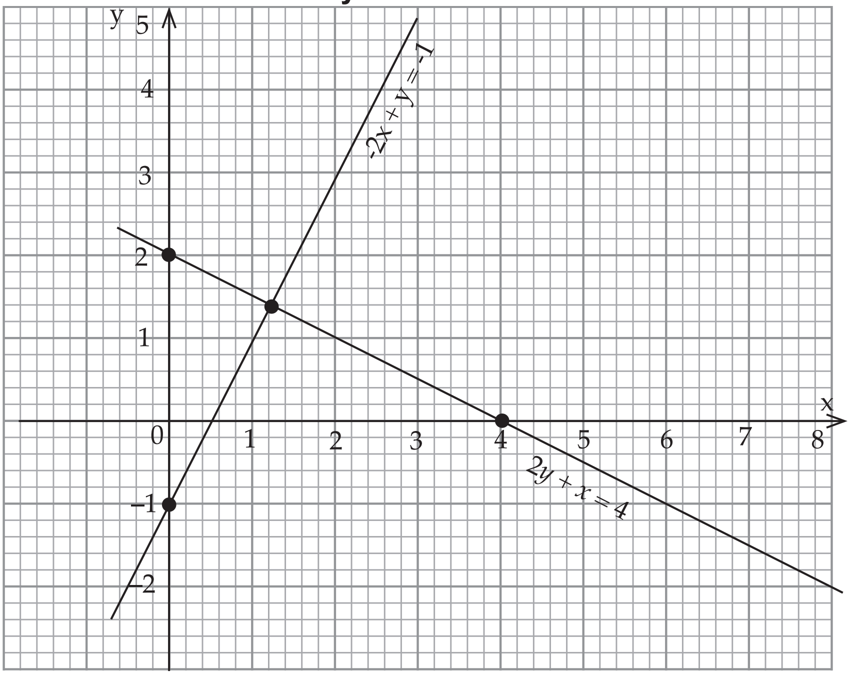
(c) Ask learners to form simultaneous equations using the situation described in the Student's Book. Ask them to solve the equations using different methods.

(d) Ask the learners to show the given inequalities in one graph and complete the activity as instructed in the Student's Book.

  - Using the learners observations, give them an overview of the concepts and skills that are going to be developed in this unit.
  - It would be ideal if this activity could be given as home work, to create more time when it comes to the class discussion on the same activity.

Answers to the unit focus activity

1a) (i)



(b) Point of intersection (1.2, 1.4).

This means $x = 1.2$ and $y = 1.4$ satisfy both equations simultaneously.

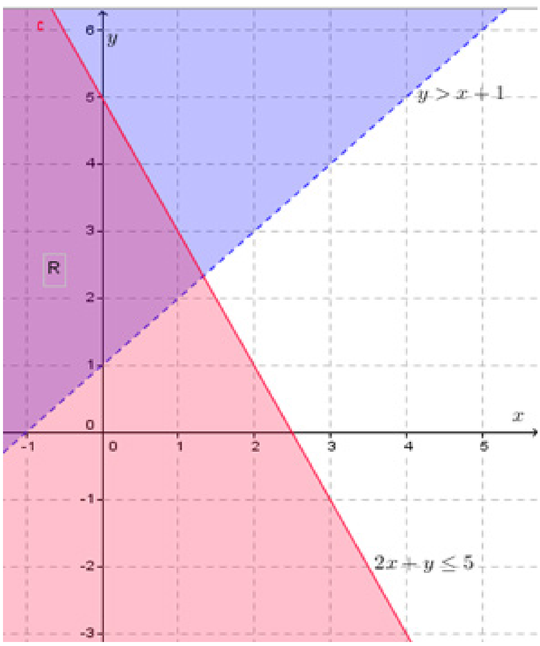
2. (a) $x = \frac{y+1}{2}$ (b) $2y + \frac{y+1}{2} = 4$

(c) $y = \frac{7}{5}$ (d) $x = \frac{6}{5} = 1\frac{1}{5}$

3. a) $3x+2y=4000$ (i) $x + 5y = 3500$ (ii)

$x = 1000$, $y = 500$ (x FRW is amount earned by men and y FRW amount earned by women.

4.



R is the required region.

The line $y = x + 1$ is broken because the inequality does not include the points on the line For the region $2x + y \leq 5$, the line is solid because the inequality includes the line.

Attention to special needs

- Ensure that learning is inclusive. All learners must feel part of class activity. Some learners may have physical needs, emotional and even psychological needs. Be alert and help them accordingly.
- In case of fast learners, identify them and provide additional more challenging work so that they have no opportunity of getting bored and getting into mischief.
- For slow learners, provide remedial work and guide them through to help them understand the concepts and so feel cared for.
- You may find some learners sleep in class, lack concentration, are generally unhappy etc. talk to them well and try to find out what is happening to them. You could find they are dealing with an issue they cannot handle alone.

List of lessons

Lesson No	Lesson Title	No. of previous
1	Introductory activity	2
2	Solving simultaneous linear equations graphically	3
3	Forming and solving simultaneous linear equations from word problems	2
4	Graphical representation of linear inequalities in one unknown	3
5	Simultaneous linear inequalities in one unknown	2
6	Linear inequalities in two unknowns	2
7	Simultaneous linear inequalities in two variables	2
8	Forming linear inequalities from graphs of inequalities	2
9	Unit assessment	1

Lesson Development

Lesson 1: Solving simultaneous linear equations graphically

Lesson objective

By the end of the lesson, the learner should be able to draw graphs of linear equations, form equations and inequalities from given situations.

Teaching Aids

- Classroom board with square grid, chalk including coloured chalk.
- Graph books/papers.
- Other writing materials

Introduction to the lesson

Introduce the lesson by asking learners to individually draw the graph of two simultaneous equation and use them to solve equations in the Student's Book. Check the learners' work to ensure that they are able to follow the instructions given in the activity.

This activity is important because most of the activities to follow in this unit will involve drawing of lines whose equations are given, and also representing inequalities graphically. Ensure that the learners have the skills and knowledge of drawing linear graphs, form equations and inequalities from given situations, represent inequalities graphically. This activity introduces the learners to the concepts to be developed in this unit.

Learning Activities

Activity 4.1

- Organise the learners to do Activity 4.1 in the Student's Book.
- Ask all the learners to make a table of values for the equations given in the activity and use the values to draw the graphs.
- Go round checking that they have drawn correct graph as instructed and that they are making the correct observation.
- As the groups to report their findings to the rest of the class.
- Through class discussion help the learners to relate the coordinates of the point of intersection of the lines with the solutions of the given equations.
- Take learners through the points to note listed after the activity.

Synthesis

Having done the activity the activity helps the learners to understand that;

- The intersection of two linear graphs gives the solution set of the equations.
- Guide the learners through the Example 4.1 in the Student's Book.

Assessment

- Ask the learners to do Exercise 4.1 in the Student's Book.

Answers to Activity 4.1

Tables 4.1 and 4.2 represent the required tables of values for $y = 2x + 1$ and $2y = x - 4$ respectively.

For $y = 2x + 1$

x	–3	–2	–1	0	1	2	3
y	–5	–3	–1	1	3	5	7

Table 4.1

$2y = x - 4$	x	–3	–2	–1	0	1	2	3
	y	–3.5	–3	–2.5	–2	–1.5	–1	–0.5

Table 4.2

- Fig 4.3 shows the graphs of the given equations.

2.

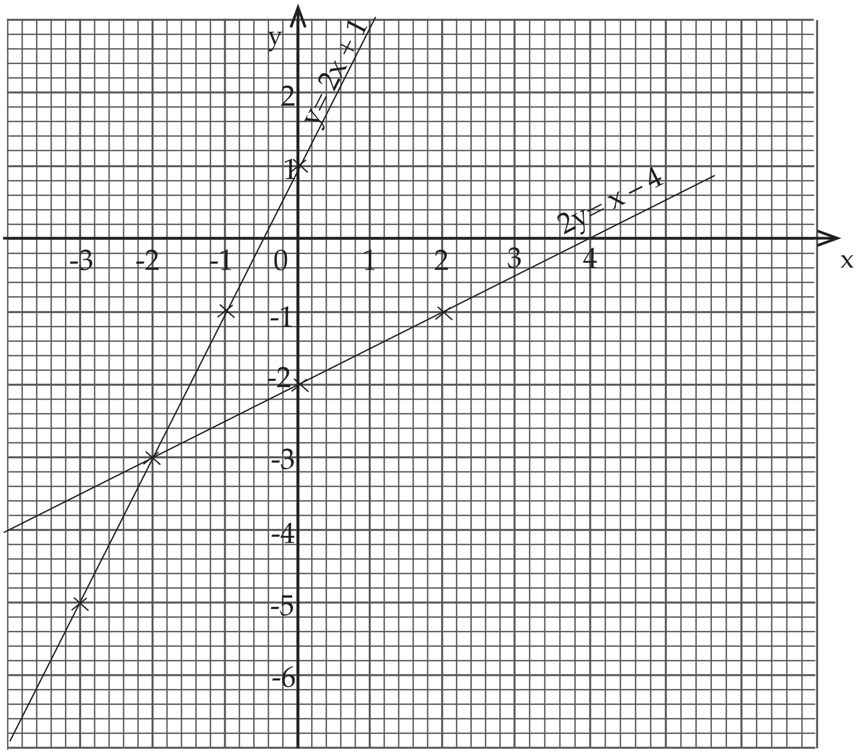


Fig 4.3

Note that for a linear graph, the table of values need not to contain more than three points any line is defined by two points only.

- The lines intersect at the point (–2, –3)
- At the point of intersection, $x = -2$, and $y = -3$
- The values of x and y at the point of intersection represent the solution of the simultaneous equations $y = 2x + 1$ and $2y = x - 4$.

Lesson 2: Solving problems involving simultaneous equations

Lesson objective

By the end of the lesson, the learner should be able to form and solve simultaneous linear equations from word problems.

Teaching Aids

- Graph books/papers
- Geometric set

Introduction to the lesson

Introduce the lesson by inviting the learners to create simple simultaneous equations using everyday examples. Emphasize the need to choose appropriate variables use their examples, discuss their examples, affirm the correct ones and use this opportunity to ensure that the correct concept has been understood.

Learning Activities

Activity 4.2

- Organise the learners in groups to do Activity 4.2 in the Student's Book.
- When the activity is done, ask the groups to present their findings to the class allowing them some time to deliberate.
- Ensure that all learners are actively involved in the activity and that their findings are accurate and if not correct them.

Synthesis

- Having done the activity under your supervision, guide them through a class discussion.
- Take the learners through the points necessary to develop the concept of the activity effectively.
- Take learners through Example 4.2 in the Student's Book, in a class discussion.

Assessment

- Ask the learners to do Exercise 4.2 in the Student's Book.

Answers to Activity 4.2

1. The two numbers x and y add up gives 90.

   Therefore, $x + y = 90$ is the relation connecting x, y and 90.

2. Since $x < y$, x is the smaller number.

   Therefore a third of the smaller is $\frac{1}{3}x = \frac{x}{3}$

   Thus, y is the larger number.

   Therefore, a seventh of the larger number = $\frac{y}{7}$

3. The required two equations are:

   $x + y = 90$ (i) and

   $\frac{1}{3}x = \frac{1}{7}y$ (ii)

4. The equations can be solved by substitution method.

Using equation (ii), $\frac{1}{3}x = \frac{1}{7}y$, (multiplying both sides by 3)

$x = \frac{\mathbf{3}}{\mathbf{7}}y$ (iii)

Substituting equation (iii) in equation (i),

$x + y = 90$ becomes $y + \frac{\mathbf{3}}{\mathbf{7}}y = 90$

$\therefore 7x\, y + 7 \times \frac{3}{7}\, y = 90 \times 7$ (multiplying both sides by 7 to eliminate the denominator 7)

$3y + 7y = 630$

$10y = 630$

$y = \frac{630}{10} = 63$

Now substituting 63 for y in equation (iii),

$x = \frac{3}{7}y$ becomes

$x = \frac{3}{7} \times 63$

$x = 27$

Solution set {(27,63)}

Therefore, the two numbers x and y are 27 and 63 respectively.

It is possible to verify our working.

Check $x + y = 90 \Rightarrow 27 + 63 = 90$ True

and $\frac{1}{3}x = \frac{1}{7}y \Rightarrow \frac{1}{3} \times 27 = \frac{1}{7} \times 63$ True

$9 = 9$

LHS = RHS

Note: the two equations can be solved graphically.

For $x + y = 90$

x	0	90	50	40
y	90	0	40	50

For $\frac{1}{3}x = \frac{1}{7}y$

x	0	21	9
y	0	49	21

Fig. 4.4 shows the graphs of the two equations. At the point of intersection, x = 27 and y = 63.

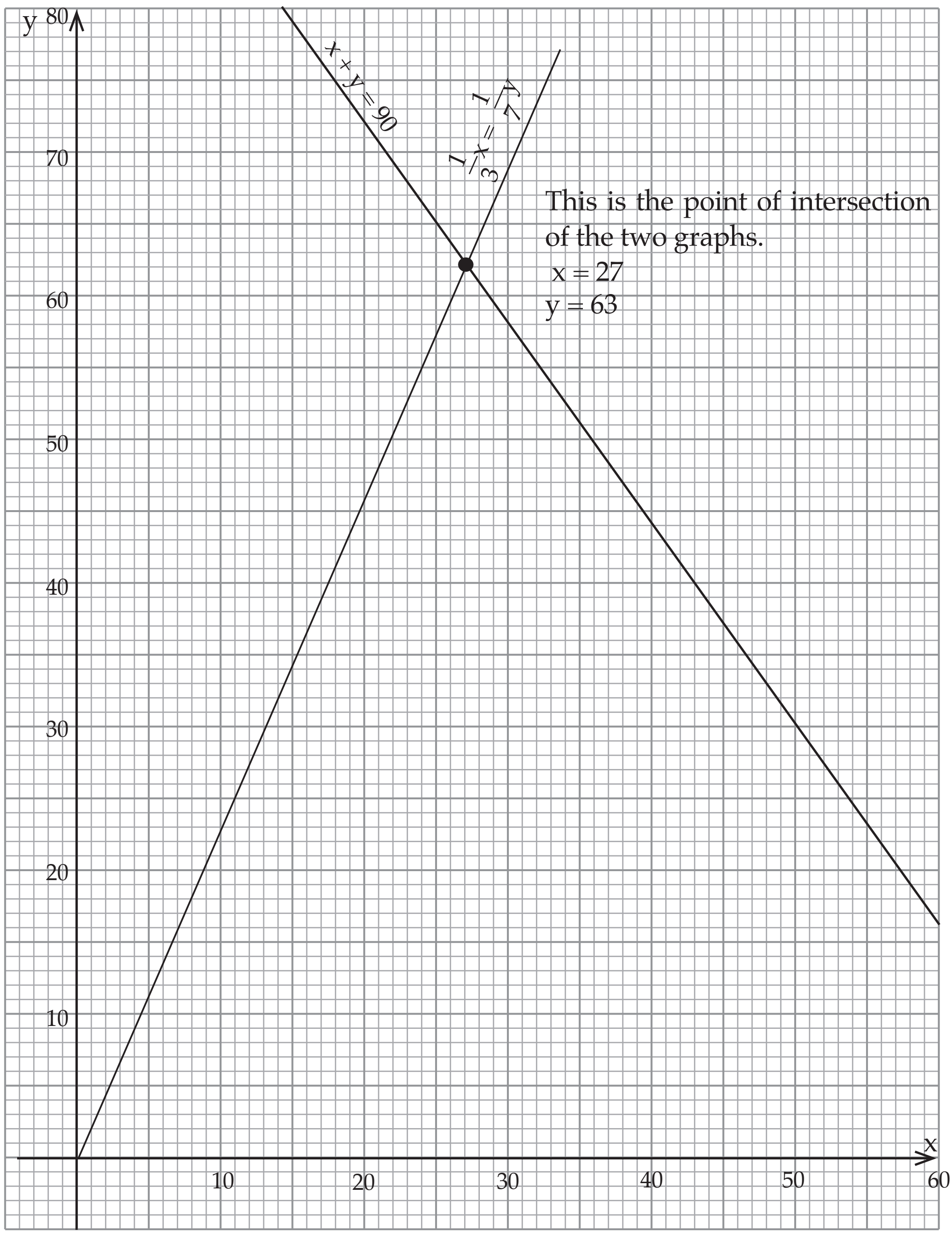


Fig 4.4

Lesson 3: Graphical representation of linear inequalities

Lesson objective

By the end of the lesson, the learner should be able to describe a given region in relation to a line such as $x = k$, and $y = c$ where c and k are constants. He/she should be able to represent simple inequalities graphically.

Teaching Aids

- A classroom board with square grid. (Draw the grids on chalkboard or manilla paper.)
- Graph books/papers
- Mathematical instruments

Introduction to the lesson

Introduce the lesson with a brief class discussion relating numbers using inequality symbols. Using the Cartesian plane and any line on it, involve the learners in describing:

(i) The number of regions (areas) into which the line divides the plane.

(ii) The sets of points into which the line divides the plane.

This discussion should be based on the introduction given in the Student's Book.

Learning Activities

Activity 4.3

- Organise the learners in groups to do Activity 4.3 in the Student's Book.
- Ask learners to present their findings in a class discussion demonstrating their findings on the board.

Synthesis

- Summarise the learners' observations emphasizing the conventional method of representing regions graphically i.e. the significance of drawing a broken line, solid line, shading the unwanted region, the meaning of the symbols $>$, $\geq$, $<$, $\leq$ in inequalities etc.
- Through a class discussion guide the learners through the highlights in the Student's Book immediately after Activity 4.3.

Answers to Activity 4.3

1. The line $x = 3$ divides the plane into **three** sets of points.

   B: points on the line

   A: points to the left of the line

   C: points to the right of the line

2. (i) the points on the line are such that the x co-ordinate is always equal to 3, y can take any value. These points satisfy the equation $x=3$.

   (ii) For the points to the left of the line, the x value is always less than 3, y can take any value. These points satisfy the inequation (inequality) $x < 3$.

   (iii) For the points, to the right of the line the x co-ordinate is always greater than 3, y can take any value. These points satisfy the inequation (inequality) $x>3$.

3. The line $x=3$ divides the plane into two areas; the area to the left and the area to the right. These areas are also called regions.

Assessment

- Ask the learners to do Questions 1 to 5 from Exercise 4.3 in the Student's Book.

Lesson 4: Forming inequalities from given regions

Lesson objective

By the end of the lesson, the learner should be able to describe a given region in relation to a line such as $x = k$, and $y = c$ where c and k are constants.

Teaching Aids

- A classroom board with square grid.(you and raw grids on chalkboard or manilla paper)
- Graph books/papers
- Mathematical instruments

Introduction to the lesson

Introduce the lesson with a brief class discussion relating numbers using inequality symbols. Using the Cartesian plane and any line on it, involve the learners in describing: Finding the equation of a simple equation in terms of x and y.

(i) The number of regions (areas) into which the line divides the plane.

(ii) The sets of points on either side of a given line with refrence to the equation of the line.

This discussion should be based on the introduction given in the Student's Book.

Learning Activities

- Organise the learners in groups to do Activity 4.4 in the Student's Book.
- Ask learners to present their findings in a class discussion demonstrating their findings on the board.

Synthesis

- Summarise the learners' observations emphasising the conventional method of representing regions graphically i.e. the significance of drawing a broken line, solid line, shading the unwanted region, the meaning of the symbols $>$, $\geq$, $<$, $\leq$ in inequalities the meaning of a boundary line etc.
- Through a class discussion guide the learners through the highlights in the Student's Book immediately after Activity 4.4.

Answers to Activity 4.4

(i) The line that defines the region passes through the point (4,0) and it is parallel to the y-axis.

(ii) The equation of the boundary line is $x = 4$.

(iii) The region that is to the right of the line $x = 4$ is the required region.

(iv) Any point to the right of the line has x-coordinates greater than 4.

$\therefore$ it satisfies the inequality $x > 4$. We use the symbol $>$ because the line is drawn broken, therefore it is part of the required region.

Assessment

- Ask the learners to do question 6 from Exercise 4.3 in the Student's Book.

Lesson 5: Simultaneous linear inequalities with one unknown

Lesson objective

By the end of the lesson, the learner should be able to solve linear inequalities in one unknown i.e should be able to identify and describe compound inequalities represented on a Cartesian plane and also represent the same graphically.

Teaching Aids

- Square grid board / manilla papers(These lines can be drawn.)
- Graph books / papers
- Mathematical instruments

Introduction

Introduce this lesson by asking the learners to recall the properties and concepts that were learned in the previous lesson. This is best done in a class discussion.

Learning activities

Explain to the learners the meaning of simultaneous inequalities by demonstrating some on the board. Help the learners understand the meaning of **intersection** of **regions** as compared to the intesection of lines in solcing simultaneous equations.

Synthesis

Individually ask learners to create a pair of simple simultaneous inequalities and illustrate them in their books. Ask for volunteers to demonstrate the same on the board. Ensure that they all understand the concept and can apply it.

Assessment

Ask the learners to do Exercise 4.4 in Student's Book.

Lesson 6: Linear inequalities in two unknowns

Lesson objective

By the end of the lesson, the learner should be able to represent inequalities in two unknowns graphically and be able to identify the required region.

Teaching Aids

- Square grid board/manilla papers (you can draw grids yourself)
- Graph books/papers.
- Mathematical instruments.

Introduction to the lesson

Introduce the lesson by asking learners to explain how they can create a region given an inequality in two variables. Use their suggestions to refine this concept i.e. forming equation of a line from a given inequality, drawing the line 'broken' or 'solid' depending on the given inequality etc. This is not a completely new concept since they have done the same using one variable.

Learning Activities

Activity 4.5

- Organise the learners in groups to do Activity 4.5 in the Student's Book 55.
- Move round the class as learners work so that you could verify that they are doing the right thing.
- Ask the groups to represent their findings to the rest of the class. Ensure that this presentation is demonstrated on the board for all to see and therefore be able to give their contribution.

Synthesis

- In a class discussion, summarize their observations, as you verify findings and conclusions.
- Take learners through the discussion immediately after the activity.

Answers to Activity 4.5

1. The line divides the Cartesian plane into two regions.
2. We can use any point provided it is not on the line. Let us use point (0, 0). Using (0, 0) in equation $y = \frac{3}{2}x + 3$.

   LHS $= 0$ and

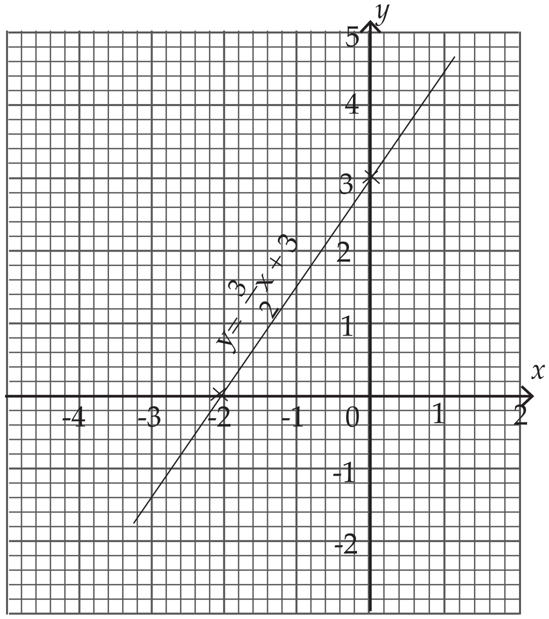
   RHS $= \frac{3}{2} \times 0 + 3 = 3$

   Thus LHS is less than RHS

   i.e. $0 < 3$

This means for any point on the same side of the line as (0, 0) the value on the LHS will be less than the value on the RHS. Thus this region is described by the inequality $y < \frac{3}{2}x + 3$. Since the line is drawn solid, the region includes the line, written as $y \leq \frac{3}{2}x + 3$

Fig 4.5 shows the line whose equation is $y = \frac{3}{2}x + 3$.



3. We can use any point on the opposite side of the origin i.e. (-4, 2).

Using $y = \frac{3}{2}x + 3$

LHS = y = 2

RHS = $\frac{3}{2}$ (-4) + 3

= 3x – 2 + 3

= –6 + 3

= –3

Since LHS = 2 and RHS = –3 then LHS > RHS

$\therefore y > \frac{3}{2}x + 3$

The two regions on either side of the line satisfy different inequalities.

Assessment

- Ask the learners to do Exercise 4.5 in the Student's Book.

Lesson 7: Graphical Solutions to simultaneous linear inequalities in two unknowns

Lesson objective

By the end of the lesson, the learner should be able to represent two or more inequalities in the same graph, identify the common region and therefore state the solution.

Teaching Aids

- Square grid board/manilla papers (draw grids yourself)
- Graph books/papers
- Mathematical instruments

Introduction to the lesson

Introduce the lesson by asking learners to explain the meaning of solution of simultaneous equations. Distinguish between simultaneous equations and simultaneous inequalities. Use the learners responses to explain the concepts in this lesson.

Learning Activities

Activity 4.6

- Individually ask the learners to do Activity 4.6 in the Student's Book.
- Move round the class ensuring that they are doing the right thing.
- Taking one line at a time, guide the learners through the process of identifying the required region, all the time shading the unwanted region. Remind them how to identify the required by substitution of x and y values in the appropriate inequality '(x and y)' must be specific values defining the coordinates (x, y) of a point not on the line.
- Now ask the learners to compare their graph with those of other members of the class.
- Invite volunteers to demonstrate their work on the board.

Synthesis

- Emphasise to the learners the following:
  (i) The required region must be clean (no shading).
  (ii) Where the inequality does not include the line, the line must be drawn broken otherwise, the line is drawn solid.
- If the learners need any clarification, do not tire to explain.
- Take the learners through Example 4.3 in the Student's Book.

Answers to Activity 4.6

1. The lines having equations: (i) $x + y = 3$

   (ii) $3x + 3y = 12$

2. Required graph is shown below.

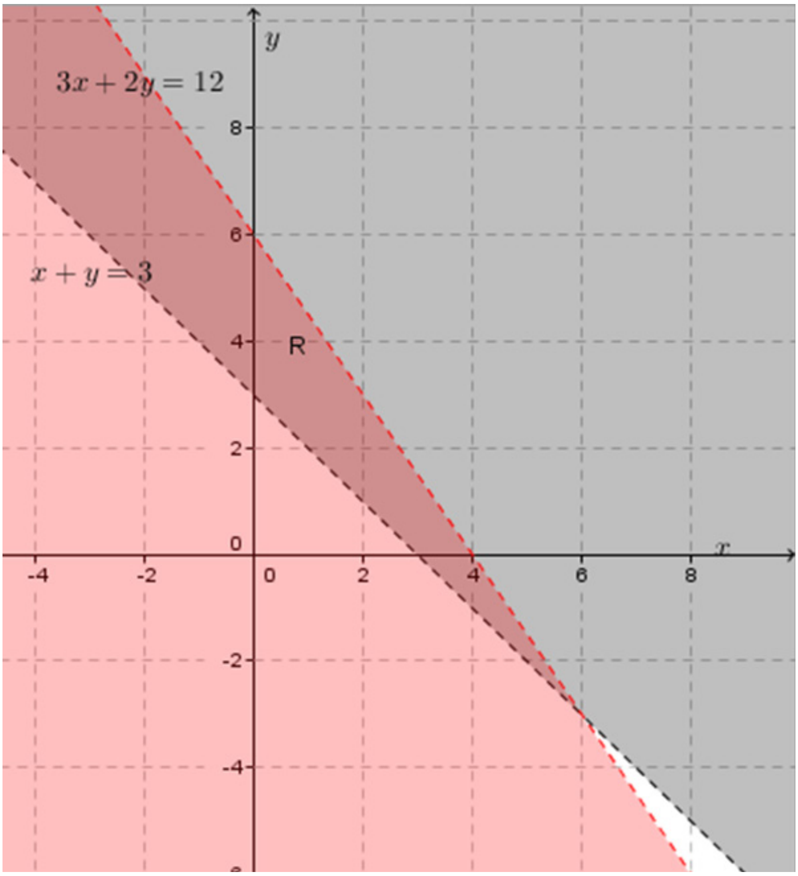


Fig 4.6

3. The required region is shown by shown by shading the unwanted region.
4. The region is denoted by letter R.
5. This unshaded region satisfies the inequalities $x + y > 3$ and $3x + 2y < 12$.

Note:

The line $3x + 2y = 12$ is drawn as a broken line to emphasize the fact that it is not part of the required region.

Assessment

- Ask the learners to do questions 1 to 5 and 6 to 11 in class and Exercise 4.6 as homework.

Lesson 9: Linear inequalities from inequality graphs

Lesson objective

By the end of the lesson, the learner should be able to find simultaneous inequalities given the solution set.

Introduction to the lesson

Introduce this lesson by first considering simple inequalities in one unknown, progressing into inequalities in two variables. Remind them that a line divides a plane into 3 sets of points i.e. points on the line, points on either side of the line. Similarly, line divides the same plane into two regions i.e. regions I and II one on either side of the line.

Since to define a region must be with reference to a line, ensure that they can find the equation of a line by whatever method.

Learning Activities

Activity 4.7

- Organise class to work in groups to do Activity 4.7 in the Student's Book.
- Ensure that they are able to describe the three sets of points in Fig 4.24 and describe the two regions A and B with reference to the line L.
- Ask the groups to present their findings and conclusions in a class discussion.

Synthesis

Using the learners observations, emphasise the following procedures

- Procedure of finding equation of a line.
- Procedure of testing for the appropriate inequality for each of the regions,
- Stating the solution set appropriately.
- Now guide the learners through Example 4.4 in the Student's Book.

Answers to Activity 4.7

- Some suitable points on the line are;

  (4,0), (3,2), (2,4), (1,6), (0,8)

- We can use any two points to find the gradient of the line.

  For example, using (4,0) and (3,2)

  $$\text{Gradient} = \frac{2-0}{3-4}$$

  $$= \frac{2}{-1} = -2$$

Let the gradient be denoted by letter m, a general point P on the line by (x,y) and a specific point on the line say, (3,2).

Therefore gradient in terms of m, x, and y is

$\frac{y-2}{x-3} = \text{m}$ but m = –2.

$\therefore \frac{y-2}{x-3} = -2$

$\frac{y-2}{x-3}(x-3) = (-2)\ (x-3)$ multiplying both sides by $(x\text{-}3)$

$y - 2 = -2x + 6$

$y + 2x = 8$

The equation of the line l is

$y + 2x = 8$ **or** $y = -2x + 8.$

- Using a point from region A, say (2,1) and substituting for x and y in the equation

  $y = -2x + 8$, we obtain

  LHS = 1

  RHS = –2 (2) + 8

  = –4 + 8 = 4

$\therefore$ LHS < RHS

But the line is drawn solid.

$\therefore y \leq -2x + 8$

Similarly the points in region B are such that $y \geq -2x + 8$.

Assessment

- Ask the learners to do Exercise 4.7 in the Student's Book.

Lesson 10: Unit Summary

Summary of the unit

- Ask the learners to take the whole class through the different concepts highlighted in the unit summary given in the Student's Book. Take the learners through a question and answer session to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit. Let them discuss and solve the unit focus activity stated at the beginning of this unit. Ask them probing questions to gauge their understanding on this topic.

Unit assessment

Ask the learners to do the Unit 4 Test at the end of the unit in the Student's Book. Mark their work to assess their areas of weaknesses and strengths. From the result, give extra remedial exercise to the weaker learners while giving the extended exercise to the learners who are comfortable with the rest of the exercise.

Additional information to the teacher

Simultaneous equations are sometimes called **systems** of **linear equations**. To solve a systems of equation means to find the value of x and y that satisfy both equations. Ensure that the learners are familiar with this reference to the simultaneous equations.

- If the equations $x + y = 8$ and $x - y = 12$ are solved simultaneously, the solution set is $x = 10$, $y = -2$.

This solution set can be expressed as an ordered pair. Thus $(x,y) = (10,-2)$ is another way of expressing the solution of the system of equations $x + y = 8$ and $x - y = 12$

- The skills of finding the coordinates of the point of intersection are useful throughout this unit.
- A system of two equations in two variables is **equivalent** to another system if both systems have the same solution set. There are three types of systems of equations.

1. If the system has only one point of intersection, it means this system has only one solution. Such a system is said to be **consistent** and independent. $x - 2y = 0$ and $2x + y = 5$ is an example of this class of system of equations. You can also ask them to solve the equations using any method of solving simultaneous equations.
2. If the system of equations do not intersect, then the equations have no solutions and the lines are parallel. Such a linear system of equations is said to be **inconsistent**. Consider the equations $x - y = 4$ and $x - y = 2$ and verify that the equations are truly inconsistent i.e. no solution. You can ask the learners to represent the equations graphically to verify. You can also ask them to solve the equation using any other method of solving simultaneous equations.
3. If the lines representing a system of equations are coincident, then any solutions of one satisfy the other. The equations are said to have an infinite solutions set. Such a set of equation is said to be consistent and dependent. Verify by any means that the equation $2x - 6y = -12$ and $x - 3y = 6$ are consistent and dependent.
5. Unlike in a system of equation a system of linear inequalities defines a region and not a point.

   In a system of inequalities to find the solution set means to find all the points (x,y) that satisfy all the given inequalities. A region may be defined by one or more boundary lines. To define a region, you must know the equation(s) of the boundaries lines before you can mark out the region. Ensure that the leaners are also able to find the equation of a line. The procedures to identify the region are simple;

   - From the given inequalities, state the equation of the boundary line (s).
   - Mark the boundaries using a solid or broken line according to the inequality sign used.
   - Shade out the unwanted regions to show the required regions. Ensure that the learners remembered how to test for the require region. Remember also, shading the unwanted region is just a convention so that the required region is left clean.
   - Ensure that learner can now solve systems of equations using all the algebraic methods learned earlier. They should also be able to solve linear inequalities graphically and analyse the regions to deduce required findings.

Remedial Activities

1. Find the co-ordinaties of the point of intersection of each of the following pairs.

   (a) $y - 2x = 2$, $x + y = 4$ (b) $3x + y = 8$, $x - y = 7$ (c) $3x - y = -7$, $x - 2y = 1$

   How can you check your answers to this question?

2. Use graphical method to solve the following systems of equations.

   (a) $x + y = 2$, $2x - y = 7$ (b) $x = 3$, $2x - y = 7$ (c) $x = +3$, $x + y = 2$ (d) $x + y = 2$, $y = -1$

3. Solve the equations.

   (a) $x - y = 1$, $x + 2y = 5$ (b) $3x + y = 7$, $x - y = 1$ (c) $x - 3y = 3$, $2x - 3y = 6$

4. The length of a rectangle is 2cm more than its breadth. The perimeter of the rectangle is 8 cm. If the length of the rectangle is x cm and its width is y cm, form two simultaneous equations relating to x and y. Use graphical method to find the dimensions of the rectangle.

5. Two books have a total of 500 pages and one book has 350 more than the other. Choose two suitable variables and use them to form a pair of equations and solve them to find the number of pages in each books.

6. On separate diagrams represent the inequalities.

   (a) $y \leq 4$ and $y > -2$ (b) $x \geq -3$ (c) $x < 4$

Extended Exercise

1. Show the region described by the inequalities $x + y \geq 1$ and $y > 2x - 4$.

2. Describe the region bounded by lines.

   $x = 0$, $y = 0$ and $2x - 3y = 6$

   Show the region by shading the unwanted region.

3. Draw the region defined by each of the systems of inequalities.

   (a) $2x - y \geq 3, x + y \leq 3$ (b) $y - x \leq 4$, $y - 3x \geq 0$, $x + y > 1$

4. Describe the closed region bounded by the lines whose equations are.

   $x - y = -2$

   $x + 2y = 4$

   $y = -3$

5. (a) Use graphical method to solve the simultaneous equations in the questions below.
(b) Classify each system of equations.

(a) $3x + 2y = 6$

$x - 2y = 2$

(b) $2x - 3y = -6$

$2x - 3y = 15$

(c) $3x - 2y = 5$

$6x - 4y = 10$

(d) $2x - y = -3$

$x + y = 0$

Answers

Exercise 4.1

1. $x = 2, y = 1$
2. $x = 3, y = -1$
3. $x = -2, y = 2$
4. $x = \frac{-4}{3}, y = \frac{19}{6}$
5. $x = -2, y = -3$
6. $x = \frac{-1}{2}, y = -2$
7. $x = \frac{-1}{4}, y = 2$
8. $x = \frac{12}{29}, y = \frac{115}{29}$
9. $x = 2, y = -1$
10. $x = \frac{14}{3}, y = \frac{-4}{3}$
11. $x = 1, y = 3$
12. $x = 11, y = 2$
13. (i) (ii) (iii), the corresponding answers should be identical to the above.

Exercise 4.2

1. (a) $(60x + 100y)$ km (b) $(5x + 10y)$km
2. (a) $5x + 6y = 80450$ (b) $8x - 9y = 10{,}000$
3. (a) $x + y = 48$ (b) $x - y = 5$
   (c) $w + l = 96$ (d) $x + y = 36$ years
4. $\left.\begin{aligned} x + y &= 84 \\ 3x - 2y &= 62 \end{aligned}\right\}$; The two numbers are 38 and 46.
5. $5x + 10y = 580$
   $x + y = 76$
   40: 10 dollar bills
   36: 5 dollar bills

6. $\left.\begin{array}{l} x + y = 4800 \\ 10y - 9x = 4300 \end{array}\right\};$ 2300 FRW at 9% and 2500 FRW at 10%

7. $\left.\begin{array}{r} 40x + 10y = 185 \\ x + y = 5.75 \end{array}\right\};$ 15Km

8. 80 FRW

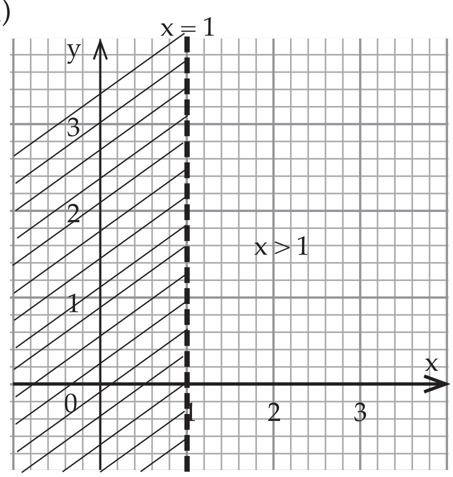
9. The numbers are 20 and 36

10. 4483 chemists and 651 engineers

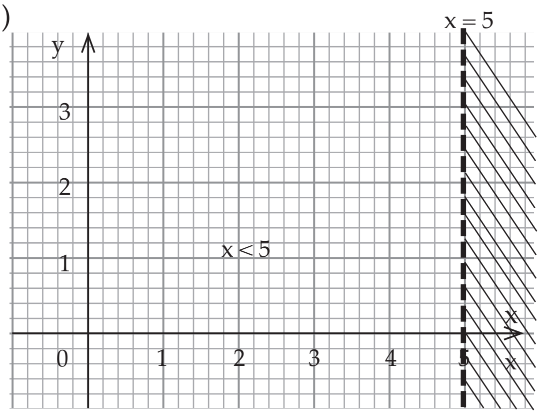
11. 125 sacks of been seed.

Exercise 4.3

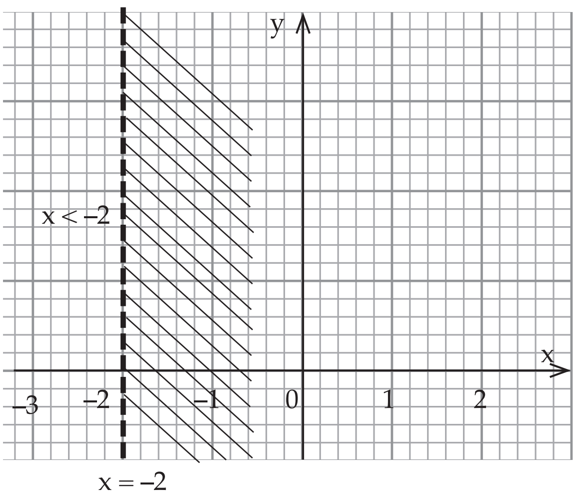
1. (a)



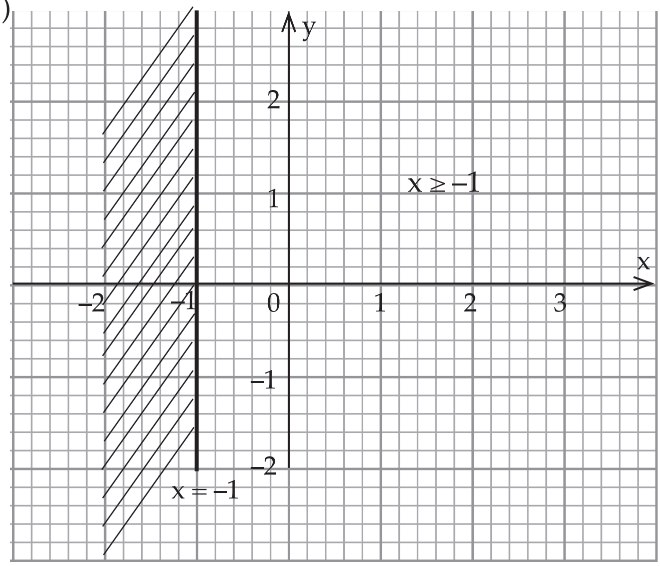
(b)



2. (a)

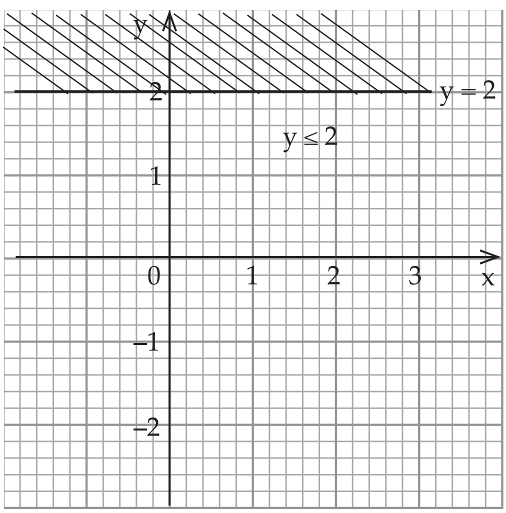


(b)



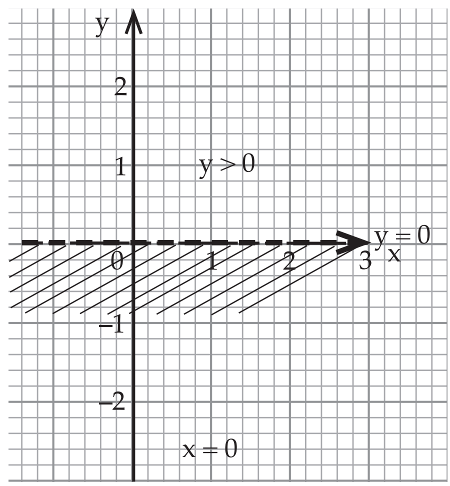
y
2
1
x ≥ –1
x
–2
–1
0
1
2
3
–1
–2
x = –1

3. (a)



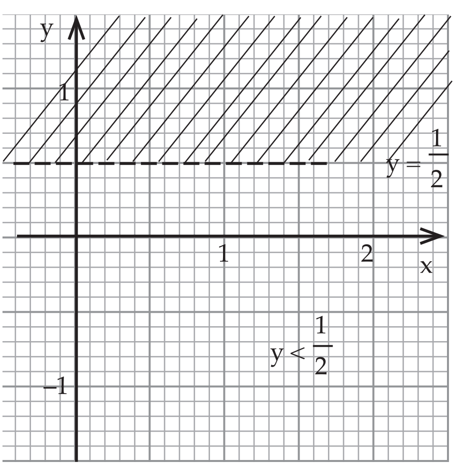
y
2
y = 2
y ≤ 2
1
0
1
2
3
x
–1
–2

(b)



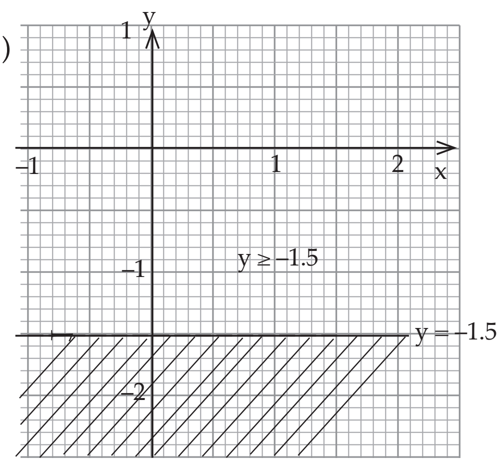
y
2
1
y > 0
y = 0
0
1
2
3
x
–1
–2
x = 0

4. (a)



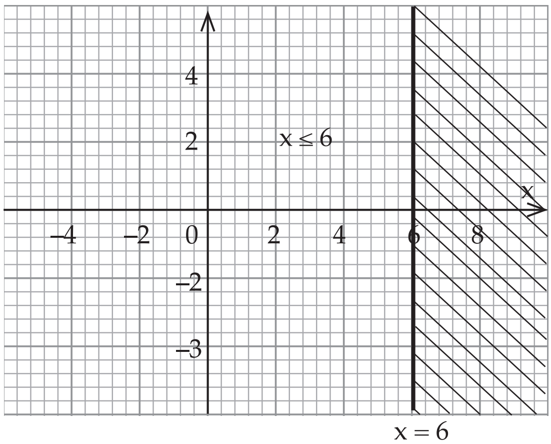
y
1
y = 1/2
1
2
x
y < 1/2
–1

(b)



1
y
–1
1
2
x
–1
y ≥ –1.5
y = –1.5
–2

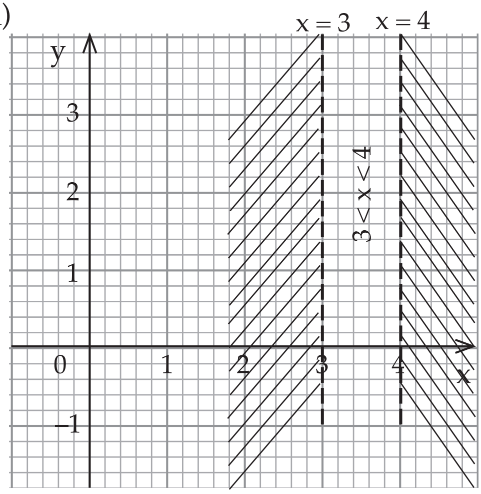
5. $4x - x^2 \le x(1 - x) + 18 \Rightarrow 3x \le 18 \Rightarrow x \le 6$



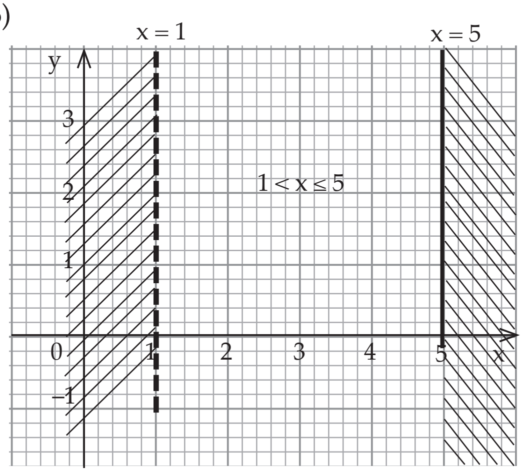
6. (a) $y \le 1$ (b) $x \le -1$ (c) $y < 0$ (d) $x < -1$
(e) $y > 0.6$ (f) $x < 3$

Exercise 4.4

1. (a)

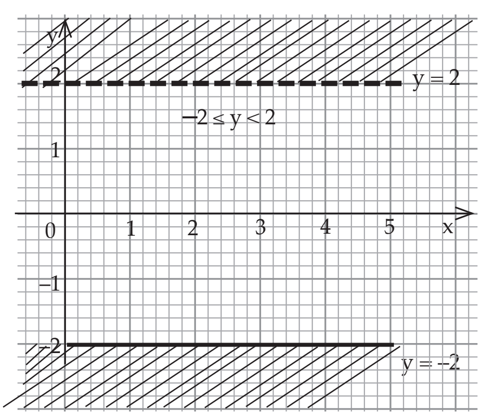


(b)

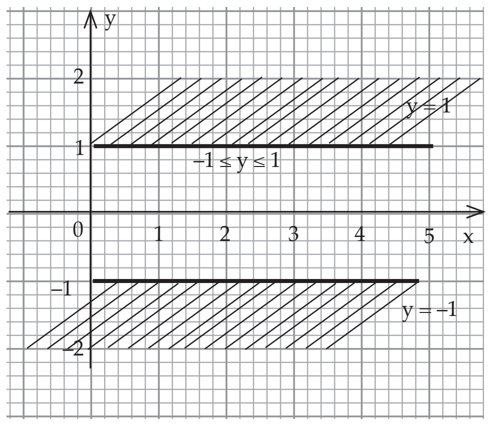


Required region $3 < x < 4$.

2. (a)



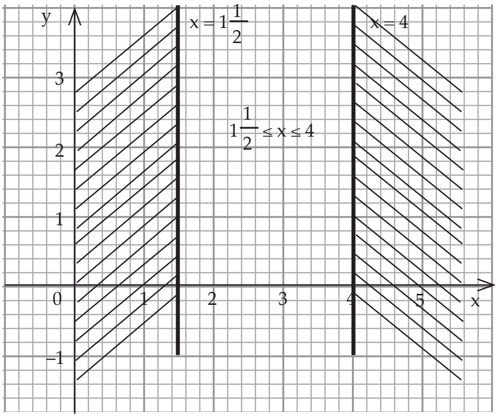
(b)



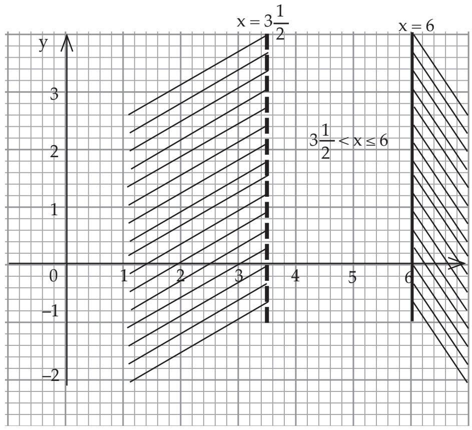
3. (a) $1\frac{1}{2}x \leq x \leq 4$

$1\frac{1}{2} \leq x$ and $x \leq 4$

$x \geq 1\frac{1}{2}$ and $x \leq 4$



(b)

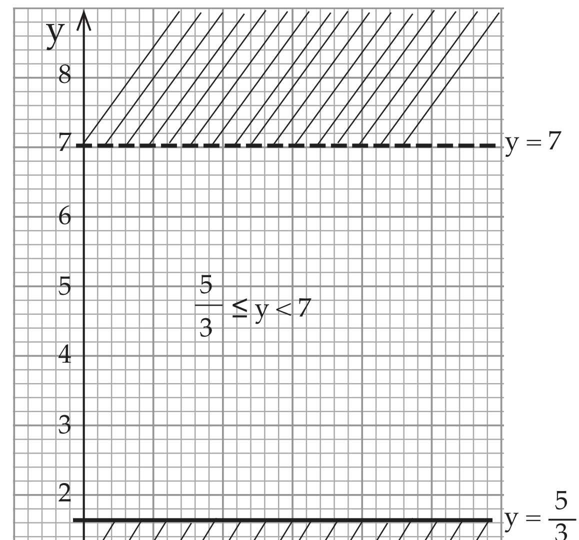


4. $y + 5 \leq 4y < 2y + 14$

$y + 5 \leq 4y$ and $4y < 2y + 14$

$y \leq 3y \qquad 2y < 14$

$y \geq \frac{5}{3} \qquad y < 7$

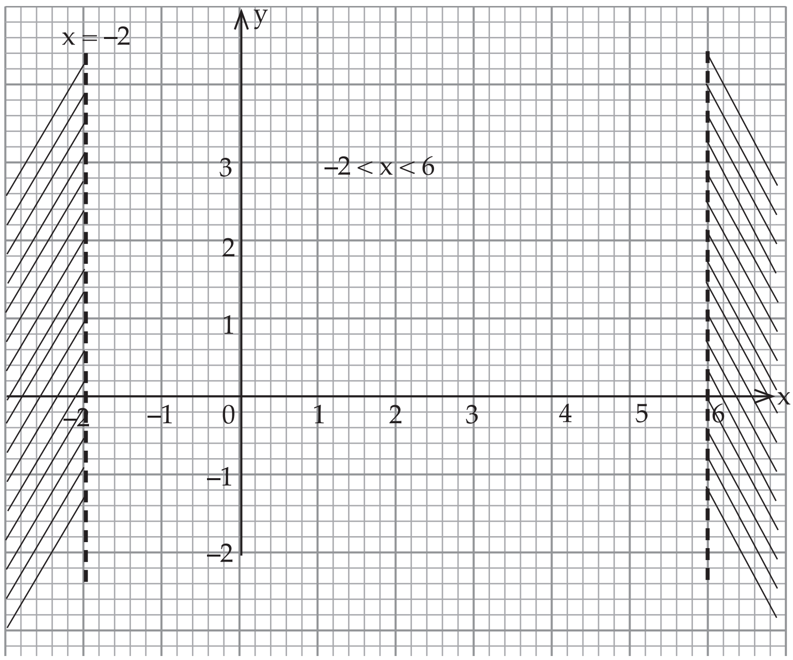


5. (a) $-1 \le x < 3$ (b) $-2 \le y \le 1$

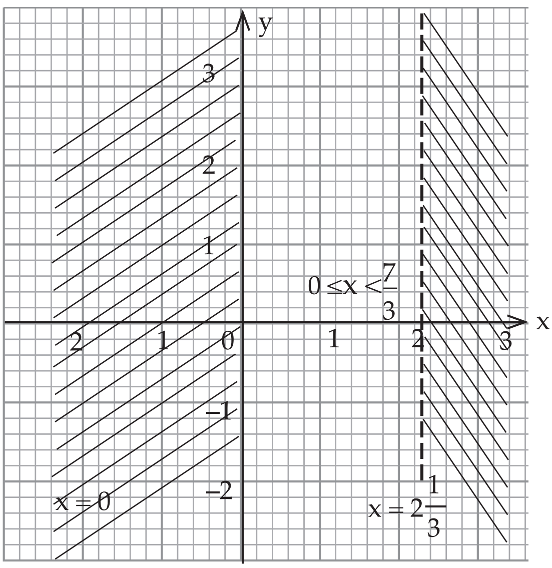
(c) $-1 < x < 2\frac{1}{2}$ (d) $-1 \le x < 2$ and $-2 < y \le 2$

Rectangle

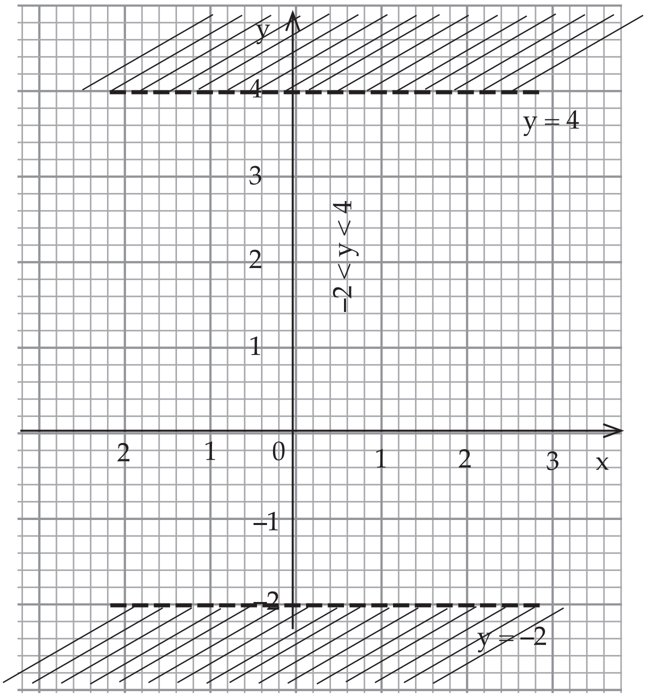
6.



7.



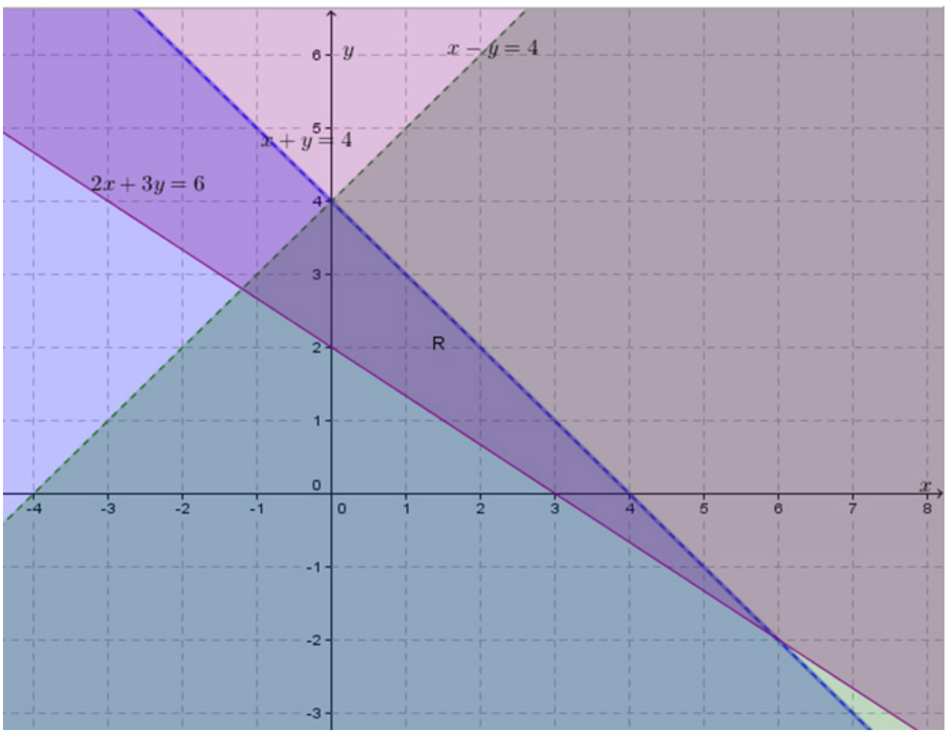
8.



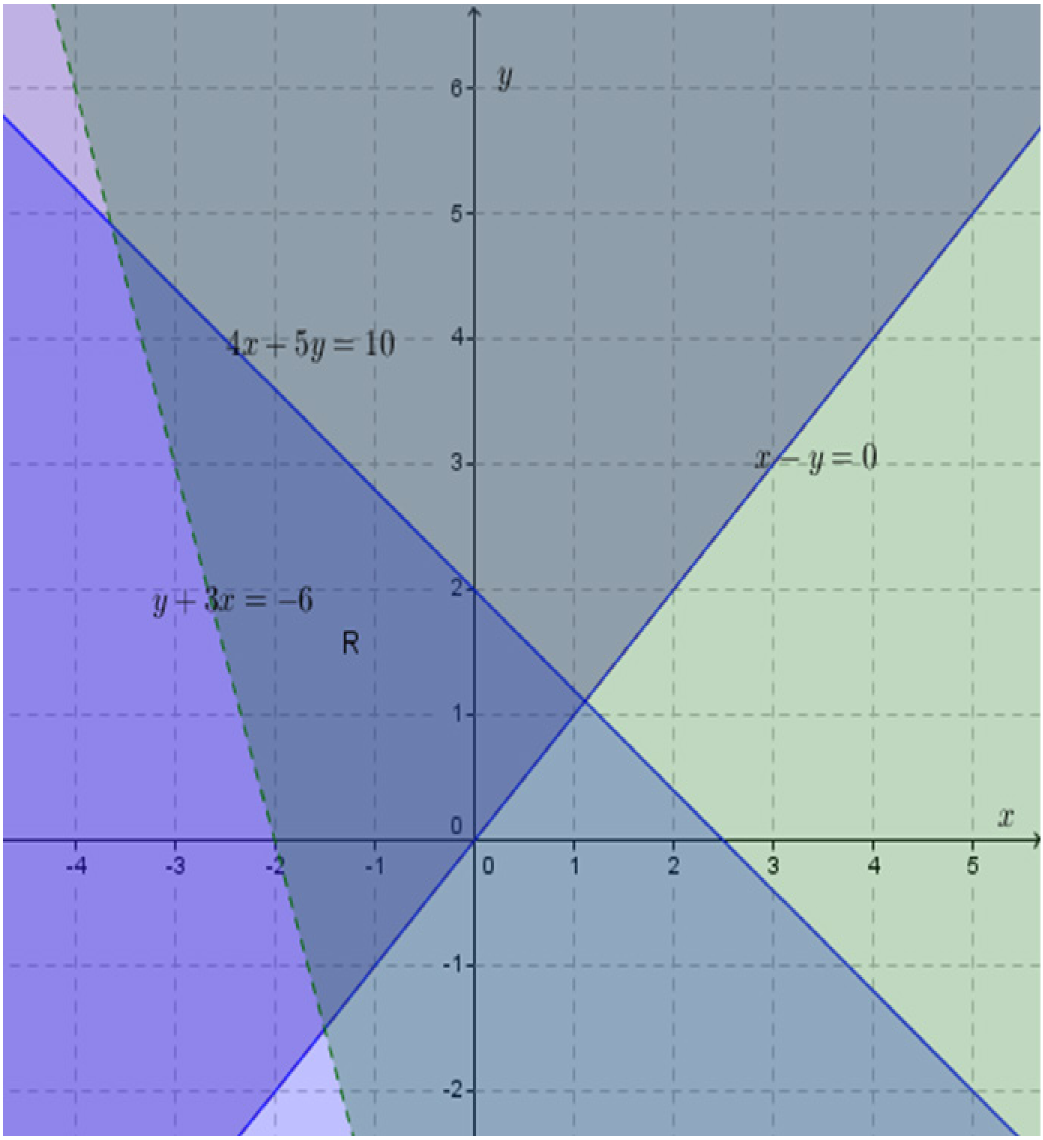
Exercise 4.5

R is required region $x + y < 4$, $y - x < 4$, $2x + 3y \geq 6$.

1. (a)



2.



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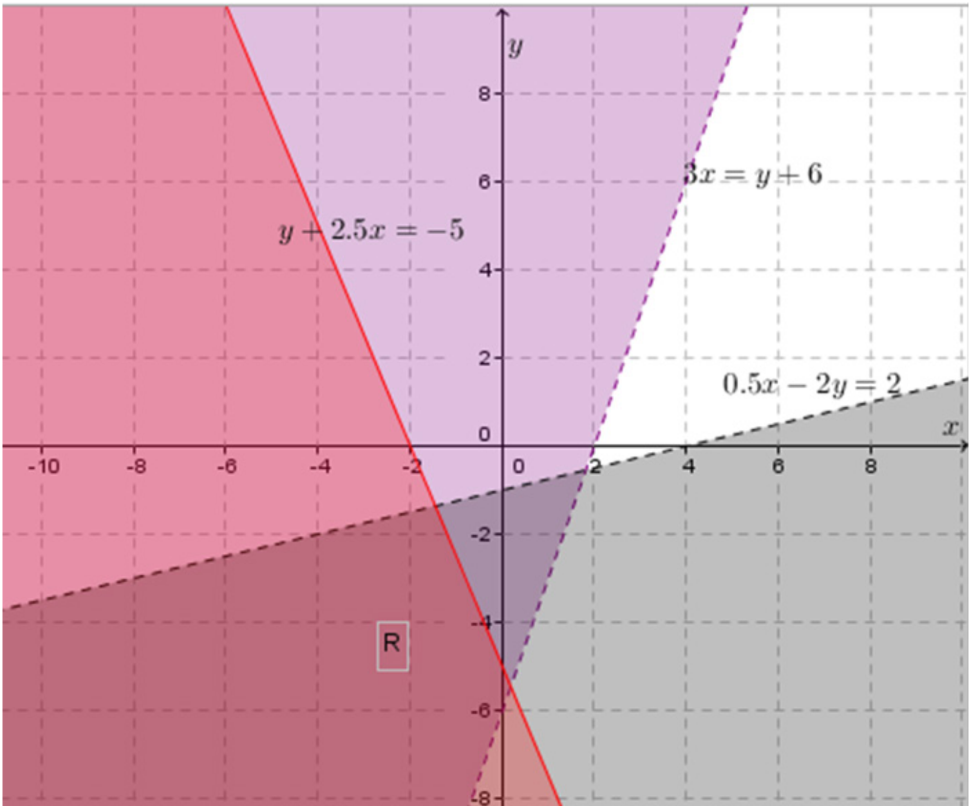

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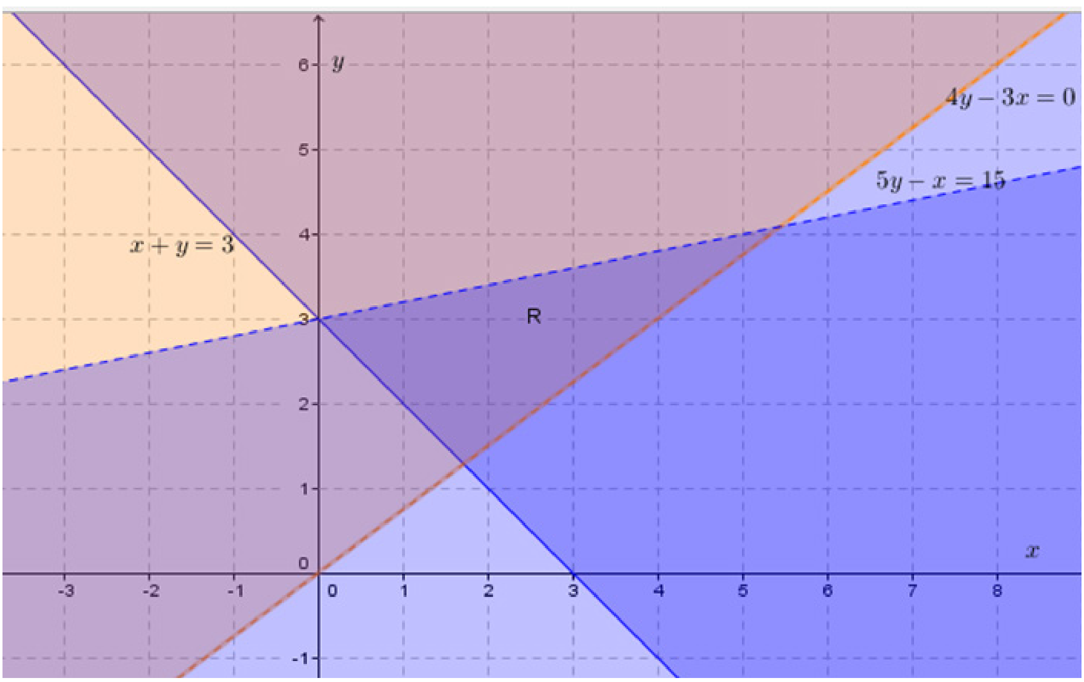
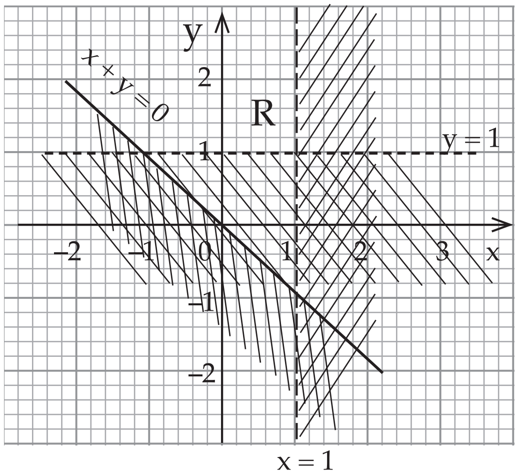


Exercise 4.6

1.



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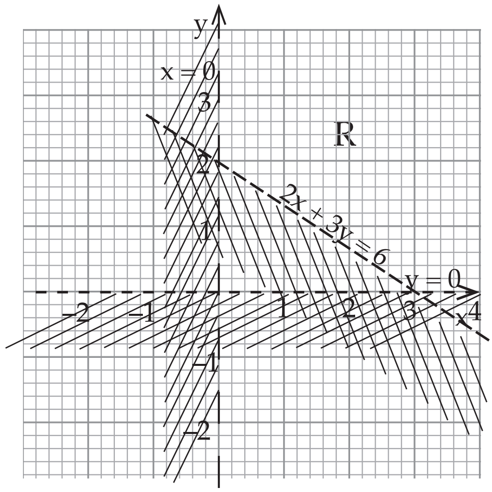

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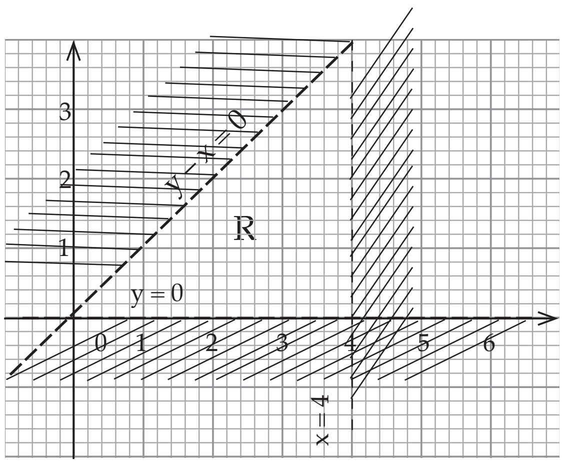

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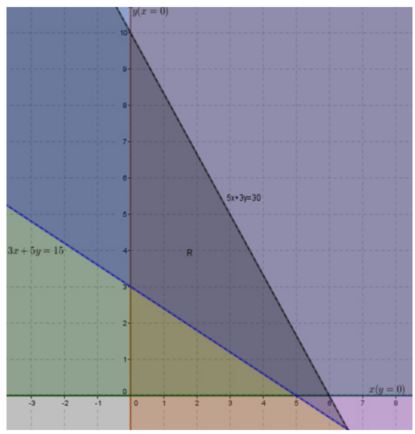

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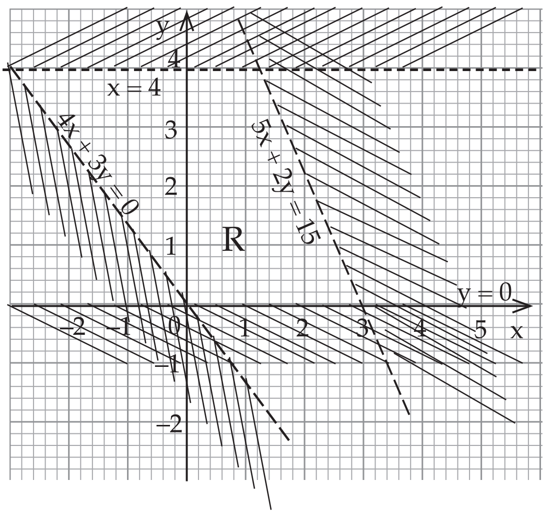

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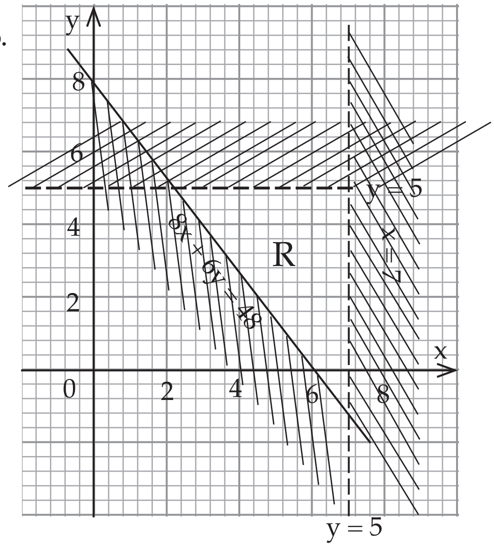
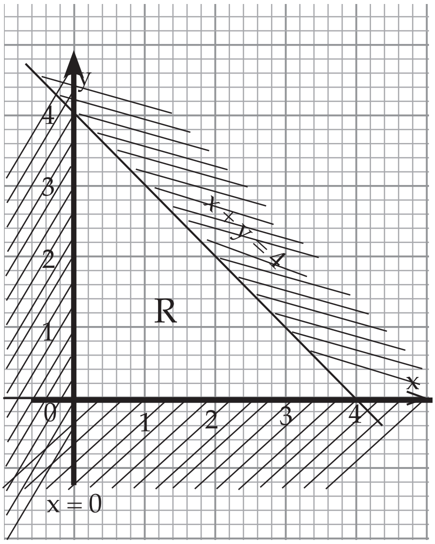
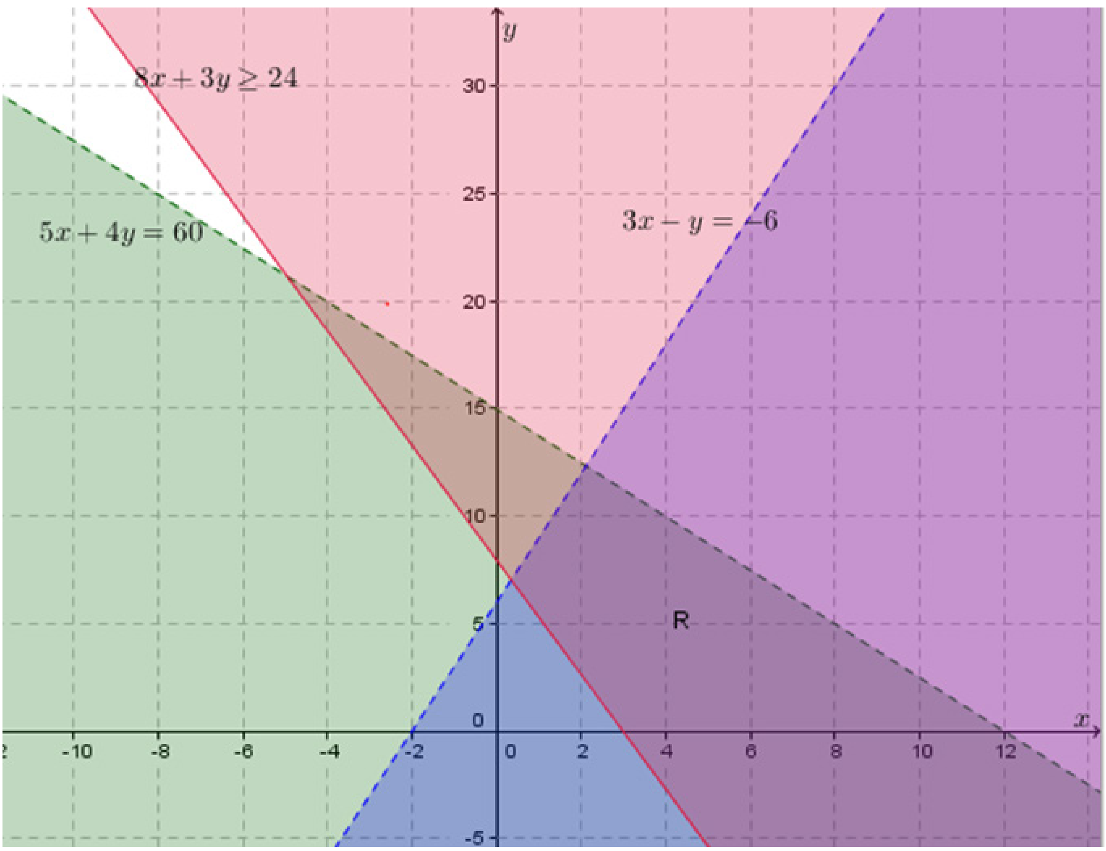


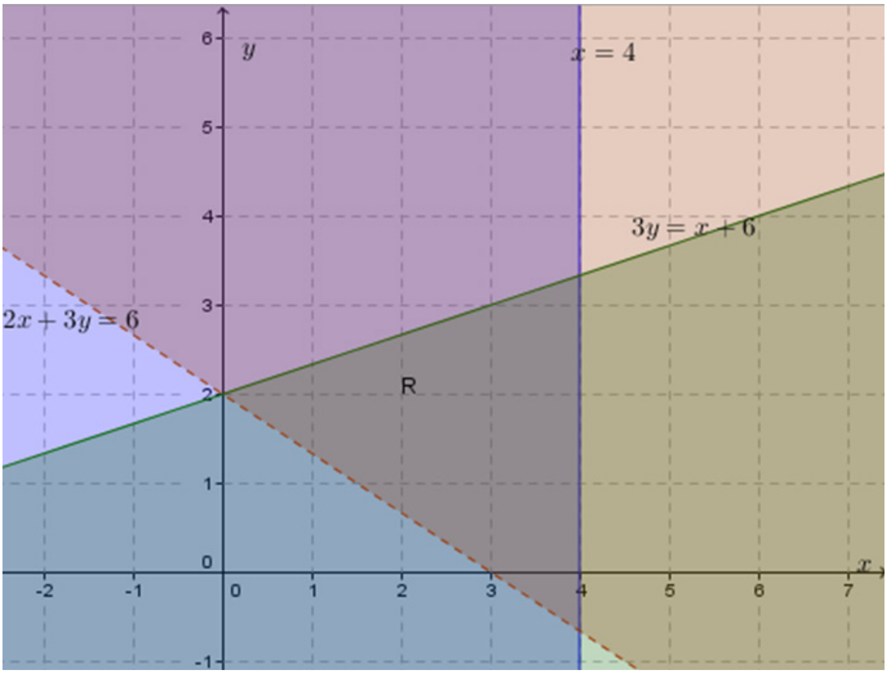
7.


y
4
3
2
1
0
1
2
3
4
R
x + y = 4
x
y = 0
x = 0

8.

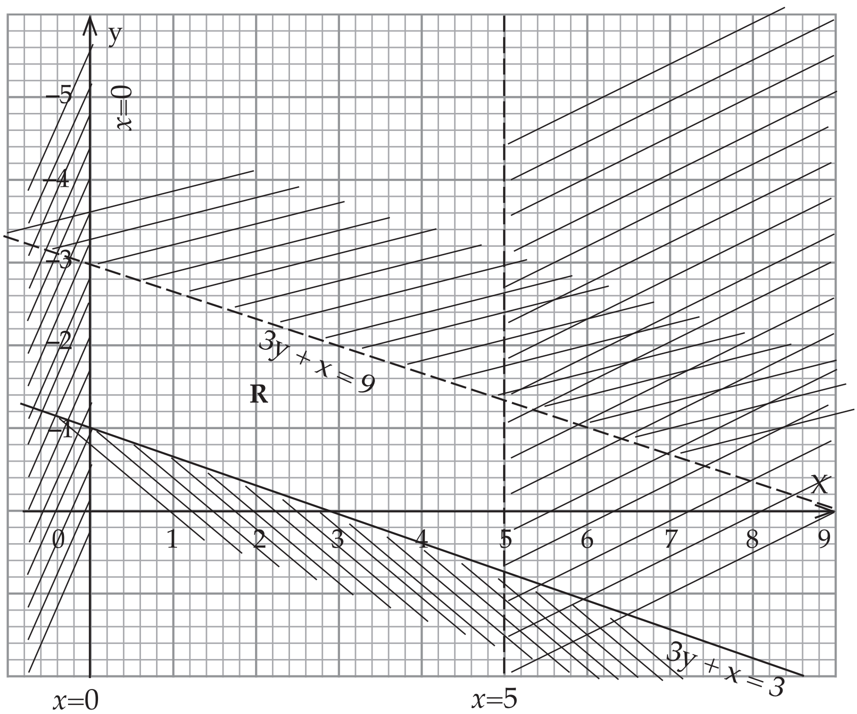

y
$8x + 3y \geq 24$
$5x + 4y = 60$
$3x - y = -6$
30
25
20
15
10
5
0
-5
R
x
-10
-8
-6
-4
-2
0
2
4
6
8
10
12

9.

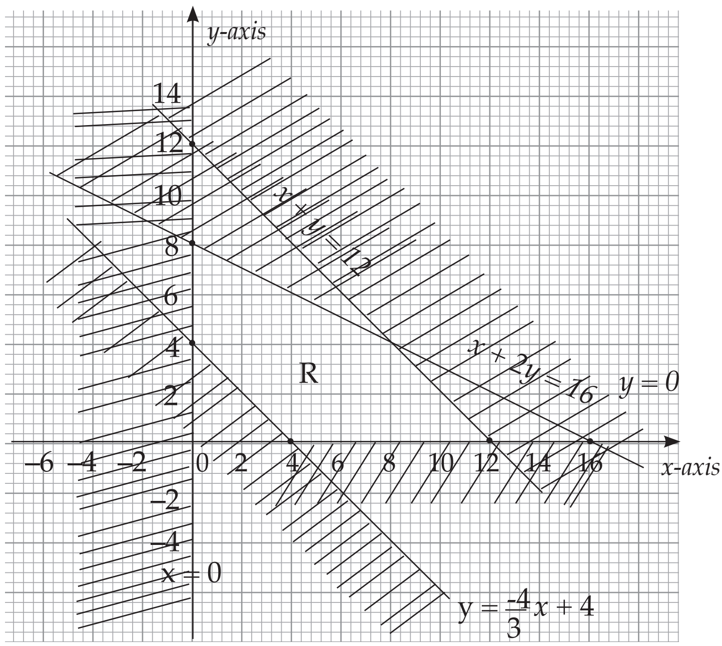


Solution set (1, 2) (2, 1) (2, 2) (3, 1) (3, 2) (4, 0) (4,1) (4,2) (4,3)

10.

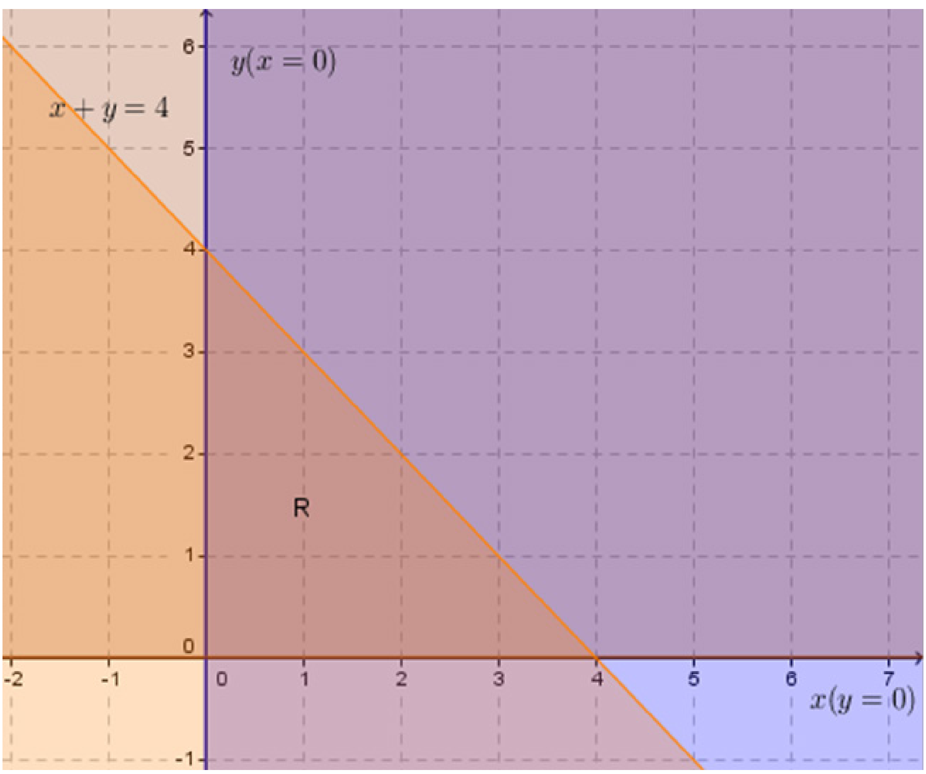


11.



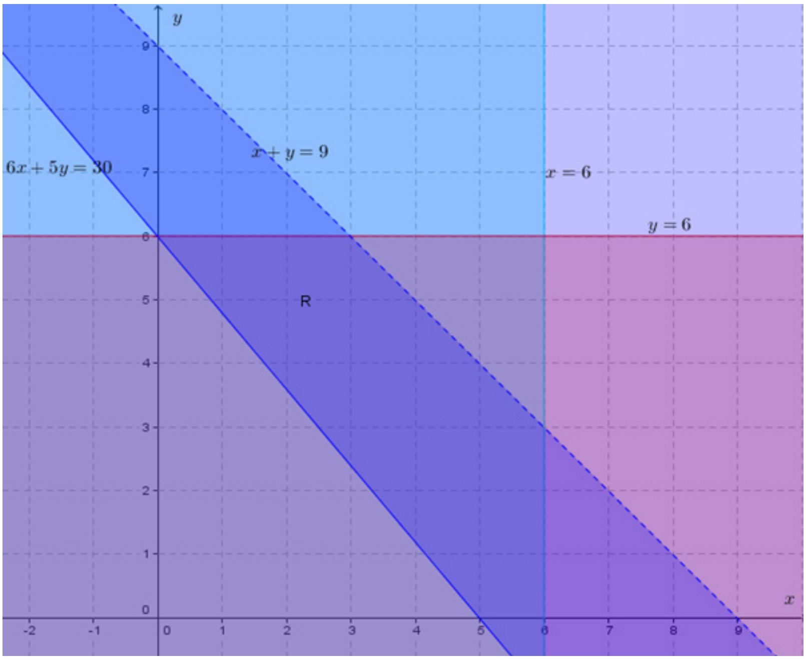
y-axis
14
12
10
8
6
4
2
R
x + y = 12
x + 2y = 16
y = 0
−6 −4 −2 0 2 4 6 8 10 12 14 16
x-axis
−2
−4
x = 0
y = -4/3 x + 4

12.



6
5
4
3
2
1
0
-1
y(x = 0)
x + y = 4
R
-2 -1 0 1 2 3 4 5 6 7
x(y = 0)

13.



Required point.

(0,6) (1,6) (2,6) (1,5) (2,5) (3.5)

(2,4) (3,4) (4,4)(3,3) (4,3) (5,3)

(4, 2) (5, 2) (6,2) (5,1) (6,1) (5,0)

(6,0) (6-1)

Exercise 4.7

1. $y \geq 0,\ 3y + x < 24,\ y + x > 8$

2. $y < -4x + 40,\ y > \frac{4}{3}x,\ y < 4x$

3. (a) $x \geq 0,\ y > \frac{-2}{3}x + 2,\ y + 2x < 8$

   (b) $x \geq 0,\ 5y \geq -13x + 26,$

   $y > \frac{-2x}{5} + 4,\ y < \frac{-4}{3}x + 8,\ y \leq \frac{-5}{5}\ x + 5$

   (c) $x < 4.8,\ y < 5,\ y > -\frac{5}{3}x + 8,\ y < \frac{-4.4}{5}\ x + 2$

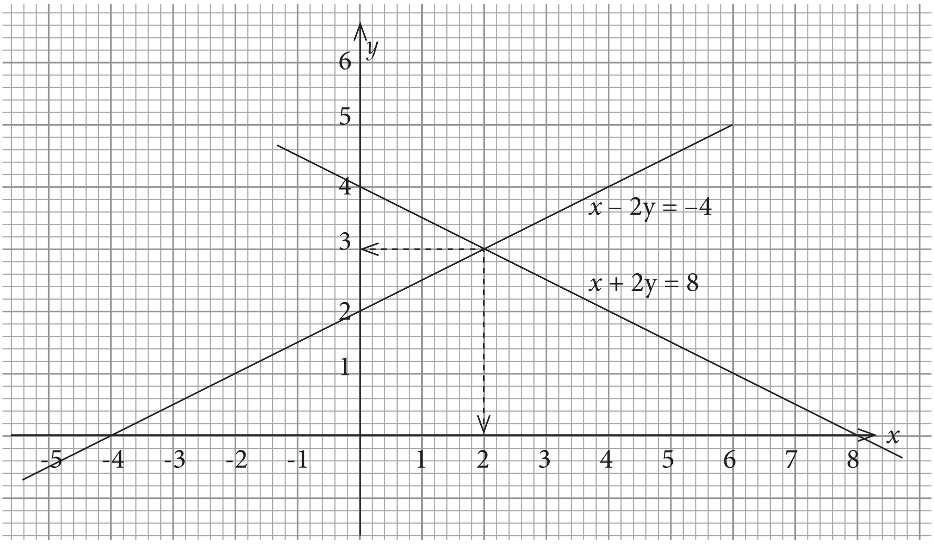
Unit 4 Test

1. (a) $x = 4,\ y = 4,$ (b) $x = 5,\ y = 2$ (c) $x = 2.5,\ y = -0.5$

2. (a) $x + y = 1$ (c) $3x = 5 - 2y$

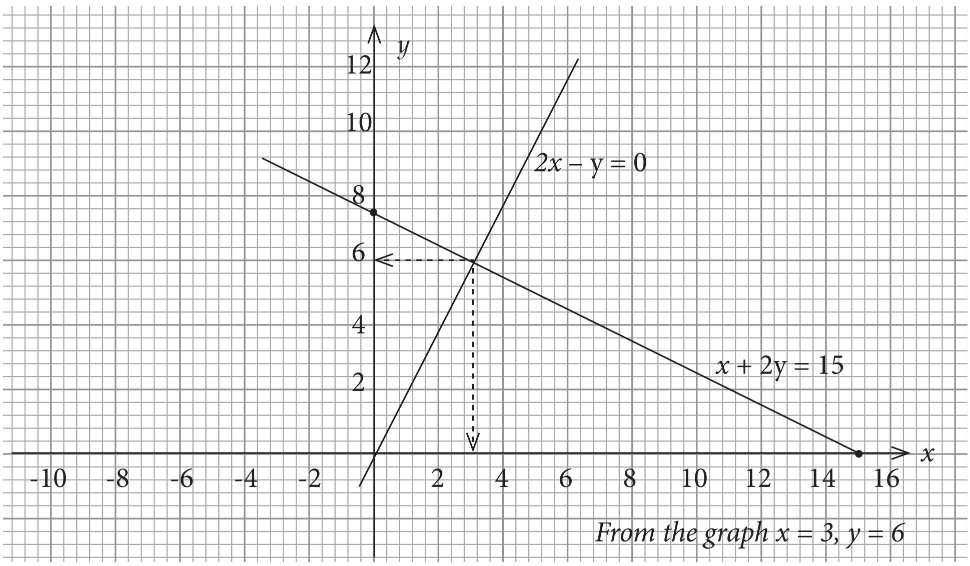
   $x - y = 5$ $x + y = 1$

3.

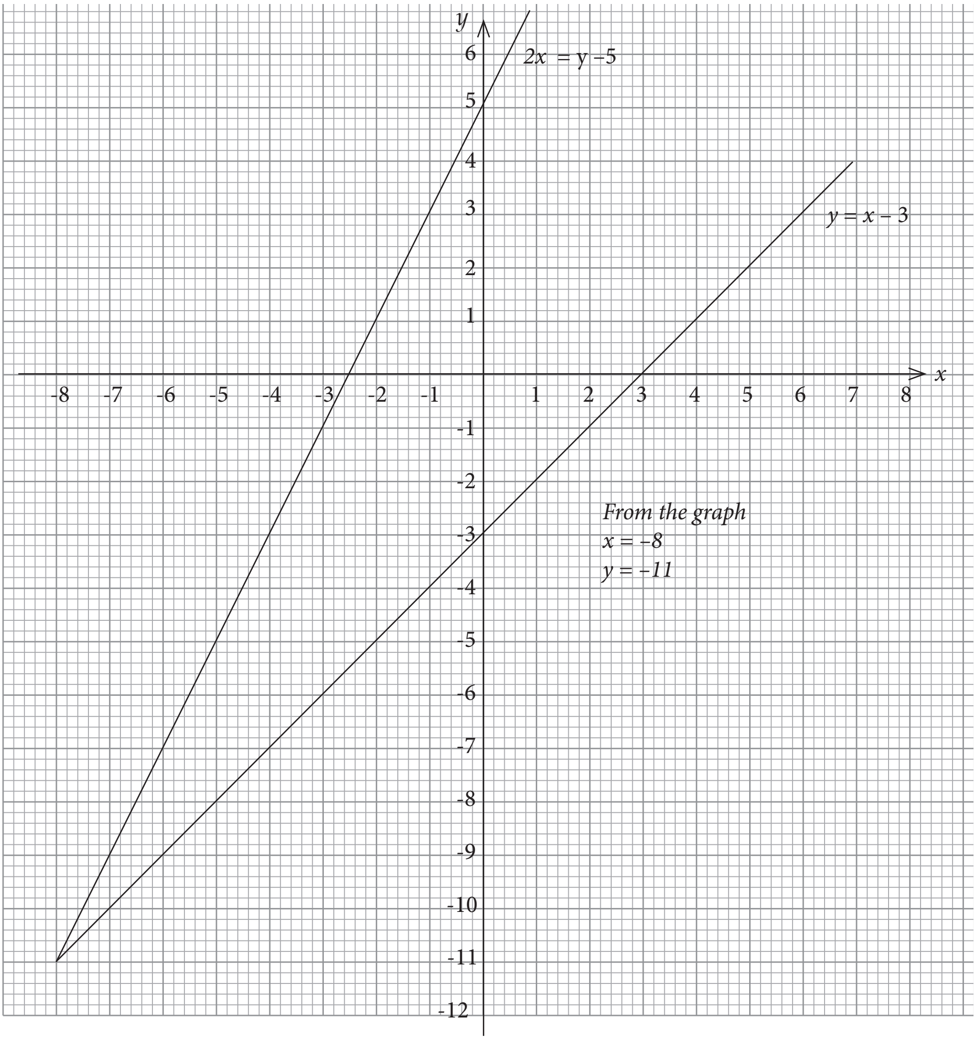

y
x
$x - 2y = -4$
$x + 2y = 8$
6
5
4
3
2
1
-5
-4
-3
-2
-1
1
2
3
4
5
6
7
8

$x = 2, y = 3$

4. (a)


y
x
$2x - y = 0$
$x + 2y = 15$
12
10
8
6
4
2
-10
-8
-6
-4
-2
2
4
6
8
10
12
14
16
From the graph $x = 3, y = 6$

(b)



5. Equation; $x + y = 34$

$$2x + 3y = 86$$

Ages: 16 and 18 years

James' present age is 16 years

David's Present age is 18 years

6. Equations: $5x + 3y = 1205$ ....(i)

$3x + 5y = 1107$ ....(ii)

x = 169, y = 120

Therefore,

1 box of sweet, costs 169FRW

1 bag of candles cost 120 FRW

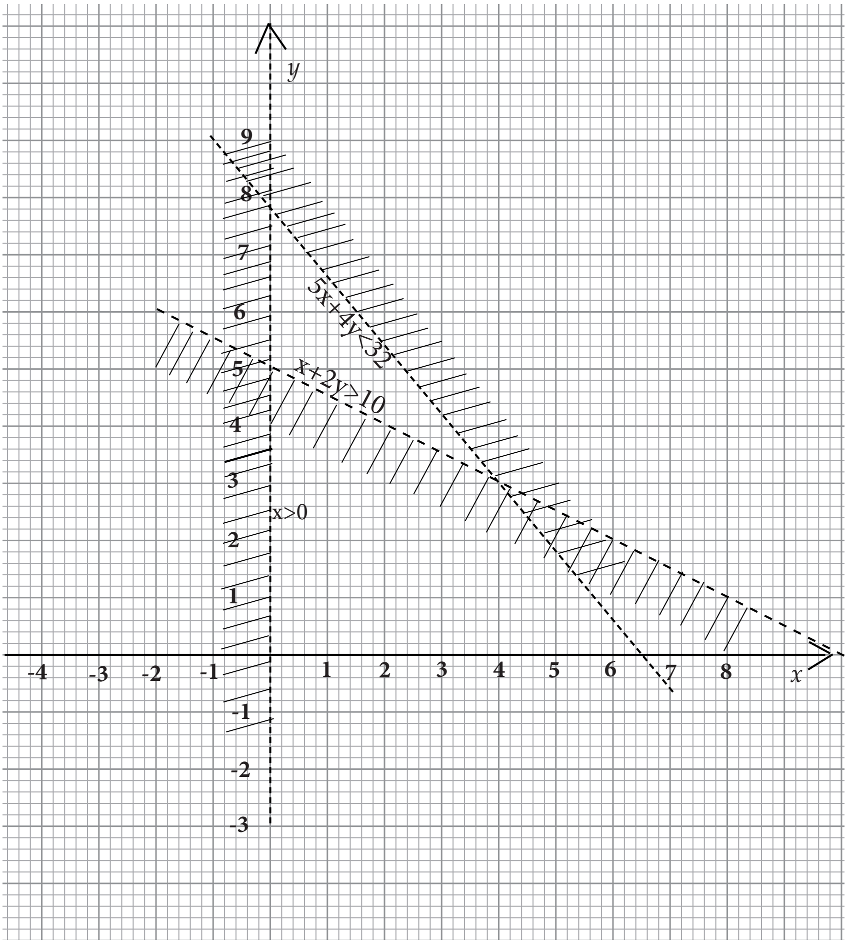
7. (a) $0 \leq y \leq 2$

(b) $y \geq x - 1$

(c) $\{y \geq -x + 2\} \cap \{y \leq x - 2\}$

(d) $\{y \geq -2x + 4\} \cap \{y \leq -\frac{2}{5}x + 4\} \cap \{y \geq x - 3\}$

8. (a)



(b) (4,3) (0, 5) (0,8)

Remedial Activities - Answers

1. (a) (2,2) (c) (-3,-2) (b) $\left(\frac{15}{4}, \frac{-13}{4}\right)$

3. (a) x = -3, y = -4

(b) x = 2, y = 1

(c) x =3, y = 0

4. Equtions (i) $\boxed{x + (x + 2)}$

(ii) $4x + 4 = 8$

Dimension: length = 3cm

Breathe = 1cm

5. One Book has 75 pages

second book has 425 pages

Equations: (i) $x + y = 500$ .... (i)

$x = y + 350$ or $x - y = 350$

$$\begin{cases} x + y = 500 \text{ ....(i)} \\ x - y = 350 \text{ ....(ii)} \end{cases}$$

UNIT 5

QUADRATIC EQUATIONS

Key Unit competence

By the end of this unit, learners should be able to define a quadratic equation, factorise quadratic equations, solve quadratic equations by factorisation, completing squares, graphical and formula methods, and apply quadratic equations in real life problems.

Learning objectives:

Knowledge and understanding

- Define quadratic equation
- Apply quadratic equations in problems.

Skills

- Solve quadratic equation by formula, graphical, completing squares and model problems involving quadratic equations.

Attitudes and values

- Appreciate how to solve quadratic equations and solve mathematical word problems involving them.
- Develop clear logic and coherent thinking while solving quadratic equations.
- Show patience, mutual respect tolerance, team spirit and curiosity in group practices while solving and discussing mathematical situations involving quadratic equations.

Cross-cutting issues to be addressed in this unit.

- **Financial Education**: This unit will involve learners working with some financial related problems. Kindly guide the learners how they can manage their finances.
- **Inclusive education**: A teacher should provide for learning of all learners including those with special needs. A teacher should use sign language for those with hearing difficulty and those who are dumb.
- **Environment and sustainability**: Since the unit will involve the learners working with graph papers, sensitize the learners on the importance of disposing such materials once safely once they are no longer needed to avoid polluting the environment.

Generic competences to be addressed in this unit

- **Critical thinking:** This competence will be achieved when the teacher involve learners in activities that involve opening up forming quadratic equations, and critically analyzing their solutions

- **Problem solving**: This competence will be achieved when the teacher involve learners in activities and exercises on determining the solutions to the quadratic equations with different methods.
- **Communication skills and listening**: This competence will be achieved when learners will be allowed to express themselves while presenting the discussion results to the whole class.
- **Leadership skills**: This competence will be achieved because during group discussions, each group will be having a leader and a secretary.

Vocabulary/keywords

In the course of learning the concepts in this unit, the learners will discover the meaning of the following new words:

- Horners rule
- Perfect square
- Factorise
- Quadratic formula

Guide the learners to understand the meanings of these words and construct and speak out mathematical statements involving them in order to master their meaning and usage.

Guidance on the problem statement

In order to focus the learner's attention into the general direction of the unit, the teacher should organize learner to do the following introductory activity in groups.

Guidelines to focus activity

- In order to give the learners a challenge that will motivate them to actively participate in the lessons for this unit in order to acquire the skills and knowledge envisaged in this unit, facilitate them to do the Unit focus Activity outlined in the Student's Book.
- The activity entails solving a real life problem on a group of people fund raising to start a self-help group. The question involves forming a quadratic equation, simplifying and solving it to get the required number of people.
- Let the learners do the activity in groups of three through a discussion and organise their working in the best way possible
- Ask them to present their findings to the rest of the class through a class discussion, as you and the rest of the groups evaluate how well they have worked out each step of the problem.
- It is most likely that only a few or no group will get the answer correctly. Appreciate all the pupils for the attempt then help them to understand why it very important to be active in all the lessons for this unit, in order to learn how to and form and solve problems quadratic equations using a verity of methods

- Use the setting of the activity to sensitize the learners on the importance of saving and borrowing money for investment. This will enhance their financial literacy.

Answers to unit focus activity

(a) Let the number of people be x.

Initial contribution: $\frac{800\,000}{x}$

When 4 people pull out: $\frac{800\,000}{x-4}$

To get extra 10,000 extra contribution

To get extra 10,000 extra contribution

$$\frac{800\,000}{x-4} - \frac{800\,000}{x} = 10\,000$$

$$\frac{800\,000\,x - (x-4)\,(800\,000)}{(x-4)\,x} = \frac{10\,000}{1}$$

$800\,000x - 800\,000x + 3{,}200\,000 = 10\,000\,(x^2 - 4x)$

$3\,200\,000 = 10\,000x^2 - 40000x$

Dividing by 10000 through

$320 = x^2 - 4x$

$x^2 - 4x - 320 = 0$

This can be solved by factor method.

$x^2 - 20x + 16x - 320 = 0$

$x\,(x-20) + 16(x-20) = 0$

$(x+16)\,(x-20) = 0$

Either: $x - 20 = 0 \Rightarrow x = 20$

or $x + 16 = 0 \Rightarrow x = -16$.

Since x represents the number of people it can not be a negative value. So, original number of people were 20.

(b) Amount of money contributed after withdrawal is $\frac{800\,000}{20-4} = \frac{800\,000}{16} = 50\,000$ FRW

Attention to special needs

- You should provide for learning of all learners including those with special needs.
- The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Prepare additional more challenging questions for these learners.
- For the slow learners, organise remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

List of lessons

Lesson No.	Lesson Title	No. of periods
1.	Definitions and examples of quadratic equations	1
2.	Factorising Quadratic equations	2
3.	Solving Quadratic Equations by factorization method	3
4	Solving Quadratic equations by graphical method	3
5	Perfect Squares	1
6.	Completing squares and solving quadratic equations	3
7.	Solving quadratic equations by formula method	2
8.	Solving quadratic equations by synthetic division method	2
9.	Problems involving quadratic equations	2
10.	Equations reducible to quadratic equations	2
11.	Unit assessment	3

Lesson development

Lesson 1: Definition and examples of quadratic equations

Lesson objective

By the end of the lesson, the learner should be able to understand and define quadratic equations.

Teaching Aids

- Books
- Pens and Pencils
- Chalks and Classroom chalkboard
- Calculators and other geometric instruments

Learning Activities

Activity 5.1

- Organise the learners in pairs. One learner must work as a secretary.
- Let the learners do Activity 5.1 in the Student's Book.
- Allow the learners to present their findings on the chalkboard through the group secretaries.
- Guide the learners where they go wrong after the presentations.
- Go ahead to briefly describe what a quadratic equation is to the whole class.

Answers for Activity 5.1

1. (i) $15x + 18$ (ii) $x^2 - 7x + 10$ (iii) $x^3 - 2x^2 - 5x + 6$

2. (i) 1 (ii) 2 (iii) 3
3. Expression (ii) $x^2 - 7x + 10$ is quadratic

Assessment of the lesson

- Allow the learners to work on Exercise 5.1 in the Student's Book and present their answers for marking.
- Correct those who are wrong by giving them more guidance.

Lesson 2: Factorising quadratic expressions

Lesson Objective

By the end of the lesson, the learner should be able to understand how quadratic equations can be solved.

Teaching Aids

- Books,
- Chalks and classroom chalkboard
- Pens and pencils
- Calculators and other geometric instruments

Learning Activities

Activity 5.2

- Organise the learners in groups of three and instruct them to do Activity 5.2 in the Student's Book.
- Make sure one learner is selected as the secretary to record down the findings.
- Let the groups present their findings through their secretaries.
- Observe the presentation and correct the learners where they are wrong.

Answers to Activity 5.2

1. To factorise means to remove the common factors and move them together in the same expression.
2. (a) $x(x + 1)$ (b) $2x(2x + 1)$ (c) $(x - 2)(x + 2)$
3. To factorise we move factors that are common together.

   So $x^2 + 2x + 3x + 6 = (x^2 + 2x) + (3x + 6)$

   $= x(x + 2) + 3(x + 2)$

   $= (x + 2)(x + 3)$
4. $x^2 - x - 12 = x^2 - 4x + 3x - 12$

   $= x(x - 4) + 3(x - 4)$

   $= (x - 4)(x + 3)$

Synthesis

- By observing the errors made by learners during presentation of Activity 5.2, explain to the learners the steps that are taken to factorise a quadratic expression.
- Guide the learners to explain more about factorization by doing Examples 5.1, 5.2 and 5.3 in the Student's Book.

Assessment of the lesson

- Having passed through Examples 5.1, 5.2 and 5.3, allow the learners to do observations in Example 5.4.
- Let the learners do Exercise 5.2 in the Student's Book. Correct them and assess whether learning objectives have been achieved.

Lesson 3: Solving quadratic equations by factorisation method

Lesson objective

By the end of the lesson, the learner should be able to acquire the knowledge of solving quadratic equations by factorisation method.

Teaching Aids

- Books
- Pens and pencils
- Chalks and classroom chalkboard
- Calculators and other geometric instruments

Learning Activities

Activity 5.3

- Organise the learners in groups of three and ask them to do Activity 5.3 from Student's.
- Make sure one learner acts as the secretary to record the findings.
- Let the learners present their findings in groups through their secretaries.
- Observe their presentations and correct the errors made.

Synthesis

- Explain to the learners how quadratic equations can be solved by factorization method by looking at the factors after factorization using Examples 5.5 to 5.10 in the Student's Book.
- Ensure that learners understand how these examples are calculated.

Answers to Activity 5.3

1. $x - 2 = 0$ and $x + 3 = 0$
   $x = 2$ or $x = -3$
   We get
2. $(x + 4)(x + 6) = 0$
   We have $x + 4 = 0$ or $x + 6 = 0$
   hence $x = -4$ or $x = -6$

3. $4x^2 - 2x = 0$ gives $2x(2x - 1) = 0$

then $2x = 0$ or $2x - 1 = 0$

hence $x = 0$ or $x = \frac{1}{2}$

Assessment of the lesson

- Let the learners do Exercise 5.3 in the Student's Book and correct them where they are wrong. This exercise will help you to know whether learning objectives have been achieved.
- Make sure all learners understand how this exercise is solved.

Lesson 4: Solving quadratic equations by graphical method

Lesson objectives

By the end of the lesson, the learner should be able to understand how to solve quadratic equations by graphical method.

Teaching Aids

- Calculators
- Exercise books
- Geometrical instruments
- Chalks
- Graph papers/graph books

Learning Activities

Activity 5.4

- Organise learners in pairs to work on Activity 5.4 in the Student's Book
- Make sure one learner acts as the secretary.
- Let the learners present their findings on the chalkboard through the secretaries
- Make clear observations and correct each group where they are wrong at the end of the presentation.

Synthesis

- Explain to the learners how tables of results are filled.
- Guide them through Examples 5.11, 5.12, 5.13, and 5.14 in the Student's Book for students to understand how concepts are being applied.

Answers to Activity 5.4

1. (a)

x	–3	–2	–1	0	1	2
y	7	0	–3	–2	3	12

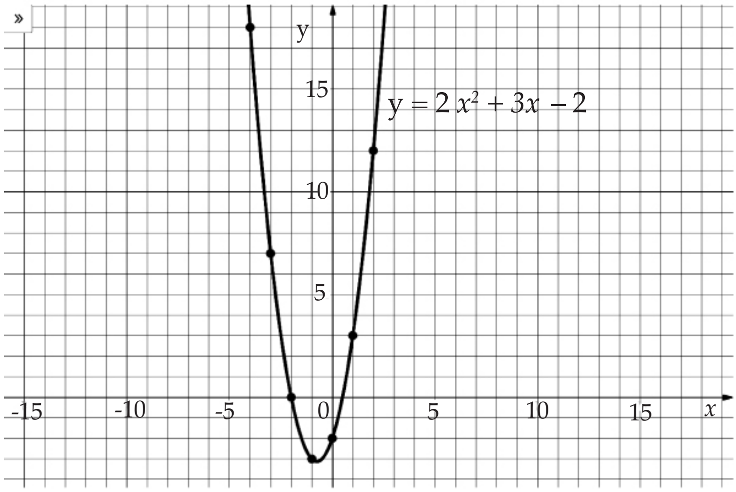
(b)

x	–3	–2	–1	0	1	2
y	6	4	4	6	10	16

2. (–3,7), (–2,0), (–1,–3), (0,–2), (1,3), (2,12)

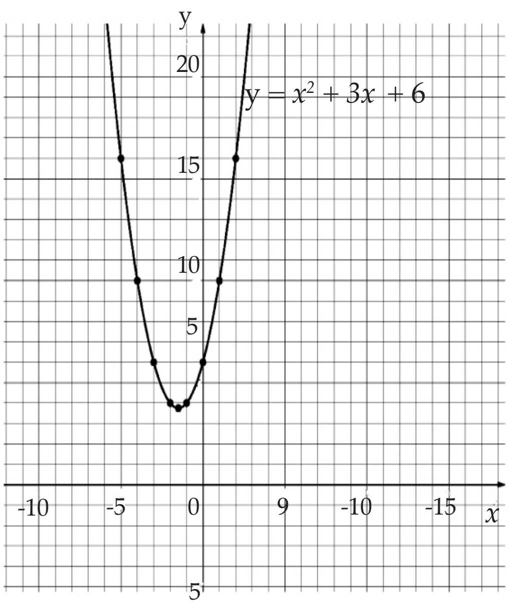
(–3,6), (–2,4), (–1,4), (0,6), (1,10), (2,16)

3. (a)



y
15
10
5
-15
-10
-5
0
5
10
15
x
y = 2 x² + 3x − 2

(b)



y
20
15
10
5
-10
-5
0
9
-10
-15
x
5
y = x² + 3x + 6

Assessment of the lesson

- Test the learners with Exercise 5.4.
- Make sure you correct learners where are wrong and encourage individual participations in doing the exercise.
- Let the learners do the rest of the questions in the exercise as the homework.

Lesson 5: Perfect squares

Lesson Objectives

By the end of the lesson, the learner should be able to understand the meaning of a perfect square and how to make a certain equation a perfect square.

Teaching Aids

- Calculators
- Exercise books, pens and pencils or any other writing materials.
- Chalks and class chalkboard.

Learning Activities

Activity 5.5

- Organise learners in pairs to work on Activity 5.5 in the Student's Book.
- Make sure one learner acts as the secretary.
- Let the learners present their findings on the chalkboard through the secretaries
- Make clear observations and correct each group where they are wrong at the end of the presentation.

Synthesis

- Explain how completing squares is done by using the general form of a quadratic equation.
- Guide the learners through Examples 5.15, 5.16 and 5.17 in the Student's Book for students to understand how concepts are being applied in perfect squares.

Answers to Activity 5.5

(a) Equation $x^2 + 8x + 16$ is a perfect square because $x^2 + 8x + 16 = (x + 4)^2$

(b) $x^2 + 10x + 12$ is not a perfect square.

(c) $9x^2 + 6x + 1$ is a perfect square because $9x^2 + 6x + 1 = (3x + 1)^2$

(d) $36x^2 – 8x + 4$ is not a perfect square.

Assessment of the lesson

- Having passed through the activity and examples, allow the learners with first four questions Exercise 5.5 in the Student's Book.
- Make sure you correct learners where are wrong and encourage individual participations in doing the exercise.
- Let the learners do the rest of the questions in the exercise as the homework.

Lesson 6: Completing squares and solving quadratic equations

Lesson objective

By the end of the lesson, the learner should be able to understand the how perfect squares are used in completing squares to solve quadratic equations.

Teaching Aids

- Calculators
- Exercise books, pens and pencils or any other writing material
- Geometrical instruments
- Chalks and classroom chalkboard

Learning Activities

Activity 5.6

- Organise learners in pairs to work on Activity 5.6 in the Student's Book.
- Make sure one learner acts as the secretary.
- Let the learners present their findings on the chalkboard through the secretaries
- Make clear observations and correct each group where they are wrong at the end of the presentation.

Synthesis

- Pass through the learners' presentations by explaining to them what a completing squares is and how it can be achieved.
- Ask the learners to discuss Examples 5.18 to 5.21 in Student's Book for the learners to digest the concepts.
- Ask learners to discuss Example 5.22 in the Student's Book while elaborating on the solution using completing square method when the coefficient of x^2 is not equal to one.

Answers to Activity 5.6

1. (a) Add 4, then $x^2 + 4x + 4 = (x+2)^2$

   (b) add $6x$, then $x^2 + 6x + 9 = (x+3)^2$

   (c) add $20x$, then $4x^2 + 20x + 25 = (2x+5)^2$

2. (a) $-3 + \sqrt{5}$ or $-3 - \sqrt{5}$ (b) The equation has no real roots.

Assessment of the lesson

- Test the learners with questions of Exercise 5.6 from the Student's Book.
- Make sure you correct learners where they are wrong and encourage individual participations in doing the exercise.
- Let the learners do the rest of the questions in the exercise as the homework.
- Ask the learners to attempt questions from Exercise 5.7 Student's Book.

Lesson 7: Solving quadratic equations by using formula method

Lesson objective

By the end of the lesson, the learner should be able to understand how to solve quadratic equations by formula method.

Teaching Aids

- Chalk
- Calculators
- Exercise books
- Geometrical instruments

Learning Activities

Activity 5.7

- Organise the learners in groups of three to do Activity 5.7 in the Student's Book.
- Let one of the learners to act as the class secretary to record the findings of the group.
- Let the class present their findings through the secretary and note the errors made.

Synthesis

- By borrowing knowledge from completing squares method, derive the quadratic formula for the learners to understand what they have to use in the lesson.
- Make sure the learners understand how the formula is derived and how it can be useful to them.
- Discuss with the learners Examples 5.23 and 5.24 in the Student's Book.
- Make emphasis on the signs of the coefficients for the quadratic equation since these signs can decide the answer one is supposed to get.

Answers to Activity 5.7

1. Make sure the coefficient of x^2 is one. This is done by dividing p throughout the given equation, $\mathrm{p}x^2 + qx + \mathrm{r} = 0$

$\frac{px^2}{p}+\frac{qx}{p}+\frac{r}{p}=\frac{0}{p}$..........(i) $\quad x^2+\frac{q}{p}x=\frac{-\mathrm{r}}{\mathrm{p}}$ ....(iii)

$x^2+\frac{q}{p}x+\frac{r}{p}=0$ ....(ii) $\quad x^2+\frac{q}{p}x+\left(\frac{q}{2p}\right)^2=\left(\frac{q}{2p}\right)^2-\frac{\mathrm{r}}{p}$....(iv)

Let L.H.S in (iv) be perfect square:

$$\left(\mathrm{x}+\frac{q}{2p}\right)^2=\frac{-r}{p}+\left(\frac{q}{2p}\right)^2 \qquad \text{(v)}$$

$$\sqrt{\left(\mathrm{x}+\frac{q}{2p}\right)^2}=\sqrt{\frac{-r}{p}+\left(\frac{q}{2p}\right)^2} \qquad \text{(vi)}$$

$$x+\frac{q}{2p}=\pm\sqrt{\left(\frac{q}{2p}\right)^2-\frac{r}{p}} \qquad \text{(vii)}$$

$$x = \frac{-q}{2p} \pm \sqrt{\frac{q^2}{4p^2} - \frac{r}{p}} \qquad \text{(viii)}$$

Simplifying the expression under the square root using LCM and taking square root of denominator we get

$$x = \frac{-q}{2p} \pm \sqrt{\frac{q^2 - 4pr}{4p^2}} = -\frac{q}{2p} \pm \frac{\sqrt{q^2 - 4pr}}{2p} \qquad \text{(ix)}$$

$$x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$$

2. By substituting the values of p, q and r, we get the values of x as 1 and 1.5.

3. The results obtained by factorization must be similar to the results obtained by completing squares.

   $2x^2 - 5x + 3 = 0$

   $(x - 1)(2x - 3) = 0$

   $\therefore x = 1 \text{ or } x = 1.5$

Assessment of the lesson

- Let the learners do questions of Exercise 5.8 in the Student's Book.
- Make sure they do the exercise in your presence so that you can correct their mistakes.

Lesson 8: Solving quadratic equations by synthetic division method

Lesson objective

By the end of the lesson, the learner should be able to know how to use synthetic division to solve quadratic equations.

Teaching aids

- Chalk
- Exercise books, pens, pencils and other writing materials.
- Calculators
- Geometrical instruments
- Internet connection

Learning Activities

Activity 5.8

- Organise the learners in pairs and provide them with internet connection.
- Let the learners do research on Activity 5.8 in the Student's Book.
- Let the learners present their findings through secretaries.
- Observe the presentations and be able to correct the errors made by the learners.

Synthesis

- Explain to the learners the steps of synthetic division.
- Use Examples 5.25 and 5.26 in the Student's Book to explain how the steps are performed to reach the final result of the equation.

Answers to Activity 5.8

Refer to the Student's Book step 5 of Activity 5.8.

Assessment of the lesson

- Let the learners individually do Exercise 5.9 in the Student's Book.
- Make sure you pass around the class marking and checking the errors made by the learners.
- Correct them where they are wrong and make sure each and every learner is satisfied with the methods.

Lesson 9: Problems involving quadratic equations

Lesson objective

By the end of the lesson, the learner should be able to formulate (model) problems involving quadratic equations and provide necessary solutions.

Teaching aids

- Chalk
- Calculators
- Geometrical instruments
- Exercise books

Learning Activities

Activity 5.9

- Organise the learners in groups of three to do Activity 5.9 in the Student's Book.
- Make sure one learner is the secretary to record down the findings.
- Let the learners observe the problem and record down their findings to the class through the class secretaries.
- Observe how learners are presenting their findings with the real–life problem modelled out of the activity.

Synthesis

- Explain well to the learners the steps they can take to modal a quadratic equation out of real life problems.
- Use Examples 5.27 and 5.28 in the Student's Book to explain how quadratic equations find their way in real life problems.
- Remember to tell learners that some problems involve real–life issues and they can never give negative quantities as seen in Examples 5.27 and 5.28.

Answers to Activity 5.9

There are two speeds to think about: the speed the boat makes in the water, and the speed relative to the land:

Let x = the boat's speed in the water (km/h)

Let y = the speed relative to the land (km/h)

Because the river flows downstream at

(a) When going upstream, v = $x - 2$ (its speed is reduced by)

(b) When going downstream, v = $x + 2$ (its speed is increased by)

(c) We can turn those speeds into times using:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

And we know the total time is 3 hours:

Total time = time upstream + time downstream = 3 hours

Put all that together:

$$\text{Total time} = \frac{15}{(x+2)} + \frac{15}{(x-2)} = 3 \text{ hours}$$

(d) Now we use our algebra skills to solve for "x".

First, get rid of the fractions by multiplying through by $(x - 2)(x + 2)$

$3(x - 2)(x + 2) = 15(x + 2) + 15(x - 2)$

Expand everything:

$3(x^2 - 4) = 15x + 30 + 15x - 30$

Bring everything to the left and simplify:

(e) The name of the expression obtained is **Quadratic expression**.

(f) To get the ships speed and upstream journey, we need to get the value of x.

Solving, $3x^2 - 30x - 12 = 0$ we get $x = -0.39$ and $x = 10.39$

$x = -0.39$ makes no sense for this real world question because there are no negative speed but $x = 10.39$ is just perfect!

Boat's Speed = 10.39 km/h (to 2 decimal places)

And so the upstream journey = $\frac{15}{(10.39-2)}$ = 1.79 hours = 1 hour 47 minutes

And the downstream journey = $\frac{15}{(10.39+2)}$ = 1.21 hours = 1 hour 13 minutes

Assessment of the lesson

- Let the learners do questions of Exercise 5.10 in the Student's Book on individual basis.

- Make sure you pass around the class checking the errors made by individuals and giving them necessary guidance where necessary.

Summary of the unit

- Asking different learners to take the different concepts highlighted in the unit summary given in the Student's Book.
- Ask the class probing questions to help them recall the concepts correctly.

End of unit assessment

- A teacher should ask learners to do unit 5 test in the Student's Book to assess their understanding of the whole unit.
- A teacher should take time to correct the unit test to find out whether the concepts of the whole unit were properly digested.

Additional information to the teacher

- A teacher should know the concepts concerning different types of equations to assist him/her in making learners understand this unit.
- A teacher should as well teach the learners well how to use their geometrical instruments especially the calculators in this unit.

Remedial activities

For the slow learners, a teacher should organize remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

Remedial Exercise

1. The length of a rectangle exceeds the width by 7 cm. If the area is 60 cm^2, find the length of the rectangle.
2. Solve the quadratic equation $200x^2 + 1000x - 10\,000 = 0$ with the aid of factorization method.
3. Solve the following by synthetic division method.

   (a) $x^2 + 10x = 24$ (b) $v^2 - 36 = 0$

   (c) $x^2 = 4x - 3$ (d) $4x^2 - 9 = 0$
4. Solve the following by completing squares method.

   $x^2 - 2x - 8 = 0$ (b) $x^2 - 5x + 2 = 0$

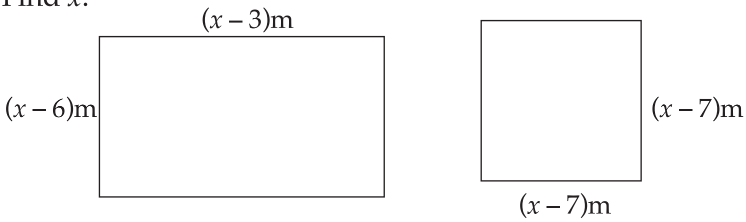
Extended activities

The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Ask learners to do the following questions.

Extended Exercises

1. The area of the rectangle exceeds the area of the square by 28 m^2.

   Find x.



2. The sum of two numbers is 27 and their product is 50. Find the numbers.
3. The length of a rectangle is 5 cm more than its width and the area is 50 cm^2. Find the length, width and the perimeter.
4. The three sides of a right-angled triangle are x, $x + 1$ and 5 cm. Find the value of x and the area, if the longest side is 5.
5. The product of two numbers is 24 and the mean is 5. Find the numbers.
6. A ball is thrown upwards from a rooftop, 80 m above the ground. It will reach a maximum vertical height and then fall back to the ground. The height of the ball from the ground at time t is h, which is given by, $h = -16t^2 + 64t + 80$.

   (a) What is the height reached by the ball after 1 second?

   (b) What is the maximum height reached by the ball?

   (c) How long will it take before hitting the ground?

Solutions to unit exercises, unit test, remedial exercises and extended exercises

Exercise 5.1

1. (a) $x^2 + 4x + 3$: quadratic (b) $x^3 - 3x^2 - 6x + 8$: Not quadratic

   (c) $x^3 - 5x^2 - 17x + 21$: Not quadratic (d) $x^3 - 7x^2 + 12x$: Not quadratic

   (e) $a^3 - 3a^2 + 8a - 24$: Not quadratic (f) $p^2 - 3p - 10$: quadratic
2. b, d, e, f
3. (a) False (b) True (c) True (d) True (e) True

   (f) True (g) True (h) True
4. (a) $x^2 + 5x - 50 = 0$ (b) Quadratic expression
5. (a) $x^2 + 21x + 108$ (b) Quadratic expression

Exercise 5.2

1. (a) $(x + 4)(x + 2)$ (b) $(x - 5)(x - 3)$ (c) $(x + 7)(x - 5)$

   (d) $(x - 3)(x + 3)$ (e) $(x - 7)(x + 7)$ (f) $(x + 4)(x - 3)$
2. (a) Yes (b) Yes (c) No (d) No
3. (a) $x^2 + 5x - 5 = 0$

(b) Quadratic expression

(c) $(x + 10)(x - 5) = 0$

4. (a) $w^2 - 35w - 300 = 0$ (b) $(w - 15)(w - 20) = 0$

Exercise 5.3

1. (a) –4 or –2 (b) 5 or 3 (c) 5 or –7 (d) $x = 3$ *or* -3
2. (a) –7 or –2 (b) 12 or –1 (c) –3 (d) 7 or –6
   (e) 1 (f) 0 or –8
3. (a) True, –12 and 2 (b) True, 3 and 1 (c) true, $\frac{5}{2}$ and $\frac{7}{3}$ (d) False

Exercise 5.4

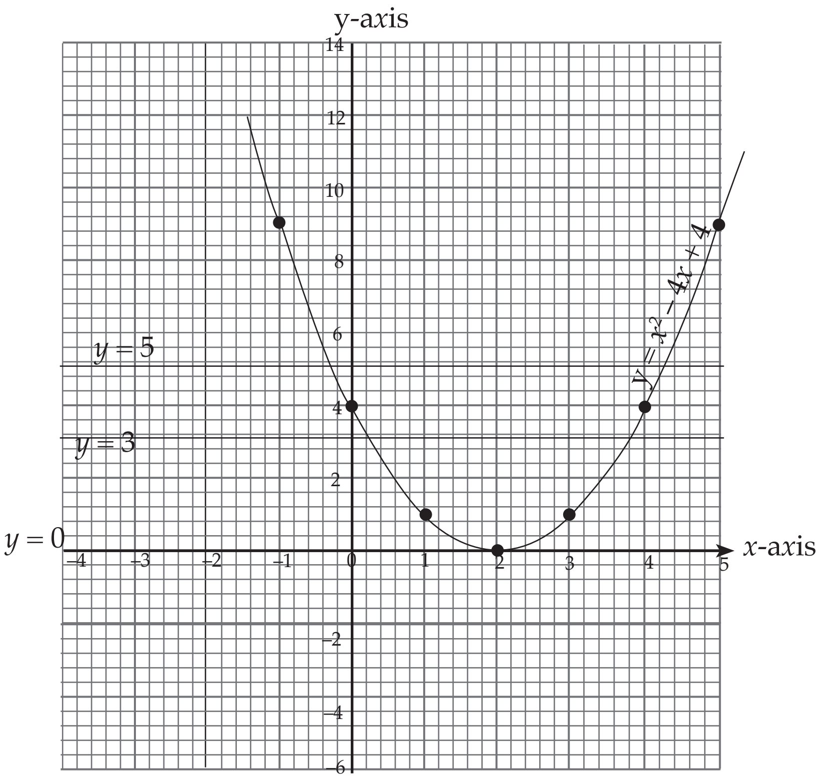
1. (a)

Table of values

x	–1	0	1	2	3	4	5
$y = x^2 - 4x + 4$	9	4	1	0	1	4	9

Co-ordinates

(–1,9), (0,4), (1,1), (2,0), (3,1,), (4,4) and (5,9)

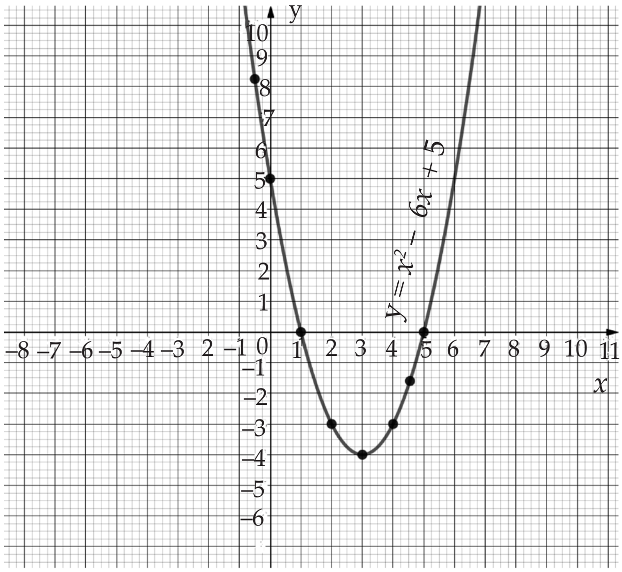


(a) $x = 2$

(b) $x = 0.3$ or $x = 3.8$

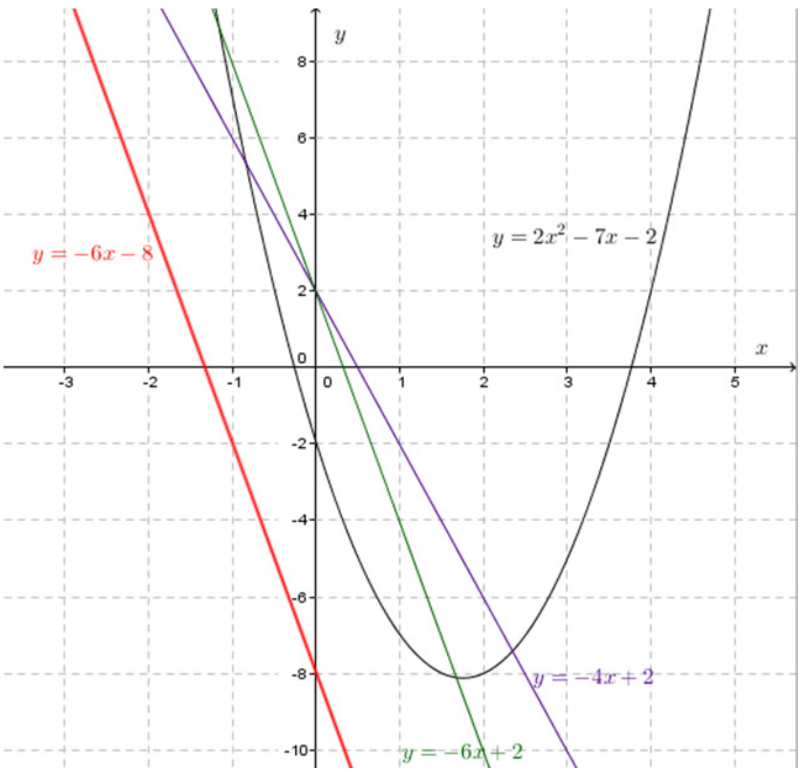
(c) $x = 4.2$ or -0.2

2.



The roots are $x = 1$ and $x = 5$ because those are the values where the graph meets the x-axis.

3.

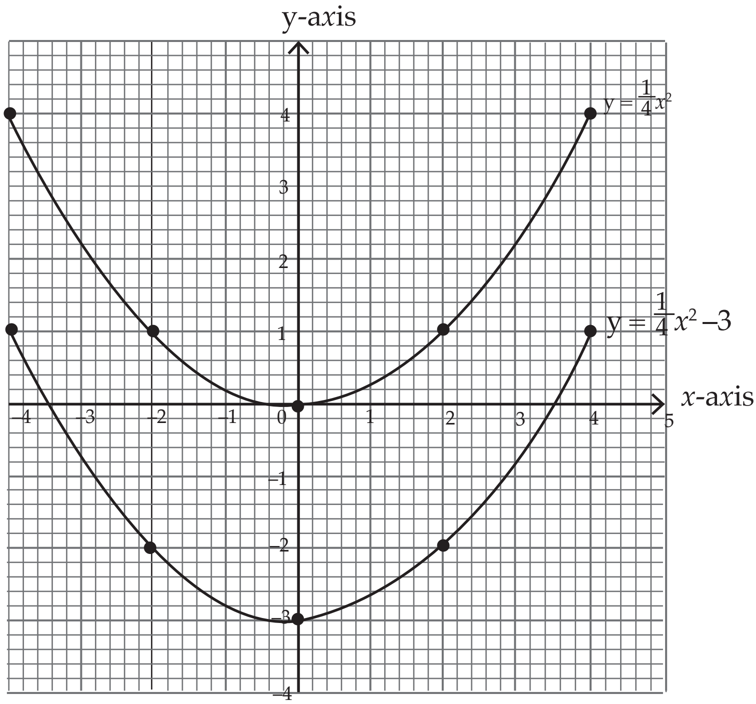


(a) 1.6 or –1.3

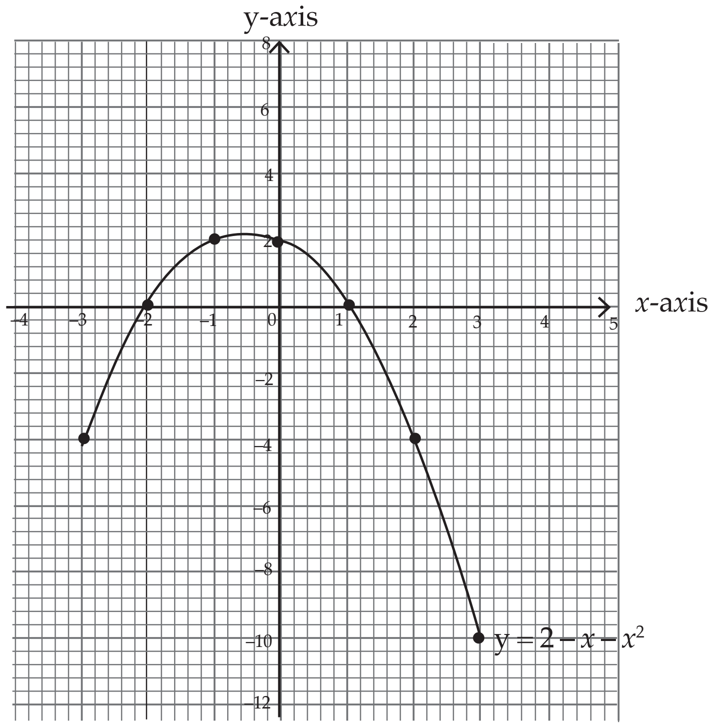
(b) No real solution

(c) 2.4 or –0.8

4.

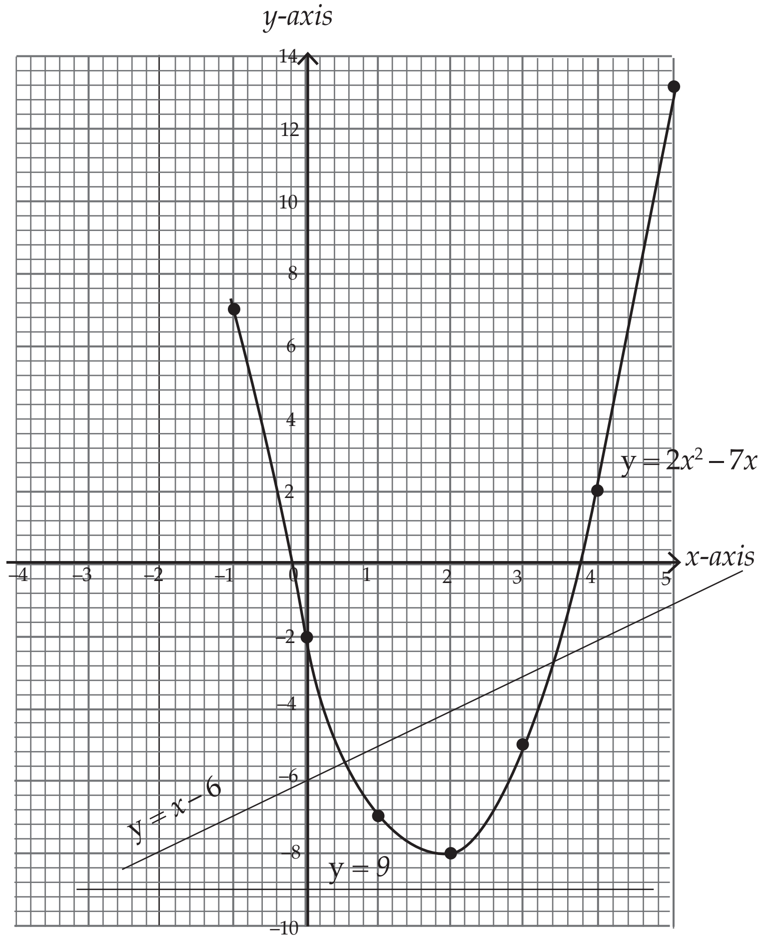


5.



$x = -2$ or 1.

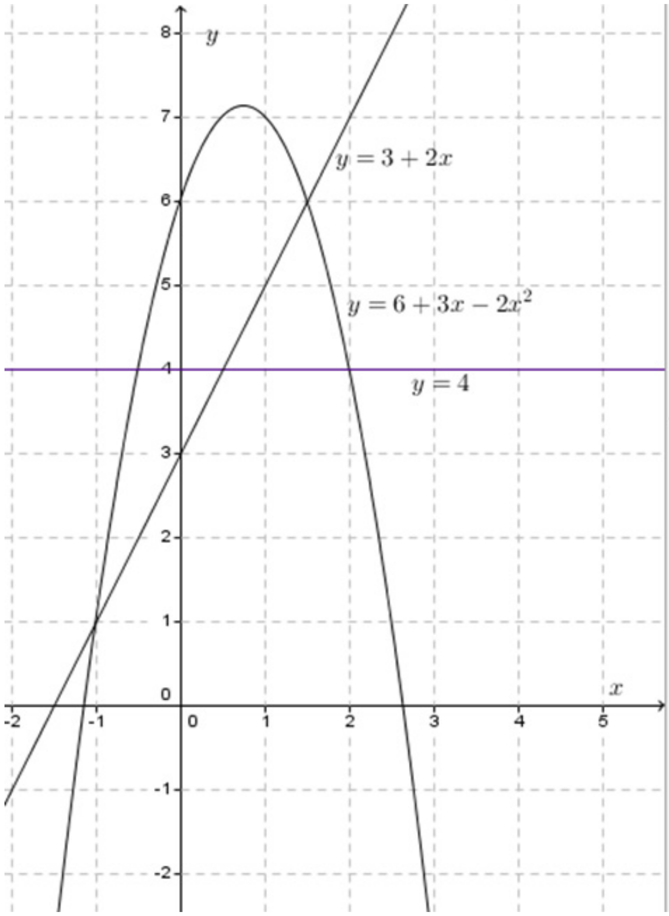
6.



(a) 3.8 or – 0.3 (b) x = 3.4, or x = 0.6 (c) No real values

7.

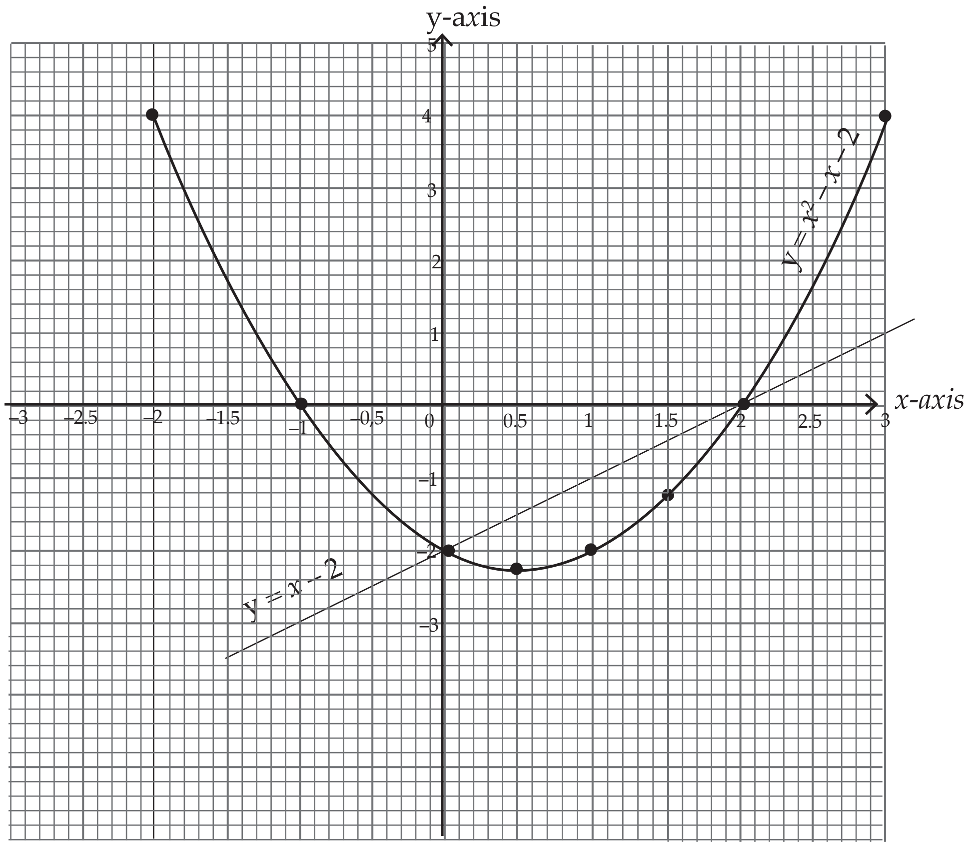
x	–2	–1	0	0.5	1	2	3
$3x$	–6	–3	0	1.5	3	6	9
6	6	6	6	6	6	6	6
$-2x^2$	–8	–2	0	–0.5	–2	–8	–18
y	–8	1	6	7	7	4	–3

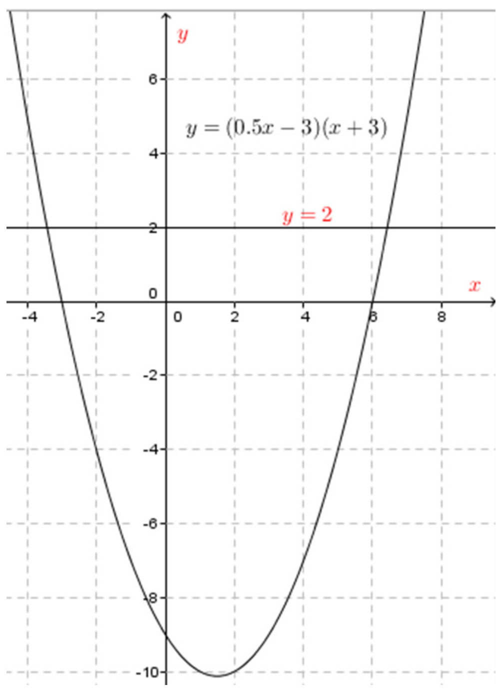


(a) 2.63 or –1.1 (b) 2 or –0.5 (c) –1 or 1.6

8.

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3
x^2	4	2.25	1	0.25	0	0.25	1	2.25	4	6.25	9
$-x$	2	1.5	1	0.5	0	–0.5	–1	–1.5	–2	–2.5	–3
–2	–2	–2	–2	–2	–2	–2	–2	–2	–2	–2	–2
y	4	1.75	0	–1.25	–2	–2.25	–2	–1.25	0	1.75	4

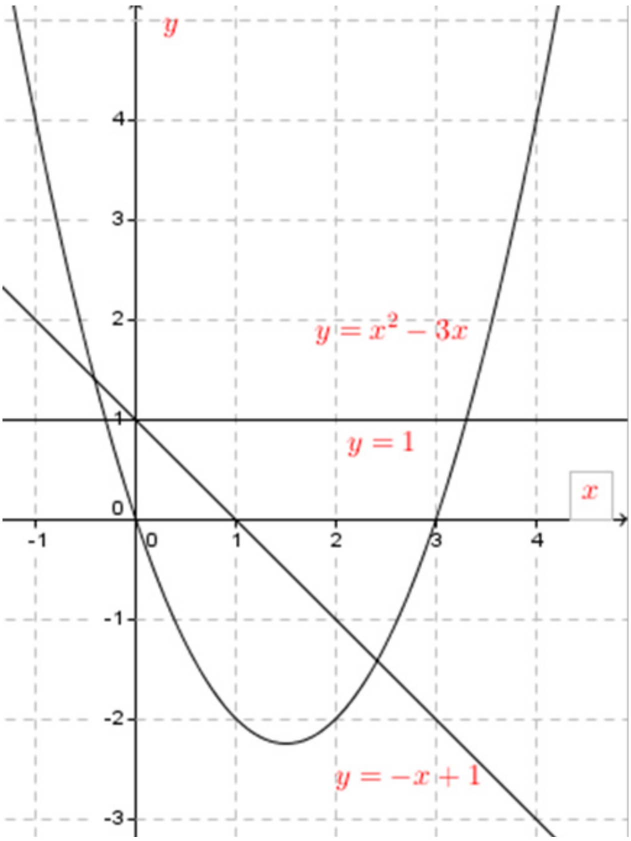


9. 

–3.5 or 6.5

10.

x	–1	0	1	2	3	4
$y = x^2 - 3x$	4	0	–2	–2	0	4

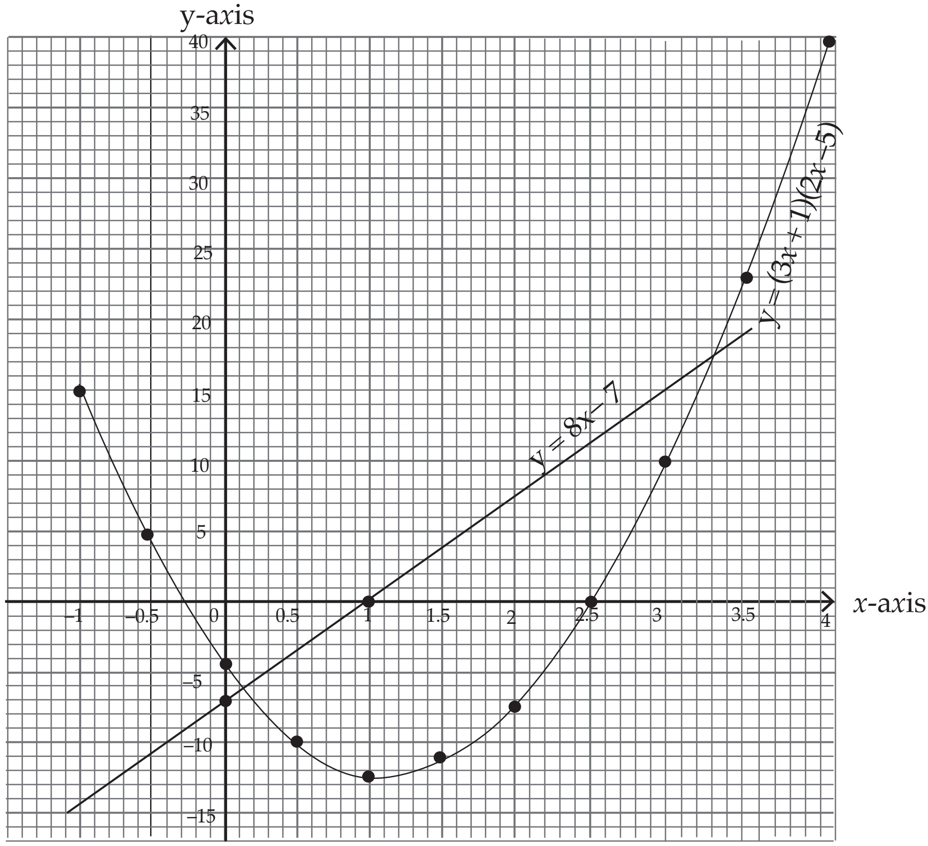


(a) from 0 to 3.

(b) -0.4 or 2.4 and 3.3 or –0.3.

11. For missing values of y.
-5, -10, -7, 0, 10 and 39

Solution for $6x^2 - 21x + 2$ is 3.4 or 0.1



Exercise 5.5

1. (a) $x^2 + 10x + 25$ (b) $x^2 - 12x + 36$ (c) $9 - 6x + x^2$ (d) $y^2 + 8y + 16$
   (e) $4x^2 + 36x + 81$ (f) $4y^2 - 28y + 49$ (g) $16x^2 - 8xy + y^2$ (h) $25y^2 - 40yz + 16z^2$
   (i) $x^2 + 2 + \frac{1}{x^2}$
2. (a) Yes, $(x+3)^2$ (b) No (c) Yes, $(x-4)^2$ (d) Yes, $(x-1)^2$ (e) No
   (f) No (g) No (h) Yes, $(t+1)^2$ (i) No (j) Yes, $(5p-7q)^2$

Exercise 5.6

1. (a) 25, $(y+5)^2$ (b) 16, $(x-4)^2$ (c) $9q^2$, $(p+3q)^2$ (d) $\frac{25}{4}, \left(d+\frac{5}{2}\right)^2$
   (e) $36b^2$, $(a-6b)^2$, (f) $\frac{49}{4}, \left(q-\frac{7}{2}\right)^2$ (g) $\frac{1}{4}, \left(n+\frac{1}{2}\right)^2$ (h) $\frac{n^2}{4}, \left(m-\frac{n}{2}\right)^2$
2. (a) 7 or –3 (b) –5 or 2 (c) $\frac{-3+\sqrt{53}}{2}$ or $\frac{-3-\sqrt{53}}{2}$
   (d) $-2+\sqrt{8}$ or $-2-\sqrt{8}$ (e) $\frac{-3+\sqrt{17}}{2}$ or $\frac{-3-\sqrt{17}}{2}$ (f) 5
   (g) $7+\sqrt{47}$ or $7-\sqrt{47}$ (h) $\frac{15+\sqrt{241}}{2}$ or $\frac{15-\sqrt{241}}{2}$

Exercise 5.7

1. (a) 4 or –2 (b) 0.44 or 4.56 (c) 2 or –4 (d) 0.41 or –2.41

(e) 5 or 0 (f) –2.82 or –0.18

2. a) $x^2 - 8x + 16 = 7 + 16$

b) $x^2 + \frac{1}{2}x + \frac{1}{16} = 3 + \frac{1}{16}$

c) $x^2 - \frac{5}{2}x + \frac{25}{16} = -1 + \frac{25}{16}$

d) $x + 4x + 4 = 0 + 4$

e) $x^2 - \frac{2}{3}x + \frac{4}{36} = 2 + \frac{4}{36}$

f) $x^2 + \frac{1}{2}x + \frac{1}{16} = 0 + \frac{1}{16}$

3. (a) $x = 3$ or -1.50 (b) $x = 1.71$, $x = 0.29$ c) No real value

d) $x = 1.87$, or $x = 0.13$ e) 1.5 or 0.5 f) $x = 2.32$, $x = -0.32$

g) $x = 2.76$, or $x = 0.24$ h) $a = 4.16$, $a = -0.16$ i) $x = -0.07$, $x = -2.93$

j) $z = -0.56$, $z = -4.44$

Exercise 5.8

1. (a) $\frac{-1}{2}$ or –5 (b) –0.67 or –3 (c) $\frac{-2}{3}$ or $\frac{-1}{2}$ (d) 0.33 or 3
   (e) 1 or 0.40 (f) 1.50 or 0.33 (g) –0.63 or –2.37
   (h) –0.27 or –3.73
2. (a) (ii) (b) (iv) (c) (i)
3. (a) true (b) false (c) False (d) true (e) False

Exercise 5.9

1. –4 or 3 2. –2 or –7 3. –3 4. 1 5. –12 or 2
6. –6 or 6 7. 12 or –1 8. 7 or –6 9. 0 or –8 10. 3 or 1
11. –1.5 or 1.5

Exercise 5.10

1. 11 and 8 or –8 and –11 2. 11 and 13 or –11 and –13 3. 12cm 4. 6cm

5. 11m

6. 24cm by 10cm

7. 8km north, 15km east

8. 287 Eggs

9. 4

10. 5 and 2

Unit 5 Test

1. $\frac{40}{x}$ hours, $\frac{40}{x-2}$, 10

2. 60km/h

3. 156.524km

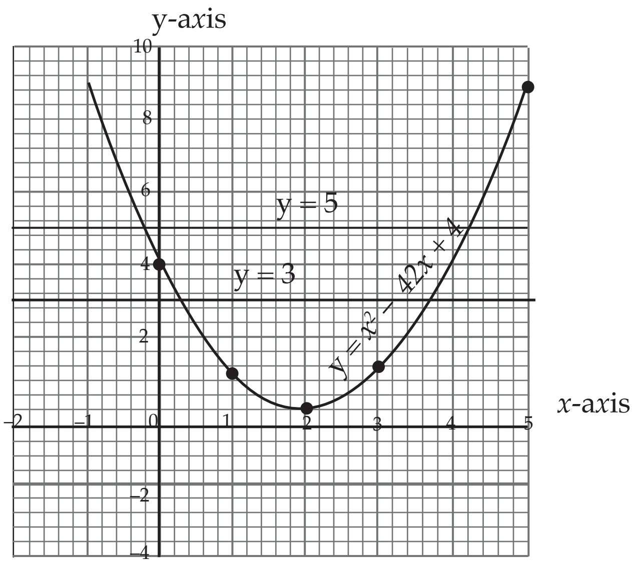
4. 2cm

5. $\frac{3}{4}$

6. 9cm

7. 14.2cm

8.



(a) The value of x is 2.

(b) Plotting the line y = 3 on the same axes with the graph above will meet the graph at 3.7 and 0.3 and these are the values of x.

(c) 4.2 or –0.2

9. (a) –4 or 3 (b) –7 and –2

Answers for Remedial Activities

1. Length of the rectangle is 12cm and width is 5cm.
2. $x = 5$ or $x = -10$
3. (a) $x = 2$ or $x = -12$ (b) $V = -6$ or $V = 6$

   (c) $x = 3$ or $x = 1$ (d) $x = \frac{-3}{2}$ or $x = \frac{3}{2}$
4. (a) $x = 4$ or $x = -2$
5. $\frac{5 + \sqrt{17}}{2}$ or $\frac{5 - \sqrt{17}}{2}$

Extended Activites

1. $x = \frac{59}{5}$
2. Two numbers are 2, 25 or 25, 2
3. Width 5cm, length 10cm, perimeter 30cm.
4. $x = 3$ cm

   so, sides are; 3cm, 4 cm, 5cm

   Area = $\frac{1}{2}$ bh = $\frac{1}{2} \times 3 \times 4 = 6cm^2$
5. Numbers are 4 and 6 or 6 and 4
6. (a) After 1 second, h = 128 m

   (b) Maximum height is 144m

   (c) 5 seconds.

UNIT
6 LINEAR AND QUADRATIC FUNCTIONS

Key unit competence

By the end of this unit, learners should be able to solve problems involving linear or quadratic functions and interpret the graphs of quadratic functions.

Knowledge and understanding

- Learners should be able to define a Cartesian equation of a straight aline. Linear equations.
- Define quadratic function.
- List the characteristics of linear or quadratic function.
- Differentiate linear from quadratic functions.

Skills

- Determine Cartesian equations of straight lines, coordinates of vertices, and the equation of axis of symmetry.
- Determine the intercepts of a quadratic function.
- Sketch and draw graphs from a given function.
- Use linear or quadratic function to solve problems in various situations and interpret the results. Part of this is seen in unit 5 of this book.

Attitude and values

- Appreciate how to solve linear and quadratic functions and mathematical problems involving the functions.
- Develop patience, mutual respect, tolerance and team spirit in teamwork while solving and discussing mathematical problems involving the functions.

Generic competences addressed in this unit

The specific generic competences to be addressed in this unit are:

- **Critical thinking**: This competence will be achieved when the teacher involves learners in activities that involve finding slopes, lines, vertices and sketching.
- **Problem solving**: This competence will be achieved when the teacher involves learners in activities and exercises on determining the slopes, equations of straight lines, parallel and perpendicular lines, vertices and sketching without table of values
- **Communication and listening skills**: This competence will be achieved because learners will be used to present their findings to class through discussions. This will develop self-confidence within the learners
- **Leadership skills**: This competence will be achieved because each group will have the chairperson and the secretary who will exercise the leadership skills.

Cross-cutting issues addressed in this unit.

The following cross-cutting issues are addressed in this unit.

- **Peace and value education**: this will be achieved by encouraging learners to work together without discriminating each other.
- **Gender studies**: This will be achieved through ensuring gender balance in group discussions in class. Some parts of exercises talk about gender issues.
- **Inclusive education**. attention will be given to learners with special needs inorder to make them integrated in the class.

Vocabulary/keywords

In the course of learning the concepts in this unit, the learners will discover the meaning of the following new words:

- Gradient or slope
- Intercepts
- Vertex
- Parallel
- Perpendicular

Guide the learners to understand the meanings of these words and construct and speak out mathematical statements involving them in order to master their meaning and usage.

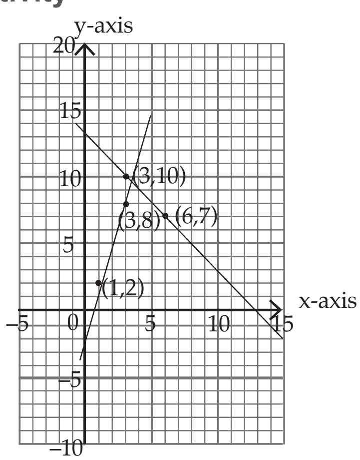
Guidance on the problem statement

In order to focus the learner's attention into the general direction of the unit, the teacher should organise learner to do the following introductory activity in groups.

Unit Focus activity

- Organise the learners in groups of three and make sure one leader is working as the secretary to record the findings.
- Let the learners do unit focus activity in Student's Book.
- Allow the group leaders to present their findings through class discussions.
- Observe the presentations and note down the errors they make.
- The unit focus activity will draw the attention of the learners and they will be ready to digest each and every idea coming along their way in this unit.

Answers to unit focus activity



1. From the line, Gradient AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{7-10}{6-3} = \frac{-3}{3} = -1$

Gradient CD = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{3-1} = \frac{6}{2} = 3$

Line CD is steeper.

2. (a) Learners should appreciate that a line meets x-axis when y = 0 and y axis when x=0.

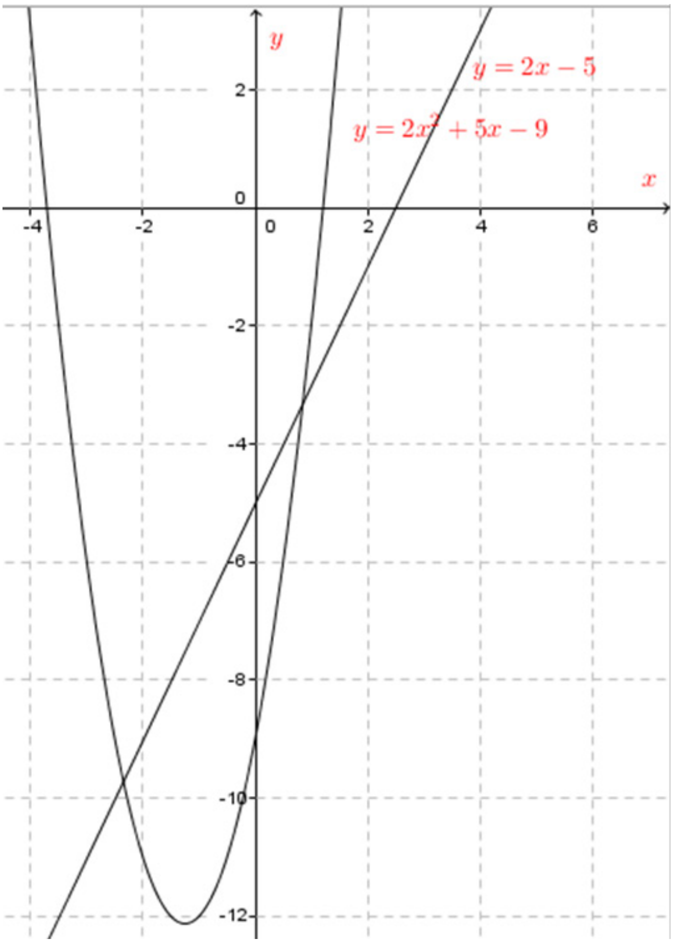
When x = 0, $y = 9$ and we get (0,9)

When y = 0, we get x= –6 and so, (–6,0)

(b) = $A : y = \frac{-2}{3}x + 9$

(c) = $B : y = \frac{3}{2}x + 9$

3.



(a) $x = 1.21$, $x = -3.71$

(b) line $x = -1.25$

(c) $x = 0.85$, $x = -2.35$

Attention to special needs

- You should provide for learning of all learners including those with special needs.
- The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Prepare additional more challenging questions for these learners.
- For the slow learners, organise remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

List of lessons

Lesson No.	Lesson Title	No. of periods
1.	Introduction	1
2.	Slopes/gradients of a linear function	2
3.	General form of Cartesian equation of a straight line introduction	1
4.	Finding equation of a straight line given gradient and one point on the line	2
5.	Equation of a straight line joining two points	2
6.	Parallel lines	2
7.	Perpendicular lines	2.
8.	Quadratic functions	1
9.	Table of values	3
10.	Vertex of a quadratic function and axis of symmetry	2
11.	Intercepts, vertices and sketching a quadratic function	3
12.	Unit assessment	3

Lesson development

Lesson 1: Definition of linear functions

Lesson objective

By the end of the lesson, the learner should be able to plot the graph of a linear function at the end of this lesson.

Teaching Aids

- Rulers

- Chalks ad class chalkboard
- Graph papers
- Pencils, pens and other writing materials
- Calculators and other geometrical instruments

Learning Activities

Activity 6.1

- Organise the class in groups of three and make sure that one of them is acting as the secretary to record down the findings
- Provide the learners with graph papers.
- Ask learners to carry out Activity 6.1 given in the Student's Book i.e. to complete the table given and plot them on graph papers.
- When the learners have completed the activity, ask secretary to present their findings in a class discussion and allow other members of the class to point out any errors in the presentation.

Synthesis

- Point out the errors made during presentation and guide the learners accordingly.
- Summarise the presentation by emphasizing on the key points when plotting a graph and explaining to the learners what a linear function is.

Answers to Activity 6.1

x	–3	–2	–1	0	1	2	3
$y = x^2 - 1$	8	3	0	–1	0	3	8

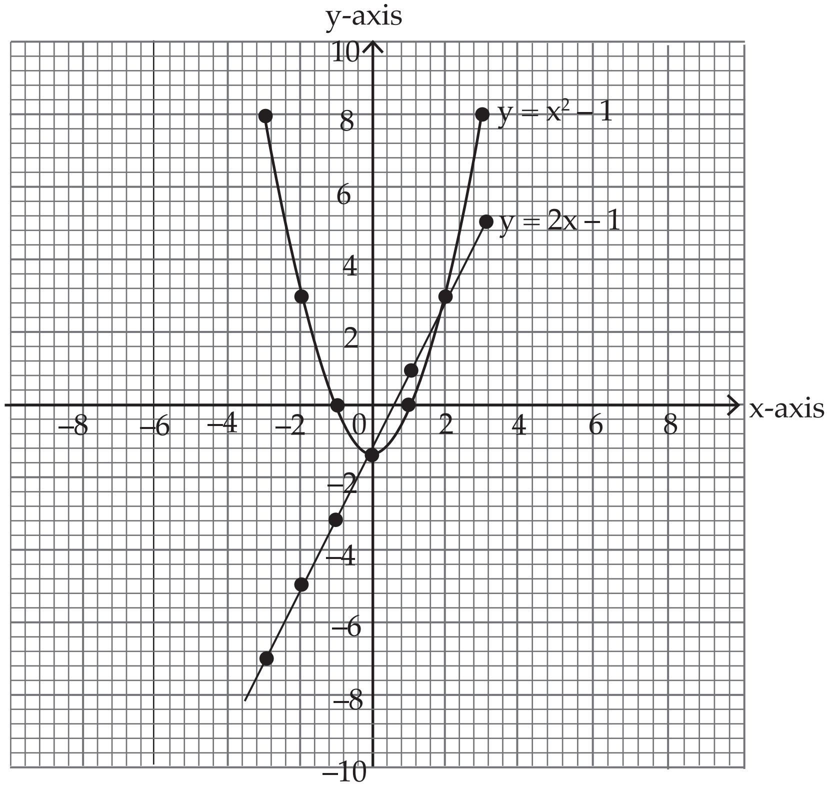
x	–3	–2	–1	0	1	2	3
$y = 2x - 1$	–7	–5	–3	–1	1	3	5

Coordinates are:

(–3,8), (–2,3), (–1,0), (0, –1), (1,0), (2,3), (3,8)

(–3,–7), ((–2,–5), (–1,–3), (0,–1), (1,1), (2,3), (3,5)

Use these co-ordinates to plot the graphs on same axes.



3. The graph of $y = 2x - 1$ is a straight line.

   The graph of $y = x^2 - 1$ is a u-shaped because it is quadratic in nature.

Lesson 2: Slope/Gradient of a linear function

Lesson objective

By the end of the lesson, the learner should be able to understand the gradient of a straight line.

Teaching Aids

- graph paper
- Chalks and classroom chalkboard
- pencil pens
- chats
- calculators and other geometrical instruments

Learning Activities

Activity 6.2

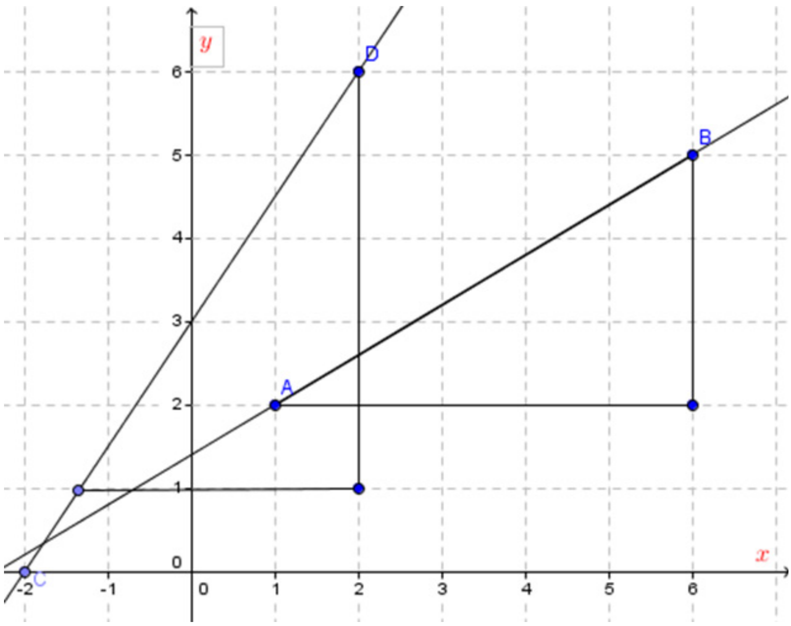
- Organise the learners in groups of three and tell them to do Activities 6.2 and 6.3 Student's Book.

- Provide the learners with graph papers.
- Make sure one learner acts as the secretary to record and present the findings.
- Allow the learners to do the Activities 6.2 and 6.3 and let the secretaries present the results through class discussion.

Synthesis

- Use the opportunity to explain the meaning of slope and what slope contributes to the steepness of the line.
- Explain the formula for finding the slope and use the formula to do Examples 6.1, 6.2 and 6.3 in the Student's Book for the learners to understand the content.

Answer to Activity 6.2



1. Line CD is steeper than AB by observation.
2. With the aid of gradient calculation,

   Gradient AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{6-2} = \frac{4}{4} = 1$

   Gradient CD = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{1--2} = \frac{6}{3} = 2$

   Gradient CD is greater than gradient AB hence CD is steeper than AB.

Answers to Activity 6.3

1. A(2,6), B(1,4) , C(-1,0)
2. Gradient AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-6}{1-2} = \frac{-2}{-1} = 2$

   Gradient BC = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{-1-1} = \frac{-4}{-2} = 2$

3. Gradient AB = Gradient BC (same straight line)

Assessment of the lesson

- Let the learners do Exercise 6.1 in the Student's Book.
- Correct them where they are wrong to ensure that objectives of the lesson are achieved.

Lesson 3: General form of Cartesian equation of a straight line

Lesson objective

By the end of the lesson, the learner should be able determine the general form of a Cartesian equation of a straight line.

Teaching Aids

- Chalks and classroom chalkboard
- chats
- pencil pens
- calculators and other geometrical instruments

Learning Activities

Activity 6.4

- Organise the learners in groups of three and allow them to do Activity 6.4 from the Student's Book.
- Make sure one student from each group acts as the secretary to record and present the findings at the end of the discussion.
- As the groups are presenting, allow fellow learners to correct the errors from other groups.

Synthesis

- Use the opportunity at the end of the presentation to tell the learners the meaning of slope and intercepts with axes of a straight line.
- Do Examples 6.4 and 6.5 in Student's Book to ensure that learners have mastered the content.

Answers to Activity 6.4

1. (a) Gradient 3, y-intercept 4

   (b) Gradient -2, y-intercept 5

   (c) Gradient -5, y-intercept 6

   (d) Gradient 7, y-intercept 0

2.

Equation	Form y=mx+c	m	c	Gradient	y-intercept
$x - 2y = -6$	$y = \frac{1}{2}x + 3$	$\frac{1}{2}$	3	$\frac{1}{2}$	3
$2x + 3y = 6$	$y = \frac{-2x}{3} + 2$	$-\frac{2}{3}$	2	$-\frac{2}{3}$	2
$4x + 2y = 5$	$y = -2x + \frac{5}{2}$	-2	$\frac{5}{2}$	-2	$\frac{5}{2}$
$2y = x - 4$	$y = \frac{1}{2}x - 2$	$\frac{1}{2}$	-2	$\frac{1}{2}$	-2
$y = -1$	$y = 0x - 1$	0	-1	0	-1
$4x - 0.5y = 6$	$y = 8x - 12$	8	-12	8	-12

Assessment of the lesson

- Allow the learners to do Exercise 6.2 in Student's Book in your presence and on individual basis.
- Correct them to ensure that they have mastered the content.

Lesson 4: Finding the equation of a straight line given gradient and a point on the line

Lesson objective

By the end of the lesson, the learner should be able to find the equation of a straight line given gradient and one point on the line.

Teaching aids

- graph paper
- pencil pens
- Chalks and classroom chalkboard
- chats
- calculators and other geometrical instruments.

Learning Activities

Activity 6.5

- Organise the learners in groups of three to carry out Activity 6.5 from the Student's.
- Make sure one student is working as the secretary to record the findings of the group and to present to the class at the end of the discussion.
- Let the secretaries present their findings at the end of the discussion.
- Allow other learners to correct their fellows when they are wrong.

Synthesis

- Use the opportunity to summarise the steps taken to find equation of a straight line given gradient and one point on the line after presentations have been made by the learners.
- Use Example 6.6 in the Student's Book to explain the findings and to make sure that learners have mastered the content.

Answers to Activity 6.5

Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

1. $\frac{y+1}{x-3} = 2$ 2. $y + 1 = 2(x - 3) \Rightarrow y = 2x - 7$

Assessment of the lesson

- Allow learners to do Exercise 6.3 from Student's Book.
- Correct them and guide them where they are wrong to ensure that the lesson is successful.

Lesson 5: Equation of a straight line given two points

Lesson objective

By the end of the lesson, the learner should be able to determine equation of a straight line passing through two given points.

Teaching Aids

- Chalks and classroom chalkboard
- pencil pens
- chats
- calculators and other geometrical instruments

Learning Activities

Activity 6.6

- Organise the learners in groups of three to do Activity 6.6 from the Student's Book.
- Make sure one learner acts as the secretary to record and present the findings at the end of the discussion.
- Observe their presentations and note the errors made.

Synthesis

- Use the opportunity to explain how equation of the straight line can be determined using two points.
- Use Examples 6.7, 6.8 and 6.9 in the Student's Book to explain and to ensure that learners have mastered the content.

Answers to Activity 6.6

1. Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{-1-3} = \frac{-1}{-4} = \frac{1}{4}$

2. $\frac{y-1}{x+1}$ = gradient

3. $\frac{1}{4} = \frac{y-1}{x+1} \Rightarrow 4y - 4 = x + 1 \Rightarrow y = \frac{1}{4}x + \frac{5}{4}$

4. $\frac{y-2}{x-3} = \frac{1}{4}$

   $\Rightarrow y = \frac{1}{4}x + \frac{5}{4}$

5. The lines are the same

Assessment of the lesson

- Let the learners do Exercise 6.4 in the Student's Book to ensure that they have mastered the concepts.
- Correct their exercises and guide those who are wrong.

Lesson 6: Parallel lines

Lesson objective

By the end of the lesson, the learner should understand the difference between parallel and perpendicular line.

Teaching Aids

- Chalks and classroom chalkboard
- pencil pens
- chats
- calculators and other geometrical instruments

Learning Activities

Activity 6.7

- Organise the learners in groups of three to do Activity 6.7 in the Student's Book.
- Make sure one learner acts as the secretary to record the findings and present the results at the end of the group discussion
- Let the learners do the activity and let the secretaries present their findings through class discussion.
- Allow other groups to provide the corrections to errors made by other groups.

Synthesis

- Use the opportunity to explain the condition which makes the lines parallel and perpendicular.
- Use Example 6.10 in the Student's Book to ensure that the learners master the content.

Answers to Activity 6.7

2. l_2, l_3 and l_5 are parallel

3. For l_2, Gradient = $\frac{2-0}{0--3} = \frac{2}{3}$

For $l_{3,}$ Gradient = $\frac{2-0}{3-0} = \frac{2}{3}$

For l_5, Gradient = $\frac{-2-0}{0-3} = \frac{-2}{-3} = \frac{2}{3}$

Parallel lines have the same gradients.

Assessment of the lesson

- Let the learners do question 1 of Exercise 6.5 questions in the Student's Book.
- Correct them where they are wrong and guide them well to ensure that the objectives are achieved.

Lesson 7: Perpendicular lines

Lesson objective

By the end of the lesson, the learner should be able to understand the concept of perpendicular lines.

Teaching Aids

- Books
- Chalks and classroom chalkboard
- pencil pens
- chats
- calculators and other geometrical instruments

Learning Activities

Activity 6.8

- Organise the learners in groups of three to do Activity 6.8 from the Student's Book.
- Make sure one learner acts as the secretary to record the findings and to present the results at the end of the group discussions to the whole class.
- Let the learners do the activity and present their findings through secretaries.
- Let other learners eliminate the errors from their fellow students during the discussion.
- Observe the presentations well as the teacher and make sure you record the errors made by the learners.

Synthesis

- Use the opportunity to explain the meaning of perpendicular lines.
- Use Example 6.11 in the Student's Book to make sure the learners master the content.

Answers to Activity 6.8

2. Perpendicular lines

l_5, l_6 l_1, l_3 l_2, l_4

Pair	M_1	M_2	M_1 x M_2
L_1L_2	1	4	4
L_1L_3	1	-1	-1
L_1L_4	1	-0.25	-0.25
L_1L_5	1	-2	-2
L_1L_6	1	0.5	0.5
L_2L_3	4	-1	-4
L_2L_4	4	-0.25	-1
L_2L_5	4	-2	-8
L_2L_6	4	0.5	2
L_3L_4	-1	-0.25	0.25
L_3L_5	-0.25	-2	0.5
L_3L_6	-1	0.25	-0.25
L_4L_5	-0.25	-2	0.5
L_4L_6	-0.25	-0.5	0.125
L_5L_6	-2	0.5	-1

The product of two gradients of perpendicular lines gives –1.

Assessment of the lesson

- Let the learners do question 2 and all other questions of Exercise 6.5 on individual basis from the Student's Book to ensure that they have mastered the content.
- Correct them and make corrections where they are wrong.

Lesson 8: Quadratic functions

Lesson objective

By the end of the lesson, the learner should understand the quadratic expressions and their properties.

Learning Aids

- Books
- pencil pens
- Chalks and classroom chalkboard
- calculators and other geometrical instruments

Learning Activities

Activity 6.9

- Organise the learners in groups of three students to do Activity 6.9 from the Student's Book.
- Make sure one learner acts as the secretary to record the findings and present to the group during class discussions.
- Let the learners present their findings about which functions are quadratic according to the activity.

Synthesis

- Guide the learners and emphasize on the format of the quadratic function i.e. Learners should know the general form of a quadratic function.
- Learners should also be guided about the properties of quadratic equations.

Answers to Activity 6.9

(a) Not quadratic, highest degree is not 2

(b) Quadratic, highest degree is 2

(c) Not quadratic, highest degree is not 2.

(d) Quadratic, highest degree is 2

(e) Not quadratic highest degree is not 2. It is quotient function.

Lesson 9: Table of values

Lesson objective

By the end of the lesson, the learner should be able to know how to fill the table of values for any given quadratic function.

Learning Aids

- Books
- Chalks and classroom chalkboard
- pencil pens
- calculators and other geometrical instruments

Learning Activities

Activity 6.10

- Organise the learners in pairs and let them do Activity 6.10 in the Student's Book.
- Make sure one learner acts as the secretary to record down the findings.
- As learners are doing the activity, do a follow up to know which group is getting difficulties in filling the table of values.
- Make sure that correct tables are presented and correct coordinates are stated.

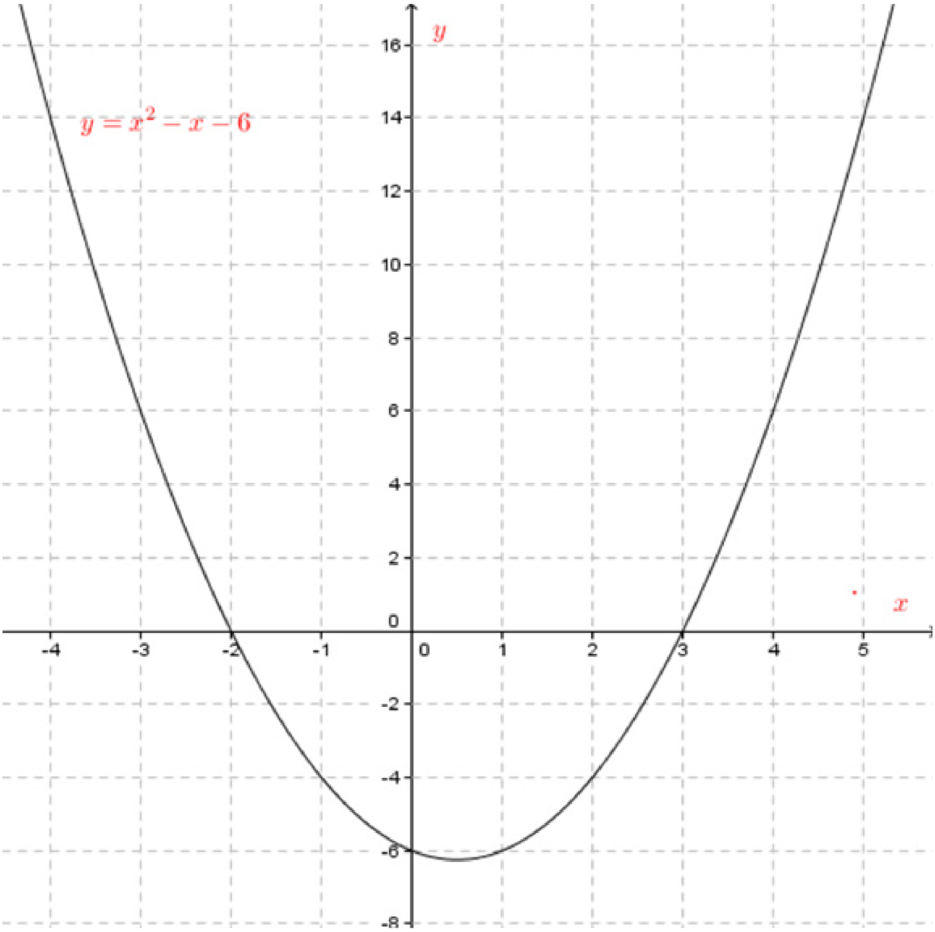
Synthesis

- Emphasize the learning objective by discussing Examples 6.12 and 6.13 in the Student's Book.
- Make sure you guide learners how table of values can be filled and how coordinates can be written down.

Answers to Activity 6.10

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-x$	4	3	2	1	0	–1	–2	–3	–4
-6	–6	–6	–6	–6	–6	–6	–6	–6	–6
$y = x^2 - x - 6$	14	6	0	– 4	– 6	–6	–4	0	6

Co-ordinates to be plotted:
(-4,14), (-3,6), (-2,0), (-1,- 4), (0,- 6), (1,– 6), (2,- 4), (3,0), (4,6)



Assessment of the lesson

- Let the learners do Exercise 6.6 in the Student's Book.
- Correct their errors and guide them to ensure they get all the concepts.

Lesson 10: Determining Vertex of a quadratic function and axis of symmetry from the graph

Lesson objective

By the end of the lesson, the learner should be able to understand how to find the vertex of a quadratic function and the axis of symmetry.

Teachings Aids

- Books
- Pens and pencils
- Calculators and other geometrical instruments.
- Chalks and class Chalkboard
- Internet connection or dictionary

Learning Activities

Activity 6.11

- Organise the learners in pairs, provide internet connection or dictionaries and let them do Activity 6.11 of the Student's Book.
- Make one learner acts as a secretary to present the findings at the end of the discussion.
- Let the group leaders present their findings and make sure you note the errors they make during presentations.

Synthesis

- Correct the errors made during the presentations and give the right definitions of the terms given in the activity.
- Establish the formulae for the vertex and axis of symmetry to the learners.
- Apply the concept from the formulae to do Examples 6.14 and 6.15 in the Student's Book.

Answers to Activity 6.11

(a) Line of symmetry is the line that divides the curve into two equal halves from the vertex point.

It is also defined as the line that passes through the vertex of the quadratic curve vertically.

(b) Maximum point is the point where the curve turns downwards.

(c) Minimum point is the point where the curve turns upwards.

Assessment of the lesson.

- Let the learners do questions of Exercise 6.7 in the Student's Book.
- Correct their errors and guide them where they are wrong.

Lesson 11: Determining the intercepts, vertices and sketching a quadratic functions

Learning objective

By the end of the lesson, the learner should be able to obtain intersection points and integrate them with vertices to sketch the quadratic function.

Teaching Aids

- Graph papers/graph books
- Books
- Pens and pencils
- Calculators
- Geometrical instruments.

Learning Activities

Activity 6.12

- Organise the learners into groups of three.
- Make sure one acts as the secretary to record the findings
- Let the learners do Activity 6.12 in the Student's Book.
- Let the learners present their findings through their secretaries.

- Make sure you note the errors made during presentations.

Synthesis

- Establish the formulae for vertex, intercepts and guide learners how to sketch a quadratic function.
- Guide learners through Examples 6.16 to 6.19 in the Student's Book to ensure that they have mastered the content.

Answers for Activity 6.12

1. Vertex is (4,16)
2. Axis of symmetry is $x = 4$
3. The graph cuts the x-axis at (–0.25, 0) and (8,0) and cuts y-axis at (0, 2).

Assessment of the lesson

- Assess whether learners understood the concepts by evaluating them in Exercise 6.8 in the Student's Book.
- Correct the learners and guide them where they go wrong.
- Prepare challenging questions for quick learners and remedial work for slow learners.

Summary of the unit

- Ask different learners to take the different concepts highlighted in the unit summary given in the Student’s Book.
- Ask the class probing questions to help them recall the concepts correctly.

End of unit assessment

- A teacher should ask learners to do unit 6 test in the Student's Book to assess whether learners understood the whole unit.
- A teacher should take time to correct the unit test to find out whether the concepts of the whole unit were properly digested.

Additional information to the teacher

- A teacher should know the concepts concerning different types of equations to assist him/her in making learners understand this unit.
- A teacher should as well teach the learners well how to use their geometrical instruments especially the calculators in this unit.

Remedial activities

For the slow learners, a teacher should organise remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

Remedial exercises

1. (a) Find equation of the line that passes through the points (1,4) and (2,6).
   (b) Find also the equation of the line whose gradient is –0.5 and passes through the point (2, –1).
   (c) Find the points where the equations in (a) and (b) cut x-and y-axes.
   (d) Find the points where the equations in (a) and (b) meet.
   (e) Sketch the lines obtained in (a) and (b) on the same axes.
2. Given the function $f(x) = -2x^2 + 4x - 4$:
   (a) Identify the type of function and explain why
   (b) Find the vertex of the function.
   (c) Find the intercepts of the function with axes.
   (d) Sketch the graph of the function on a Cartesian plane.
3. Line L_1 passes through the point (–1, 3) and has a gradient of –3. Line L_2 passes through the point (2, 3) and (0, 6).
   (a) Find the equations of the two lines.
   (b) Draw the lines L_1 and L_2 on the same pair of axes.
   (c) State the intercepts of the lines with axes.
4. Given that the line $y = 3x + w$ passes through (–1, -4), find the value of w.

Extended activities

The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Give learners the questions below.

Extended exercises

1. Find the equation of the line that passes through the points (a,0) and (0, –a)
2. Find the equation of the line that is parallel to another line whose equation $x + 2y + 8 = 0$ is and passes through the point (a, –b).
3. The perimeter of a rectangle is 42 cm. If the diagonal is 15 cm:
   (a) Form an expression that relates the diagonal, length and width of the rectangle
   (b) What special name is given to the expression in 3(a) above?
   (c) Factorise the expression obtained in 3(b) above.
   (d) Solve the equation obtained.
   (e) State the dimensions of the rectangle.
4. A 3-hour ship cruise goes 15 km upstream and then back again.

The water in the river has a speed of 2 km/h. Let x represent the speed of the ship.

(a) Write down an expression for the speed of the ship upstream.

(b) Write down an expression for the speed of the ship downstream.

(c) Find expression for the time taken by the ship in to move up and downstream.

(d) Write down and simplify an expression for the total time taken by the ship to move up and downstream.

(e) What name can be given to the expression obtained in (d) above?

(f) What is the ship's speed and how long was the upstream journey?

Answers to the unit exercises, Unit test, remedial and extended activities.

Exercise 6.1

1. (a) 3, 2, gradient $\frac{2}{3}$ (b) 5, –2 gradient $-\frac{2}{5}$ (c) 4, 2 gradient $\frac{1}{2}$

   (d) 2, 10 gradient 5 (e) 4, –4, gradient –1 (f) 0, 5, No gradient

   (g) –4, 16 gradient –4 (h) 5,0, gradient 0

2. (a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{3}{7}$ (d) $-\frac{1}{2}$ (e) –1

   (f) $\frac{2}{7}$ (g) $-\frac{1}{3}$ (h) 0 (i) $\frac{5}{2}$

   (j) No gradient

3. (a) $(0,-8), \left(\frac{8}{3}, 0\right)$ (b) $(0,-3), \left(\frac{3}{2}, 0\right)$ (c) (0,1), (1,0) (d) (10,0), (0,10)

   (e) (–4,3), (–4,7) (f) (1,3), (2,3)

4. $l_1 \Rightarrow y = 3x$; m = 3 $l_2 \Rightarrow 3y = x - 3$; m = $\frac{1}{3}$ $l_3 \Rightarrow y = 2.5$; m = 0

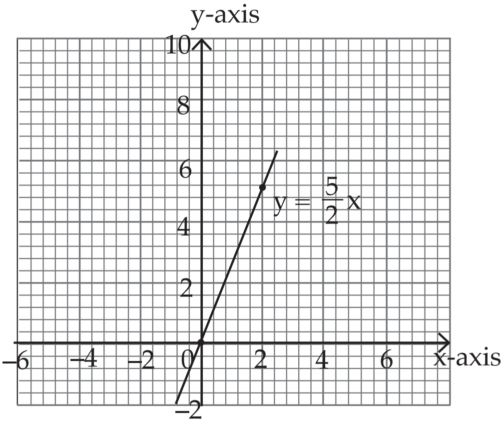
   $l_4 \Rightarrow y = -2x + 4$; m = –2 $l_5 \Rightarrow x = 1.5$; no gradient.

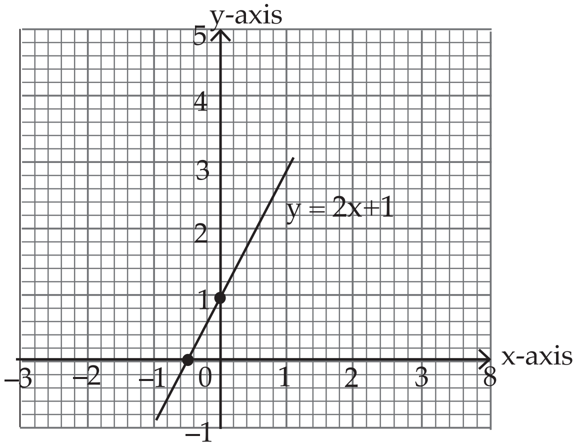
Exercise 6.2

1. (a) $y = \frac{5}{2}x$, gradient = $\frac{5}{2}$ (b) $y = 2x + 1$, gradient = 2

   gradient = $\frac{5}{2}$, y-intercept = 0 gradient = 3 ,y - intercept = 1

   y - intercept = 0 y - intercept = 1



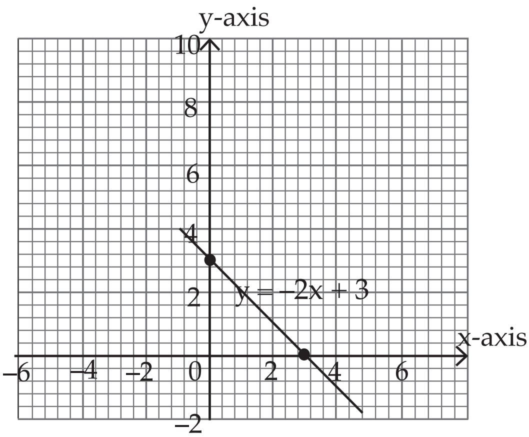


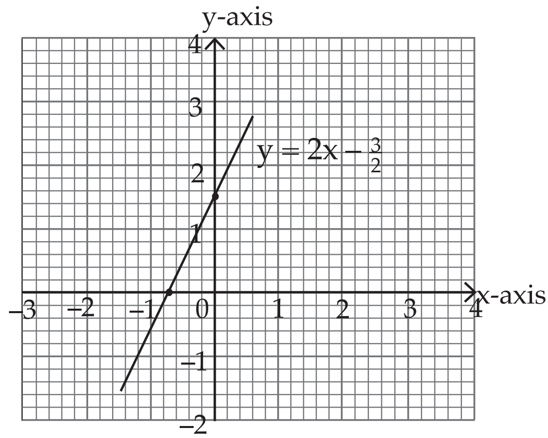
(c) $y = -2x + 3$

gradient −2, y-intercept = 3

(d) $y = 2x + \frac{3}{2}$

gradient = 2, y-intercept = $\frac{3}{2}$

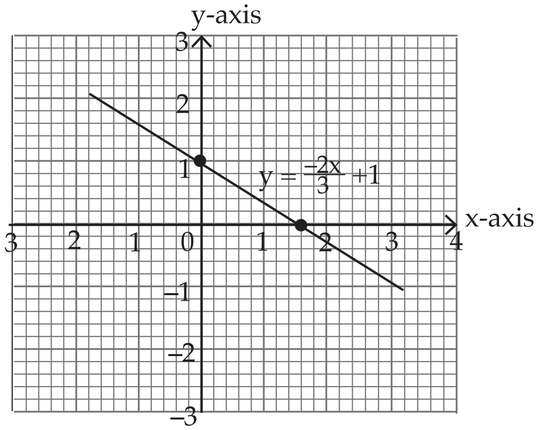




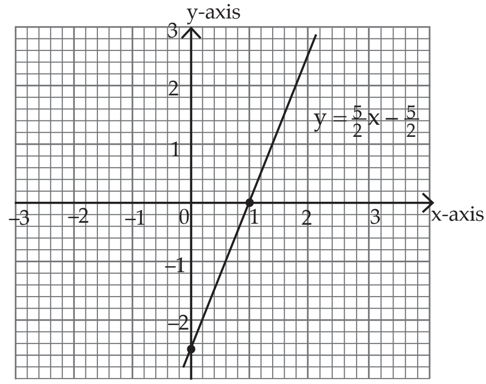
(e) $y = \frac{-2}{3}x + 1$

(f) $y = \frac{5}{2}x - \frac{5}{2}$

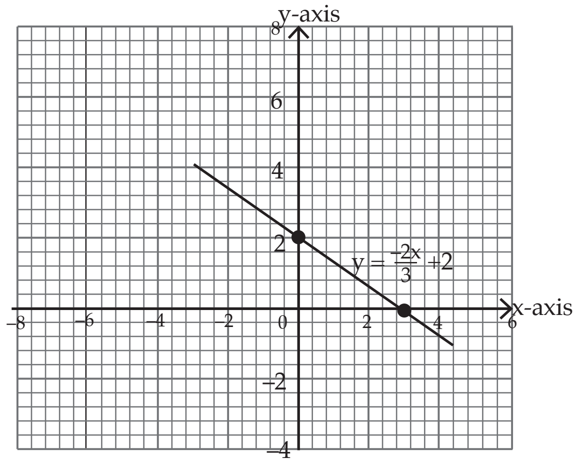
gradient $\frac{-2}{3}$, y-intercept 1



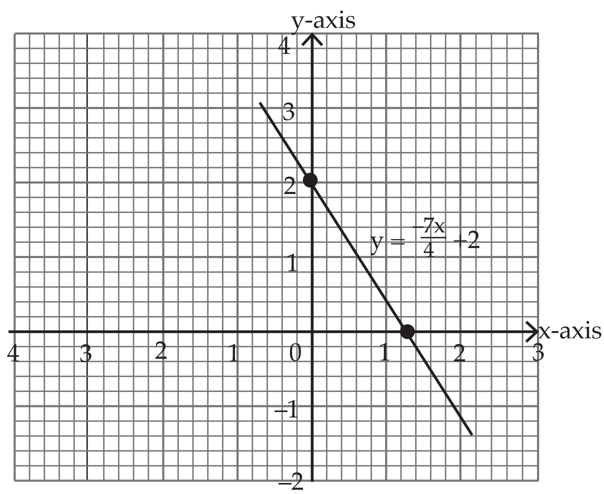
gradient $\frac{5}{2}$, y-intercept $\frac{-5}{2}$



(g) $y = -\frac{2}{3}x + 2$ gradient $\frac{-2}{3}$, y-intercept 2



(h) $y = \frac{-7}{4}x + 2$ gradient $\frac{-7}{4}$, y-intercept 2



2. (a) $m = 3, \; c = 0$ (b) $m = \frac{5}{2}, c = \frac{19}{3}$ (c) $m = -2, c = 18$ (d) $m = -\frac{7}{4}, c = \frac{9}{2}$

   (e) $m = 0, c = -2$ (f) No gradient, no y-intercept

3. line a: $y = \frac{2}{3}x + 2$, line b: $y = \frac{2}{3}x - 2$,

   line c: $y = -\frac{4}{3}x + 4$, line d: $y = 0x + 2$,

Exercise 6.3

1. (a) $y = 3x - 8$ (b) $y = \frac{1}{2}x + 3$ (c) $y = -x + 1$

   (d) $y = -\frac{1}{5}x + 6$ (e) $y = 3$ (f) $x + 4 = 0$

   (g) $y = \frac{2}{3}x + 2$ (h) $y = \frac{-2}{3}x + 2$

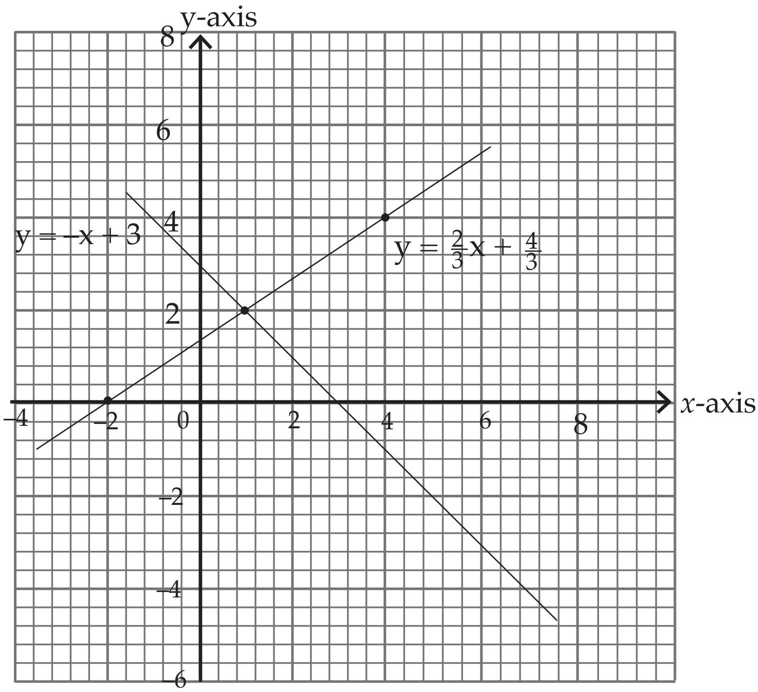
2. (a) $y = -3x$ (b) $y = \frac{2}{3}x$

   (c) $y = -\frac{5}{2}x + 10$ (d) $y = 3$

3. (a) $y = -2x + 1$ (b) $y = -\frac{1}{5}x + 4$

4. (a) $y = \frac{-1}{2}x + 4$ (b) $y = 3x - 3$

5. $L_1 : y = -x + 3$ $L_2 : y = \frac{2}{3}x + \frac{4}{3}$



The point of intersection is (1,2).

Exercise 6.4

1. (a) $y = \frac{1}{4}x + \frac{5}{4}$ (b) $y = -\frac{1}{3}x + \frac{13}{3}$ (c) $y = 5$ (d) $y = 4x - 22$

   (e) $y = \frac{1}{12}x + \frac{5}{2}$ (f) $x - 2 = 0$ (g) $y = -x + \frac{7}{12}$ (h) $y = \frac{10}{7}x - \frac{29}{70}$

2. (a) $y = -x + 7$

   (b) $m = \frac{0-a}{a-0} = \frac{-a}{a} = -1$

   $\therefore\ y - a = -1\,(x - 0)$

   $y - a = -x$

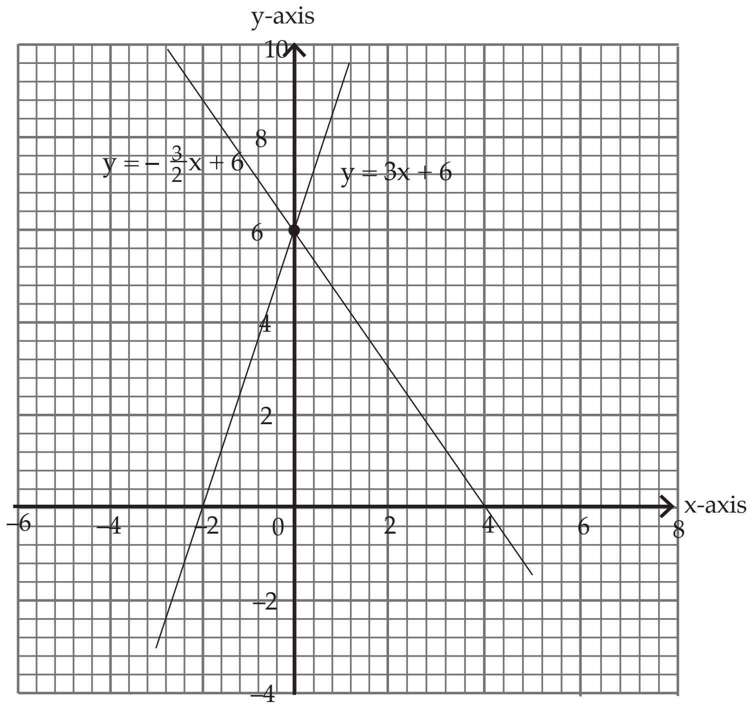
   $y + x = a$

3. AC : $y = \frac{3}{4}x + \frac{3}{2}$; BC : $y = 3$ AB: $y = 3x + 6$

4. (a) $k = \frac{-5}{3}$ (b) $y = \frac{-5}{54}x - \frac{35}{27}$

5. (a) $y = -\frac{1}{3}x$ (b) $y = -x - 5$ (c) $y = \frac{5}{4}x - 5$

6. $L_1 : y = 3x + 6$ L2 : $y = -\frac{3}{2}x + 6$



Point of intercept is (0,6).

7. (a) Gradient 8, y-intercept. 1 (b) Gradient 1, y-intercept 0

   (c) Gradient -2, y-intercept 3 (d) Gradient –1, y-intercept 0

   (e) Gradient $-\frac{1}{3}$, y-intercept 3 (f) Gradient $-\frac{2}{5}$, y-intercept –2

   (g) Gradient $-\frac{2}{3}$,y-intercept 4 (h) Gradient $-\frac{5}{4}$,y-intercept $-\frac{25}{2}$

8. The point (-1,-4) balances the line $y = 3x + 1$ so it lies on it.

9. (0, k), (k, 0)

$\frac{\Delta y}{\Delta x} = \frac{0-k}{k-0} = \frac{-k}{k} = -1$

(k, 0); –1

$\frac{y-0}{x-k} = -1$

$y = -x + 1$ k or $y + x =$ k

10. $a = 1$

Exercise 6.5

1. (a) Gradient 2, Parallel (b) Gradient 0, Parallel

(c) Gradient 2, 4, not parallel (d) Gradient $-\frac{3}{5}$, parallel

(e) Gradient $-\frac{1}{2}$, $-\frac{2}{3}$, not parallel (f) Gradient –2, 3, not parallel

(h) Gradient 2 parallel (i) Gradient $\frac{5}{3}$ parallel

(j) Gradient $\frac{1}{5}$, $\frac{1}{4}$ not parallel (g) Gradient $-\frac{1}{2}$, $-\frac{1}{3}$ not parallel

2. (a) Perpendicular (b) Not perpendicular (c) Perpendicular,

(d) Not perpendicular (e) Not perpendicular (f) Perpendicular

3. a = 21

4. y = 4x – 6

5. $y = -\frac{5}{4}x + 15$

6. $y = \frac{2}{5}x - \frac{11}{5}$

7. (a) $y = \frac{4}{3}x + \frac{2}{3}$ (b) $y = -\frac{4}{3}x$ (c) $y = -\frac{2}{3}x - \frac{2}{3}$ (d) $y = 5x - 10$

Exercise 6.6

1. (a)

x	–3	–2	–1	0	1	2	3
$-2x^2$	–18	–8	–2	0	2	8	18
x	–3	–2	–1	0	1	2	3
1	1	1	1	1	1	1	1
$y = 1 + x - 2x^2$	–20	–9	–2	1	0	–5	–14

Co-ordinates: (–3, –20), (–2,–9), (–1,–2) , (0,1), (1,0), (2, –5), (3, –14)

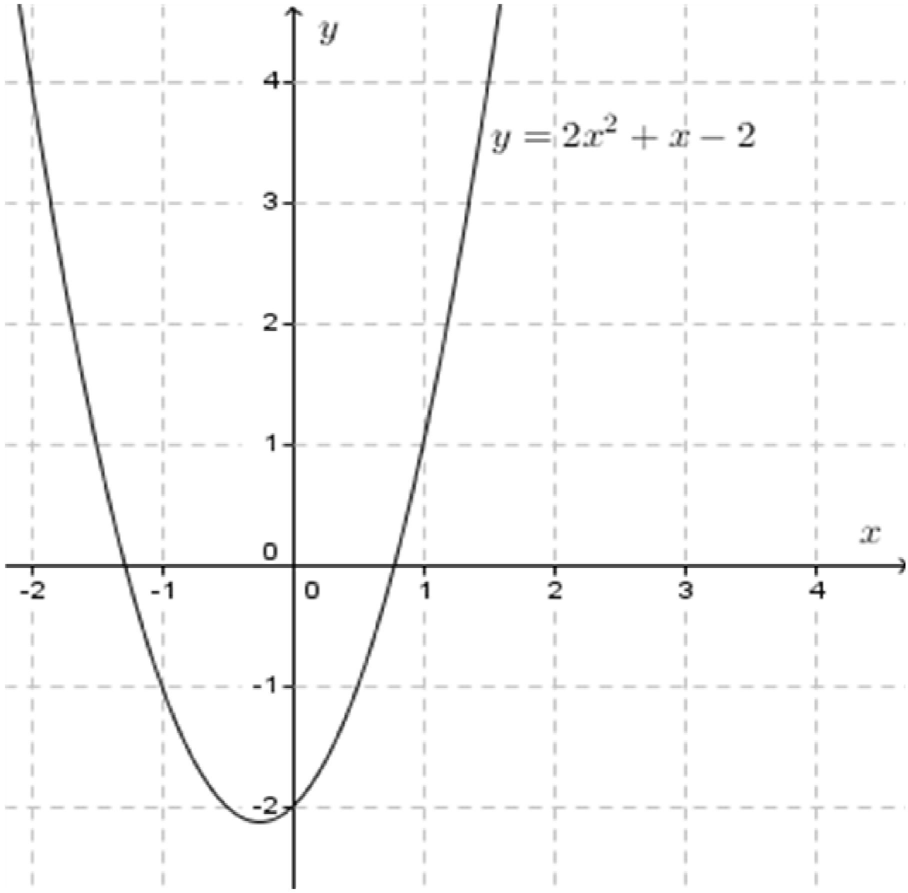
(b)

x	–3	–2	–1	0	1	2	3
$y = 2x - 5$	–11	–9	–7	–5	–3	–1	1

Co-ordinates: (–3, –11), (–2, –9), (–1,–7), (0, –5), (1, –3) , (2, –1), (3, 1)

2.

x	–3	–2	–1	0	1	2
$y = 2x^2 + x - 2$	13	4	–1	–2	1	8



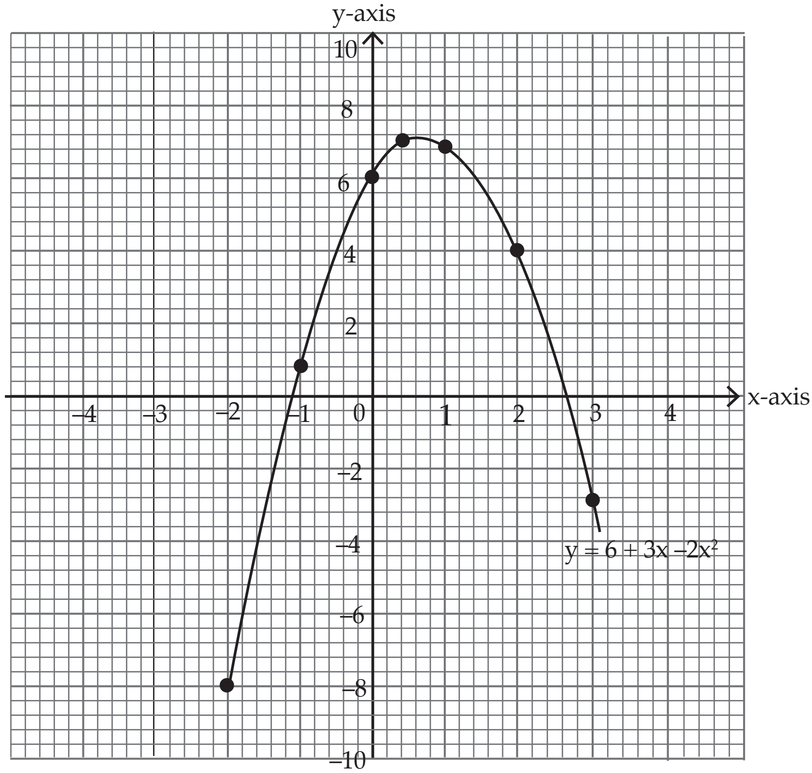
Co-ordinates: (–3,13) , (–2,4) (–1,–1), (0,–2), (1,1), (2,8)

Roots are $x = -1.2$ or $x = 0.8$

3.

x	–2	–1	0	0.5	1	2	3
6	6	6	6	6	6	6	6
3x	–6	–3	0	1.5	3	6	9
$-2x^2$	–8	–2	0	–0.5	–2	–8	–18
y	–8	1	6	7	7	4	–3

Co-ordinates: (–2,–8), (–1,1), (0,6), (0.5,7), (1,7), (2,4), (3,–3)



4.

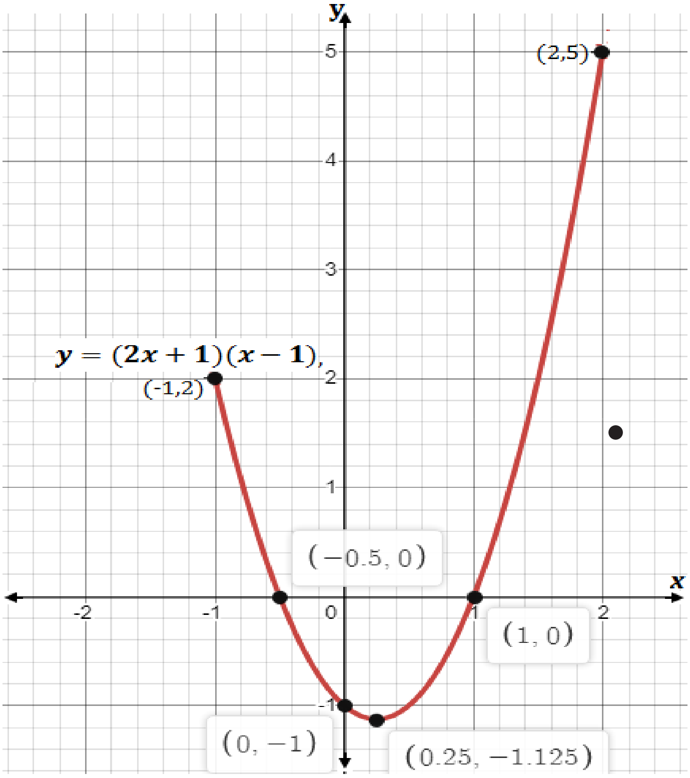
x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3
x^2	4	2.25	1	0.25	0	0.25	1	2.25	4	6.25	9
$-x$	2	1.5	1	0.5	0	0.5	–1	–1.5	–2	–2.5	–3
–6	–6	–6	–6	–6	–6	–6	–6	–6	-6	–6	–6
y	0	–2.25	–4	–5.25	–6	–6.25	–6	–5.25	-4	–2.25	0

Co-ordinates: (–2,0) , (–1.5, –2.25), (–1,–4), (-0.5,–5.25), (0,–6), (0.5,–6.25), (1,–6), (1.5,–5.25), (2,–4), (3,0)

5.

x	–1	–0.5	0	0.25	1	2
$2x + 1$	–1	0	1	1.5	3	5
$x - 1$	–2	–1.5	–1	–0.75	0	1
y	2	0	–1	–1.125	0	5

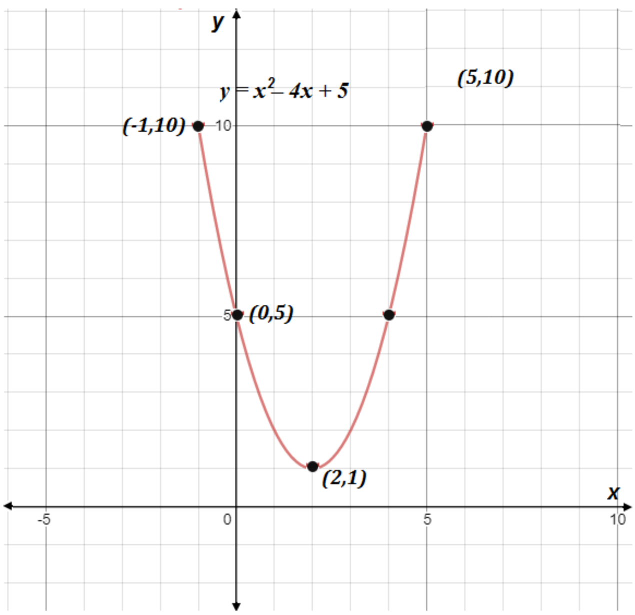
The coordinates are: (–1,2), (–0.5,0), (0,–1), (0.25,–1.125), (1,0), (2,5)



Exercise 6.7

1. (a) The curve does not cut x-axis at all and draw graph.

(b) Axis of symmetry is x = 2 (c) Vertex (2,1) (d) When x = 3, y = 2

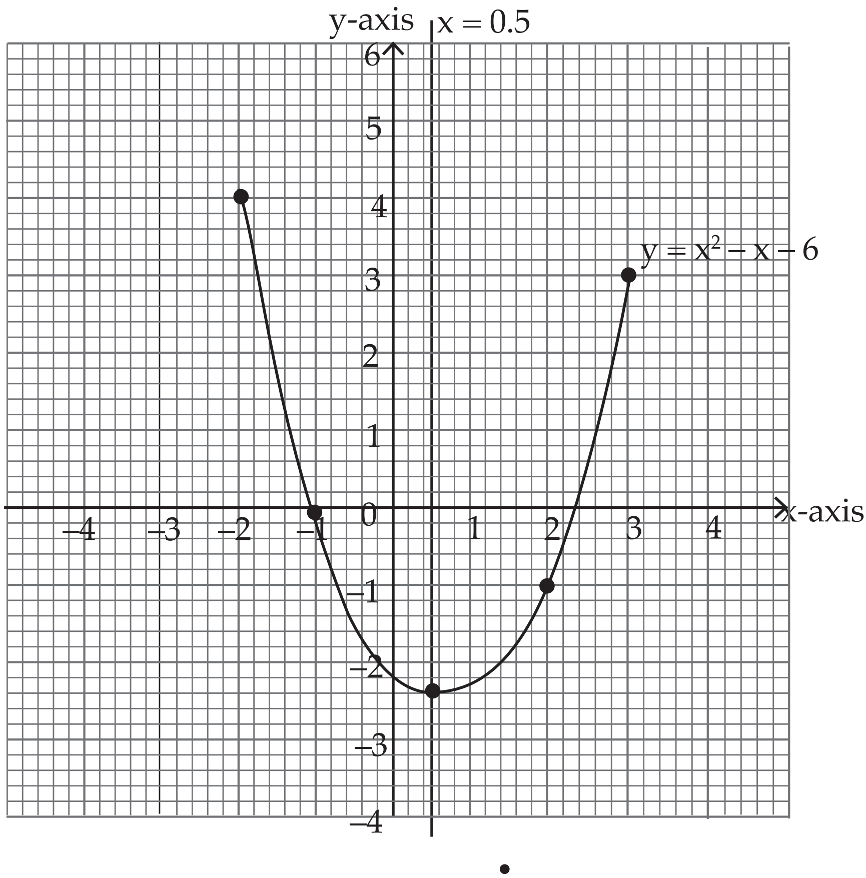


2.

x	x^2	$-x$	–2	y
–2	4	02	–2	4
–1	1	1	–2	0
0	0	0	–2	–2
0.5	0.25	–0.5	–2	–2.25
1	1	–1	–2	–2
2	4	–2	–2	0
3	9	–3	–2	4

(a) Co-ordinates: (–2,4), (–1, 0), (0,–2), (0.5, –2.25), (1,–2), (2, 0), (3,4)

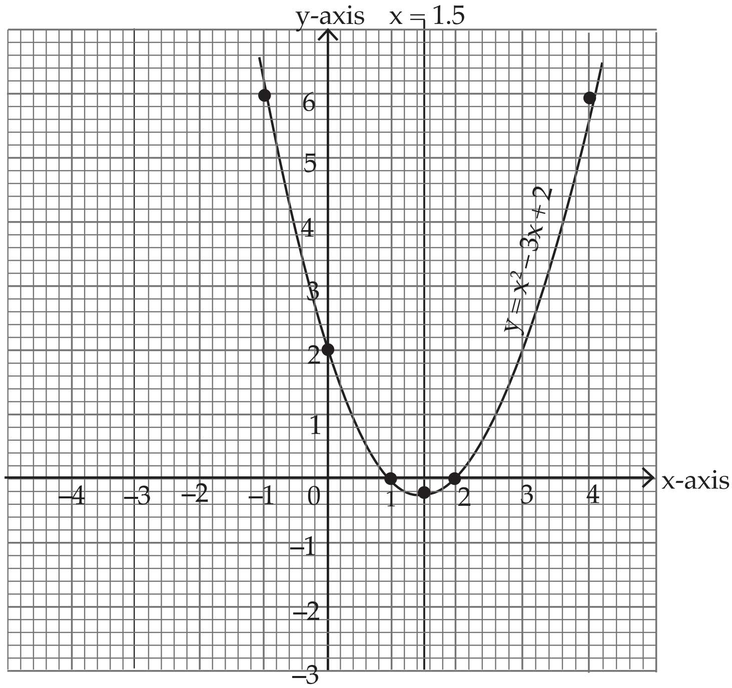
(b)



(c) Axis of symmetry is $x = 0.5$.

(d) Vertex: (0.5, –2.25)

3. (a) *Plot these coordinates to generate a graph. (-1,6), (0,2), (1,0), (1.5, –0.25) (2,0), (3,2), (4,6)*

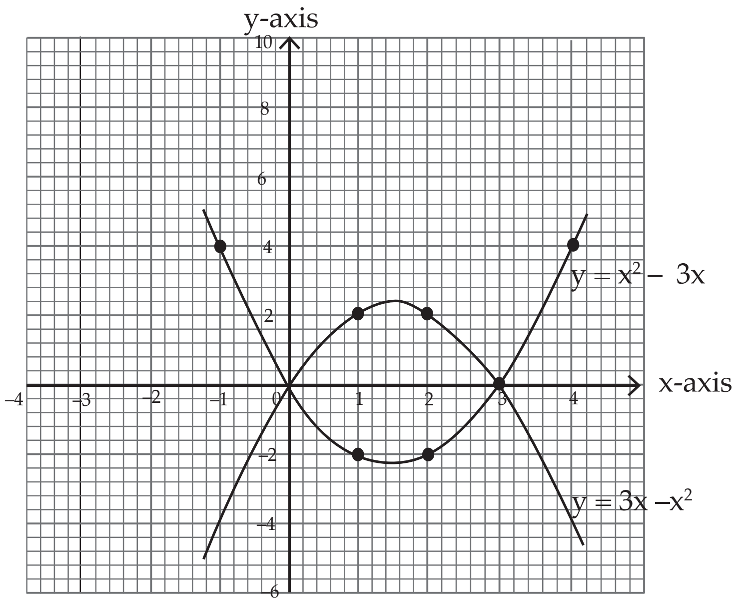


(b) Axis of symmetry is x=1.5

(c) Vertex: (1.5, -0.25)

(d) $x = 2$ or $x = 1$

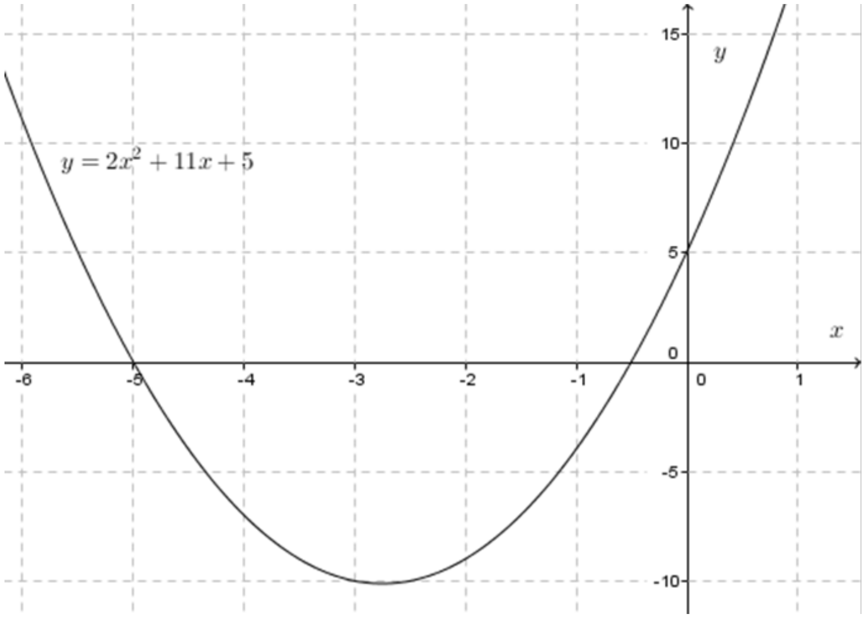
4.



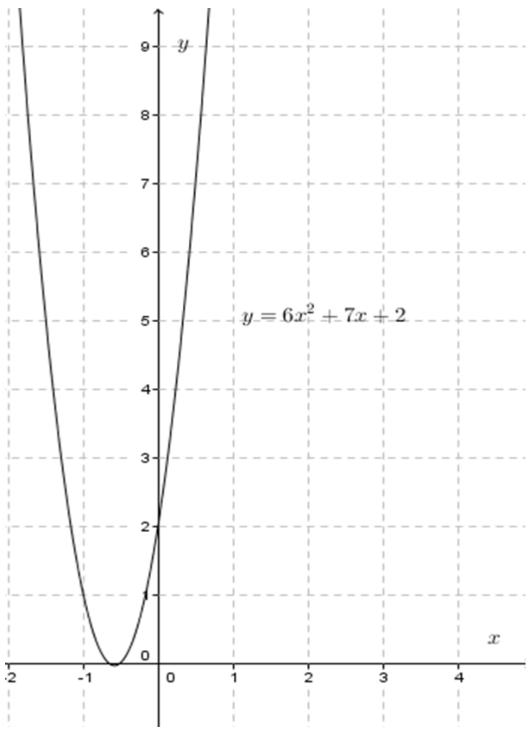
(b) The equations $y = 3x - x^2$ and $y = x^2 - 3x$ have the same solution.

Exercise 6.8

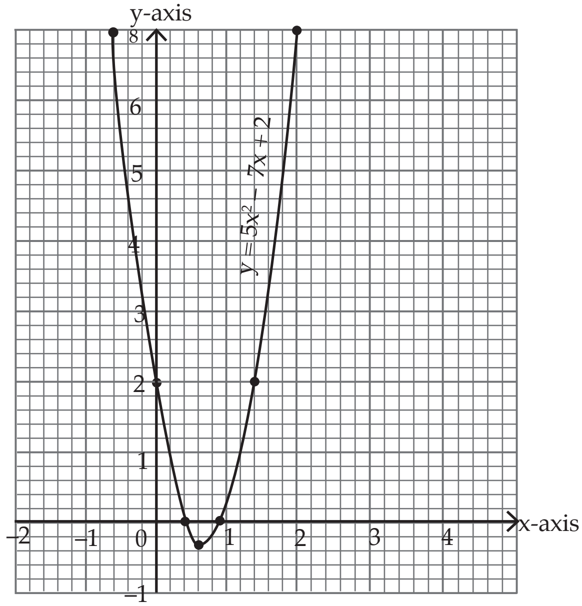
1. (a) $y = 2x^2 + 11x + 5$



(b) $y = 6x^2 + 7x + 2$



(c)

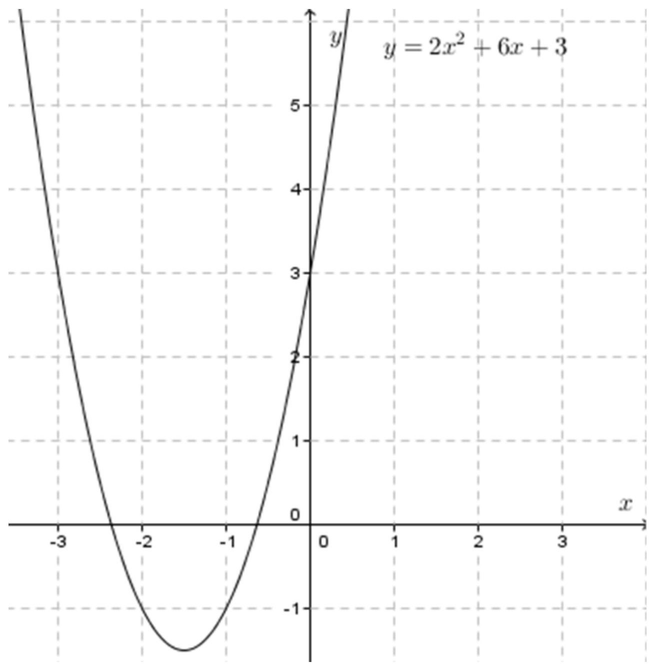


2. (a) Intercepts are (-2.36, 0), (–0.63, 0) and (0, 3).

   (b) Vertex: (–1.5, –1.5)

   (c) Axis of symmetry is $x = -1.5$

   (d)

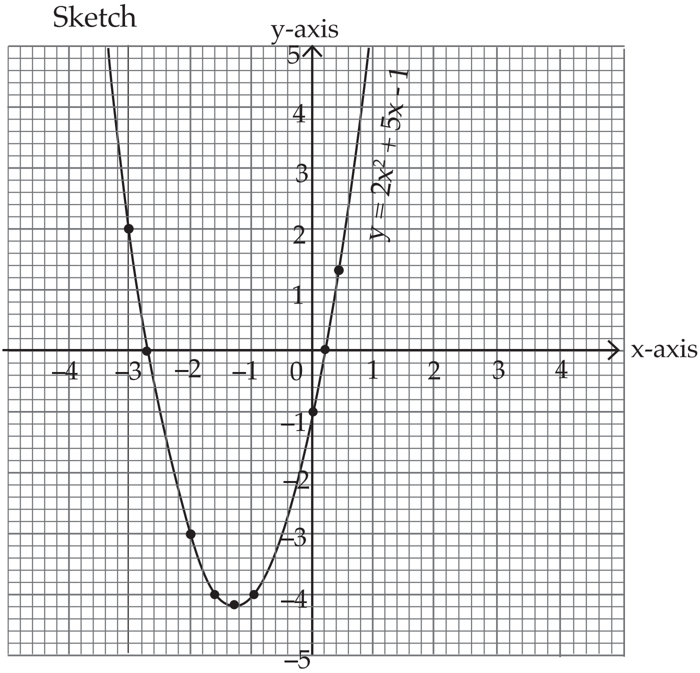


3. (a) Equation: $y = 2x^2 + 5x - 1$

   Vertex: (–1.25, – 4.125), intercepts (–2.68,0) and (0.186,0) and (0,–1)

Axis of Symmetry x=–1.25

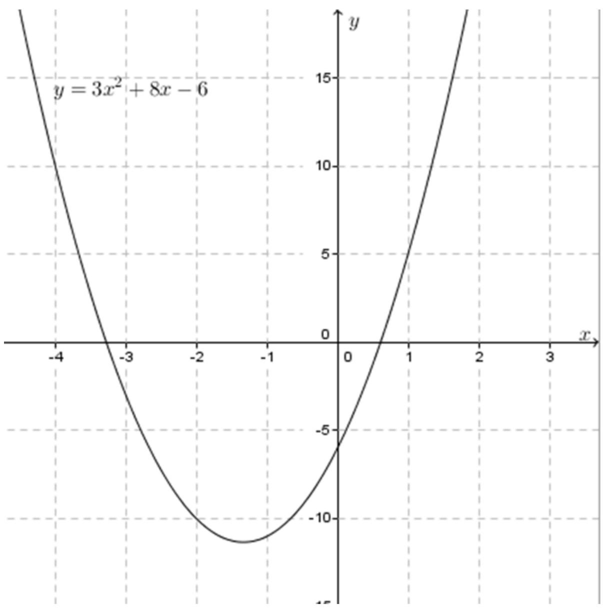
Sketch



(b) Equation. $y = 3x^2 + 8x - 6$

Vertex (–1.33, –11.33), Axis of symmetry x = –1.33, intercepts (–3.27,0), (0.61,0); (0,–6)

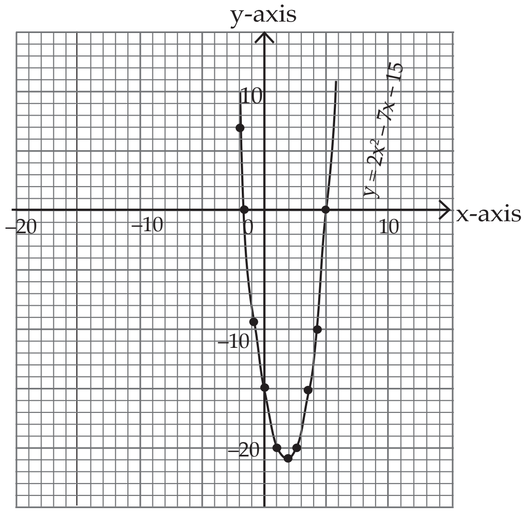
Sketch



(c) Equation is y = $2x^2 - 7x - 15$.

- Vertex (1.75, –21.125), Intercepts: (5,0); (–1.5,0) and (0,–1.5)
- axis of symmetry $x = 1.75$

Sketch



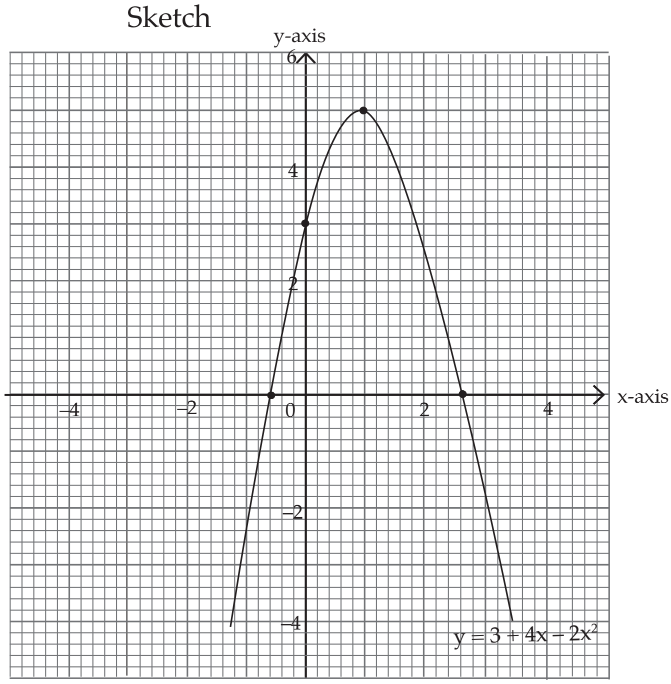
(d) Equation $y = 3 + 4x - 2x^2$

Vertex (1,5)

Intercepts with x-axis (2.58,0), (-0.58,0)

Axis of symmetry x=1, intercepts: (2.58,0); (-0.58,0) and (0,3)

Sketch



Unit 6 Test

1. $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - p}{p - 0}$$

$$m = \frac{-p}{p} = -1$$

$$\therefore \; y - y_0 = m\,(x - x_0)$$

$y - p = -1(x - 0)$, by $P(0, p)$

$$y - p = -x$$

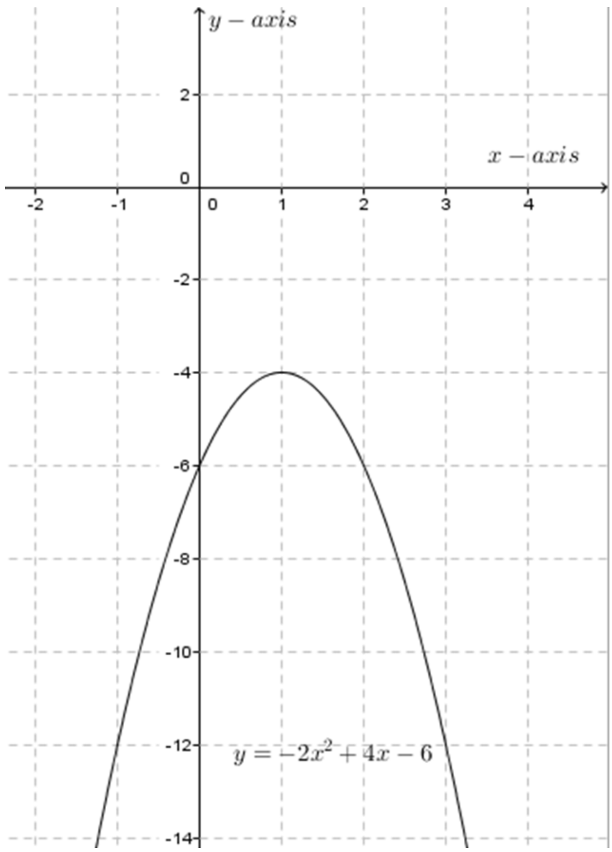
hence $y + x = p$

2. (a) Quadratic function because the highest degree of the unknown variable is two.

   (b) Vertexs is (–1, 8), intercepts: (-3,0); (1,0) and (0,6)

   (c) No x-intercept, (0,6) is y-intercepts.

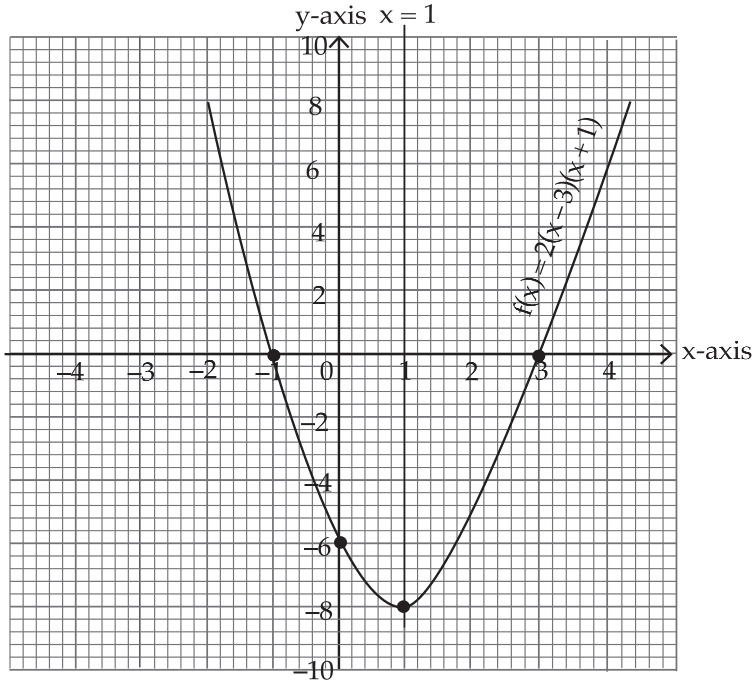
   (d)



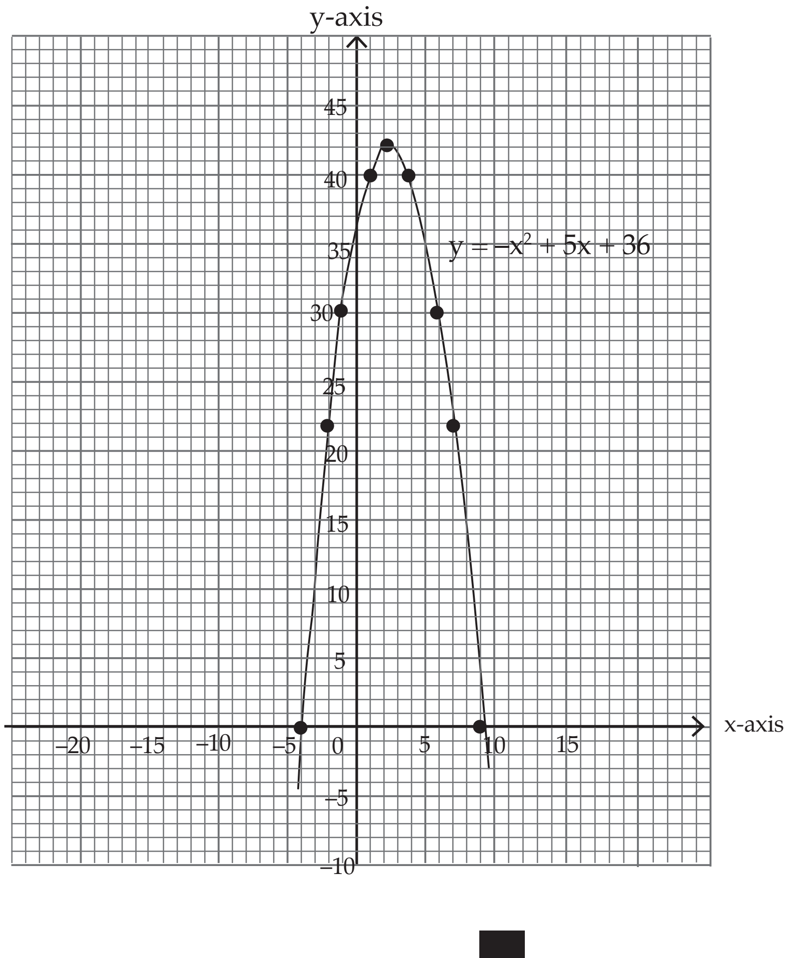
3. (a) The curve is open upwards because the coffecient of the highest degree is positive.

   (b) Vertex (1,-8), x-intercept (3,0), (–1,0), y-intercept (0,–6)

   (c) Axis of symmetry x=1 (d) sketch



4.



5. $y = -\frac{5}{4}x + 15$

6. (a) Point (-1,-4), line: y = 3x - 1.

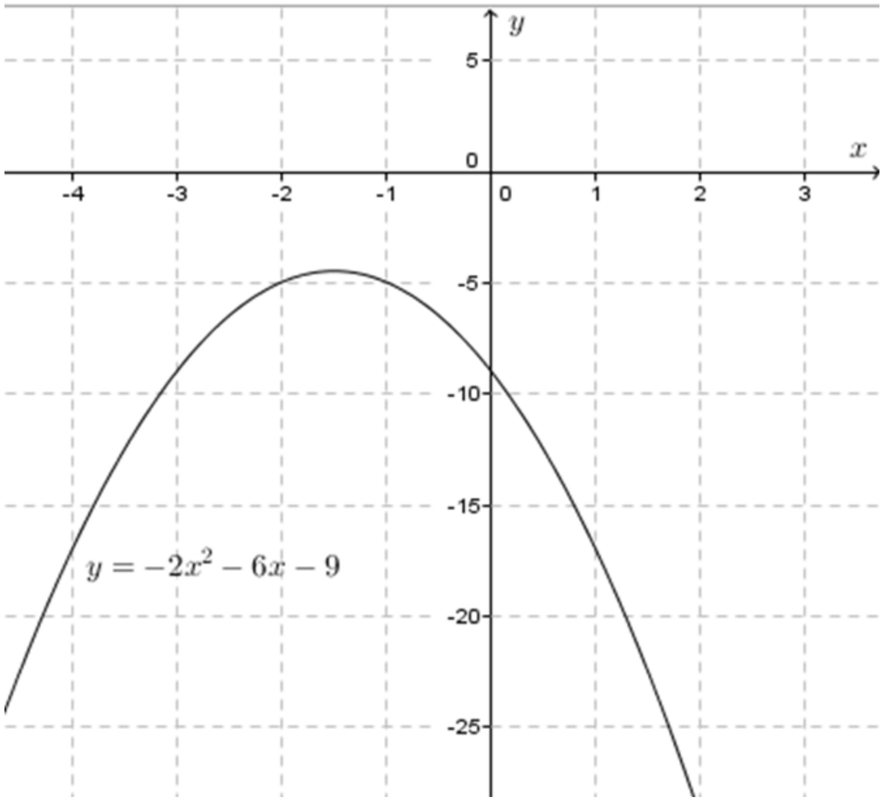
$$-4 = 3(-1) - 1$$
$$-4 = -3 - 1$$
$$-4 = -4$$

Hence the point lies on the line

(b) $y = -\frac{1}{2}x - 4$

7. (a) $a = 1$

(b)



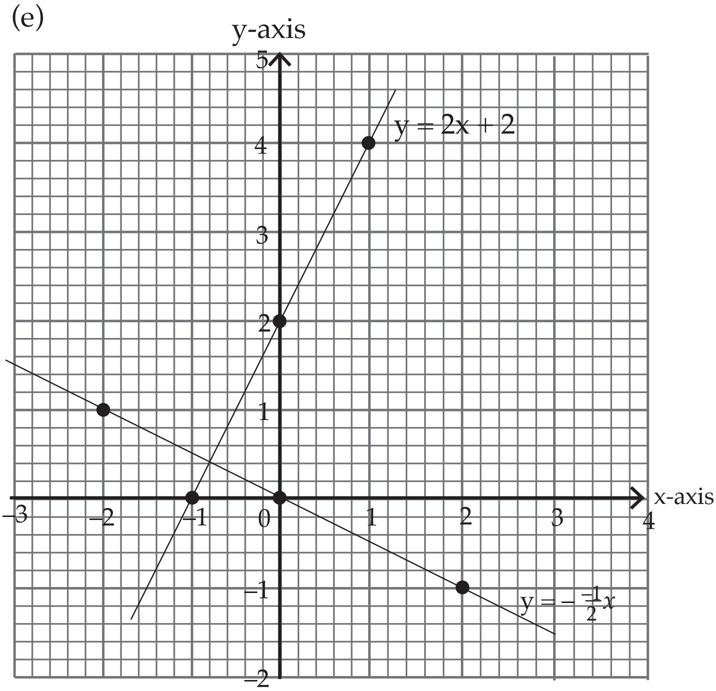
Remedial exercises

1. (a) $y = 2x + 2$ (b) $y = -\frac{1}{2}x$ (c) (0,0) for $y = \frac{-1}{2}x$

(0,2) and (–1,0) for y = 2x + 2

(d) $\left(-\frac{4}{5}, \frac{2}{5}\right)$

(e)

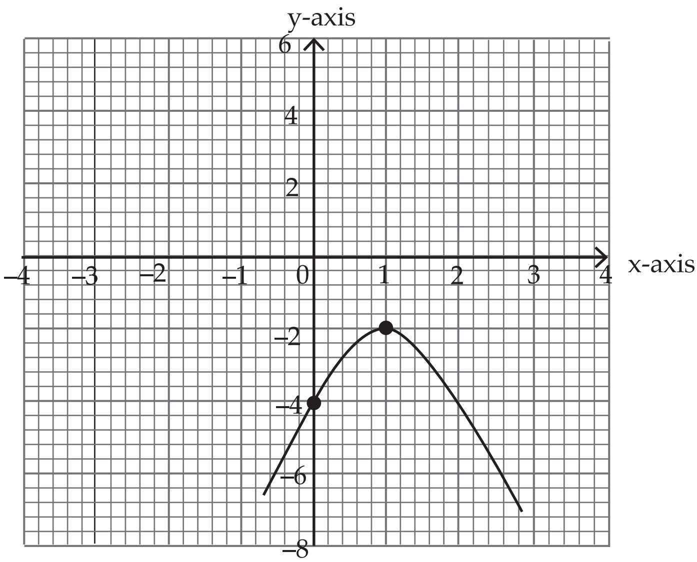


2. (a) Quadratic fruction

(b) Vertex (1,–2) because the highest degree of the unkown is 2

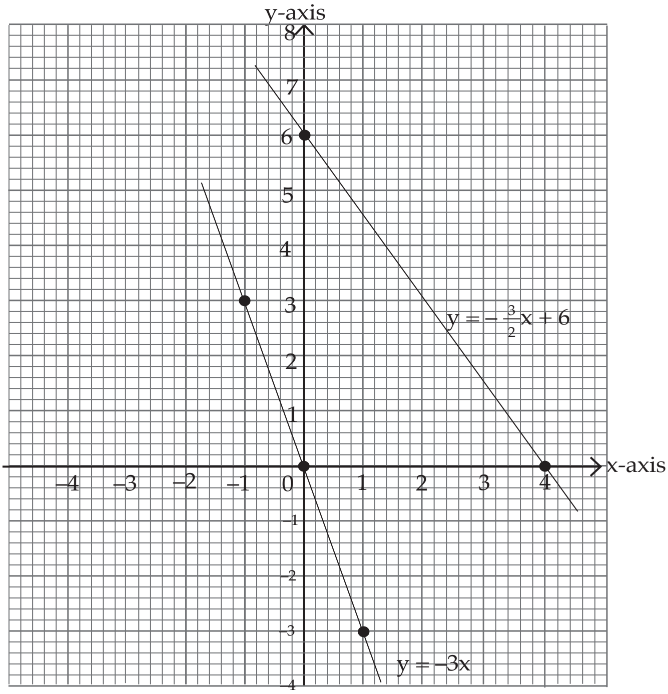
(c) No x-intercepts, (0,–4) is y-intercept

(d) Graph drawn



3. (a) L_1: $y = -3x$ L2: $y = \frac{-3}{2}x + 6$

(b) Draw graph on paper.



(c) intercepts are;

x intercepts (0,0) for L_1 and y-intercept (0,0) for L_2

while y intercepts is (0,6) and x-intercept is (4,0) for L_1.

4. w = –1

Extended exercises

1. $y = x - a$
2. $y = -\frac{1}{2}x + \frac{a - 2b}{2}$
3. (a) $x^2 + y^2 = 225$ which gives, where x length, y is width

   $x^2 - 21x + 108 = 0$ on simplifying, where from perimeter $y=21-x$

   (b) Quadratic equation (c) $(x - 12)(x - 9) = 0$

   (d) $x = 9$ or $x = 12$

   (e) Dimension: Length Length 12cm width 9cm

4 (a) $(x - 2)$ km/hr (d) $3x^2 - 30x - 12 = 0$

(b) $(x + 2)$km/hr (e) Quadratic equation

(c) $\frac{15}{(x-2)} + \frac{15}{(x+2)} = 3$ (f) Speed 10.39 km/h

Upstream Journey takes 1.79 hours.

UNIT 7 COMPOUND INTEREST, REVERSE PERCENTAGE, AND COMPOUND PROPORTIONAL CHANGE

Key unit competence

By the end of this unit, learners should be able to solve problems involving compound interest, reverse percentage and proportional change using multipliers.

Learning Objectives

Knowledge and Understanding

- Define compound interest, reverse percentage, compound proportional change, and continued proportional.
- Find reverse percentages in a given mathematical problem.
- Determine a compound interest in a given mathematical problem.
- Simplify ratio in their simplest form.

Skills

- Solve problems involving reverse percentages and compound interest.
- Apply compound interest in solving mathematical problems involving savings or calculations in any other financial activity.
- Apply reverse percentage and compound proportional change in solving real life mathematical problems.

Attitudes and Values

- Appreciate the role of compound interest in banking and financial activities.
- Appreciate that in the case of compound interest, saving and investing money can increase the value of wealth.
- Show concern for paying taxes and being honest in daily activities involving money.
- Develop logical and critical thinking while solving problems involving compound interest, reverse change, and continued proportional change.

Pre-requisite to this Unit

For learners to acquire the knowledge, skills, attitudes and values that are envisaged in this unit without any difficulty, they in need to acquire the following;

- Knowledge on percentages: this involves percentage increases and decrease on products and calculations of percentages. Learners were introduced to these concepts in S1. Using the question and answer method, the teacher should make sure that all the learners can remember these concepts before they are introduced to the concept of reverse percentage.

- The skills in finding simple interests: Simple interest is the initial stage of calculating any interest rate. Learners are introduced to this topic area back in S1 and therefore the teacher is expected to remind the learners on the simple interest formula and to use short written questions and answers to ascertain that they still have these skills before you can introduce them to compound interest. The teachers should guide those with challenges.

Generic competence to be addressed in this unit

- **Problems solving skills:** this will be achieved by learners by solving complexed problems

Cross cutting issues to be addressed

The specific cross cutting issues to be addressed in this unit include;

- **Financial education**: this unit directly deals with finance related issues and learners should be able to gain knowledge about it.
- **Peace and value education**: learners should learn to pay other people debts and loans can avoid a lots of interests.

Vocabulary/Keywords

In the course of learning, the learner will come across the following concepts that are explained in the book. These concepts include:

- Reverse percentage
- Proportional change
- Compound interest
- Principle

Guide the learners to understand the meaning of these words and how they are used. You also need to speak out mathematical statements involving these words in order to master their meanings, usage and application.

Guidance on the problem statement

- In order to focus the attention of the learners into the general direction of this unit, the teacher should guide the learners on how to conduct unit focus activity at the beginning of this unit of the Student's Book.

Unit Focus Activity

- Ask learners to form groups of five looking into this unit focus activity in the Student's Book. In these groups, let them maintain gender balance then let them name some of the financial institutions in our country.
- Ask the learners to research in their groups and give the meaning of interest and some of two methods used by the financial institutions to calculate interest. Let them compare the two methods of calculating interest and give the best method.
- From their analysis on the best interest method, let them advice Lucie appropriately on the best method she should consider in investing her money.

Answers to unit Focus Activity

(a) (i) simple interest is simple method of calculating interest on loan/deposit .

$$I = \frac{P \times R \times T}{100}$$

(ii) Compound interest is the addition of interest to the principle deposit is stead of paying it out.

(b) Compound interest.

(c) 31, 000 FRW

Attention to special needs

- You should provide for learning materials for all learners including those with special needs i.e. physically challenged, fast and slow learners, and those with impaired hearing.
- For the slow learners, you need to organise some remedial exercises (tasks) where you need to guide them through the activities once again slowly to help them understand the needed concepts.
- The calculations in this unit may not be challenging to the specially gifted learners and they may finish ahead of the others then get bored and start distracting others. You therefore need to prepare additional and more challenging questions for these learners as extended exercises.

List of lessons

Lesson No.	Lesson title	Number of periods
1	Reverse percentage	4
2	Compound interest – step by step method	6
3	The compound interest formula	5
4	Compound proportional change	5

Lesson Development

Lesson 1: Reverse Percentage

Lesson objective

By the end of the lesson, the learner should be able to define reverse percentage and work out mathematical problems involving reverse percentages.

Introduction/Pre-requisite

Introduce the lesson by reminding the learners on percentages and how they are calculated. Use the knowledge the students have in these areas to introduce reverse percentages. Remember that reverse percentage involves the working out the original price of a product backwards after the increase or decrease in its price. Gauge the learners understanding on percentages by asking them random questions. This is important because most of the work and activities they will be carrying out later involves percentages entirely. Therefore, it is important to confirm that learners have the required skills and knowledge on percentages.

Learning Activities

Activity 7.1

- Organise the learners in appropriate groups. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Ask the learners to do Activity 7.1 in the Student's Book as you guide them through every step stated.
- Guide the learners in calculating the original price of the iron box whose original price had been increased by 25%.
- Let the learners understand that with that 25% increase in the original price of the iron box, the buying price is higher than the original/marked price.
- Ask the learners to discuss this concept in their group and choose any member from any group at random to present the findings of their group to the whole class.

This activity will promote in learners among other comptences;

1. Communication skill through the numerous group discussions
2. Cooperation and interpersonal skills through the activities in the Student's Book
3. Critical thinking and problem solving skills

Synthesis

- Having done the activity, guide the learners through a class discussion to understand that reverse percentage involves the working out of the original price of a product backward after increase in its price. The method is mainly applies when given a quantity after a percentage increase or decrease and you are expected to find the original price.
- Guide the learners through Examples 7.1 and 7.2 in the Student's Book as you gauge their understanding of the topic area.

Assessment

- Ask the learners to do Exercise 7.1 in the Student's Book as an assignment.

Answer to Activity 7.1

1. Original price can be determined from the formula $\frac{x}{100} \times 125\% = 450$
   Where x = original price.

2. The current price is higher than the original price because it has been increased by 25%.

Lesson 2: Defination of Compound Interest

Definition of Compound Interest and Step-by-Step Method of Calculating Compound Interest

Learning objective

By the end of the lesson, the learner should be able to define compound interest and work out mathematical problems involving compound interest and its application in the financial sector.

Introduction/Pre-requisite

Remind the learners on the simple interest, its formula and how to solve mathematical problems involving simple interest. Then, use this idea to introduce compound interest to them and where it can be applied in the real life situation. Leaners should be able to remember the formula of calculating simple interest.

Learning activities

Activities 7.2 and 7.3

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader amongst themselves.
- Before starting the activity, take the learners through some of the mathematical problems involving simple interest. This will give them an insight of what is expected of them in this section.
- Let the learners do Activity 7.2 in the Student's Book on research on compound interest, definition and the difference that exist between it and the simple interest.
- Similarly, guide the learners through Activity 7.3 Student's Book on the step-by-step method of calculating compound interest.
- Ask learners random questions to gauge their understanding on the topic area.

Synthesis

- After completing the activity, discuss with the leaners and let them understand that compound interest is that interest that is calculated on the initial principal and also on the accumulated interest of previous periods of a deposit or loan.
- Guide the learners through Examples 7.3 to 7.6 in the Student's Book as you enlighten them on the two methods of calculating compound interest, step-by-step method and the compound interest formula.

Assessment

Let the learners do Exercise 7.2 in the Student's Book as a class assignment following the completion of the section and proper understanding of the topic area by the students.

Answers to Activity 7.2

1. Interest is the amount charged for using another person's property for money.

Areas for application include:

(a) When paying back loan

(b) Fee charged for recovering the cost of service.

(c) When payment as paid for larger period interest is charge.

(d) Earned when a deposit is made in a bank for saving.

2. Step by step method and Compound interest formula.
3. Compound interest is calculated on the initial principal and also on the accumulated interest of previous periods of a deposit or loan. While simple interest is calculated based on the principal amount multiplied by the interest rate and the period in a loan.

Answers to Activity 7.3

After one year: $I_1 = \frac{p \times r \times t}{100} = \frac{500{,}000 \times 8 \times 1}{100}$

$I_1 = 40{,}000$

Total after one year is: $A_1 = P + I$

$A_1 = (500{,}000 + 40{,}000)$ FRW

$A_1 = 540{,}000$FRW

After 2 years: $I_2 = \frac{A_1 \times r \times t}{100} = \frac{540{,}000 \times 8 \times 1}{100}$

$I_2 = 43{,}200$ FRW

Total after 2 years is: $A_2 = A_1 + I_2$

$A_2 = (540{,}000 + 43{,}200)$ FRW

$A_2 = 583{,}200$ FRW

After 3 years: $I_3 = \frac{A_2 \times r \times t}{100} = \frac{583{,}200 \times 8 \times 1}{100}$

$I_3 = 46{,}656$ FRW

Total after 3 years: $A_3 = A_1 + I_3$

$A_3 = 629{,}856$ FRW

After 4 years $I_4 = \frac{A_3 \times r \times t}{100} = \frac{629{,}856 \times 8 \times 1}{100}$

$I_4 = 50{,}388$ FRW

Total interest: $I = I_1 + I_2 + I_3 + I_4$

$I = (40{,}000 + 43{,}200 + 46{,}656 + 50{,}388)$ FRW

$I = 180{,}244$ FRW

Lesson 3: Compound Interest Formula

Lesson objective

By the end of the lesson, the learner should be able to state compound interest formula and use it in solving mathematical problems involving compound interest.

Introduction/Pre-requisite

Use the concept learned earlier on compound interest to give the learners a glimpse of what is to be covered in this section and to make them be able to understand the required concept faster.

Learning activities

Activity 7.4

- Ask the learners to organise themselves into appropriate groups. Ensure that all the groups observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader amongst themselves.
- Let the learners do Activity 7.4 in the Student's Book where they are expected to use step by step method in calculating the amount of money accumulated yearly.
- This section will promote in learners among other comptences;

1. Communication skill through the numerous group discussions
2. Cooperation and interpersonal skills through the activities in the Student's Book
3. Critical thinking and problem solving skills

Synthesis

- Through a class discussion, guide the learners through the two methods used in solving the problem provided in the activity i.e. step by step method and the compound interest formula.

- In the compound interest formula, help them to understand that compound interest is given by;

  $A = P\,(1 + r/100)^n$

  Where n is the number of interest period, r is the rate of interest, A is the accumulated amount and P is the principal. The formula is conveniently used in solving problems of compound interest especially those involving long periods of investments or payments.
- Discuss with the learners Examples 7.7 to 7.10 in the Student's Book by doing one and letting other students guide others through the remaining examples.
- During the discussion, ask them questions involving compound interests at random to gauge their understanding of the topic area.

Assessment

Ask the learners do Exercise 7.3 in the Student's Book.

Answers to Activity 7.4

Assemble all the $A = P\left(1 + \frac{r}{100}\right)^n$

$A = 10{,}000\left(1 + \frac{5}{100}\right)^3$

$A = 11576.25$ FRW

Step by step method:

$I_1 = \frac{10000 \times 5 \times 1}{100} = 500\text{Frw}$

$A_1 = P + I_1 = (10000 + 500)\text{Frw} = 10500\text{ Frw}$

$I_2 = \frac{10500 \times 5 \times 1}{100} = 525\text{Frw}$

$A_2 = A_1 + I_2 = (10500 + 525)\text{Frw} = 11025\text{ Frw}$

$I_3 = \frac{11025 \times 5 \times 1}{100} = 551.25\text{Frw}$

$A_3 = A_2 + I_3 = (11025 + 551.25)\text{ Frw} = 11576.25\text{ Frw}$

Lesson 4: Compound proportional change

Lesson objective

By the end of the lesson, the learner should be able to solve problems involving compound proportional change i.e. problems involving rate of work and other similar problems that often contains quantities that are in compound proportions.

Introduction/Pre-requisite

Assemble all the materials required for this activity, organise the learners in appropriate groups and make sure that all the learners are able to access all of them in their respective groups. Remind the learners of the topic of mixtures and proportions learned in S1.

Learning activities

Activity 7.5

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader amongst themselves.
- Let the learners do Activity 7.5 in the Student's Book on compound proportional change.
- In these groups, let the learners consider three people working at the same rate on a piece of land, the number of these people is then increased to five but the piece of land remains the same.
- Discuss with the learners the effect of the increase of the work force in this case. They should be able to understand that the increase in the work force means the time taken to complete this work will reduce.
- Guide them on how to perform similar problems mathematically in order to get an exact value required.
- This activity will promote in learners among other comptences:
    1. Communication skill through the numerous group discussions.
    2. Cooperation and interpersonal skills through the activities in the Student's Book.
    3. Critical thinking and problem solving skills.

Synthesis

- After class discussion and through question and answer method, let the learners understand that compound proportions are proportions that involves two or more quantities.
- Guide the learners through Example 7.11 and let one student guide the others through Example 7.12 both in the Student's Book. Ensure that they are able to tackle most of these problems on their own and with ease.

Assessment

Let the learners do Exercise 7.4 in the Student's Book as class assignment.

Answers to Activity 7.5

- If three people take 3 days to plough 2 acres, the number of people are not increased but days are increased to 5 days

  So, ratio of acres to days is $\frac{2}{3}$ or 2:3
- For 5 days, we have $\frac{2}{3} \times 5 = 3.33$ acres

If days are increased to 5, then 3.33 acres are ploughed

Summary of the Unit

Summarise the unit by:

- Asking a learner at random to take the whole class through the different concepts highlighted in the unit summary given in the Student's Book. Take the learners through a question and answer method to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit like the difference between simple and compound interest and also the application of the compound interest formula in the real life situation within the financial and banking institutions.

Additional information to the teachers

It is important for the teacher to understand that:

- Simple interest is calculated only on the principal amount of a loan while compound interest is calculated on the principal amount and also on the accumulated interest of previous periods, and can thus be regarded as "interest on interest."
- In the real world, simple interest is rarely used. When you deposit money into an interest-bearing account, or take out a line of credit, the interest that accumulates is added to the principal, and the next interest calculation is done on both the principal and the interest.
- Compound interest gives a high return as compared to simple interest.
- In Simple Interest, the principal remains constant while in the case of Compound Interest the Principal changes due to the effect of compounding.
- The growth rate of Simple Interest is lower than the Compound Interest.
- Calculation of simple interest is easy while the calculation of compound interest is complex.

End of Unit Assessment

- Ask the learners to do Unit 7 Test Student's Book as class assessment and encourage all of them to actively participate.

Remedial Activities

- Suggest activities, questions and answers for the slow learners to help them understand the unit better.

Remedial exercise

1. In 2003 the ratio of male to female population in a certain town was 7 : 9. Between 2003 and 2005, the male population increased by 3%. If the total population in 2005 was 186 000 and the overall population increase during the period was 2.5%, calculate the female population in 2005.

2. What does sh. 200 000 invested at 10% simple interest per annum for 3 years amount to?
3. An investor can invest money at 10% simple interest or 8% compound interest per annum. If she invests sh 1 040 000 for 3 years, calculate which is the more profitable investment and find the difference between the two.
4. John bought shares worth sh. 1 000 000 in a certain company. If the shares appreciated at 18 % every year, how much were they worth after 2 years?

Extended Activities

- Give more complex questions for the gifted / fast learners.

Extended exercise

1. In the year 2001, the price of a sofa in a shop was 12 000 FRW.
   (a) Calculate the amount of money received from the sales of 240 sofa sets that year.
   (b) (i) In the year 2002, the price of each sofa set increased by 25% while the number of sets sold decreased by 10%. Calculate the percentage increase in the amount received from the sales.
   (ii) If at the end of year 2002 the price of each sofa set changed in the ratio 16 : 15, calculate the price of each sofa set in the year 2003.
   (c) The number of sofa sets, sold in the year 2003 was less P% the number sold in the year 2001. Calculate the value of P, given that the amount received from sales in the two years were equal.

2. For three years, a lady deposited 100 000 FRW each year in a bank which paid compound interest at a rate of 20% per annum. She then left the money in the bank for a further eight years. Ignoring any bank charges, calculate:
   (a) the amount of money the lady had at the end of the first three years.
   (b) the amount of interest the money earned over the entire period.
3. A van costs 950 000 FRW. It was depreciated at 5% per annum for the first two years, which was revised to 15% per year for subsequent years.
   (a) Calculate the worth of the van after five years.
   (b) After five years, the van was sold, through a dealer, at 25% more than the value obtained in (a). Taking the dealers sales price as the correct value after depreciation, calculate the average monthly rate of depreciation for the five years.
4. A shirt whose marked price is 800 FRW is sold to a customer after allowing him a discount of 13%. If the trader makes a profit of 20%, find how much the trader paid for the shirt.

Answers

Exercise 7.1

1. 400 FRW
2. 2 000 FRW
3. 25 000 FRW
4. 120 000 FRW

Exercise 7.2

1. (a) Amount = 8 999 FRW
   Compound interest = 999 FRW
   (b) Amount = 53 933 FRW
   Compound interest = 5933 FRW
   (c) Amount = 39690 FRW
   Compound interest = 3 690 FRW
2. Compound interest = 914 FRW
   Simple Interest = 900 FRW
   Difference = 14 FRW
3. 7 378 FRW
4. 8 596 FRW
5. 248 FRW

Exercise 7.3

1. (a) 20 157.1 FRW (b) 9 336.37 FRW
2. 4 867.20 FRW
3. 4 years
4. 5 062.50 FRW
5. 65 563.62 FRW

Exercise 7.4

1. 115 m
2. 18 workers
3. 2 3040 FRW
4. 6 days
5. 2.1 Tonnes
6. 79.38 km/h
7. 22 days
8. 105.35km
9. 9 hours

UNIT 7 Test

1. 1438.26 FRW
2. 4082 FRW
3. 112808.22kg
4. 18052.20 FRW
5. 13506 FRW
6. 1487.50 FRW
7. (a) 5% (b) 400 FRW
8. 11106 FRW
9. 5387.42 FRW
10. 11358.24 FRW (The monthly interest is 0.67%.)
11. 6.32 years (Quarterly interest is 1.75%)
12. The tank will be half full, 1 takes $2\frac{1}{2}$ min to fill the tank.
13. 3 days
14. $1\frac{1}{2}$ hours
15. $3\frac{3}{5}$days
16. (a) $\frac{2}{3}$, 6 days (b) $\frac{1}{2}$, 4 days (c) 12 days

Answers to Remedial Activities

1. 104 228
2. 260 000 FRW
3. Amount are 1352 000 FRW and 1310100.48 FRW.
   Difference is 41899.52; 10% SI is more profitable.
4. 1 392 400 FRW.

Answers to extended activities

1. (a) 2 880 000 FRW (b) (i) 12.5% (ii) 16 000 FRW
   (c) P = 62.4%
2. (a) 436 800 FRW (b) 2014960.05 FRW
3. (a) 526 400 FRW (b) 0.62%
4. 580

UNIT 8

RIGHT-ANGLED TRIANGLES

(Student's Book pages 116 – 143)

Key Unit Competence

By the end of this unit, learners should be able to find lengths and angles in right-angled triangle using trigonometric ratios.

Learning Objectives

Knowledge and Understanding

- Give and define the elements of a right-angled triangle.
- Show relationship between the elements of a right-angled triangle.

Skills

- Use Pythagoras' theorem to find the relationship between the elements of a right-angled triangle.
- Solve problems about right-angled triangle using the properties of elements of a right-angled triangle and Pythagoras' theorem.

Attitudes and Values

- Appreciate the importance of right-angled triangles in various situations.
- Promote teamwork.
- Show patience, mutual respect, tolerance, and curiosity in solving and discussing problems involving right-angled triangles.

Pre-requisite to this Unit

For learners to acquire the knowledge, skills, attitudes and values that are envisaged in this unit without any difficulty, they need to acquire the following;

- Knowledge on different types of triangles and the use of Pythagoras theorem to solve problems involving these triangles learned in primary. Here they are expected to have enough knowledge on the different parts of a right-angled triangle.
- Knowledge and skills on the derivation of the Pythagoras theorem and its application in finding the other sides of a right-angled triangle given two other sides.
- Learners should be able to have the knowledge to show the relationship between the elements of a right-angled triangle.
- Knowledge of calculating lengths of sides and angles of a right-triangled triangle using trigonometric ratios and be able to solve word problems involving right triangles and trigonometric ratios.

Cross cutting issues to be addressed

The specific cross cutting issues to be addressed in this unit includes;

- **Inclusive education**: This involves ensuring that all the learners are engaged in this unit and that other students welcome them so that everyone can achieve their potential. This inclusiveness embraces all individuals regardless of their gender and abilities including those with physical disabilities and learning difficulties.

  Peace and value education: Through discussions, sharing ideas and materials, it will help learners work peacefully. Hearing out each others opinion also contributes to peace and value education among the learners.
- **Standardization culture**: This will be through drawing and measuring angles accurately.

Generic competences to be addressed

- **Leadership skills**: Since the unit will involve the learners working in groups and each group having a group leader.
- **Critical thinking**: This competence will be achieved when the teacher involve learners in activities that involve finding lengths, heights and angles.
- **Problem solving**: This competence will be achieved when the teacher involve learners in activities and exercises on determining lengths, heights and angles in real life.

Vocabulary/Keywords

Learner will come across the following concepts in the course of their learning, which are explained in the book. Some of these concepts include:

- Pythagoras theorem
- Median theorem
- Altitude theorem
- Leg theorem
- Trigonometric ratios

Guide the learners to understand the meaning of these words and how they are used. You also need to speak out mathematical statements involving these words in order to master their meaning, usage and application.

Guidance on the problem statement

- In order to focus the attention of the learners into the general direction of this unit, the teacher should guide the learners on how to conduct unit focus activity at the beginning of this unit in the Student's Book page 116.

Unit Focus Activity

- With your guidance, ask learners to form appropriate groups in looking into this unit focus activity on in the Student's Book page 116. In these groups, let them maintain gender balance and ask them if there is any one who knows what a truss is.
- Ask the learners to research and name of the right-angled triangle theorems that they know as they relate them to the roofing truss.

Answers to unit focus Activity.

1. (a) EB = 5 by pythagoras theorem ($EB^2 = EH^2 + HB^2$)
   (b) AB = 8.66cm, using cosine
   (c) EH = 2.5 cm, Using sine
2. (a) 4.5 cm, using Altitude theorem.
   (b) KL = 10cm, using pythagoras theorem

Attention to special needs

- You should provide for learning materials for all learners including those with special needs i.e. physically challenged, fast and slow learners, and those with impaired hearing.
- For the slow learners, you need to organise some remedial lessons where you need to guide them through the activities once again slowly to help them understand the needed concepts.
- The calculations in this unit may not be challenging to the specially gifted learners and they may finish ahead of the others then get bored and start distracting others. You therefore need to prepare additional and more challenging questions for these learners.

List of lessons

Lesson No.	Lesson title	Number of periods
1	Review of Pythagoras theorem	1
2	Median theorem of a right-angled triangle	2
3	Altitude theorem of a right-angled triangle	1
4	Leg theorem of a right-angled theorem	2
5	Introduction to trigonometry	1
6	Tangent of an acute angle	2
7	Using calculators to find tangent of angles	2
8	Using tangents to solve triangles	2
9	Sine and cosine of an acute angle	1
10	Finding sine and cosine using calculators	1
11	Using sine and cosine to solve right angled triangles	1
12	Application of trigonometry	2

Lesson Development

Lesson 1: Review of Pythagoras theorem

Lesson objective

By the end of the lesson, the learner should be able to remember the Pythagoras theorem formula that states that $a^2 + b^2 = c^2$. Similarly, learners are expected to define right-angled triangle and be able to show different elements of a right-angled triangle.

Introduction/Pre-requisite

The teachers should remind the learner on the parts of a right-angled triangle and label all the parts. Learners should be able to identify the two shorter sides of the triangle and the longer side. The longer side is the hypotenuse. Gauge the understanding of the learners on Pythagoras theorem by asking them random questions. This is important because most of the work and activities they will be carrying out later involves the right-angled theorems entirely. Therefore, it is important to confirm that learners have the required skills and knowledge on right-angled triangle theorems.

Learning Activities

Activity 8.1

- Organise the learners in appropriate groups. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Ask the learners to do Activity 8.1 in the Student's Book page 116 on the Pythagoras theorem as you guide them through every step stated. Encourage all the learners to participate actively in the lesson and all the activities.
- Guide the learners through the activity and make sure that they draw the right-angled triangle accurately and label the parts.

Synthesis

- Discuss with the learners Fig. 8.3 in the Student's Book page 116 that illustrates the different elements and parts of a right-angled triangle.
- Learners should know that a right-angled triangle is any triangle that has got a right angle in it. Remind the learners also that a triangle has three sides and that the total degree in a triangle is 180°.
- Summarise the section by letting the learners understand that the elements of a right-angled triangle including its three parts; the longer side called the hypotenuse and the two shorter sides called the legs.
- Remind the learners the Pythagoras theorem that states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides i.e. $a^2 + b^2 = c^2$ where a and b are the legs and c is the hypotenuse of the triangle.

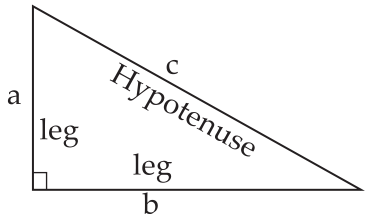
- Discuss with the learners Example 8.1 in the Student's Book page 117 as you show them the different steps involved in handling these problems.

Assessment

- Let the learners do Exercise 8.1 in the Student's Book pages 117 and 118.

Answer to Activity 8.1

- The longer side of the right-angled triangle is the hypotenuse while the two shorter sides are the legs. The square root of the sum of the square of the two shorter sides is equal to the square root of the square of the hypotenuse.



$$a^2 + b^2 = c^2$$

Lesson 2: Median theorem of a right-angled triangle

Lesson objective

By the end of the lesson, the learner should be able to state and understand the median theorem that states that the median from the right-angled vertex to the hypotenuse is half the length of the hypotenuse i.e. the median subdivides the right-angled triangle into two similar isosceles triangles.

Introduction/Pre-requisite

Prior knowledge on Pythagoras theorem is necessary in this case. They are expected to be able to use ruler and pair of compasses effectively to help them in handling the activities that follows and the lesson at large. Knowledge on the properties of right angle is also important across the unit since most of the work and activities they will be carrying out later involves the right-angled theorems entirely.

Teaching Aids

- Ruler
- Pencil
- Chart

Learning Activities

Activities 8.2 and 8.3

- Organise the learners in appropriate groups ensuring that all the groups observe gender balance and comprises learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.

- Let the learners carryout Activities 8.2 and 8.3 in the Student's Book on pages 118 and 119 respectively on the median theorem of a right-angled triangle.
- Guide the learners through all the steps stated in the activity without skipping any step stated. In the process, let them observe as they note down the important points to be used during discussions.
- Ask them to compare their findings with the proof that is well illustrated provided in the Student's Book.
- This section will promote in learners among other comptences;
  1. Communication skill through the numerous group discussions
  2. Cooperation and interpersonal skills through the activities in the Student's Book
  3. Critical thinking and problem solving skills.

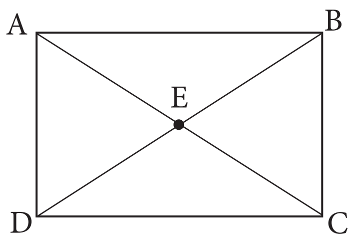
Synthesis

- Take the learners through a rigorous class discussion and demonstrate to them how the theorem is derived. Through these discussions, learners see the real life application of this topic area.
- Guide learners through Examples 8.2 and 8.3 in the Student's Book pages 119–120. Choose two students' at random to discuss with the whole class Examples 8.3 and 8.4 in the Student's Book on page 120.
- After the discussion, learners should be able to understand that the median theorem that states that the median from the right-angled vertex to the hypotenuse is half the length of the hypotenuse i.e. the median subdivides the right-angled triangle into two similar isosceles triangles.
- Activity 8.3 is used for confirming results obtained from Activity 8.2.

Assessment

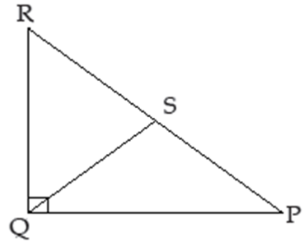
- Let the learners do Exercise 8.2 in the Student's Book page 120.

Answer to Activity 8.2



Answers to Activity 8.3

1 and 2.



3. QS is a half of PR (QS = $\frac{1}{2}$ PR)
4. All are equal i.e QS = PS = RS
5. The segments BE and ED are equal since the median AE subdivides the right angled triangle into two similar isosceles triangles, the median theorem.
6. (a) BE = DE. (b) 1:1 E is midpoint of DB (c) median (d) AE = $\frac{1}{2}$ DB

Lesson 3: Altitude Theorem of right-angled triangle

Lesson objective

By the end of the lesson, the learner should be able to understand the meaning of the word altitude and be able to realise that the altitude theorem of a right angle states that the altitude to the hypotenuse of a right-angled triangle is the mean proportion between the segments into which it divides the hypotenuse.

Introduction/Pre-requisite

Prior knowledge on Pythagoras theorem is necessary in this case. They are expected to be able to use ruler and pair of compasses effectively to help them in handling the activities that follows and the lesson at large. Knowledge on the properties of right angle is also important across the unit since most of the work and activities they will be carrying out later involves the right-angled theorems entirely.

Teaching Aids

- Ruler
- Pair of compasses
- Pencil
- Protractor
- Chart

Learning Activities

Activities 8.4 and 8.5

- Ask the learners to form groups of between 3. Ensure that all these groups observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Let the learners do Activities 8.4 and 8.5 in the Student's Book pages 121 – 122 both showing and explaining the derivation of the altitude theorem of a right-angled triangle.

- Ask learners to draw a right-angled triangle ABC with dimension of their choice on a manila paper as shown in the illustration in Fig. 8.16 in the Student's Book page 122.
- Let the group have a discussion on their observation and ask the group leader to note down the points discussed.
- This lesson will promote in learners among other comptences;
  1. Communication skill through the numerous group discussions
  2. Cooperation and interpersonal skills through the activities in the Student's Book.
  3. Critical thinking and problem solving skills.

Synthesis

- Guide the learners as you discuss on how they are expected to drop a perpendicular line from a right-angled apex that intersects the hypotenuse at a point. Let them understand that the perpendicular line they have just dropped from the apex of that triangle is the altitude of the triangle.
- Discuss with the learners Examples 8.5 and 8.6 in the Student's Book pages 122 and 123 elaborating further the workings of problems involving the altitude theorem. Pose some random questions during the working to gauge their understandings.
- Let the learners understand that the altitude to the hypotenuse of a right-angled triangle is the mean proportion between the segments into which it divides the hypotenuse.

Assessment

Ask the learners to do Exercise 8.3 in the Student's Book Page 123-124.

Answers to Activity 8.5

The ratios of $\frac{NC}{AN}$ and $\frac{NB}{NC}$ is a constant. This is because the altitude to the hypotenuse of a right-angled triangle is the mean proportion between the segments into which it divides the hypotenuse.

Lesson 4: Leg Theorem of right-angled triangle

Lesson objective

By the end of the lesson, the learner should be able to state and explain the leg theorem of a right-angled triangle and be able to solve problems involving the same.

Introduction/Pre-requisite

Prior knowledge on Pythagoras theorem is necessary in this case. They are expected to be able to use ruler and pair of compasses effectively to help them in handling the activities that follows and the lesson at large. Knowledge on the properties of right angle is also important across the unit since most of the work and activities they will be carrying out later involves the right-angled theorems entirely.

Teaching Aids

- Scissor
- Writting materials
- Minira paper

Learning Activities

Activities 8.6 and 8.7

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader amongst themselves.
- Let the learners do Activities 8.6 and 8.7 in the Student's Book pages 124-125 on leg theorem of a right-angled triangle.
- Let the learners observe and note down the observation before making any deduction. Ask them to discuss the findings amongst themselves within the group before every group produces on representative to report their findings to the whole class.
- Guide learners through the demonstration in Activity 8.7 on pages 124 – 125 and make sure they follow all the instructions to the latter without skipping any step stated.

Answers to Activity 8.6

4. Δs ADC and DBC are similar to the mother triangle.

$$\frac{AC}{AB} = \frac{AD}{AC}, \frac{BC}{AB} = \frac{BD}{BC}$$

Synthesis

- Discuss with the learners Examples 8.7 and 8.8 in the Student's Book pages 125-126 and then request one student to discuss with the whole class Example 8.8 both in the Student's Book on page 125-126.
- Formulate different questions on this section and ask the learners to answer randomly to gauge their understanding and to check if the objective has been reached.
- Let the learners understand that leg theorem states that the leg of a right-angled triangle is the mean proportional between the hypotenuse and the projection of the leg on the hypotenuse.

Assessment

- Let the learners do Exercise 8.4 in the Student's Book pages 126 – 127.

Answers to Activity 8.7

(a) (i) $\frac{EF}{EG} = \frac{EH}{EF}$ (ii) $\frac{QR}{PQ} = \frac{QS}{QR}$

(b) The leg of a right-angled triangle is the mean proportional between the hypotenuse and the projection of the leg on the hypotenuse.

5. Projection of a line defines the straight line of the plane connecting the feet of the perpendiculars let fall from the extremities of the given line.

Lesson 5: Introduction to trigonometry

Lesson objective

By the end of the lesson, the learner should be able to define trigonometry and state trigonometric ratios. Also, the learner should be able to use trigonometry in finding the other sides of a right-angled triangle.

Introduction/Pre-requisite

Learners are expected to have prior knowledge on right-angled triangles and also be able to name different parts of a right-angled triangle. This knowledge on the sides of a right-angled triangle is key inn this topic area and important across the unit since most of the work and activities they will be carrying out later involves the right-angled triangle and the positioning of its sides.

Teaching Aids

- Internet
- Chart

Learning Activities

Activity 8.8

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader amongst themselves.
- Guide the learners through Activity 8.8 in the Student's Book page 127-128 the identification of different sides of a right angled triangle that is used in this topic area, trigonometry.
- Guide the learners through the signs used in trigonometry that are provided in the Student's Book.
- Allow the learners to discuss the findings and observations amongst themselves, and then ask either the group leader or any group member at random to present the findings of their groups to the whole class. This will allow other members to contribute in pointing out errors and adding any omission to the presented facts on sliding and static frictions.
- This section will promote in learners among other comptences:
    1. Communication skill through the numerous group discussions
    2. Cooperation and interpersonal skills through the activities in the Student's Book.
    3. Critical thinking and problem solving skills

Synthesis

- Discuss with the learners in their groups the mean of trigonometry and how to identify different sides of a right-angled triangle that are used in trigonometry. Also, let the learners' understand and master some of the symbols used in trigonometry.
- Discuss with the learners Example 8.9 in the Student's Book page 128 on identification of different sides of a right-angled triangle based on trigonometry ratios.
- Let the learners understand that trigonometry defines a branch of mathematics that is concerned with the relationship between the sides and the angles of triangles.

Assessment

- Ask the learners to do Exercise 8.5 in the Student's Book page 129.

Answers to Activity 8.8

(a) Adj = AB, Opp = BC, Hyp = AC

(b) Adj = AB, Opp = BC, Hyp = AC

(c) Adj = BC, Opp = AB, Hyp = AC

Lesson 6: Sine and Cosine of an Acute Angle

Lesson Objective

By the end of the lesson, the learner should be able to derive these trigonometric ratios (sine and cosine) and use it appropriately to find the other sides of an acute angle of a right-angled triangle.

Introduction/Pre-requisite

Assemble all the requirements required for this activity, organise the learners into appropriate groups where all can access all the requirements needed in this section.

Teaching Aids

- Calculator

Learning Activities

Activity 8.9

- Ask the learners to organise themselves into appropriate groups of between four and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.
- Guide the learners through the steps and procedure stated in the Student's Book that are followed in deriving both the sine and cosine trigonometric ratios.

- Given the right-angled triangles, the learner should be able to find the ratios as stated and be able to comprehend and relate in getting the ratios required here. Guide the learners effectively until all are able to handle the problems with ease.
- Let the learners do Activity 8.9 in the Student's Book page 129 demonstrating how to find the sine and cosine of an angle. Also, the activity demonstrates that these ratios are constant for a particular angle irrespective of the size of the triangle.
- This section will promote in learners among other comptences;
  1. Communication skill through the numerous group discussions
  2. Cooperation and interpersonal skills through the activities in the Student's Book.
  3. Critical thinking and problem solving skill

Synthesis

- Learners must have realised that given a right-angled triangle:
  - ***Sin*** θ = Opposite side/Hypotenuse
  - ***Cos*** θ = Adjacent side/Hypotenuse
- Discuss with the learners Example 8.10 in the Student's Book pages 130-131 as you gauge their understanding by asking them random relevant questions on this section.

Assessment

- Let the learners do Exercise 8.6 in the Student's Book page 131 as home assignment.

Answers to Activity 8.9

4.

i	1	2	3	4	5
OA_i (cm)	1.1	2.2	3.3	4.4	5.5
OB_i (cm)	1	2	3	4	5
A_iB_i (cm)	0.6	1.2	1.8	2.4	3.0
A_iB_i/OA_i	0.5455	0.5455	0.5455	0.5455	0.5455
OB_i/OA_i	0.9090	1.9091	2.9091	3.9091	4.9091

5. $\frac{A_iB_i}{OAi}$ = constant $\frac{OB_i}{OAi}$ = constant

Lesson 7: Finding sine and cosine using calculator

Lesson Objective

By the end of the lesson, the learner should be able to use calculator in finding both sine and cosine and to be able to solve the related questions.

Introduction/Pre-requisite

Assemble all the requirements required for this activity, organise the learners into appropriate groups where all can access all the requirements needed in this section.

Teaching Aid

- Calculator

Learning Activities

Activity 8.10

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.
- Let the learners do Activity 8.10 in the Student's Book page 131 in finding sine and cosine using calculator.
- After the end of the activity, ask the groups to present their results to the class each at a time as the other groups compare with other groups results.

Synthesis

- Let the learners discuss amongst themselves Examples 8.11 and 8.12 in the Student's Book page 131–132 as you ask them random questions on their understanding the topic area.
- Guide the learners through the steps of finding sine and cosine using the calculator and the use of sine and cosine in solving right-angled triangles.
- Guide the learners to understand that as the angle increases from 00 to 900 their sine increases from 0 to 1 and their cosine decrease from 1 to 0.
- Make sure learners have understood the activity very well.

Assessment

- Let the learners do Exercise 8.7 in the Student's Book page 132.

Answers to Activity 8.10

(a) $\sin 32° = 0.5299$ and $\cos 32° = 0.8480$

(b) $\sin 17.89° = 0.3072$ and $\cos 17.89° = 0.9516$

(c) $\sin 73.5° = 0.9588$ and $\cos 73.5° = 0.2840$

Lesson 8: Using sines and cosines to find angles and lengths of sides of right-angled triangles

Lesson Objective

By the end of the lesson, the learner should be able to use sine and cosine to solve right-angled triangles.

Introduction/Pre-requisite

Assemble all the requirements required for this activity, organise the learners into appropriate groups where all can access all the requirements needed in this section.

Teaching Aid

- Calculator

Learning Activities Activity 8.11

- Organise the learners into pairs where they are supposed to work with their immediate neighbours or according to the sitting arrangement in class.
- Let the learners do Activity 8.12 in the Student's Book page 134 on using sine and cosine to solve right-angled triangles.
- Allow the learners to discuss the findings and observations amongst themselves in their respective pairs, and then ask either of them at random to present their findings to the whole class. This will allow other members to contribute in pointing out errors and adding any omission to the presented facts air resistance.

Synthesis

- Guide the learners through the steps of finding of using sine and cosine in solving right-angled triangles.
- Wind up the activity summary and do Examples 8.13 and 8.14 in the Student's Book pages 133–134.
- Make sure the learners have understood the activity very well.

Assessment

- Ask the learners to do Exercise 8.8 in the Student's Book pages 134.

Answers to Activity 8.11

(a) $x = 7.28$ cm, $y = 5.29$ cm, $\theta = 36°$

(b) $y = 4.38$ cm, $\theta = 42.4°$

Lesson 9: Defination of a tangent of an Acute Angle

Lesson objective

By the end of the lesson, the learner should be able to know how to find the tangent of an acute angle and to solve all related questions. Similarly, the learner should be able to derive this trigonometric ratio and use it appropriately to find the other sides of an acute angle of a right-angled triangle.

Introduction/Pre-requisite

Prior knowledge on trigonometry and right-angled triangles is necessary. Similarly, the teacher needs to remind learners on the topic of angles learned in S1 where they are expected to know and be able to identify an acute angle. Gauge the learners understanding on angles by asking them random questions. This is important because most of the work and activities they will be carrying out in this topic area entirely involves acute angles and right-angled triangles. It is therefore important to confirm that learners have the required skills and knowledge on angles.

Teaching Aids

- Internet
- Chart

Learning Activities

Activity 8.12

- Organise the learners in appropriate groups. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Ask the learners to do Activity 8.12 in the Student's Book pages 134-135 as you guide them through every step stated.
- Guide the learners on the process of constructing a perpendicular line from a point as stated in the activity.
- Let the learners make observations and take measurements of their findings. All the measurements they are supposed to feed in the table 8.2 provided in the Student's Book within the same activity.
- Ask the learners to discuss this concept in their group and choose any member from any group at random to present the findings off their group to the whole class.
- Make sure that all the learners participates actively and make sure all are able to handle the exercise.
- This activity will promote communication skill through the numerous group discussions, cooperation and interpersonal skills through the activities in the Student's Book and also critical thinking and problem solving skills amongst the learners in learners among other comptences.

Synthesis

- Discuss with the learners Example 8.15 in the Student's Book page 135. Request one student to discuss with the whole class Example 8.16 in the Student's Book page 135-136.
- Formulate different questions on this section and ask the learners to answer randomly to gauge their understanding and to check if the objective has been reached.
- Let the learners understand that the tangent of each of the acute angles in the two

figures is equal to the opposite side divided by the adjacent side of the triangle.

$$\textbf{Tan } \theta = \frac{\textbf{Opposite side}}{\textbf{Adjacent side}}$$

where θ is one of the adjacent sides of the acute angle of a right-angled triangle.

- Guide the learners to prove that the tangent of an angle is constant irrespective of the size of the triangle.

Assessment

- Let the learners do Exercise 8.9 in the Student's Book page 136.

Answers to Activity 8.12

5.

i	1	2	3	4	5
A_iB_i (cm)	0.7	1.4	2.1	2.8	3.5
OB_i (cm)	1	2	3	4	5
A_iB_i / OB_i	0.7	0.7	0.7	0.7	0.7

6. $\frac{A_iB_i}{OB_i}$ is a constant

Lesson 10: Use calculators to find tangent of angles and angles given their tangents

Lesson Objective

By the end of the lesson, the learner should be able to know and understand the steps involved in using a calculator to find the tangent of angles and to be able to solve questions involving the same.

Introduction/Pre-requisite

Assemble all the requirements required for this activity, organise the learners into appropriate groups where all can access all the requirements needed in this section and guide them on how to find tangent of angles by use of calculators.

Teaching Aids

- Calculator

Learning Activities

Activity 8.13

- Ask the learners to organise themselves in pairs. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.
- Before taking the learners through the activity stated in the Student's Book, take the learners through the requirements of the derivation as stated in in the previous activity in the Student's Book.

- Take the learners through Activity 8.13 in the Student's Book page 136 on the use of calculators in finding the tangent of an acute angle of a right-angled triangle.
- This section will promote in learners among other competences;
  1. Communication skill through the numerous group discussions
  2. Cooperation and interpersonal skills through the activities in the Student's Book.
  3. Critical thinking and problem solving skills.

Synthesis

- Guide and discuss with the learners the use of calculators in finding the tangent of an acute angle of a right-angled triangle. By comparing the use of a calculator and the manual method, the learners must have realised that it is easier to use a calculator compared to the manual method.
- Discuss with the learners Examples 8.17 and 8.18 in the Student's Book page 136 and then ask them random questions to gauge their understanding of the topic area.

Assessment

- Let the learners' do Exercise 8.10 in the Student's Book page 137.

Answers to Activity 8.13

(a) 0.1228 (4 dp)

(b) 0.2419 (4 dp)

(c) 1.1918 (4 dp)

Lesson 11: Using tangents to find lengths of sides and angles of right-angled triangles

Lesson Objective

By the end of the lesson, the learner should be able to know and understand the use of tangents to solve triangles most specifically in finding the missing side of a right-angled triangle given the two sides and an angle.

Introduction/Pre-requisite

Assemble all the requirements required for this activity, organise the learners into appropriate groups where all can access all the requirements needed in this section and remind them of the formula of finding the tangent of an acute angle of a right-angled triangle.

Teaching Aids

- Calculator

Learning Activities

Activity 8.14

- Organise the learners in appropriate groups. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Let the learners do Activity 8.14 in the Student's Book page 137 demonstrating the use of tangents in solving triangles, in terms of the sides and the angles of the triangle given.
- Let the learners conduct a discussion within their groups on their observation as they note down the points discussed.
- Ask any other member from the group at random, to present their findings to the whole class and allow other members to contribute in pointing out errors and adding any omission to the presented facts.
- This activity will promote communication skill through the numerous group discussions, cooperation and interpersonal skills through the activities in the Student's Book and also critical thinking and problem solving skills amongst the learners in learners among other comptences

Synthesis

- Discuss with the learners Example 8.19 and 8.20 in the Student's Book page 137-138 as you ask them random question to gauge their understanding of the topic area.
- The learners, in their respective groups, must have found that using the expression for the tangent of an acute angled right-angled triangle, they are able to find the values of unknown sides and angles in the triangle.

Assessment

- Let the learners do Exercise 8.11 in the Student's Book page 138.

Answers to Activity 8.14

1. (a) 5.6 (b) $\alpha = 36.9°$ and $\theta = 53.1°$

Lesson 12: Application of trigonometric ratios (sine, cosine and tangent)

Lesson objective

By the end of the lesson, the learner should be able to relate trigonometry to the real life situation and its application.

Introduction/Pre-requisite

Prior knowledge on Pythagoras theorem is necessary in this case. They are expected to be able to use ruler and pair of compasses effectively to help them in handling the activities that follows and the lesson at large. Knowledge on the properties of right angle is also important across the unit since most of the work and activities they will be carrying out later involves the right-angled theorems entirely.

Teaching Aids

- Calculator

Learning Activities

Activity 8.15

- Organise the learners in appropriate groups ensuring that all the groups observe gender balance and comprises learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Let the learners do Activity 8.15 in the Student's Book pages 138-139 on the application of trigonometry in the real-life situation.
- Guide the learners through all the steps stated in the activity without skipping any step stated. In the process, let them observe as they notes down the important points to be used during discussions.
- Ask them to compare their findings with the proof that is well illustrated provided in the Student's Book.
- This section will promote in learners among other comptences;
  (a) Communication skill through the numerous group discussions
  (b) Cooperation and interpersonal skills through the activities in the Student's Book.
  (c) Critical thinking and problem solving skills.

Synthesis

- Make sure that learners can relate trigonometry to the real life situation and give out some of its applications.
- Discuss with the learners Examples 8.21 and 8.22 in the Student's Book pages 139-140.

Assessment

- Let the learners do Exercise 8.12 in the Student's Book pages 140 – 141.

Answers to Activity 8.15

$$\tan/3 = \frac{x}{AC} \Rightarrow AC= \frac{x}{\tan\beta}$$

$$\tan\alpha = \frac{x}{AB} \Rightarrow AB= \frac{x}{\tan\alpha}$$

$$\alpha = \frac{x}{\tan\beta} - \frac{x}{\tan\alpha} \qquad \alpha = AC - AB$$

Summary of the Unit

Summarise the unit by:

- Asking the learners to take the whole class through the different concepts highlighted in the unit summary given in the Student's Book page 141-142. Take the learners through a question and answer method to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit like the difference between simple and compound interest and also the application of the compound interest formula in the real life situation within the financial and banking institutions.

Additional information to the teachers

It is important for the teacher to understand that:

- In Mathematics, the Pythagorean theorem, also known as Pythagoras's theorem, is a fundamental relation in Euclidean geometry among the three sides of a right triangle.
- The Pythagorean theorem takes its name from the ancient Greek mathematician Pythagoras, who was perhaps the first to offer a proof of the theorem. But people had noticed the special relationship between the sides of a right triangle long before Pythagoras.

End of Unit Assessment

- Ask the learners to do Unit Test 8 in the Student's Book pages 142-143 as a class assessment and encourage all of them to participate actively.

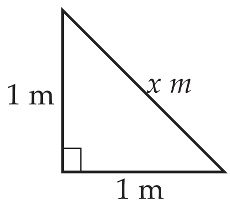
Remedial Activities

- For the slow learners, a teacher should organize remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

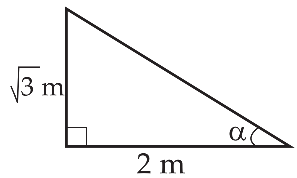
Remedial Exercises

1. In the triangles below lengths are given in metres. Find the values of the unknown.

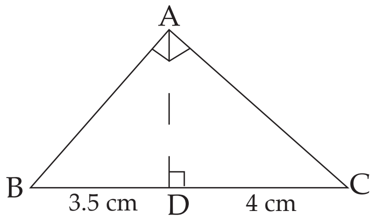
(a)



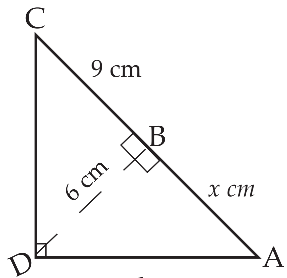
(b)



2. Find the length of AB in the triangle ABC below.



3. In triangle ADC below, find the length of segment AB.



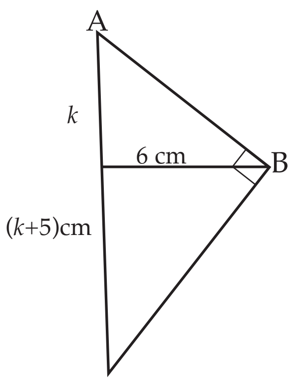
4. Find the values of the angles in the following:

a) Sin $x = 0.5$ b) Sin $y = 0.7071$ c) Tan $x = \frac{\sqrt{3}}{3}$ d) Cos $x = \frac{\sqrt{3}}{2}$

Extended Exercise

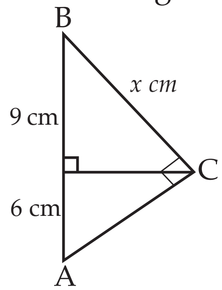
- The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Give learners the questions below.

1. Find the value of k in the figure below.



2. The altitude to the hypotenuse of a right – angled triangle is 10 cm long. If the hypotenuse is 22 cm long, what are the lengths of two segments of the hypotenuse?

3. In the triangle ABC below, find the lengths represented by letters.



4. Given that $\sin a = \frac{x}{y}$, what is the value of $\tan a$ in terms of x and y?

Answers

Exercise 8.1 (Student's Book pages 117 – 118)

1. (a) 10.91 cm (b) 1.73 cm (c) 2.65 cm (d) 1.66 cm
   (e) 1.41 cm (f) 6.71 cm
2. (a) 4.6m (b) 5m
3. 8.245 m 4. 6.44m

Exercise 8.2 (Student's Book page 120)

1. 9 cm
2. (a) 15 cm (b) 9 cm
3. 3.75 cm
4. (a) 24° (b) 48°
5. x = 4cm, median = 10cm

Exercise 8.3 (Student's Book pages 123 – 124)

1. 12.5 cm 2. 36 cm 3. 6 cm
4. (a) 6 cm (b) 10 cm
5. 4 cm, 16 cm 6. 13.42 cm 7. 4 cm
8. EH = 5 cm, HG = 20 cm 9. 4 cm and 25 cm 10. 6 cm

Exercise 8.4 (Student's Book pages 126 – 127)

1. 10.39 cm 2. 9.17 cm 3. 2.5 cm 4. BD = 5.33, AC = 14.42
5. 10.58 cm 6. 1.8 cm 7. XV = 5.54 cm, XW = 5.77 cm
8. (a) KN = 9.6 cm (b) NL = 5.4 cm (c) NM = 7.2 cm
9. (a) QR = 9.8, PQ = 6.93 cm, SU = 5.66 cm
10 (a) 10.4 cm (b) 13.75 cm (c) 15.87 cm

Exercise 8.5 (Student's Book page 129)

1. α – Adj = PQ, Opp = PS, Hyp = QS
   β –Adj = PS, Opp = PQ, Hyp = QS

θ –Adj = PQ, Opp = QR, Hyp = PR

Υ –Adj = QR, Opp = PQ, Hyp = PR

ϕ – Not in right-angled triangle hence not applicable.

2. (a) 4 cm (b) 6.32 cm

3. (a) AD, AB
   AD = 16 cm
   AB = 20 cm
   (b) BC, AC
   BC = 15 cm
   AC = 25 cm

Exercise 8.6 (Student's Book page 131)

1. (a) 0.3, 0.9 (b) 0.6, 0.7 (c) 0.9, 0.4 (d) 0.9, 0.2

2. (a) A = 42° (b) B = 49° (c) C = 37° (d) D = 71° (e) E = 65°
   (f) F = 44° (g) G = 25° (h) H = 57° (i) I = 41° (j) J = 77°
   (k) K = 70° (l) L = 56°

Exercise 8.7 (Student's Book page 132)

1. (a) 0.052 3 (b) 0.225 0 (c) 0.939 7 (d) 0.891 0
   (e) 0.228 4 (f) 0.740 8 (g) 0.982 3 (h) 0.999 9

2. (a) 0.951 1 (b) 0.891 0 (c) 0.656 1 (d) 0.342 0
   (e) 0.942 6 (f) 0.802 8 (g) 0.213 0 (h) 0.106 3

3. (a) 23.4° (b) 52° (c) 68.1° (d) 85.1°
   (e) 71.6° (f) 66.6°

4. (a) 23.33° (b) 55.66° (c) 82.91° (d) 64.4°
   (e) 28.35° (f) 56.25°

Exercise 8.8 (Student's Book page 134)

1. (a) 3.009 cm (b) 4.815 cm (c) 3.126 cm (d) 5.446 cm
   (e) 79.864 cm (f) 6.84 cm (g) 93.969 cm (h) 8.097 cm

2. (a) x = 5.00 cm, y = 7.47 cm (b) p = 7.04 cm, q = 5.40 cm

Exercise 8.9 (Student's Book page 136)

1. (a) 0.8391 (b) 1.0000 (c) 1.1918 (d) 0.4663
   (e) 0.6494 (f) 1.7321

2. (a) 14.0° (b) 21.8° (c) 23.2° (d) 51.3°
   (e) 50.2° (f) 60.3° (g) 69.4° (h) 45°
   (i) 71.6°

Exercise 8.10 (Student's Book page 137)

(a) 0.1228 (b) 0.2419 (c) 1.1918 (d) 1.0990

(e) 5.9758 (f) 0.0386 (g) 0.3338 (h) 1.4648

(i) 2.3547 (j) 1.2685 (k) 2.0732 (l) 3.3759

Exercise 8.11 (Student's Book page 138)

1. (a) 12.6 cm (b) 13 .7 cm (c) 9.99 cm (d) 17.4 cm
2. (a) 2.18 cm (b) 11.9 cm (c) 46.4 cm (d) 12.4 cm
3. (a) 28° (b) 22.62° (c) 28.03° (d) 56.02° (e) 42.83°

Exercise 8.12 (Student's Book pages 140–141)

1. 2.75 m
2. 49.45 m
3. Length = 9.06 m; width = 4.23 m
4. 14.84 km.
5. 17.46° (2d.p.)
6. 24.62° (2 d.p.)
7. 1 562 m
8. (a) Boat A travelled 2.0 km east.
   Boat B travelled 4.6 km east.
   Boat B by 2.6 km
   (b) 7.3 km
9. 147.2 m
10. 39.81° (2 d.p.)
11. 46.9 m (1 d.p.)
12. 384.9 m (1 d.p.)
13. 86.6 m (1 d.p.)
14. (a) 84.8 cm (b) 65 cm

Unit 8 Test (Student's Book pages 142–143)

1. (a) a = 5.8 cm (b) b = 9.9 cm
2. 4. 47cm ≈ 4.5 cm
3. 12.8 cm
4. x=10 cm, Hyp = 46 cm
5. (a) m = 4.9 cm, n = 5.75 cm
6. 6.25 cm, 3.75 cm
7. 16 cm, 4 cm
8. 24 cm
9. 237 m
10. A = 92 m, B= 69 m
11. (a) 9.4 cm (b) a = 24.1 cm, b = 42.7 cm, c = 18.86 cm
    (c) x = 3.9 m, h = 7.3 m

Remedial Exercises

1. (a) $x = 1.414$ m (b) $a = 40.9°$
2. AB = 5.12 cm
3. AB = 4 cm
4. (a) $x = 30°$ (b) 45° (c) 30° (d) 30°

Extended Exercises

1. K = 4 cm
2. 15.6 cm and 6.4 cm
3. $x = 11.62$ cm
4. $\tan a = \frac{x}{\sqrt{y^2 - x^2}}$

UNIT 9

CIRCLE THEOREM

(Student's Book pages 144–174)

Key unit competence

By the end of this unit, the learner should be able to construct mathematical arguments about circles and use circle theorems and discs to solve related problems.

Learning Objectives

Knowledge and Understanding

- Recognize and identify the elements of a circle.
- Identify angle properties in a circle.

Skills

- Find the length of elements of a circle.
- Calculate the area of disk and its sector.
- Use the angle properties of lines in circles to solve problems.
- Use tangent properties to solve circle problems.

Attitudes and Values

- Develop clear, logical, and coherent thinking.
- Appreciate the importance of circle theorems in dividing into sectors.
- Promote teamwork when working in groups.
- Show patience, mutual respect, tolerance, and curiosity in solving and discussion problems involving circle theorems and disk.

Pre-requisite to this Unit

Learners need the following to acquire the knowledge, skills, attitudes and values that are envisaged in this unit without any difficulty:

- Properties of circles: here the learners should be able to remember some of the properties of circles that were learned in S1. Using the question and answer method, the teacher should make sure that all the learners can remember these concepts before they are introduced to the concept of circle theorem.
- Learners are introduced to basic concepts about circles back in P5, their properties and to get their areas and circumference. The teacher is therefore expected to remind the learners on some of the basic elements of a circle and to use short written questions and questions and answers to ascertain that they still have these skills before you can introduce them to all the theorems derived from the circle. The teachers should guide those with challenges.

- The understanding of the unit by the learners is fundamental as it cuts across several areas and topics of mathematics. For better understanding of the topic, you should constantly engage the learners in the several practical activities provided in the Student's Book. Similarly, the learner should be made to understand and relate these activities with their daily lives to make them appreciate and enjoy the effects and applications of circle theorem in making our lives easier and better in overcoming some of the daily challenges we constantly face in life.

Generic competences to be addressed

- **Problem solving skills**: this competence will be gained when leaners involve themselves in solving complexed problems.
- **Critical thinking**: this will be though engagement in different activities of this unit
- **Leadership skills**: this is though leaners working as group leaders.

Cross-cutting issues to be addressed

The specific cross cutting issues to be addressed in this unit includes;

- **Inclusive education**: this involves ensuring that all the learners are engaged in education and that they are welcomed by other students so that everyone can achieve their potential. This inclusiveness embraces all individuals regardless of their gender and abilities including those with physical disabilities and learning difficulties. The aim of inclusive education and curriculum is ensuring participation in education of learners with varied learning styles and other difficulties.
- **Peace and value education**: This will be though encouraging leaners to work in groups. Peace is critical for the society to flourish and for every individual to focus on personal development and achievements and their contribution to the success of the nation. Value education forms a key element for the strategy for ensuring young people recognise the importance of contributing to the society, working for peace and harmony and being committed to avoiding conflict.

Vocabulary/Keywords

Learner will come across the following concepts in the course of their learning, which, are explained in the book. Some of these concepts include:

- Circle
- Chord
- Tangent
- Segment
- Secant
- Sector

Guide the learners to understand the meaning of these words and how they are used. You also need to speak out mathematical statements involving these words in order to master their meaning, usage and application.

Guidance on the problem statement

- In order to focus the attention of the learners into the general direction of this unit, the teacher should guide the learners on how to conduct unit focus activity at the beginning of this unit in in the Student's Book page 144.

Unit Focus Activity

- Organise the learners into appropriate groups of three and guide them through the focus activity on page 144 on the Student's Book at the beginning of the unit. The groups should consider gender balance and any disability if any.
- From their reference books, ask them to research the meaning of angle theorems for circles.
- Still in their groups, ask them to apply the theorems found above to find the values of angles given in Fig. 9.1 in the Student's Book page 144. Let them state the reasons for their answers supporting the theorems in each case.
- From their analysis and research, let them compare their answers with those from other groups and ask one member from every group to present their findings to the whole class.

Answers to Unit Focus Activity

1. Angles at the centre and circumference of a circle
   (a) Angles in a semi circle
   (b) Angles in same segment
   (c) Angles in a cyclic quadrilateral
   (d) Tangent to a circle
   (e) Angles in alternate segment.
   (f) Perpendicular bisector of a chord
2. (a) <MLN=40^{o} Angles subtended to the circumference by same cord.
   (b) <OLN =25^{o} Angles subtended to the circumference by the diameter add up to 90^{o}
   (c) <LNP = <LKN = 65^{o}, Angles in alternate segment are equal.
   (d) <MPN = 10^{o}

Attention to special needs

- You should provide the learning materials for all learners including those with special needs i.e. physically challenged, fast and slow learners', and those with impaired hearing.
- For the slow learners', you need to organise some remedial lessons where you need to guide them through the activities once again slowly to help them understand the needed concepts.
- The calculations in this unit may not be challenging to the specially gifted learners and they may finish ahead of the others then get bored and start distracting others. You therefore need to prepare additional and more challenging questions for these learners.

List of lessons

Lesson No.	Lesson title	Number of periods
1	Elements of a circle	2
2	Theorem 1 – Angles at the centre and circumference of a circle	2
3	Theorem 2 – Angles in a semi circle	2
4	Theorem 3 – Angles in the same segment	2
5	Theorem 4 – Angles in a cyclic quadrilateral	2
6	Theorem 5 – Tangent to a circle	2
7	Constructing a tangent at any given point on the circle Constructing a tangent to a circle from a common point	2
8	Theorem 6 – Angles in alternate segments	2
9	Theorem 7 – Properties of chords – Perpendicular bisector of a chord	2

Lesson Development

Lesson 1: Elements of a circle and disk

Lesson objective

By the end of the lesson, the learner should be able to state and label different elements of a circle.

Introduction/Pre-requisite

In S1, the learners had already learned about the properties of circles and solving problems on the finding of area and circumference of the same. Remind the learners that a circle defines a line forming a complete loop with every point on which is a fixed distance from a centre point. Use the activity provided to let them understand the elements of both circles and discs and later the circle theorems.

Teaching Aids

- Pair of compasses,
- Ruler
- Pencil

Learning Activities

Activity 9.1

- Ask the learners to form groups of between 3 and 5. Ensure that all these groups observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Let the learners do Activity 9.1 in the Student's Book page 144 on the elements of a circle and disc.
- Ask learners draw a circle of any radius then using a ruler and a pencil draw a straight line that passes through the centre of the circle and measure the distance

from the centre of the circle to both ends of the straight line. Then let them name the different parts of the circle.

- Let the group have a discussion on their observation and ask the group leader to note down the points discussed.
- This lesson will promote in learners among other comptences;
  1. Communication skill through the numerous group discussions.
  2. Cooperation and interpersonal skills through the activities in the Student's Book.
  3. Critical thinking and problem solving skills.

Synthesis

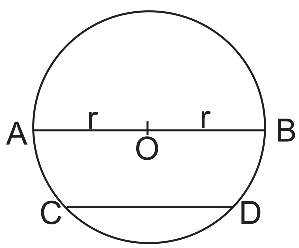
- Let the learners understand that both the distances from the centre to the circumference of the circle are equal. These distances are the radius of the circle.
- Formulate different questions on this section and ask the learners to answer randomly to gauge their understanding and to check if the objective has been reached.
- Let the learners observe figures 9.2 to 9.11 in the Student's Book pages 144–146 illustrating and defining different parts of a circle.
- Summarise by letting the learners understand all the elements of a circle and the circle theorems.

Assessment

Use question and answer method to prompt the learners' understandings on the different elements of a circle.

Answers to Activity 9.1

1.



2. r = 2cm
3. AB = Diameter

   r = radius

   O = centre

   CD = chord

- The teacher to guide learners on how to draw a circle using a pair of compass.
- The teacher to go round and ensure that learners are measuring the correct diameter.

- Parts of a circle include: centre, diameter, radius, circumference, a chord, segment, sector, arc, secant line, tangent line etc. .

Circle Theorems

Lesson 2: Theorem 1 - Angles at the centre and circumference of a circle

Lesson objective

By the end of the lesson, learners should be able to understand and demonstrate that the angle subtended at the centre of a circle is twice the angle subtended by the same on the circumference of the same circle on the same side.

Introduction/Pre-requisite

From the concept of the elements of a circle and disc learned early, the learner should be able to have the understanding and to know the connection and relationship between first circle theorems stating that the angle subtended at the centre of the circle is twice the angle at the circumference. Similarly, in S1, the learner had already learned some of these types of circle theorems.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activity 9.2

- Organise the learners into appropriate groups. Ensure that all these groups observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.
- Ask the learners do Activity 9.2 in the Student's Book on page 146 that demonstrates the angles at the centre and the circumference of the circle.
- Let the learners draw a circle centre O with any convenient radius, mark two points A and B on the circumference, then mark point P that is then joined to the two marked points.
- Allow the learners to discuss the findings and observations amongst themselves, and then ask either the group leader or any group member at random to present the findings of their groups to the whole class. This will allow other members to contribute in pointing out errors and adding any omission to the presented facts on sliding and static frictions.
- Guide the learners through the proof of the theorem stated in the Student's Book page 149 using the two figures given.
- Still in the same groups, discuss with the learners the proof of the theorem as in the Student's Book.
- This section will promote in learners among other comptences;
    1. Communication skill through the numerous group discussions.
    2. Cooperation and interpersonal skills through the activities in the Student's Book.

3. Critical thinking and problem solving skills.

Synthesis

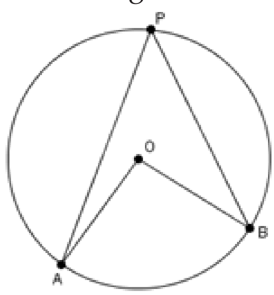
- Let the learners discuss Examples 9.1 to 9.3 in the Student's Book pages 147-148 as you guide them through as you shed more light on the applications of this theorem.
- Summarise the section by letting the learners know that the angle subtended at the centre of a circle is twice the angle subtended by the same on the circumference of the same circle on the same side.

Assessment

- Formulate different questions on this section and ask the learners to answer randomly to gauge their understanding and to check if the objective has been reached.
- Let the learners do Exercise 9.1 in the Student's Book pages 148-149.

Answers to Activity 9.2

1. The teacher to go round and ensure that <APB drawn by learners is on the circumference and <AOB is at the centre.
2. The teacher to go round and ensure learners are measuring angles <AOB and <APB correctly.
3. Angle AOB is twice angle APB.
4. For all learners angle AOB should be twice <APB.



< AOB = 2 APB

Lesson 3: Theorem 2 - Angle in a semicircle

Lesson objective

By the end of the lesson, the learner should be able to understand and demonstrate that the angle in a semicircle is a right angle at the circumference.

Introduction/Pre-requisite

Learners are required to have knowledge on the concept of the elements of a circle and disc and the first circle theorem learned early. From here, they should be in a position to connect and relate these concepts to the second circle theorem that states that the angle the angle in a semicircle is a right angle at the circumference.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activity 9.3

- Still in the same groups of Activity 9.3, ask the learners to carry out Activity 9.3 in the Student's Book page 149 that are based on the angle in a semicircle.
- Here, the learners are expected to draw any circle centre O with any convenient radius. Then draw a diameter AB and mark another point C on the circumference and join A to C and B to C.
- Ask the learner to observe and measure angle ACB and compare their results with the findings of members of other groups.
- Allow the learners to discuss the findings and observations amongst themselves, and then ask either the group leader or any group member at random to present the findings of their groups to the whole class. This will allow other members to contribute in pointing out errors and adding any omission to the presented facts on sliding and static frictions.
- This section will promote in learners among other comptences:
  1. Communication skill through the numerous group discussions.
  2. Cooperation and interpersonal skills through the activities in the Student's Book.
  3. Critical thinking and problem solving skills.

Synthesis

- Ask the learners random questions to gauge their understanding and to check if the objective has been reached.
- Let the learners know that the angle in a semicircle is a right angle at the circumference.
- Let the learners discuss Examples 9.4 and 9.5 in the Student's Book page 150 as you guide them through as you explain more on the applications of this theorem.

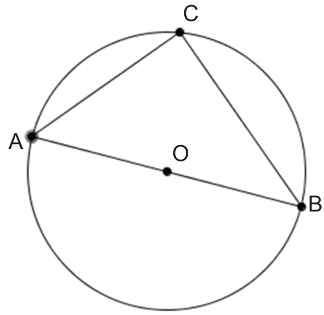
Assessment

- Ask the learners random questions based on the activity to gauge their understanding on the topic area.
- Let them attempt Exercise 9.2 Student's Book pages 150-151.

Answers to Activity 9.3

Guide the learners through the activity as you go around checking if they are following the instructions to the latter.

4. $\angle ACB = 90^{o}$ for all learners



Lesson 4: Theorem 3 - Angle in the same segment

Lesson Objective

By the end of the lesson, the learner should be able to understand and demonstrate that angles subtended by the same chord on the same side of the chord are equal.

Introduction/Pre-requisite

From the knowledge and concept on angle properties and the elements of a circle and disc learned early, the learner should be able to connect and relate these concepts to the third circle theorem stating that angles subtended by the same chord on the same side of the chord are equal.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activities 9.4 and 9.5

- Organise the learners into pairs where they are supposed to work with their immediate neighbours or according to the sitting arrangement in class.
- Let the learners do Activities 9.4 and 9.5 in the Student's Book pages 151 and 152 respectively demonstrating that angles subtended by the same chord on the same side of the chord are equal.
- Ask the learner to draw the same circle as in the other previous activities but here they have to mark four points, A, B, C, and D such that chord AB has the same length as CD. Guide them through all the instructions and procedures stated within the two activities.
- Let the learners draw angle APB and CQD as shown in the Student's Book then allow them to measure the angles, Activity 9.5. Let the learners comment on the size of the angles.

- Allow the learners to discuss the findings and observations amongst themselves in their respective pairs, and then ask either of them at random to present their findings to the whole class. This will allow other members to contribute in pointing out errors and adding any omission to the presented facts air resistance.

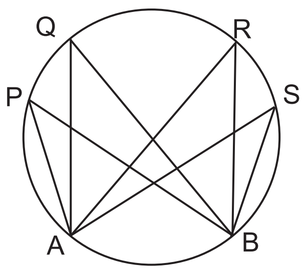
Synthesis

- Let the learners understand that the two angles are equal and that angles subtended on the circumference by the same chord in the same segment are equal.
- Similarly, learners should be able to understand that equal arcs of the same circle or of equal circles subtend equal angles at the circumference. Also, equal chords of the same circle or of equal circles cut off equal arcs.
- Still in pairs, allow the learners to discuss Example 9.6 to 9.8 in the Student's Book page 152-153 as you elaborate to them further on angles subtended by the same chord. This will enable you to gauge if the objectives have been met.

Assessment

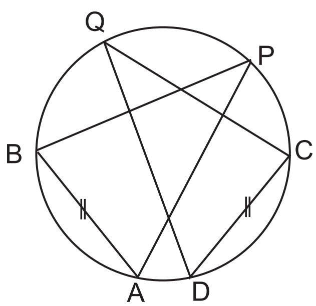
- Let the learners do Exercise 9.3 in the Student's Book pages 153-154.

Answers to Activity 9.4



<APB = <AQB = <ARB = <ASB. All classmates should have the same observation.

Answers to Activity 9.5



<APB = <CQD 7. The length of arc AB is equal CD.

Lesson 5: Theorem 4 - Angles in a cyclic quadrilateral

Lesson Objective

By the end of the lesson, the learner should be able to understand that the opposite interior angles of a cyclic quadrilateral are supplementary (add up to 180°).

Introduction/Pre-requisite

Knowledge on the properties off angles and subsequent circles learned back in S1 and S2 is necessary in handling this section and therefore, the learner should be able to connect and relate these concepts and knowledge to the fourth circle theorem stating that the opposite interior angle o a cyclic quadrilateral are supplementary (add up to 180^0).

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activity 9.6

- Organise learners into appropriate groups of between four and six. In these groups, ensure that all these groups observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.
- Let the learners do Activity 9.6 in the Student's Book page 154.
- Let the learners draw a circle centre O of any convenient radius then mark points A, B, C and D in that order then join them to form a quadrilateral. Ask the learners observe and measure the sizes of angles ABC and ADC then find their sum.
- Let the group have a group discussion on their observation and note them down before comparing the result with those from other groups.
- This section will promote in learners among other comptences;
  1. Communication skill through the numerous group discussions
  2. Cooperation and interpersonal skills through the activities in the Student's Book
  3. Critical thinking and problem solving skills.

Synthesis

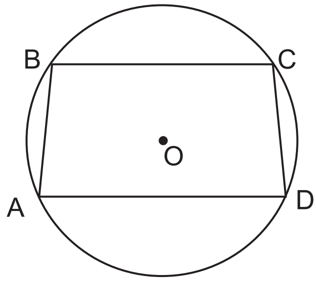
- Guide the learners through the proof of the theorem that is stated in the Student's Book page 155-156. From these proofs, learners should be able to understand that; the opposite interior angles of a cyclic quadrilateral are supplementary i.e. they add up to 180° and that the exterior angle is equal to the interior opposite angle.
- Summarise the section by formulating some of the questions that are related to the section and elaborating more on the proof that the opposite interior angles of a cyclic quadrilateral are supplementary.

- Let the learners do Examples 9.9 to 9.12 then ask one student to guide others on the board in the Student's Book on pages 156-157.

Assessment

- Ask learners to do Exercise 9.4 in the Student's Book pages 157-158.

Answers to Activity 9.6



- The sum of angle BAD and ∠BCD is equal to 180^o.
- The pairs of angles are opposite.
- For all the members of the class the sum of two pairs of opposite angles should add up to 180^o.
- An exterior angle ∠B is equal to opposite interior angle ADC.

Lesson 6: Theorem 5 – Defination of tangent to a circle

Lesson Objective

By the end of the lesson, the learner should be able to understand that a tangent to a circle is perpendicular to the radius drawn through the point of contact. Also, a perpendicular to a tangent at its point of contact passes through the centre of the circle.

Introduction/Pre-requisite

Introduce the lesson by asking every learner to use a pair of compasses and a pencil to draw any circle of any convenient radius and then draw a line that touches the circle at the circumference from a point outside the circle. Check their work if they have followed the instructions to the later. The use of pairs of compasses in this lesson is key and the learner should be able to use the instruments appropriately and be able to take accurate measurements.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activities 9.7 and 9.8

- Organise your learners into appropriate groups depending the availability of suggested learning materials. Ensure that the groups formed are gender sensitive (in case your class has boys and girls) and of different abilities. Working in groups will enhance teamwork and cooperation among learners.

- With your guidance, ask them to do Activity 9.7 and 9.8 in the Student's Book pages 158 respectively that demonstrates the theorem of the tangent to a circle.
- The activity may be enjoyable and amazing to the learners, but let them not lose the main concept of the activity i.e. learners tend to enjoy using pair of compass to draw circular figures, this should not distract them from the objective of the lesson.
- Let the learners discuss their observation from their activity. This will promote communication skills among your learners.

Synthesis

Having done the activity, guide the learners through a class discussion on their finding. Let them understand that:

1. A line that cuts a circle at one distinct point is called a secant
2. A line that has only one point in contact with the circle is called a tangent.
3. A tangent to a circle is perpendicular to the radius drawn through the point of contact
4. A perpendicular to a tangent at its point of contact passes through the centre of the circle.

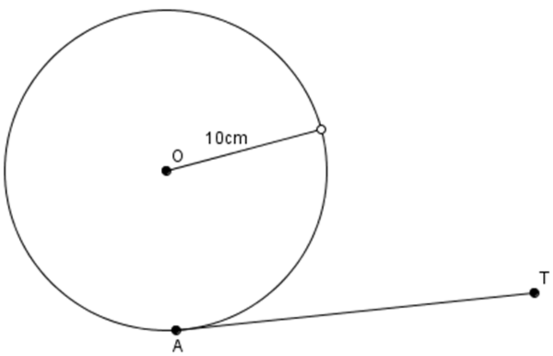
Guide the learners through Example 9.13 in the Student's Book page 151. Choose any student at random to guide others through Example 9.14 in the Student's Book pages 159-160.

Assessment

- Let the learners do Exercise 9.5 in the Student's Book pages 160-161.

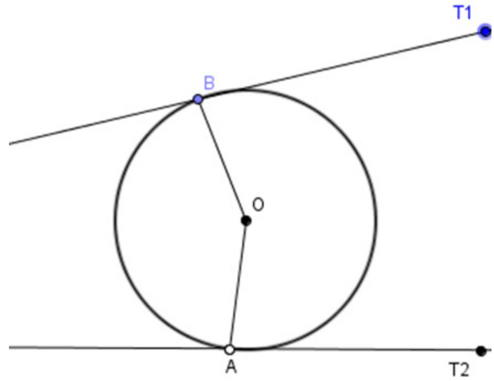
Answers to Activity 9.7

The line is called a tangent.



AT is called a tangent.

Answers to Activity 9.8



The angle between the tangent and the radius is 90°.

Lesson 7: Constructing a tangent at any given point on a circle

Lesson Objective

By the end of the lesson, the learner should be able to construct a tangent at any given point of the circle and constructing a tangent to a circle from a common point.

Introduction/Pre-requisite

Leaners are expected to be able to use a pair of compasses and a pencil to draw any circle of any convenient radius. They should have the knowledge that a tangent is perpendicular to the radius at the point of contact. Check their work if they have followed the instructions to the later. The use of pairs of compasses in this lesson is key and the learner should be able to use the instruments appropriately and be able to take accurate measurements.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activities 9.9, 9.10, and 9.11

- In this activity, you may decide to use the groups formed in Activity 9.7 and 9.8 or form the new groups. Note that when learners are working in groups, competences such as cooperation, teamwork, and leadership skills among others are enhanced.
- Let them do Activities 9.9, 9.10 and 9.11 in the Student's Book page 161, that is, to construct a tangent at any given point on a circle and to construct a tangent to a circle from a common point respectively. This activity involves the use of hands, construction by the use of pair of compass and a pencil. Those learners with any hand challenges should be involve by being asked to observe keenly and contribute by giving their observations. If they are able, they can also be asked to record down the observations obtained from the activity. By doing so, you have ensured that all learners are involved in the learning activity.

- Allow them to discuss their findings from the activity. This will promote teamwork, cooperation, communication skills critical thinking among other competences in learners.
- Go around to ensure that the main objective of this section (i.e. constructing a tangent) is realised by learners.

Synthesis

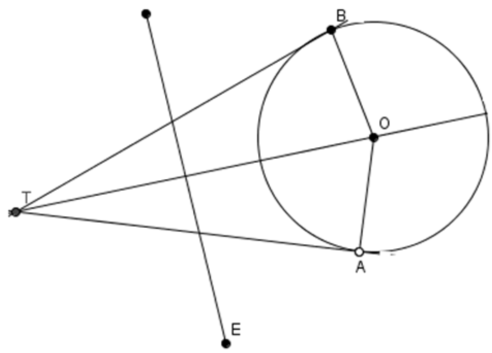
- Guide the learners through Examples 9.15 and 9.16 in the Student's Book page 162–163.
- Leaners should get to understand that in constructing a tangent to a circle, we use the fact that a tangent is perpendicular to the radius at the point of contact.
- Similarly, if two tangents are drawn to a circle from a common point, then:
  - (a) The tangents are equal.
  - (b) The tangent subtends equal angles at the centre.
  - (c) The line joining the centre to the common point bisects the angles between the tangents.

Assessment

Let the learners do Exercise 9.6 in the Student's Book pages 163–164.

Answers to Activity 9.11

5. (a) AT is equal to BT (b) <ATO is equal to <BTO
   (c) <AOT is equal to <BOT



Lesson 8: Theorem 6 - Angles in alternate segment

Lesson Objective

By the end of the lesson, the learner should be able to demonstrate that the angle between a chord and a tangent is equal to the angle in the alternate segments.

Introduction/Pre-requisite

Use the content learned earlier on elements of a circle and the properties of angles on alternate segments to kick-start this section and for quicker grasp by the learners.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activity 9.12

- Organise the learners in appropriate groups ensuring that all the groups observe gender balance and comprises learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Let the learners do Activity 9.12 in the Student's Book page 164 on angles in an alternate segment.
- Guide the learners through all the steps stated in the activity without skipping any step stated. In the process, let them observe as the group leader notes down the important points to be used during discussions, Fig. 9.72 on page 164 in the Student's Book.
- Ask them to compare their findings with the proof that is well illustrated provided in the Student's Book.
- This section will promote in learners among other competences;
  1. Communication skill through the numerous group discussions.
  2. Cooperation and interpersonal skills through the activities in the Student's Book.
  3. Critical thinking and problem solving skills.

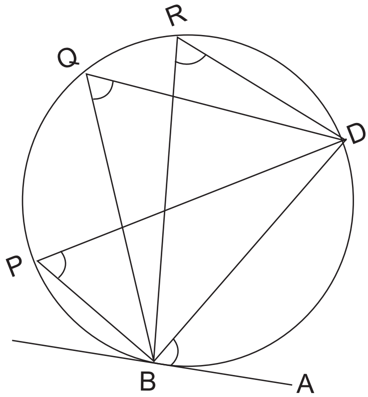
Synthesis

- Let the learners understand that the angle between a chord and a tangent is equal to the angle in the alternate segment.
- Guide the learners through the analytic proof of the alternate segment theorem stated in the Student's Book pages 164–165.
- Summarise by asking the learners to do Examples 9.17–9.19 provided in the Student's Book pages 165–166.
- Take the learners through a question and answer session where you set a question then choosing the learner to answer randomly to gauge the understanding of the learners and if the content has been understood.

Assessment

- Let the learners do Exercise 9.7 in the Student's Book pages 166–168.

Answers to Activity 9.12



5. <ABD = <BPD = <BQD = <BRD

Lesson 9: Theorem 7 - Perpendicular bisector of a chord

Lesson Objective

By the end of the lesson, the learner should be able to understand and demonstrate that the perpendicular lines from the centre of the circle bisect the chord.

Introduction/Pre-requisite

Use the content learned earlier to give the learners some highlights on what is to be covered in this section and to give guidelines on what is expected within this section.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Learning Activities

Activities 9.13 and 9.14

- Ask the learners to maintain the same groups of Activity 9.12 in this activity.
- Guide them through Activities 9.13 and 9.14 in the Student's Book pages 168–170 respectively i.e. demonstrating that the perpendicular lines from the centre of the circle bisect the chord. This will also help answer the question you asked them in the previous activity.
- Let the learners follow all the steps given in the activity as they draw appropriate circle with a convenient radius, centre A.
- Ask the learners to measure the angles formed as stated in the Student's Book. What do they notice about these angles, ask them to explain their answers
- Ask the learners to form discussion groups to revise the provided at the end of the unit.

- This section will promote in learners among other comptences;
    1. Communication skill through the numerous group discussions
    2. Cooperation and interpersonal skills through the activities in the Student's Book.
    3. Critical thinking and problem solving skills

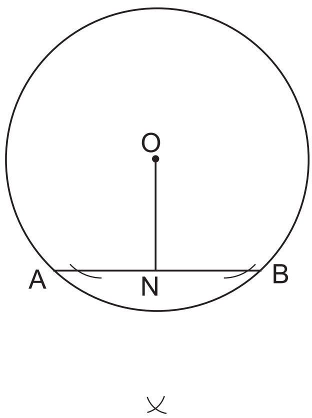
Synthesis

- Guide the learners through Examples 9.20 to 9.23 in the Student's Book pages 168-170. Ask them random questions to gauge their understanding of the topic area.
- From the activity, the learners should have realised that:
    1. A perpendicular drawn from the centre of a circle to a chord bisects the chord.
    2. A perpendicular bisector of a chord passes through the centre of the circle.
    3. If two chords of a circle are parallel, then the perpendicular bisector of one is the perpendicular bisector of the other.
    4. The midpoints of parallel chords of a circle lie on a diameter.
    5. If two chords of a circle are equal, they are equidistant from the centre.
    6. If two parts of a circle are equidistant from the centre, then the lengths are equal.
    7. If two chords of a circle are equal, then the angles they subtend at the centre are equal.
    8. If two angles at the centre of a circle are equal, then they are subtended by equal chords

Assessment

- Ask the learners to do Exercise 9.8 in the Student's Book page 169. Similarly, let the learners do Exercise 9.9 in the Student's Book page 171.

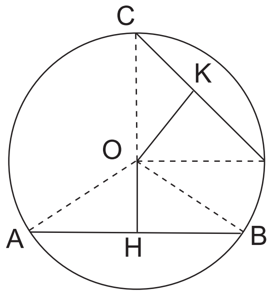
Answers to Activity 9.13



1. AN and NB are equal in length.
2. The perpendicular bisector of CD passes through the centre of the circle.

Answers to Activity 9.14

1. $OTR = 90^o$; $RT = TS$ hence OT is a perpendicular bisector of RS.
2. $OH = OK$; $<AOB = <COD$.



Summary of the Unit

Summarise the unit by:

- Asking the learners to take the whole class through the different concepts highlighted in the unit summary given in the Student's Book page 111. Take the learners through a question and answer method to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit like the difference between simple and compound interest and also the application of the compound interest formula in the real life situation within the financial and banking institutions.

Additional information to the teachers

It is important for the teacher to understand that:

End of Unit Assessment

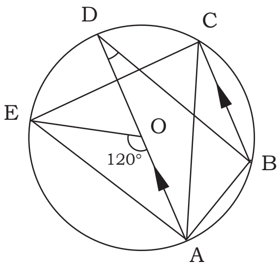
- Ask the learners to do Unit 9 Test pages 173-174 as class assessment and encourage all of them to participate actively.

Remedial Activities

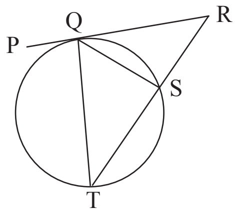
- Suggest activities and questions and answers for the slow learners to help them understand the unit better.

Remedial exercise

1. In the figure below, O is the centre of the circle, AD is a diameter, BC //AD *$\angle AOE = 120°$, and $\angle ADB = 25°$. Find the sizes of the angles of quadrilateral ABCE.*



2. In the following figure, PR is a tangent to the circle.



(a) If ∠PQT = 66°, find ∠QST.

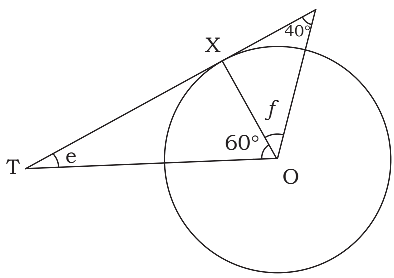
(b) If ∠QTS = 38° and ∠QRS = 30°, find ∠QST.

(c) If ∠QTS = 35° and ∠TQS = 58°,find ∠QRS.

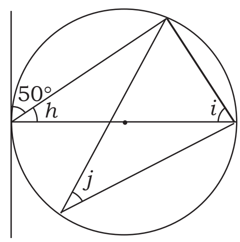
(d) If ∠PQT = 50° and ∠PRS = 30°, find ∠SQT.

3 Calculate the angles marked by letters in Fig. 8.26.

(a)



(b)



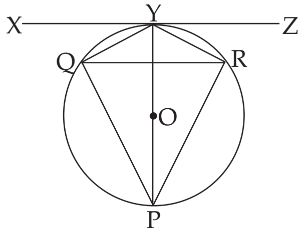
4. A chord of a circle of radius 14cm is 4 cm from the centre. Find the lengtth of the chord.

Extended Activities

- Get more complex questions for the gifted/fast learners.

Extended exercise

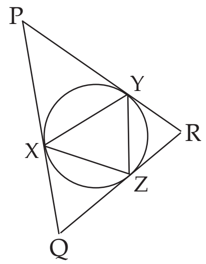
1. In the following figure PY is a diameter of the circle and XYZ a tangent.



Find (a) (i) $\angle QYP$ if $\angle QPY = 42°$ (ii) $\angle QPY$ if $\angle QYX = 42°$

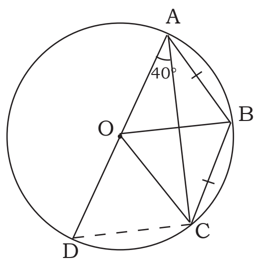
(b) $\angle QYX$ if $\angle QRY = 38°$

2. In the following figure $\angle QPR$, $\angle PQR$ and $\angle PRQ$ are in the ratio 3 : 4 : 5.

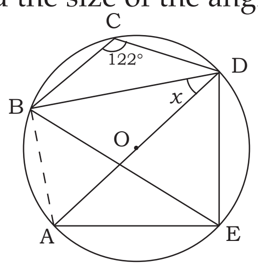


Find $\angle XYZ$, $\angle YXZ$ and $\angle XZY$.

3. In figure below, O is the centre of the circle ABCD and AOD is a straight line. If AB = BC and $\angle DAC = 40°$, calculate $\angle BAC$.



4. Find the size of the angle marked x in following figure.



5. ABCDE is a regular pentagon M is the midpoint of AB. DM intersects EB at N. Sketch the pentagon and find $\angle BAE$, $\angle BED$ and $\angle BNM$

Answers

Exercise 9.1 (Student's Book pages 148–149)

1. (a) 48^o (b) 59^o (c) 38^o (d) 62^o (e) $x = 2a$
   (f) $b = 90^o - a$
2. $d = 47^o$, $p = 133^o$ 3. 60^o 4. $72°$

Exercise 9.2 (Student's Book pages 150–151)

1. (a) 54^o 2. (a) $q = 72^o$, (b) $f = 144^o$
3. (a) 25^o (b) 40^o (c) 45^o

Exercise 9.3 (Student's Book pages 153–154)

1. $x = 84°$, $y = 24°$ 2. $a = 52°$, $b = 35°$
3. (a) <A = 50°; < C = 65°; < B=50° (b) arc AC = arc BC
4. $x = 20°$, $y = 50°$ 5. $x = 80°$, $y = 40°$ 6. $x = 40°$, $y = 30°$

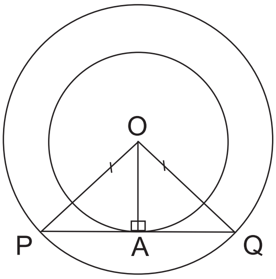
Exercise 9.4 (Student's Book pages 157–158)

1 (a) <AEB, <ADB (b) <BED, <BAD (c) 65°
2. (a) 70° (b) 95°
3. (a) $a = 98°$, $b = 105°$, $c = 98°$; $d = 105°$
4. (a) 32° (b) 90°
5. 108°

Exercise 9.5 (Student's Book pages 160–161)

1. (a) <OAB is the angle between tangent and radius.
   (b) 4cm
2. (a) 42.5° (b) 64° (c) 108° (d) 146°
3. (a) 57° (b) 30° (c) 65° (d) 142°

4.



since PO = OQ, radius of big circle.

$$(PO)^2 = (PA)^2 + (AO)^2$$

$$PA = \sqrt{(PO)^2 - (AO)^2}$$

$$(QO)^{\square} = (QA)\ + (AO)$$

$$AQ = \sqrt{(PO)^2 - (AO)^2}; \quad where\ PO = QO$$

Therefore $PA = AQ$

5. 16.61 cm 6. 15 cm

Exercise 9.6 (Student's Book pages 163–164)

1. 68^o 2. 43^o
3. (a) 56^o (b) 29^o (c) 75^o (d) 16^o
4. (a) 5.4 cm (b) 50^o
5. (a) 24.5 cm (b) 26.8 cm (c) 15.3 cm
6. 13.4 cm (to 1d.p) 7. 7.43 cm 8. 11.12 cm
9. (a) 4.5 cm (b) 96^o 10. 41^o

Exercise 9.7 (Student's Book pages 166–168)

1. 19^o
2. (a) 79^o (b) 62^o
3. (a) 66^o (b) 68^o (c) 52^o (d) 110^o
4. $\angle EDF = 59.5^o$, $\angle DEF = 52.5^o$ and $\angle EFD = 68^o$
5. 57^o
6. $\angle ABD + \angle CBD = 180^0 \Rightarrow \angle ABD = 180^0 - CBD$

$\angle BAD + \angle ADB + \angle ABD = 180^0$

But $\angle BAD = \angle CDB$

So $\angle CDB + \angle ADB + \angle ABD = 180^0$

Remember that $\angle ABD = 180^0 - \angle CBD$

$\angle CDB + \angle ADB + (180^0 - \angle CBD) = 180^0$

$\angle CDB + \angle ADB = -(180^0 - \angle CBD) + 180^0$

$\angle CDB + \angle ADB = \angle CBD$

Also $\angle CDB + \angle ADB = \angle ADC$

$\therefore \angle ADC = \angle CBD$

7. 78^o

8. $\angle ABE + \angle BEA + \angle BAE = 180^0$……..(i)

But $\angle ABE = \angle DBE + \angle ABD$

And $\angle BAE = \angle BAD = \angle DBE$

$\angle ABD = \angle BEA$ (Angles in alternate segment)

Substituting the above results in ….(i)

$\angle DBE + \angle ABD + \angle ABD + \angle DBE = 180^0$

$2(\angle DBE + \angle ABD) = 180^0$

$\angle DBE + \angle ABD = 90^0$

Also $\angle DBE + \angle ABD = \angle ABE$

$\angle ABE = 90^0 \Rightarrow BE \perp AC$

Therefore, BE is a Diameter

9. 30^o

10. 144^o

11. 36^o

Exercise 9.8 (Student's Book pages 169)

1. 24 cm	2. 16.12 cm	3. 13 cm	4. 3.32 cm
5. 10.39 cm	6. 9.80 cm	7. 14.42 cm	8. 0.09 cm
9. 6.36 cm			

Exercise 9.9 (Student's Book page 171)

1. (a) 2.92 cm (b) 4.52 cm

2. 24 cm

3. 0.5 cm, 3.5 cm

6. (a) 120^o (b) 3.46 cm

7. 7.15 cm

8. 2 cm and 14 cm

9. 7 cm

Unit 9 Test (Student's Book pages 173–174)

1. $a = 30°$, $b = 50°$, $c = 60°$,
   $d = 40°$, $e = 50°$
2. $a = 30°$, $b = 30°$, $c = 40°$, $d = 40°$
3. $x = 110°$, $y = 20°$
4. (a) BC= 12 cm (b) 73, 74° (c) 12.87 cm (d) 16.38 cm^2
5. <PQU = 62°, <QSU = 62°, <QUS = 75°, <QRS=105°
6. Radius = 8.66 cm, arc = 18.2 cm
7. (a) $x = 25°$, $y = 60°$, $\angle ADC = 60°$, $\angle ABC = 120°$, $\angle DCB = 75°$
   (b) $x = 35.6°$, $\angle QRS = 112.8°$, $\angle QTS = 67.2°$
8. $x = 24.6°$, $y = 8.2°$ Angles are 147.6°, 73.8°, 32.8°, 106.6°
9. 32°
10. (a) 25° (b) 130° (c) 60° (d) 135° (e) 45°

Answers to Remedial Exercises

1. $\angle ABC = 115°$, $\angle BCE = 85°$, $\angle AEC = 65°$, $\angle BAE = 95°$
2.
3. (a) 66° (b) 68° (c) 52° (d) 110°
4. e = 30°, f = 50°, h = 40°, i = 50°, j = 40°
5. 26.83 cm

Answers to Extended Exercises

1. (a) (i) 48° (ii) 42° (b) 38°
2. 52.5°, 60°, 67.5°
3. BAC = 25°
4. 32°
5. (a) 104° (b) 25° (c) 152° (d) 42°

UNIT
10 COLLINEAR POINTS AND ORTHOGONAL VECTORS

(Student's Book pages 175 – 184)

Key Unit Competence

By the end of this unit, learners should be able to apply properties of collinearity and orthogonality to solve problems involving vectors.

Learning Objectives

Knowledge and understanding

- State the conditions and properties of collinearity and orthogonality.

Skills

- Use definition and properties to show whether three given points are collinear or not and whether two vectors are orthogonal or not.

Attitudes and Values

- Appreciate the use of properties of collinearity and orthogonality to solve problems about vectors in two dimensions.
- Show patience, mutual respect, tolerance, and curiosity in solving and discussing problems involving vectors in two dimensions.

Pre-requisite to this unit

- Learners should be able to state the conditions and properties of collinearity and orthogonality.
- Learners should be able to use definition and properties to show whether three given points are collinear or not and whether two vectors are orthogonal or not.
- Knowledge of knowing whether points lie on a single straight line or not should be sufficient enough to decide collinear and non-collinear points.
- Knowledge of ratio theorem where points divide lines should also be sufficient enough for the learners to digest the content in this unit.
- Skills on finding the product of two vectors which can help to determine orthogonality of vectors. The teacher should give the learners a short written quiz to ascertain that they have these skills before introducing them to find the collinear and orthogonal points. The teacher should guide those with challenges.

Cross-cutting issues to be addressed in this unit

- **Inclusive education**: A teacher should provide for learning of all learners including those with special needs. A teacher should use sign language for those with hearing difficulty and those who are dumb

- **Standardization culture**: This will be through sharing knowledge by working together in groups, to determine accurately collineality and vector orthogonality.

Generic competences to be addressed

- **Leadership skills**: Since the unit will involve the learners working in groups and each group having a group leader.
- **Critical thinking**: This competence will be achieved when the teacher involve learners in activities that involve finding whether lines are orthogal and collinear.
- **Problem solving**: This competence will be achieved when the teacher involve learners in activities and exercises on determining collinearity and orthogonality.
- **Communication**: This competence will be achieved when the learners are presenting their findings.

Vocabulary/keywords

In the course of learning the concepts in this unit, the learners will discover the meaning of the following new words:

- Collinear
- Orthogonal
- Vector
- Ratio

Guide the learners to understand the meanings of these words and construct and speak out mathematical statements involving them in order to master their meaning and usage.

Guidance on the Problem Statement

- In order to focus the learner's attention into the general direction of the unit, the teacher should organise learners in groups of 5 students to do focus activity in Student's Book page 175.
- Encourage them to use straight-line theory and from unit 6 of Student's Book 3.
- Let the learners tell whether the mentioned points are collinear or not as given in the activity through class discussion.
- After this, let the whole class know that the concepts involved in the activity form the content of the unit hence by keenly participating in all the lessons for this unit, they will discover how to do this unit focus activity. This will motivate them and make them eager to learn all the concepts in this unit

Answers to Focus Activity

(a) Points P,Q and R are collinear if all the points lie on the same line.

$$\frac{4-2}{1-{}^{-}1} = \frac{2}{2} = 1$$

Equation of line PQ is y =mx + c. $y = x + c$; at (4,1)

$4 = 1 + c \quad c = 3$ So, $y = x + 3$

Testing point R on the line,

$6 = 3 + 3$

Point R lies on line PQ hence PQR are all collinear points.

(b) We have LQN, we get line LQ and test whether N lies of that line,

Gradient LQ $= \frac{0 - -2}{1 - 3} = \frac{2}{-2} = -1$

Line LQ: $y - 0 = -1(x - 1)$

We get $y = -x + 1$.

Testing point N (0,3).

$3 = 0 + 1$ and

The points L,Q and N are not collinear.

Advantages of having objects on line.

(i) Helps in some principles eg physics where light travels in a straight line.

(ii) Easy to measure their length or width

(iii) Very flexible in geometry.

2. (a) $(5 \times -3) + (3+5) = 15 + -15 = 0$. Vectors are orthogonal.

(b) $(-2 \times 12) + (5 \times -8) = -24 - 40 = -64$. Vectors are not collinear.

Attention to Special Needs

- You should provide for learning of all learners including those with special needs.
- The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Prepare additional more challenging questions for these learners.
- For the slow learners, organize remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

List of lessons

Lesson No.	Lesson Title	No. of periods
1.	Introduction	1
2.	Verifying collinearlity of points using vector laws	1
3.	Applications of collinearlity in proportional division of lines	2
4	Orthogonal vectors	1
5	Unit assessment	1

Lesson development

Lesson 1: Defination of collinearity

Lesson objective

By the end of the lesson, the learner should be able to discover collinear and non-collinear points through construction of lines.

Teaching Aids

- Books
- Pens and pencils
- Geometrical instruments
- Calculators
- Chalks and Classroom chalkboard

Learning Activities

Activity 10.1

- Organise the learners in pairs to do Activity 10.1 in the Student's Book page 175.
- Make sure one learner is working as the secretary to record the findings.
- Move around the class to see how learners are performing in the activity.
- Let the learners present their findings through secretaries.

Synthesis

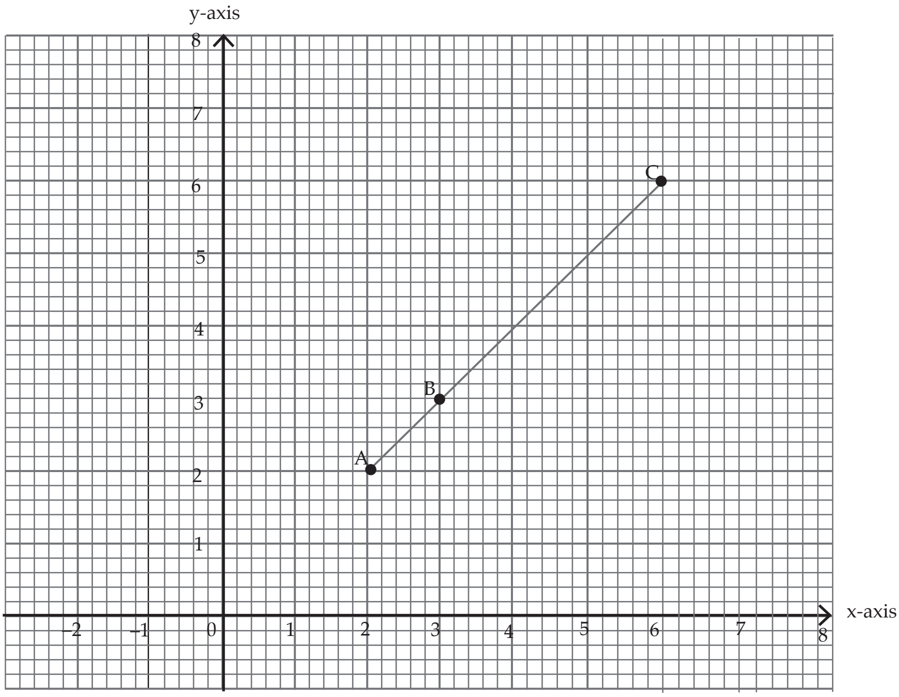
- Summarise the presentations by defining what a collinear point is by explaining the observations made from the activity.
- Deeply explain the meaning of collinear and non-collinear points.
- At this rate, learners should have understood the meaning of collinear and non-collinear points.

Assessment of the lesson

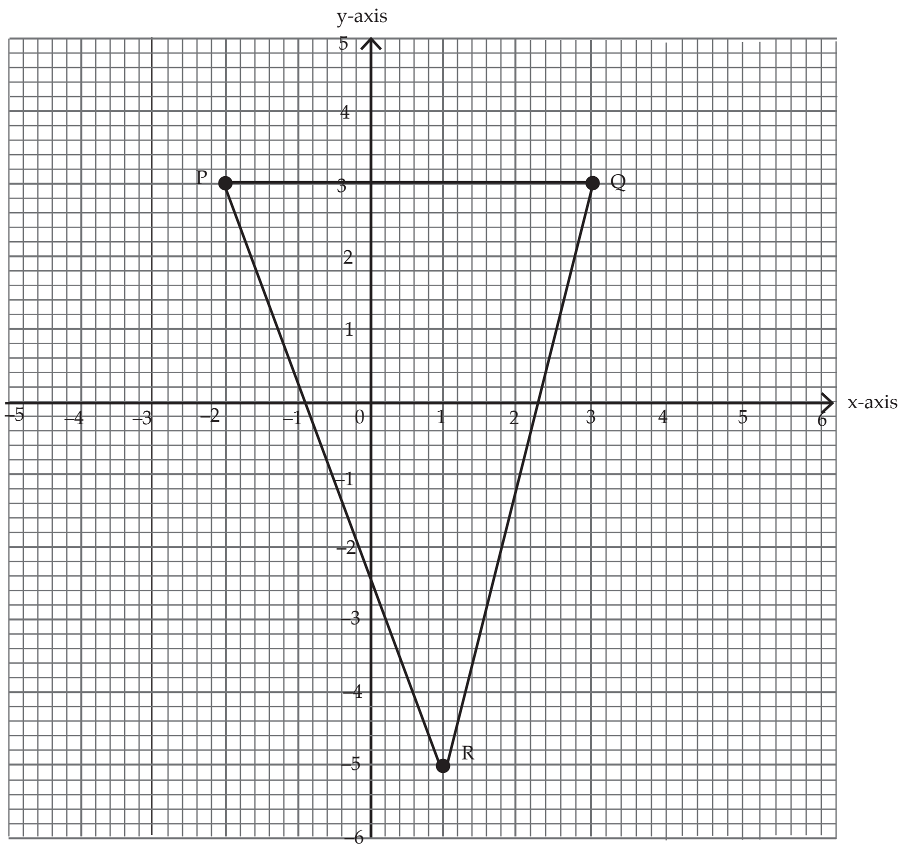
- Instruct the learners to do question one of Exercise 10.1 on page 177 in the Student's Book (By drawing method) to verify whether the given points are collinear or not.
- Correct their errors where they go wrong.

Answers to Activity 10.1

1.



2.



3. A, B and C are collinear while P, Q and R are not collinear.
4. Without drawing, we can determine using straight-line theory where two points are used and the third point should balance the equation if they are collinear. If they are not collinear, the third point cannot balance the equation of the line obtained from the two points.

Lesson 2: Verifying Collinearlity of Points Using Vector Laws

Lesson objectives

By the end of the lesson, the learner should be able to verify whether two or more points are collinear by using vector laws.

Teaching Aids

- Books
- Pens and pencils
- Calculators
- Geometrical instruments
- Chalks and classroom chalkboard.

Learning Activities

Activity 10.2

- Organise the learners into groups of three.
- Make sure one student's works as a secretary to record the findings.
- Let the learners do Activity 10.2 in the Student's Book page 176.
- As they are doing the activity, pass around checking the groups and give little guidance where necessary but allow the learners to think on their own.
- Let the learners present their group findings through group secretaries. The teacher should observe the errors the learners are making.

Synthesis

- Give further explanation and establish the formula for the collinearlity between the points
- Explain the collinear point calculations to the learners by doing Examples 10.1 and 10.2 from the Student's Book pages 176-177to ensure the mastery of the content.

Assessment of the lesson

- Assess whether the concepts are digested by instructing the learners to do Exercise 10.1 in the Student's Book page 177.
- Correct the learners to see whether the concepts have been digested.

Answers to Activity 10.2

1. Column Vector **PQ** $= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Column Vector **QR** = $\begin{pmatrix}2\\2\end{pmatrix}$

2. $\mathbf{PQ} = \frac{1}{2}\mathbf{QR}$

3. **PQ** = k. **QR,** where k is a scalar

Lesson 3: Applications of Collinearity in Proportional Division of Lines

Lesson objective

By the end of the lesson, the learner should be able to establish the relationship between collinearity and proportional division of lines.

Teaching Aids

- Books
- Pens and pencils
- Calculators
- Geometrical instruments
- Chalks and classroom chalkboard

Learning Activities

Activity 10.3

- Organise the learners into groups of three.
- Make sure one student's works as a secretary to record the findings.
- Let the learners do Activity 10.3 in the Student's Book page 177.
- Let the learners present their group findings through group secretaries. The teacher should observe the errors the learners are making.

Synthesis

- Correct the errors learners made in presentation of the activity and deeply how proportional of lines are related to collinearlity.
- Explain to the learners calculation of the collinear point calculations by doing Examples 10.3, 10.4 and 10.5 from the Student's Book pages 178 —179 to ensure that the content is properly digested.

Assessment of the lesson

- Assess whether the concepts are digested by instructing the learners to do question 2 of Exercise 10.1 in the Student's Book pages 177.
- Learners should also do Exercise 10.2 in Student's Book page 179—180.
- Correct the errors made by learners in the exercise.
- Learners should do the rest of the exercise as homework.

Answers to Activity 10.3

(a) $\mathbf{AX} = \frac{2}{3}\mathbf{AB}$ (b) $\mathbf{XB} = \frac{1}{3}\mathbf{AB}$ (c) $\mathbf{AX} = 2\mathbf{XB}$

Lesson 4: Orthogonal Vectors

Lesson objective

By the end of the lesson, the learner should be able to ensure that learners digest the meaning and calculations involving orthogonal vectors.

Teaching Aids

- Books
- Pens and pencils
- Geometrical instruments
- Calculators
- Chalks and classroom chalkboard

Learning Activities

Activity 10.4

- Organise the learners into groups of three.
- Make sure one student’s works as a secretary to record the findings
- Let the learners do Activity 10.4 in the Student's Book page 180 –181.
- Let the learners present their group findings through group secretaries. The teacher should observe the errors the learners are making.

Synthesis

- Correct the errors learners made in the activity presentation and deeply explain the orthogonal vector meaning and establish the formula for calculating orthogonal vectors.
- Use Examples 10.7, 10.8 and 10.9 from the Student's Book page 181 to explain to the learners the concept of orthogonal vectors.

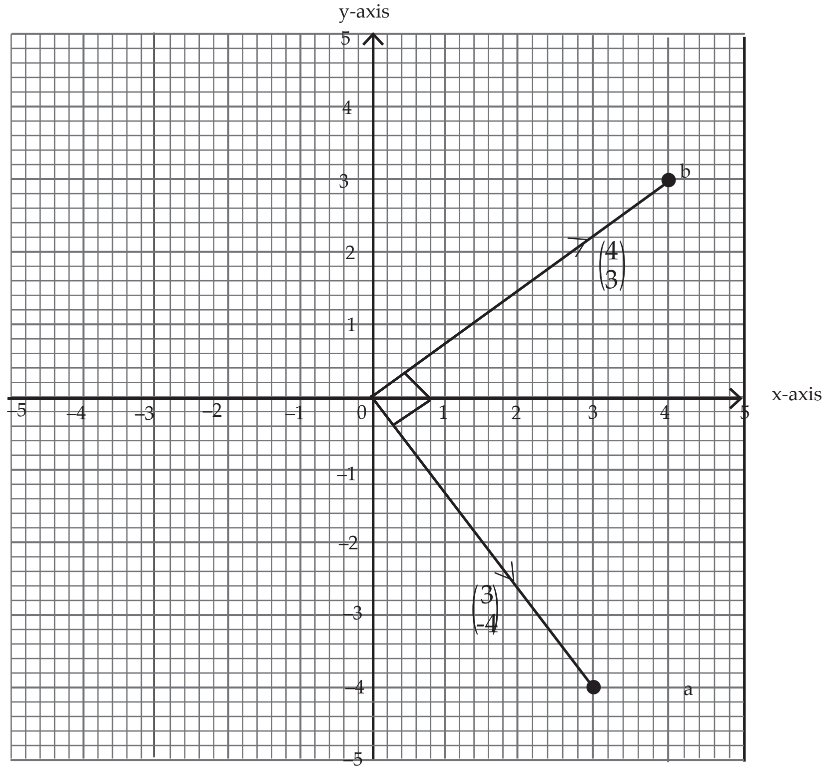
Assessment of the lesson

- Assess whether the concepts are digested by instructing the learners to do Exercise 10.3 in the Student's Book page 182.
- Correct the learners to ensure that they have understood the concepts.

Answers to Activity 10.4

1. Multiply the x coordinates together; multiply the y-coordinates together and then find the sum of the results.
2. (a) $(3 \times 4) + (-4 \times 3) = 12 - 12 = 0$

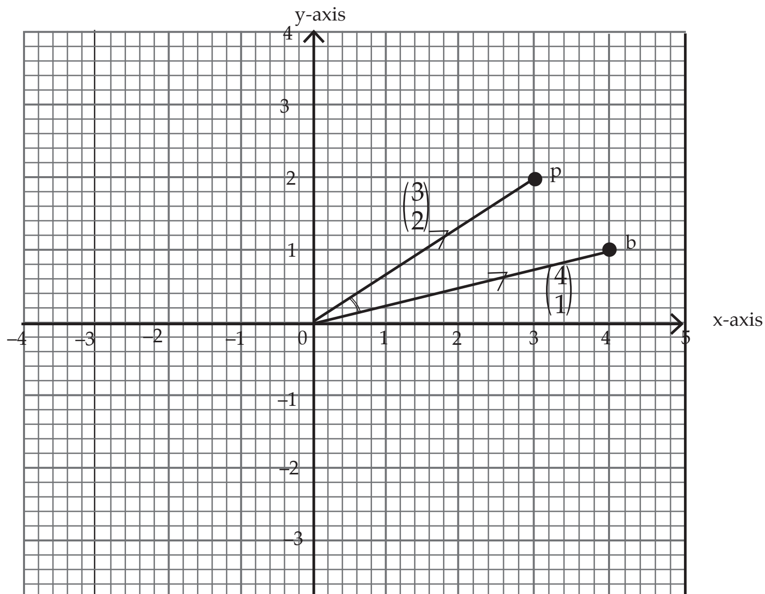
(b)



The angle between them is 90°.

3. (a) Product $(4 \times 3) + (2 \times 1) = 12 + 2 = 14$

(b)



Angle not easy to measure with eyes.

4. The angle between two perpendicular vectors is 90° and their product is 0.

Summary of the unit

- Asking different learners to take the different concepts highlighted in the unit summary given in the Student's Book page182—183.
- Ask the class probing questions to help them recall the concepts correctly.

End of unit assessment

- A teacher should ask learners to do unit test in Student's Book in page 183 - 184 assess whether learners understood the whole unit.

Additional information to the teacher

- A teacher should know the concepts of vectors taught in senior two in order to apply the knowledge and solve problems in this unit.

Remedial activities

For the slow learners, a teacher should organise remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

Remedial Exercise

1. Find the sum of products of tops and bottoms in the following vectors and state whether the given vectors are orthogonal.

   (a) $\mathbf{u}=\begin{pmatrix}1\\4\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}0\\-8\end{pmatrix}$ (b) $\mathbf{u}=\begin{pmatrix}-7\\-2\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}10\\1\end{pmatrix}$

   (c) $\mathbf{u}=\begin{pmatrix}1\\-1\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}2\\2\end{pmatrix}$ (d) $\mathbf{u}=\begin{pmatrix}2\\1\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}4\\6\end{pmatrix}$

2. Let $\mathbf{a}=(k, 1)$ and $\mathbf{b}=(6, 3)$ be two vectors in R^2. Find the values of k if; **a** and **b** are perpendicular.
3. Verify whether the following points are collinear.

   (a) (2,3) , (2,4), (5,4) (b) (–1,1) , (3,1) , (2, 10)

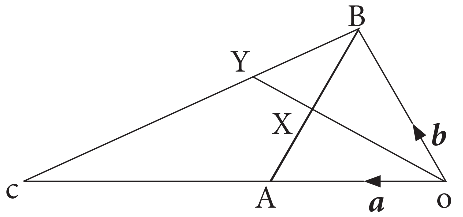
   (c) (–2, 5) , (7, 1) , (1 , 4)

Extended activities

The calculations in this unit may not be challenging to the specially gifted learners hence they finish doing the task so fast then get bored and start distracting others. Give learners the questions below.

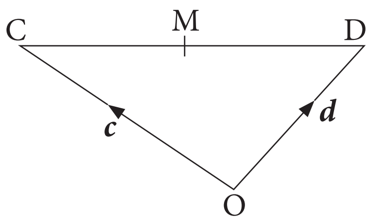
Extended Exercises

1. In the figure below, **OA** = $\boldsymbol{a}$ and **OB** = $\boldsymbol{b}$.



   (a) Express **AB** in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.
   (b) If X is the mid-point of **AB**, express **AX** interms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.
   (c) Given that **OC** = 3$\boldsymbol{a}$, express **BC** in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.
   (d) Given that Y is the midpoint of **BC**, express **OY** in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

2. In Figure below OCD, **OC** = $\underset{\sim}{c}$ and **OD** = $\underset{\sim}{d}$. C, M and D lie on the same straight line.



   (a) Express **CD** in terms of vectors $\boldsymbol{c}$ and $\boldsymbol{d}$.
   (b) M is the midpoint of **CD**. What is **CM** in terms of $\boldsymbol{c}$ and $\boldsymbol{d}$?
   (c) Using your answers to (a) and (b), find **OM** in terms of **c** and $\boldsymbol{d}$.

Answers to exercises, unit test, remedial activities and extended activities

Exercise 10.1 (Student's Book page 177)

1. (a) Not collinear (b) Not collinear (c) Collinear (d) Colliner
2. (a) (i) 2.8 (ii) 5 (iii) 4
   (b) Points are collinear as in (i), (ii) and (iii) but when plotted, combined points cannot lie on the same straight line.

Exercise 10.2 (Student's Book pages 179—180)

1. (a) $\overrightarrow{\mathbf{OD}} = 2a$ $\overrightarrow{\mathbf{OE}} = 4b$ $\overrightarrow{\mathbf{BA}} = \boldsymbol{a} - \boldsymbol{b}$ $\overrightarrow{\mathbf{ED}} = \mathbf{2}a - \mathbf{4}b$
   (b) (i) $3\boldsymbol{a} - 2\boldsymbol{b}$ (ii) **EC**$= 3a - 6b$
   (c) $\overrightarrow{\mathbf{ED}} = 2\overrightarrow{\mathbf{DC}}$
2. $\overrightarrow{\mathrm{OM}} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$
3. (a) (i) $\frac{1}{2}\boldsymbol{a}$ (ii) $\boldsymbol{c}$ (iii) $\boldsymbol{c} - \boldsymbol{a}$ (iv) $7(\boldsymbol{c} - \boldsymbol{a})$

(b) $7c - \frac{13}{4}a$ **(c)** $\frac{1}{2}a + \mathbf{p}c$ **(d)** $7q\,\mathbf{c} - \frac{13}{2}\mathbf{q}a$ **(e) 7:6**

4. (a) **t** (b) **–r** (c) $-\frac{1}{4}$r (d) $t - \frac{1}{4}r$ (e) $\frac{3}{4}r + t$

5. (a) $b - a$ (b) $\frac{1}{2}a + \frac{1}{2}b$ (c) $\frac{1}{2}b - \frac{1}{2}a$

6. (a) $\boldsymbol{OY} = 2b$ $\boldsymbol{OX} = \frac{5}{2}a$ $\boldsymbol{AB} = b - a$ $\boldsymbol{XY} = -\frac{5}{2}a + 2b$

   (b) $\boldsymbol{OC} = 6b - 5a$ $\boldsymbol{XC} = 6b - \frac{15}{2}a$ (c) $\boldsymbol{XC} = 3\boldsymbol{XY}$

7. (a) $\boldsymbol{AB} = b - a$, $\boldsymbol{AP} = 3(b - a)$, $\boldsymbol{MA} = \frac{1}{2}a$, $\boldsymbol{MP} = 3b - \frac{5}{2}a$

   (b) $\boldsymbol{MX} = \frac{1}{2}a + 2b$ (c) MPX is not a straight line

Exercise 10.3 (Student's Book page 182)

1. (a) (1×6)+(2x–8)=–10 Are not othorgonal (b) (–7×0)+(–3×1) = –3 Are not othorgonal

(c) (1×8)+(–3×–2)= 14 Are not othorgonal (d) (5×–4)+(1×20) Are othorgonal

2. (i) $\begin{pmatrix}3\\4\end{pmatrix}$ $\begin{pmatrix}5\\-2\end{pmatrix}$ (ii) $15-8 \neq 0$ Are not orthogonal

3. $-\frac{3}{4}$

4. (a) False (b) False (c) False (d) True (e) True

   (f) False (g) False

5. (a) PQ=$\begin{pmatrix}7\\7\end{pmatrix}$ RS=$\begin{pmatrix}8\\8\end{pmatrix}$ (b) Not Orthogonal

Unit 10 Test (Student's Book pages 183–184)

1. $\frac{2}{5}$ 2. $\frac{1}{2}(\boldsymbol{b} - \boldsymbol{a})$

3. $\overrightarrow{OP} = \frac{1}{3}(2\boldsymbol{a} + \boldsymbol{b})$ $\overrightarrow{BQ} = -b + \frac{1}{3}a$

   (a) h=$\frac{3}{7}$, k=$\frac{6}{7}$ (b) OC=$\frac{1}{7}b + \frac{2}{7}a$ (c) 6:1

4. **OA**= $\frac{1}{4}y$, $\boldsymbol{XY} = y - x$, $\boldsymbol{XB} = \frac{2}{5}(y - x)$, **OB** = $\frac{1}{5}$(3x +2y), AX=$-\frac{1}{4}y+x$

   (*a*) OC = $\frac{1}{5}$h(3x + 2y) (*b*) **OC** = $(1 - k)\frac{1}{4}y + kx$,*h*= $\frac{5}{13}$ *and k*=$\frac{3}{5}$

   ,OC:CB=5:8 and AC:CX=3:2

5. AC:CP=1:16 ,BC:CQ=12:5

6. (a) 0 (b) Orthogonal

7. AB= $\frac{2}{3}$**BC** 8. a) False b) False c) False

9. (a) $\frac{30}{19}$

10. $\frac{114}{13}$ 11. $-\frac{36}{13}$ 12. $\frac{266}{17}$

13. –18 14. 0

Remedial Exercise

1. (a) –32, Not orthogonal (b) –72, Not orthogonal (c) 0, Are orthogonal
   (d) 14, Not orthogonal
2. $-\frac{1}{2}$
3. (a) Not collinear (b) Not collinear (c) Not collinear

Extended Exercises

1. (a) $\mathbf{AB} = \mathbf{b} - \mathbf{a}$ (b) $\mathbf{AX} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$ (c) $\mathbf{BC} = 3\mathbf{a} - \mathbf{b}$
   (d) $\mathbf{OY} = \frac{1}{2}(3\mathbf{a} + \mathbf{b})$
2. (a) $\mathbf{CD} = \mathbf{d} - \mathbf{c}$ (b) $\mathbf{CM} = \frac{1}{2}(\mathbf{d} - \mathbf{c})$ (c) $\mathbf{OM} = \frac{1}{2}(\mathbf{c} + \mathbf{d})$

UNIT 11

ENLARGEMENT AND SIMILARITY IN 2D

Key Unit Competence

By the end of this unit, learners should be able to solve shape problems about enlargement and similarities in 2D.

Learning Objectives

Knowledge and Understanding

- Define enlargement.
- Define similarity.
- Identify similar shapes.
- List properties of enlargement and similarities.

Skills

- Determine the linear scale factor of an enlargement.
- Find the centre of an enlargement.
- Construct an image of an object that has been enlarged.
- Use properties of enlargement and similarities to transform a given shape.
- Find lengths of sides, area, and volume of similar shapes.
- Construct an image of an object under composite and inverse enlargement.

Attitudes and Values

- Appreciate the importance of enlargement and similarities to transform shapes.
- Show patience, mutual respect, tolerance, and teamwork while solving and discussing problems involving enlargement and similarities.

Pre-requisite to this unit

For learners to acquire the knowledge, skills, attitudes and values that are envisaged in this unit without any difficulty, they in need to acquire knowledge on shapes and geometry off different figures. Similarly, in S2, the learner also learned the Thales theorem. This unit introduces the learner to similarity and enlargement in 2D. The learner is introduced to the definition of enlargement and similarity, their properties, construction and identification off similar shapes and enlargement construction procedures. The unit also introduces the learners to the field of scale factor of enlargement, centre of enlargement, and both the volume and area of enlargement off similar objects and shapes.

Cross cutting issues to be addressed

The specific cross cutting issues to be addressed in this unit includes;

- **Inclusive education**: this involves ensuring that all the learners are engaged in in this unit, so that everyone can achieve his/her potential.This inclusiveness embraces all individuals regardless of their gender and abilities including those with physical disabilities and learning difficulties.

- **Peace and value education**: Achieved by allowing leaders to work in groups without hatred and discrimination. Peace is critical for the society to flourish and for every individual to focus on personal development and achievements and their contribution to the success of the nation. Value education forms a key element for the strategy for ensuring young people recognise the importance of contributing to the society, working for peace and harmony and being committed to avoiding conflict.

Generic competences to be addressed

- **Leadership skills**: Since the unit will involve the learners working in groups and each group having a group leader.
- **Critical thinking**: This competence will be achieved when the teacher involve learners in activities that involve finding linear scale factor, area scale factor, and volume scale factor.
- **Problem solving**: This competence will be achieved when the teacher involve learners in activities and exercises on determining similarity and enlargement.

Vocabulary/Keywords

Learner will come across the following concepts in the course of their learning, which, are explained in the book. Some of these concepts include:

- Similarity
- Enlargement
- Polygon
- Scale factor
- Linear scale factor
- Area scale factor
- Volume scale factor

Guide the learners to understand the meaning of these words and how they are used. You also need to speak out mathematical statements involving these words in order to master their meaning, usage and application.

Guidance on the problem statement

- In order to focus the attention of the learners into the general direction of this unit, the teacher should guide the learners on how to conduct unit focus activity at the beginning of this unit in the Student's Book.

Answers to unit Focus Activity

1. Ratios of correponding sides. Ratios of corresponding angles.
2. 1.8 m long, 0.9 m wide and 0.45 m high.
3. Answers are same
4. The company should recycle the used up cartons.

Attention to special needs

- You should provide for learning materials for all learners that including those with special needs i.e. physically challenged, fast and slow learners', and those with impaired hearing.

- For the slow learners', you need to organise some remedial lessons where you need to guide them through the activities once again slowly to help them understand the needed concepts.
- The calculations in this unit may not be challenging to the specially gifted learners and they may finish ahead of the others then get bored and start distracting others. You therefore need to prepare additional and more challenging questions for these learners.

List of lessons

Lesson No.	Lesson Title	Number of Periods
1	Introduction, definition and properties of similarity	1
2	Similar polygons	2
3	Similar triangles	2
4	Calculating lengths of sides of similar shapes using similarity and Thales theorem	2
5	Definition of enlargement	1
6	Positive scale factor	2
7	Negative scale factor	2
8	Locating the centre of enlargement and finding scale factor	2
9	Enlargement in the Cartesian plane	2
10	Area scale factor	2
11	Volume scale factor	2

Lesson 1: Similar Triangles

Lesson objective

By the end of the lesson, the learner should be able to understand similarity in similar triangles.

Introduction/Pre-requisite

Teaching Aids

- Books
- Pens and pencils
- Geometrical instruments
- Calculators
- Chalks and Classroom chalkboard

Learning Activities

Activities 11.1, 11.2, 11.3 and 11.4

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.

- Let the learners do Activities 11.1 to 114 in the Student's Book in showing similarity in similar triangles.
- After the end of the activity, ask the groups to present their results to the class each at a time as the other groups compare with other groups results.
- This activity will promote communication skill through the numerous group discussions, cooperation and interpersonal skills through the activities in the Student's Book and also critical thinking and problem solving skills amongst the learners in learners among other comptences.

Synthesis

- Discuss with the learners the workings on the Student's Book considering the two triangles given in explaining similarity in similar triangles.
- Discuss with the learners Example 11.1 in the Students Book on the proof of similarity in similar triangles.
- Learners should be able to understand that two triangles are similar either if the ratio of the corresponding sides is equal or if the corresponding angles are equal.
- Let the learners discuss as you assist them in showing the proofs in this case. Any group member should thereafter be able to submit and explain their work to the whole class.

Assessment

- Ask the learners to do Exercise 11.1 in the Student's Book.

Answers to Activity 11.1

1. Similarity is the state or fact of being similar.
4. Corresponding angles are equal.
5. Ratios of corresponding sides are equal and constant.

Answers to Activity 11.2

The ratios of the corresponding sides of the two triangles are constant/equal i.e

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = 1.5.$

Therefore, triangle ABC and PQR are similar.

Answers to Activity 11.3

2. Angle ABC = Angle DEF = 40^{o}
3. The ratios of the corresponding sides and angles are equal hence the two triangles are similar.

Answers to Activity 11.4

Triangle ABC is not similar to triangle DEF since the ratios of their corresponding sides and angles are not constant.

Lesson 2: Similar Polygons

Lesson objective

By the end of the lesson, the learner should be able to understand similarity in regular polygons.

Introduction

Teaching Aids

- Books
- Geometrical instruments
- Chalks and Classroom chalkboard
- Pens and pencils
- Calculators

Learning Activities

Activities 11.5 and 11.6

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader amongst themselves.
- Let the learners do both Acivities 11.5 and 11.6 in the Student's Book demonstrating similarity in similar and regular polygons.
- Guide the learners through all the steps stated in the activity without skipping any step stated. In the process, let them observe as they notes down the important points to be used during discussions.
- Ask them to compare their findings with the proof that is well illustrated provided in the Student's Book.
- This section will promote in learners among other comptences;
  (a) Communication skill through the numerous group discussions
  (b) Cooperation and interpersonal skills through the activities in the Student's Book
  (c) Critical thinking and problem solving skills.

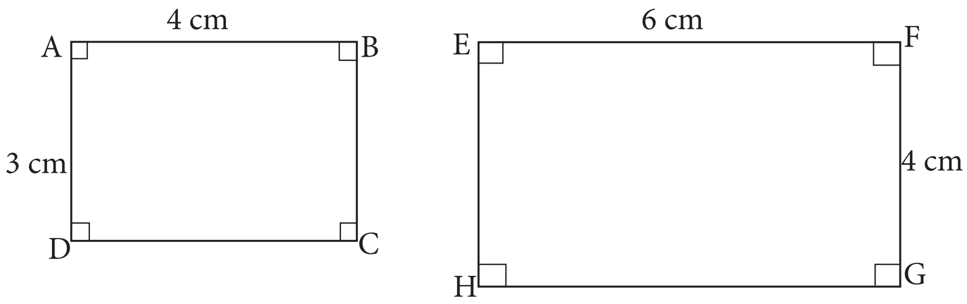
Synthesis

- Discuss with the learners Examples 11.2 to 11.5 in the Student's Book on the processes and the procedures followed when locating the centre of enlargement and finding the scale factor.
- Let the learners discuss as you assist them in showing the proofs in this case. Any group member should thereafter be able to submit and explain their work to the whole class.
- Let the learners understand that two polygons are similar if and only if the ratio of corresponding sides is constant and if the corresponding angles are equal.

Assessment

- Ask the learners to do Exercise 11.2 in the Student's Book.

Answers to Activity 11.5

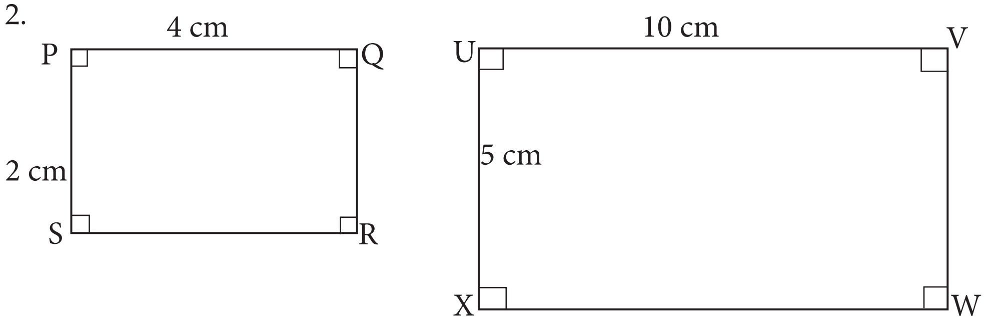


1. By drawing the two rectangles, you will notice that the corresponding circles are equal and constant but the ratio of corresponding sides are not equal i.e $\frac{DC}{HQ} = \frac{AB}{EF} = \frac{4}{6} = \frac{2}{3}$

   $\frac{AD}{EH} = \frac{BC}{FG} = \frac{3}{4}$ Thus $\frac{AB}{EF} \neq \frac{BC}{FG}$

   the rectanglar are not similar.

2.



$\frac{PQ}{UV} = \frac{4}{10} = \frac{2}{5} = \frac{SR}{XW}$ Also, $\frac{PS}{UX} = \frac{2}{5} = \frac{QR}{VW}$

Rectangles PQRS and UVXW are simular.

Answers to Activity 11.6

a and *d* are similar because the ratios of their corresponding sides are equal.

b and *c* are not similar to (a) since ratio of corresponding sides are not equal.

Lesson 3: Calculating lengths of sides of similar shapes using similarity and Thales theorem

Lesson Objective

By the end of the lesson, the learner should be able to calculating lengths of sides of similar shapes using similarity and Thales theorem.

Lesson objective

By the end of the lesson, the learner should be able to use Thales' theorem to calculate lengths of sides of similar shapes.

Teaching Aids

- Books
- Pens and pencils
- Geometrical instruments
- Calculators
- Chalks and Classroom chalkboard

Learning Activities

Activity 8.7

- Organise the learners into appropriate groups and let them do Activity 11.7 Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Discuss with the learners Examples 11.6 to 11.10 in the Student's Book as you ask them random questions to gauge their understandings.
- Guide the learners through the working on the calculation of lengths of sides of similar shapes with the use of Thales theorem and similarity.
- Ensure that all the learners participate actively.

Synthesis

- Learners should be reminded and let to understand that Thales theorem states that if a line is drawn parallel to one side of a triangle intersecting the other two sides, it divides the two sides in the same ratio.

Assessment

- Let the learners do Exercise 11.3 in the Student's Book.

Answers to Activity 11.7

a) $\frac{SP}{ZW} = \frac{2}{4} = \frac{1}{2}$; Also, $\frac{PQ}{WX} = \frac{4}{8} = \frac{1}{2}$

b) The two rectangles are similar

Lesson 4: Introduction, Definition and Properties of Similarity

Lesson objective

By the end of the lesson, the learner should be able to define similarity and give its properties.

Introduction/Pre-requisite

Teaching Aids

- Books
- Pens and pencils
- Geometrical instruments
- Calculators
- Chalks and Classroom chalkboard

Learning Activities

Activity 11.8

- Organise the learners in appropriate groups. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Let the learners do Activity 11.8 in the Student's Book on definition and properties of similarity.
- Let the learners conduct a discussion within their groups on their observation as they write down some of the points on the properties of similar figures/objects.
- Ask the any other member of the group chosen at random, to present their findings to the whole class and allow other members to contribute in pointing out errors and adding any omission to the presented facts.

Synthesis

- Discuss with the learners in groups some of the properties of similarity.
- Discuss with the learners Examples 11.11 and 11.12 in the Student's Book.
- Let the learners understand that two solids/figures/objects are only similar if the ratio of the lengths of their corresponding sides is constant and if the corresponding angles are equal.

Assessment

- Summarise the lesson by asking the learners to do Exercise 11.4 in the Student's.

Answers to Activity 11.8

2. $\frac{B'C'}{BC} = \frac{1.3}{1} = 1.3 \quad \frac{C'D'}{CD} = \frac{1.3}{1} = 1.3, \quad \frac{DE'}{DE} = \frac{1.3}{1} = 1.3, \quad \frac{D'F'}{DF} = \frac{1.3}{1} = 1.3$

1. All the ratios of the corresponding sides are equal / constant.
2. Corresponding angles are equal.

Lesson 5: Definition of Enlargement

Lesson objective

By the end of the lesson, the learner should be able to define and relate enlargement.

Learning Aids

- Books
- Pens and pencils
- Geometrical instruments
- Calculators
- Chalks and Classroom chalkboard

Learning Activities

Activity 11.9

- Organise the learners in pairs to do Activity 11.9 in the Student's Book
- Make sure one learner is working as the secretary to record the findings.
- Move around the class to see how learners are performing in the activity.
- Let the learners present their findings through secretaries.

Synthesis

- Summarise the presentations by defining what enlargement is by explaining the observations made from the activity.
- Deeply explain the meaning of enlargement and its area of application.
- Discuss with the learners the working in the Student's Book elaborating enlargement.

Assessment

- Ask learners random questions to gauge their understanding of the topic area.

Answers to Activity 11.9

1. The two shapes are similar though of different sizes
2. 2 times bigger
3. Enlargement
4. By measuring and comparing the corresponding sides of the pictures.
5. They both have the same centre of enlargement and on the same side of the centre of enlargement.

Lesson 6: Constructing objects and Images under enlargement – Positive scale factor>1

Lesson objectives

By the end of the lesson, the learner should be able to construct images and objects under the positive scale factor and fractional scale factor.

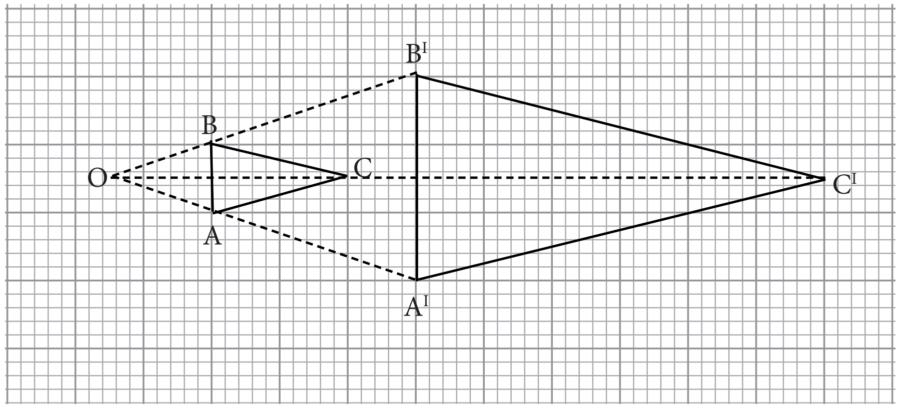
Teaching Aids

- Books
- Pens and pencils
- Calculators
- Geometrical instruments
- Chalks and classroom chalkboard.

Learning Activities

Activities 11.10 and 11.11

- Organise the learners into groups of three
- Make sure one student's works as a secretary to record the findings
- Let the learners do Activities 11.10 and 11.11 in the Student's Book respectively.
- As they are doing the activity, pass around checking the groups and give little guidance where necessary but allow the learners to think on their own.
- Let the learners present their group findings through group secretaries. The teacher should observe the errors the learners are making.



$\Delta A'B'C' = 3\Delta ABC$

Synthesis

- Explain the positive scale factor construction and calculations to the learners by doing Examples 11.13 and 11.14 from the Student's Book to test the mastery of the content. Discuss with the learners Examples 11.15 and 11.16 on in the Student's Book explaining the fractional scale factor.
- Let the learners' understand that under enlargement of scale factor greater than 1, the object and the image are on the same side of the centre of enlargement, the image is larger than the object and any line on the image is parallel to the corresponding line on the object.

Assessment

- Assess whether the concepts are well understood by instructing the learners to do Exercises 11.5 and 11.6 in the Student's Book.
- Correct the learners where necessary.

Lesson 7: Constructing Objects and Images under Enlargement – Negative Scale Factor

Lesson objective

By the end of the lesson, the learner should be able to construct images and objects under the negative scale factor

Teaching Aids

1. Books
2. Pens and pencils
3. Calculators
4. Geometrical instruments
5. Chalks and classroom chalkboard

Learning Activities

Activity 11.12

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.

- Let the learners do Activity 11.12 in the Student's Book.
- Let the learners present their group findings through any of the group members. The teacher should observe the errors the learners are making.
- This section will promote in learners among other competences;
  (a) Communication skill through the numerous group discussions
  (b) Cooperation and interpersonal skills through the activities in the Student's Book
  (c) Critical thinking and problem solving skills.

Synthesis

- Explain the positive scale factor construction and calculations to the learners by doing random examples to ensure the masterly of the content.
- Let the learners' understand that under enlargement of negative scale factor, the object and the image are on the opposite sides of the centre of enlargement, the image is smaller or larger than the object depending on whether the scale factor is greater than 1 or negative. Finally any line on the image is parallel to the corresponding line on the object but the image is inverted relative to the object.
- Assess whether the concepts learned are well understood by discussing with the whole class Example 11.17 in the Student's Book.

Assessment of the lesson

- Assess whether the concepts are well understood by instructing the learners to do Exercise 11.7 in the Student's Book
- Correct the learners to see whether the concepts are well understood by marking their work.

Lesson 8: Locating the centre of enlargement and finding the Scale Factor

Lesson objective

By the end of the lesson, the learner should be able to locate the centre of enlargement and also be able to find the scale factor.

Teaching Aids

- Books
- Pens and pencils
- Geometrical instruments
- Calculators
- Chalks and classroom chalkboard

Learning Activities

Activity 11.13

- Ask the learners to form groups of between 3 and 5. Ensure that all these groups observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader.
- Let the learners do Activity 11.13 in the Student's Book on locating the centre of enlargement and the scale factor of enlargement.
- Let the learners present their group findings through group secretaries. The teacher should observe the errors the learners are making.

- The learners should join A to A′, B to B′ and C, to C′. The point intersec is the centre of enlargement. Scale factor is $\frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$.

Synthesis

- Discuss with the learners Example 11.19 in the Student's Book illustrating how learners are expected to locate the centre of enlargement and in finding the scale factor.
- Learners are expected to understand that any enlargement is fully described if both the centre and scale factors of enlargement are defined.
- Discuss with the learners the properties of enlargement stated in the Student's Book.

Assessment

- Let the learners do Exercise 11.8 in the Student's Book.

Lesson 9: Enlargement in the Cartesian plane

Lesson objective

By the end of the lesson, the learner should be able to understand and be able to perform enlargement in the Cartesian plane.

Teaching Aids

- Ruler
- Cartesian plane
- Pencil

Pre-requisite/Introduction

Use the content learned earlier on both the negative and positive scale factors to highlight the learner of what is to be covered in this section for easier and faster understanding of this section.

Learning Activities

Activities 11.14, 11.15 and 11.16

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities. Let them choose a group leader amongst themselves.
- Let the learners do Activities 11.14, 11.15 and 11.16 in the Student's Book to demonstrate properties of enlargement in the Cartesian plane.
- Direct all the learners to follow all the steps provided in the activity promptly.
- After drawing the quadrilateral with the given vertices, let the learners fill table 11.1 and 11.2 provided in the Student's Book.
- This section will promote in learners among other competences;
  (a) Communication skill through the numerous group discussions.
  (b) Cooperation and interpersonal skills through the activities in the Student's Book

(c) Critical thinking and problem solving skills.

Activity 11.15

Scale factor	A	B	C	D	P
2	(0,6)	(4,6)	(6,2)	(6,–4)	(2a,2b)
0.5	(0,1.5)	(1,1.5)	(1.5,0.5)	(1.5, –1)	(0.5a,0.5b)
–2	(0,–6)	(–4,–6)	(–6,–2)	(–6,4)	(–2a,–2b)
–0.5	(0,–1.5)	(–1,–1.5)	(–1.5,0.5)	(–1.5,1)	(–0.5a,–0.5b)

Activity 11.16

Scale factor	A′	B′	C′	D′
2	(–2,5)	(2,5)	(4,1)	(4,–5)
$\frac{1}{3}$	(1.4, 1.6)	(2,1.66)	(2.33,1)	(2.5,0)
–2	(6,–3)	(2,–4)	(0,1)	(1,4)
$-\frac{1}{3}$	(2.6,1.4)	(2,0.34)	(1.67,1)	(1.6,2)

Synthesis

- Discuss with the learners Examples 11.20 and 11.21 in the Student's Book demonstrating the steps in performing enlargement in a Cartesian plane.
- Summarise by letting the learners know that an enlargement with centre (0,0) and scale factor k maps a point *P(a,b)* onto *P'(ka,kb).*
- Similarly, the learners should be able to understand that it is never possible to generalise for *P(a,b)* when the centre is not the origin and therefore we must carry out a complete construction in order to obtain the require image.
- Summarise the section by letting the learners understand that if the scale actor of enlargement is 1 or -1, the object and the image have the same size, they are congruent.

Assessment

- Let the learners do Exercise 11.9 in the Student's Book as an assignment.

Lesson 10: Area Scale Factor

Lesson Objective

By the end of the lesson, the learner should be able to area scale factor in enlargement.

Introduction/Pre-requisite

Assemble all the requirements required for this activity, organise the learners into appropriate groups where all can access all the requirements needed in this section.

Teaching Aids

- Ruler
- Cartesian plane
- Pencil

Learning Activities

Activity 11.17

- Ask the learners to organise themselves into groups of between three and five. Within the groups, ensure that they all observe gender balance and comprise learners of different abilities. All the learners should participate actively in the lesson and all the activities.
- Let the learners do Activity 11.17 in the Student's Book on finding area scale factor.
- After the end of the activity, ask the groups to present their results to the class each at a time as the other groups compare with other groups results.

Original rectangle	Scale factor	Image dimension	L.S.F	A.S.F
2 cm by 3 cm	2	4 cm by 6 cm	2	4
2 cm by 3 cm	3	6 cm by 9 cm	3	9
2 cm by 3 cm	–2	4 cm by 6 cm	2	4
2 cm by 3 cm	$\frac{1}{2}$	1 cm by 1.5cm	0.5	0.25

A.S.F = $(\text{L.S.F})^2$

Synthesis

- Let the learners discuss amongst themselves Examples 11.22 and 11.23 in the Student's Book as you ask them random questions on their understanding of the topic area.
- Learners should be able to understand that area scale factor is the area of the object divided by the area of the image.
- Make sure learners have understood the activity very well.

Assessment

- Let the learners do Exercise 11.10 in the Student's Book.

Lesson 11: Volume Scale Factor

Lesson objective

By the end of the lesson, the learner should be able to understand volume scale factor and be able to solve related questions.

Introduction/Pre-requisite

Use the content learned earlier to give the learners some highlights on what is to be covered in this section and to give guidelines on what is expected within this section.

Teaching Aids

- Pair of compasses
- Ruler
- Pencil

Teaching guidelines

Activity 11.18

- Ask the learners to maintain the same groups of Activity 11.17.
- Let the learners follow all the steps given in the Activity as they demonstrate appropriate steps of finding the volume scale factors.
- This section will promote in learners among other comptences
  1. Communication skill through the numerous group discussions
  2. Cooperation and interpersonal skills through the activities in the Student's Book.
  3. Critical thinking and problem solving skills.

Answers to Activity 11.18

1. 16 cm by 10 cm by 8 cm
2. Vol. of object = 160 cm^3

   Vol. of image = 1280 cm^3

   $\frac{\text{Vol. image}}{\text{Vol. of object}} = \frac{1280}{160} = 8$
3. V.S.F = $(\text{L.S.F})^3$
4. 24 cm by 15 cm by 12 cm

   Vol = 4320 cm^3

   V.S.F = $(\text{L.S.F})^3$

   If L.S.F = $\frac{1}{2}$, V.S.F = $\frac{1}{8}$
5. V.S.F = $(\sqrt{\text{A.S.F}})^3$

   A.S.F = $(\sqrt[3]{\text{A.S.F}})^2$

Synthesis

- Guide the learners through Examples 11.24, 11.25 and 11.26 in the Student's Book. Ask them random questions to gauge their understanding of the topic area.
- Learners should be able to understand that, volume scale factor = $(\text{linear scale factor})^3$.

Assessment

- Ask the learners to do Exercise 11.11 in the Student's Book and later summarise the topic by discussing Unit 11 Test that is provided in the Student's Book.

Summary of the Unit

Summarise the unit by:

- Asking the learners to take the whole class through the different concepts highlighted in the unit summary given in the Student's Book. Take the learners through a question and answer method to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit like the difference between simple and compound interest and also the application of the compound interest formula in the real life situation within the financial and banking institutions.

Additional information to the teachers

It is important for the teacher to understand that:

- The other method of performing enlargement is by the use of matrices.

End of Unit Assessment

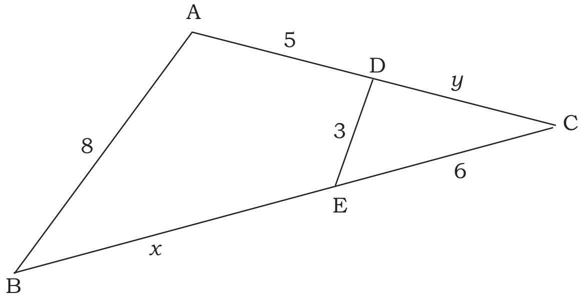
- Ask the learners to do Unit 11 Test in the Student's Book as a class assessment and encourage all of them to participate actively.

Remedial Activities

- For the slow learners, a teacher should organize remedial lessons where he/ she guides them through the activities once again and more slowly to help them understand the concepts.

Remedial exercise

1. The corresponding sides of two similar rectangles are 8 cm and 12 cm.Find the ratio of their areas, giving your answer in the simplest form.
2. The floor of a house measures 8 m long by 6 m wide. A scale model of the house is 8 cm long. Calculate:
   (a) the width of the model house.
   (b) the height of the house given that the model house is 9 cm high.
   (c) the floor area of the model.
   (d) the volume of the house in m^3 if the model has a volume of 900 cm^3.
3. Find the image of ΔABC $A(2, 4)$, $B(4, 2)$, $C(5, 5)$, under enlargement centre $(0, 0)$ scale factor 2 and state the co-ordinates of A', B' and C', the images of points A, B and C.
4. In the figure below, AB//DE. Identify two similar triangles and use them to find the values of x and y.



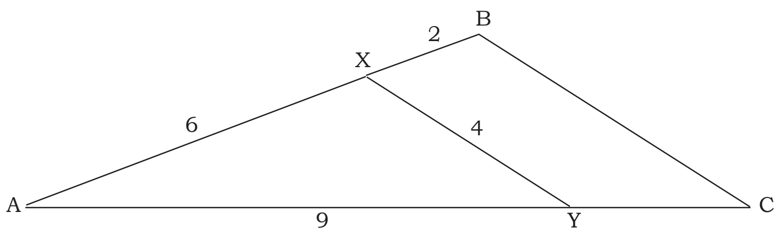
5. Two cylinders are similar, and their circumferences are 30 cm and 24 cm. Find the
   (a) ratio of their surface areas. (b) ratio of their volumes.

Extended Activities

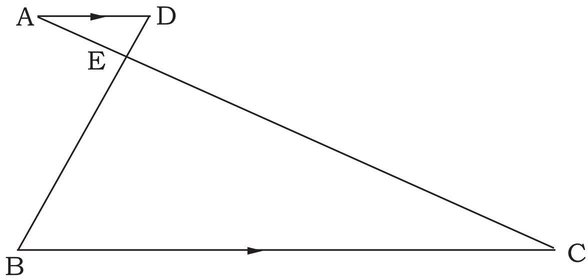
- The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Give learners the questions below.

Extended exercise

1. Draw a sketch of ΔABC with AB = 8 cm, BC = 6 cm and $\angle ABC = 90°$. Given that BX is an altitude of ΔABC:
   (a) calculate the length of AC.
   (b) show that ΔABX and ΔCAB are similar.
   (c) write down the ratio of corresponding sides of ΔABX and ΔCAB.
   (d) calculate the ratio of the area of ΔABX to the area of ΔCAB.
2. In the following figure ΔABC is an enlargement of ΔAXY. Find the scale factor of the enlargement and hence, calculate the length of BC.



3. Consider two similar containers having capacities of 6.4 l and 2.7 l respectively. If the smaller container has a height of 18 cm, what is the corresponding height of the larger one?
4. In the following figure AD/ / BC. AC and BD intersect at E. Given that AE: EC = 1 : 5 and BD = 12 cm, calculate the length of DE.



Exercise 11.1

1. (a) Similar because the ratio of the corresponding sides and angles are equal.
   (b) Similar because the ratio of the corresponding sides and angles are equal.
2. Yes, No, Yes
3. Triangle ABC is similar to triangle XYZ.

Exercise 11.2

1. (a) Similar (b) Not similar (c) Similar
2. Depends on the nature and size of the desk.
3. None of the triangles are similar.

Exercise 11.3

1. (a) $x = 16$ cm (b) $x = 25$ cm (c) $x = 6$ cm (d) $x = 20$ cm
2. AC = 6 cm PQ = 25 cm
3. (a) $x = 6$, $y = 4$ (b) $x = 14$, $y = 5.5$
4. BD = 13.5 cm
5. (a) $x = 6.6$ cm; (b) $x = 16$ cm
6. ΔABD and ΔACD, $x = 6.7$ cm, y = 2
7. $x = 48$ m

Exercise 11.4

1. 9.24 m 2. 101.25 cm^2 3. 2.25 m 4. Yes, Yes, No
5. Not true since the ratio of corresponding sides are not equal.

Exercise 11.5

2. (a) P (b) $\frac{4}{3}$

Exercise 11.6

2. Enlargement scale factor = $\frac{1}{3}$,
   centre (1, 5)
3. (a) $\frac{2}{5}$ (b) 1.2 cm
4. (a) (i) 3 times longer (ii) 3 times shorter
   (b) (i) unchanged (ii) unchanged
   (c) (i) unchanged (ii) unchanged
   (d) (i) unchanged (ii) unchanged

Exercise 11.7

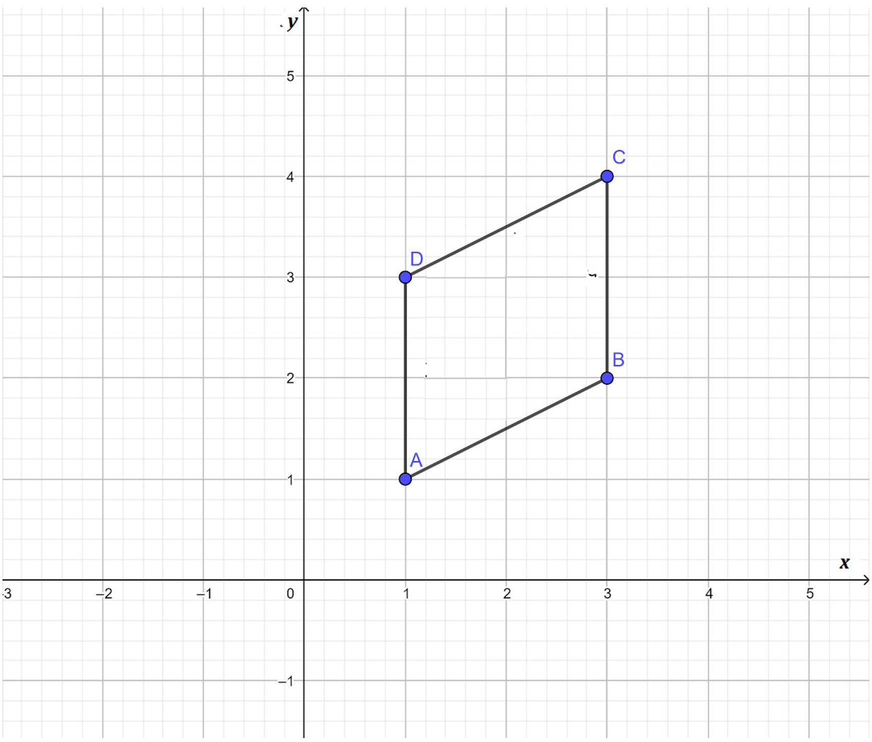
Check students' accuracy in construction and measurement in this exercises.

Exercise 11.8

1. (a) 0 (b) $\frac{9}{5}$ (c) 5.4 cm , 3.2 cm
2. (a) $\frac{-2}{3}$; (b) (i) R (ii) S (iii) p (c) 10 cm
5. (a) 1 : 100 (b) 5.2 m
6. 15 cm 7. $\frac{1}{400}$, 4
8. Check student's accuracy in construction and measurement.

Exercise 11.9

1. (a) A′(8, 4), B′(−4, 4), C′(−4, 16), D′(8, 16)
   (b) A′(4 , 2), B′(−2 , 2), C′(–2 , 8), D′(4 , 8)
   (c) A′(2 , 1), B′(−1 , 1), C′(–1 , 4), D′(2 , 4)
   (d) A′($\frac{1}{2}$, $\frac{1}{4}$), B′($\frac{-1}{4}$, $\frac{1}{4}$), C′($\frac{-1}{4}$, 1), D′($\frac{1}{2}$, 1)
2. (a) Scale factor = $\frac{1}{3}$ (b) centre (–3, 1)
3. A′(−3, −6) B’(−9, −6) C′(−9, −12)
4. (a)



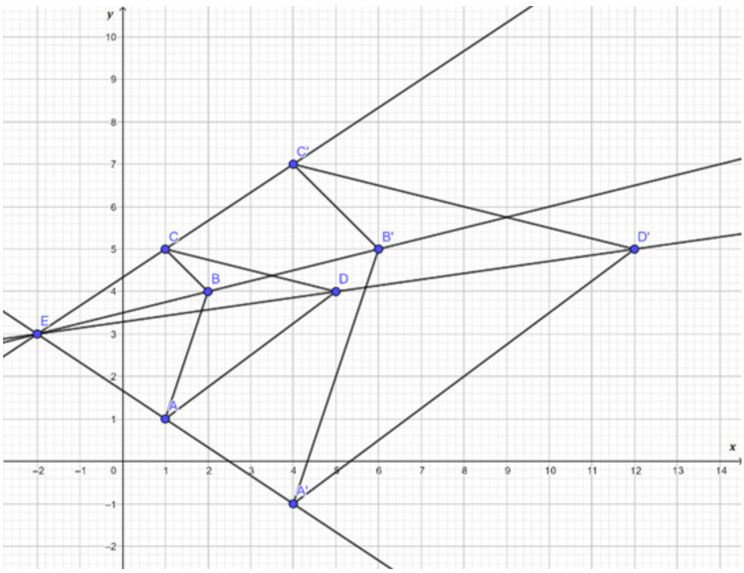
(b) (i) A′(–0.5, 1.5), B′(2.5, 3), C′(2.5, 6), D′(–0.5, 4.5)
(ii) A′(11.2, –2.4), B′(6.4, –4.8), C′(6.4, –9.6), D′(11.2, –7.2)

5. centre ($^-$1, 4) scale factor = 3
6. E(1.5, 2) centre (0, $^-$1)
7.

s.f	A′	B′	C′
3	(12,0)	(0,9)	(12, 9)
1.5	(6, 0)	(0, 4.5)	(6, 4.5)
$^-$1	($^-$4, 0)	(0 $^-$3)	($^-$4, $^-$3)
$\frac{1}{2}$	(2, 0)	(0,1.5)	(2, 1.5)

8. (i) A′($^-$6, $^-$4), B′($^-$2, $^-$8), C′($^-$8, $^-$8)
(ii) A′(1.5, 1), B′(0.5, 2), C′(2, 2)

9.



10. A′(1, $^-$2), B′($^-$5, $^-$2), C′($^-$2, 1)

11. (a) centre of enlargement ($^-$0.5, 2.5) scale factor $^-$2

(b) X′′(–9,6), Y′′(–19, 11), Z′′(–19, 1)

Exercise 11.10

1. 162 cm^2
2. (a) 2.56 (b) 100 cm^2
3. (a) $\frac{5}{4}$ (b) 35 cm
4. 15 cm by 12 cm
5. 10.5 m^2 6. 192 cm
7. 96 500ha 8. Packet = 5 976cm^2, Box = 95 616 cm^2

Exercise 11.11

1. 3, 9
2. 616 cm^3 ; 2079 cm^3
3. 15 cm
4. 154 cm^2
5. (a) 174.96 tonnes (b) 162 m^2
6. 417 960 litres
7. 168.75 cm^3
8. 9:12, 256 litres
9. (a) 18 cm (b) 3:1 (c) 76° (d) 216 cm^3

Unit 11 Test

1. ΔADC ; and ΔBAC ; a = 3.2 cm ; b = 1.6 cm
2. (–5, 5) ; 2
3. A'(4,8) , B'(8, 4) , C'(10, 10)
4. (a) 125 : 27 (b) 25 : 9 ; 144 cm^2
5. (a) 49° (b) 9 cm (c) 4:9 (d) 6:5
6. (a) ΔABE and ≈ ΔCDE are equiangular
   (b) BO : EB = 4:3
7. (a) 128 cm (b) 2516 cm^2 (c) 16.8 Litres
8. x = 10, y = 3

Answers to remedial activities

1. 4:9
2. (a) 6 cm (b) 9 m (c) 48 cm^2 (d) 900 m^3
3. A′(4,8) B′(8,4) C′(10,10)
4. ΔABC ≈ ΔDEC. $x = 10$, $y = 3$
5. (a) 25:16 (b) 125:64

Answers to extended activities

1. (a) 10 cm (b) Check student prove
2. Scale factor $\frac{4}{3}$; BC = $5\frac{1}{3}$ units
3. 24 cm
4. 2 cm

UNIT
12 INVERSE AND COMPOSITE TRANSFORMATION IN 2D

Key unit competence

By the end of this unit, the learner should be able to solve shape problems involving inverse and composite transformation

Learning objectives:

Knowledge and understanding

- State and explain properties of composite and inverse transformations in 2D.
- Identify type of transformation used in given drawings in 2D.
- Show an image of an object from different transformed shapes in 2D.

Skills

- Construct an image of an object under composite and inverse transformation in 2D
- Solve problems involving inverse and composite transformations in 2D.

Attitudes and values

- Appreciate the importance of inverse and composite transformations to transform shapes.
- Show patience, mutual respect, tolerance, and curiosity in solving and discussing problems involving inverse and composite transformations.

Pre-requisites to this unit

For learners to acquire the concepts of this unit, they need to have skills of the following

- Vector algebra (addition and subtraction of vectors) studied in S2.
- The effect of using mirrors to reflect some objects (borrowing knowledge from Physics unit of optics/light).
- The effect of rotating objects in clockwise and anticlockwise directions.
- The equation of straight line theory studied in book three unit 6.

Cross-cutting issues to be addressed in this unit.

- **Inclusive education**: A teacher should provide for learning of all learners including those with special needs. A teacher should use sign language for those with hearing difficulty and those who are dumb.

- **Standardization**: This unit will involve learners in drawing graphs accurately to get clear and correct images.
- **Environment and sustainability**: Since the unit will involve the learners working with graph papers, sensitize the learners on the importance of disposing such materials that may be poorly used once safely once they are nolonger needed to avoid polluting the environment.

Generic competences to be addressed in this unit

- **Critical thinking**: This competence will be achieved when the teacher involve learners in activities concerning transformations.
- **Problem solving**: This competence will be achieved when the teacher involve learners in activities and exercises on determining the solutions to the problems involving transformations.
- **Communication skills and listening**: This competence will be achieved when learners will be allowed to express themselves while presenting the discussion results to the whole class.
- **Leadership skills**: This competence will be achieved because during group discussions, each group will be having a leader and a secretary.

Vocabulary/keywords

In the course of learning the concepts in this unit, the learners will discover the meaning of the following new words:

- rotation
- reflection
- translation

Guide the learners to understand the meanings of these words and construct and speak out mathematical statements involving them in order to master their meaning and usage.

Guidance on the problem statement

Unit focus Activity

In order to focus the learner's attention into the general direction of the unit, the teacher should organise learners in groups of 5 students to do focus activity in the Student's Book.

- Let the learners present their findings in class discussions as others critique and ask questions to seek clarifications.
- Use this observation to help learners appreciate the need to participate actively in all the lessons organised for this unit in order to discover how to do the unit focus activity correctly and with ease.

Attention to special needs

You should provide for learning of all learners including those with special needs as follows.

- Ensure the graph papers being used to represent transformations have small squares that are visible to all students including those longsighted and the colour blind ones.
- Prepare additional more challenging questions for the fast learners.
- Organise remedial lessons for the slow learners.

List of lessons

Lesson No.	Lesson Title	No. of periods
1.	Composite translation in two dimensions	4
2.	Composite reflection in two dimensions	4
3	Composite rotation in two dimensions	5
4	Mixed composite transformations in two dimensions.	4
5.	Inverse transformations in two dimensions	4
6.	Unit test	3

Lesson development

Lesson 1: Composite translations in two dimensions

Lesson objective

By the end of the lesson, the learner should be able to understand the composite translation and how to apply it in different shapes and objects in two dimensions.

Teaching Aids

- Books,
- Pens and Pencils
- Chalks and Classroom chalkboard
- Calculators and other geometric instruments
- Graph papers

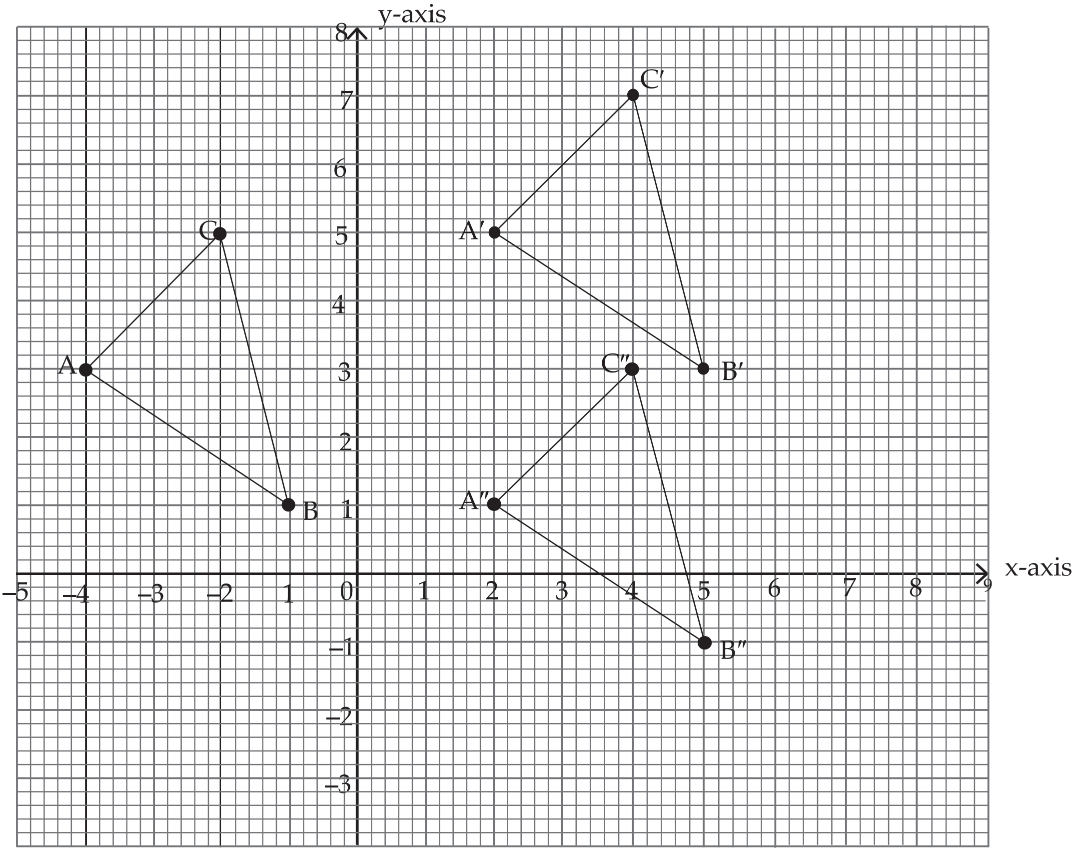
Learning Activities

Activity 12.1

- Organise the learners in pairs. One learner must work as a secretary.
- Let the learners do Activity 12.1 in the Student's Book.

- Allow the learners to present their findings on the chalkboard through the group secretaries.
- Observe the presentations and write down the errors made by the learners.

Answers for Activity 12.1



A' (2,5),	B'(5,3),	C'(4,7)
A''(2,1),	B''(5,-1)	C''(4,3).

Assessment of the lesson

- Guide the learners through Examples 12.1 and 2.12 in the Student's Book to ensure that they have digested the concepts of composite translation.
- Allow the learners to attempt Exercise 12.1.
- Correct those who are wrong by giving them more guidance.

Lesson 2: Composite reflections in two dimensions

Lesson Objective

By the end of the lesson, the learner should be able to understand how composite reflections can be applied to obtain the images of different shapes and objects in two dimensions.

Teaching Aids

- Books,
- Chalks and classroom chalkboard
- Pens and pencils
- Calculators and other geometric instruments
- Graph papers

Learning Activities

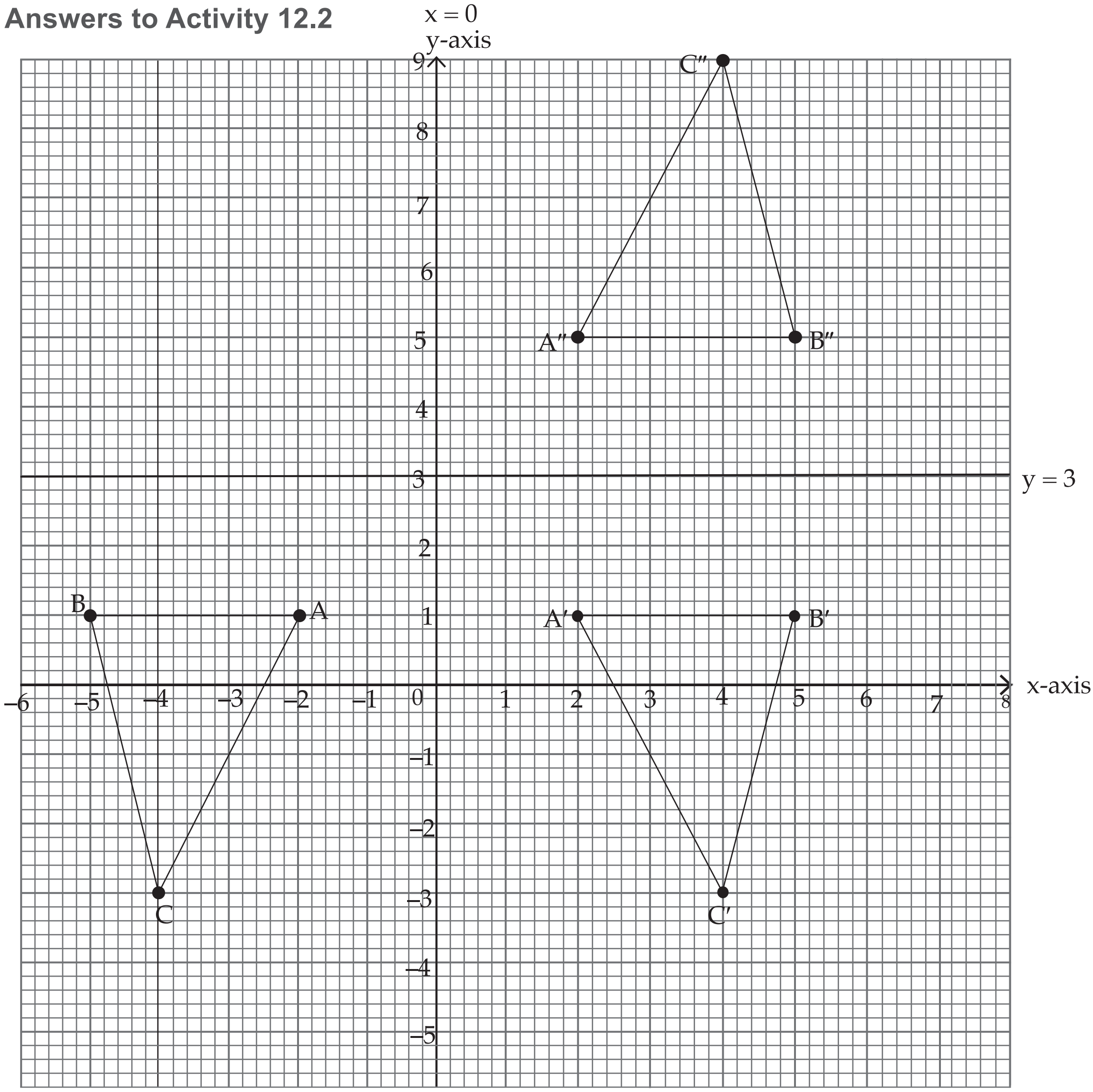
Activity 12.2

- Organise the learners in groups of three and instruct them to do Activity 12.2 in the Student's Book.
- Make sure one learner is selected as the secretary to record down the findings.
- Provide each group with a plane mirror from the school laboratory and pieces of plane papers.
- Let the groups present their findings through their secretaries.
- Observe the presentation and correct the learners where they are wrong.

Synthesis

- Explain different reflections in different axes to the learners as indicated in the Student's Book.
- Guide the learners through Examples 12.3, 12.4, 12.5 and 12.6 in the Student's Book to ensure that they understand the concepts.

Answers to Activity 12.2



A' (2,1),	B'(5,1),	C'(4,-3)
A''(2,5),	B''(5,5)	C''(4,9).

Assessment

- Having passed through Examples 12.3— 12.6, allow the learners to do Exercise 12.2 in the Student's Book.
- Let the learners present their work for marking, correct them where they are wrong.

Lesson 3: Composite rotations in two dimensions

Lesson objective

By the end of the lesson, the learner should be able to understand how composite rotations are applied to obtain the images of different shapes and sizes of the objects.

Teaching Aids

- Calculators
- Exercise books
- Geometrical instruments
- Chalks
- Graph papers/graph books

Learning Activities

Activity 12.3

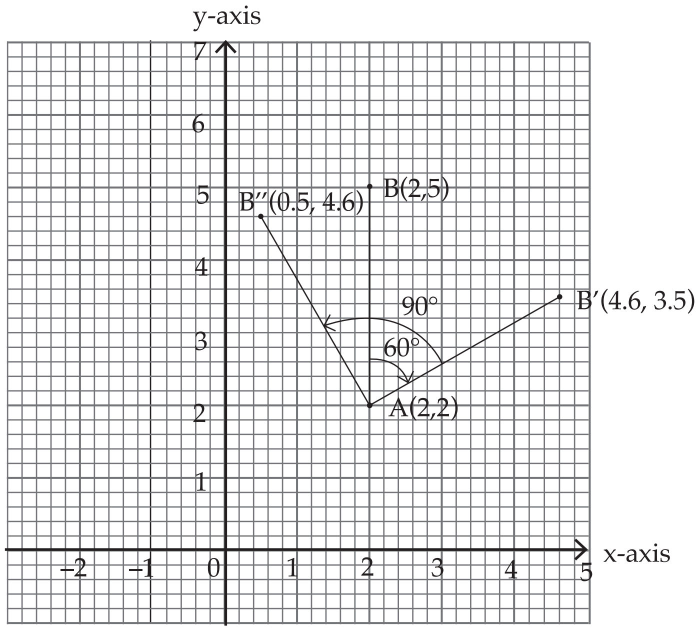
- Organise learners in pairs to work on Activity 12.3 in the Student's Book.
- Make sure one learner acts as the secretary to record down the findings.
- Let the learners present their findings on the chalkboard through the secretaries
- Make clear observations and record down the errors made by the learners during the class presentations.

Synthesis

- Explain different rotation types i.e clockwise and anticlockwise to the learners.
- Do Examples 12.7 and 12.8 in the Student's Book for students to understand how concepts are being applied.

Answers to Activity 12.3

a)



(b) B′(4.6, 3.5) (c) B″(0.5, 4.6)

2. (a) 

(b) 

Assessment of the lesson

- Test the learners with questions from Exercise 12.3 in the Student's Book.
- Make sure you correct learners where are wrong and encourage individual participations in doing the exercise.

Lesson 4: Mixed composite transformations in two dimensions

Lesson Objectives

By the end of the lesson, the learner should be able to apply mixed transformations to obtain the images of different objects.

Teaching Aids

- Calculators
- Exercise books, pens and pencils or any other writing materials.
- Chalks and class chalkboard
- Graph papers

Learning Activities

Activity 12.4

- Organise learners in pairs to work on Activity 12.4 in the Student's Book.
- Make sure one learner acts as the secretary.
- Let the learners present their findings on the chalkboard through the secretaries
- Make clear observations and correct each group where they are wrong at the end of the presentation.

Synthesis

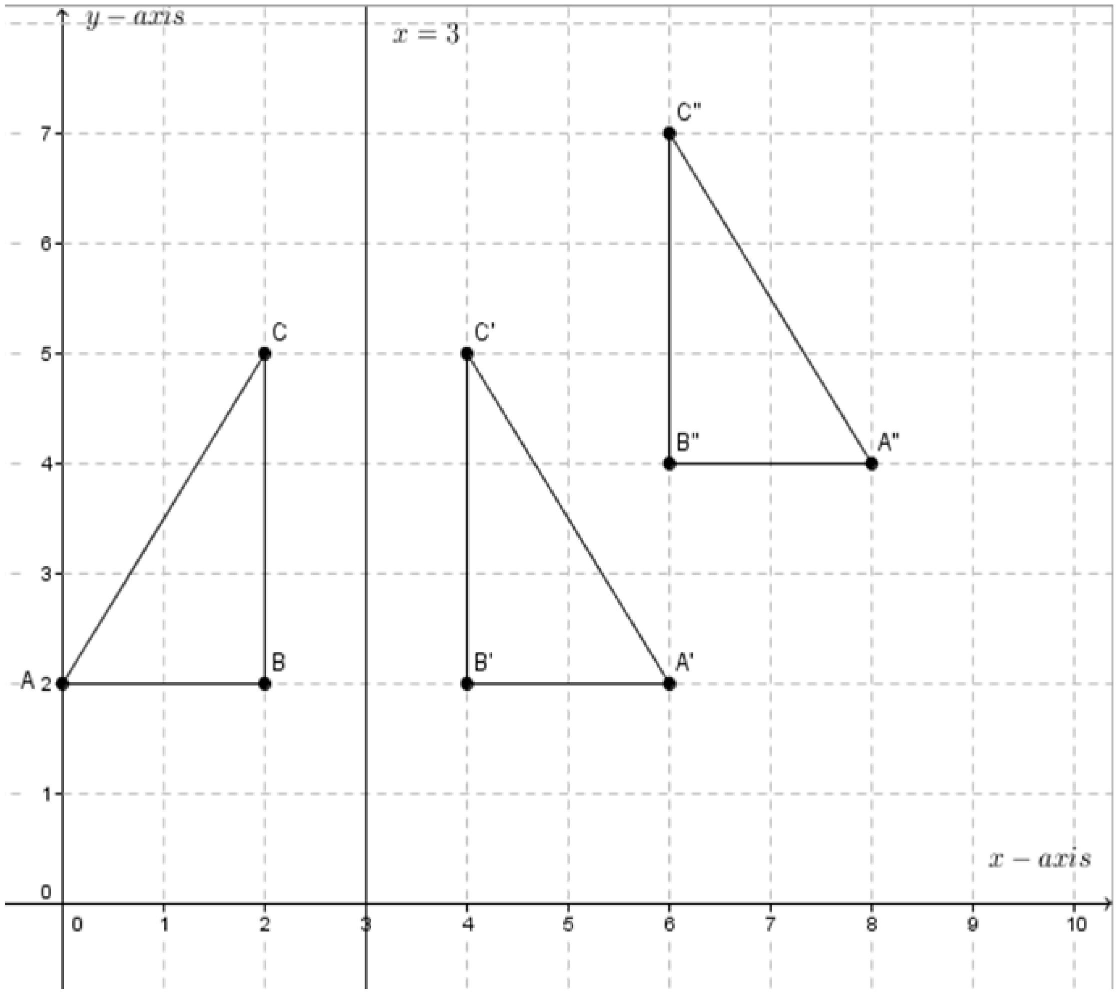
- Explain to the learners how mixed composite transformations are carried out.
- Pass the learners through Examples 12.9 and 12.10 in the Student's Book and make them understand the concepts deeply.

Assessment of the lesson

- Having passed through the activity and examples, allow the learners to do Exercise 12.4.
- Make sure you correct learners where are wrong and encourage individual participations in doing the exercise.

Answers to Activity 12.4

(a)



(b) Image after reflection is A'(6,2), B'(4,2), C'(4,5).

(c) Image after translation is A''(8,4), B''(6,4), C''(6,7).

Lesson 5: Inverse transformations in two dimensions

Learning objective

By the end of the lesson, the learner should be able to understand the how perfect squares are used in completing squares to solve quadratic equations.

Teaching Aids

- Calculators
- Exercise books, pens and pencils or any other writing material
- Geometrical instruments
- Chalks and classroom chalkboard
- Graph papers

Learning Activities

Activity 12.5

- Organise learners in groups of three to work on Activity 12.5 in the Student's Book.
- Make sure one learner acts as the secretary to record down the findings.
- Let the learners present their findings through class discussions through the secretaries.
- Make the observations as they are presenting and note down the errors made.

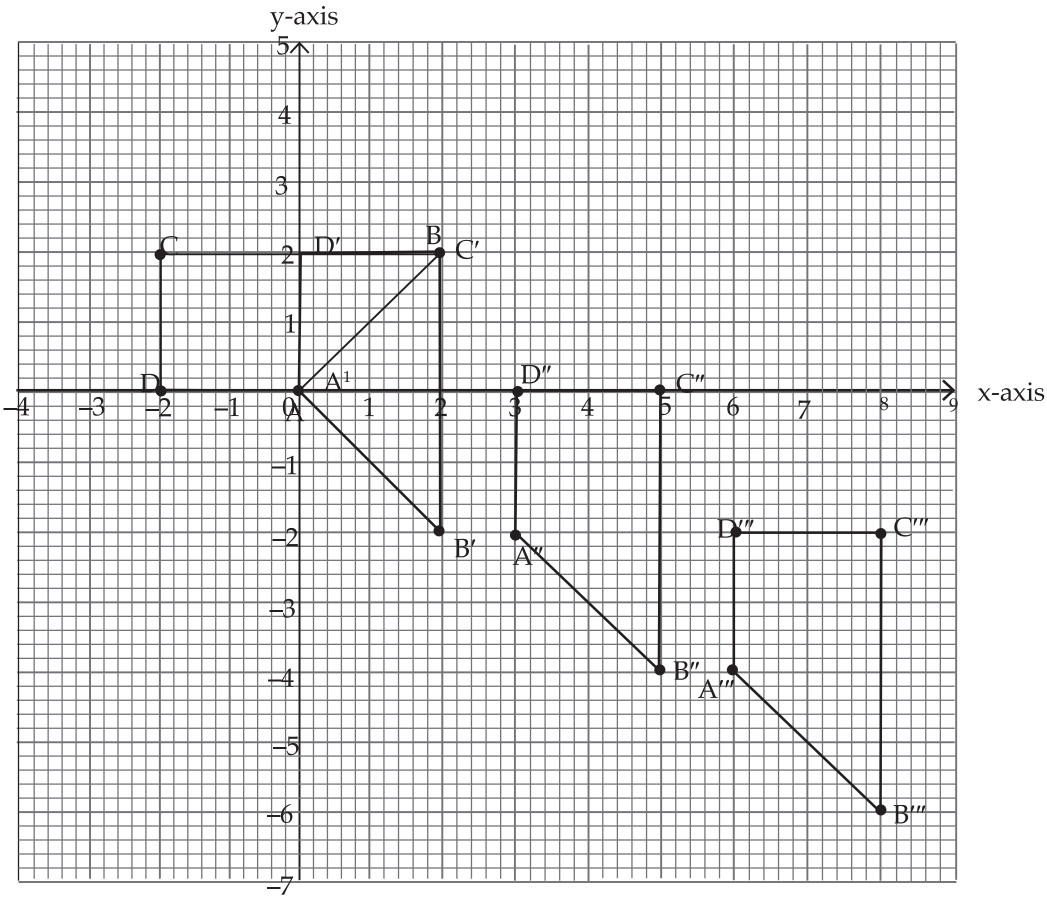
Synthesis

- Explain to the learners how inverse transformations are achieved.
- Allow the learners to ask more questions about this part where they don't understand.
- Ask the learners to discuss amongst themselves Examples 12.12 and 12.13 in the Student's Book.

Assessment of the lesson

- Test the learners with questions of Exercise 12.5 from the Student's Book.
- Make sure you correct learners where are wrong and encourage individual participations in doing the exercise.

Answers to Activity 12.5



Summary of the unit

- Asking different learners to take the different concepts highlighted in the unit summary given in the Student's Book.
- Ask the class probing questions to help them recall the concepts correctly.

End of unit assessment

- A teacher should ask learners to do unit 12 test in the Student's Book to assess whether learners understood the whole unit.
- A teacher should take time to correct the unit test to find out whether the concepts of the whole unit were properly digested.

Additional information to the teacher

- A teacher should know the concepts of isometries taught in S2 to make learners understand the concepts of rotation.

- A teacher should as well teach the learners well how to use their geometrical instruments especially the calculators in this unit.

Remedial activities

For the slow learners, a teacher should organise remedial lessons where he/she guides them through the activities once again and more slowly to help them understand the concepts.

Remedial exercises

1. Given the vertices of a triangle ABC as A (-4, 3), B (-1, 1) and C (-2, 5).
   (a) Plot the points ABC on a Cartesian plane.
   (b) Join the points A, B and C to form triangle ABC.
   (c) Move point A horizontally to the right by 6 units and then vertically upwards by 2 units. Locate the final position of A as A″.
   (d) Repeat procedure (c) for points B and C.
   (e) Join the points A″, B″ and C″.
   (f) What do you observe about the size and shape of ABC and A″B″C″?
2. Triangle XYZ is such that X (-2, 1), Y (-3, 2) and Z (-3, 4). It is given a translation of vector $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$. Find the coordinates of its image, $X^1Y^1Z^1$.
   Plot the object and the image on the same Cartesian plane.

Extended activities

The calculations in this unit may not be challenging to the specially gifted learners hence they finish the doing the task so fast then get bored and start distracting others. Give learners the questions below.

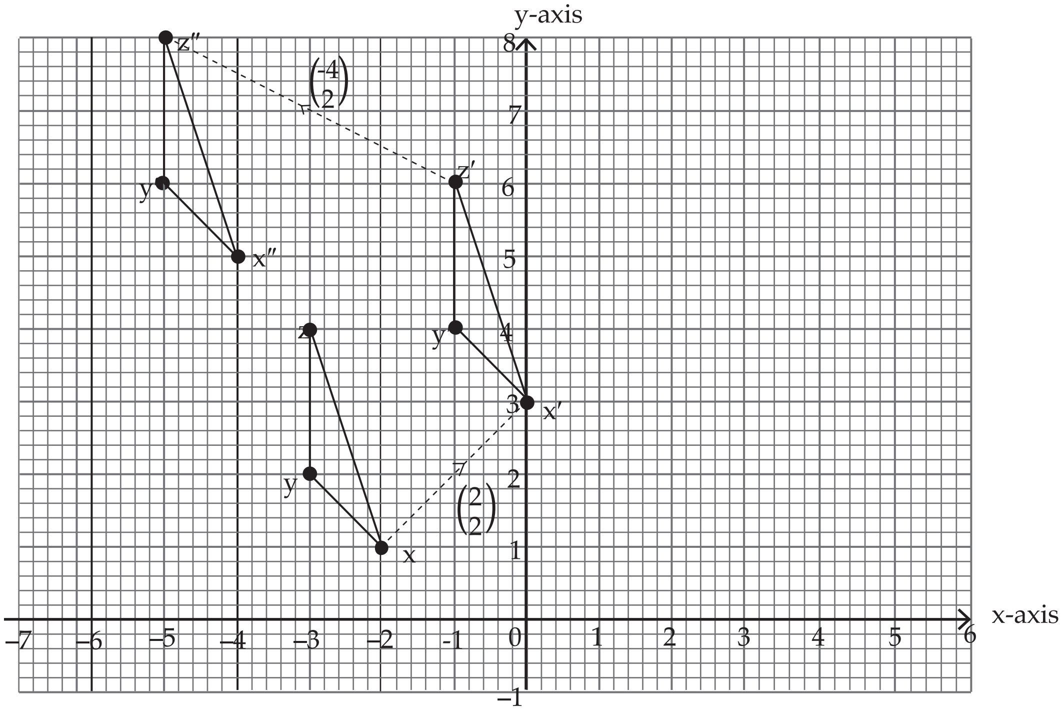
Extended exercises

1. Points A (3, 4), B (3, 7), C (6, 7) and D (9, 4) which represent a trapezium.
   (a) Find the image of trapezium ABCD under translation represented by vector $\begin{pmatrix} -6 \\ -5 \end{pmatrix}$.
   (b) The Image obtained in (a) above is further translated by vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. What are the coordinates of the new image?
   (c) Plot ABCD and its images on the same Cartesian plane.

Answers

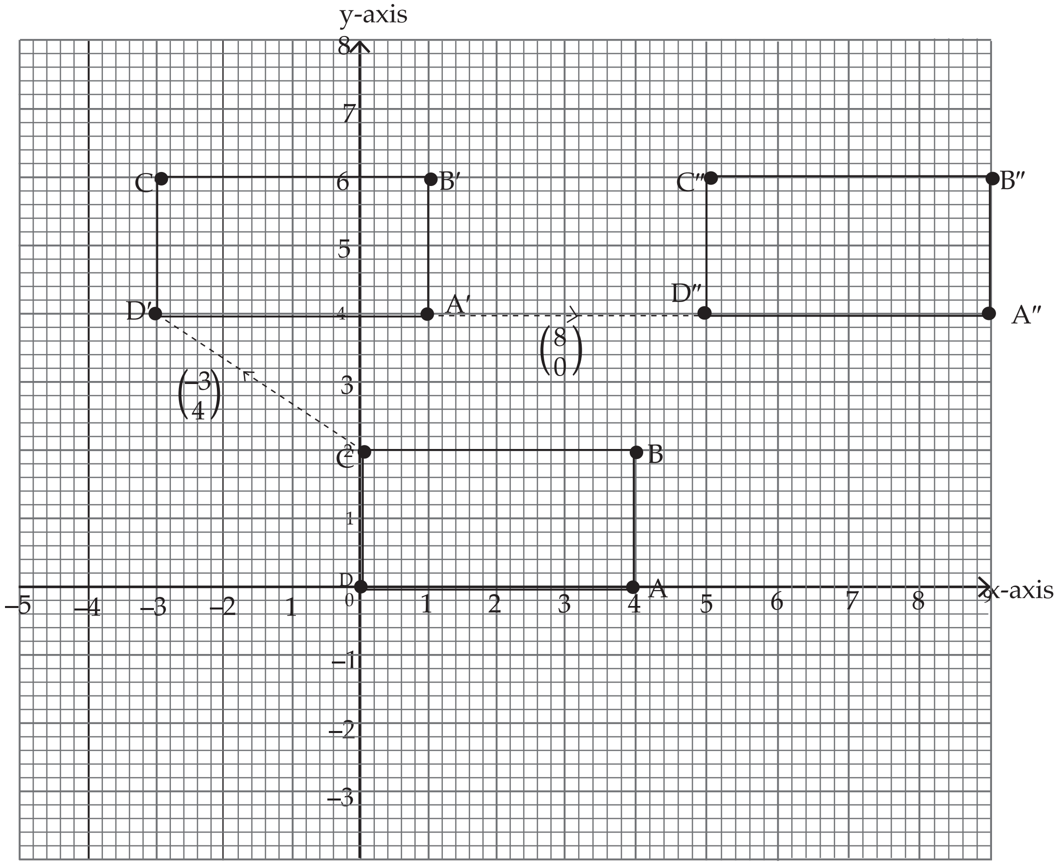
Exercise 12.1

1. (a) X′(0,3) ,Y′(–1,4), Z′(-1,6) (b) X′′(– 4,5), Y′′(–5,6), Z′′(–5,8)



2. (a) A′(1,4), B′(1,6), C′(– 3,6), D′(– 3,4)
   (b) A′′(9,4) B′′(9,6), C′′(5,6), D′′(5,4)

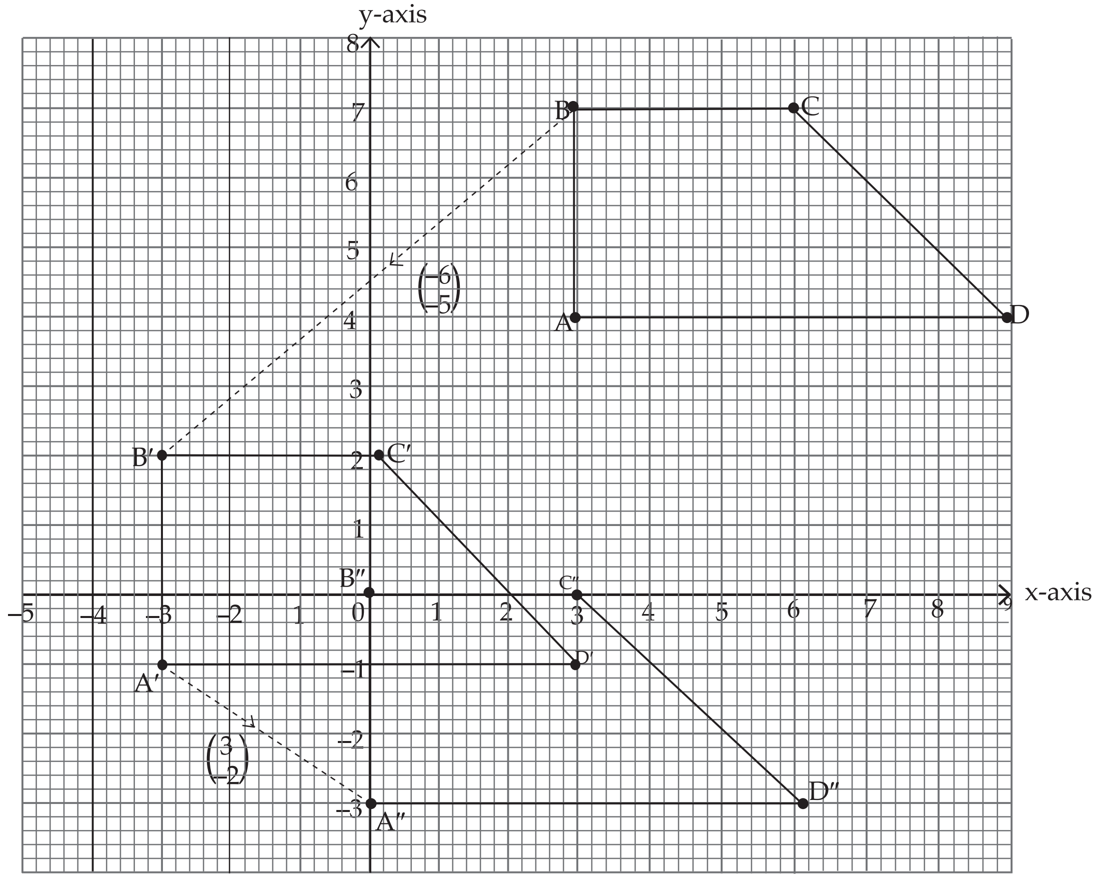
(c) Done on the graph.



3. (a) A′(–3,–1), B′(–3,2), C′(0,2) D′(3,–1)

(b) A″(0,–3), B″(0,0), C″(3,0), D″(6,–3)

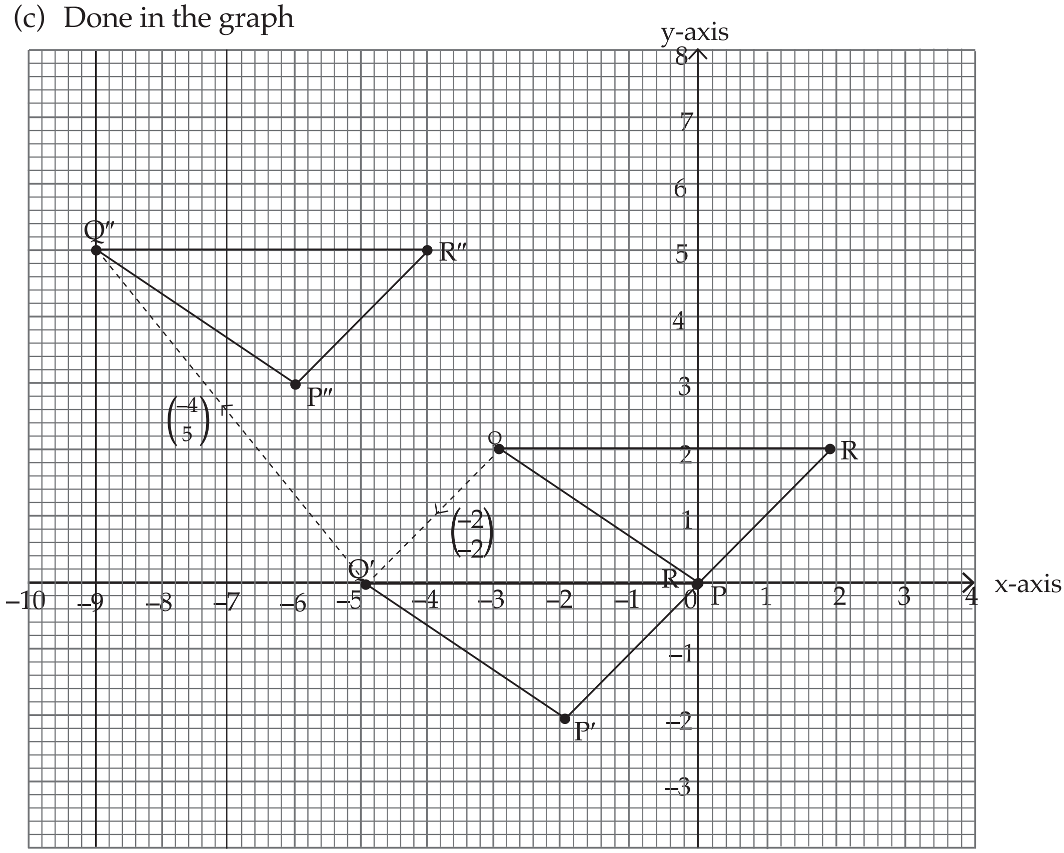
(c) Done on the graph



4. (a) P′(–2,(–2), Q′(–5,0), R′(0,0)

(b) P″(– 6,3), Q″(– 9,5), R″(– 4,5)

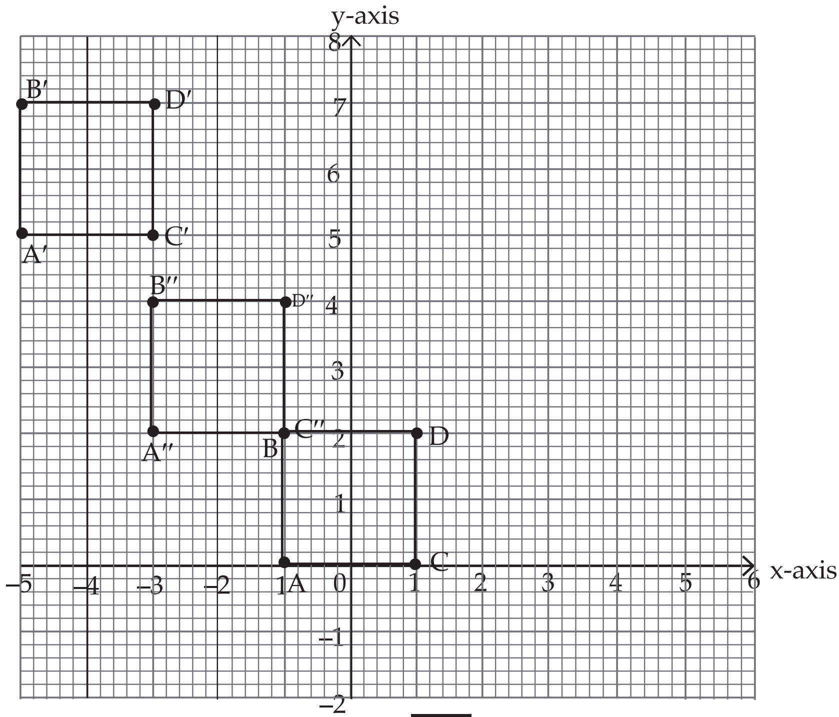
(c) Done in the graph



5. (a) A′(–5,5), B′(–5,7), C′(–3,5) D′(–3,7)

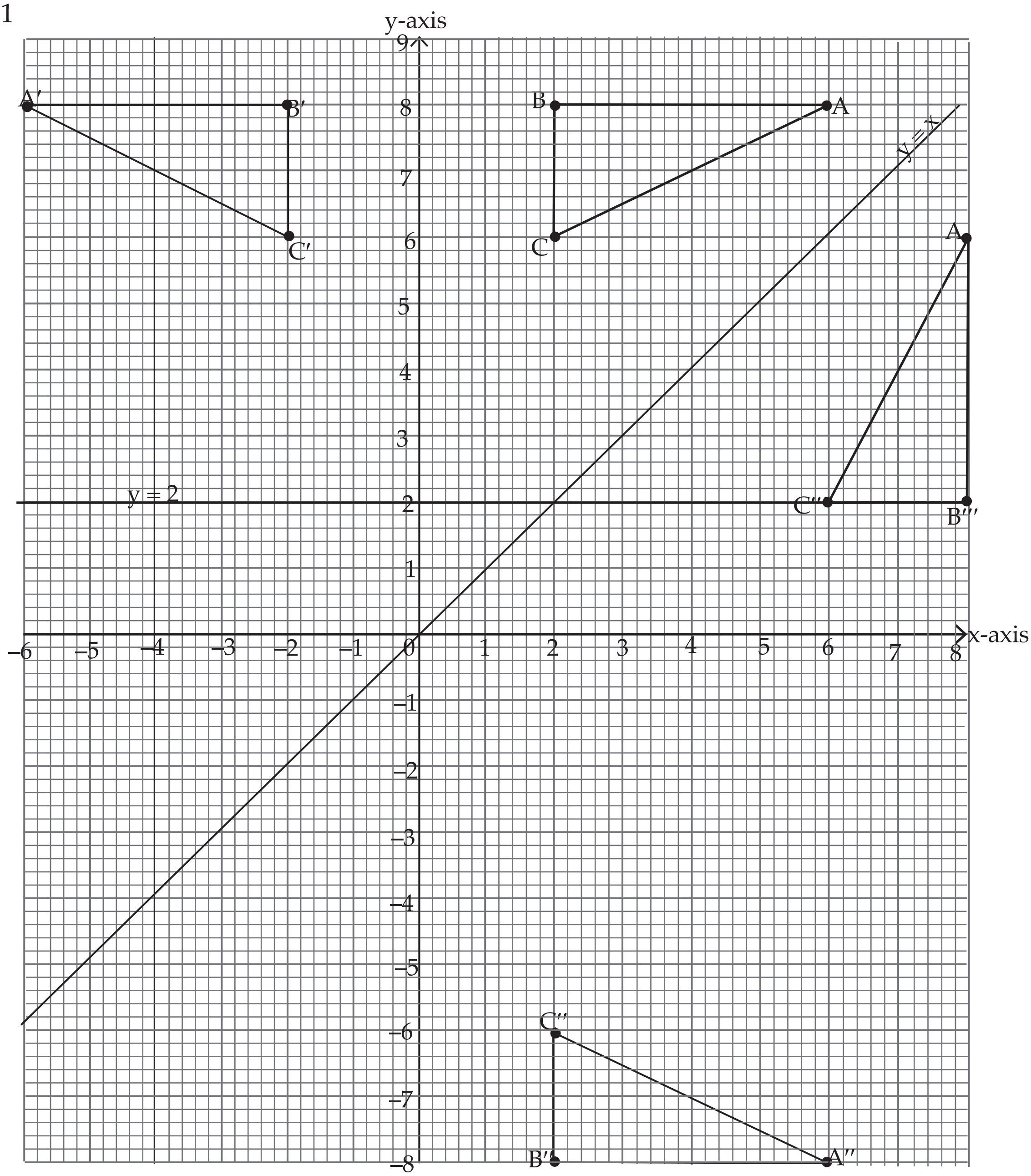
(b) A″(–3,2), B″(–3, 4), C″(–1,2) D″(–1,4)

(c)



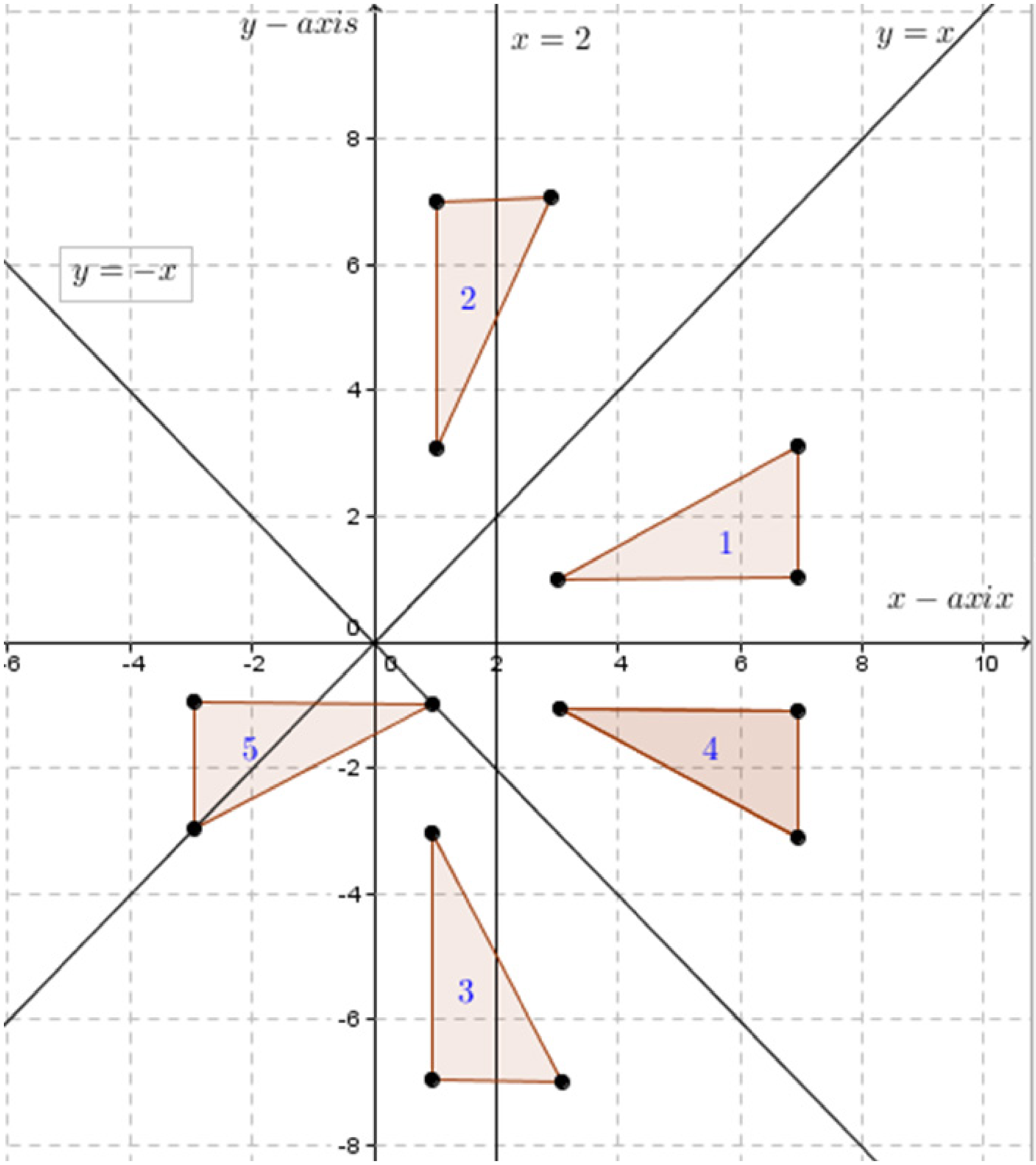
Exercise 12.2

1



$\therefore$ (i) A′(-6,8), (ii) A″(6,– 8) (iii) A‴(8,6)

2.

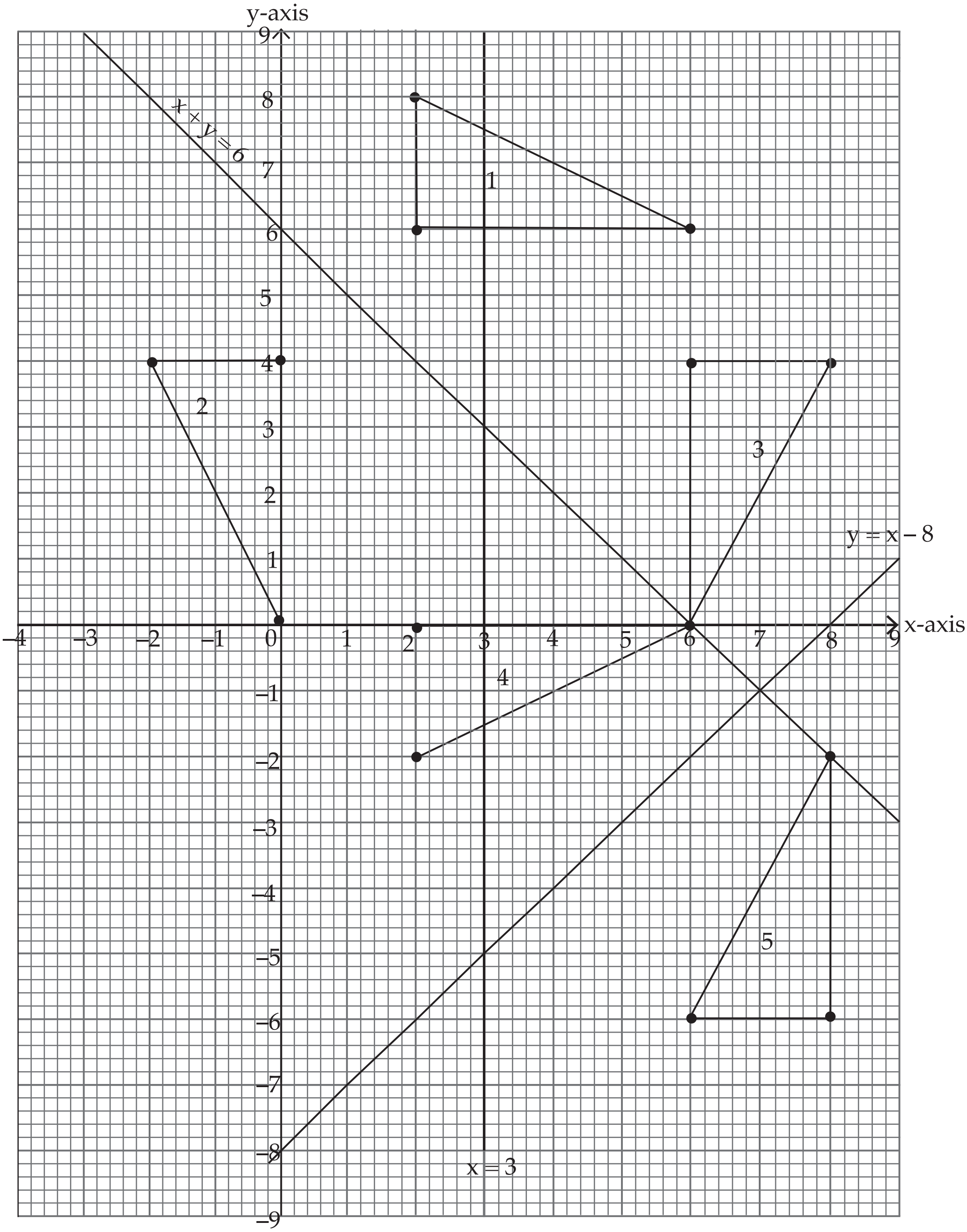


(f) The co-ordinates of triangle 5 are (– 3, –1), (–3,–3) and (1,–1).

3. Triangle 2: (0,4), (–2,4), (0,0)

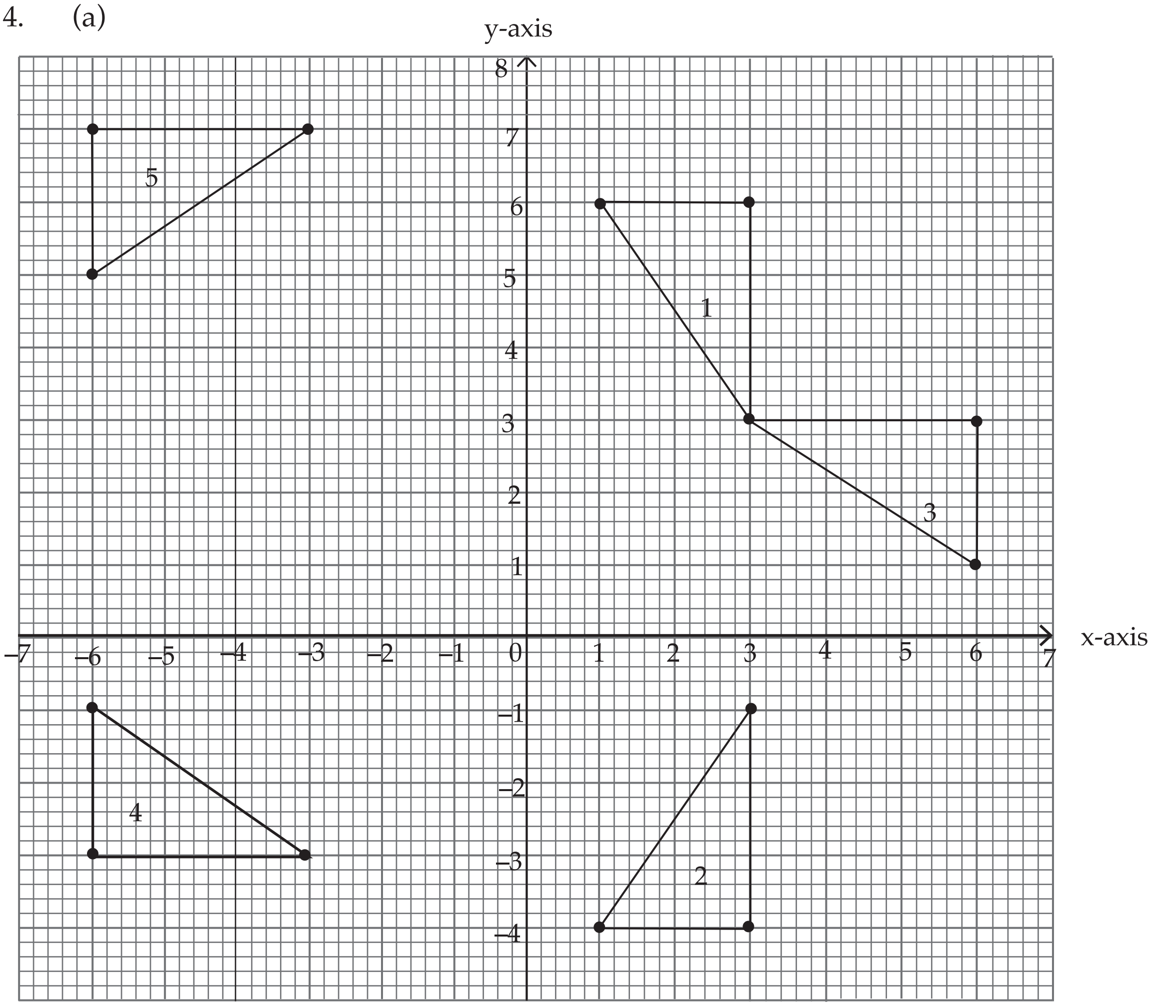
Triangle 3: (6,4) (8,4), (8,0)

Triangle 4: (2,0), (2,,–2), (6,–2)



(f) The co-ordinates of triangle 5 are (8, – 6), (8,– 2) and (6,– 6).

4. (a)



(b) (i) $y = 1$

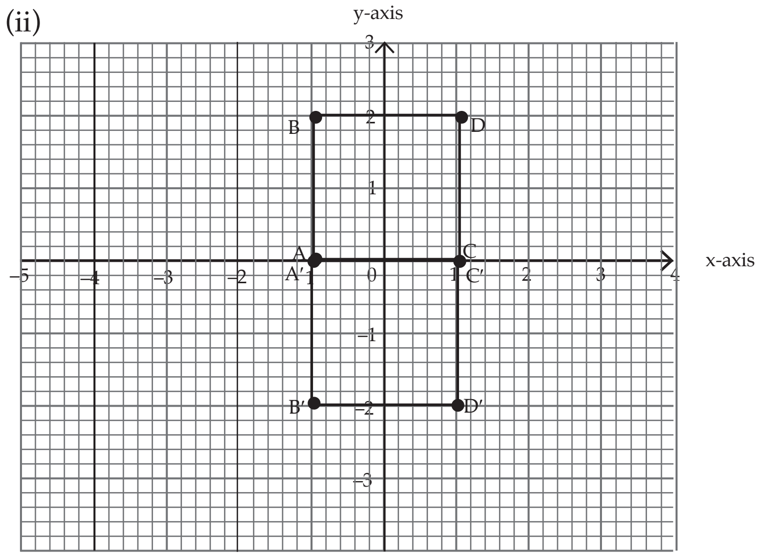
(ii) $y = x$

(iii) $y = -x$

(iv) $y = 2$

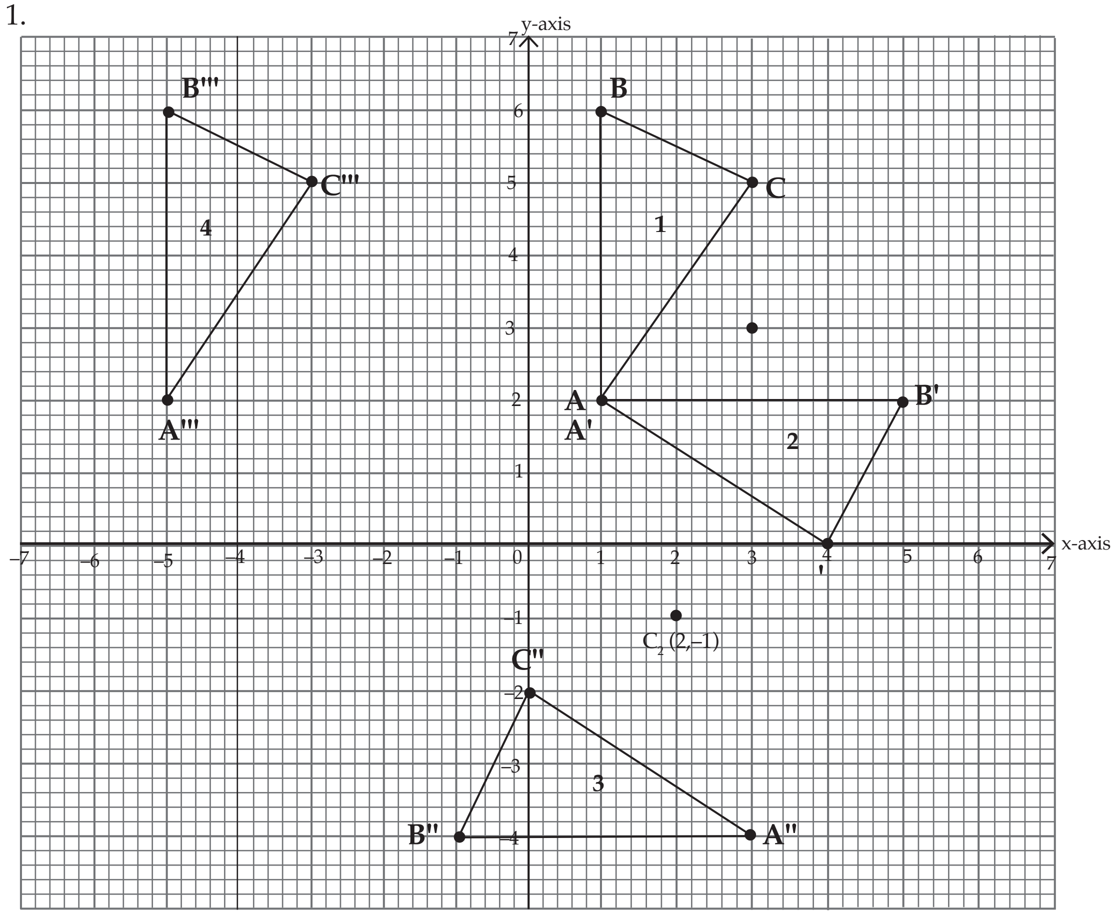
5. (i) A′ (–1,0), B′(–1, –2), C′(1,0), D′(1,–2)

(ii)



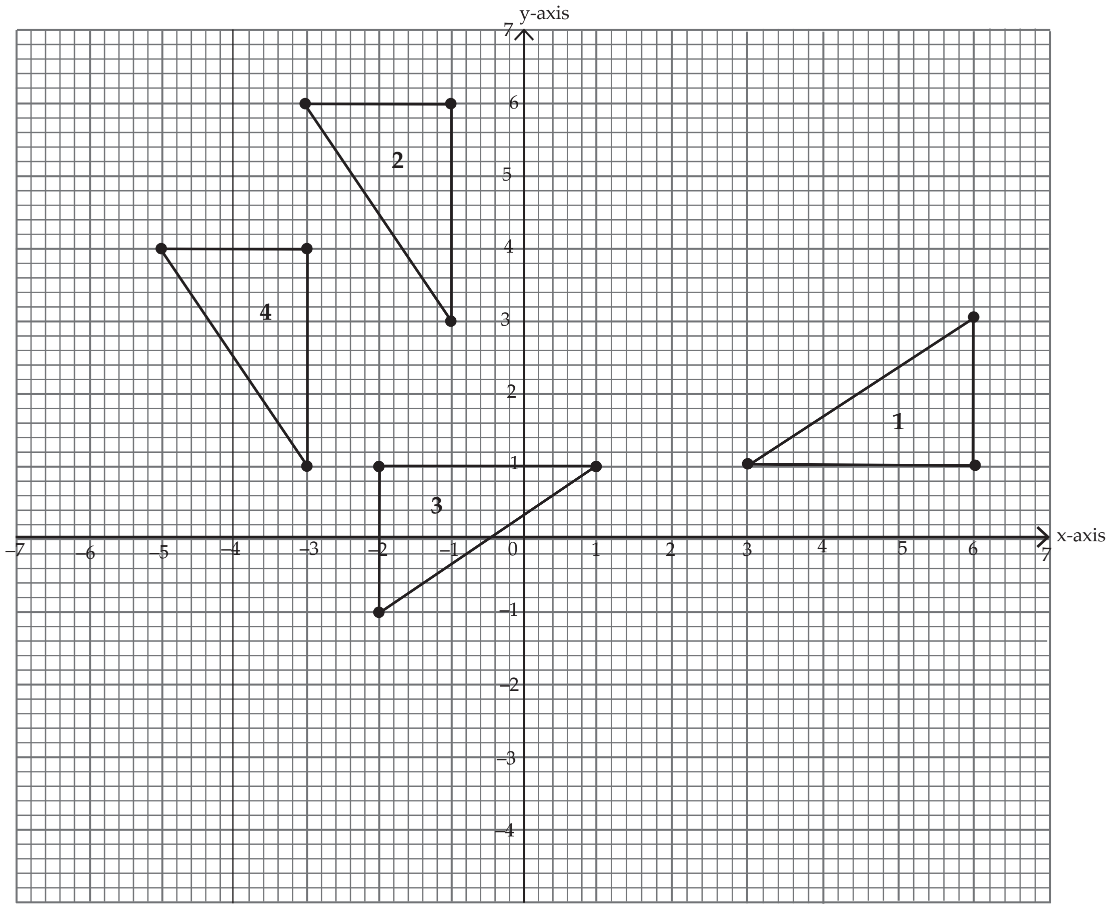
Exercise 12.3

1.



(e) The co-ordinates of the fourth triangle are: A'''(-5, 2), B'''(-3, 5), C'''(-5, 6).

2. (a)



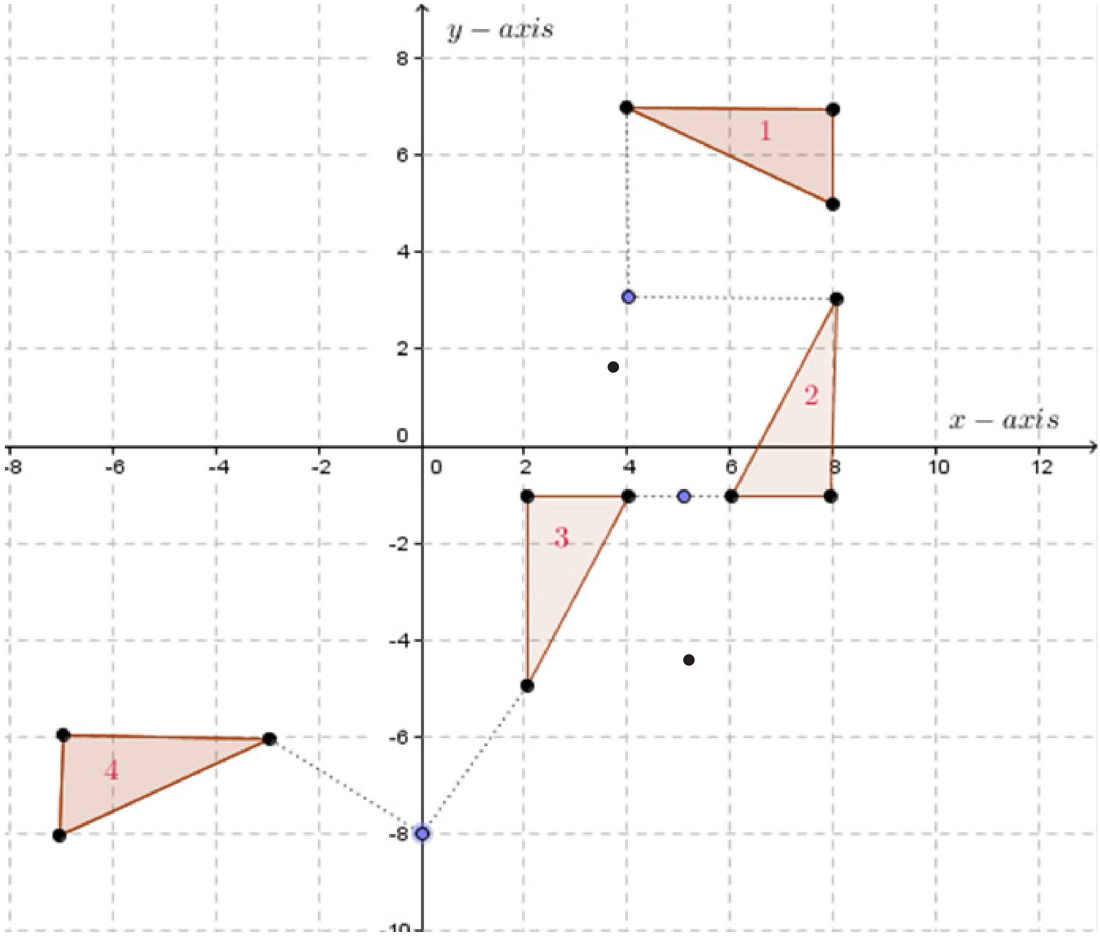
(b) (i) 90° anticlockwise about origin

(ii) 180° about (2,1)

(iii) 90° anticlockwise about (0, -2)

(iv) 90° clockwise about (1,3)

3.

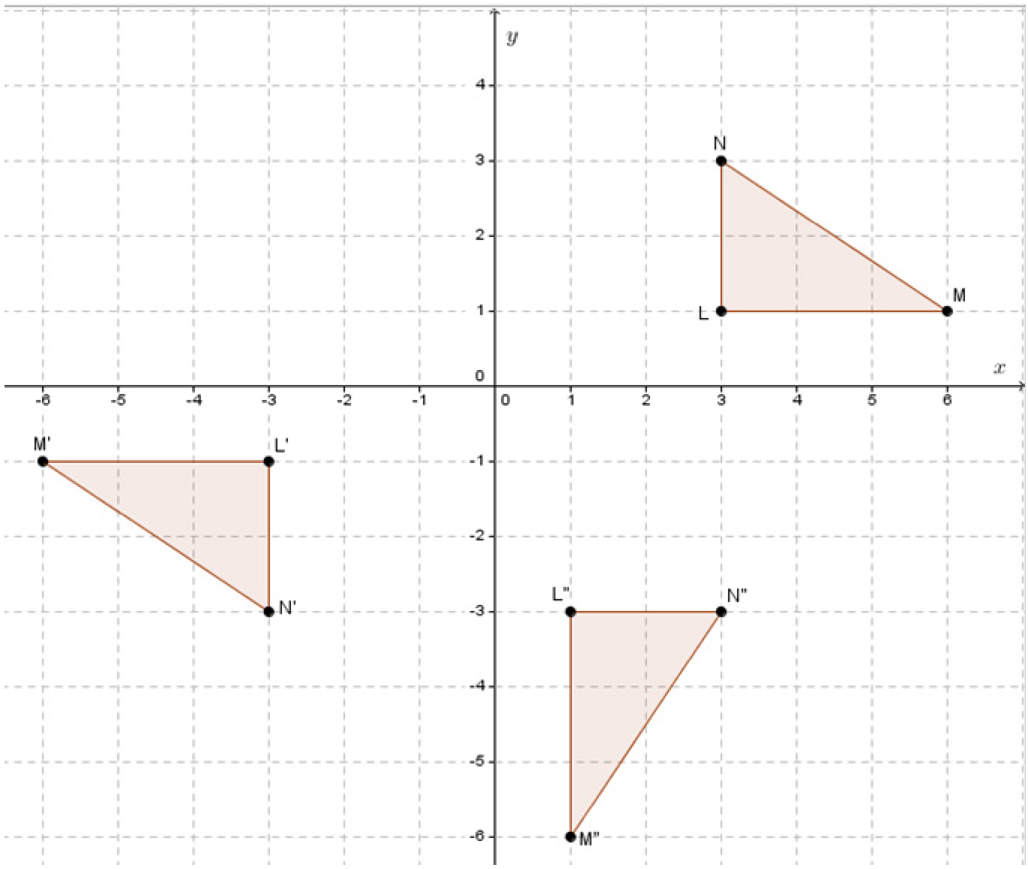


(e) (i) translation by $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$

(ii) 90° rotation about (–2,4)

4. L′ (–3,–1) M′(–6,–1) N′(–3,–3)

L″ (–1,3) M″(–1,6) N″(–3,3)



Exercise 12.4

1.	(a)	(2,-3)	(b)	(6,4)	(c)	(0,0)	(d)	(3,–2)
	(e)	(3,-2)	(f)	(0,4)	(g)	(4,–6)	(h)	(–6,–4)
	(i)	(3,2)	(j)	(0,0)				

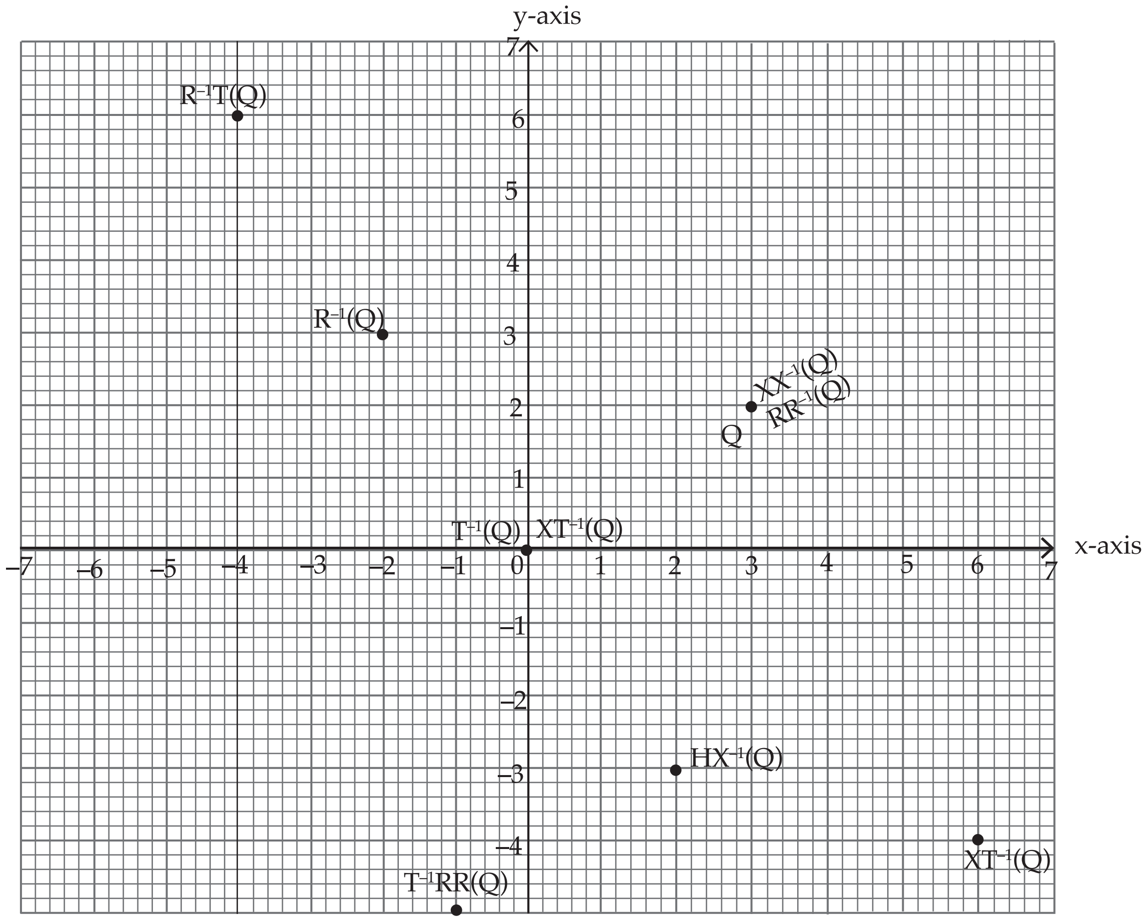
2.	(a)	L"(4,6)	M"(0,6)	N"(0,8)
	(b)	L"(2,–2)	M"(2,–6)	N"(4,–6)
	(c)	L"(0,0)	M"(0,–4)	N"(4,–6)
	(d)	L"(0,4)	M"(–4,2)	N"(–4,6)
	(e)	L"(2,–2)	M"(–2,–2)	N"(–2,–4)
3.	(a)	P"(–2,5),	Q"(–6,5),	R"(–6,7)
	(b)	P"(–4,0),	Q"(–4,–4)	R"(–2,–4)
	(c)	P"(2,–2),	Q"(2,–6)	R"(0,–6)
	(d)	P"(6,2),	Q"(10,2)	R"(6,4)
4.	(a)	X"(2,4)	Y"(2,1)	Z"(4,1)
	(b)	X"(–4,–2)	Y"(–4,1)	Z"(–2,1)

	(c)	X''(4,6)	Y''(1,6)	Z''(1,8)
5.	(a)	O''(0,0)	P(2,0)	Q''(2,–3)
	(b)	O''(–2,5)	P''(–2,7)	Q''(1,7)
	(c)	O''(–2,6)	P''(–4,6)	Q''(–4,3)
	(d)	O''(3,4)	P''(5,4)	Q''(5,1)
6.	(a)	X''(5,–4)	Y''(5,–8)	Z''(3,–8)
And		X''(–7,0)	Y''(–7,–4)	Z''(–9,–4)
	(b)	X''(–7,0)	Y''(–7,–4)	Z''(–9,–4)

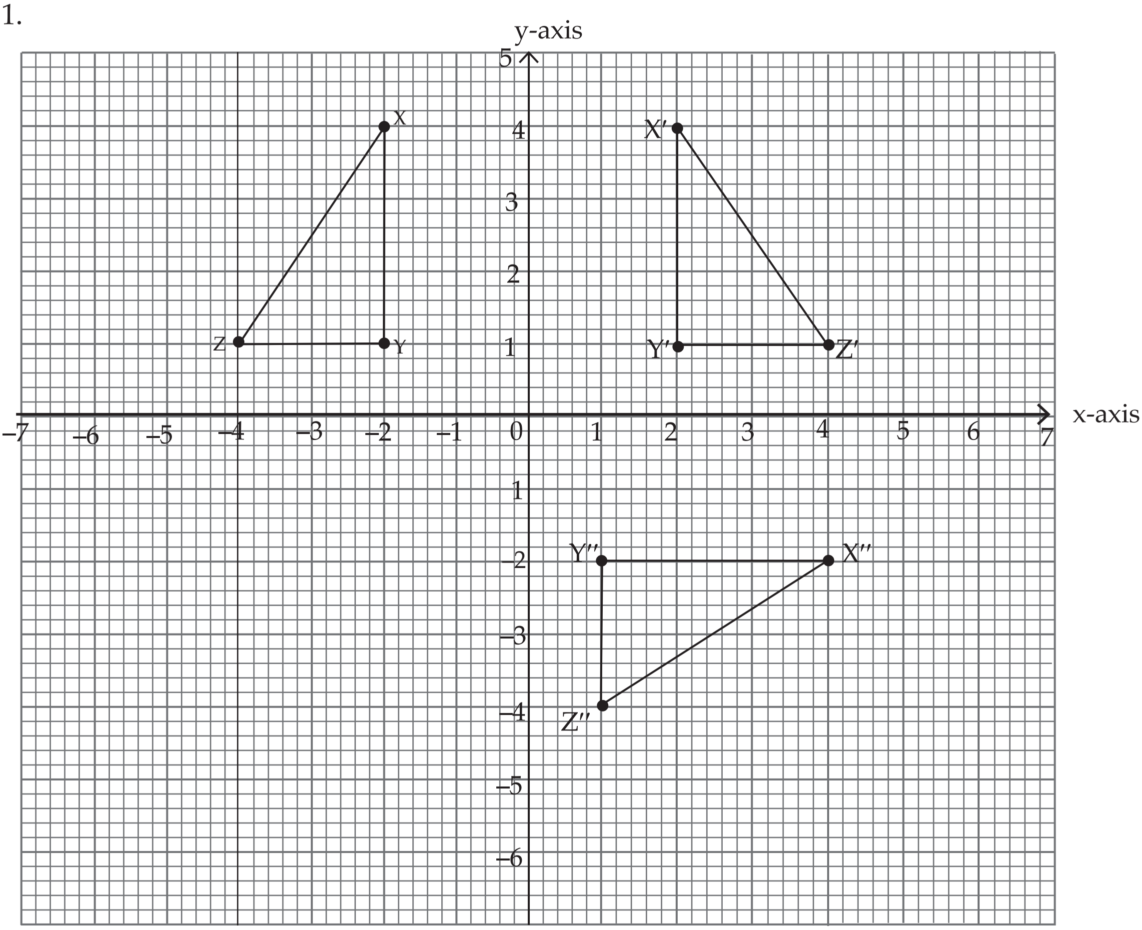
(c) 180° rotation about (0,0) clockwise direction.

Exercise 12.5

1. $R^{-1}(Q) = (-2,3)$	2. $T^{-1}(Q) = (0,0)$	3. $(T^{-1})^3H(Q) = (-12,-8)$
4. $HX^{-1}(Q) = (2,-3)$	5. $RR^{-1}(Q) = (3,2)$	6. $T^{-1}R(Q) = (-1,-5)$
7. $R^{-1}T(Q) = (-4,6)$	8. $XT^{-1}(Q) = (0,0)$	9. $XX^{-1}(Q) = (3,2)$
10. $XT^{-1}H(Q) = (6,-4)$		



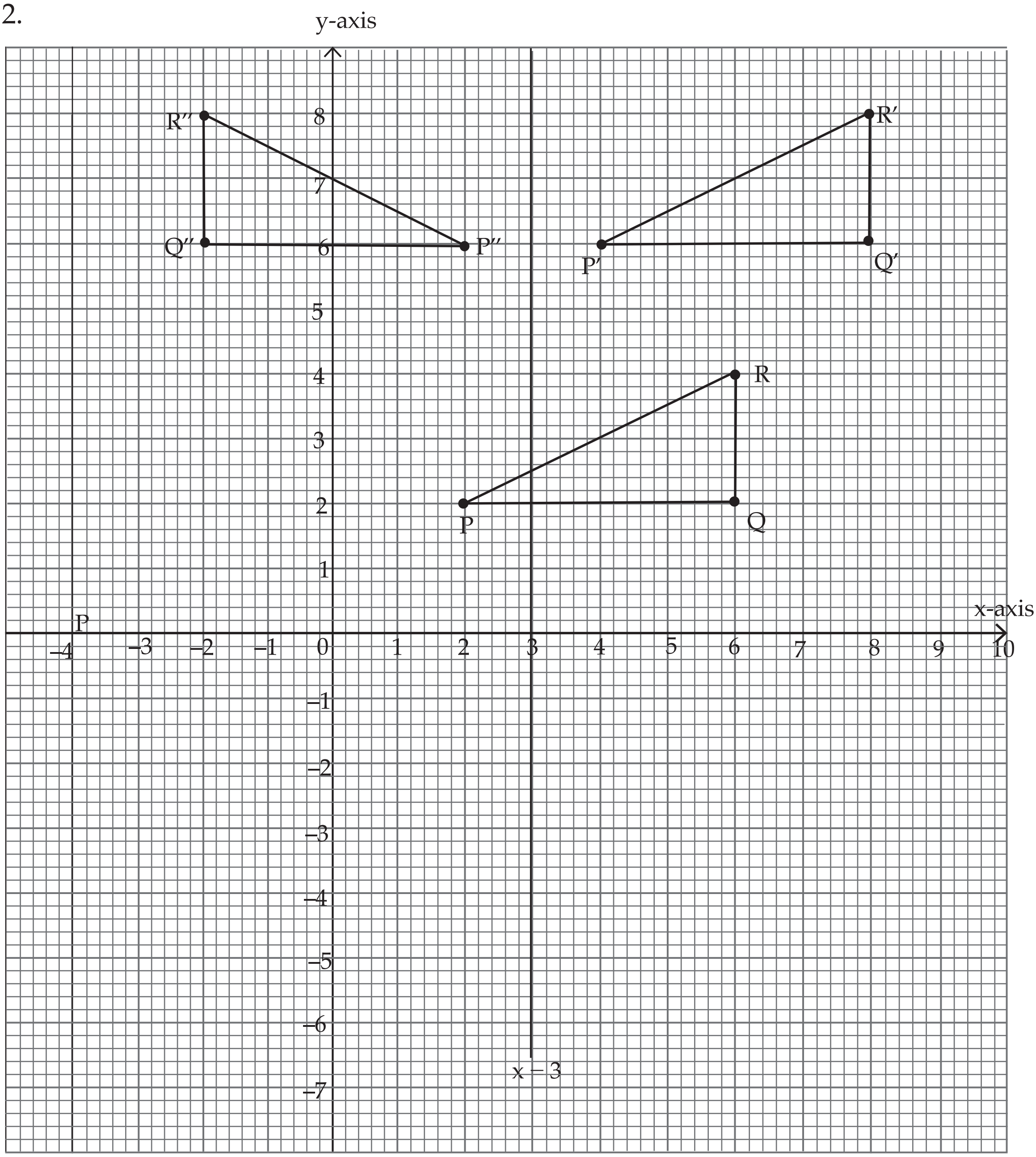
1.



(a) x′ (2, 4), y′(2, 1) and z′(4, 1)

(b) x″(4, –2), y″(1, –2) and z″(1,–4)

2.


y-axis
x-axis
R''
Q''
P''
R'
P'
Q'
R
P
Q
P
x = 3
8
7
6
5
4
3
2
1
–1
–2
–3
–4
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–6
–7
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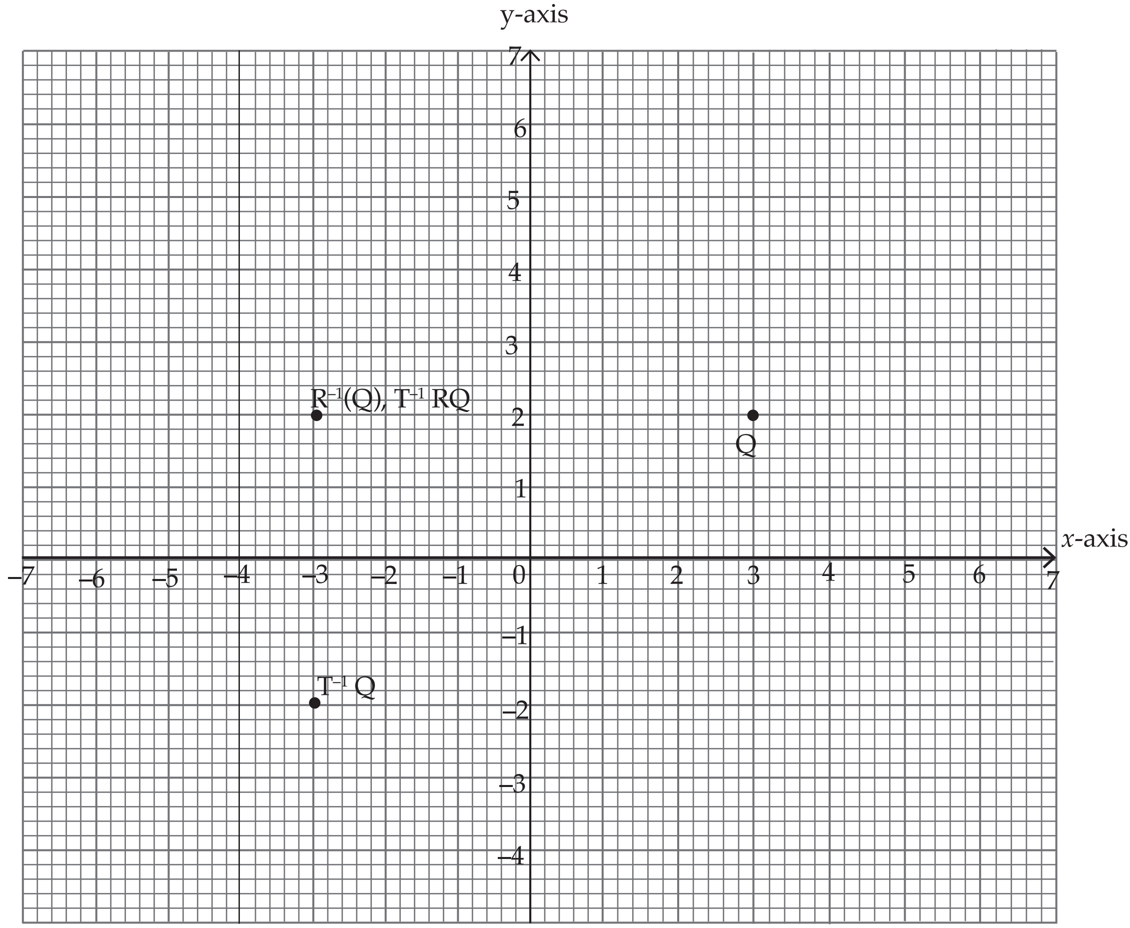
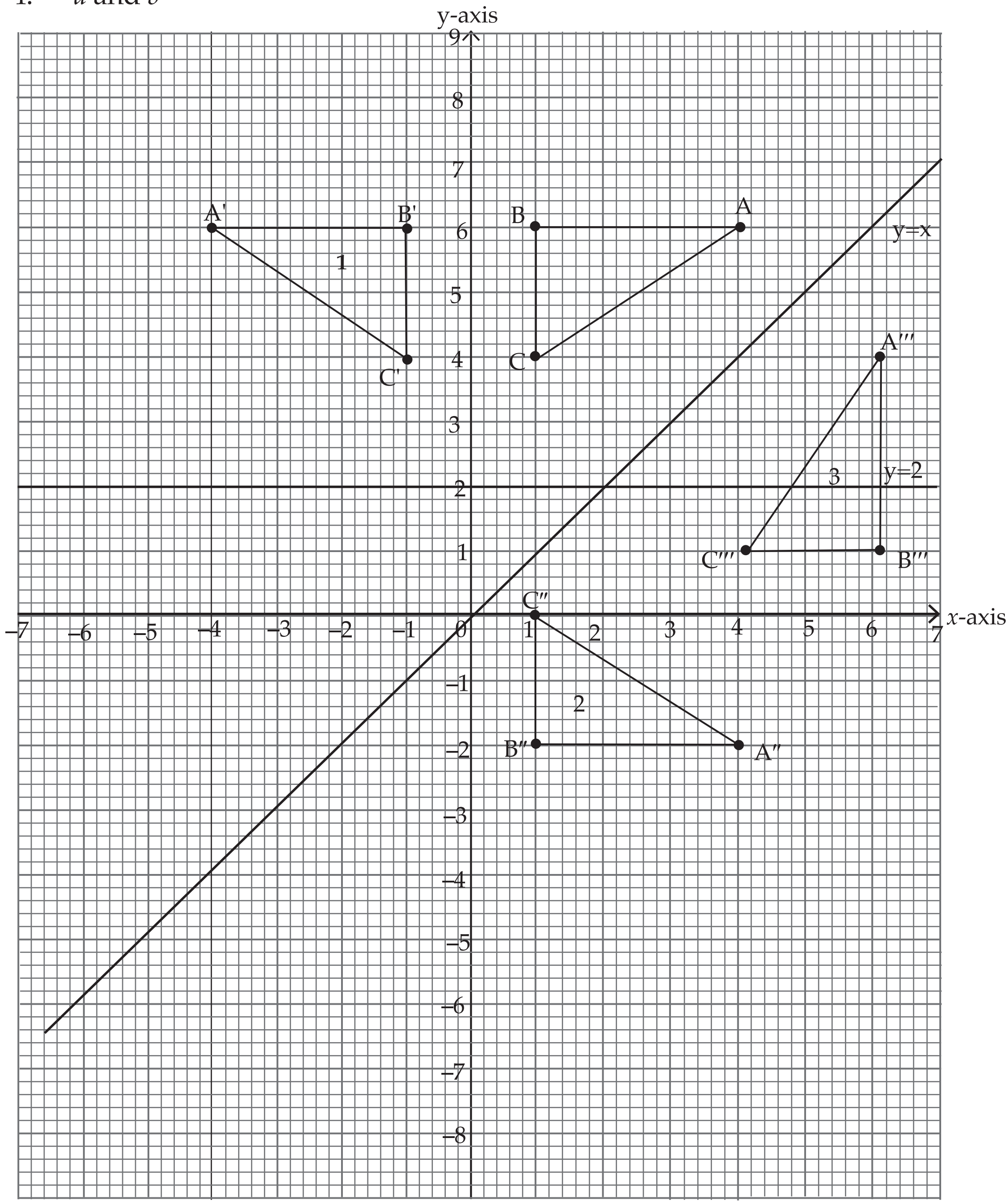
10

P''(2,6)

3.



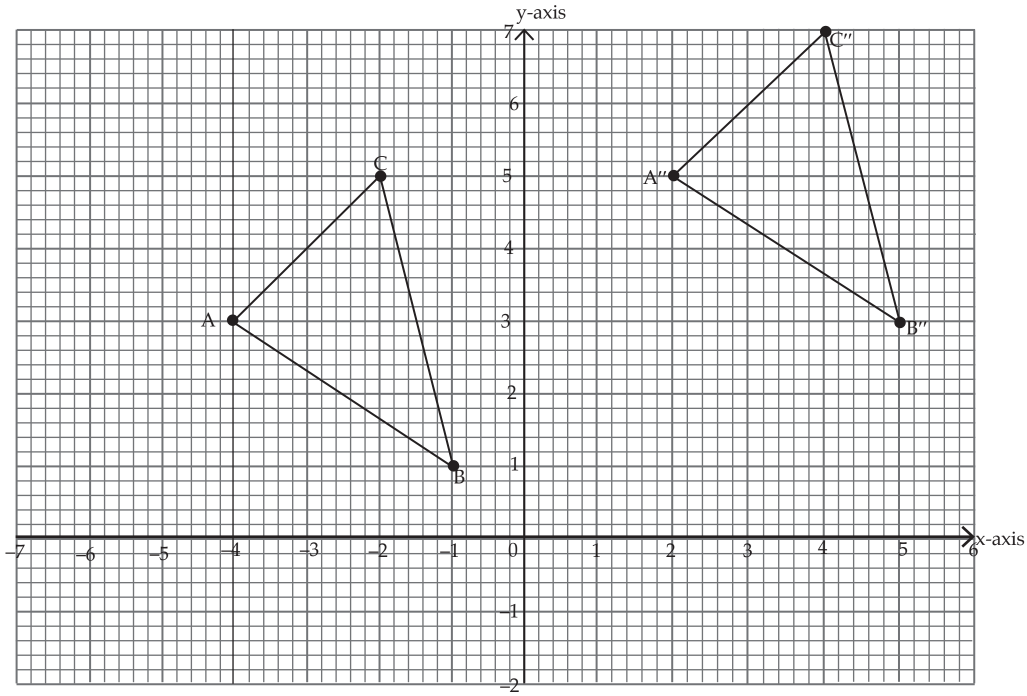
4. *a* and *b*



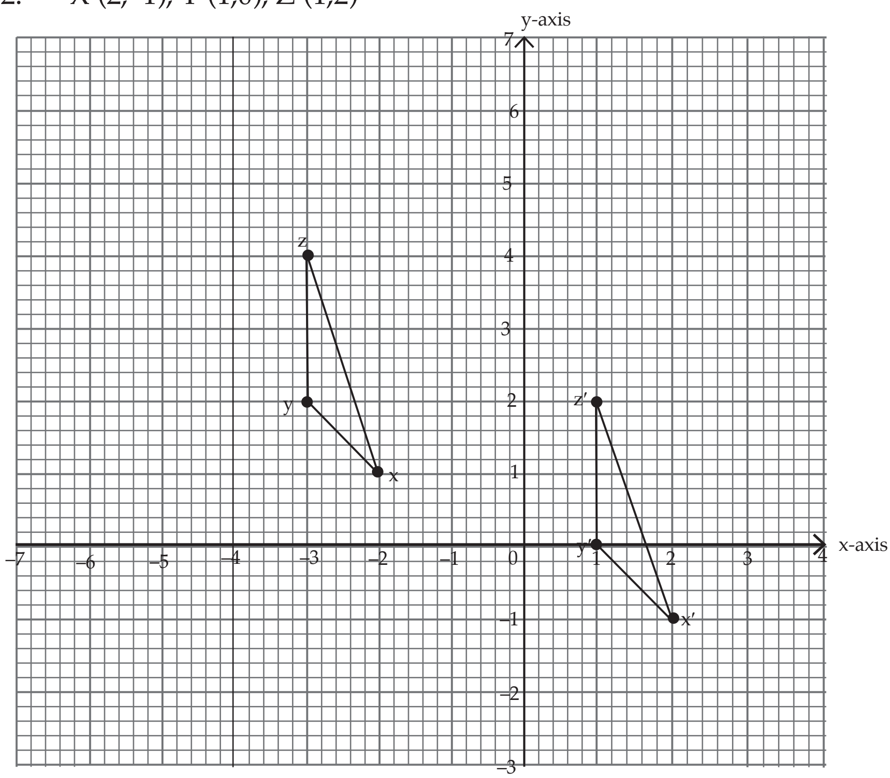
(c) A′(–4, 6), A′′(4, –2), A′′′(6, 4)

Remedial Exercise

1. (f) same size



2. X′(2,–1), Y′(1,0), Z′(1,2)

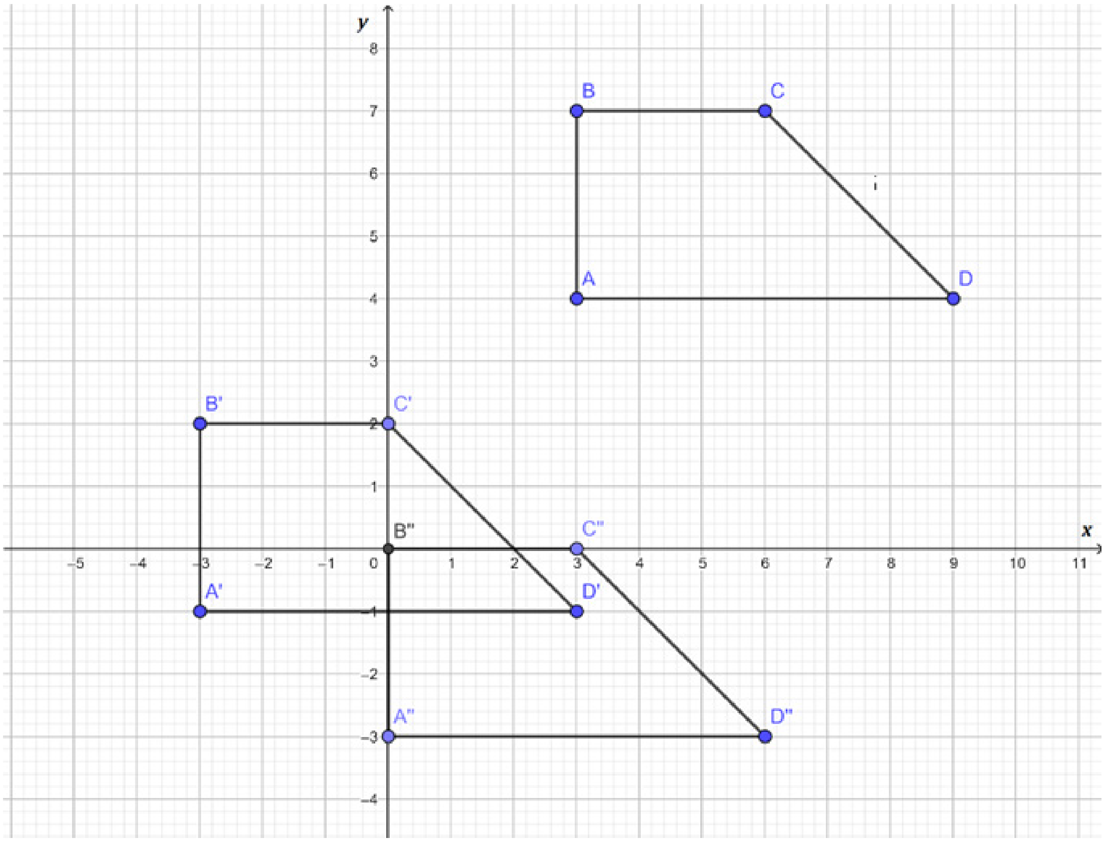


Answers to Extended exercises

(a) A' (-3,-1), B' (-3,-2), C' (0,2), D' (3,-1)

(b) A'' (0,-3), B'' (0,0), C'' (3,0), D'' (6,-3)

(c)



UNIT
13 STATISTICS (Bivariate data)

Key unit competence

By the end of this unit, the learner should be able to collect, represent and interpret bivariate data.

Learning objectives

Knowledge and understanding

- Define bivariate data.
- Make a frequency distribution table of collected bivariate data.
- Interpret scatter diagrams.
- Identify type of correlation on a scatter diagram.

Skills

- Draw scatter diagram for bivariate data and indicate the type of correlation.
- Analyze a scatter diagram and infer conclusion.

Attitudes and values

- Develop clear, logical, and coherent thinking while drawing conclusion related to bivariate data or scatter diagrams.
- Appreciate the use of scatter diagrams to represent information.
- Show patience, mutual respect, tolerance, and curiosity in collecting, representing, interpreting bivariate data.

Pre-requisites to this unit

For the learners to acquire the knowledge and attitudes and values suggested in this unit with ease and effectively, they need to have mastered the following:

- Knowledge and skills to collect, represent and interpret quantitative discrete data. These concepts were dealt with in detail in S1 under statistics and probability, unit 8 and in S2 under the same topic area unit 10. Using question and answer method in a class discussion or revision tests, the teacher should ensure that the learners remember these concepts before introducing them to the new concept of **bivariate data**.
- Skills on collecting, organising and representing ungrouped and grouped data.
- Interpret statistical data.
- Calculating measures of central tendency such as mean, median, measures of central tendencies etc.

The teacher should give the learners some written quizzes to ascertain that the learners are ready to be introduced to the new unit.

Cross-cutting issues to be addressed

Environmental conservation

Learners should be sensitised about the dangers of cutting trees and swamps. they should know that such causes drought which dangerous.

Financial and business education

Some questions will involve use of statistics in planning and laying strategy for successful business ventures.

HIV/AIDS and interpretations in this unit can be applied to the impact or effect of some social traumas such as HIV and AIDS.

Inclusive education should be embraced by both learners and the teacher. Learners with difficulties should be helped, fit in environment and acquire edneation like others.

Generic comptences

The specific generic comptences to be addressed in this unit are: -

- **Team work, sharing and mutual respect**. These comptences will be achieved when the teacher involves learners in activities that require learners to work together.
- **Research skills and techniques**. These comptences will be achieved when the teacher involves learners in using internet and other literature to meanings of new terms and words and using own words to summarize findings of activities.
- **Communication skills**: This is achieved when the learners are given a chance to report group findings.
- **Problem solving**: This competence is achieved through activities and exercises involving statistical methods.

Vocabulary/Key words

In the course of learning the concepts in this unit, learners will discover the meaning of the following new words or phrases;

- Bivariate data.
- Scatter diagram.
- Best line of fit.
- Correlation.
- Variate.

Guide the learners to understand the meanings of these words or phrases so that the learners can use the words appropriately.

Guidance on the problem statement

In order to show the general direction of this unit, the teacher should organise learners to do the introductory unit focus activity in the Student's Book.

Unit Focus Activity

Materials

- Calculator , geometrical instruments
- Graph books/papers
- Square grid board.
- Access to internet.
  - Ask learners to use any method of their choice to determine which of the two performances was better.
  - It is most likely most learners might use a method of calculating the mean mark in each paper and compare the two mean marks.
  - Use this opportunity to explain why a graph representing the data might be a better way to see the trend of the points representing the marks, especially if combined with the mean marks or scores.

Synthesis

Having done the unit focus activity guide the learners in a class discussion responding to all the parts of the activity as is required. If there is any new concept that cannot be discussed effectively at this point, assure the learners that you will tackle the same later, at an appropriate time.

Answers to unit focus Activity

- The response to part (a) is best done after the points have been plotted in order to see the trend.
- Fig 13.1 below shows the required points on the graph.

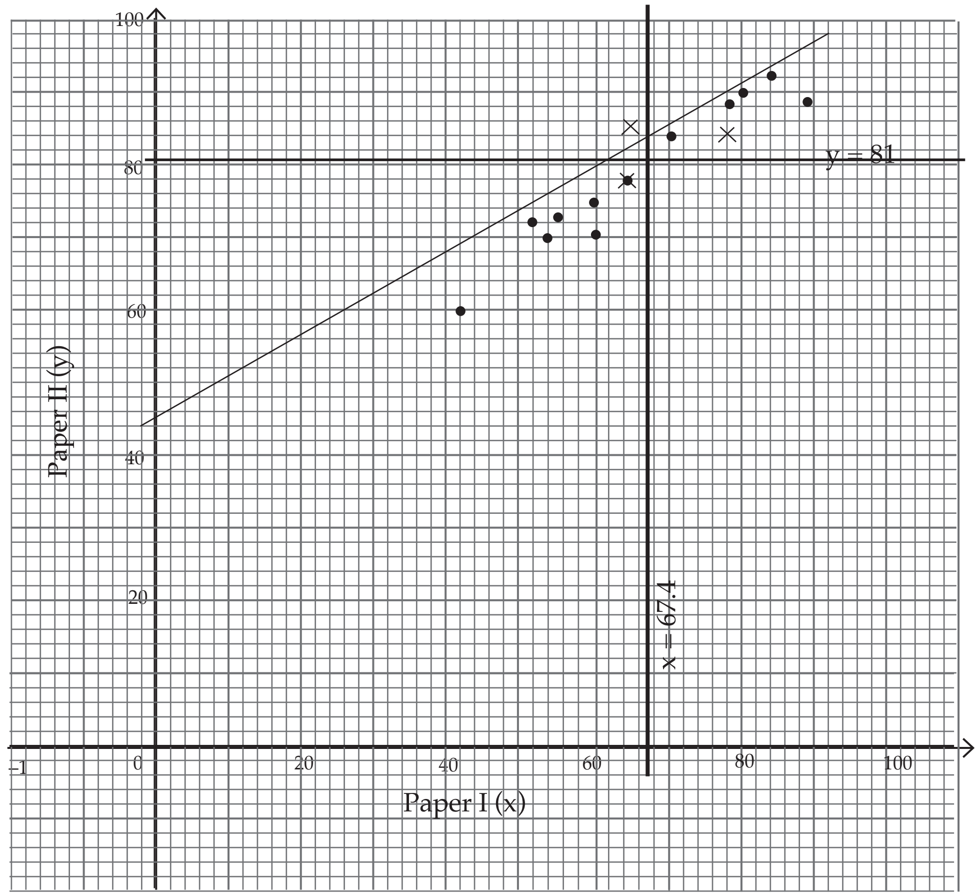


Figure 13.1

(a) It is not possible for us to tell how the two performances relate because points are scattered with no evident a pattern.

(b) The resulting graph is neither a line nor a curve. The points cluster in a certain direction.

(c) It is not possible to obtain a rule connecting x and y. Points have no defined pattern. Use this opportunity to help the learners appreciate the need to establish a way of approximating points into a line.

(d) The mean mark for paper I is 67.4 and that of paper II is 81.

The tines representing them have the equations $y = 81$ and $x = 67.4$. With reference to the lines, we can see some pattern in the performance.

i.e In paper I, 8 people are below average,

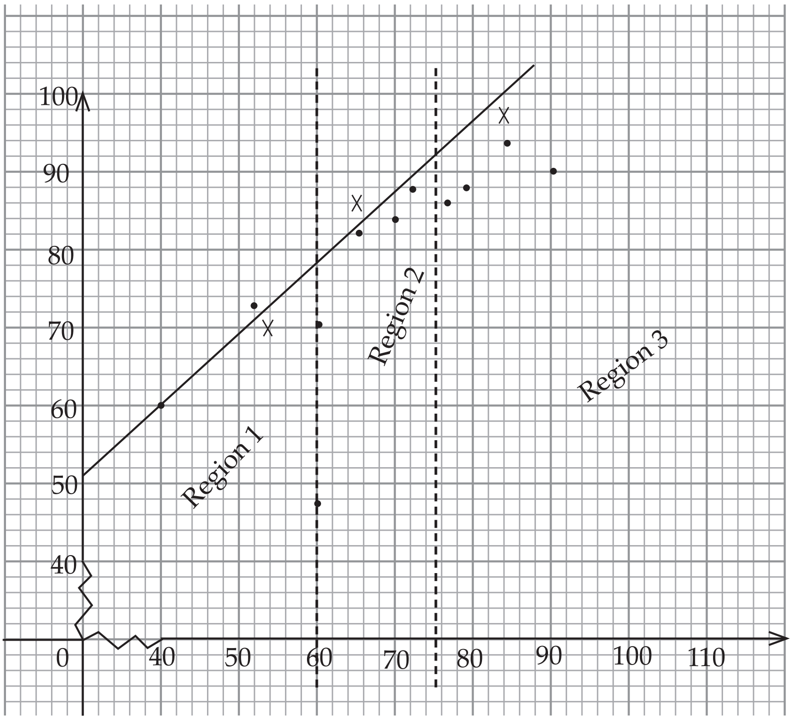
in paper II, 7 are below average

This means 7 people are below average in both papers.

(e) No, beacause all points can not be on the same line

(f) $m_x = 66$ $\qquad m_y = 84$

(g)



- For the first region the median of the *x*-coordinates is 54 the median of the *y* coordinate is 70. Therefore first median point is (54,70)
- For the second region the *x*-coordinate 9 is 66

  The y-coordinate is 85

  Therefore the second median points is (66, 85)
- For the region 3 *x* coordinate (*x*-median) is 83.

  The y-coordinate (*y*-median) 99.

  There third median point is (83,99).

  Ensure that the learners are able to plot the points, identify the median coordinates and divide the graph into the three regions as described in the activity and demonstrated in the graph above.

  (i) $m_{x_1} = 54$, $m_{y_1} = 70$ $m_{x_2} = 66$, $m_{y_2} = 85$, $m_{x_3} = 83$ $m_{y_3} = 99$

  (ii) Coordinates: (54,70), (66,85) (83,99)

  (iii) The line does not pass through the three points but we can draw an approximate line to represent the three points.

Attention to Special Needs

- You should be able to identify possible special needs with your learners. Such needs could be physical, psychological or emotional. Such needs could include challenges in hearing, sight, lack of concentration, sadness etc. Identify such and take appropriate action.
- Special need could be a specially gifted child or even a slow learner. Ensure that you move with your whole class.
- For the fast learners, provide extra work so as not to bore them.

For the slow learners, arrange for remedial program so that all feel wanted, loved and cared for.

List of lessons

Lesson No.	Lesson Title	No. of Periods
	Review of statistical data	
1	Bivariate data	2
2	Presentations of bivariate data	2
3	Scatter diagram the best-line of fit	3
4	Use the median averaging method to draw the best line of fit	2
5	Correlation between bivariate data	3
6	Unit assessment	

Lesson 1: Bivariate data

Lesson objective

By the end of the lesson, the learner should be able to define and identify bivariate data and formulate examples of the same and present them in tables based on information obtainable in class.

Teaching Aids

- Dictionaries (English/Mathematics)
- Mathematics sets.
- Calculator/electronic materials
- Graph books/paper
- Board with square grid

Introduction to the lesson

To introduce this lesson, remind the learners of the concepts and skills developed and achieved in their earlier work on Statistics in S1 unit 8, S2 unit 10.

Make the introduction a quick and brief session and relevant to this lesson.

Learning Activities

Activities 13.1 and 13.2

To introduce bivariate data encourage the learners to use dictionary or any other means to define the term bivariate.

- Organise class to work in groups to do Activities 13.1 and 13.2 Student's Book.
- Ask the groups to present their findings and observations in a class discussion.
- Summarise their findings in a class discussion emphasizing on the key points and their meanings i.e. bivariate data, collecting and recording such data. Help learners to identify bivariate data using examples in their class.
- Help them to formulate and present such data in tables of values.
- In a class discussion, take learners through the analysis of the example given in the pupil's book after the activity.

Synthesis

- In a class discussion, use the learners observations to highlight the key concepts or properties of bivariate data i.e. the two variates belong to the same entry etc.
- Ask the learners to think of examples of bivariate data that give information about each member of their class i.e. age (years) and mass (kg); marks % in two subjects etc. Encourage learners to arrange such data in separate tables of values etc.

Answers to the Activities 13.1 and 13.2

For every member of your class, the following data can be obtained:

(a) Height (x), and mass (y)

(b) Age (x) and height (y)

(c) Marks scored in Mathematics (x) and marks scored in physics (y) or any other pair of subject other sources of such data could be

- The amount of rainfall (x) and the group of the vegetation grass).
- Height of son (x) and height of father (y) over a period of time.
- A person's age (x) and blood pressure (y) for a group of people of different age groups.

These are just a few examples. You can now think about some more examples. Note that answers to these two activities will vary.

Assessment

Ask the learners to collect sets of bivariate data concerning members of the class. Let them work individually so that you are able to tell anybody having challenges with the concept.

Answers to the assessment will depend on the information gathered.

Lesson 2: Presentation of data

Lesson objective

By the end of the lesson, the learner should be able to represent bivariate data.

Teaching Aids

- Graph books/papers
- Board with square grid
- Pen/pencil/ruler

Introduction to the lesson

Introduce the lesson by asking the learners what they require to produce a good accurate graph.

Learning Activities

Activity 13.3

- Organise learners in groups to do Activity 13.3 in the Student's Book.
- Remind them that a good scale appropriate scale and correct plotting produces a good graph.
- Ask the learners to present their findings to the rest of the class and encourage them to ask questions and also to point out mistakes or errors. Allow them to demonstrate their observations on the board.
- Since the graph is neither a line or a curve, probe the learners about the name of the graph. See if they can come up with the name 'scatter diagram' by relating it to the scattered pattern of the points.

Synthesis

- Take the learners through the analysis immediately following Activity 13.3 explaining the points given.
- In a class discussion, take learners through the observations made from Activity 13.3
- In a class discussion take the learners through Example 13.1 in the Student's Book.

Answers to Activity 13.3

The required ordered pairs are (2, 1) (4,4) (3,1) (7,5) (5,6) (2, 1) (4,4) (3,1) (7,5) (5,6)

H. Scale 1cm :1 unit

V Scale 1cm : 1 unit

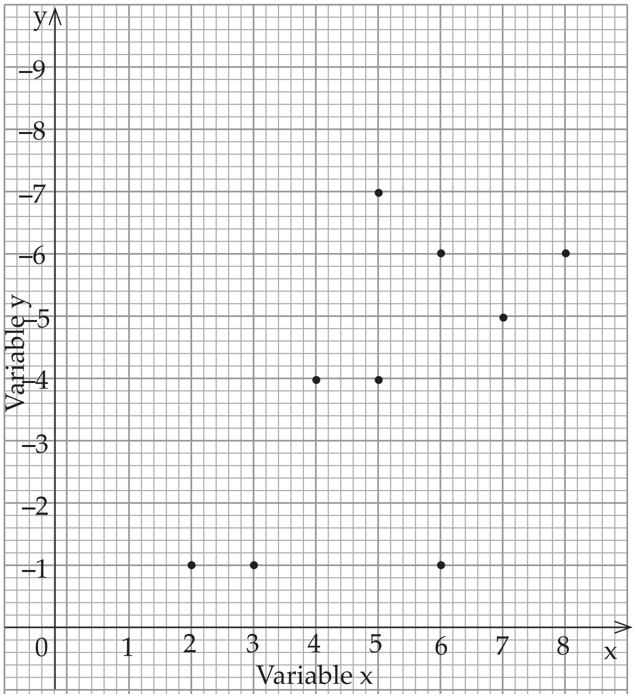


Figure 13.2

The points on the graph are scattered around and displays no pattern. It makes no sense to join the points because they can not be on the same straight line. a graph like this is called a **scatter diagram** or a **scatter plot**.

Assessment

Ask learners to do Exercise 13.1 in the Student's Book.

Lesson 3: Definition of scatter diagrams

Lesson objective

By the end of the lesson, the learner should be able to represent bivariate data, draw and use a line of best fit.

Teaching Aids

- Square grid board.
- Mathematical instruments.
- Graph book/paper.
- Chalk board ruler,.

Introduction to the lesson

Since points in a scatter diagram contains scattered points, it means to represent such points by a line means to approximate them into a line. Ask learners to explain what it means to approximate. Use their suggestions to explain its meaning and explain why and how it is done.

Learning Activities

Activities 13.4 and 13.5

- Organise the learners into groups to do Activities 13.4 and 13.5 in the Student's Book. Ensure that each member of each group is able to plot ordered pairs.
- Ask groups to present their observations to the rest of the class. Use the learner's observation/presentations to describe the resulting graphs accurately, describe the trend or pattern of the points.
- Help the learners to determine what group(s) of society would be interested in such findings.
- Ask the same groups to do Activity 13.5 in the Student's Book.
- Ask the learners to use the graphs they have just drawn in Activity 13.4. Move round the room to ensure that everybody is doing the activity.
- Ask groups to demonstrate their findings on the board as they explain. Use this opportunity to ensure that the graphs are as accurate as they can possibly be.

Synthesis

- In a class discussion guide the learners through the Activities 13.4 and 13.5. Discuss the learners' observations, affirming the correct ones, and taking time to correct the erroneous ones.
- Discuss the summary that follows immediately after Activity 13.4. Discuss also the observation that follows immediately after Activity 13.5.
- In a class discussion take learners through Example 13.2 in the Student's Book.

Assessment

Ask the learners to do Exercise 13.2 in the Student's Book.

Answer to Activity 13.4

Fig 13.2 shows the required graph. The points are scattered along a line but no meaningful conclusion can be drawn since the point display no pattern.

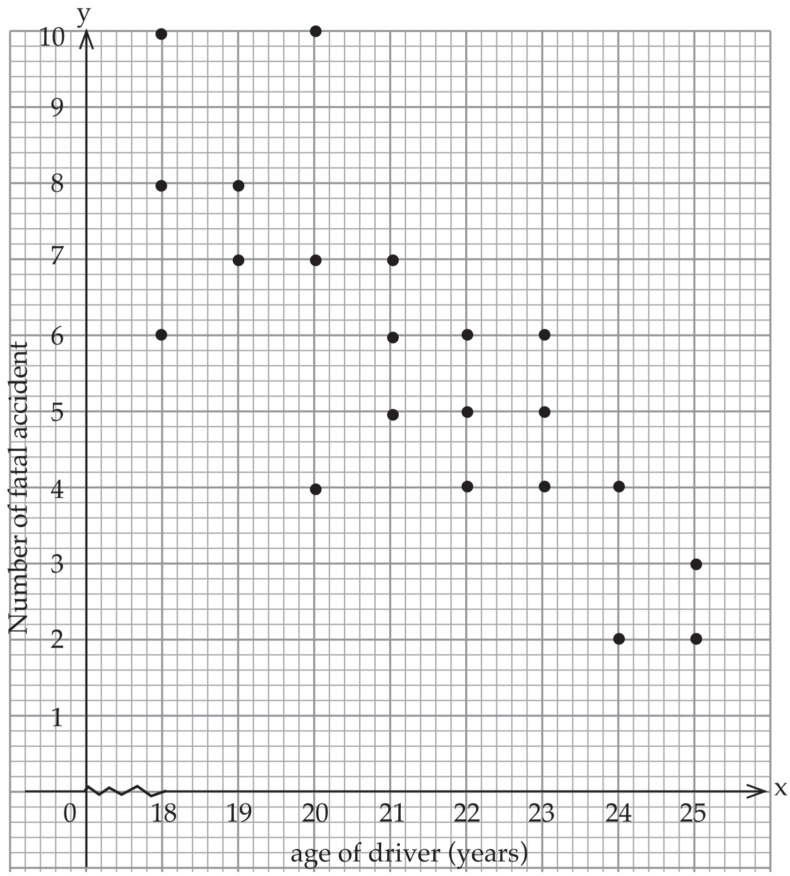


Fig 13.2

Traffic police, motor insurance agents, driving instructors etc. might be interested in the findings of this activity to be able to establish any relationship between the data, we need to analyze the data further.

Answer to Activity 13.5

There are no specific answers to this activity.
The activity is best demonstrated on the chalk board. This activity forms a very good class discussion in which every learner can participate in observing the trend and draw the conclusions from the graph.

Demonstrate this activity on the chalk board especially the estimation of the line of best fit.

Lesson 4: Best line of fit using the median method

Lesson objective

By the end of the lesson, the learner should be able to calculate medians and quartiles of bivariate data and use them to estimate the line of best fit.

Teaching Aids

- Calculator
- Graph book/paper
- Mathematical set

Introduction to the lesson

Introduce the lesson by asking the learners to explain how to find median and quartiles of ungrouped data. Help those who have challenges in the concept so that you may move through the lesson with all of them.

Learning Activities

Activity 13.6

- In a class discussion guide the learners through Activity 13.6 as guided in the Student's Book.
- This activity requires that you ensure that the learners are doing the calculations suggested on their books.
- Once the calculations are done and the points plotted ask the learners to report their findings to the rest of the class.
- Verify that the coordinates and the graphs are accurately done.
- Summarise the group findings emphasizing the fact that no apparent pattern can be identified, no meaningful conclusions can be drawn in a graph of this nature. Hence the need to engage other approaches or alternative methods that can be used.

Synthesis

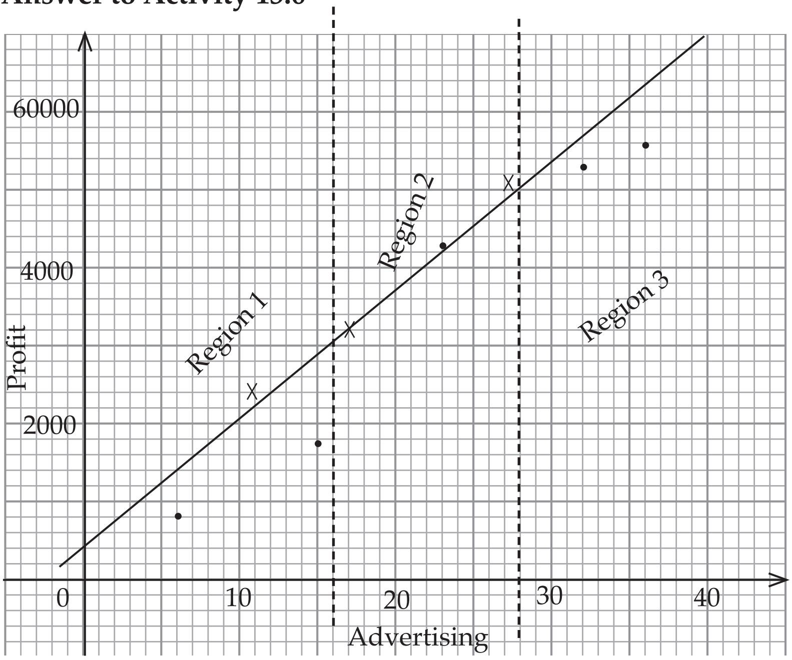
- Having done the activity, take learners through the findings of the learners in a class discussion.
- This activity helps the learners to apply some concept they learned earlier in this course. It gives you an opportunity to emphasize how different concepts relate with other concepts that seem not related.
- Discuss the analysis given immediately after the two activities.

Note: The line is also an approximation. It does not exactly pass through the three points.

- Take the learners through Examples 13.3 and 13.4 in Student's Book

The graph shows an upward trend and the line of best fit has a positive gradient. This means that the more the money spent on advertising, the higher the profit.

Answer to Activity 13.6



For the first region the median of the *x*-co-ordintes of 30, 15 and 17 is 17 the median of the *y* coordinate 4040, 1740, 3250 is 3250, the first median point is (17, 3250).

- For the second region the x coordinate of 7, 32 and 9 is 9
- The y-co-ordinates of 880, 5570 and 2090 is 2090 , the third median point is – (26, 4990).

Assessment

Ask learners to do Exercise 13.3 in the Student's Book.

Lesson 5: Correlation

Lesson objective

By the end of the lesson, the learner should be able to define the word correlation, distinguish between direct/positive and negative/inverse correlation.

Teaching Aids

- Chalkboard with square grid
- Graph book/paper
- Access to internet or dictionary.
- Long ruler
- Mathematical instruments

Introduction to the lesson

Introduce the lesson by asking the learners to use their dictionaries or any other reference to find the meaning of the word correlation. Ask the learners to suggest how to obtain correlation and what use it is.

Use their responses to explain the meaning of the word, what is it, who uses it etc.

This question may motivate the learners to want to learn about it.

Learning Activities

Activities 13.7 and 13.8

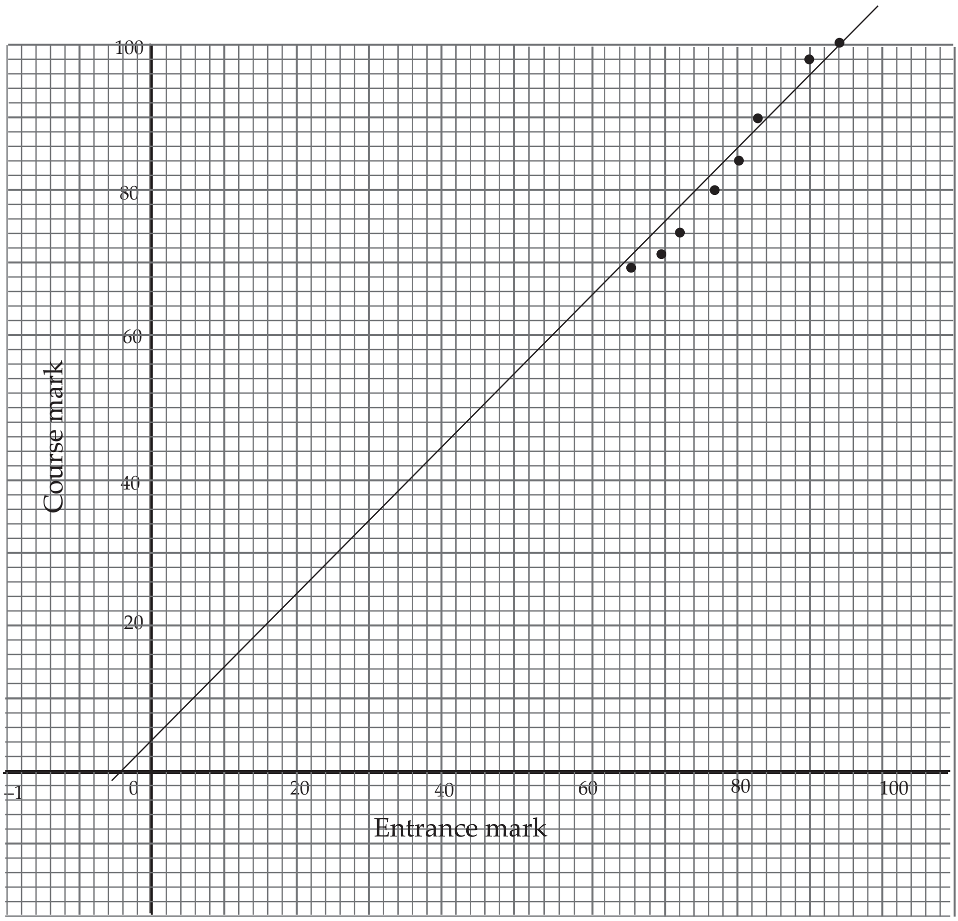
- Organise the learners in groups to do the Activity 13.7 in the Student's Book .
- Ensure that all the participating actively by moving round the class, affirming those who are doing well and helping those who are challenged.
- When the activity is done, and the line of best fit drawn, ask the learners to connect on the resulting graph. This part is best done as a class discussion so that all may get a chance to contribute and compare notes.
- Demonstrate the graph on the board so that the learners may compare their graphs with those of other members of the class and therefore identify any possible error.
- Ask the learners to summarise their findings in a class discussion. Use this opportunity to clarify and emphasize the key points.
- Discuss with the learners Activity 13.8 in the Student's Book.

Synthesis

- Having done the activity, guide the learners through the discussion following behind the activity. Analyse the graph and describe the correlation evident in the graph.
- Define corrrelation and give real life examples where correlation is applied for the learners to digest the concepts
- Activity 13.8 can be used to make learners understand the types of correlation ie. positive and negative correlation.
- Take learners through Example 13.5 in the Student's Book in a class discussion. This example will help the learners to understand fully the concept of correlation. You can also guide them through Example 13.6 Student's Book.

Answers to Activity 13.7

According to the concise oxford English dictionary, the word correlation means a mutual relation between two or more things. It shows or describes interdependence of variable quantities.



Corresponding sets of marks compare well the representing nearly the same abilities. However the set of marks for the first candidates represent very different abilities. On the graph, the point representing the set of marks is very far from the rest of the points.

To be able to compare given variables it is necessary to be able to do the following: -

- Plot points on a Cartesian plane.
- Draw the line of best fit if points are not collinear.
- Describe the trend of graphs and analyze possible conclusions.

These points are fully discussed in the Student's Book under the same activity.

Answers to Activity 13.8

(a) Positive correlation
(b) Positive correlation
(c) Negative correlation
(d) Negative correlation
(e) Negative correlation

Assessment

Ask learners to do Exercise 13.4 in the Student's Book.

Lesson 6: Unit Assessment

Lesson objective

By the end of the lesson, the learner should be given summary questions in order to assess how well the concepts and skills have been developed and achieved.

Summary of unit

Summarise the unit by:

- An oral exercise from all the areas of the unit. This will help the learners to recall the concepts correctly.
- Prepare written quiz that covers all areas of the unit. This will help you to assess to determine to what extent the objectives of the unit have been achieved.
- Take the whole class through the different concepts highlighted in the unit summary given in the Student's Book. Take the learners through a question and answer method to help them recall the learnt concepts correctly.
- Ask the learners to describe the solutions to problems they faced at the beginning of the unit. Let them discuss and solve the unit focus activity stated at the beginning of this unit. Ask them probing questions to gauge their understandings on this topic.

Unit assessment

Ask the learners to do the Unit 13 Test at the end of the unit. Mark their work to be able to get their areas of weaknesses and strengths. From the result, give extra remedial exercise to the weaker students while giving the extended exercise to the learners who are comfortable with the rest of the exercise.

Additional information to the teacher

The activities provided in the Student's Book in this unit are not exhaustive. In the interest of time, only data that is readily and easily available in the learners environmental could be used. You can organise learners out of class time, as in a mathematics clubs, to do some projects to enhance the statistical processes. Let the learners understand that the concepts involving statistics are used in many fields in everyday life.

For example,

- How can a manufacturer predict for which product there will be an increase in demand?
- How does a couch determine a winning combination of the team players?
- How does a fast food company determine the best location for a new restaurant?

These and many other situations use statistics to plan appropriately. Below find some processes used to expand statistical concepts;

- It is important for learners to know of different methods of use to collect data ;
- Personal interviews (door to door, at the shopping malls
- Polls conducted by telephone
- Questionnaires
- Experiments designed to collect data
- Measurements taken to check quality of products
- Data from past records etc.

You can also ask learners to suggest some other possible cases where they think statistics can be used.

After collection and display and data analysis we can use the analysis to interpret data in a useful way.

Economics and business graphs help us to forecast and make predictions as we did with scatter diagrams, and best line of fit among other processes. Let learners know that of two sets data may be related but no pattern may be obvious.

As we did with scatter diagram, we can analyse data recording them as ordered pairs and drawing a graph that may display a trend.

In such a case we can go a step further and use the resulting graph to estimate a line of best fit.

In statistics as in many other fields especially in mathematics we continue building new skills on the earlier learned skills.

Remedial Activity

1. Ten people were asked a set of test questions designed to measure their attitude towards television as a news medium, and a further set to measure their attitude to newspapers. A higher overall score shows greater satisfaction. The scores are shown in the table below.

Person	1	2	3	4	5	6	7	8	9	10
TV Score (x)	5	0	3	1	2	4	5	3	5	4
Newspaper score (y)	1	2	1	3	3	2	3	1	0	2

   (a) Use the scores (x,y) to draw a scatter diagram to represent the scores.

   (b) Use the scatter diagram to draw the best line of fit to represent this data.

   (c) Is there a correlation between the TV score and the newspaper scores? Describe the type of correlation.

2. For the sets of data in this question,

   (a) Draw a scatter diagram.

   (b) Use the scatter diagram to assess whether there appears to be correlation between the two measurements labelled x and y. The data represents the latest available figures of vehicles and road deaths in a group of ten African countries.

Country	Vehicles per 100 population (x)	Road deaths per 100,000 population (y)
1	31	14
2	32	29
3	30	22
4	47	32
5	30	25
6	19	20
7	36	21
8	40	22
9	47	30
10	58	35

Extended Exercise

1. The mass (kg) and average daily food consumption were measured for 12 obese adolescent girls and boys.

Mass (kg) x	84	93	81	61	95	86	90	78	85	72	65	75
Food consumption (y) (100 calories per day)	32	33	33	24	39	32	34	28	33	27	26	29

(a) Use the data to draw a scatter diagram.

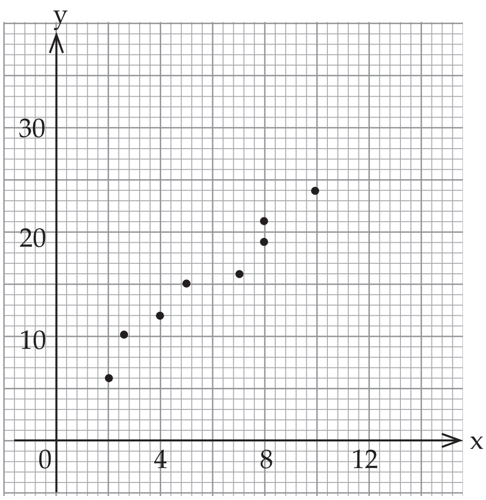
(b) Use the scatter diagram to assess whether there appear to be a correlation between x and y.

(c) Describe the correlation.

Answers

Exercise 13.1

1.



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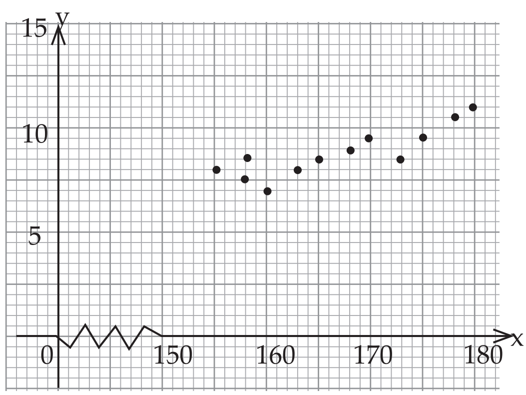
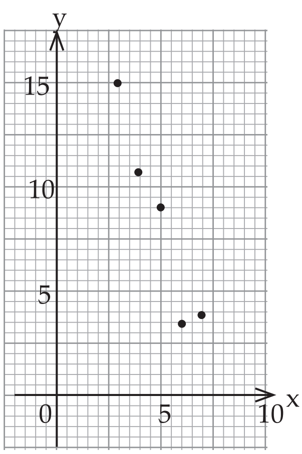
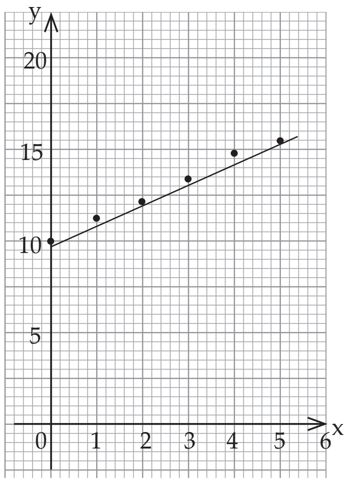


3.

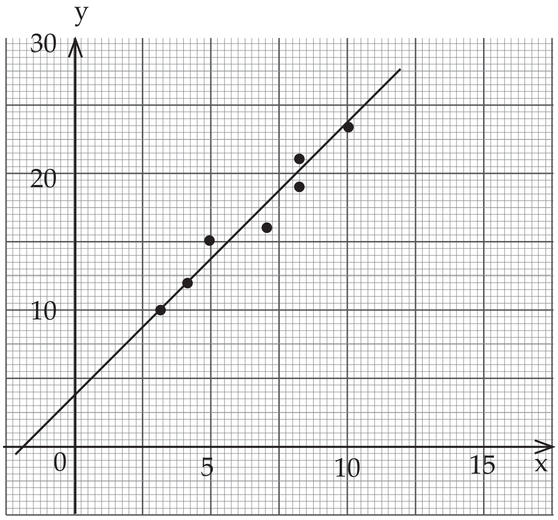


4.

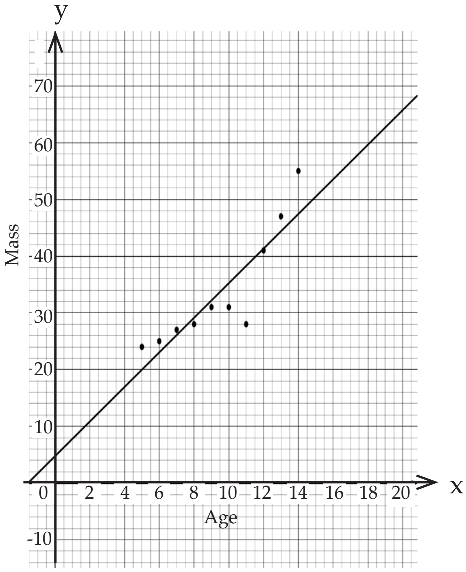


Exercise 13.2

1.



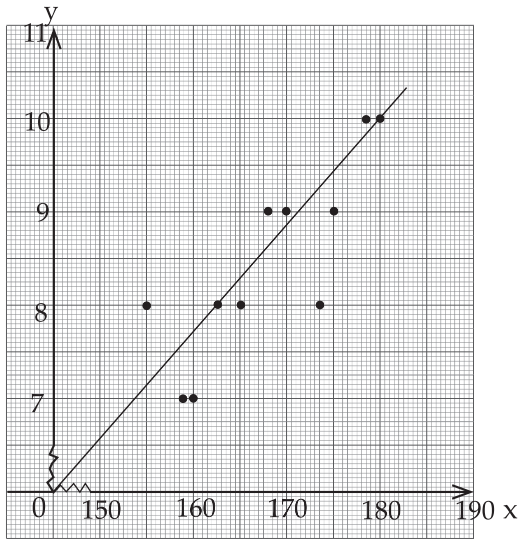
2.



(b) The line has a positive gradient

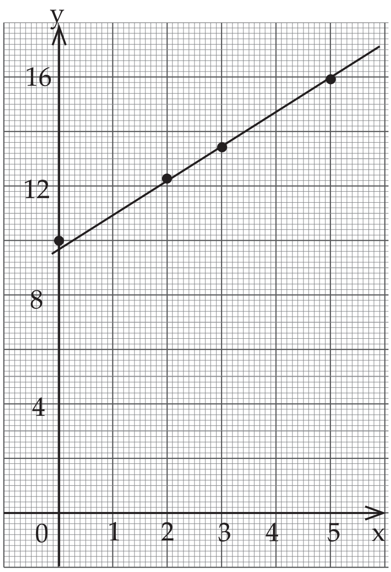
$5y = 19x - 12$

3.



171cm tall ⇒ shoe size 9 (By approximation)

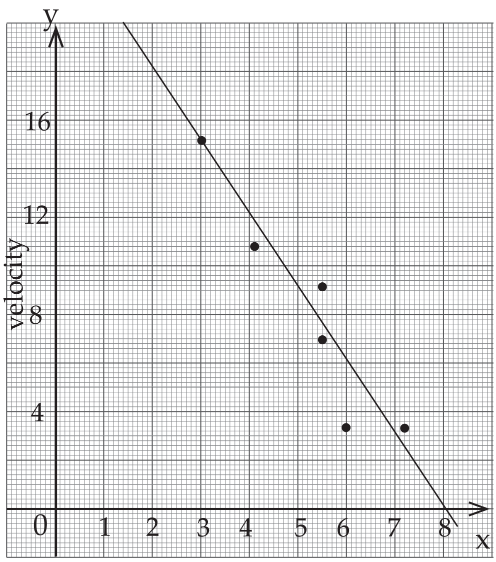
4. (a)



(a) $y = \frac{2.3}{2}x + 10$

(b) 7.1 N

5.



velocity

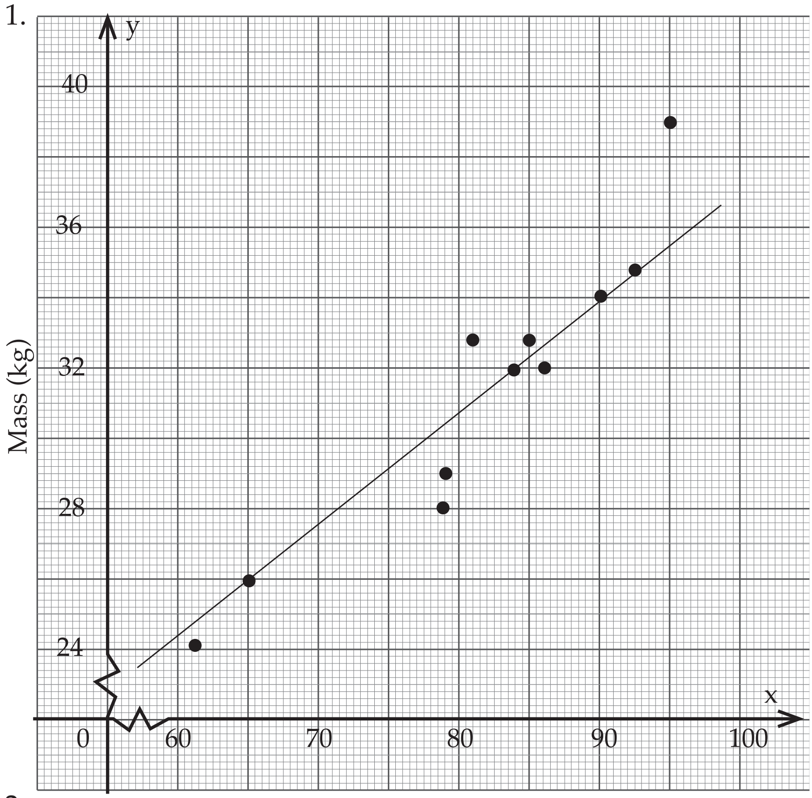
(a) 0.2 volts

(c) gradient is negative

(d) $y = -8x + 39$

Exercise 13.3

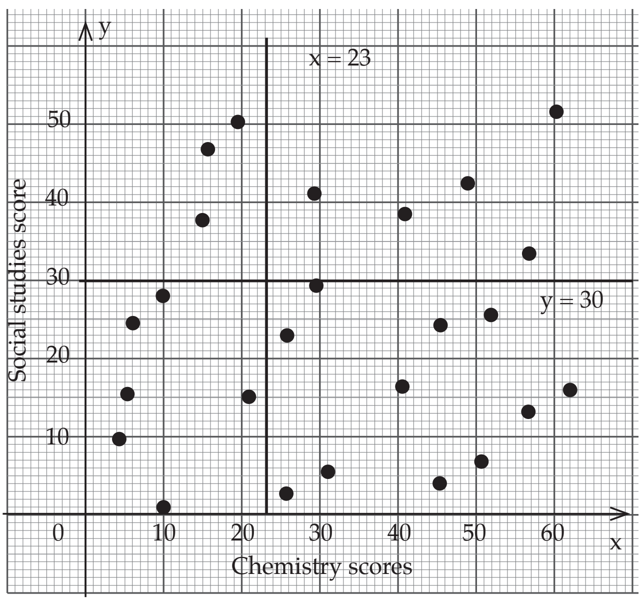
1.



(c) line has a positive gradient

Food consumption (x 100 cal. per day)

2.



Both subjects are poorly performed.

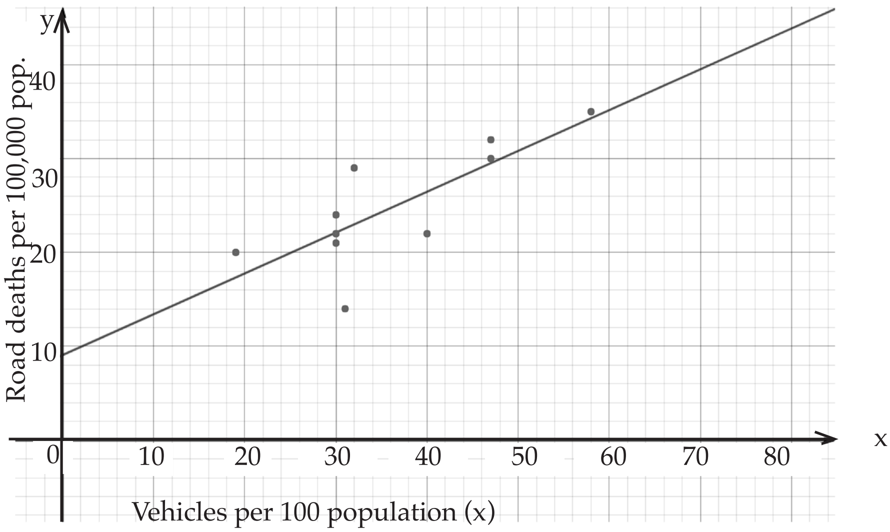
Less than a third of the class scored above average in both subject.

Exercise 13.4

Note some answers in this question will vary

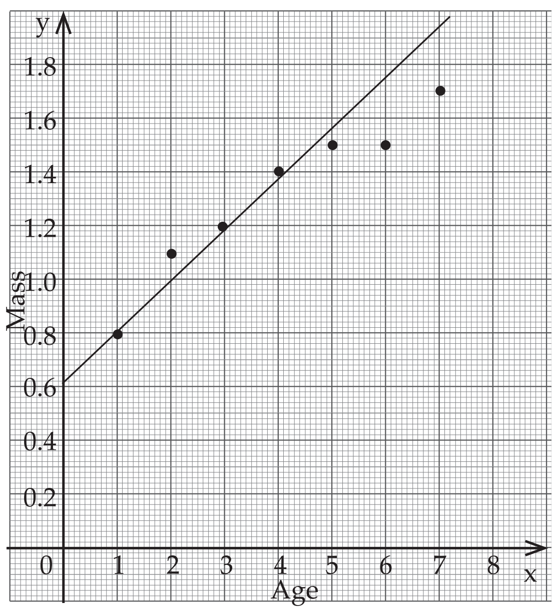
Question 1, 2, 3, 5 have positive correlation i.e gradient of lines of best-fit is positive question 4 has negative correlation since the gradient of line of best fit in negative.

1.



Data have positive correlation.

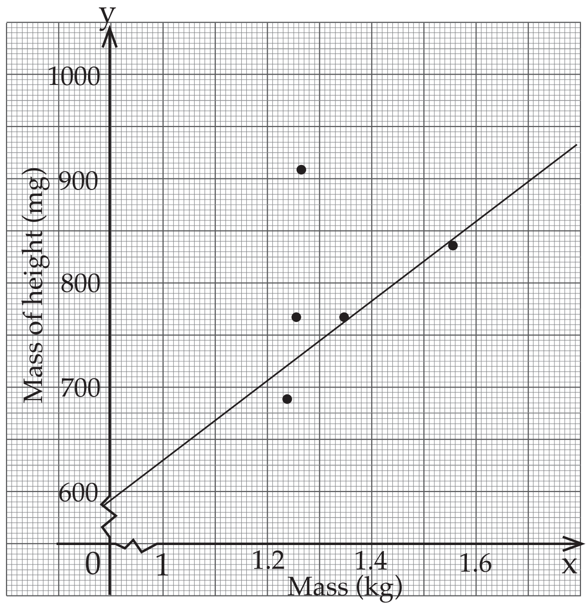
2.



(c) Mass of rabbit 8 weeks old ≈ 2.05

(d) The data have a positive correlation.

3.



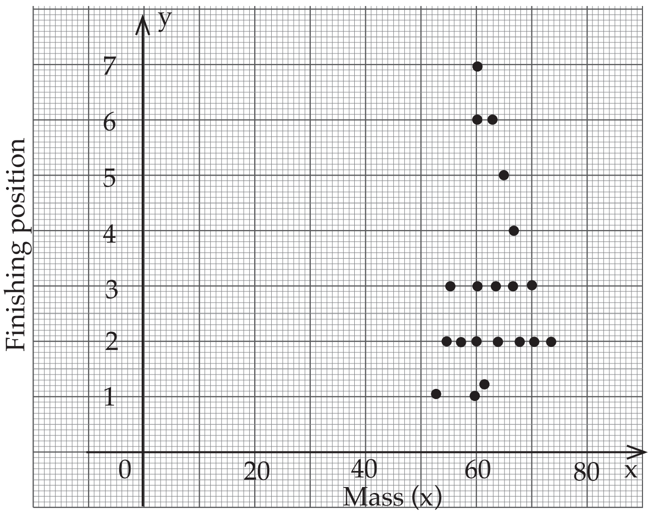
(b) Ignore the extreme point on the diagram then draw the market best line of fit.

(c) $y = \frac{150}{1.3}x + 600$

(d) Substitute the appropriate variable in the equation.

(e) Positive correlation.

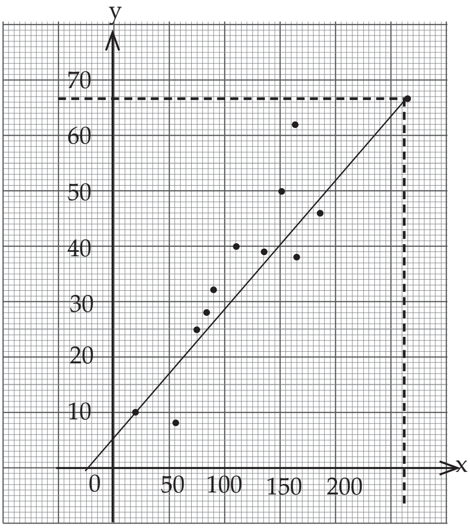
4.



No correlation between the sets of data.

5. The graph below shows the required scatter diagram and the line of best fit.

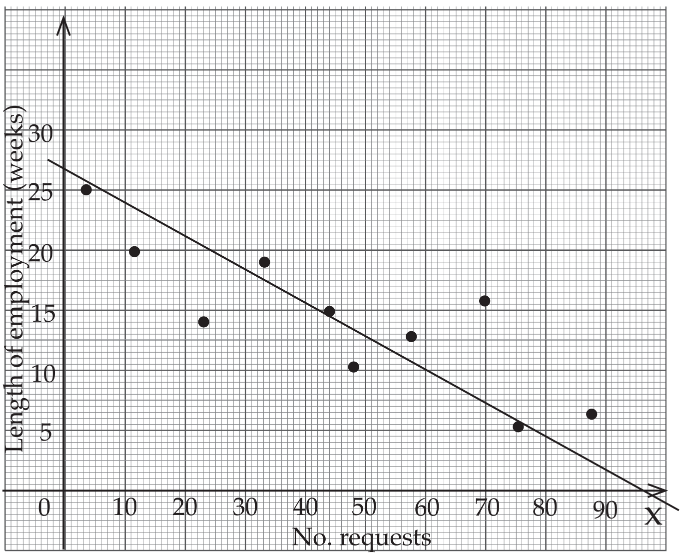
(a)



(b) There is positive correlation.

(c) 213 minutes (appropriately)

6.

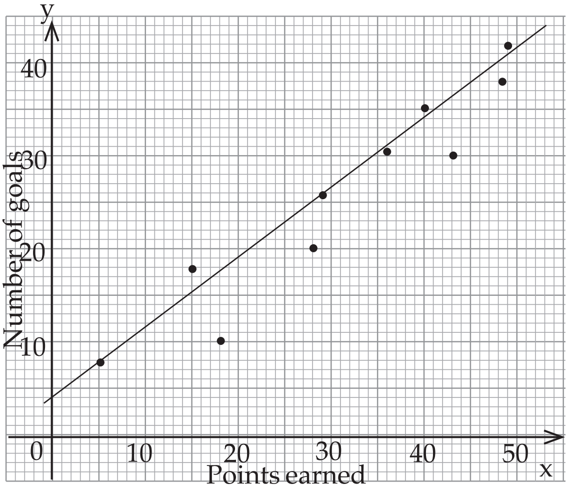


(b) negative correlation

(c) the longer the period of employment, the lesser the consultation ie.
E has worked for 3 weeks consulted 25 times.
J worked for 87 weeks consulted the least 6 times.

Unit Test

1. (a) The graph below shows the required scatter diagram and line of best fit.



b.(i)y = 29 FRW

(c) (i) 29 points (ii) 36 goals

(d) If a line runs from the bottom left hand corner to the top right hand corner, its means the vertical change is positive and the horizontal change is positive therefore $\frac{\text{vertical change}}{\text{horizontal change}}$ = positive change. Thus gradient is positive.

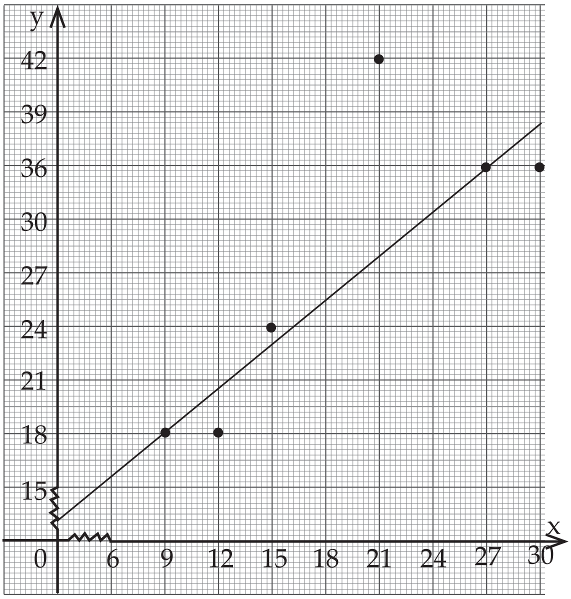
(e) 15y = 11x + 60

(f) Positive correlation

(g) This relationship means the more the goals scored, the higher the number of points earned therefore, the better the player.

2. Let x represent price in may 2016 and y represent the price in june 2016.

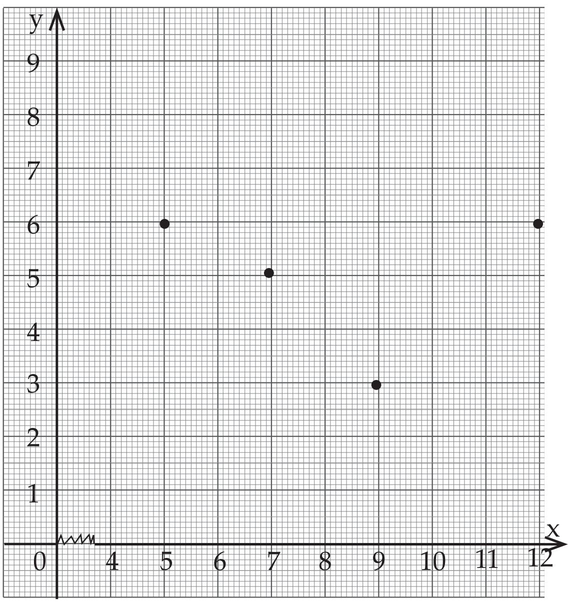
a). (i)$\overline{x}$ = 19 FRW b.(i) $\overline{y}$ = 29 FRW b).



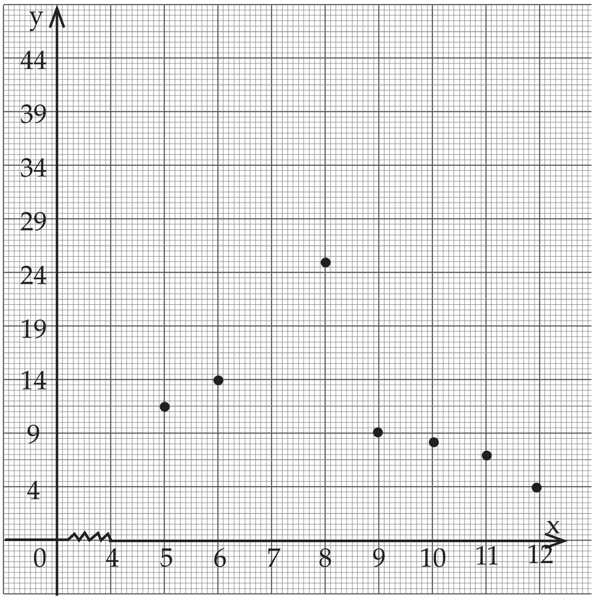
(c). There was an increase in the price for both months because of positive gradient.

3. a). (i) $\tilde{x} = 8.25$ (ii) $\tilde{y} = 5$

b).



4. a)



b) It is difficult to draw the line of best fit since all the points are scattered.

c) Negative correlation since the more the learners, the less the number of days they remain at school and vice versa.