USER GUIDE FOR PRACTICAL ACTIVITIES AND LABORATORY EXPERIMENTS

MATHEMATICS

SENIOR SIX (S6)

Kigali, 2022

FOREWORD

Dear teacher,

Rwanda Basic Education Board (REB) is honoured to present the user guide for Practical Activities and Laboratory Experiments in Mathematics for Senior Six(S6). This user guide will supplement competence-based teaching and learning, to ensure consistency and coherence in the learning of Mathematics.

In this user guide, special attention was paid to practical activities that facilitate the learning process in which students can manipulate concrete materials, develop ideas, and make new discoveries during activities carried out individually or in pairs/ small groups.

In a competence-based curriculum, practical activities open students' minds and provide opportunities to interact with the world, use available tools, collect data, and effectively model real-life problems.

For efficient use of this user guide, your role as a teacher is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize groups for students considering the importance of social constructivism.
- Engage students through active learning methods.
- Provide supervised opportunities for students to develop different competences by giving tasks which enhance critical thinking, problem-solving, research, creativity and innovation, communication, and cooperation.
- Support and facilitate the learning process by valuing students' contributions to the practical activities.
- Guide students towards the conclusion of the results of the experiments.
- Encourage individual, peer, and group evaluation of the work done and use appropriate competence-based assessment approaches and methods.

To facilitate your teaching activities, the content of this guide is self-explanatory so that you can easily use it. It is divided into three parts:

- Part I: Structure of this guide and the general introduction on the role of practical activities and laboratory experiments in the implementation of CBC.
- Part II: List of some Mathematics materials distributed to schools.
- Part III: Selected practical activities and laboratory experiments and how you can facilitate them in lessons.

Even though this guide contains practical activities and laboratory experiments, they are not enough; teachers can guide students to carry out more practical activities using improvised teaching resources.

I wish to sincerely extend my appreciation to the people who contributed towards the development of this guide; The African Institute for Mathematical Sciences, Teacher Training Program (AIMS - TTP) in partnership with Mastercard Foundation who provided technical and financial support and REB staff particularly those from the Mathematics and Science Subjects Unit in the Curriculum Teaching and Learning Resources Department who organized the wholeprocess from its inception.

Special appreciation goes also to teachers and independent experts in education who supported the exercise throughout the process. Any comment or contribution would be welcome for the improvement of this booklet for next versions.

Dr. MBARUSHIMANA Nelson

Director General, REB

ACKNOWLEDGEMENT

I wish to express my appreciation to the people who played a major role in the development and editing of the user guide for Practical Activities and Laboratory Experiments in Mathematics for Senior Six (S6). It would not have been successful without the active participation of different education stakeholders.

Special thanks are given to AIMS staff, IEE, Independent people, teachers, illustrators, designers and all other individuals whose efforts in one way or the other contributed to the success of the development of this user guide.

I owe gratitude to the Rwanda Basic Education Board staff particularly those from Mathematics and Science subjects Unit in the CTLR Department who were involved in the whole process of the development work of this user guide.

Finally, my word of gratitude goes to AIMS - TTP in partnership with Mastercard Foundation for their support in terms of human and financial resources towards the development of this guide which will strengthen STEM teaching hence improving the quality of education in Rwandan schools.

Joan MURUNGI

Head of CTLR Department

LIST OF ACRONYMS

AIMS : African Institute for Mathematical Sciences

CBC : Competence-Based Curriculum

ICT : Information and Communication Technology

KBC : Knowledge Based Curriculum

Lab : Laboratory

SET : Science and Elementary Technology

STEM : Science Technology Engineering and Mathematics

UR-CE : University of Rwanda- College of Education

TTP : Teacher Training Program

STRUCTURE OF THE USER GUIDE

This user guide for Practical activities and Laboratory experiments in Mathematics for Senior Four is divided into 3 parts:

Part I: General introduction on the role of practical activities and lab experiments in the implementation of CBC.

Part II: List of main Mathematics kit items distributed in schools.

Part III: Practical activities or laboratory experiments and how to facilitate them in lessons.

Table of Contents

FOREWORD .... **iii**

ACKNOWLEDGEMENT .... **v**

LIST OF ACRONYMS .... **vi**

STRUCTURE OF THE USER GUIDE .... **vii**

PART I: GENERAL INTRODUCTION .... **xi**

1.1. Background xi

1.2 Why Mathematics Practical Activities and Laboratory Experiments? xii

1.3 Type of laboratory experiments xiii

1.4 Organization, analysis, and interpretation of data xiv

1.5 Organising laboratory experiments xiv

1.6 Role and responsibilities of teacher, laboratory technician, and students in the laboratory experiment xvi

1.7 Safety rules, and precautions during lab experiments xvii

1.8. Guidance on the Management of lab materials: Storage Management, repairing and disposal of Lab equipment xxii

1.9. Student Experiment Work Sheet xxiv

1.10. Report template for students xxiv

PART II: LIST OF MAIN KIT ITEMS DISTRIBUTED IN SCHOOLS .... **xxvi**

PART III: PRACTICAL ACTIVITIES AND LABORATORY EXPERIMENTS FOR SENIOR SIX (S6) **1**

PRACTICAL ACTIVITY 1: Illustration of 8th roots of unit on the Argand diagram and determination of the length of the side and the area of the formed figure 2

PRACTICAL ACTIVITY 2: Plotting the curve of the Logarithmic function $f(x) = \log_2 x$ on graph paper 10

PRACTICAL ACTIVITY 3: Domain and range of logarithmic and exponential functions 14

PRACTICAL ACTIVITY 4: Explore a mortgage problem and decide the type of loan you can take from a bank 17

PRACTICAL ACTIVITY 5: Depreciation model of a banana. 22

PRACTICAL ACTIVITY 6: Estimation of the trigonometric number of an angle 26

PRACTICAL ACTIVITY 7: Using definite integral to determine the area of the right triangle made under a linear function 30

PRACTICAL ACTIVITY 8: Using the appropriate function and the definite integral properties, determine the area of a circle....36

PRACTICAL ACTIVITY 9: Using the appropriate function and the definite integral properties to determine the area of an ellipse 38

PRACTICAL ACTIVITY 10: Determining the shapes by rotating a graph of a function around the x-axis or y-axis. 41

PRACTICAL ACTIVITY 11: Using definite integral of the appropriate function to determine the volume of a cylinder 45

PRACTICAL ACTIVITY 12: Using the definite integral of the appropriate function to determine the volume of a cone. 48

PRACTICAL ACTIVITY 13: Using differential equations to explore the simple harmonic motion 53

PRACTICAL ACTIVITY 14: Using differential equations to explore the population growth 58

PRACTICAL ACTIVITY 15: Introduction to conics 62

PRACTICAL ACTIVITY 16: Explore how to practically draw an ellipse and identify its elements 67

PRACTICAL ACTIVITY 17: Determine the probability density function of a given situation 69

REFERENCE **73**

Annex 1: Name of commonly hazard symbols useful in the laboratory . 74

PART I: GENERAL INTRODUCTION

1.1. Background

To effectively implement a competence-based curriculum (CBC) Students should apply what they have learnt, in different situations by developing skills, attitudes, and values in addition to knowledge and understanding. This learning process is learner-focused, where a learner is engaged in active and participatory learning activities, and Students finally build new knowledge from prior knowledge. Since 2015, the Rwanda Education system has changed from knowledge-based competence (KBC) to CBC for preparing students that meet the national and international job market requirements and job creation. Therefore, implementing the CBC education system necessitates qualitative laboratory practical works for mathematics and science as more highlighted aspects.

In addressing this necessity, laboratory experiments play a major role. A child is motivated to learn mathematics by getting involved in handling various concrete manipulatives in various activities. In addition to activities, games in mathematics also help the child's involvement in learning by strategizing and reasoning.

For learning mathematical concepts through the above-mentioned approach, child-centred Mathematics kits have been developed for the students of primary and secondary schools. The kits include various kit items along with a manual for performing activities.

The kit broadly covers the activities in the areas of algebra, geometry, trigonometry, and measurement.

The kit has the following advantages:

- Availability of necessary and common materials in one place
- Multipurpose use of items
- The economy of time in doing the activities
- Portability from one place to another
- Provision for teacher's innovation
- Low-cost material and use of indigenous resources.

Apart from the kit, the user guide for laboratory and practical activities to be used by teachers was developed. This laboratory experiment user guide is designed to help mathematics and science teachers to perform high-quality lab experiments for mathematics and science. This user guide structure induces learners' interest, achievement, and motivation through the qualitative mathematics and science lab experiments offered by their teachers and will finally lead to the targeted goals of the CBC education system, particularly in the field of mathematics and science.

In CBC, students hand on the materials and reveal the theory behind the experiment done. Here, experiments are done inductively, where experiments serve as an insight towards revealing the theory. Thus, the experiment starts, and theory is produced from the results of the experiment.

1.2 Why Mathematics Practical Activities and Laboratory Experiments?

Mathematics plays an important role in our daily activities. It provides the vital underpinning of the knowledge of the economy. Mathematics is essential in the physical sciences, technology, business, financial services and many areas of ICT (Roohi, 2015). As a the basis of most scientific and industrial research and development, the teaching and learning of mathematics has to be given much attention by utilizing all possible means to help students to acquire knowledge, skills and understanding of different concepts that are mostly abstract.

The concept of a mathematics laboratory has become very popular in recent years due to its important role in clarifying abstract concepts using real materials. The Mathematics laboratory is a room wherein we find a collection of different kinds of materials and teaching/learning aids, needed to help the students understand the concepts through relevant, meaningful, and concrete activities. These activities may be carried out by the teacher or the students to explore the world of mathematics, to learn, discover and develop an interest in the subject (Maheshwari, 2018).

The majority of students view mathematics as a dull, boring, and stereotyped subject. They think that mathematics is about getting the right or wrong answer. When they get it wrong, they think that they are not good enough for Mathematics and lose interest in learning. Mathematics laboratory helps students understand the principal idea behind Mathematics concepts. Although Mathematics is not an experimental science in the way in which physics, chemistry and biology are, a Mathematics laboratory can contribute greatly to the learning of mathematical concepts and skills. The benefit of using Mathematics laboratories in teaching and learning among others (Adenegan & Balogun, 2010):

- Arousing interest and motivating learning.
- Cultivating favourable attitudes towards mathematics.
- Enriching and varying instructions.
- Encouraging and developing creative problems solving ability.
- Allowing for individual differences in the manner and speed at which students learn.

- Making students see the origin of mathematical ideas and contributing to mathematics innovation.
- Allowing students to engage in the doing rather than being passive observers or recipients of knowledge in the learning process.

In the Mathematics laboratory, the activities help students to visualize, manipulate and reason. They provide an opportunity to make conjectures and test them and generalize observed patterns. Students learn to deal with problems while doing a concrete activity, which lays down a base for more abstract thinking.

1.3 Type of laboratory experiments

The goal of the experiment defines the type of experiment and how it is organized. Therefore, before doing practical work, it is important to have a clear idea of the objective.

The three types of practical works that correspond with its three main goals are:

1. **Equipment-based practical works:** the goal is for students to learn to handle scientific equipment like using a compass, a set square, a thermometer, a protractor, etc.
2. **Concept-based practical works:** the goal is to clarify the new concepts.
3. **Inquiry-based practical works:** the goal is for students to learn process skills. Examples of process skills are the following: defining the problem and good research question(s), installing an experimental setup, observing, measuring, processing data in tables and graphs, identifying conclusions, defining limitations of the experiment, etc.

Note:

- To learn the new concept through practical work, the lesson should start with the practical work, and the theory can be explained afterwards (explore – explain). Starting by teaching the theory and then doing the practical work to prove what they have learned is demotivating and offers little added value for student learning.
- Try to avoid complex arrangements or procedures. Use simple equipment or handling skills to make it not too complicated and keep the focus on learning the new concept.
- The experiments should be useful for all Students and not only for aspiring scientists. Try to link the practical work as much as possible with their daily life and preconceptions.

1.4 Organization, analysis, and interpretation of data

Once data are collected, they must be ordered in a form that can reveal patterns and relationships and allows results to be communicated to others. We list goals for analysing and interpreting data.

By the end of secondary education, students should be able to:

- Analyze data systematically, look for relevant patterns or test whether data are consistent with the initial hypothesis.
- Recognize when data conflict with expectations and consider what revisions in the initial model are needed.
- Use spreadsheets, databases, tables, charts, graphs, statistics, mathematics, and ICT to compare, analyze, summarize, and display data and explore relationships between variables, especially those representing input and output.
- Evaluate the strength of a conclusion that can be inferred from any data set, using appropriate grade-level mathematical and statistical techniques.
- Recognize patterns in data that suggest relationships worth investigating further. Distinguish between causal and correlational relationships.
- Collect data from physical models and analyze the performance of a design under a range of conditions.

1.5 Organising laboratory experiments

i. Methods of organizing a practical work

There are 3 methods of organizing practical work:

- **Each group does the same experiments at the same time**

All Students can follow the logical sequence of the experiments, but this implies that a lot of material is needed. The best group size is 3, as all Students will be involved. With bigger groups, you can ask to experiment twice, where Students change roles.

- **Experiments are divided among groups with group rotation**

Each group does the assigned experiment and moves to the next experiment upon a signal from the teacher. At the end of the lesson, each group has done every experiment. This method saves material but is not perfect when experiments are ordered logically. In some cases, the conclusion of an experiment provides the research question for the next experiment. In that case, this method is not very suitable.

The organization is also more complex. Before starting the lesson, the materials for each experiment should be placed in the different places where the groups will work. Also, the required time for each experiment should be about the same. Use a timer to show Students the time left for each experiment. Provide extra exercise for fast groups.

- **All experiments are divided among groups without group rotation**

Each group does only one or two experiments. The other experiments are done by other groups. Afterwards, the results are brought together and discussed with the whole class. This saves time and materials, but it means that each learner does only one experiment and 'listens' to the other experiments' descriptions. The method is suitable for experiments that are optional or like each other. It is not a good method for experiments that all Students need to master.

ii. Preparation of a practical work

When preparing practical work, do the following:

- Have a look at the available material at school and make a list of what you can use and what you need to improvise.
- Determine the required quantities by determining the method to apply (see above).
- Collect all materials for the experiments in one place. If the Students' group is small, they can come to get the materials on that spot, but it is better for each group to prepare a set of materials and place it on their desk.
- Test all experiments and measure the required time for each experiment.
- Prepare a nice but educational extra task for Students who are ready before the end of the lesson.
- Write on the blackboard how groups of Students are formed.

iii. Preparation of a lesson for practical work

In the lesson plan of a lesson with practical work, there should be the following phases:

1. The introduction of the practical work or the 'excite' phase consists of the formulation of a key question, discrepant event, or a small conversation to motivate Students and make connections with daily life and Students' prior knowledge.
2. The discussion of safety rules for the practical work. For example,
   - Students must work at the assigned place.
   - Long hairs should be tied together, and safety eyeglasses should be worn when dealing with chemical experiments.

- Only the material needed for the experiment should be on the table.

3. Set the practical work instructions: how groups are formed, where they get the materials, special treatment of materials (if relevant), what they must write down, etc.

4. Set how to conduct practical work:
   - Students do the experiments, while the teacher coaches by asking questions (Explore phase).
   - The practical work should preferably be processed immediately with an explain phase. If not, this should happen in the next lesson.

5. Set how to conclude the lesson of practical work:
   - Students refer to instructions and conduct the experiment,
   - Students record and interpret recorded data,
   - Cleaning the workspace after the practical work (by the Students as much as possible).

1.6 Role and responsibilities of teacher, laboratory technician, and students in the laboratory experiment

1.6.1. The roles and responsibilities of teacher during a laboratory experiment

Before conducting an experiment, the teacher will:

- Decide how to incorporate experiments into class content best,
- Prepare in advance materials needed in the experiment,
- Prepare protocol for the experiment,
- Perform in advance the experiment to ensure that everything works as expected,
- Designate an appropriate amount of time for the experiment - some experiments might be adapted to take more than one class period, while others may be adapted to take only a few minutes.
- Match the experiment to the class level, course atmosphere, and your students' personalities and learning styles.
- Verify the status of lab equipment before lab practices.
- Provide the working sheet and give instructions to Students during lab sessions.

During practical work, the teacher's role is to coach instead of helping with advice or questions. It is better to answer a learner's question with another question than to immediately give the answer or advice. The additional question should help Students to find the answer themselves.

- Prepare some pre-lab questions for each practical work, no matter what the type is.
- Try and start the practical work: start with a discrepant event or questions that help define the problem or questions that link the practical work with students' daily life or their initial conceptions about the topic.
- Use coaching questions during the practical work: 'Why do you do this?', 'What is a control tube?', ''What is the purpose of the experiment?', 'How do you call this product?', 'What are your results?' etc.
- Use some questions to end the practical work: 'What was the meaning of the experiment?', 'What did we learn?', 'What do we know now that we didn't know at the start?', 'What surprised you?' etc.
- Announce the end of the practical work 10 minutes before giving students enough time to finish their work and clean their space.

1.6.2. The Role of a lab technician during a laboratory-based lesson

In schools having laboratory technicians, they assist the science teachers in the following tasks:

- Maintaining, calibrating, cleaning, and testing the sterility of the equipment,
- Collecting, preparing, and/or testing samples,
- Demonstrating procedures.

1.6.3. The students' responsibilities in the laboratory work

During the lab experiment, both students have different activities to do; the table barrow summarizes them. General learner's activities are:

- Experiment and obtain data themselves,
- Record data using the equipment provided by the teacher,
- Analyse the data often this involves graphing it to produce the related graph,
- Interpret the obtained results and deduct the theory behind the concept under the experimentation,
- Discuss the error in the experiment and suggest improvements,
- Cleaning and arranging material after a lab experiment.

1.7 Safety rules, and precautions during lab experiments

Regardless of the type of lab you are in, there are general rules enforced as safety precautions. Each lab member must learn and adhere to the rules and guidelines set, to minimize the risks of harm that may happen to them within the working environment. These encompass dress' code, use of personal protection equipment, and general behaviour in the lab. It is important to know that some laboratories contain certain inherent dangers and hazards. Therefore, when working in a laboratory, you must learn how to work safely with these hazards to prevent injury to yourself and other lab mates around you. You must make a constant effort to think about the potential hazards associated with what you are doing and think about how to work safely to prevent or minimize these hazards as much as possible. Before doing any scientific experiment, you should make sure that you know where the fire extinguishers are in your laboratory, and there should also be a bucket of sand to extinguish fires. You must ensure that you are appropriately dressed whenever you are near chemicals or performing experiments. Please make sure you are familiar with the safety precautions, hazard warnings, and procedures of the experiment you perform on a given day before you start any work. Experiments should not be performed without an instructor in attendance and must not be left unattended while in progress.

1.7.1 Hygiene plan

A laboratory is a shared workspace, and everyone has the responsibility to ensure that it is organized, clean, well-maintained, and free of contamination that might interfere with the lab members' work or safety.

For waste disposal, all chemicals and used materials must be discarded in designated containers. Keep the container closed when not in use. When in doubt, check with your instructor.

1.7.2 Hazard warning symbols

To maintain a safe workplace and avoid accidents, lab safety symbols and signs need to be posted throughout the workplace.

Chemicals pose health and safety hazards to personnel due to innate chemical, physical, and toxicological properties. Chemicals can be grouped into several different hazard classes. The hazard class will determine how similar materials should be stored and handled and what special equipment and procedures are needed to use them safely.

Each of these hazards has a different set of safety precautions associated with them. Annex 1 shows hazard symbols found in laboratories and the corresponding explanations.

1.7.3 Safety rules

Safety is the number one priority in any laboratory. All students are required to know and comply with good laboratory practices and safety norms; otherwise, they will be asked to leave the laboratory. Make sure you understand all the safety precautions before starting your experiments, and you are requested to help your Students to understand too.

The following are some general guidelines that should always be followed:

- **Lab coat**

While working in the lab, everyone must always wear a lab coat (Figure 1) to prevent incidental and unexpected exposures to the skin and clothing. The primary purpose of a lab coat is to protect against splashes and spills.

The lab coat must be wrist-fitted and must always keep buttoned.

A lab coat should be non-flammable and should be easily removed.

- **Safety glasses**

For eyes protection, goggles must always be worn over by all persons in the laboratory while students are working with chemicals. Safety glasses, with or without side-shields, are not acceptable.

The eyes protection safety indicates the possibility of chemical, environmental, radiological, or mechanical irritants and hazards in the laboratory.

– **Breathing Masks**

Respirators are designed to prevent contamination from volatile compounds that may enter in your body through the respiratory system. “Half mask” respirators (Figure3) cover just the nose and mouth; “full face” respirators cover the entire face, and “hood” or “helmet” style respirators cover the entire head.

The breathing mask safety sign lets you know that you are working in an area with potentially contaminated air.

– **Eye Wash Station**

Eyes wash stations consist of a mirror and a set of bottles containing saline solution that can be used to wash the injured eye with water. The eye wash station is intended to flood the eye with a continuous stream of water.

Eyes wash stations provide a continuous, low-pressure stream of aerated water in laboratories where chemical or biological agents are used or stored and in facilities where non-human primates are handled.

The eyewash stations should easily be accessed from any part of the laboratory, and if possible, located near the safety shower so that, if necessary, the eyes can be washed while the body is showered.

– **Footwear**

Shoes that cover entirely the toes, heel, and top of the foot provide the best general protection (Figure 1.5). Closed shoes must always be worn while in the laboratory, regardless of the experiment or curricular activity. Shoes must fully cover your feet up to the ankles, and no skin should be shown.

Socks do not constitute a cover replacement for shoes. Sandals, backless and open shoes are unacceptable.

– **Gloves**

When handling chemical, physical, or biological hazards that can enter the body through the skin, it is important to wear the proper protective gloves.

Butyl, neoprene, and nitrile gloves are resistant to most chemicals, e.g., alcohols, aldehydes, ketones, most inorganic acids, and caustics.

– **Hair dressing**

If hair is long, it must be tied back. It is good to report all accidents including minor incidents to your instructor immediately.

– **Eat and drink**

Never drink, eat, taste, or smell anything in the laboratory unless you are allowed by the lab instructor.

– **Hot objects**

Never hold very hot objects with your bare hands.

Always hold them with a test tube holder, tongs, or a piece of cloth or paper.

1.8. Guidance on the Management of lab materials: Storage Management, repairing and disposal of Lab equipment

Keeping and cleaning up

Working spaces must always be kept neat and cleaned up before leaving. Equipment must be returned to its proper place. Keep backpacks or bags off the floor as they represent a tripping hazard. Open flames of any kind are prohibited in the laboratory unless specific permission is granted to use them during an experiment.

Management of lab materials

A science laboratory is a place where basic experimental skills are learned only by performing a set of prescribed experiments. Safety procedures usually involve chemical hygiene plans and waste disposal procedures. When providing chemicals, you must read the label carefully before starting the experiment. To avoid contamination and possibly violent reaction, do never return unwanted chemicals to their container. In the laboratory, chemicals should be stored in their original containers, and cabinets should be suitably ventilated. It is important to notify students that chemicals cannot be stored in containers on the floor. Sharp and pointed tools should be stored properly. Students should always behave maturely and responsibly in the laboratory or wherever chemicals are stored or handled.

Hot equipment and glassware handling

Hazard symbols should be used as a guide for the handling of chemical reagents. Chemicals should be labelled as explosives, flammable, oxidizers, toxic and infectious substances, radioactive materials, corrosives, etc. All glassware should be inspected before use, and any broken, cracked, or chipped glassware should be disposed of in an appropriate container. All hot equipment should be allowed to cool before storing it.

All glassware must be handled carefully and stored in its appropriate place after use. All chemical glass containers should be transported in rubber or polyethylene bottle carriers when leaving one lab area to enter another. When working in a lab, do never leave a hot plate unattended while it is turned on. It is recommended to handle hot equipment with safety gloves and other appropriate aids but never with bare hands. You must ensure that hands, hair, and clothing are kept away from the flame or heating area and turn heating devices off when they are not in use in the laboratories.

Waste disposal considerations

Waste disposal is a normal part of any science laboratory. As teachers or students perform demonstrations or laboratory experiments, chemical waste is generated.

These wastes should be collected in appropriate containers and disposed of according to local, state, and federal regulations. All schools should have a person with the responsibility of being familiar with this waste disposal. In order to minimize the amount of waste generated and handle it safely, there are several steps to consider.

Sinks with water taps for washing purposes and liquid waste disposal are usually provided on the working table. It is essential to clean the sink regularly. Notice that you should never put broken glass or ceramics in a regular waste container. Use a dustpan, a brush, and heavy gloves to carefully pick-up broken pieces, and dispose of them in a container specifically provided for this purpose. Hazardous chemical waste, including solvents, acids, and reagents, should never be disposed of down sewer drains. All chemical waste must be identified properly before it can be disposed of. Bottles containing chemical waste must be labelled appropriately. Labelling should include the words "hazardous waste." Chemical waste should be disposed of in glass or polyethylene bottles. Plastic-coated glass bottles are best for this purpose. Aluminium cans that are easily corroded should not be used for waste disposal and storage.

Equipment Maintenance

Maintenance consists of preventative care and corrective repair. Both approaches should be used to keep equipment in working order. Records of all maintenance, service, repairs, and histories of any damage, malfunction, or equipment modification must be maintained in the equipment logs. The record must describe hardware and software changes and/or updates and show the dates when these occurred. Each laboratory must maintain a chemical inventory that should be updated at least once a year.

1.9. Student Experiment Work Sheet

There should be a sheet to guide students about how they will conduct the experiment, the materials to be used, the procedures to be followed, and the way of recording data. The following is the structure of the student experiment worksheet. It can be prepared by the teacher or be availed from the other level.

1. Date
2. Name of student/group
3. The title of the experiment
4. Type of experiment (concept, equipment, and inquiry-based)
5. Objective(s) of the experiment
6. Key question(s)
7. Materials (equipment/instrument, resources, etc…)
8. Procedures & Steps of experiment
9. Schematic reference if required.
10. Data recording and presentation

Number of tests	Variables	Results	Comments/Observations
1			
2			
3			
Etc			

11. Reflective questions and answers

    Question1

    Question 2

    Question 3

12. Answer for the key question.

1.10. Report template for students

After conducting a laboratory experiment, students should write a report about their findings and the conclusion they took.

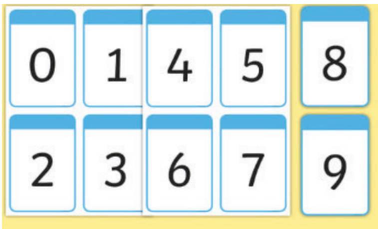
The report to be made depends on the level of students. The report done by primary school Students is not the same us the one to be made by secondary school Students.

The following is a structure of the report to be made by a group of secondary school Students.

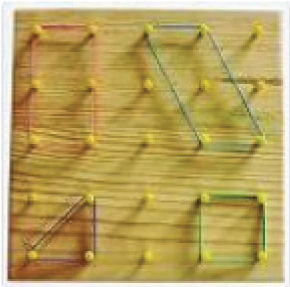
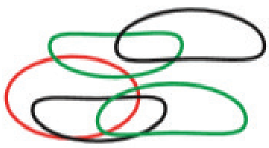
1. Introduction (details related to the experiment: Students identification, date, year, topic area, unit title, and lesson).
2. The title of the experiment.
3. Type of experiment (concept, equipment, and inquiry-based)
4. Objective(s) of the experiment.
5. Key question(s)
6. Materials (equipment/instrument, resources, etc…)
7. Procedures & Steps of experiment
8. Schematic reference if required.
9. Data recording
10. Data analysis and presentation (Plots, tables, pictures, graphs)
11. Interpretation/discussion of the results, student alternative ideas from observation.
12. Theory or main concept, formulas, and application.
13. Conclusion (answer reflective questions and the key question).

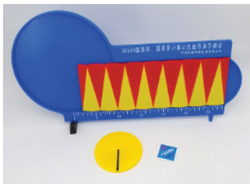
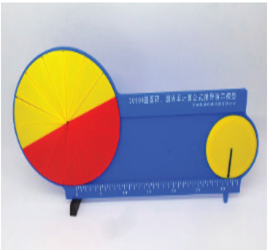
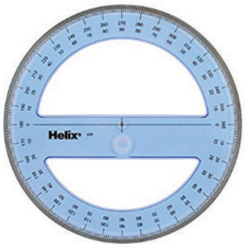
In conclusion, there are safety rules and precautions to consider before, during, and at the end of a practical activity and lab experiment. We hope tutors are inspired to conduct a Mathematics practical activity in a conducive environment as expected by the Competence Based Curriculum.

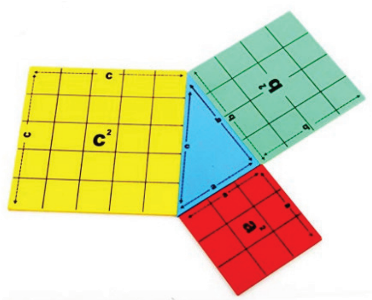
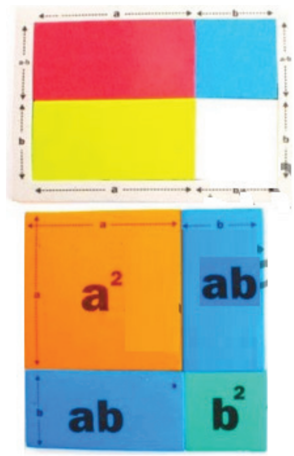
PART II: LIST OF MAIN KIT ITEMS DISTRIBUTED IN SCHOOLS

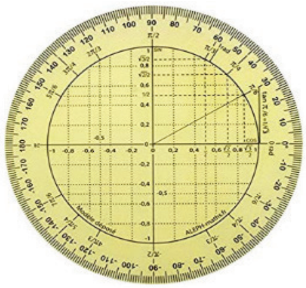
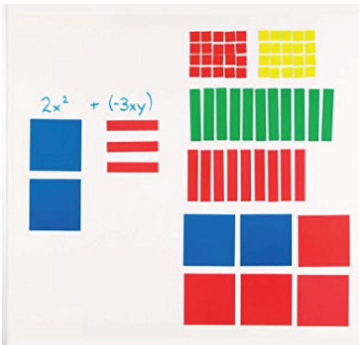
#	Item and description	Picture	Description
1	**Laminated number cards** ***Use**: Used in game for composition, sorting, factorization of numbers, etc.*		A pack of laminated cards numbered from 0 to 9 (9 cards from an A4 paper).
2	**Circle set fraction** ***Use**: Used for exploring "Area of Circle "and activities related to "Fractions"* and area of a circle		7 Blue (or any other color) colored circular plastic having 3mm thickness and diameter 160 mm. divided into 4, 6, 8, 12, 16 and 32 equal sectors. Each piece is magnetic.
3	**Clock** ***Use**: To learn to tell the time according to the 24 hours international convention.*		1 plastic teaching clock

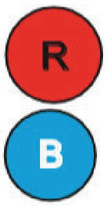
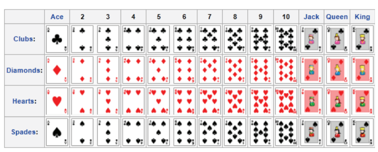
4	**Mathematical set for teachers:** Full circle protractor, meter rule, compass, tape measure, T-square, rope, decameter.		Wooden or plastic
5	**Mathematical set for students**: 2 Metal Study Compasses, 2 T-squares, Ruler, Protractor, Pencil for Compass, Pencil Sharpener, Eraser, Lead Refill.		Geometry 10 Piece Set,
6	**Fest night Stainless Steel 180 Degree Protractor**		Angle Finder Both Arms Stainless Steel Protractor with 0-180 Degrees, Angle10 inch,250mm, 30cm Scale Angle Finder Ruler. Smooth surface, convenient to use, easy to read. 0-180 degree arbitrary rotation. Adjustable screw design, easy operation for fixed reading.

7	**Basic geometric solids**		**6** *pieces* **of** *wooden solids* Includes cube, cylinder, sphere, cone, triangular prism, pyramid . *Use: To demonstrate geometry solids (3D).*
8	**Geoboard** *Geoboard is used to represent planar shapes/ figures and also to find the approximate areas as well as to learn different geometric figures using a rubber band.*		*1 geographic board of 33.5 cm × 53.5 cm. It is printed with 187 grids 3 cm × 3 cm each in alternated colors. Copper pins are nailed on each crossing point of the grids.*
9	**Rubber bands** *for use with geoboard.*		120 rubber bands in 6 colors come with the boards.

10	**Transparent geometric 3D-shapes plus their corresponding fold-up nets:** cylinder, square pyramid, cube, rectangular prism, cone, hexagonal prism, triangular pyramid, and triangular prism. ***Use****: Used to make solid shape.*		Transparent geometric shapes plus their corresponding fold-up net inserts. 16-piece set (8 transparent and 8 folding shapes)
11	**Circle-Area and Diameter Demonstrator**	 	1 plastic demonstrator board of 48 cm × 25 cm. It consists of 17 sectors: 15 sectors that are equal to 1/16 of the cylinder volume; and 2 sectors that are equal to 1/32 of the cylinder volume. Use: To learn how to measure the area and diameter of circles.
12	**Full Protractor**		Helix Professional 360 Degree Protractor 15cm As per sample

13	**Cut Outs for Pythagoras** **Theorem**.		1 plastic right angled triangle. Measure: 3” x 4” x 5” & 3 different size square equal to sides of triangle.
14	**Cut outs for algebraic** **Identities**.		Made up of 3mm thick wooden colored cardboard and includes (1) Square cut-out of side 76 mm (2) 3 cut-outs obtained from another square of side 76mm out of which one is a square of side 38mm and the remaining two are trapezium of dim.38 x76 mm^2 (3) Square cut-out s of side 80 mm and 45mm (4) Rectangular cut-out of dimension 80x45mm^2

15	**Circular Trigonometric Protractor**		CIRCULAR Protractor ROBUST: shatterproof and scratch resistant translucent plastic. Use: direct reading of the angles in degrees and radians and cosine and sine near 0.05.
16	**Algebraic tiles** a) $x^2, x, 1$ b) $-x^2, -x, -1$		Made up of plastic cardboard in different sizes: 40 (20red+20 blue) squares of side 10mm known as unit tiles. 20 (10 red +10 blue) rectangles of 50 x10mm^2 dimension known as x or -x tiles. 10 (5 red + 5 blue) squares of side 50mm known as x^2 or $-x^2$ tiles, etc.

17	**Cubic dice** From 1 sided to 6 sided.	 	6 plastic dices with different edges and different shapes: 8mm, 12mm, 16mm, 19mm and 25mm.
18	**Counters: (20)** ***Use***: *Used in activity "Addition and Subtraction of Integers".*		A set of 20 Plastic pieces or lamineted transparent counters whose one side is blue and other side is red.
19	**Scientific Calculator**		Casio Fx-991es Plus Scientific Calculator
20	**Playing cards** to be used in probability		A set of 52 playing cards
21	**The container**: a box in metal to contain all these materials per kit.		A container in metal which can contain or these materials

PART III: PRACTICAL ACTIVITIES AND LABORATORY EXPERIMENTS FOR SENIOR SIX (S6)

As discussed here above, where possible, every concept developed in Mathematics should start by a practical activity as the concrete stage of learning.

Practical activities given here below were selected as a sample. The teacher will guide students to do more activities depending on the new concept to be developed in a given lesson. The title of the lesson and the unit in which the practical activity takes place are given in the rationale of each practical activity.

Therefore, the teacher and students will verify in this part (rationale) if the given practical activity is selected from the content of their syllabus as expected.

UNIT: 1 COMPLEX NUMBERS

PRACTICAL ACTIVITY 1: PRACTICAL ACTIVITY 1: ILLUSTRATION OF 8^{th} ROOTS OF UNIT ON THE ARGAND DIAGRAM AND FINDING THE LENGTH OF THE SIDE AND THE AREA OF THE FORMED FIGURE

a) Rationale:

This activity is done when teaching the n^{th} roots of a complex number to be learnt in Unit1 of S6. Students will engage in the activities of drawing, measuring, and observing. This will make mathematics more fun and practical than following demonstrations done by the teacher blindly.

This is basically an inquiry-based practical work. In real life, complex numbers are used in solving problems of other sciences such as in physics (mechanics, electronics, alternating current, Sound waves) or engineering (**control theory, signal analysis, relativity, and fluid dynamics**). n^{th} roots can also be observed using the wheels of a bicycle. This is observed in daily life.

b) Objective:

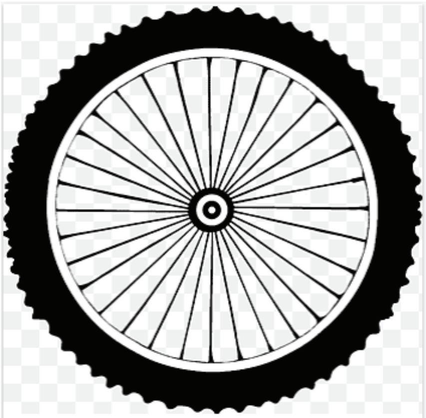
To illustrate the 8^{th} roots of unit as a complex number, represent them on the Argand diagram and calculate the length of side, and the area of the geometric figure formed by roots.

c) List of required materials:

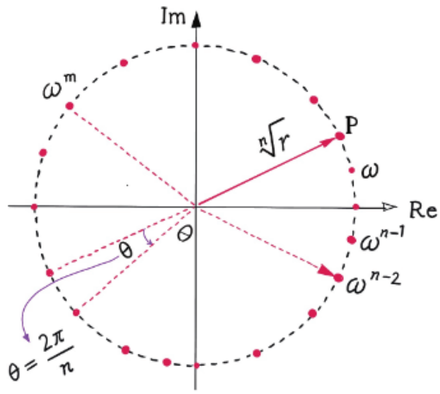
Manila paper, pen/pencil, ruler, compass, protractor, and scientific calculator.

d) Illustration or set up

Practical usefulness of learning the n^{th} root of a number (A wheel of bicycle)



The n^{th} roots of a complex number



n^{th} root of a complex number

Suppose n is a positive integer and a complex number z_k is n th root of z, then

we have $z_k^n = z$.

Let $z = \rho\ (\cos\phi + i\sin\phi)$;

Since z_k is the n^{th} root of z, then

$z_k^n = z \Rightarrow \rho^n\ (\cos n\phi + i\sin n\phi) = r\ (\cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi)),\ k \in \mathbb{Z}$.

Comparing the moduli and arguments, we get

$$\rho^n = r \text{ and } n\phi = \theta + 2k\pi, k \in \mathbb{Z}$$

$$\rho = r^{\frac{1}{n}} \text{ and } \phi = \frac{\theta + 2k\pi}{n}, k \in \mathbb{Z}$$

Therefore, the values of z_k are as follows

$$z_k = r^{\frac{1}{n}}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right), k \in \mathbb{Z} \qquad \text{where } k = 0,1,2,3,\cdots,n-1.$$

e) Procedures

Step 1: Express z in polar form: $z = r(\cos\theta + i\sin\theta) = \text{rcis}\,\theta$ where r is the modulus and θ the principal argument of the complex number z =1.

Step 2: Consider the case of the complex number $z = 1$, calculate the 8^{th} roots

using the formula

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right), k = 0,1,2,3,\cdots,n-1. \text{ and n = 8}$$

Step 3: Consider the case of n=8 and find 8^{th} roots

Step 4: Draw a well labelled Argand diagram on a manila paper using a pencil, ruler, compass, and protractor and represent the 8^{th} roots of z =1.

Step 5: Join points to obtain a geometric figure and use the ruler to measure the length of its sides and determine its area

Reflective questions:

- What is the name of the obtained figures?
- How many sides and vertices does it have?
- Can you generalize the result to be obtained when representing geometrically the n^{th} roots of a complex number z?

f) Data recording

Complex number	Polar form of z	Form of the nth roots	Number of roots	Geometric figure obtained	Number of sides for the figure
$z = \rho\ (\cos\phi + i\ \sin\phi\ )$					
z =1					

Use a ruler to measure the perimeter of the obtained geometric figure for n =8, How can you calculate it without measuring?

How do we get the area of the obtained geometric figure for n =8?

g) Interpretation of results and conclusion

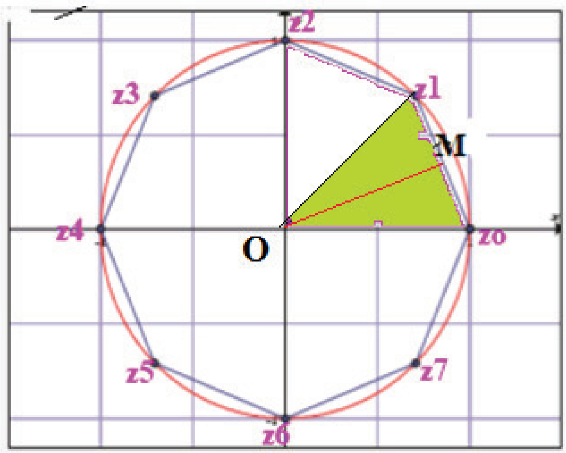
Expected answer

The figure obtained for n = 8 is a polygon named octagon and the eighth roots can be obtained using the formula:

$$z_k = r^{\frac{1}{n}}\left(\cos\frac{\theta+2k\pi}{n}+i\sin\frac{\theta+2k\pi}{n}\right), k=0,1,2,\ldots,7$$

We have n =8, it means that the angle of the centre is $\frac{2\pi}{n}=\frac{2\pi}{8}=\frac{\pi}{4}$ and the radius R is a unit.

The drawing gives an octagon.



After drawing octagon, we can find the length of its side.

Let s_8 be the side of an octagon. Its lengh is the same as the distance between $z_0(1,0)$ and $z_1\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$. $OM = a_8$ is the apothem.

The area of one triangle **OZ$_0$Z$_1$** is $\frac{a_8\times s_8}{2}$

Therefore, the side $s_8=\sqrt{2-\sqrt{2}}$, the apothem $a_8=\frac{1}{2}\sqrt{2+\sqrt{2}}$.

The total area is $A_8 = 8\times A_8 = 8\frac{a_8\times s_8}{2}=2\sqrt{2}.$

If the radius of the circle R is different from 1 i.e. $(R\neq 1)$,

then $S_8=\sqrt{2-\sqrt{2}}\,\mathrm{R}, a_8=\frac{1}{2}\sqrt{2+\sqrt{2}}R$ and $A_8=2\sqrt{2}R^2$.

Information for teacher

You can ask learners to consider the case n = 4, n = 5 and n = 12. It appears that graphically the 4$^{\text{th}}$ roots of $z=1$ are the vertices of a regular polygon of 4 equal sides and 4 equal angles which is a square.

In general, graphically the n[th] roots of a complex number $z=1$ are the vertices of a regular polygon of n sides.

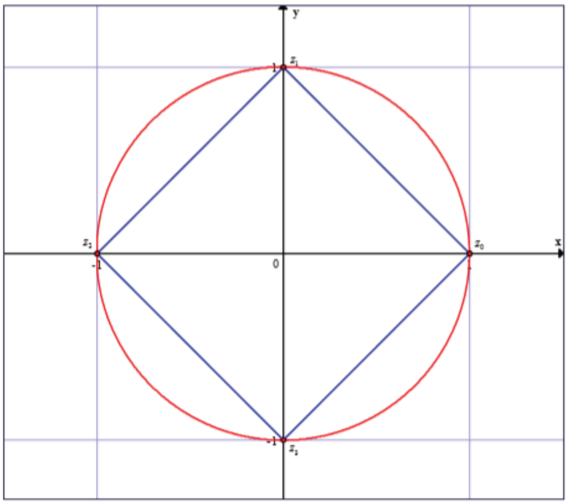
Example: Graphic representation of the 4^{th} roots of the unit $z=1$

We have four vertices:

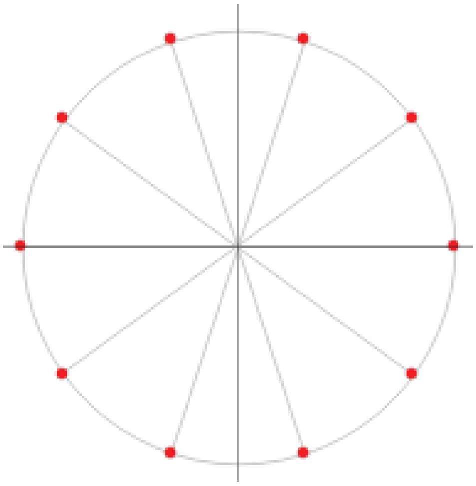
$$z_k = cis\frac{2k\pi}{4} = cis\frac{k\pi}{2}; k = 0,1,2,3.$$

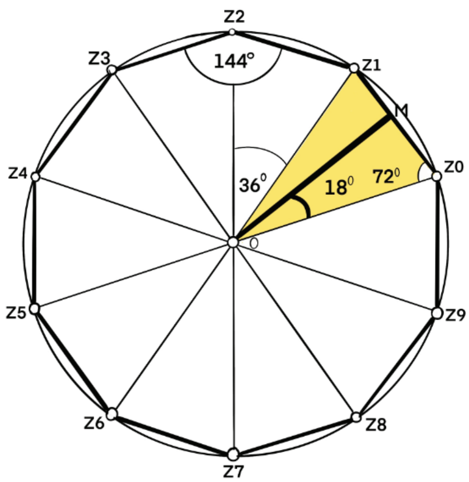
And then,

$$z_0 = cis0 = 1,\ z_1 = cis\frac{\pi}{2} = i, z_2 = cis\pi = -1, z_3 = cis\frac{3\pi}{2} = -i.$$



ii) For n =10, we get a decagon





The angle of the centre is $\frac{2\pi}{10} = \frac{\pi}{5}$ and to calculate $z_0 = cis\frac{\pi}{5}$ is difficult.

The teacher uses the trigonometric relation of the right angle to calculate the length side and the length of the apothem $a_{10} = OM$.

Consider the right-angled triangle OMz_0.

Then $\cos\frac{\pi}{5} = \frac{s_{10}}{Hypotenuse}$ where hypotenuse $z_0 = 1$.

$$\cos\frac{2\pi}{5} = \frac{s_{10}}{2} \Leftrightarrow 2\cos\frac{2\pi}{5} = s_{10}$$

Remember that the sum of n^{th} roots is equal to zero

Then $z_0 + z_1 + z_2 + z_3 + z_4 + ... + z_9 = 0$

The teacher asks the student to show that $\cos\frac{2\pi}{5} = \frac{-1+\sqrt{5}}{4}$ and the length of a side of the decagon is given by $s_{10} = \frac{-1+\sqrt{5}}{2}$.

To calculate the length of the apothem we use Pythagoras' theorem for a right-angled triangle

$$(Hypotenuse)^2 = \left(\frac{s_{10}}{2}\right)^2 + (a_{10})^2$$

$$\Leftrightarrow 1 = \left(\frac{s_{10}}{2}\right)^2 + (a_{10})^2$$

$$\Leftrightarrow a_{10} = \frac{1}{4}\sqrt{10+2\sqrt{5}}$$

The area is $A_{10} = \frac{10}{2} \times s_{10} \times a_{10}$

If the radius of circle R differs from 1, $(R \neq 1)$ then

$$s_{10} = \frac{-1+\sqrt{5}}{2} R$$

$$a_{10} = \frac{1}{4}\sqrt{10+2\sqrt{5}} \times R$$

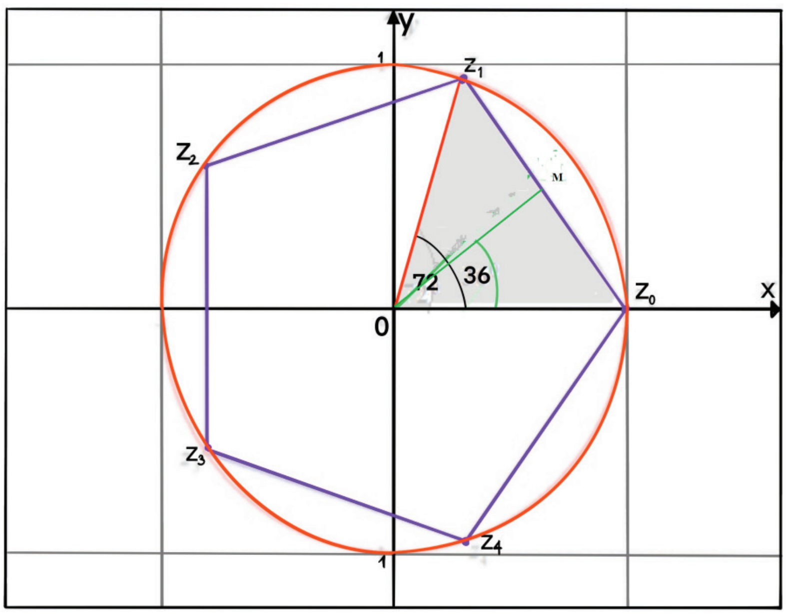
The area is $A_{10} = \frac{10}{2} \times s_{10} \times a_{10}$

iii) Pentagon

We have five vertices: $z_k = cis\frac{2k\pi}{5}; k = 0,1,2,3,4.$

$$z_0 = cis0 = 1;\ z_1 = cis\frac{2\pi}{5}$$

$$z_2 = cis\frac{4\pi}{5};\ z_3 = cis\frac{6\pi}{5};\ z_4 = cis\frac{8\pi}{5}$$



Consider M midpoint of z_1 and z_0. Then OM is apothem of pentagon

Consider the right OMZ_0, we have $\sin\frac{\pi}{5} = \frac{\frac{S_5}{2}}{hypotenuse} \Rightarrow \sin\frac{\pi}{5} = \frac{S_5}{2}$

We know that $\cos\frac{2\pi}{5}=\frac{-1+\sqrt{5}}{4}$ and $\sin\frac{2\pi}{5}=\frac{1-\cos\frac{2\pi}{5}}{2}$,

This implies that $\sin\frac{2\pi}{5}=\frac{1}{4}\sqrt{10-2\sqrt{5}}$

Then $S_5=\frac{1}{2}\sqrt{10-2\sqrt{5}}$

- To calculate the apothem a_5 we use the Pythagoras theorem and we found $a_5=\frac{1}{4}\sqrt{6+2\sqrt{5}}$
- The area of Pentagon is $A_5=\frac{5}{2}\times a_5\times S_5$

Generally, to determine the length side s_n, the length apothem a_n and the area A_n of the regular polygon of n sides like square, triangle, hexagon and octagon we use the following formula $S_n=\frac{1}{2}\left|1-z_1\right|$, $a_n=\frac{1}{2}\left|1+z_1\right|$ *and* $A_n=\frac{n}{2}\times S_n\times a_n$ where $z_1=cis\frac{2\pi}{n}$.

UNIT: 2 LOGARITHMIC AND EXPONENTIAL FUNCTIONS

PRACTICAL ACTIVITY 2: PLOTTING THE CURVE OF THE LOGARITHMIC FUNCTION $f(x)=\log_2 x$ ON GRAPH PAPER

a) Rationale:

This activity is done when teaching the graph of logarithmic functions learnt in Unit 2 of S6. Students engage more in using geometric materials, plotting graphs and sharing ideas with the teacher and coming up with a presentation and report of the activity.

In real life, logarithmic functions are used when finding the values for variables that contributed the result of a phenomena expressed by an exponential function.

This is an Inquiry-based practical activity is **that will encourage students to ask questions and investigate real-world problems while plotting the functions.**

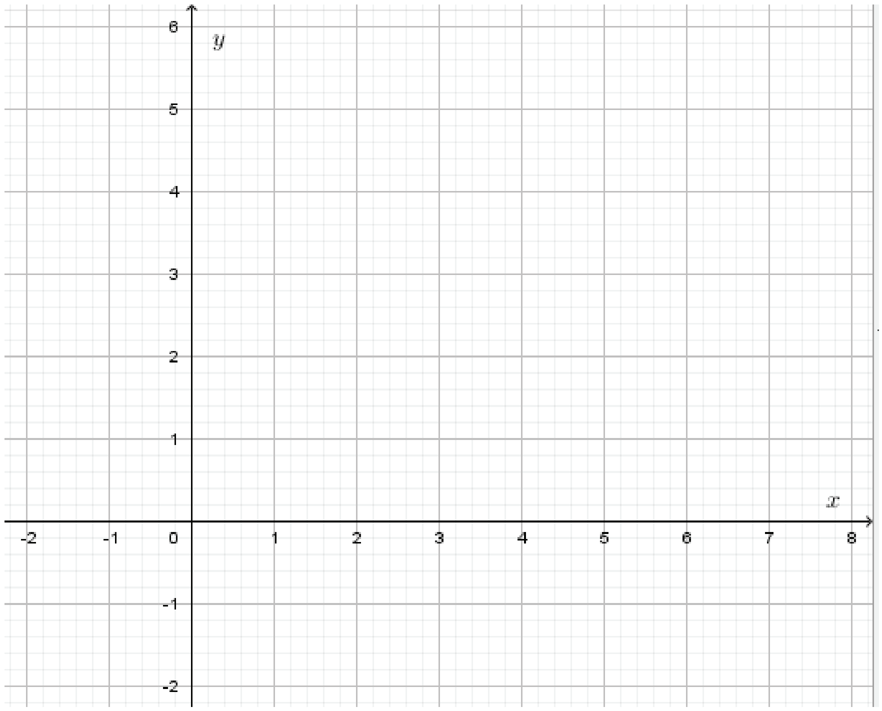
b) Objective:

To represent the graph of a given logarithmic function $f\left(x\right)=\log_2 x$ on the Cartesian plane drawn on the manila papers and compare the result with the graph produced by a mathematics software such as GeoGebra and interpret the results.

c) List of required materials:

Graph paper, ruler, pencil, eraser, scientific calculator, and Geometric software such as GeoGebra installed in computers.

d) Set up of the activity:



e) Procedures and data recording

Step1: Choose values of x and use the calculator to get their image values for $\log_2 x$

Step 2: Record data in a table for

x	0.2	0.5	0.8	1	2	3	4	5	6	7	8	9
$\log_2 x$												

Step 3: Plot the points in a Cartesian plane on the graph paper using geometric instruments.

Step 4: Join the different points to draw the curve of f(x) = $\log_2 x$

Step 5: Discuss the behaviour of the graph obtained.

- Does it increase or decrease?
- Can you predict what happens if x increases towards infinity?
- Can you predict what happens when x decreases towards 0?

Step 6: Use geometric software such as GeoGebra, enter $\log_2 x$ and find the graph. Compare your graph and the one given by the software.

Are they the same? What are the sources of difference if you observed any?

f) Recorded data:

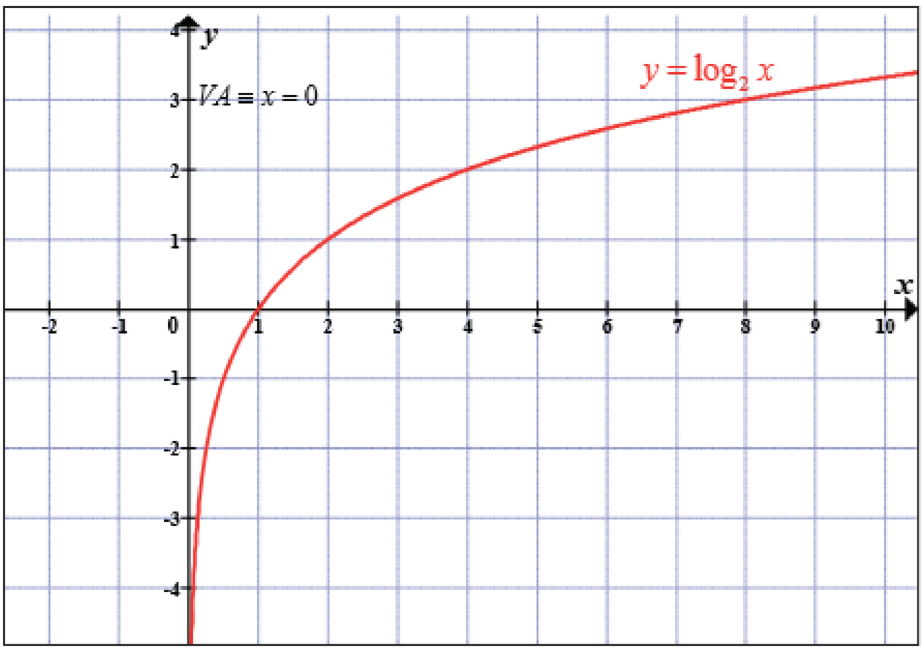
x	0.2	0.5	0.8	1	2	3	4	5	6	7	8	9
$\log_2 x$												

To be completed using a calculator

g) Interpretation of results and conclusion

Graph:

The logarithmic function of base 2, y= $\log_2 x$



- The logarithmic function of base 2 is an increasing function.
- The y-axis is a vertical asymptote to the curve of the function y = $\log_2 x$
- The concavity is turned downward.

g) Information for teacher

$\log_2 x$ is defined only when x > 0;

The GeoGebra software can be downloaded through (This link)

h) Guidance on evaluation

- Propose various exercises in which $a > 1$ and $0 < a < 1$
- Plot the graph of the following functions manually and using Geogebra then compare the results: 1) $f(x) = \log_{\frac{1}{2}} x$ 2) $f(x) = \log_2 \sqrt{x+1}$

PRACTICAL ACTIVITY 3: DOMAIN AND RANGE OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

a) Rationale:

This activity can be done when teaching the domain and range of logarithmic and exponential functions learnt in Unit 2 of S6**.** Logarithmic and exponential functions help model many real-world situations, such as population growth, decay, compounded interest, and model of the spread of an epidemic (There is a constant percentage each period). This is a concept-based practical activity that mainly focuses on domain and range of logarithmic and exponential functions.

b) Objectives:

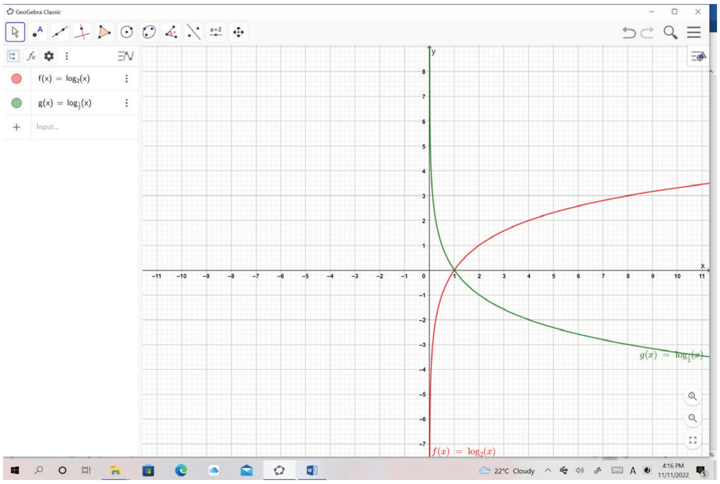
To determine domain and range of logarithmic and exponential functions.

c) List of required materials:

Graph paper, ruler, scientific calculator, Computer (Mathematics software: GeoGebra).

d) Illustration of the activity:

Logarithmic function of base 2and 1/2 respectively $y = \log_2 x$, and $\mathrm{y} = \log_{\frac{1}{2}} x$



e) Procedures

Step 1: Choose input values of x and use the calculator to get their image values for both functions $y=\log_2 x$ and $y=\log_{\frac{1}{2}} x$.

Step 2: Record data in a table as follows:

x	-1	$\frac{1}{2}$	1	2	3	4	5	6	7	8
$y=\log_2 x$										
$y=\log_{\frac{1}{2}} x$										

Step3: Plot the found points in Cartesian plane on the graph paper.

Step4: Join the different points for each function to draw the respectively curves of $y=\log_2 x$ and $y=\log_{\frac{1}{2}} x$

Step5: Discuss the behaviour of the two graphs obtained. Are the functions defined for every value of x ? Can you predict what happens when tends to zero for both functions? Can you predict what happens when x tends to positive infinity for both functions? what can you conclude about the line y-axis?

Expected answer

x	-1	0.3	$\frac{1}{2}$	1	2	3	4	5	6	7	8
$y=\log_2 x$	Not defined		-1	0	1	1.58	2	2.32	2.58	2.81	3
$y=\log_{\frac{1}{2}} x$	Not defined		1	0	-1	-1.58	-2	-2.32	-2.58	-2.81	-3

Observations:

1. When $y=\log_2 x$ is tending to positive infinity, $y=\log_{\frac{1}{2}} x$ is tending to negative infinity.
2. There is no intersection with y-axis.

Step 6: By using GeoGebra software, enter $y=\log_2 x$ and $y=\log_{\frac{1}{2}} x$ in algebraic view, and find the graphs.

Step 7: Observe the two outputs and compare your graphs and the one given by the GeoGebra software. Are the results the same through observation? Are there any similarities for what you have done and the one obtained by using GeoGebra?

f) Interpretation of the results and conclusion

The logarithmic functions of base 2 and ½ are not defined for negative values of x

(Domf: $x \in]0,+\infty[$

The range of logarithmic functions of base 2 and ½ is the set of real numbers: $]-\infty,+\infty[$

The y-axis is a vertical asymptote to both curves of the functions $y=\log_2 x$ and $y=\log_{\frac{1}{2}} x$

g) Information for teacher

Students should know that $y=\log_a u(x)$ is defined only for when $u(x)>0$, $and\ a>0$

h) Guidance for evaluation

Propose various activities in which $u(x)>0$, $and\ a>0$. For example,

Plot the graph of 1. $y=\log_2 3x$ 2. $y=\log_{\frac{1}{2}} 4x$ $y=\log_{\frac{1}{2}}\sqrt{x^2-1}$ and find the values where the function is defined and the limits at the boundaries of domain.

PRACTICAL ACTIVITY 4: EXPLORE A MORTGAGE PROBLEM AND DECIDE THE TYPE OF LOAN YOU CAN TAKE FROM A BANK

a) Rationale:

This activity can be done when teaching the application of exponential functions in real life learnt in Unit 2 of S6. Students engage in real-life situations of thinking about how getting a loan of constructing a house or any other loan will be linked to mathematics, instead of taking it as a strange problem.

When a person gets a loan (mortgage) from the bank, the mortgage amount M, the number of months required to repay the total amount of the loan **n**, the monthly amount of the payment P, and the interest rate r, it is proved that all the 4 components are related by the following formula:

$$P = \frac{M \times i \times (1+i)^n}{(1+i)^n - 1} \text{ or } P = M\left[\frac{i}{1-(1+i)^{-n}}\right] \text{ is the monthly amount to pay.}$$

Where: **M** is the mortgage amount of the loan, **i** is the monthly interest rate per payment period and **n** is the number of months required to repay the total amount of the loan. This is a concept-based practical activity of monthly payment of a mortgage, the total amount to pay, the related interest to be taken by the bank and decide the type of loan you can take from a bank.

b) Objective:

To calculate the monthly payment of a mortgage, the total amount to pay, the related interest to be taken by the bank and decide the type of loan you can take from a bank.

c) List of required materials:

Paper, Pencil or pen, scientific calculator, computer (excel sheet for loan amortization).

d) Illustration of the activity:

Formula to be used: $P = M\left[\frac{i}{1-(1+i)^{-n}}\right]$

Excel sheet to be used: Note that the dollar sign ($) in the excel sheet below does not stand for currency. It is by default; ignore it when you want to use Rwanda Francs (Frw) as our currency.

Loan Amortization Schedule

Enter values	
Loan amount	$ 9,045,000.00
Annual interest rate	18.50 %
Loan period in years	5
Number of payments per year	12
Start date of loan	11/15/2021
Optional extra payments	$ -

Loan summary	
Scheduled payment	$ 232,150.86
Scheduled number of payments	60
Actual number of payments	60
Total early payments	$ -
Total interest	$ 4,884,051.72

Lender name:

Pmt. No.	Payment Date	Beginning Balance	Scheduled Payment	Extra Payment	Total Payment	Principal	Interest	Ending Balance	Cumulative Interest
1	12/15/2021	$ 9,045,000.00	$ 232,150.86	$ -	$ 232,150.86	$ 92,707.11	$ 139,443.75	$ 8,952,292.89	$ 139,443.75
2	1/15/2022	$ 8,952,292.89	$ 232,150.86	$ -	$ 232,150.86	$ 94,136.35	$ 138,014.52	$ 8,858,156.54	$ 277,458.27
3	2/15/2022	$ 8,858,156.54	$ 232,150.86	$ -	$ 232,150.86	$ 95,587.62	$ 136,563.25	$ 8,762,568.93	$ 414,021.51
4	3/15/2022	$ 8,762,568.93	$ 232,150.86	$ -	$ 232,150.86	$ 97,061.26	$ 135,089.60	$ 8,665,507.67	$ 549,111.12
5	4/15/2022	$ 8,665,507.67	$ 232,150.86	$ -	$ 232,150.86	$ 98,557.62	$ 133,593.24	$ 8,566,950.05	$ 682,704.36
6	5/15/2022	$ 8,566,950.05	$ 232,150.86	$ -	$ 232,150.86	$ 100,077.05	$ 132,073.81	$ 8,466,873.00	$ 814,778.17
7	6/15/2022	$ 8,466,873.00	$ 232,150.86	$ -	$ 232,150.86	$ 101,619.90	$ 130,530.96	$ 8,365,253.10	$ 945,309.13
8	7/15/2022	$ 8,365,253.10	$ 232,150.86	$ -	$ 232,150.86	$ 103,186.54	$ 128,964.32	$ 8,262,066.55	$ 1,074,273.45
9	8/15/2022	$ 8,262,066.55	$ 232,150.86	$ -	$ 232,150.86	$ 104,777.34	$ 127,373.53	$ 8,157,289.22	$ 1,201,646.98
10	9/15/2022	$ 8,157,289.22	$ 232,150.86	$ -	$ 232,150.86	$ 106,392.65	$ 125,758.21	$ 8,050,896.56	$ 1,327,405.18

e) Procedures

Step 1: Read the problem for the old man who wants to take a loan of 9,045,000Frw but he is not sure if he will pay more interest to the bank when the number of periods (instalments) increases.

What monthly payment is necessary to pay off a loan of 9,045,000Frw at 18.5% per annum? in 2 years, in 3 years? And in 5 years.

What total amount is paid out for each loan?

What interest will the bank earn for each loan?

What advice can you give your colleague about the number of years and the interest to be given to the bank?

Step 2: Calculate the monthly payments using the formula and complete in the table of data recording.

Step 3: Use the excel sheet and find the monthly payments and the total interest and complete them in the table of data recording

Step 4: Evaluate the total amount paid and the total interest for each case.

Step 5: Consider the monthly payments and the interest to pay and then take the decision on whether you can choose a long-term loan or a short-term loan.

f) Data recording

	n=24	n=36	n=60
M			
i			
$1+i$			
$(1+i)^{-n}$			
$1-(1+i)^{-n}$			
$\dfrac{1}{1-(1+i)^{-n}}$			
P			
$P\times n$			
$(P\times n)-M$			

Does the calculation give the same results of P and total interest as the output of the excel sheet of the bank?

What advice can you give your colleague about the number of years and the interest to be given to the bank?

Expected answers

n	12	24	36	60
Loan	M	9045000	9045000	**9,045,000.00**
	i	0.015416667	0.015416667	0.015416667
	$1+i$	1.015416667	1.015416667	1.015416667

	$(1+i)^{-n}$	0.692687101	0.576508415	0.399339943
	$1-(1+i)^{-n}$	0.307312899	0.423491585	0.600660057
	$\frac{1}{1-(1+i)^{-n}}$	0.050166025	0.036403714	0.025666209
Monthly payment	P	453751.6994	329,271.60	**232,150.86**
Total	$P\times n$	10890040.79	11,853,777.45	**13,929,051.72**
Total of interest	$(P\times n)-M$	1,845,040.79	2,808,777.45	**4,884,051.72**

The calculation gives the same results as those shown by the excel sheet of the bank.

g) Interpretation of results and conclusion

- In a long-term loan, the monthly payment is low, but the total interest is high. This means that when your salary is low, you are obliged to take a long-term loan. For a loan of 9,045,000Frw in 12 months, the borrower is asked to pay 453,731.7Frw per month but in 60 months, the borrower will need to pay 232,150.9Frw per month.
- In a short-term loan, the monthly payment is high, but the total interest is low.
- The borrower will take a decision considering how much money he is able to pay per month and the total interest he do not want to give the bank.

h) Information for teacher

When r is the interest rate, n the number of instalments per year, t the number of year, obove formula can be transformed into

$P=\dfrac{M\times\frac{r}{n}}{1-\left(1+\frac{r}{n}\right)^{-nt}}$. This is the formula seen in ordinary level

Taking $i=\frac{r}{n}$ we come back to the formula $P=M\left[\dfrac{i}{1-(1+i)^{-n}}\right]$ where nt stands

for the total number of instalments (number of all months).

i) Guidance on evaluation

The teacher will propose to learners the exercises involving the mortgage, the interest rate, and the period of payment referring to the above-given model example.

Example:

What monthly payment is necessary to pay off a loan of \$800 at 10% per annum? in 2 years, in 3 years? What total amount is paid out for each loan?

Solution:

a) For the 2-year loan, $P = \$800, \quad n = 24, \quad and\ i = \dfrac{0.10}{12}$. The monthly payment P is

$$P = M\left[\frac{i}{1-(1+i)^{-n}}\right] = 800\left[\frac{\frac{0.10}{12}}{1-\left(1+\frac{0.10}{12}\right)^{-24}}\right] = 36.92$$

The excel sheet gives the following:

Loan Amortization Schedule

Enter values	
Loan amount	\$ 800.00
Annual interest rate	10.00 %
Loan period in years	2
Number of payments per year	12
Start date of loan	11/15/2021
Optional extra payments	\$ -

Loan summary	
Scheduled payment	\$ 36.92
Scheduled number of payments	24
Actual number of payments	24
Total early payments	\$ -
Total interest	\$ 85.98

Lender name:

Pmt. No.	Payment Date	Beginning Balance	Scheduled Payment	Extra Payment	Total Payment	Principal	Interest	Ending Balance	Cumulative Interest
13	12/15/2022	\$ 419.90	\$ 36.92	\$ -	\$ 36.92	\$ 33.42	\$ 3.50	\$ 386.48	\$ 66.39
14	1/15/2023	\$ 386.48	\$ 36.92	\$ -	\$ 36.92	\$ 33.70	\$ 3.22	\$ 352.79	\$ 69.61
15	2/15/2023	\$ 352.79	\$ 36.92	\$ -	\$ 36.92	\$ 33.98	\$ 2.94	\$ 318.81	\$ 72.55
16	3/15/2023	\$ 318.81	\$ 36.92	\$ -	\$ 36.92	\$ 34.26	\$ 2.66	\$ 284.55	\$ 75.21
17	4/15/2023	\$ 284.55	\$ 36.92	\$ -	\$ 36.92	\$ 34.54	\$ 2.37	\$ 250.01	\$ 77.58
18	5/15/2023	\$ 250.01	\$ 36.92	\$ -	\$ 36.92	\$ 34.83	\$ 2.08	\$ 215.18	\$ 79.66

b) For the 3-year loan, $M = \$800, \quad n = 36, \quad and\ i = \dfrac{0.10}{12}$.

The monthly payment P is $P = M\left[\dfrac{i}{1-(1+i)^{-n}}\right] = 800\left[\dfrac{\frac{0.10}{12}}{1-\left(1+\frac{0.10}{12}\right)^{-36}}\right] = 25.81$

The excel sheet gives the following:

Loan Amortization Schedule

Enter values	
Loan amount	$ 800.00
Annual interest rate	10.00 %
Loan period in years	3
Number of payments per year	12
Start date of loan	11/15/2021
Optional extra payments	$ -

Loan summary	
Scheduled payment	$ 25.81
Scheduled number of payments	36
Actual number of payments	36
Total early payments	$ -
Total interest	$ 129.29

Lender name:

Pmt. No.	Payment Date	Beginning Balance	Scheduled Payment	Extra Payment	Total Payment	Principal	Interest	Ending Balance	Cumulative Interest
13	12/15/2022	$ 559.41	$ 25.81	$ -	$ 25.81	$ 21.15	$ 4.66	$ 538.25	$ 73.83
14	1/15/2023	$ 538.25	$ 25.81	$ -	$ 25.81	$ 21.33	$ 4.49	$ 516.93	$ 78.32
15	2/15/2023	$ 516.93	$ 25.81	$ -	$ 25.81	$ 21.51	$ 4.31	$ 495.42	$ 82.63
16	3/15/2023	$ 495.42	$ 25.81	$ -	$ 25.81	$ 21.69	$ 4.13	$ 473.73	$ 86.75
17	4/15/2023	$ 473.73	$ 25.81	$ -	$ 25.81	$ 21.87	$ 3.95	$ 451.87	$ 90.70
18	5/15/2023	$ 451.87	$ 25.81	$ -	$ 25.81	$ 22.05	$ 3.77	$ 429.82	$ 94.47
19	6/15/2023	$ 429.82	$ 25.81	$ -	$ 25.81	$ 22.23	$ 3.58	$ 407.59	$ 98.05

c) For the 2-year loan, the total amount paid out is $(36.92)(24) = \$886.08$;

For the3-year loan, the total amount paid out is $(25.81)(36) = \$929.16$.

We see that the calculation and the excel sheet used by the banks give the same results.

PRACTICAL ACTIVITY 5: DEPRECIATION MODEL OF A BANANA.

a) Rationale:

This activity is related to Unit 2: Logarithmic and Exponential functions in syllabus of senior six. It is conducted during teaching and learning activities for explaining how the exponential and logarithmic functions are used to solve depreciation (or decay) problems. The banana was used as the real-life object to show the lifetime of decaying as an exponential function. The following practical activity is a concept based experiment.

b) Objective:

Determining depreciation (or decay) model of a banana using exponential function and logarithmic function.

c) List of required materials:

Table, Banana, scientific calculator, a white paper, and markers of different colours.

d) Illustration of the activity:

 **Figure 1:** Fresh banana	 **Figure 2:** Rotten banana	 **Figure 3:** Completely rotten banana
 **Figure 4:** Decay progress		

e) Procedures

Step1: Read the illustration problem on decay:

If we start a biology experiment with 5,000,000 cells of banana and 45% of the cells are dying every minute, how long will it take to have less than 1,000 cells?

Step 2: Place three bananas of different states on the table [fresh banana **(Figure 1)**, rotten banana but not completely **(Figure 2)** and rotten banana completely **(Figure 3)**]. The banana decays daily **(Figure 4)** and direct the students observe all bananas,

Step 3: Arrange the bananas from fresh bananas to rotten bananas completely**.**

Step 4: Assume that the fresh banana **(Figure 1)** is having 5,000,000 cells.

Step 5: Take 45% as the rate of the cells dying every minute.

Step 6: Determine how long it will take to have less than 1,000 cells as shown in figure 3.

Step 7: Knowing that If V_0 is the value of a certain time, and r s the rate of depreciation per period, consider that depreciation (or decay) is a negative growth and consequently the value V_t at the end of t periods is $V(t) = V_0(1-r)^t$ and deduce the time t.

Expected answer

Using $V(t) = V_0(1-r)^t$

Given that $V(t) = 1000$, $V_0 = 5000000$ and $r = 0.45$

We have the equation : $1000 = 5000000(1-0.45)^t$

$$\frac{1000}{5000000} = (1-0.45)^t$$

Then, $0.0002 = (0.55)^t$

Applying logarithm on both sides, we get:

$$\ln(0.0002) = t\ln(0.55)$$

$$t = \frac{\ln(0.0002)}{\ln(0.55)} \approx 14.2$$

It will take about 14.2 minutes for the cell populations of a banana to drop below a 1,000 count.

f) Interpretation of results and conclusion

The decay of the banana is negative growth. If $V_0 = 5,000,000$ is the value of a certain time, and $r\% = 0.45$ is the rate of depreciation per period, the value $V(t) = 1000$ at the end of t periods is

$$V(t) = V_0(1-r)^t$$

g) Information for teacher

From the model $V(t)=V_0(1-r)^t$, students can calculate the number of cells, V remaining after t seconds. This will depend on the given data.

h) Guidance on evaluation

Give students more examples to show how the exponential function is applied in depreciation (or decay) models. Source: Ngezahayo E. & Icyingeneye P. (2017). *Advanced Mathematics for Rwandan Schools: Learner's Book, Senior Six.* page 132-141

UNIT: 3 TAYLOR AND MACLAURIN SERIES

PRACTICAL ACTIVITY 6: ESTIMATION OF THE TRIGONOMETRIC NUMBER OF AN ANGLE

a) Rationale:

This activity can be done when teaching the estimation of trigonometric number of an angle. It is taught in Unit 3 of S6. There are various **applications of trigonometry** in science and in everyday life. One of the most notable examples of this is in mathematics since it intervenes in all its fields. Other of its most outstanding applications are shown in navigation, geography, astronomy, architecture and in all fields of engineering. The following is a concept based practical activity of estimating trigonometric number of an angle.

b) Objective:

To estimate the trigonometric number of an angle

c) List of required materials:

Paper, A pen, a scientific calculator

d) Illustration of the activity:

Given that when x is a small angle expressed in radians, we can approximate the trigonometric number by using the series of trigonometric functions. Estimate the value of $\cos\frac{\pi}{6}$ to 3 decimal places.

e) Procedures

Step 1: Consider the formula for Maclaurin series:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

$$=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\dots+\frac{f^{(n)}(0)}{n!}x^n+\dots$$

Step 2: Find derivatives of the function $f(x)=\cos x$

Step 3: Use the derivatives to find factors of Maclaurin Series for $f(x)=\cos x$

Step 4: Put $x=\frac{\pi}{6}$ into the Maclaurin Series for $f(x)=\cos x$.

Step 5: Use the general term and estimate the value of $\cos\frac{\pi}{6}$ when $n=2$

f) Data recording:

Function	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}(x)$	$f^{(n)}(x)$
$f(x)=\cos x$					
$f^{(n)}(0)$					

Use the obtained data to estimate $\cos x$

Expected answer

The Maclaurin series of order n for $f(x)=\cos x$

$f(x)=\cos x$, then $f(0)=\cos 0=1$

$f'(x)=-\sin x$, then $f'(0)=0$

$f''(x)=-\cos x$, then $f''(0)=-1$

$f'''(x)=\sin x$, then $f'''(0)=0$

$f^{(4)}(x)=\cos x$, then $f^{(4)}(0)=1$

In the table, we have:

Function	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{(4)}(x)$	$f^{(n)}(x)$
$f(x)=\cos x$	$\cos x$	$-\sin x$	$-\cos x$	$\sin x$	$\cos x$	

$f^{(n)}(0)$	1	0	-1	0	1	

$$\cos x = 1 - \frac{0x}{1!} - \frac{x^2}{2!} + \frac{0x^3}{3!} + \frac{x^4}{4!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + (-1)^n \frac{x^{2n}}{(2n)!}$$

We first need to obtain the Maclaurin series for $f(x) = \cos x$ where $x = \frac{\pi}{6}$.

Taking $x = \frac{\pi}{6}$, We have:

$$\cos\frac{\pi}{6} = 1 - \frac{\left(\frac{\pi}{6}\right)^2}{2!} + \frac{\left(\frac{\pi}{6}\right)^4}{4!} + \ldots + (-1)^n \frac{\left(\frac{\pi}{6}\right)^{2n}}{(2n)!}$$

The general term is $(-1)^n \frac{\left(\frac{\pi}{6}\right)^{2n}}{(2n)!}$

Since we need the value of $\cos\frac{\pi}{6}$ to 3 decimal places, we need the value of n such that

$$\left| \frac{\left(\frac{\pi}{6}\right)^{2(n+1)}}{(2n+1)!} \right| < 0.001$$

Here, n = 2 because $\frac{\left(\frac{\pi}{6}\right)^{2(2+1)}}{(2\bullet 2+1)!} = 0.00002 < 0.001$

$$\cos\frac{\pi}{6}=1-\frac{\left(\frac{\pi}{6}\right)^2}{2!}+\frac{\left(\frac{\pi}{6}\right)^4}{4!}+\ldots+(-1)^n\frac{\left(\frac{\pi}{6}\right)^{2n}}{(2n)!}$$

Implies that $\cos\frac{\pi}{6}=1-\frac{\left(\frac{\pi}{6}\right)^2}{2!}=1\text{- }0.137=0.86$

f) Interpretation of results and conclusion

When x is expressed in radians, we can approximate the value of any trigonometric number using the series of trigonometric functions

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots+(-1)^n\frac{x^{2n}}{(2n)!}$$

Helps us to get $\cos\frac{\pi}{6}=1-\frac{\left(\frac{\pi}{6}\right)^2}{2!}+\frac{\left(\frac{\pi}{6}\right)^4}{4!}+\ldots+(-1)^n\frac{\left(\frac{\pi}{6}\right)^{2n}}{(2n)!}$

$$\frac{\left(\frac{\pi}{6}\right)^{2(2+1)}}{(2\bullet 2+1)!}=0.00002<0.001$$

We get $\cos\frac{\pi}{6}=1-\frac{\left(\frac{\pi}{6}\right)^2}{2!}=1\text{- }0.137=0.86$.

g) Information for teacher

As a teacher, tell students that x must be expressed in radians. The Maclaurin series can be used for instance, to **approximate a function, find the anti-derivative of a complicated function, or compute an incomputable sum.**

h) Guidance on evaluation

Propose other exercises to students. For example, Using Maclaurin series, estimate the number $\sin\frac{2\pi}{3}$ to 3 decimal places.

UNIT: 4 INTEGRATION

PRACTICAL ACTIVITY 7: USING DEFINITE INTEGRAL TO DETERMINE THE AREA OF THE RIGHT TRIANGLE MADE UNDER A LINEAR FUNCTION

a) Rationale

This activity is related to Unit 4 of S6. It is done during teaching the application of definite integral in real life. It is easy for students to evaluate the area of the objects in form of right-angled triangles using definite integrals. Basing on calculations of integrals without allowing students to apply what they learn in real life is an outdated methodology. This is an inquiry-based practical activity.

b) Objective

Determine the area of a right triangle of base $x = 4$ and height $y = 8$ by using definite integral and extend the case for $x = b$ and height $y = h$.

c) List of required materials

Graph paper, ruler, scientific calculator, computer with GeoGebra software or excel sheet.

d) Illustration of the activity

Using ICT tools like GeoGebra software, the students can draw the graph with the help of teacher, as shown in the figure below.

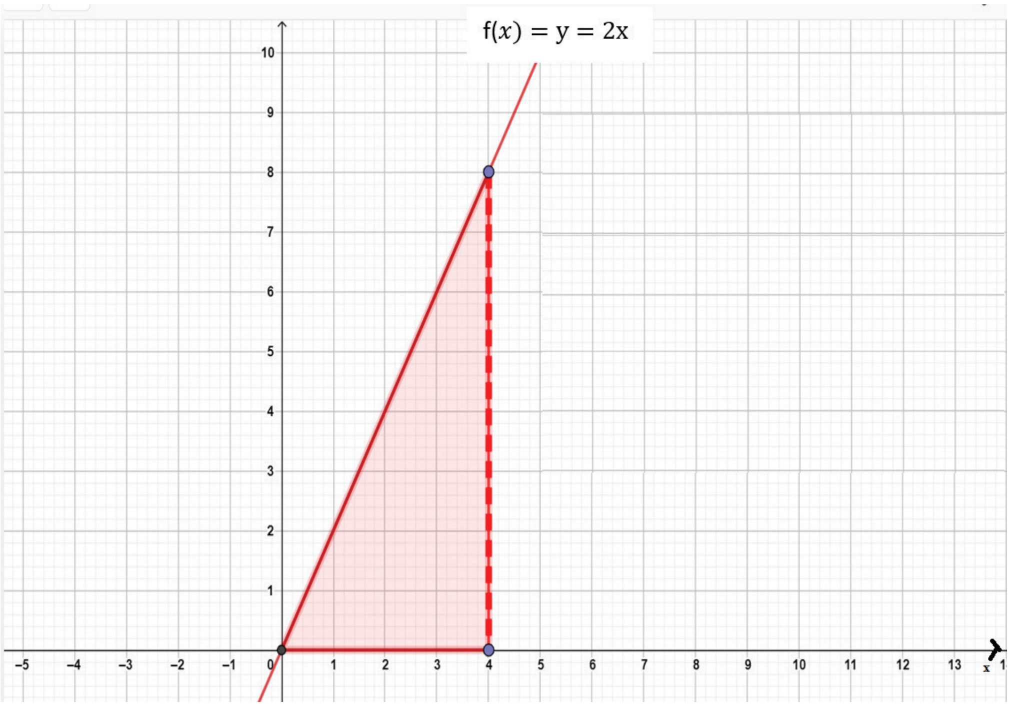


Figure: area under the curve of y = 2x

e) Procedures

Step 1: Observe the right-angled triangle OBC drawn in a Cartesian plane such that O(0,0) B(4,0) and C(4,8).

Step 2: Find the area of this triangle by using the formula as the base and the height are known. Record the answer in the table of data recording.

Step 3: Observe the other way of finding this area considering the function whose graph is the line passing by points O and C. Find the equation of this line. Is it $f(x) = 2x$?

Step 4: Consider the other boundaries of the triangle: What is the equation of the height? What is the equation of the base?

Step 5: Use the findings obtained from steps 3 and step 4 to find the area of the concerned triangle by using the appropriate integral.

Step 6: Compare the area found in step 2 and the area found in step 5. Are they equal or different?

Step 7: Change the value of the base and the height of such a triangle and verify the validity of the integral by comparing the area found by using the formula and the area found by using integration $A = \int_0^b 2x\,dx$. Is the integration applicable in real life to find the area of the geometric objects?

f) Data Recording

Ask the student to record the data in the table as follows;

$x = \textbf{\textit{base}} = \textbf{\textit{b}}$	***0***	***1***	***2***	***3***	***4***
Height =h ***0*** ***2***			***4***	***6***	***8***
$A = \frac{1}{2}bh$					
$A = \int_0^b 2x dx$					

Expected answers

Students record the data in the table as follows.

Base (b)	**0**	**1**	**2**	**3**	**4**
Height (h)	**0**	**2**	**4**	**6**	**8**
$A = \frac{1}{2}bh$	0	1	4	9	16
$A = \int_0^b 2x dx$	0	1	4	9	16

For all values of x = b considered as the base of the right angled triangle, made by the space bounded by f(x)=2x, x = b, y = f(b), the area $A = \frac{1}{2}bh$ is equal to the integral $A = \int_0^b 2x dx$.

g) Interpretation of results and conclusion

The definite integral is the area under the curve between two fixed points;

lower bound and upper bound. We see that $A=\frac{1}{2}bh$ is equal to the integral

$A=\int_0^b 2xdx$.

Therefore, definite integrals can be used to find the area of geometric figures. The answer found by using the definite integral is the same as the answer used by applying the formula of the area learnt in previous grades.

h) Information for teacher

In case the area is negative, please remember to interchange the boundaries of the functions or if the area is located in negative values of y -axis, we shall multiply by the negative sign.

$\int_a^b f(x)dx=-\int_b^a f(x)dx$ The calculation of a definite integral is to find the area of the surface under the curve.

i) Guidance on evaluation

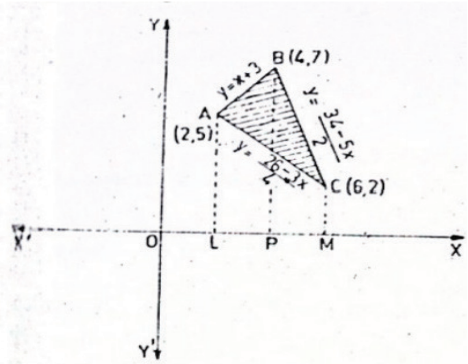
Using definite integral, give a chance to the learners to explore for finding the area of geometric figures in the Cartesian plane such as a square, rectangle, circle and sphere.

Example for deep understanding:

Using integration, find the area of the triangle ABC whose vertices have coordinates A (2, 5), B (4, 7) and C (6, 2).

Expected answer

Let A(2, 5), B(4, 7) and C(6, 2) be the vertices of the triangle.



Equation of AB is

$$y-5=\frac{7-5}{4-2}(x-2)$$

$$\Leftrightarrow y-5=x-2$$

$$\Rightarrow y=x+3$$

Equation of BC IS

$$y-7=\frac{2-7}{6-7}(x-4)$$

$$\Leftrightarrow y=\frac{-5}{2}x+10+7$$

$$\Rightarrow y=\frac{34-5x}{2}$$

Equation of AC is

$$y-5=\frac{2-5}{6-2}(x-2)$$

$$\Leftrightarrow y-5=\frac{-3}{4}(x-2)$$

$$\Leftrightarrow 4y-20=-3x+6$$

$$\Rightarrow y=\frac{26-3x}{4}$$

The area of the triangle ABC is

= Area of trapezium ABPL + Area of trapezium BCMP - Area of trapezium ACML

$$=\int_2^4 (x+3)\,dx+\int_4^6\left(\frac{34-5x}{2}\right)dx-\int_2^6\left(\frac{26-3x}{4}\right)dx \quad =7 \textit{ square units.}$$

Note: Let students know why on the third integration we are having a negative sign.

PRACTICAL ACTIVITY 8: **USING THE APPROPRIATE FUNCTION AND THE DEFINITE INTEGRAL PROPERTIES, DETERMINE THE AREA OF A CIRCLE**

a) Rationale

This activity is conducted when teaching the *application of definite integral* in real life. It is a topic for Unit 4 of S6 mathematics. In real life, people can use a circle in different games. In addition, during the construction of roads, engineer can use the different circles for creating a complete roundabout.

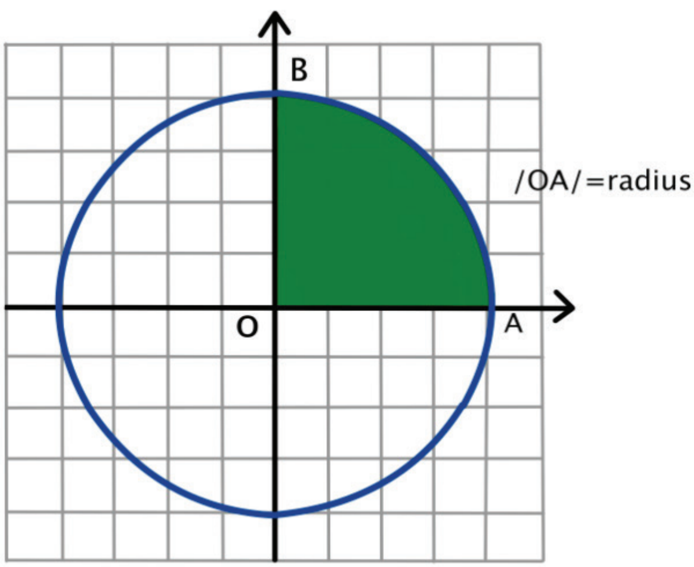
b) Objective

Use definite integral to determine the area of a circle of radius r whose centre is the origin of axis O.

c) List of required materials

Graph paper, ruler, pair of compasses, and pencil.

d) Illustration of the activity



e) Procedures

Step 1: Draw a circle of center O (0,0) and radius r =4 units of length (UL) as shown by the illustration above.

Step 2: Find the area of this circle by using the formula $(A = \pi r^2)$ considering the radius r = 4 Unit Length. Record the answer in the table of data recording.

Step 3: Observe the other way of finding this area considering the function whose graph is the fourth of a circle with radius r. Find the equation of the circle. Is it $y=\sqrt{r^2-x^2}$? Or $f(x)=\sqrt{r^2-x^2}$

Step 4: Use the findings obtained from step 3 to find the area of concerned circle by using the definite integral. It means integrating $f(x)$ from 0 to r.

Step 5: Compare the area found in step 2 and the area found at step 4.

Step 6: Repeat the procedures from step 1 to 5 for different radii. Compare the results obtained?

f) Data Recording

Radius ■	1 cm	2 cm	**5 cm**	10 cm	20cm
Area $\mathbf{A}=\pi\mathbf{r}^2$					
Area $\mathbf{A}=4\int_0^r\sqrt{\mathbf{r}^2-\mathbf{x}^2}\,\mathbf{dx}$					

Expected answer

$$\mathbf{A}=4\int_0^r\sqrt{\mathbf{r}^2-\mathbf{x}^2}\,\mathbf{dx}$$

Radius π	1 cm	2 cm	**5 cm**	10 cm	20 cm
$\mathbf{A}=\pi\mathbf{r}^2$	π	4π	25π	100π	400π
$\mathbf{A}=4\int_0^r\sqrt{\mathbf{r}^2-\mathbf{x}^2}\,\mathbf{dx}$	π	**4** π	**25** π	**100** π	**400** π

g) Interpretation of results and conclusion

The definite integral is the area under the curve between two fixed points; lower bound and upper bound. We see that the area for the circle of radius r is $A = \pi r^2$ and it is equal to the integral $\boldsymbol{A} = 4\int_0^r \sqrt{\boldsymbol{r}^2 - \boldsymbol{x}^2}\,\boldsymbol{dx}$.

Therefore, definite integrals can be used to find the area of geometric figures. The answer found by using the definite integral is the same as the answer used by applying the formula of area learnt in previous grades.

h) Information for teacher

In case the graph of the function is not a curve in which the formula of the area is known; we can't establish the comparison done in step 5. Calculating the definite integral is the only way to have the area of that surface.

- For a square $A = a\int_0^a dx = a^2$, for $0 \le x \le a$ Where ***a*** is the side of the square.
- For a rectangle use the function $A = w\int_0^l dx = wl$, $f(x) = l$ for $0 \le x \le l$

i) Guidance on evaluation

Using definite integrals, propose calculation of the area of square and rectangle, to let them understand that definite integrals can be used to solve problems from real-life situations.

PRACTICAL ACTIVITY 9: USING THE APPROPRIATE FUNCTION AND THE DEFINITE INTEGRAL PROPERTIES TO DETERMINE THE AREA OF AN ELLIPSE

a) Rationale

This activity is conducted during the teaching of the *application of definite integral* in real life. It is taught in Unit 4 (Integration) of S6 Mathematics. Finding the area of an ellipse can be done using the formula but to help students realize that the definite integrals are used in real life, There is a need to use integration to prove that the results is the same as the one found when using the formula learnt in previous grades. This also makes the lesson more understandable and enjoyable. The ellipse can be applied in technology of the two-stage oscillator, which has a great potential for practical application in various fields like in: electricity generation, water pumping, irrigation, water purification, geothermal heat pumping, oil pumping, etc.

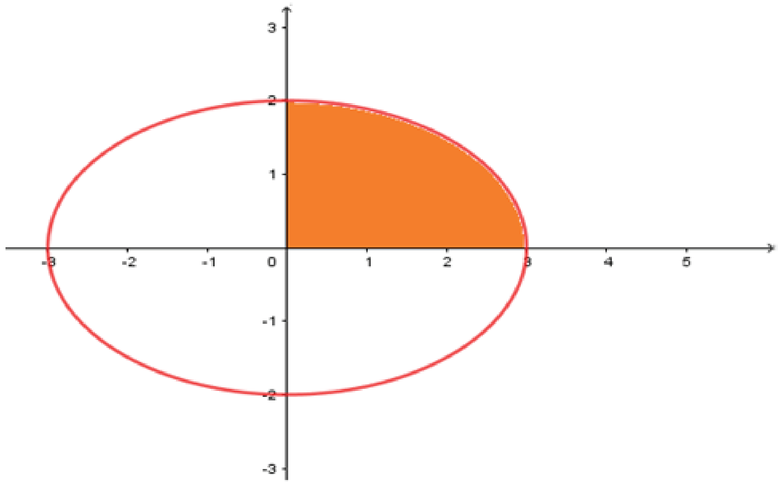
b) Objective

Use definite integral to determine the area of an ellipse.

c) List of required materials

Graph paper, ruler, pair of compasses, scientific calculator,

d) Illustration of the activity



e) Procedures

Step 1: Choose an ellipse of the semi-major axis a = 3 Unit of Length and semi-minor axis b =2 Unit of Length and calculate the area A of the ellipse using the formula learnt in previous grades.

Step 2: Find the function $f(x)$ with the variable x such that its graph is the ellipse whose centre is the origin of the axis, the semi-major axis a =5 and the semi-minor axis b =2.5.

Step 3: Calculate the area A of this ellipse using the appropriate definite integral of the function $f(x)$.

Step 4: Take different ellipses with different semi -majors axis and different semi-minor axis and then complete the table of data recording.

f) Data Recording

Semi-major axis (a)	1 cm	2 cm	5 cm	10 cm	20 cm
Semi-minor axis (b)	0.5 cm	1 cm	2.5 cm	5 cm	10 cm
Area $A=\pi ab$					
Area A= $4\frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$					

Expected answers

Semi-major axis (a)	1 cm	2 cm	5 cm	10 cm	20 cm
Semi-minor axis(b)	0.5 cm	1 cm	2.5 cm	5 cm	10 cm
$A=\pi ab$	0.5 π	2 π	12.5 π	50 π	200 π
$4\frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$	0.5 π	2 π	12.5 π	50 π	200 π

g) Interpretation of results and conclusion

If we compare the answers found in step 1 and step 3 as shown in step 3, we find:

$\boldsymbol{A = 4\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx}$ Let $x = a\sin t \quad \Leftrightarrow dx = a\cos t dt\,,$ $If\ x = 0 \quad ,t = o$ $If\ x = a \quad ,t = \frac{\pi}{2}$	A= $4\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx = 4\frac{b}{a}\int_0^{\frac{\pi}{2}} a^2\cos^2 t dt$ $= 2ab\int_0^{\frac{\pi}{2}}(1+\cos 2t)dt$ $= 2ab\left|1 + \frac{\sin 2t}{2}\right|_0^{\frac{\pi}{2}}$ $= \pi ab$ Therefore $A = \pi ab = 4\frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx$

The definite integral is the area under the curve between two fixed limits called boundaries. It can be used to find area of geometric figures for which we only know related functions.

h) Information for teacher

In case the graph of the function is not a curve in which the formula of the area is known; we can't establish the comparison done in step 3. Calculating the definite integral is the only way to have the area of that surface.

i) Guidance on evaluation

Using definite integral, derive the area of the trapezium. For the trapezium use the function $f(x) = \frac{b}{h}x + \frac{B}{2};\ \ 0 \le x \le h$ where b and B are the basis and h is the height of the trapezium.

PRACTICAL ACTIVITY 10: DETERMINING THE SHAPES BY ROTATING A GRAPH OF A FUNCTION AROUND THE X-AXIS OR Y-AXIS.

a) Rationale

This activity is conducted to engage learners in the lessons of finding volume of a solid of revolution by application of definite integral in real life. It is found in u**nit 4 (Integration) of S6 Mathematics.** The activity is based on hands on activity and the observation of the way that can be used to obtain solids easily by rotating a graph of function around an axis. A complete rotation is equal to one revolution or rotation of 360^0. This practice is fun since it arouses the interest of students on the importance of Mathematics in the production of home materials. In addition, the activity facilitates students to determine the volume of solids using definite integrals as they will be aware of the shapes they are dealing with. This is an inquiry based practical activity

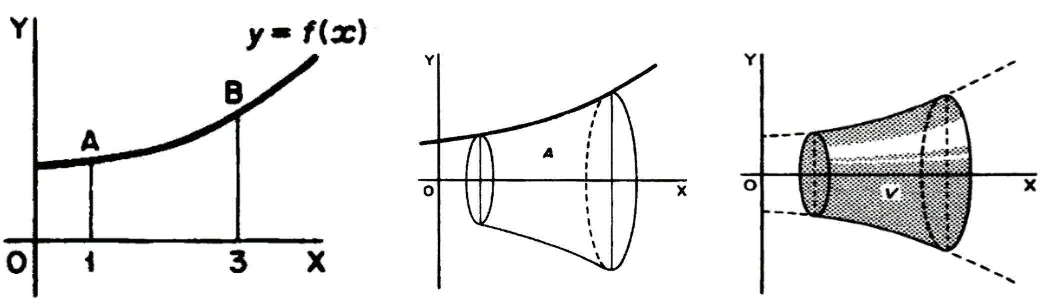
b) Objective

Determine the shapes by rotating a graph of given function through one revolution around the x-axis or y-axis.

c) List of required materials

Graph paper, douzaine of tooth sticks, ruler, pencil/ pen, mirror

d) Illustration of the activity



e) Procedures

Phase 1: Practice

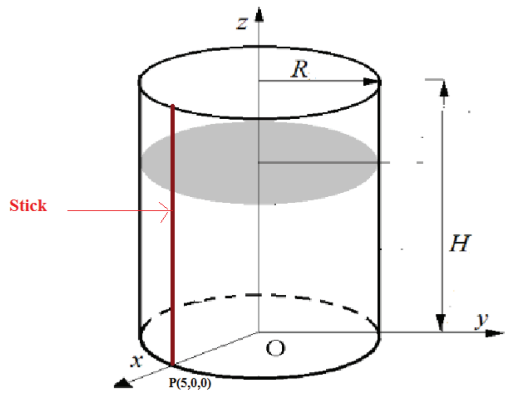
Step1: Mark a point O (0,0,0) on a drawing board fixed on a table and draw axis x and y plane or (0,x) and (o, y) axes.

Step 2: Measure a line segment of 5 units of length on the horizontal axis and mark the point P (5,0,0). Draw a circle of radius r = 5 UL on the plane of the graph paper.

Step 3: Consider the circle, and fix a vertical stick (tooth sticks) at the point P (5,0,0) paralleling with the axis OZ

Step 4: Fix as many as possible sticks around the circumference of the circle. What does obtained shape look like?

Expected results



Phase 2: Extension

Observe the following materials and explain functions that were rotated and axis of rotation

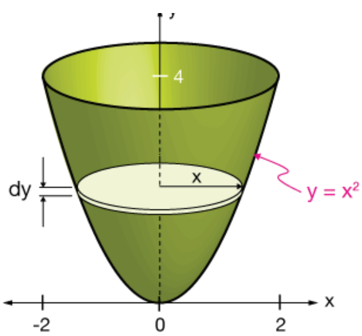


Figure (a)

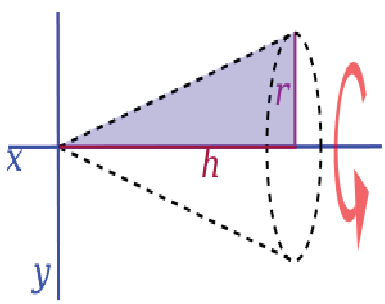


Figure (b)

Discuss what you can get if you rotate the following functions of the plane XOY:

Curve rotated	Obtained shape
A segment of horizontal line parallel to x-axis With end points (1,2) and (4,2)	
A segment of an oblique line joining (0,0) and (2,2)	

Step 6: Draw a Cartesian plane on a graph paper**.** Choose a linear function $f(x)=2+x^2$ with a predetermined interval, record the data in the table as follows, draw the graph on another graph paper and rotate the obtained curve on a revolving axis (x-axis or y-axis). It must be a one revolution.

Can you see that you obtained a new shape? If yes, are you able to give the name of new obtained shape?

x	***0***	***1***	***2***	***3***	***4***
$y=f(x)$					

Expected results

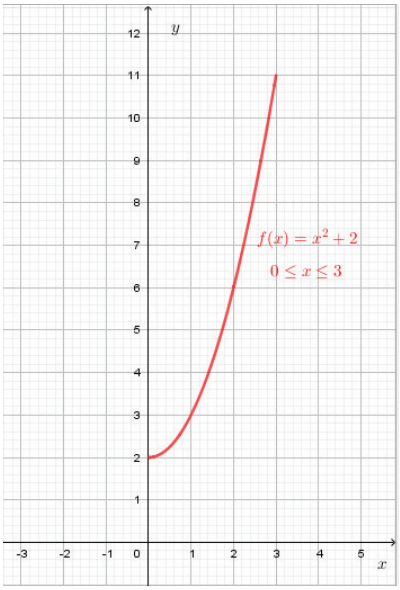
At step 5:

Curve rotated	Obtained shape
A segment of line parallel to ox and joining points (1,2) and (4,2)	A cylinder
A segment of an oblique line coming from (0,0) to (2,2)	A cone

At step 6: If we rotate $f(x)=2+x^2$, *for* $x \geq o$, we obtain the following table

x	***0***	***1***	***2***	***3***	***4***
$f(x)=2+x^2$	***2***	***3***	***6***	***10***	***18***

If we rotate $f(x)=2+x^2$, *for* $x \geq o$, we obtain the following:



- **Around OY:** We find a Cup
- **Around OX:** We find a material that looks like a hat.

f) Interpretation of results and conclusion

Rotating the given curve around an axis helps us to obtain different forms of solids. It shows us that all materials we use at home can be produced basing on the rotation of a function around an axis: Cylinder, cone, cup, hut, etc are solids of revolution.

g) Information for teacher

Teacher should remind students that they can attempt one revolution (360˚) and record the result. It is not necessarily to use y- axis or x- axis as a revolving axis. Any straight line can be considered as a revolving axis. For example: We can rotate a certain curve around x=2, y=3, x=5, etc…

Teacher can also give a critical thinking question about:

1. Rotation of a point around OX or OY
2. Rotation of a vertical/ horizontal segment around OX/OY
3. Rotation of a part of broken lines…

h) Guidance on evaluation

Give a chance to the students to explore and think of other shapes they can rotate for getting the solids. Let them try rotating what they obtained around the x-axis and y-axis and tell what they obtained as a result.

PRACTICAL ACTIVITY 11: USING DEFINITE INTEGRAL OF THE APPROPRIATE FUNCTION TO DETERMINE THE VOLUME OF A CYLINDER

a) Rationale

This activity is conducted when teaching the application of definite integral to determine the volume of revolution for cylindrical shapes. This lesson is taught in unit 4 of S6 Mathematics. In real life, we find cylindrical shapes everywhere. Even the colour pencil boxes are cylinders. Gas cylinders we have at home, oxygen cylinders in hospitals and many cosmetic boxes have the shape of a cylinder.

b) Objective

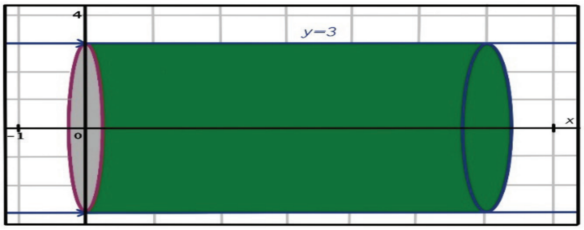
Determine the volume of revolution for a cylinder around x-axis of a horizontal segment of length h at the distance **r** from x-axis or around y-axis of a vertical segment of length height at the distabce r from y-axis. This is a concept-based activity.

c) List of required materials

graph paper, ruler, scientific calculator.

d) Illustration of the activity

Use integration to determine the volume of the solid generated when the line $L \equiv y = 3$ for $0 \le x \le 6$ is revolved around $x - axis$.



e) Procedures

Step 1: Let's consider a cylindrical object of base with radius r = 3dm, and the height h=6dm.

Step 2: Calculate the volume V of that cylinder using the formula learnt in previous grades

($V = Area\ of\ Base \times height = \pi r^2 h$).

Step 3: On a Cartesian plane XOY of origin O (0,0), draw the graph of the function $y = f(x) = 3$.

Step 4: Consider the graph of the function $f(x)$ from x=0 to x = 6 and rotate it around the x-axis at 360°. What is the solid of revolution do you get? What are its characteristics?

Step 5: Calculate the volume V of the cylinder using definite integral of the function $f(x)$ on boundaries y=0 and y=3. Hint: $V = \pi \int_0^h y^2 dx$

Step 6: Record and compare results obtained in step 2 and 5.

Step 7: Consider the cylinders having the same height with varying radii, where $V = \pi \int_0^h y^2 dx$ and complete the table. Is the integration used in our real life? Does it make Sense?

f) Data Recording

Radius r	1cm	2 cm	3 cm	4 cm	10 cm
Height	12 cm	12 cm	12 cm	12 cm	12 cm
$V = \pi r^2 h$					
$V = \pi \int_0^h y^2 dx$					

Expected answer

Radius r	**1cm**	**2 cm**	**3 cm**	**4 cm**	**10 cm**
Height	**12 cm**	**12 cm**	**12 cm**	**12 cm**	**12 cm**
$V = \pi r^2 h$	12π cm³	48π cm³	108π cm³	192π cm³	1200π cm³
$V = \pi \int_0^h y^2 dx$	12π cm³	48π cm³	108π cm³	192π cm³	1200π cm³

For each case, we see that the calculated volume using the formula is the same as the volume found after integration.

$V=\pi r^2 h=\pi\int_0^h y^2 dx$ For the given data.

g) Interpretation of results and conclusion

To find the volume of a cylinder, we can use the definite integral $V=\pi\int_0^h f^2(x)dx$ where the the function $\boldsymbol{f(x)=y_0}$ for $x_0\le x\le h$ is revolving around $x-axis$.

h) Information for teacher

The volume of the solid of revolution bound by the curve of the function $f(x)$ about the $x-axis$ calculated from $\boldsymbol{x=a}$ to is $\boldsymbol{x=b}$ given by $V=\pi\int_a^b (f(x))^2 dx$

This method is called disc method. If the region is rotated about a horizontal line, integrate with respect to x, and if the region is revolved about a vertical line, integrate with respect to y.

i) Guidance on evaluation

Propose learners to calculate the volume of the solid generated by the revolution of other functions around $x-axis$.

For example, the volume of a cone, the Frustum of a cone (Trunk of the cone) and the sphere.

- For the cone, use the function $f(x)=\frac{r}{h}x$ for $0\le x\le h$
- For the Frustum of the cone, use the function $f(x)=\frac{R-r}{h}x+r$ for $0\le x\le h$
- For the Sphere, use the function $f(x)=\sqrt{r^2-x^2}$ for $-r\le x\le r$.

PRACTICAL ACTIVITY 12: USING THE DEFINITE INTEGRAL OF THE APPROPRIATE FUNCTION TO DETERMINE THE VOLUME OF A CONE.

a) Rationale

This activity is done when teaching the application of definite integral. It is learnt in Unit 4 of S6. It sounds hard to the students when talking about definite integrals but if it is applied in real life, it will be easy for them to understand the concept. Students are already familiar with a cone and know how to calculate its volume using the formula. Letting them know that definite integrals can also be used to get the same results will be an added value and will attract their interest in the activity. This is an inquiry based practical activity

In real life, a cone is a three-dimensional shape in geometry that narrows smoothly from a flat base (usually circular base) to a point (which forms an axis to the centre of the base) called the apex or vertex. We can also define the cone as a pyramid which has a circular cross-section, unlike a pyramid which has a triangular cross-section. There are many examples of cones in real life: Christmas

The Motivating Question is:

How can we use a definite integral to find the volume of a three-dimensional solid of revolution obtained from revolving a two-dimensional function about a particular axis?

b) Objective

To determine the volume of revolution for the cone generated by the right-angled triangle around using definite integral.

c) List of required materials

graph paper/manila paper, ruler, pen/pencil, rubber, and scientific calculator.

d) Illustration of the activity

Illustration of the cone generated when the linear function is revolved around the x-axis.

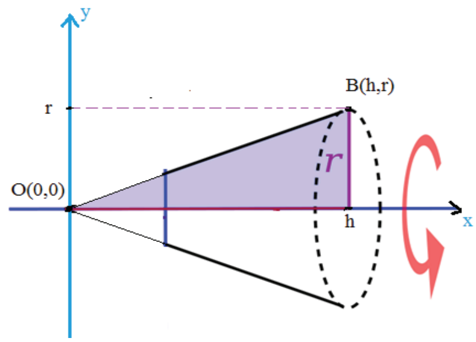


Figure 1

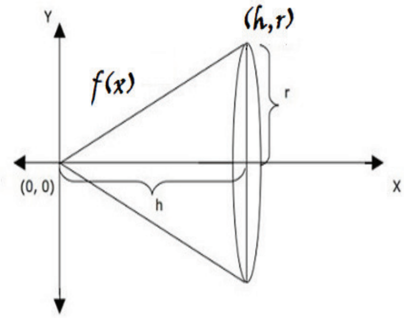


Figure 2

e) Procedures

Step1: Draw a well-labelled Cartesian plane

Step 2: Find the linear function $y = f(x)$ passing through the point $O(0,0)$ *and* $B(h,\mathrm{r})$. For more clarification, let us take B (3,4).

Step3: Revolve the triangle obtained around the x-axis (anti-clockwise). What solid do you get? Show the characteristics of the solid obtained (radius, height, edge, vertex, face)

Step 4: Calculate the volume (V) of the shape using $V = \frac{1}{3}\pi r^2 h$

Step 5: Calculate the volume (V) of the generated solid using the definite integral of the function $f(x)$. The definite integral is given by $V = \pi\int_0^h y^2 dx$ such that $y = f(x)$.

Step 6: Compare the volume obtained in step 4 and the volume obtained in step 5.

Step 7: Take different points $B(h,\mathrm{r})$ by changing the coordinates (h, r) of the point B, rotate the line segment OB around x-axis and in each case, calculate the volume of the solid obtained in the two ways mentioned in step 4 and step 5. Complete the results in the table of data recording below and in each case, compare the volume obtained.

Is the integration used in our real life? Does it make Sense?

f) Data Recording

Radius	r	1cm	2cm	3cm	4cm	8cm
Height	h	10cm	10cm	10cm	10cm	10cm
Volume formula	$V=\frac{\pi r^2 h}{3}$					
Volume -integration	$V=\pi\int_0^h y^2dx$					

Expected answer

- The line OB is expressed by the linear function $y=f(x)=\frac{r}{h}x$ obtained by finding the equation of the line passing by the points $O(0,0)$ *and* $B(h,r)$
- By rotating the line segment OB around x-axis, we find **a cone of radius r and height h.**
- The volume of such a cone is $V=\frac{Area\ of\ base\times height}{3}=\frac{\pi r^2 h}{3}$
- For the given function $y=f(x)=\frac{r}{h}x$, the integration becomes

  $$V=\pi\int_0^h y^2dx=\frac{\pi r^2 h}{3}$$

- These results $\frac{\pi r^2 h}{3}$ obtained by integration is the same as the one obtained by calculation.
- By changing the position of the point B (h, r) we have the results in the following table:

Radius	r	1cm	2cm	3cm	4cm	10cm
Height	h	10cm	10cm	10cm	10cm	10cm

Volume formula	$V=\frac{\pi r^2 h}{3}$	$\frac{10\pi}{3}$ cm^3	$\frac{40\pi}{3}$ cm^3	30π cm^3	$\frac{160\pi}{3}$ cm^3	$\frac{1000\pi}{3}$ cm^3
Volume -integration	$V=\pi\int_0^h y^2dx$	$\frac{10\pi}{3}$ cm^3	$\frac{40\pi}{3}$ cm^3	30π cm^3	$\frac{160\pi}{3}$ cm^3	$\frac{1000\pi}{3}$ cm^3

g) Interpretation of results and conclusion

- We can obtain a cone by rotating a line segment OB from the origin O (0,0) to the point B (h, r) around the x-axis.
- The volume of the cone of radius r and height h found by calculation $\frac{\pi r^2 h}{3}$ is equal to its volume obtained by using the integration $\pi\int_0^h y^2dx$.
- In real life, integration can be used to find the volume of solids.

h) Information for teacher

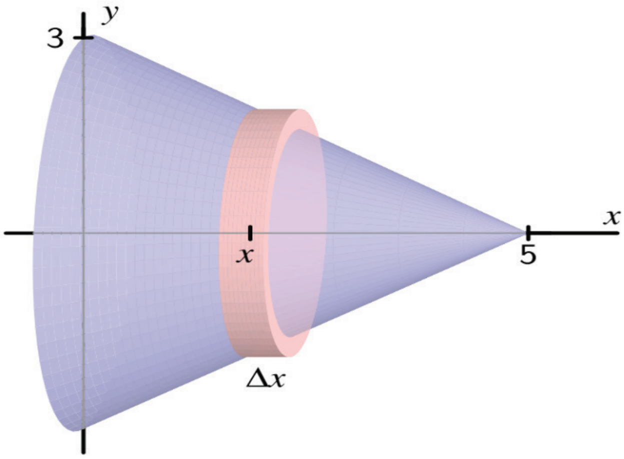
The volume of the solid of revolution bound by the curve of the function about the x- axis calculated from $\boldsymbol{x}=\boldsymbol{a}$ to is $\boldsymbol{x}=\boldsymbol{b}$ is given by $V=\pi\int_a^b (f(x))^2dx$.

This method is called the disc method. If the region is rotated about a horizontal line, integrate with respect to x, and if the region is revolved about a vertical line, integrate with respect to y.

i) Guidance on evaluation

Give learners a similar activity to find the volume of solid of revolution. For example:

Consider a circular cone of radius 3 and height 5, which we view horizontally as pictured in Figure below. Our goal in this activity is to use a definite integral to determine the volume of the cone.



The circular cone Described in the Preview

a. Find a formula for the linear function

b. What definite integral will be used to find the volumes of the full horizontal span of the cone? What is the exact value of this definite integral?

c. Compare the result of your work in (b) to the volume of the cone that comes from using the formula $V = \frac{Area\ of\ base \times height}{3} = \frac{\pi r^2 h}{3}$

Expected answer

a. $y = -\frac{3}{5}x + 3$

b. $V = \pi \int_0^5 \left(f(x)\right)^2 dx = \pi \int_0^5 \left(\frac{-3x}{5} + 3\right)^2 dx = 15\pi\ cubic\ units$

c. $V = \frac{Area\ of\ base \times height}{3} = \frac{\pi r^2 h}{3} = \frac{\pi (3)^2 (5)}{3} = 15\pi\ cubic\ units$

Therefore, the results in (b) are the same as the ones obtained in (c).

UNIT: 5 DIFFERENTIAL EQUATIONS

PRACTICAL ACTIVITY 13: USING DIFFERENTIAL EQUATIONS TO EXPLORE THE SIMPLE HARMONIC MOTION

a) Rationale

This activity can be done when teaching the application of differential equations in real-life situations. It is learnt in Unit 5 of S6. **Simple** harmonic Motion occurs when a particle or object moves back and forth within a stable equilibrium position under the influence of a restoring force proportional to its displacement. There are so many examples of simple harmonic motions that are playing a great role in our daily life as well as making our daily life activities easier. Examples of simple harmonic motions are observed in **simple pendulum, torsion pendulum, hearing process, mass loaded on a spring, etc.** This is a inquiry based practical activity.

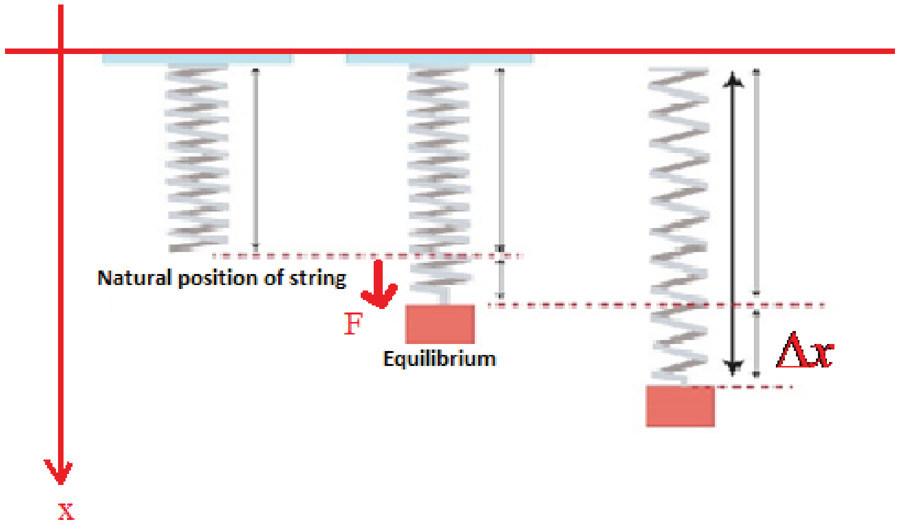
b) Objective

1. To establish and interprete the differential equation of a simple harmonic motion and
2. To solve diiferential equation of simple harmonic equation when given initial conditions.

c) List of required materials

Spring, ruler, mass, chronometer, a force gauge/ force metre.

d) Illustration of the activity



e) Procedures

Step 1: Attach a mass m at the end of a spring-suspended to a rigid support.

Step 2: Pull the mass downward with the force $\vec{F}$ at a certain length then let it move.

Step 3: Measure the time of movement from the last position to go upward and return to the same position. How is the movement of the mass?

Step 4: The particle is affected by a restoring force $\vec{F}$ opposite to the initial direction. This means that if k is a positive constant called the spring constant, the vector on the spring can be expressed by $\vec{F} = -k\vec{x}$ (resultant force).

Use the mass **m** and the acceleration of the particle to deduce the vector equation $\vec{F} = -k\vec{y}$ into a differential equation.

Step 5: The body attached to the spring has a downward force $\vec{F} = m\vec{a}$, where $\vec{a} = y''$

Step 6: Sum up the two opposite forces

Step 7: Verify if obtained differential equation looks like $my'' + ky = 0$ and find the value of ω

Step 8: If the mass is of 1kg and the spring constant k = 0.04 (k-nature of a spring), solve the differential equation obtained to indicate the position of the mass at any point in time t.

Step 9: Verify if $y = A\sin(\omega t + \phi)$ and $y = A\sin\omega t$ are the solutions of the equation and explain their difference.

Expected answer

The particle moving with following an harmonic motion has the vector equation $\vec{F} = -k\vec{y}$

Given that its mass is m, the force communicates the acceleration $\vec{a}$ such that

$m\vec{a} = -k\vec{y}$,

or $my''\vec{i} = -ky\vec{i}$ that implies

$$y'' + \frac{k}{m}y = 0$$

This is the differential equation of the motion.

Let $\omega^2 = \frac{k}{m}$, the differential equation becomes

$y'' + \omega^2 y = 0$. This is the simplified differential equation of the motion where

$\omega = \sqrt{\frac{k}{m}}$

The solution of this differential equation gives: $y(t) = A\cos(\omega t) + B\sin(\omega t)$ where A and B are constant.

We can replace the mass m by 1kg and the spring constant k by 0.04 in all expression above to get the required expressions.

Notes:

The solution $y(t) = A\cos(\omega t) + B\sin(\omega t)$ can be written in an other form. Let $y(t) = A\cos(\omega t) + B\sin(\omega t) =$

$$y(t)=\sqrt{A^2+B^2}\left[\frac{A}{\sqrt{A^2+B^2}}\cos(\omega t)+\frac{B}{\sqrt{A^2+B^2}}\sin(\omega t)\right]$$

There exist an angle ϕ such that $\frac{A}{\sqrt{A^2+B^2}}=\sin\phi$ and $\frac{B}{\sqrt{A^2+B^2}}=\cos\phi$

The solution becomes $y(t)=M\sin\phi\cos(\omega t)+\cos\phi\sin(\omega t)$

Or $y(t)=M\sin(\omega t+\phi)$ where M is called the amplitude of the osciation and ϕ is called the phase shift of the motion. In this case,

i. $x=A\sin\omega t$ this is a solution of the differential equation when the particle is in its mean position (point 0).

ii. $x=A\sin\phi$ When the particle is at the at the position different from mean position

iii. $x=A\sin(\omega t+\phi)$ when the particle is at another position at any time.

iv. This differential equation has the general solution $x(t)=c_1\cos\omega t+c_2\sin\omega t$

f) Data Recording

The particle moving with a harmonic motion has the vector equation

$\vec{F}=-k\vec{x}$

The differential equation becomes

$mx''+\omega x=0$ where $\omega=\sqrt{\frac{k}{m}}$

The solution of this differential equation is $x=A\sin(\omega t+\phi)$ or the combination of $x=A\sin(\omega t+\phi)+B\cos(\omega t+\phi)$

This explains well the reason why the motion is made by oscillations.

The above general solution gives the position of the mass at any point in time.

g) Information for teacher

The system composed of the spring and the mass is commonly called a **spring-mass system**. **Gravity** is pulling the mass downward and the restoring force of the spring is pulling the mass upward. When these two forces are equal, the mass is said to be at the **equilibrium position**. The motion of the mass is called **simple harmonic motion.**

The **period** of this motion (the time it takes to complete one oscillation) is

$T = \frac{2\pi}{\omega}$ and the frequency is $f = \frac{1}{T} = \frac{\omega}{2\pi}$

h) Guidance on evaluation

Propose to students exercises in which they can determine a characteristic of a simple harmonic motion when other data are given.

For example:

When a mass of 200g is hung from a spring, the spring is displaced downward at a distance of 1.5 cm.

i. What is the spring constant k? [F=mg, x=1.5 cm= 0.015m]
ii. An additional mass of 400g to the initial 200-g mass in. What will be the increase in the downward displacement? (ΔF is due only to the added mass).
iii. Amass at the end of a spring vibrates up and down with a frequency of 0.600Hz and an amplitude of 5cm. what is its displacement 2.56 seconds after it reaches a maximum?

Expected answer

$$i)k = \frac{F}{x} = \frac{(0.200kg)(9.8m/s^2)}{0.015m}; k = 131N/m$$

$$ii)k = \frac{\Delta F}{\Delta x}; \Delta x = \frac{\Delta F}{k} = \frac{(0.400\,\mathrm{kg})(9.8\ \mathrm{m/s^2})}{131\ N/\mathrm{m}}; \Delta x = 2.99\ cm$$

$$iii)x = A\cos\ (2\pi f\ \mathrm{t}) = (5\,\mathrm{cm})\cos[2\pi(0.6\,\mathrm{Hz})(2.56\,\mathrm{s})]; x = -4.87cm$$

PRACTICAL ACTIVITY 14: USING DIFFERENTIAL EQUATIONS TO EXPLORE THE POPULATION GROWTH

a) Rationale

This activity will be done when teaching the application of differential equations in real-life situations, concept taught in Unit 5 of S6. The growth or decay factor describes the rate at which a quantity is changing over a certain period. In everyday life, differential equations can be used to calculate rates of change for speed of moving particles, growth and decay of populations and materials, heat flow, fluid flow, and so on. In each case, we can construct models of varying degrees of sophistication to describe given situations. This is a concept based practical activity. This is a concept based practical activity.

b) Objective

Establish the differential equation of population growth and solve it when given initial conditions.

c) List of required materials

Microscope, ruler, calculator, pen, pencil, graph paper

d) Illustration of the activity

Decaying avocado and **teeth for an pig** due to bacteria duplication. This means that as the number bacteria (cells) increases, teeth become more damaged.

 **Source**: https://edgedental.com.au/teeth-like-apple-cores-terrifying-reality-extreme-tooth-decay/ Damaging teeths caused by the increase number of bacteria	 Source: https://www.google.com/search?q=decay+avocado&sxsrf Damaging avocadoes caused by the multiplication of bacteria.

e) Procedures

Step 1: Observe a rotten avocado using a microscope and model the experiment referring to the observation. For example, a colony of bacteria may double every hour. If the size of the colony after hours is given by $y(t)$, then we can express this information in mathematical language in the form of an equation: $\frac{dy}{dt} = 2y$

A quantity $y(t)$ that grows or decays at a rate proportional to its size fits in an equation of the form $\frac{dy}{dt} = ky$

If , the above equation is called the law of natural decay and if , the equation is called the law of growth.

This is an example of a differential equation because it gives a relationship between a function and one or more of its derivatives.

Step 2: Given differential equation $\frac{dy(t)}{dt} = k\,y(t)$

Step 3: Verify that $y(t) = Ce^{kt}$ is also a solution for any constant C.

Step 4: If you found that $y(t) = Ce^{kt}$ is a solution, consider $y(0) = 100$ cells, deduce the value of C and then, write the solution $y(t) = Ce^{kt}$ without the constant C

Step 5: Using the obtained solution at step 4, find the number of cells after 10 second, 20 second and 10 days if $k = 0.01$.

Step 6: Represent the obtained function graphically on a Cartesian plane**.**

Is the number of cells increasing or reducing? What can you say about the status of tooth after 10 days?

f) Data Recording

t	0	1	2	5	10	10 days
$y(t)$	100					

Expected answers

t	0	1	2	5	10	10 days
$y(t)$	100					

g) Interpretation of results and conclusion

If P is the size of a population at a time t. The law of natural growth is a good model for population growth (up to a certain point):

$$\frac{dy(t)}{dt} = k\,y(t) \text{ and } y(t) = y_0 e^{kt}$$

As the exponential function is increasing from the value y_0 (when t = 0), the number of population is increasing with time.

In our case, The number of cells increases with time from 100 cells at t=0. If toohs are not blushed, the number of bacteria increases with time and they continue to domage tooth.

h) Information for teacher

There are several well-known applications of first-order equations which provide classic prototypes for mathematical modelling. These mainly rely on the interpretation of $\frac{dy(t)}{dt}$ as a rate change of a function with respect to time .

i) Guidance on evaluation

Propose learners' exercises on population growth.

Example 1:

The population of bacteria called Mathland at the end of the year 2000 was 500. The population increases (continuously steadily) by approximately 10% per year. What is the function $P(t)$ of the size of the population after t years using the exponential model above?

a. What is the differential equation satisfying the function $P(t)$?
b. What is the value of k ?
c. What is $P(0)$?

d. Give the formula for $P(t)$,

e. What will the population be at the end of 2050 years?

Expected answer

a. $\frac{dP(t)}{dt} = kP(t)$

b. k = rate of the growth = 0.1

c. $P(0)$ =initial population size = 500

d. $P(t) = P(0)e^{kt} = 500e^{0.1t}$

e. At the end of 2050 years, we will have $t = 50$ and the population will be

$P(50) = 500e^{0.1(50)} = 500e^{5} \approx 74,206$

Example 2

The population of bacteria called Calculand was 700 in the year 2000 and 3000 in the year 2010. Using the exponential model for population growth, find an estimate of the population of Calculand in the year 2015.

Expected answer

. $P(0) = ?$ If we set $t = 0$ in year 2000, therefore $P(0) = 700$

. $\mathrm{P}(10) = ?$ when $t = 10$, the year is 2010, so $\mathrm{P}(10) = 3000$

. Find the value of k.

$P(t) = P(0)e^{kt}$

$P(10) = 700e^{k(10)}$

$\Leftrightarrow 3000 = 700e^{10k}$

For finding the value of k., we apply the logarithmic function:

$\Leftrightarrow \ln(3000) = \ln(700) + 10k\ln(\mathrm{e})$

$\Rightarrow \mathrm{k} = \frac{\ln(3000) - \ln(100)}{10} = \frac{\ln\left(30/7\right)}{10} \approx 0.147$.

For the formula for $P(t)$, use it to find $P(15)$

$P(t) = P(0)e^{kt}$

$\Rightarrow P(15) = 700e^{0.147(15)} \approx 6,210$

UNIT: 8 CONICS

PRACTICAL ACTIVITY 15: INTRODUCTION TO CONICS

a) Rationale

This activity is conducted when teaching conics. It is taught in Unit 8 of S6. In real life carrots can be used in this activity. This is because students are familiar with carrots and thus it will be fun to experiment using them. Students will get engaged in observing phenomena and/or manipulate real objects.

For instance, the paths of the planets around the sun are ellipses with the sun at one focus. Parabolic mirrors are used to converge light beams at the focus of the parabola. Hyperbolic as well as parabolic mirrors and lenses are used in systems of telescopes. This is a concept based practical activity.

b) Objective

Based on this experiment, students will determine the characteristics and the different forms of the graph of a conic like a cycle, ellipse, parabola, and hyperbola and present them in the Cartesian plane.

c) List of required materials

Table, knife, carrot, pencil/pen, graph paper, and ruler.

d) Illustration of the activity

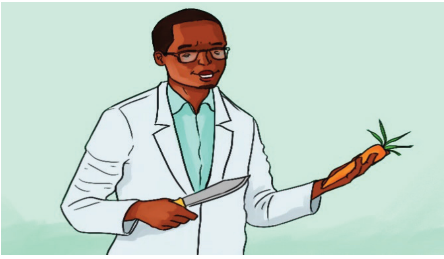


Figure 1

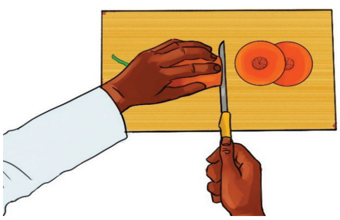


Figure 2

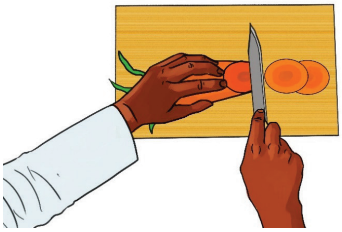


Figure 3

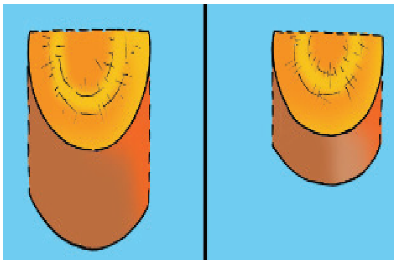


Figure 4

e) Procedures

Step 1: Take a carrot, put it on a table and cut it in a perpendicular position as shown in **Figure 2**;

Step 2: What type of a shape do we get (considering the outer surface of the cross section of the carrot)?

Step 3: Take another carrot and cut it at certain slope (in oblique way) as shown in **Figure 3**

Step 4: What a shape do we get (considering the outer surface of the cross section of the carrot)?

Step 5: Divide the obtained shape in step 3 into two parts as shown in **Figure 4**.

Step 6: What a shape do we get (considering the outer surface of the cross section of the carrot)?

Step 7: Repeating the step 3, rotate each part at 90^0.

Step 8: What a shape do we get (considering the outer surface of the cross section of the carrot)?

Step 9: Try to draw the obtained shapes on the graph paper.

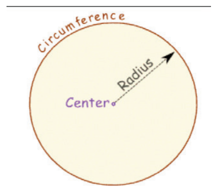
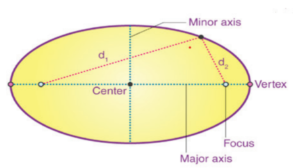
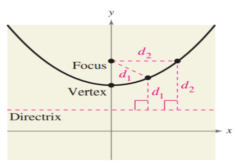
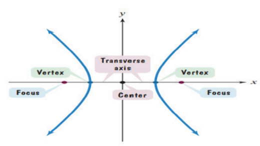
f) Data Recording

By referring to the different forms obtained, ask students to think about filling the table below, and give them time to think about different real-life examples that resemble the same shapes.

Shape	Figure
Circle	
Ellipse	
Parabola	
Hyperbola	

Is it making sense to cut the carrots? Can you find other materials that can be transformed and give the shape of a circle, an ellipse, a parabola, or an hyperbola? Fill out the table above with the asked information.

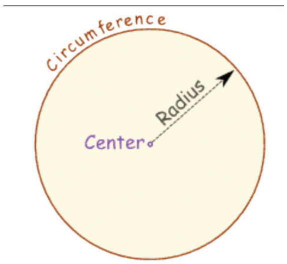
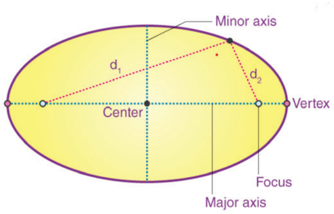
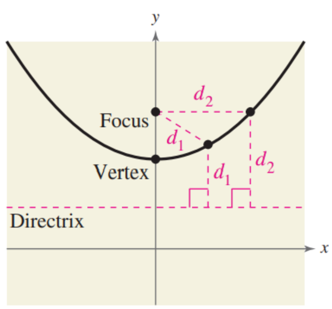
Expected answer

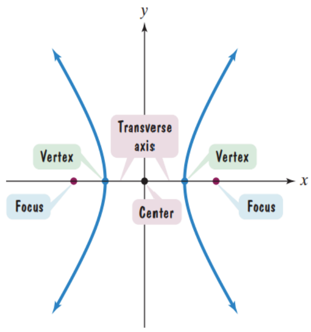
Steps	**Shape**	**Figure**
2	Circle	
4	Ellipse	
6	Parabola	
8	Hyperbola	

g) Interpretation of results and conclusion

When we cut a carrot in different ways we obtain shapes in form of a circle, an ellipse, and a parabola.

g) Information for teacher

Shape	Figure	Interpretation
Circle		**Circle**- The set of points in a plane equidistant from a given point (the center of the circle), where a **radius** is a segment from the center of the circle to a point on the circle (the distance from the center to a point on the circle.) and **circumference** is the distance around the edge of the circle.
Ellipse		An **ellipse-** is the locus of all those points in a plane such that the sum of their distances from two fixed points in the plane, is constant. The fixed points are known as the **foci** (singular focus), which are surrounded by the curve. The fixed line is directrix and the constant ratio is the eccentricity of the ellipse. Eccentricity is a factor of the ellipse, which demonstrates its elongation and is denoted by 'e'. an ellipse has also two axes: the major axis and the minor axis
Parabola		**A parabola**- is the set of all points in a plane that are equidistant from a fixed line (directrix) and a fixed point (focus) not on the line. The midpoint between the focus and the directrix is called **the vertex**, and the line passing through the focus and the vertex is called the axis of the parabola. Note: A parabola is symmetric with respect to its axis.

Hyperbola		**A hyperbola** - is the set of points in a plane the difference of whose distances from two fixed points, called foci, is constant. illustrates the two branches of a hyperbola. The line through the foci intersects the hyperbola at two points, called **the vertices**. The line segment that joins the vertices is the **transverse axis**. The midpoint of the transverse axis is the center of the hyperbola. Notice that the center lies midway between the vertices, as well as midway between the foci

i) Guidance on evaluation

Invite students to do the activity in groups and tell them to draw the shapes they got on graph paper. Let them repeat the activity several times to understand the new concepts.

PRACTICAL ACTIVITY 16: EXPLORE HOW TO PRACTICALLY DRAW AN ELLIPSE AND IDENTIFY ITS ELEMENTS

a) Rationale

This activity can be done when teaching the characteristics of an ellipse. It is taught in Unit 8 of S6. It is sometimes difficult for students to draw an ellipse. This is because it is not an easy shape that they can easily relate to other objects. The practical activity of drawing an ellipse and identifying its elements will help students know how to easily draw it without any difficulty.

In real life, examples of ellipses are orbits of celestial bodies, seeing a water mellon in the shape of a 2-dimension. Ellipses are unique both as objects of art and also for their unique reflective properties that are used in technology.

b) Objective

To draw an ellipse and identify its elements. This is a concept based practical activity.

c) List of required materials

Two nails, one stick, and a string.

d) Illustration of the activity

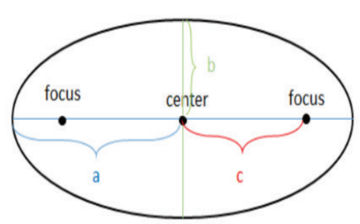


Figure 1

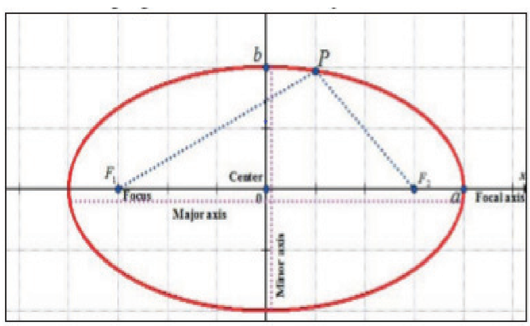


Figure 2

e) Procedures

(Consider figure 2)

Step 1: Fix two nails at a certain distance in points F_1 and F_2 (take at least 1m apart) as shown in Figure 2.

Step 2: Attach a string of length greater than **2a** on the two nails where "**a**" is a number greater than $\frac{F_1F_2}{2}$

Step 3: Place a stick on the interior side of the string and move it from a chosen point P (See Figure 2) until you return to the same point P.

Step 4: Observe the figure obtained. Where is the distance 2a maximum? Is it on the axis F1F2 or its perpendicular line passing through the center O? How are called points F1 and F2?

Expected answer

- The figure obtained by this process is a closed curve called an ellipse.
- Points F1 and F2 are called **foci** of the ellipse.
- The line through the foci is called the focal axis of the ellipse.
- The point on the focal axis halfway between the foci is called the center of the ellipse.
- The points where the ellipse crosses the focal axis are called the vertices.
- The line segment joining the two vertices is called the major axis.
- The line segment through the center and perpendicular to the major axis, with both endpoints on the ellipse, is called the minor axis.

f) Interpretation of results and conclusion

An ellipse is defined by $C = \{P(x, y) : PF_1 + PF_2 = 2a, a \in \mathbb{R}\}$

g) Information for teacher

Certain features define an ellipse, including:

- Unlike the circle, the ellipse has diameters of different lengths
- Only 2 of the ellipse's several diameters bisect it into two identical halves. The two are called axes of symmetry. They are also called the major axis and the minor axis respectively and they are also its longest and shortest diameters.
- An ellipse has two foci (called focus in the singular) which are two opposite points that lie on the major axis separated at equal distances from the center
- Eccentricity is a property that measures how much the ellipse deviates from roundness. An ellipse usually has an eccentricity value of between 0 and 1.
- When we travel from any of the focal points to any point on the ellipse and then to the second focal point, the distance covered stays constant.
- A circle is a special kind of ellipse. Both of its foci are located at some point which is its center.

h) Guidance on evaluation

Involve learners in the activity and encourage them to repeat the activity by changing the position of nails at point F1 and point F2.

UNIT: 9 RANDOM VARIABLES

PRACTICAL ACTIVITY 17: DETERMINE THE PROBABILITY DENSITY FUNCTION OF A GIVEN SITUATION

a) Rationale

This activity is done when teaching discrete and finite random variables in probability. It is learnt in Unit 9 of S6. The probability density function is used to determine the possibility of a random variable. This is a concept based practical activity.

b) Objective

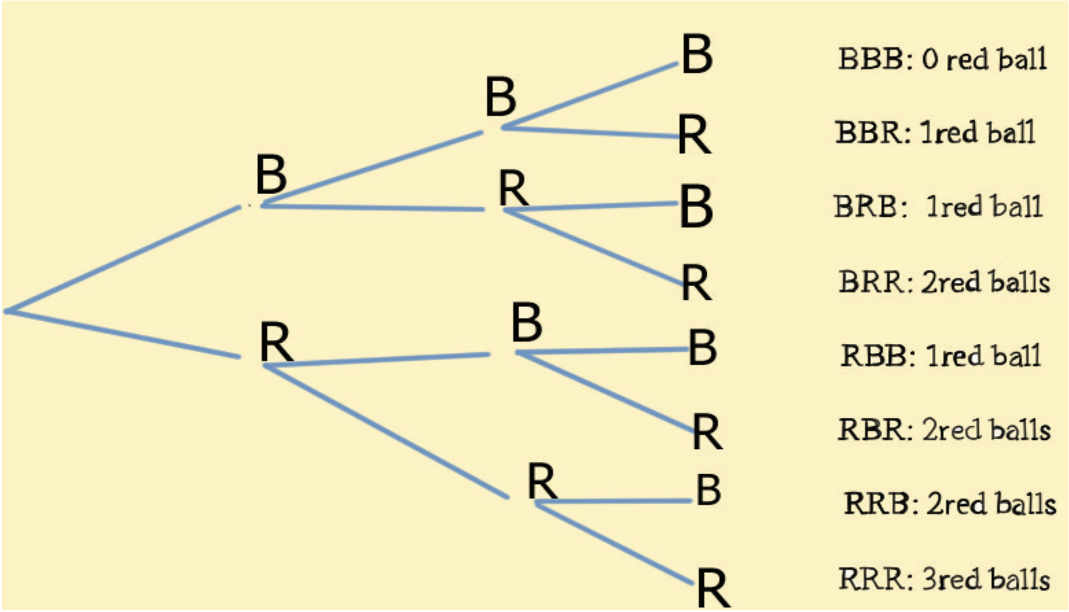
Determine the probability density function of a given situation in real life.

c) List of required materials

Paper and scientific calculators

d) Illustration of the activity

A bag contains 6 blue balls and 4 red balls. Three balls are drawn and not replaced. Determine the probability distribution for the number of red balls drawn.



e) Procedures

Step 1: Draw a tree diagram of drowning three balls from the bag without replacing.

Step 2. Record all possible outcomes

Step 3: Determine all possible probabilities of getting red balls (three red balls, two red ball, one red ball, or no red ball).

Step 4: Record in the table the probability density function (pdf).

f) Data Recording

x	0	1	2	3
P(X=x)				

X is the random variable “the number of red balls drawn”.

Expected answer

x	0	1	2	3
P(X=x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

- There is 1 possibility to draw 0 red ball: BBB
- There are 3 possibilities to draw 1 red ball: RBB, BRB, BBR
- There are 3 possibilities to draw 2 red balls: RRB, RBR, BRR
- There is 1 possibility to draw 3 red balls: RRR

g) Interpretation of results and conclusion

Let B = blue balls and R = red balls

If X is the random variable "the number of the red balls drawn", we have

$$P(X=0)=P(no\ red\ ball)=P(BBB)$$

$$=\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}=\frac{2}{12}=\frac{1}{6}$$

$$P(X=1)=P(1\ red\ ball\ and\ 2\ blue\ balls)$$

$$\Leftrightarrow P(X=1)=P(RBB)+P(BRB)+P(BBR)$$

$$=\frac{4}{10}\times\frac{6}{9}\times\frac{5}{8}+\frac{6}{10}\times\frac{4}{9}\times\frac{5}{8}+\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}=\frac{1}{2}$$

$$P(X=2)=P(2\ red\ balls\ and\ 1\ blue\ ball)$$

$$=P(RRB)+P(RBR)+P(BRR)$$

$$=\frac{4}{10}\times\frac{3}{9}\times\frac{6}{8}+\frac{4}{10}\times\frac{6}{9}\times\frac{3}{8}+\frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}=\frac{3}{10}$$

$$P(X=3)=P(no\ blue\ ball)=P(RRR)$$

$$=\frac{4}{10}\times\frac{3}{9}\times\frac{2}{8}=\frac{1}{30}$$

h) Information for teacher

The tree diagram should be dressed by the learners. In the case of the continuous random variables, the probability density function is a function with $\int_{all\ x} f(x)dx=1$.

The probability that a random variable attains values between x_1 and x_2 denoted by

$P\left(x_1 \le X \le x_2\right)$

Is given by the formula:

$$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(x)dx$$

i) Guidance on evaluation

The teacher will propose other exercises involving probability density functions referring to the above-worked example.

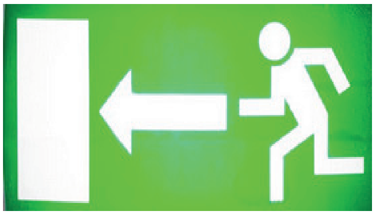
REFERENCE

- Ministry of Education and REB (2015). Mathematics Syllabus for Advanced Level (S4-S6), Kigali, Rwanda.
- UR-CE (2020). Continuous professional development certificate in innovative teaching mathematics and science (CPD ITMS), Module 1: Innovative Teaching methods for Mathematics and Science (CMS1141), Kigali: UR-CE.
- Ministry of Education and REB (2020). Advanced Mathematics, S4 Student's book, Kigali, Rwanda.
- Ministry of Education and REB (2020). Advanced Mathematics, S5 Student's book, Kigali, Rwanda.
- Ministry of Education and REB (2020). Advanced Mathematics,S6 Student's book, Kigali, Rwanda.
- Adenegan, K. E. and Balogun, F.O. (2010): Some proffered Solutions to the Challenges of Teaching mathematics in Our Schools. Unpublished Seminar Paper at Ebonyi for Pricipals.
- V.K.Maheshwari. Mathematics Laboratory and Lab Method -Activity Oriented Pedagogy in Mathematics Teaching. College, Roorkee, India.
- Roohi, A., DeMara, R.F., Khoshavi, N.: ' Design and evalutation of an ultra-area-efficient fault-tolerant QCA full adder', *Microelectr. J.*, 2015, 46, pp. 531– 542 (https://doi/org/10.1016/j.mejo.2015.03.023).

Annex 1: Name of commonly hazard symbols useful in the laboratory

S/N	Name	Hazard Symbol	Explanation
1	Flammable and combustible		The flammable and combustible symbol signifies substances that will ignite and continue to burn in air. Substances in this category may be gases, aerosols, liquids, or solids, and include many solvents and cleaning materials that are commonly used in the laboratory.
2	Oxidizing agents		The symbol for oxidizing materials indicates the presence of chemicals that readily give off oxygen or other oxidizing substances. Oxidizing materials may intensify fires and cause explosions, and also may be toxic or corrosive. Some common oxidizing liquids and solids found in laboratories are bromine, chlorates, nitrates, perchloric acid, and peroxides.

3	Toxic		A substance known to pose that is classified as posing skin corrosion or irritation; serious eye damage or eye irritation; respiratory or skin sensitization; germ cell mutagenicity; carcinogenicity; reproductive toxicity, and other toxicity is classified as hazardous or toxic substance. These substances can cause death or damage to health by inhalation, ingestion or skin absorption. Example: acid
4	Irritants		Irritants are substances that cause reversible inflammatory effects on living tissue at the site of contact.

5	Magnetic Field		Certain pieces of laboratory equipment generate strong magnetic fields. The strong magnetic field sign alerts lab members to the dangers that this type of equipment can pose. The risks are especially imminent for people wearing pacemakers and implants, which will tend to align themselves with the magnetic field lines, as will watches, clipboards, and certain tools. Magnetic fields result from the flow of current through wires or electrical devices. Examples of sources: machines, electrical wiring (such as power lines)
6	Exit		It is good to know where all of the exits are located, especially when working in a laboratory environment where you may need to get out quickly. Labs are required to mark exits routes from the area with clearly identifiable signs.

7	Fire extinguisher		Fires can happen anywhere, but lab fires can be even more dangerous due to Bunsen burners, flammable liquids, research documents, laptops, and lab equipment that might be present at any given time. It is essential that the occupants of a laboratory are fully aware of the risks and the appropriate extinguishing media. A fire extinguisher safety sign indicates the exact location of a lab's fire extinguisher.
8	Electrical hazard		The electrical hazard safety symbol, which typically includes a frayed wire and a hand with a lightning bolt across it, indicates any electrical hazards in the lab. If an electrical hazard is suspected, the device in question should be disconnected immediately and the cause determined by a qualified technician. Equipment should always be turned off and unplugged when any work is being done on it.