USER GUIDE FOR PRACTICAL ACTIVITIES AND LABORATORY EXPERIMENTS

MATHEMATICS

SENIOR FOUR (S4)

Kigali, 2022

FOREWORD

Dear teacher,

Rwanda Basic Education Board (REB) is honoured to present the user guide for Practical Activities and Laboratory Experiments in Mathematics for Senior four (S4). This user guide will supplement competence-based teaching and learning, to ensure consistency and coherence in the learning of Mathematics.

In this user guide, special attention was paid to practical activities that facilitate the learning process in which students can manipulate concrete materials, develop ideas, and make new discoveries during activities carried out individually or in pairs/ small groups.

In a competence-based curriculum, practical activities open students' minds and provide opportunities to interact with the world, use available tools, collect data, and effectively model real-life problems.

For efficient use of this user guide, your role as a teacher is to:

- Plan your lessons and prepare appropriate teaching materials.
- Organize groups for students considering the importance of social constructivism.
- Engage students through active learning methods.
- Provide supervised opportunities for students to develop different competences by giving tasks which enhance critical thinking, problem-solving, research, creativity and innovation, communication, and cooperation.
- Support and facilitate the learning process by valuing students' contributions to the practical activities.
- Guide students towards the conclusion of the results of the experiments.
- Encourage individual, peer, and group evaluation of the work done and use appropriate competence-based assessment approaches and methods.

To facilitate your teaching activities, the content of this guide is self-explanatory so that you can easily use it. It is divided into three parts:

- Part I: Structure of this guide and the general introduction on the role of practical activities and laboratory experiments in the implementation of CBC.
- Part II: List of some Mathematics materials distributed to schools.
- Part III: Selected practical activities and laboratory experiments and how you can facilitate them in lessons.

Even though this guide contains practical activities and laboratory experiments, they are not enough; teachers can guide students to carry out more practical activities using improvised teaching resources.

I wish to sincerely extend my appreciation to the people who contributed towards the development of this guide; The African Institute for Mathematical Sciences, Teacher Training Program (AIMS - TTP) in partnership with Mastercard Foundation who provided technical and financial support and REB staff particularly those from the Mathematics and Science Subjects

Unit in the Curriculum Teaching and Learning Resources Department who organized the wholeprocess from its inception.

Special appreciation goes also to teachers and independent experts in education who supported the exercise throughout the process. Any comment or contribution would be welcome for the improvement of this booklet for next versions.

Dr. MBARUSHIMANA Nelson
Director General, REB

ACKNOWLEDGEMENT

I wish to express my appreciation to the people who played a major role in the development and editing of the user guide for Practical Activities and Laboratory Experiments in Mathematics for Senior four (S4). It would not have been successful without the active participation of different education stakeholders.

Special thanks are given to AIMS staff, IEE, Independent people, teachers, illustrators, designers and all other individuals whose efforts in one way or the other contributed to the success of the development of this user guide.

I owe gratitude to the Rwanda Basic Education Board staff particularly those from Mathematics and Science subjects Unit in the CTLR Department who were involved in the whole process of the development work of this user guide.

Finally, my word of gratitude goes to AIMS - TTP in partnership with Mastercard Foundation for their support in terms of human and financial resources towards the development of this guide which will strengthen STEM teaching hence improving the quality of education in Rwandan schools.

Joan MURUNGI

Head of CTLR Department

LIST OF ACRONYMS

AIMS : African Institute for Mathematical Sciences

CBC : Competence-Based Curriculum

ICT : Information Communication Technology

KBC : Knowledge Based Curriculum

Lab : Laboratory

SET : Science and Elementary Technology

STEM : Science Technology Engineering and Mathematics

UR-CE : University of Rwanda- College of Education

TTP : Teacher Training Program

STRUCTURE OF THE USER GUIDE

This user guide for Practical activities and Laboratory experiments in Mathematics for Senior Four is divided into 3 parts:

Part I: General introduction on the role of practical activities and lab experiments in the implementation of CBC.

Part II: List of purchased mathematics kits.

Part III: Practical activities or laboratory experiments and how to facilitate them in lessons.

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PART I: GENERAL INTRODUCTION

1.1. Background

To effectively implement a competence-based curriculum (CBC) Students should apply what they have learnt, in different situations by developing skills, attitudes, and values in addition to knowledge and understanding. This learning process is learner-focused, where a learner is engaged in active and participatory learning activities, and Students finally build new knowledge from prior knowledge. Since 2015, the Rwanda Education system has changed from knowledge-based competence (KBC) to CBC for preparing students that meet the national and international job market requirements and job creation. Therefore, implementing the CBC education system necessitates qualitative laboratory practical works for mathematics and science as more highlighted aspects.

In addressing this necessity, laboratory experiments play a major role. A child is motivated to learn mathematics by getting involved in handling various concrete manipulatives in various activities. In addition to activities, games in mathematics also help the child's involvement in learning by strategizing and reasoning.

For learning mathematical concepts through the above-mentioned approach, child-centred Mathematics kits have been developed for the students of primary and secondary schools. The kits include various kit items along with a manual for performing activities.

The kit broadly covers the activities in the areas of algebra, geometry, trigonometry, and measurement.

The kit has the following advantages:

- Availability of necessary and common materials in one place
- Multipurpose use of items
- The economy of time in doing the activities
- Portability from one place to another
- Provision for teacher's innovation
- Low-cost material and use of indigenous resources.

Apart from the kit, the user guide for laboratory and practical activities to be used by teachers was developed. This laboratory experiment user guide is designed to help mathematics and science teachers to perform high-quality lab experiments for mathematics and science. This user guide structure induces learners' interest, achievement, and motivation through the qualitative mathematics and science lab experiments offered by their teachers and will finally lead to the targeted goals of the CBC education system, particularly in the field of mathematics and science.

In CBC, students hand on the materials and reveal the theory behind the experiment done. Here, experiments are done inductively, where experiments serve as an insight towards revealing the theory. Thus, the experiment starts, and theory is produced from the results of the experiment.

1.2 Why Mathematics Practical Activities and Laboratory Experiments?

Mathematics plays an important role in our daily activities. It provides the vital underpinning of the knowledge of the economy. Mathematics is essential in the physical sciences, technology, business, financial services and many areas of ICT (Roohi, 2015). As a the basis of most scientific and industrial research and development, the teaching and learning of mathematics has to be given much attention by utilizing all possible means to help students to acquire knowledge, skills and understanding of different concepts that are mostly abstract.

The concept of a mathematics laboratory has become very popular in recent years due to its important role in clarifying abstract concepts using real materials. The Mathematics laboratory is a room wherein we find a collection of different kinds of materials and teaching/learning aids, needed to help the students understand the concepts through relevant, meaningful, and concrete activities. These activities may be carried out by the teacher or the students to explore the world of mathematics, to learn, discover and develop an interest in the subject (Maheshwari, 2018).

The majority of students view mathematics as a dull, boring, and stereotyped subject. They think that mathematics is about getting the right or wrong answer. When they get it wrong, they think that they are not good enough for Mathematics and lose interest in learning. Mathematics laboratory helps students understand the principal idea behind Mathematics concepts. Although Mathematics is not an experimental science in the way in which physics, chemistry and biology are, a Mathematics laboratory can contribute greatly to the learning of mathematical concepts and skills. The benefit of using Mathematics laboratories in teaching and learning among others (Adenegan & Balogun, 2010):

- Arousing interest and motivating learning.
- Cultivating favourable attitudes towards mathematics.
- Enriching and varying instructions.
- Encouraging and developing creative problems solving ability.
- Allowing for individual differences in the manner and speed at which students learn.

- Making students see the origin of mathematical ideas and contributing to mathematics innovation.
- Allowing students to engage in the doing rather than being passive observers or recipients of knowledge in the learning process.

In the Mathematics laboratory, the activities help students to visualize, manipulate and reason. They provide an opportunity to make conjectures and test them and generalize observed patterns. Students learn to deal with problems while doing a concrete activity, which lays down a base for more abstract thinking.

1.3 Type of laboratory experiments

The goal of the experiment defines the type of experiment and how it is organized. Therefore, before doing practical work, it is important to have a clear idea of the objective.

The three types of practical works that correspond with its three main goals are:

1. **Equipment-based practical works:** the goal is for students to learn to handle scientific equipment like using a compass, a set square, a thermometer, a protractor, etc.
2. **Concept-based practical works:** the goal is to clarify the new concepts.
3. **Inquiry-based practical works:** the goal is for students to learn process skills. Examples of process skills are the following: defining the problem and good research question(s), installing an experimental setup, observing, measuring, processing data in tables and graphs, identifying conclusions, defining limitations of the experiment, etc.

Note:

- To learn the new concept through practical work, the lesson should start with the practical work, and the theory can be explained afterwards (explore – explain). Starting by teaching the theory and then doing the practical work to prove what they have learned is demotivating and offers little added value for student learning.
- Try to avoid complex arrangements or procedures. Use simple equipment or handling skills to make it not too complicated and keep the focus on learning the new concept.
- The experiments should be useful for all Students and not only for aspiring scientists. Try to link the practical work as much as possible with their daily life and preconceptions.

1.4 Organization, analysis, and interpretation of data

Once data are collected, they must be ordered in a form that can reveal patterns and relationships and allows results to be communicated to others. We list goals for analysing and interpreting data.

By the end of secondary education, students should be able to:

- Analyze data systematically, look for relevant patterns or test whether data are consistent with the initial hypothesis.
- Recognize when data conflict with expectations and consider what revisions in the initial model are needed.
- Use spreadsheets, databases, tables, charts, graphs, statistics, mathematics, and ICT to compare, analyze, summarize, and display data and explore relationships between variables, especially those representing input and output.
- Evaluate the strength of a conclusion that can be inferred from any data set, using appropriate grade-level mathematical and statistical techniques.
- Recognize patterns in data that suggest relationships worth investigating further. Distinguish between causal and correlational relationships.
- Collect data from physical models and analyze the performance of a design under a range of conditions.

1.5 Organising laboratory experiments

i. Methods of organizing a practical work

There are 3 methods of organizing practical work:

- **Each group does the same experiments at the same time**

All Students can follow the logical sequence of the experiments, but this implies that a lot of material is needed. The best group size is 3, as all Students will be involved. With bigger groups, you can ask to experiment twice, where Students change roles.

- **Experiments are divided among groups with group rotation**

Each group does the assigned experiment and moves to the next experiment upon a signal from the teacher. At the end of the lesson, each group has done every experiment. This method saves material but is not perfect when experiments are ordered logically. In some cases, the conclusion of an experiment provides the research question for the next experiment. In that case, this method is not very suitable.

The organization is also more complex. Before starting the lesson, the materials for each experiment should be placed in the different places where the groups will work. Also, the required time for each experiment should be about the same. Use a timer to show Students the time left for each experiment. Provide extra exercise for fast groups.

- **All experiments are divided among groups without group rotation**

Each group does only one or two experiments. The other experiments are done by other groups. Afterwards, the results are brought together and discussed with the whole class. This saves time and materials, but it means that each learner does only one experiment and 'listens' to the other experiments' descriptions. The method is suitable for experiments that are optional or like each other. It is not a good method for experiments that all Students need to master.

ii. Preparation of a practical work

When preparing practical work, do the following:

- Have a look at the available material at school and make a list of what you can use and what you need to improvise.
- Determine the required quantities by determining the method to apply (see above).
- Collect all materials for the experiments in one place. If the Students' group is small, they can come to get the materials on that spot, but it is better for each group to prepare a set of materials and place it on their desk.
- Test all experiments and measure the required time for each experiment.
- Prepare a nice but educational extra task for Students who are ready before the end of the lesson.
- Write on the blackboard how groups of Students are formed.

iii. Preparation of a lesson for practical work

In the lesson plan of a lesson with practical work, there should be the following phases:

1. The introduction of the practical work or the 'excite' phase consists of the formulation of a key question, discrepant event, or a small conversation to motivate Students and make connections with daily life and Students' prior knowledge.
2. The discussion of safety rules for the practical work. For example,
   - Students must work at the assigned place.
   - Long hairs should be tied together, and safety eyeglasses should be worn when dealing with chemical experiments.
   - Only the material needed for the experiment should be on the table.

3. Set the practical work instructions: how groups are formed, where they get the materials, special treatment of materials (if relevant), what they must write down, etc.

4. Set how to conduct practical work:

   - Students do the experiments, while the teacher coaches by asking questions (Explore phase).
   - The practical work should preferably be processed immediately with an explain phase. If not, this should happen in the next lesson.

5. Set how to conclude the lesson of practical work:

   - Students refer to instructions and conduct the experiment,
   - Students record and interpret recorded data,
   - Cleaning the workspace after the practical work (by the Students as much as possible).

1.6 Role and responsibilities of teacher, laboratory technician, and students in the laboratory experiment

1.6.1. The roles and responsibilities of teacher during a laboratory experiment

Before conducting an experiment, the teacher will:

- Decide how to incorporate experiments into class content best,
- Prepare in advance materials needed in the experiment,
- Prepare protocol for the experiment,
- Perform in advance the experiment to ensure that everything works as expected,
- Designate an appropriate amount of time for the experiment - some experiments might be adapted to take more than one class period, while others may be adapted to take only a few minutes.
- Match the experiment to the class level, course atmosphere, and your students' personalities and learning styles.
- Verify the status of lab equipment before lab practices.
- Provide the working sheet and give instructions to Students during lab sessions.

During practical work, the teacher's role is to coach instead of helping with advice or questions. It is better to answer a learner's question with another question than to immediately give the answer or advice. The additional question should help Students to find the answer themselves.

- Prepare some pre-lab questions for each practical work, no matter what the type is.
- Try and start the practical work: start with a discrepant event or questions that help define the problem or questions that link the practical work with students' daily life or their initial conceptions about the topic.
- Use coaching questions during the practical work: 'Why do you do this?', 'What is a control tube?', ''What is the purpose of the experiment?', 'How do you call this product?', 'What are your results?' etc.
- Use some questions to end the practical work: 'What was the meaning of the experiment?', 'What did we learn?', 'What do we know now that we didn't know at the start?', 'What surprised you?' etc.
- Announce the end of the practical work 10 minutes before giving students enough time to finish their work and clean their space.

1.6.2. The Role of a lab technician during a laboratory-based lesson

In schools having laboratory technicians, they assist the science teachers in the following tasks:

- Maintaining, calibrating, cleaning, and testing the sterility of the equipment,
- Collecting, preparing, and/or testing samples,
- Demonstrating procedures.

1.6.3. The students' responsibilities in the laboratory work

During the lab experiment, both students have different activities to do; the table barrow summarizes them. General learner's activities are:

- Experiment and obtain data themselves,
- Record data using the equipment provided by the teacher,
- Analyse the data often this involves graphing it to produce the related graph,
- Interpret the obtained results and deduct the theory behind the concept under the experimentation,
- Discuss the error in the experiment and suggest improvements,
- Cleaning and arranging material after a lab experiment.

1.7 Safety rules, and precautions during lab experiments

Regardless of the type of lab you are in, there are general rules enforced as safety precautions. Each lab member must learn and adhere to the rules and guidelines set, to minimize the risks of harm that may happen to them within the working environment. These encompass dress' code, use of personal protection equipment, and general behaviour in the lab. It is important to know that some laboratories contain certain inherent dangers and hazards. Therefore, when working in a laboratory, you must learn how to work safely with these hazards to prevent injury to yourself and other lab mates around you. You must make a constant effort to think about the potential hazards associated with what you are doing and think about how to work safely to prevent or minimize these hazards as much as possible. Before doing any scientific experiment, you should make sure that you know where the fire extinguishers are in your laboratory, and there should also be a bucket of sand to extinguish fires. You must ensure that you are appropriately dressed whenever you are near chemicals or performing experiments. Please make sure you are familiar with the safety precautions, hazard warnings, and procedures of the experiment you perform on a given day before you start any work. Experiments should not be performed without an instructor in attendance and must not be left unattended while in progress.

1.7.1 Hygiene plan

A laboratory is a shared workspace, and everyone has the responsibility to ensure that it is organized, clean, well-maintained, and free of contamination that might interfere with the lab members' work or safety.

For waste disposal, all chemicals and used materials must be discarded in designated containers. Keep the container closed when not in use. When in doubt, check with your instructor.

1.7.2 Hazard warning symbols

To maintain a safe workplace and avoid accidents, lab safety symbols and signs need to be posted throughout the workplace.

Chemicals pose health and safety hazards to personnel due to innate chemical, physical, and toxicological properties. Chemicals can be grouped into several different hazard classes. The hazard class will determine how similar materials should be stored and handled and what special equipment and procedures are needed to use them safely.

Each of these hazards has a different set of safety precautions associated with them. Annex 1 shows hazard symbols found in laboratories and the corresponding explanations.

1.7.3 Safety rules

Safety is the number one priority in any laboratory. All students are required to know and comply with good laboratory practices and safety norms; otherwise, they will be asked to leave the laboratory. Make sure you understand all the safety precautions before starting your experiments, and you are requested to help your Students to understand too.

The following are some general guidelines that should always be followed:

- **Lab coat**

While working in the lab, everyone must always wear a lab coat (Figure 1) to prevent incidental and unexpected exposures to the skin and clothing. The primary purpose of a lab coat is to protect against splashes and spills.

	The lab coat must be wrist-fitted and must always keep buttoned. A lab coat should be non-flammable and should be easily removed.

- **Safety glasses**

For eyes protection, goggles must always be worn over by all persons in the laboratory while students are working with chemicals. Safety glasses, with or without side-shields, are not acceptable.

The eyes protection safety indicates the possibility of chemical, environmental, radiological, or mechanical irritants and hazards in the laboratory.

– **Breathing Masks**

Respirators are designed to prevent contamination from volatile compounds that may enter in your body through the respiratory system. "Half mask" respirators (Figure3) cover just the nose and mouth; "full face" respirators cover the entire face, and "hood" or "helmet" style respirators cover the entire head.

The breathing mask safety sign lets you know that you are working in an area with potentially contaminated air.

– **Eye Wash Station**

Eyes wash stations consist of a mirror and a set of bottles containing saline solution that can be used to wash the injured eye with water. The eye wash station is intended to flood the eye with a continuous stream of water.

Eyes wash stations provide a continuous, low-pressure stream of aerated water in laboratories where chemical or biological agents are used or stored and in facilities where non-human primates are handled.

The eyewash stations should easily be accessed from any part of the laboratory, and if possible, located near the safety shower so that, if necessary, the eyes can be washed while the body is showered.

– **Footwear**

Shoes that cover entirely the toes, heel, and top of the foot provide the best general protection (Figure 1.5). Closed shoes must always be worn while in the laboratory, regardless of the experiment or curricular activity. Shoes must fully cover your feet up to the ankles, and no skin should be shown.

Socks do not constitute a cover replacement for shoes. Sandals, backless and open shoes are unacceptable.

- **Gloves**

When handling chemical, physical, or biological hazards that can enter the body through the skin, it is important to wear the proper protective gloves.

Butyl, neoprene, and nitrile gloves are resistant to most chemicals, e.g., alcohols, aldehydes, ketones, most inorganic acids, and caustics.

- **Hair dressing**

If hair is long, it must be tied back. It is good to report all accidents including minor incidents to your instructor immediately.

- **Eat and drink**

Never drink, eat, taste, or smell anything in the laboratory unless you are allowed by the lab instructor.

- **Hot objects**

Never hold very hot objects with your bare hands.

Always hold them with a test tube holder, tongs, or a piece of cloth or paper.

1.8. Guidance on the Management of lab materials: Storage Management, repairing and disposal of Lab equipment

Keeping and cleaning up

Working spaces must always be kept neat and cleaned up before leaving. Equipment must be returned to its proper place. Keep backpacks or bags off the floor as they represent a tripping hazard. Open flames of any kind are prohibited in the laboratory unless specific permission is granted to use them during an experiment.

Management of lab materials

A science laboratory is a place where basic experimental skills are learned only by performing a set of prescribed experiments. Safety procedures usually involve chemical hygiene plans and waste disposal procedures. When providing chemicals, you must read the label carefully before starting the experiment. To avoid contamination and possibly violent reaction, do never return unwanted chemicals to their container. In the laboratory, chemicals should be stored in their original containers, and cabinets should be suitably ventilated. It is important to notify students that chemicals cannot be stored in containers on the floor. Sharp and pointed tools should be stored properly. Students should always behave maturely and responsibly in the laboratory or wherever chemicals are stored or handled.

Hot equipment and glassware handling

Hazard symbols should be used as a guide for the handling of chemical reagents. Chemicals should be labelled as explosives, flammable, oxidizers, toxic and infectious substances, radioactive materials, corrosives, etc. All glassware should be inspected before use, and any broken, cracked, or chipped glassware should be disposed of in an appropriate container. All hot equipment should be allowed to cool before storing it.

All glassware must be handled carefully and stored in its appropriate place after use. All chemical glass containers should be transported in rubber or polyethylene bottle carriers when leaving one lab area to enter another. When working in a lab, do never leave a hot plate unattended while it is turned on. It is recommended to handle hot equipment with safety gloves and other appropriate aids but never with bare hands. You must ensure that hands, hair, and clothing are kept away from the flame or heating area and turn heating devices off when they are not in use in the laboratories.

Waste disposal considerations

Waste disposal is a normal part of any science laboratory. As teachers or students perform demonstrations or laboratory experiments, chemical waste is generated.

These wastes should be collected in appropriate containers and disposed of according to local, state, and federal regulations. All schools should have a person with the responsibility of being familiar with this waste disposal. In order to minimize the amount of waste generated and handle it safely, there are several steps to consider.

Sinks with water taps for washing purposes and liquid waste disposal are usually provided on the working table. It is essential to clean the sink regularly. Notice that you should never put broken glass or ceramics in a regular waste container. Use a dustpan, a brush, and heavy gloves to carefully pick-up broken pieces, and dispose of them in a container specifically provided for this purpose. Hazardous chemical waste, including solvents, acids, and reagents, should never be disposed of down sewer drains. All chemical waste must be identified properly before it can be disposed of. Bottles containing chemical waste must be labelled appropriately. Labelling should include the words "hazardous waste." Chemical waste should be disposed of in glass or polyethylene bottles. Plastic-coated glass bottles are best for this purpose. Aluminium cans that are easily corroded should not be used for waste disposal and storage.

Equipment Maintenance

Maintenance consists of preventative care and corrective repair. Both approaches should be used to keep equipment in working order. Records of all maintenance, service, repairs, and histories of any damage, malfunction, or equipment modification must be maintained in the equipment logs. The record must describe hardware and software changes and/or updates and show the dates when these occurred. Each laboratory must maintain a chemical inventory that should be updated at least once a year.

1.9. Student Experiment Work Sheet

There should be a sheet to guide students about how they will conduct the experiment, the materials to be used, the procedures to be followed, and the way of recording data. The following is the structure of the student experiment worksheet. It can be prepared by the teacher or be availed from the other level.

1. Date
2. Name of student/group
3. The title of the experiment
4. Type of experiment (concept, equipment, and inquiry-based)
5. Objective(s) of the experiment
6. Key question(s)
7. Materials (equipment/instrument, resources, etc...)
8. Procedures & Steps of experiment
9. Schematic reference if required.
10. Data recording and presentation

Number of tests	Variables	Results	Comments/Observations
1			
2			
3			
Etc			

11. Reflective questions and answers

Question1

Question 2

Question 3

12. Answer for the key question.

1.10. Report template for students

After conducting a laboratory experiment, students should write a report about their findings and the conclusion they took.

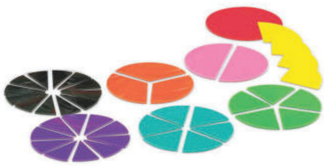
The report to be made depends on the level of students. The report done by primary school Students is not the same us the one to be made by secondary school Students.

The following is a structure of the report to be made by a group of secondary school Students.

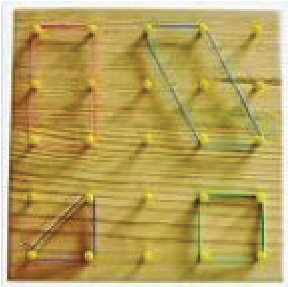
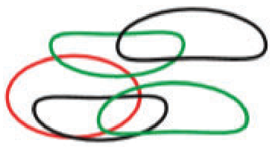
1. Introduction (details related to the experiment: Students identification, date, year, topic area, unit title, and lesson).
2. The title of the experiment.
3. Type of experiment (concept, equipment, and inquiry-based)
4. Objective(s) of the experiment.
5. Key question(s)
6. Materials (equipment/instrument, resources, etc…)
7. Procedures & Steps of experiment
8. Schematic reference if required.
9. Data recording
10. Data analysis and presentation (Plots, tables, pictures, graphs)
11. Interpretation/discussion of the results, student alternative ideas from observation.
12. Theory or main concept, formulas, and application.
13. Conclusion (answer reflective questions and the key question).

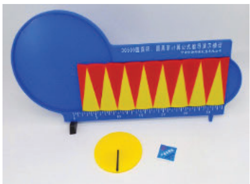
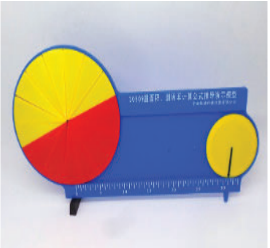
In conclusion, there are safety rules and precautions to consider before, during, and at the end of a practical activity and lab experiment. We hope teachers are inspired to conduct practical activities lab experiments in a conducive Competence Based Curriculum way.

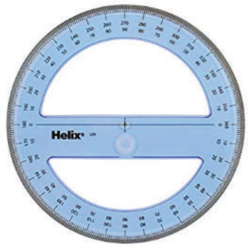
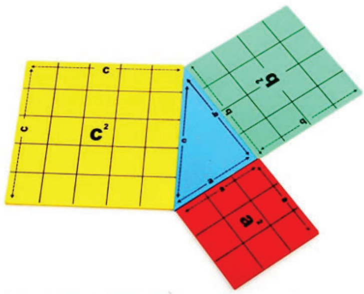
PART II: LIST OF MAIN KIT ITEMS DISTRIBUTED IN SCHOOLS

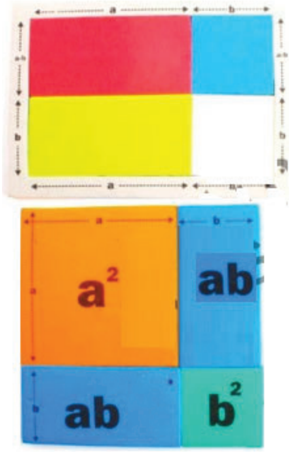
#	Item and description	Picture	Description
1	**Laminated number cards** ***Use**: Used in game for composition, sorting, factorization of numbers, etc.*		A pack of laminated cards numbered from 0 to 9 (9 cards from an A4 paper).
2	**Circle set fraction** ***Use**: Used for exploring "Area of Circle "and activities related to "Fractions"* and area of a circle		7 Blue (or any other color) colored circular plastic having 3mm thickness and diameter 160 mm. divided into 4, 6, 8, 12, 16 and 32 equal sectors. Each piece is magnetic.
3	**Clock** ***Use**: To learn to tell the time according to the 24 hours international convention.*		1 plastic teaching clock

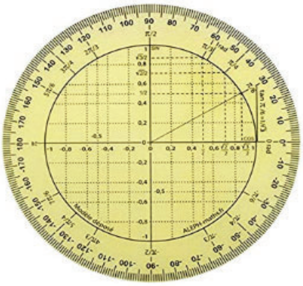
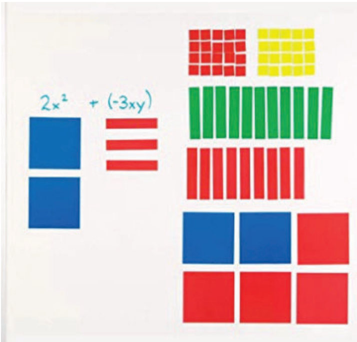
4	**Mathematical set for teachers:** Full circle protractor, meter rule, compass, tape measure, T-square, rope, decameter.		Wooden or plastic
5	**Mathematical set for students**: 2 Metal Study Compasses, 2 T-squares, Ruler, Protractor, Pencil for Compass, Pencil Sharpener, Eraser, Lead Refill.		Geometry 10 Piece Set,
6	**Fest night Stainless Steel 180 Degree Protractor**		Angle Finder Both Arms Stainless Steel Protractor with 0-180 Degrees, Angle10 inch,250mm, 30cm Scale Angle Finder Ruler. Smooth surface, convenient to use, easy to read. 0-180 degree arbitrary rotation. Adjustable screw design, easy operation for fixed reading.

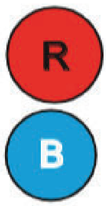
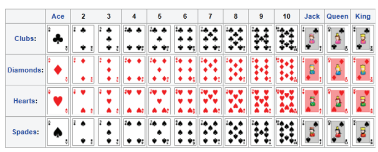
7	**Basic geometric solids**		**6** *pieces* **of** *wooden solids* Includes cube, cylinder, sphere, cone, triangular prism, pyramid . *Use: To demonstrate geometry solids (3D).*
8	**Geoboard** *Geoboard is used to represent planar shapes/ figures and also to find the approximate areas as well as to learn different geometric figures using a rubber band.*		*1 geographic board of 33.5 cm × 53.5 cm. It is printed with 187 grids 3 cm × 3 cm each in alternated colors. Copper pins are nailed on each crossing point of the grids.*
9	**Rubber bands** *for use with geoboard.*		120 rubber bands in 6 colors come with the boards.

10	**Transparent geometric 3D-shapes plus their corresponding fold-up nets:** cylinder, square pyramid, cube, rectangular prism, cone, hexagonal prism, triangular pyramid, and triangular prism. ***Use:*** *Used to make solid shape.*		Transparent geometric shapes plus their corresponding fold-up net inserts. 16-piece set (8 transparent and 8 folding shapes)
11	**Circle-Area and Diameter Demonstrator**	 	1 plastic demonstrator board of 48 cm × 25 cm. It consists of 17 sectors: 15 sectors that are equal to 1/16 of the cylinder volume; and 2 sectors that are equal to 1/32 of the cylinder volume. Use: To learn how to measure the area and diameter of circles.

12	**Full Protractor**		Helix Professional 360 Degree Protractor 15cm As per sample
13	**Cut Outs for Pythagoras** **Theorem**.		1 plastic right angled triangle. Measure: 3” x 4” x 5” & 3 different size square equal to sides of triangle.

14	**Cut outs for algebraic** **Identities.**		Made up of 3mm thick wooden colored cardboard and includes (1) Square cut-out of side 76 mm (2) 3 cut-outs obtained from another square of side 76mm out of which one is a square of side 38mm and the remaining two are trapezium of dim.38 x76 mm^2 (3) Square cut-out s of side 80 mm and 45mm (4) Rectangular cut-out of dimension 80x45mm^2

15	**Circular Trigonometric Protractor**		CIRCULAR Protractor ROBUST: shatterproof and scratch resistant translucent plastic. Use: direct reading of the angles in degrees and radians and cosine and sine near 0.05.
16	**Algebraic tiles** a) $x^2, x, 1$ b) $-x^2, -x, -1$		Made up of plastic cardboard in different sizes: 40 (20red+20 blue) squares of side 10mm known as unit tiles. 20 (10 red +10 blue) rectangles of 50 x10mm^2 dimension known as x or -x tiles. 10 (5 red + 5 blue) squares of side 50mm known as x^2 or $-x^2$ tiles, etc.

17	**Cubic dice** From 1 sided to 6 sided.	 	6 plastic dices with different edges and different shapes: 8mm, 12mm, 16mm, 19mm and 25mm.
18	**Counters: (20)** ***Use***: *Used in activity "Addition and Subtraction of Integers".*		A set of 20 Plastic pieces or lamineted transparent counters whose one side is blue and other side is red.
19	**Scientific Calculator**		Casio Fx-991es Plus Scientific Calculator
20	**Playing cards** to be used in probability		A set of 52 playing cards
21	**The container**: a box in metal to contain all these materials per kit.		A container in metal which can contain or these materials

PART III: PRACTICAL ACTIVITIES AND LABORATORY EXPERIMENTS

UNIT: 1 FUNDAMENTALS OF TRIGONOMETRY

Practical activity 1: Meaning of one radian and the relationship between radians and degrees

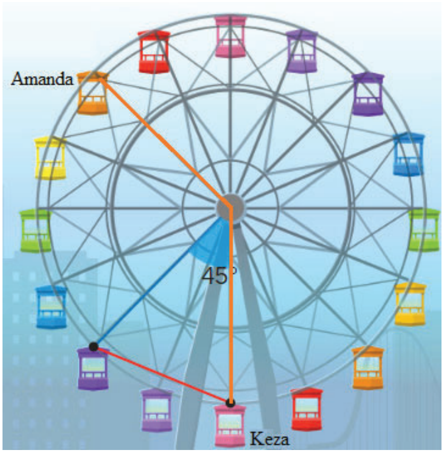
a) Rationale

This activity is done when teaching the concept or topic related to angle and its measurement. It is taught in unit 1 for S4. In real life, engineers use angle measurements to construct buildings, bridges, houses, monuments, etc. Carpenters use angle measuring devices such as protractors, compasses, and rulers, to make furniture like chairs, tables, and beds. Apart from degrees, angles can also be expressed in radians.

Radian measures are used in the following areas:

- When solving problems involving angles of rotation

Amanda sits in the orange seat at the top left of the **Ferris wheel**. Her friend, Keza, just got into the pink seat at the bottom of the **Ferris wheel**. Amanda wonders how far away he is from her friend.



To calculate the length of an arc and the area of a sector.

- To approximate the length of a chord given the central angle and radius.
- To solve problems about angular speed.

Therefore, it is necessary to explore the meaning of radian and how it is related to the degree. The **concept-based practical work will be used in this activity.**

b) Objective

Explore the meaning of one radian and establish the relationship between radian and degree.

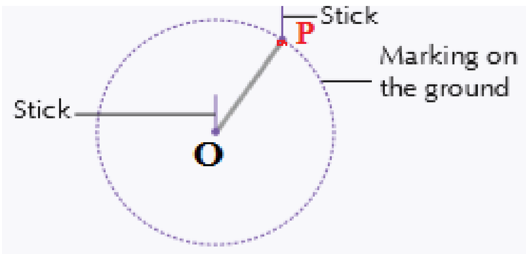
c) Materials required

Two pointed sticks, two ropes (each about 50 cm), a protractor and a metre rule.

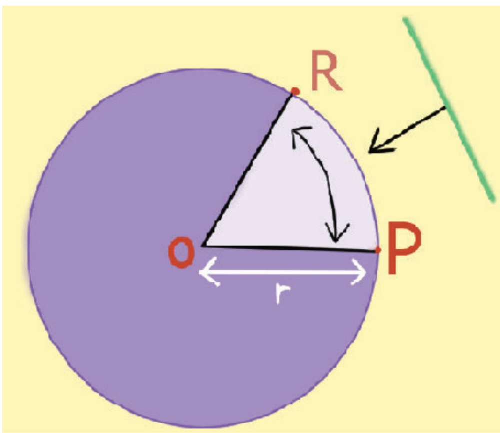
d) Procedures

Step 1: Fix the pointed sticks, one at each end of the rope.

Step 2: Place the sharp end of one of the sticks onto the ground as illustrated in the figure below

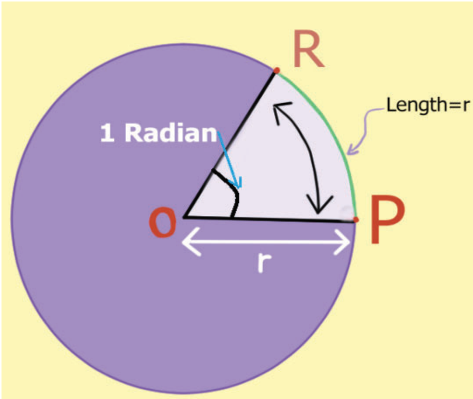


Step 3: With that point O as the centre, let the tip of the 2nd stick draw a circle with a radius of 50 cm.



Step 4: Take the 2nd rope also of length 50 cm and fit it on any part of the circumference as an arc RP.

Step 5: Take the metre rule and draw a line OP and OR (from each of the ends of the rope to the centre of the circle).



Step 6: Use the large protractor to measure the angle enclosed by the two lines OP and OR. What do you get? Expected response: Varying angles referring to the size taken by students.

Step 7: Repeat the task using ropes of different lengths, r_2= 70 cm, r_3= 40 cm, and r_4= 30 cm and record your observations.

e) Data recording

Complete your data in the following table

#	Circle	Value of angle POR formed
1	Circle 1, r = 50 cm	
2	Circle 2, r =70 cm	
3	Circle 3, r = 40 cm	
4	Circle 4, r = 30 cm	

Do the formed angles have the same value? What is the meaning of each formed angle?

If you take this angle as the unit of measurement, what can be the value of the right angle?

What can be the value of the straight angle?

f) Results, interpretation, and conclusion

The value of all angles is approximately equal, we see that in each case, the angle POR is about 57.3 degrees.

The measure of the angle POR subtended from the centre of a circle by an arc of length that is equal to the radius *r* of the circle is called the angle of 1 radian.

By comparing this angle with other angles, we find that the right-angle measures

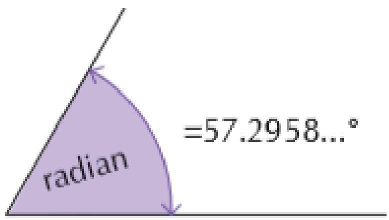
1.57 radians = $\frac{\pi}{2}$ *radians* =90 degrees,

The straight angle measures 3.14 radians = π *radians* =180 degrees.

Now establish the relationship between radians and degrees as the measurements of angles.

Expected answer:

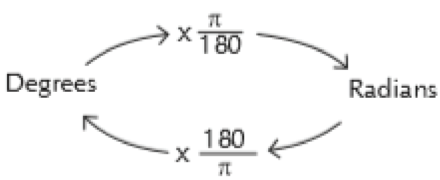
A radian is a unit of angular measure. It is defined as the value of an angle subtended at the centre of a unit circle by an arc of that circle whose length is equal to the radius r given that the radius of the circle is r.



One radian is about 57.2958 degrees.

Radius	Length of arc	Measurement of the angle
50 cm	50 cm	57.3^0
70 cm	70 cm	57.3^0
40cm	40cm	57.3^0
30cm	30cm	57.3^0

As $\frac{\pi}{2}\, radians = 90^\circ$, $\pi\ radians = 180^\circ$, the following diagram is useful for converting from radians to degrees and vice versa:



g) More information for the teacher

The radian is a measure based on the radius of the circle and is usually used by the scientific community while the degree is usually used in engineering.

The following table shows the values of some angles in radians and degrees.

Degrees	Radians (exact)	Radians (approx)
30°	$\frac{\pi}{6}$	0.524
45°	$\frac{\pi}{4}$	0.785
60°	$\frac{\pi}{3}$	1.571
90°	$\frac{\pi}{2}$	3.142
180°	π	3.142
270°	$\frac{3\pi}{2}$	4.712
360°	2π	6.283

h) Guidance on evaluation

Provide exercises on the conversion from degrees measures to radians measures and vice versa.

Practical activity 2: Exploring cosine and sine of different angles

a) Rationale

This activity is conducted while teaching trigonometric ratios. It is conducted in unit 1 of S4. Trigonometric ratios are used in a variety of fields, including civil engineering, architecture, mechanical engineering, medical imaging, electronics, electrical engineering, astronomy, chemistry, and geometry, to measure angles between two locations and the distance between them.

Additionally, they aid in determining the heights of any large mountains or towers. They are extensively employed in all fields of geometry-related study, including geodesy, solid mechanics, celestial mechanics, and many others.

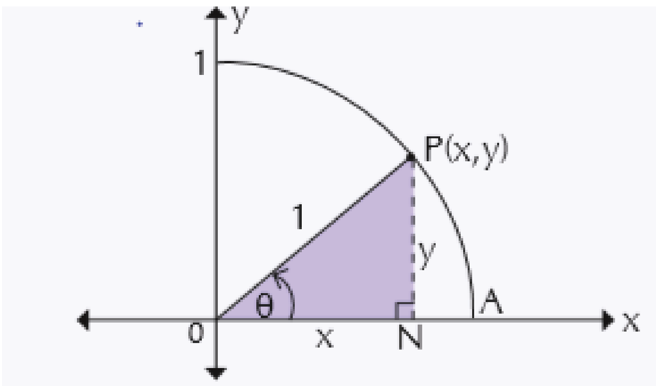
b) Objective

To Explore the meaning of sine and cosine of given angles in the unit circle.

c) List of required Materials

Manila paper, pair of compasses, protractor, ruler, scientific calculator.

d) Illustration of the activity



e) Procedures:

Step 1: Draw a unit circle on the Cartesian plane: you can take a circle with **1 dm** of radius. If you are outside, you can draw a circle with **1m** radius.

Step 2: Consider a point $P(x, y)$ which lies on the unit circle in the first quadrant the line segment OP makes the angle θ with the $x-axis$.

Step 3: Use the protractor to measure the value of this angle in degrees.

Step 4: Use the scientific calculator, check the functions cosine and sine; Then, find value of $\cos\theta$, the value of $\sin\theta$ and record them in the table of data recording.

Step 4: Find the point N image of the point P on x-axis under the parallel projection with direction of the vertical axis.

Step 5: Find the point M image of the point P on Y-axis under the parallel projection with direction of the horizontal axis (OX).

Step 6: Measure the lengths of the segments ON and OM in dm and complete them in the table of data recording.

Step 7: Repeat the same process by considering different angles in different quadrants of the circle and complete the results in the table of data recording given below.

Step 8: Compare the value for ON and OM and the values obtained by on the

f) Recording of data

i. $\cos\theta$

Angle θ	20^o	30^o	45^o	60^o	80^o
Length ON (without unit of length)					
$\cos\theta$					
Is ON= $\cos\theta$?					

What is the meaning of cosine of an angle? At which axis do we read cosine of the angle? What is the sign of cosine of angles depending on the quadrant they are located in?

ii. $\sin\theta$

Angle θ	20^o	30^o	45^o	60^o	80^o
Length OM (without unit of length)					
$\sin\theta$					
Is ON= $\cos\theta$?					

What is the meaning of sine of an angle? At which axis do we read sine of the angle? What is the sign for sine of angles depending on the quadrant they are located in?

Expected answers

Measuring ON and OM when the angle varies, we may obtain the table below:

Angle θ	20^{o}	30^{o}	45^{o}	60^{o}	80^{o}
Length ON (without unit of length)	0.93	0.87	0.70	0.50	0.17
$\cos\theta$	0.9394	0.8660	0.7170	0.5	0.1776

Angle θ	20^{o}	30^{o}	45^{o}	60^{o}	80^{o}
Length OM (without unit of length)	0.34	0.50	0.70	0.87	0.98
$\sin\theta$	0.3420	0.5	0.7071	0.8660	0.9848

g) Interpretation of results and conclusion

For more precision, we can consider the values with 2 decimal places

- **The cosine of the angle** is measured on the x-axis from the origin (0,0) to the point N(x,0) projection of the point P(x,y) given that OP is the second line segment of the angle.
- In each case, we find that length ON is equal to $\cos\theta$
- The values of cosine of angles are between -1 and 1.

The maximum value of $\cos\theta$ **is 1, the minimum value is -1. Other values are between -1 and +1.**

For points located on a unit circle, the x-coordinate is called the **cosine of the angle** θ and denoted $\cos\theta$.

The sine of the angle is measured on the y-axis from the origin (0,0) to the point M(0,y) projection of the point P(x,y) given that OP is the second line segment of the angle.

- In each case, we find that length OM is equal to $\sin\theta$
- The values of sine of angles are between -1 and 1.

The maximum value of $\cos\theta$ **is 1, the minimum value is -1. Other values are between -1 and +1.**

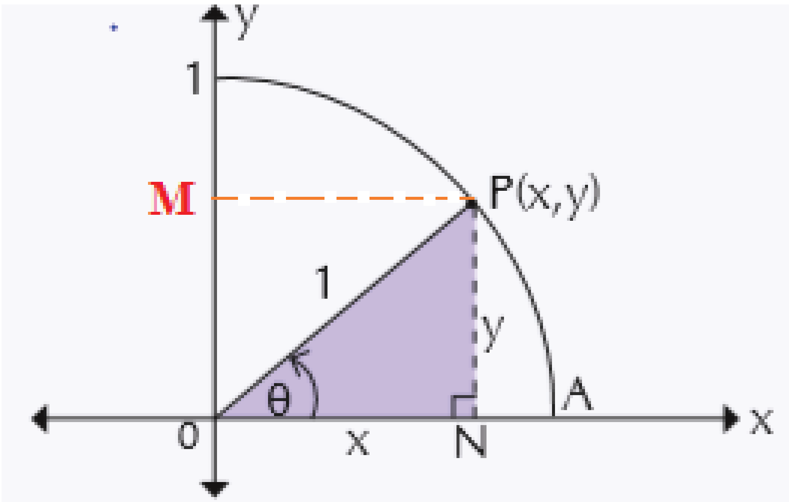
For points located on a unit circle, the y-coordinate of a point is called the **cosine of the angle** θ and denoted $\cos\theta$

Eventual source of errors:

- When measuring the length, Students can commit errors by reading the centimeters. Tell them that this is the reason why they may have different values.
- When measuring angles, the same error can happen when measuring degrees.
- Students can ask the case of using centimeters instead of decimeters. In this case, they use the circle with 10 cm instead of 1 dm. Tell them that when 10cm are represented by 1 unit, each cm is represented by 1/10 which is 0.1. This is the case for any value, it must be divided by 10 or 100 depending on the precision we need. It will be easy because Students are familiar with **scale drawing**.

g) Conclusion

For our figure of unit circle,



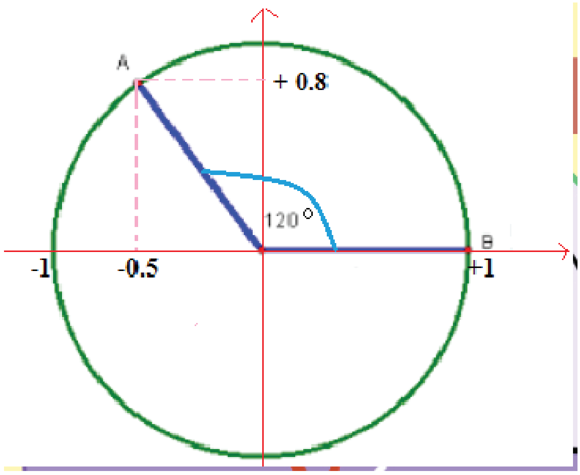
For each point P(x,y) located on a unit circle, the y-coordinate of a point is called the **sine of the angle** θ and is denoted $\sin\theta$. This means that P(x,y), $x = \cos\theta, \quad y = \sin\theta$.

In each case, we find that length ON is equal to $\cos\theta$ and length OM is equal to $\sin\theta$

h) Guidance on evaluation

Invite Students to represent for example the sine and the cosine of 120 degrees.

Solution



$$x = \cos 120^\circ = -0.5 \text{ and } y = \sin 120^\circ = 0.86$$

Practical activity 3: Finding sine, cosine and tangent of the angle with 45 degrees in a right-angled triangle

a) Rationale

This practical activity is done when teaching the concept related to trigonometric ratios in a right-angled triangle. It is taught in unit 1 of S4. Like in other trigonometric ratios seen previously, are used in measuring the distance between two points. They are used in other sciences such as civil engineering, architecture, mechanical engineering, medical imaging, electronics, electrical engineering, astronomy, and chemistry. They are used in finding heights of towers or any big mountains. They are widely used in all sciences that are related to geometry, such as navigation, solid mechanics, celestial mechanics, and geodesy.

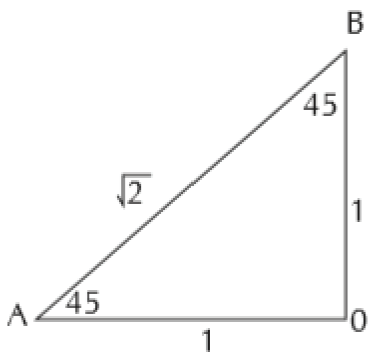
a) Objective

To use the angle of 45 degrees to explore sine, cosine and tangent of an angle in aright angled triangle.

b) List of required Materials:

Manilla paper, set square, pair of compasses, protractor, scientific calculator, etc.

c) Illustration of the activity (one)



d) Procedures:

Step 1: Draw an isosceles right -angled triangle OAB where the two equal sides OA = OB are 1 unit length (for example 1 dm or 1m, etc).

Step 2: Use a protractor to get the value of the angle OAB and the angle OBA

Step 3: Use Pythagoras' theorem to calculate the hypotenuse AB.

Step 4: Use the calculator to find cosine of 45 degrees.

Step 5: Determine the value $\frac{OA}{AB}$. How can you express $\frac{OA}{AB}$ in your own words in the triangle AOB?

Step 6: Compare the value obtained on step 3 and step 4.

e) Data recording

	Value	Are the two values equal?
$\frac{OA}{AB}$		
$\cos 45^0$		
$\frac{OB}{AB}$		
$\sin 45^0$		
$\frac{\frac{OB}{AB}}{\frac{OA}{AB}}$		
$\tan 45^0$		

- Define $\cos 45^0$ using expressions: adjacent side of 45 degrees and hypotenuse
- Deduce the definition of $\cos\theta$ in a right-angled triangle.
- Define $\sin 45^0$ using expressions: opposite side of 45 degrees and hypotenuse
- Deduce the definition of $\sin\theta$ in a right-angled triangle.

If we define $\tan 45^0$ as $\frac{\sin 45^0}{\cos 45^0}$; Find its value and explain how we can get it using the following expressions: angle of 45 degrees, opposite side of 45 degrees and adjacent side of 45 degrees. Deduce the definition of $\tan\theta$ in a right-angled triangle.

f) Interpretation of result and conclusion

Basing on the results found here above, we have

Results	meaning	For 45 degrees
$\frac{OA}{AB} = \cos 45^0$	$\cos\theta = \frac{adjacentside}{Hypotenuse}$	$\cos 45^0 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\frac{OB}{AB} = \sin 45^0$	$\sin\theta = \frac{opposite\,side}{Hypotenuse}$	$\sin 45^0 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\frac{OB}{OA} = \tan 45^0$	$\tan\theta = \frac{opposite\,side}{adjacent\ side}$	$\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = 1$

For the angle of we find the same values of the sine and cosine.

g) Guidance on evaluation

Ask students to draw an angle of 60 degrees in a right-angled triangle. Invite them to use what they learnt to give the meaning of tangent of the angle of 60 degrees.

Expected response:

Results	meaning	For 45 degrees
$\frac{OA}{AB} = \cos 60^0$	$\cos\theta = \frac{adjacentside}{Hypotenuse}$	$\cos 60^0 = \frac{1}{2}$
$\frac{OB}{AB} = \sin 60^0$	$\sin\theta = \frac{opposite\,side}{Hypotenuse}$	$\sin 60^0 = \frac{\sqrt{3}}{2}$
$\frac{OB}{OA} = \tan 60^0$	$\tan\theta = \frac{opposite\,side}{adjacent\ side}$	$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

Practical activity 4: Determination of sine, cosine, and tangent of angles with 30° and 60°

a) Rationale:

This practical activity is done when teaching the concept related to trigonometric ratios in a right-angled triangle. It is taught in the unit 1 of S4. In real life, trigonometric ratios for angles with 30 and 60 degrees are used in the calculation of length of materials in civil engineering, architecture, mechanical engineering, medical imaging, electronics, electrical engineering, astronomy, and chemistry. They are used in navigations and construction where there is a need to find heights of towers or any big mountains.

b) Objective:

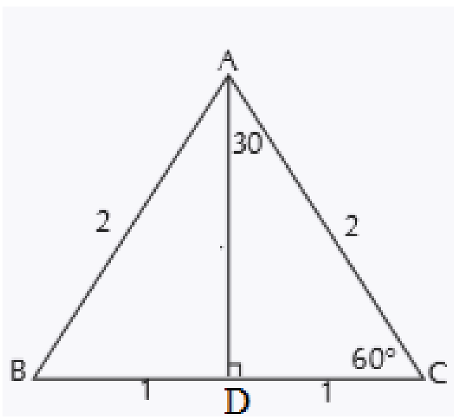
To use the meaning of trigonometric ratio to determine the exact values of trigonometric ratios of angles with 30^{o} and 60^{o}.

c) List of required Materials:

Manilla paper, set square, pair of compasses, protractor, scientific calculator, etc.

d) Set up

The angle with 60 degrees and the angle with 30 degrees



e) Procedures

Step 1: Draw equilateral triangle ABC of sides 2 units of length

Step 2: Draw AD from A perpendicular to BC.

What is the length of BD and CD?

The line segment AD bisects BC; so, BD = CD =1unit.

Step 3: Use Pythagoras theorem to calculate the length of AD.

Step 4: Deduce the 3 values of sine, cosine, and tangent of 30^0 and 60^0 and complete the table of data recording.

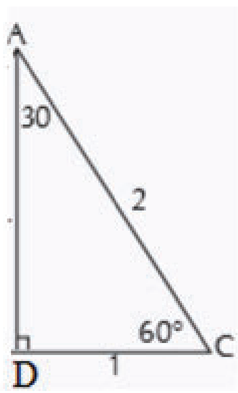
f) Data recording

θ	30°	60°
$\sin\theta$		
$\cos\theta$		
$\tan\theta$		

Expected results

The height AD:

Let us consider the right-angled triangle ADC rectangle in D.



The Pythagoras theorem gives $|AD|^2 + 1^2 = 2^2$ and $|AD| = \sqrt{3}$

Trigonometric ratios:

Given that $\cos\theta = \frac{adjacentside}{Hypotenuse}$, $\sin\theta = \frac{opposite\,side}{Hypotenuse}$ and $\tan\theta = \frac{opposite\,side}{adjacent\,side}$, we get:

θ	30^0	60^0
$\sin\theta = \dfrac{opposite\,side}{Hypotenuse}$	$\dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$
$\cos\theta = \dfrac{adjacentside}{Hypotenuse}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{2}$
$\tan\theta = \dfrac{opposite\,side}{adjacent\,side}$	$\dfrac{\sqrt{3}}{3}$	$\sqrt{3}$

g) Interpretation of result and conclusion

The sine and cosine of $30°$ and $60°$, the sine of $30°$ are given in the table above. The cosine of $60°$ and the sine of 30 degrees are equal. The sine for 60 degrees and the cosine of 30 degrees are equal.

h) Guidance on evaluation

Ask students to illustrate angles of 30°, 45°, and 60° and give the value for their sine, cosine, and tangent.

Expected answer: Illustration

θ	30^0	45^0	60^0
$\sin\theta = \dfrac{opposite\,side}{Hypotenuse}$	$\dfrac{1}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$
$\cos\theta = \dfrac{adjacentside}{Hypotenuse}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$
$\tan\theta = \dfrac{opposite\,side}{adjacent\,side}$	$\dfrac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Practical activity 5: Trigonometric ratio of an angle and the variation of lengths of the sides for the triangle

a) Rationale:

This activity is conducted when teaching the Concept or topic related to Trigonometric ratios in a right- angled triangle. It is taught in the unit 1 of S4. Trigonometry can be used to when preparing triangular parts of roof of a house, to make the roof inclined (in the case of single individual bungalows) and the height of the roof in buildings etc. It is used in naval and aviation industries.

b) Objective:

To verify that the values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle.

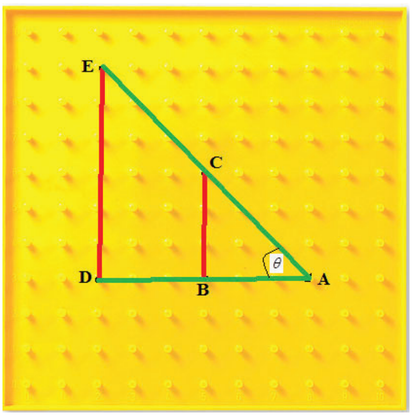
c) Materials required:

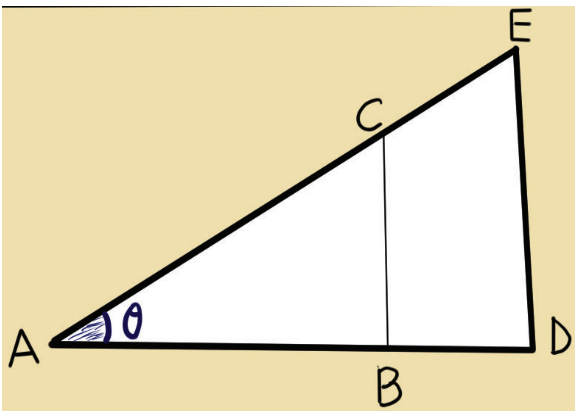
Geoboard, rubber bands, ruler, pencils, and pens.

d) Procedures and diagram:

Step 1: Consider 5 geoboard pins on the geoboard at suitable points A, B, C, D and E and join them with 2 rubber bands of different colours to represent two similar right triangles

as shown in the following figure:





Step 2: Measure length of AB, BC, AD, DE, AC and AE using a ruler.

Step 3: Find:

$$\frac{BC}{AC}, \frac{AB}{AC}, \frac{BC}{AB}, \frac{DE}{AE}, \frac{AD}{AE}, \frac{DE}{AD}$$

Step 4: Repeat the activity by making other pairs of similar right triangles and complete the following table:

No	BC	AC	DE	AE	$\frac{DE}{AE}$	$\frac{BC}{AB}$	AB	AD	$\frac{DE}{AE}$	$\frac{BC}{AB}$
θ_1										
θ_2										
θ_3										

e) Results, interpretation, and conclusion

Compare the length of BC and DE, what are different ways in which you can determine the sine of the angle θ? What about cosine of the angle θ?

Let θ denotes the angle CB = ED

Expected answers

(1) $\frac{BC}{AC} = \frac{DE}{AE}$ (2) $\frac{AB}{AC} = \frac{AD}{AE}$ 3) $\frac{BC}{AB} = \frac{DE}{AD}$

Equality (1) shows that the sine of the angle doesn't change with the length of the sides.

Equality (2) shows that the cosine of the angle doesn't change with the length of the sides.

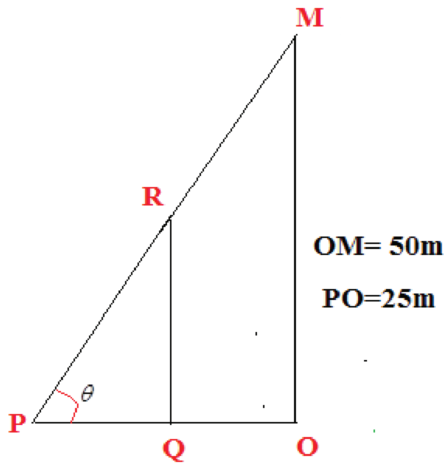
Equality (3) shows that the tangent of the angle doesn't change with the length of the sides.

Note: The trigonometric ratios of an acute angle in a right- angled triangle depend only on the value of its amplitude.

Therefore, the values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle.

f) Guidance on the evaluation:

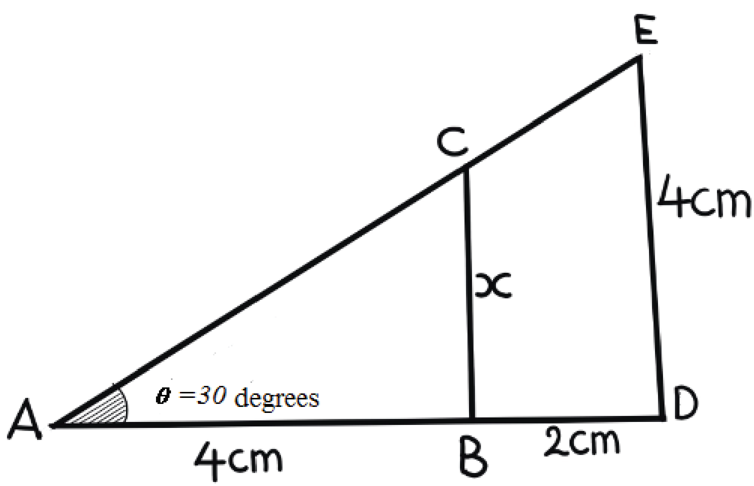
1. Ask students to use a right-angled triangle whose sides were increased. Invite them to use a ruler to determine the tangent of its angle θ.



2. Do the following activity

Construct the triangle ABC with the angle DE=30°, such that the length AD=6cm and DE=4cm. Draw a line CB//DE such that AB= 4cm. Find the length of the side BC.

Solution



$$\tan\theta = \frac{ED}{Ad} = \frac{BC}{AB}$$

Thus, $\frac{4}{6} = \frac{x}{4}$, $x \approx 2.7$

Practical activity 6: Exploring the angle of elevation in the air navigation.

a) Rationale:

This activity can be done when teaching the concept or topic related to Triangle and applications. It is taught in the unit 1 of S4. In real life, this concept is used in air navigation. Through the concept of the angle of elevation in air navigation as illustrated in the Figure 4, it is possible to calculate the speed that an airplane has used to move from one place to another.

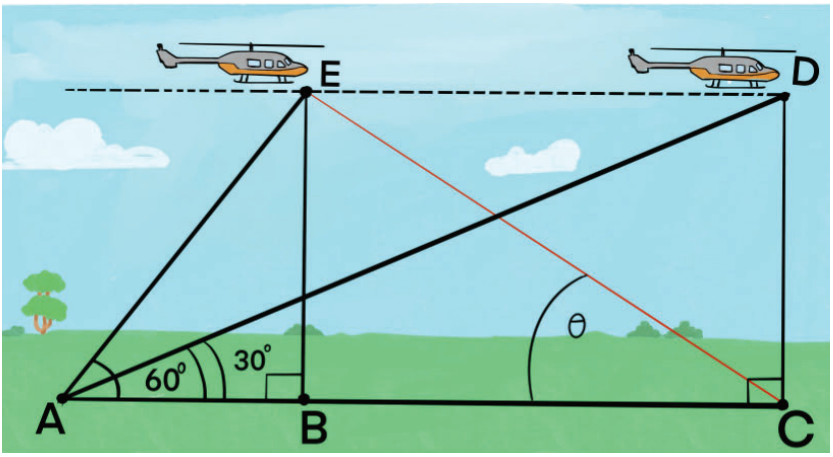
b) Objective:

To explore the angle of elevation in the air navigation.

c) List of required Materials:

Manilla paper, table, meter ruler, small box or another object that can be considered as a plane, protractor, and scientific calculator.

d) Illustration of the activity



e) Procedures:

Step 1: Using nail or pin, fix the position point A of the observer on the ground opposite to one side of rectangular table BCDE

Step 2: From point A, attach the string to small box (or other object) considered as airplane and choose the point E on one side of rectangular table as initial position of our airplane.

Step 3: Measure the angle $E\hat{A}B$ of elevation in E

Step 4: Push forward the object considered as airplane to the position D

Step 5: Measure the angle $D\hat{A}C$ of elevation in D.

Step 6: Comment on your observation

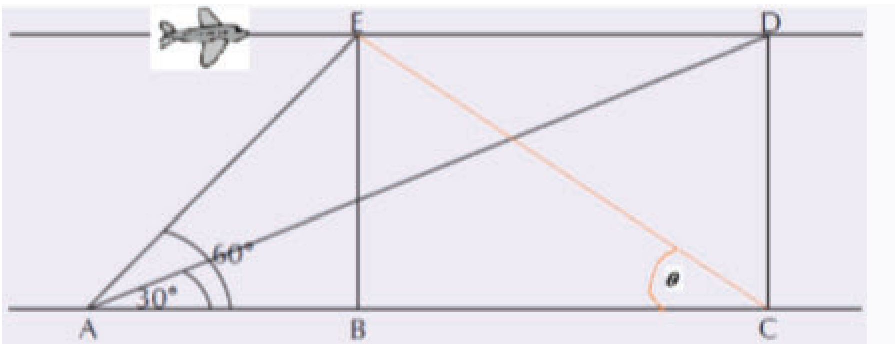
Step 7: Consider the case in which E is the first position of the aero plane and D be the position after 10 seconds.

EB = 1 km and DC = 1 km. Angle EAB = 60°. Angle DAC = 30°.

i. Find the distance travelled by the airplane and its uniform velocity.
ii. What is the angle of depression for the pilot of when the airplane arrives in E. Gve the meaning of angle of elevation and the angle of depression when you are observing two objects: one above and the other on the soil.

f) Results, interpretation, and conclusion

The following figure illustrates the work to be done



i. Let E be the first position of the aero plane and D be the position after 10 seconds. EB = 1 km and DC = 1 km. Angle EAB = 60°. Angle DAC = 30°.

We have:

$$\tan 60° = \frac{BE}{AB} \Rightarrow AB = \frac{BE}{\tan 60°} = \frac{1}{\sqrt{3}}$$

$$\tan 30° = \frac{CD}{AC} \Rightarrow AC = \frac{CD}{\tan 30°} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

Distance travelled by an airplane is

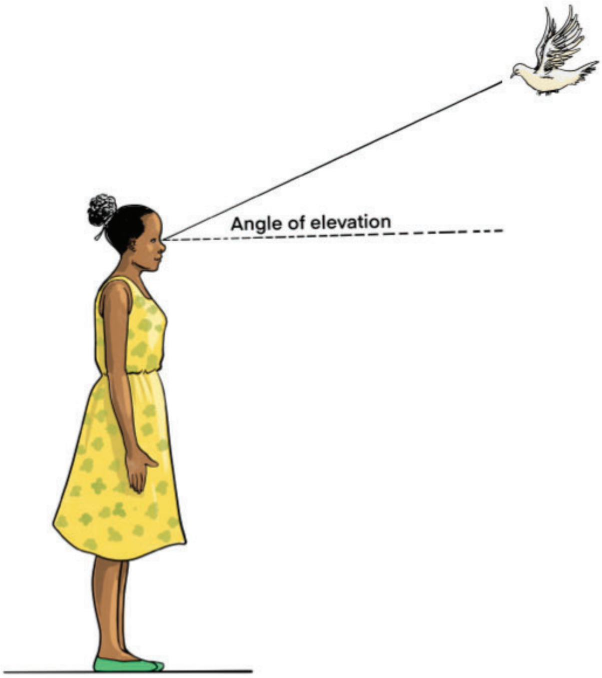
$$ED = BC = AC - AB = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Uniform speed of the Airplane is:

$$\text{Velocity} = \frac{Distance}{Time} = \frac{2}{\frac{\sqrt{3}}{10}} = \frac{2}{\sqrt{3}} \times \frac{1}{10} = \frac{1}{5\sqrt{3}} = \frac{\sqrt{3}}{15}\text{ km/h}$$

$$\Rightarrow \frac{\sqrt{3}}{15} \times 3600 km/h = 415.69\text{km/h}$$

ii. An angle of elevation is the "upward" angle from the horizontal to a line of sight from the object to a given point.



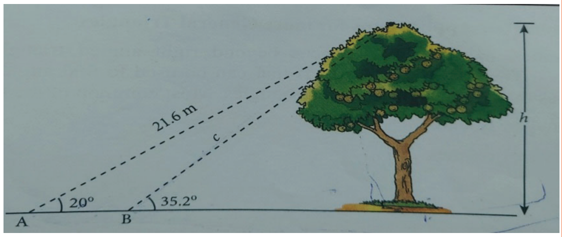
If you look up at an object, the angle your line of sight makes with a horizontal line is called the angle of elevation.

Note: The angle of elevation is formed when it is between the line of sight and the horizontal line.

g) Guidance on evaluation

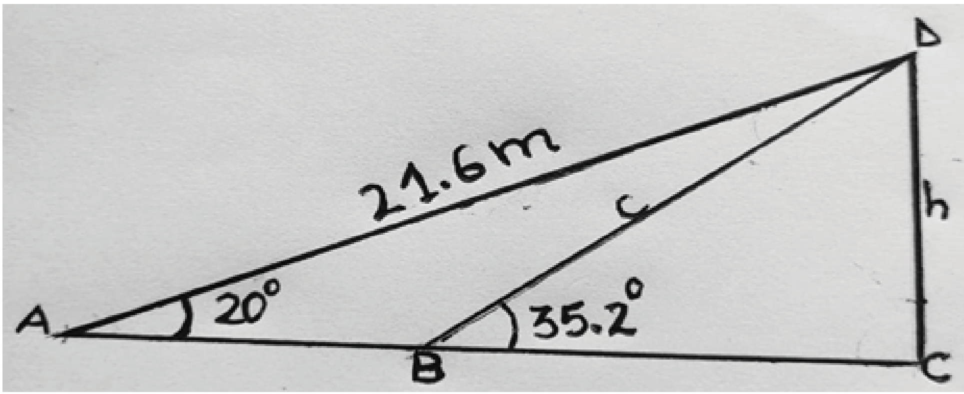
Ask students to do different activities related to this practical work. For example,

Two forestry workers in Nyungwe Forest are standing on points A and B. If the angle of elevation from point A is 20° and that from B is 35.2° to the top of the tree and the distance from point A to the top of the tree is 21.6m, find the:

a) Distance from A to the bottom of the tree b) Height of the tree c) Distance from B to the tree d) Distance from point B to the top of the tree e) Distance between A and B	

Solution

Figure 6



a) $\cos\theta = \dfrac{AC}{AD}$

Thus, $AC = AD \times \cos\theta$

$$AC = 21.6 \times \cos 20° = 20.29m$$

b) $\dfrac{DC}{AC} = \tan 20°$

Thus, $DC = AC \times \tan 20°$

$$= 20.3 \times 0.36 = 7.39m$$

The height of the tree is 7.39m

c) $\dfrac{DC}{BC} = \tan 35.3$

Thus, $BC = \frac{DC}{\tan 35.3} = \frac{7.39}{0.70} = 10.56m$

The distance from B to the tree is then 10.56m

d) $\frac{DC}{BD} = \sin 35.2$

Thus, $BD = \frac{DC}{\sin 35.2} = \frac{7.39}{0.58} = 12.74$

The distance from point B to the top of the tree is then 12.74m

e) AB=AC-BC=20.3-10.56=9.74m

The distance between A and B is then 9.74m

Practical activity 7: Exploring the angle of elevation on objects

a) Rationale:

This activity is done when teaching the concept or topic related to Triangle and its applications. It is taught in the unit 1 of S4 Mathematics.

Through the concept of the angle of elevation, as illustrated in the Figure below, it is possible to calculate the height of objects.

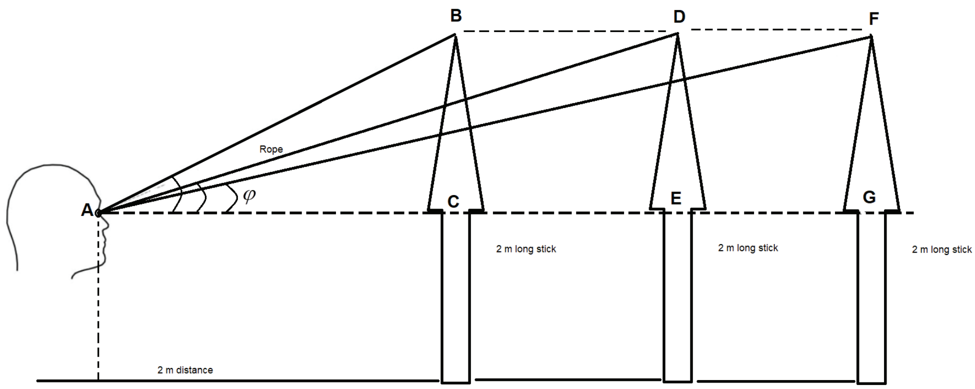
b) Objective:

To explore the angle of elevation.

c) List of required Materials:

Manilla paper, nail**,** table, meter ruler, small box or another object that can be considered as a plane, protractor, and scientific calculator.

d) Illustration of the activity



e) Procedures:

Step 1: Pick a flat place where to conduct this activity like a playground. If a playground or a large flat space is not available, you can opt to use the class. It is flat and spacious.

Step 2: Place a chair somewhere on the flat ground and make sure that from it you can find at least a 10 m long free distance or more.

Step 3: Pick one volunteer (observer) to sit on the chair.

Step 4: Ask the person on the chair to sit straight and use a meter rule or decameter to measure the position of his/her eye from the ground and call it x.

Step 5: Using a calculator, calculate the height $H = Ls - x$.

Step 6: Using ruler or another measuring instrument, measure the 2m distance from the position of the chair as shown in the figure.

Step 7: At the 2m distance (measured in step 4), place vertically a stick of known length ($Ls = 2m$ or more).

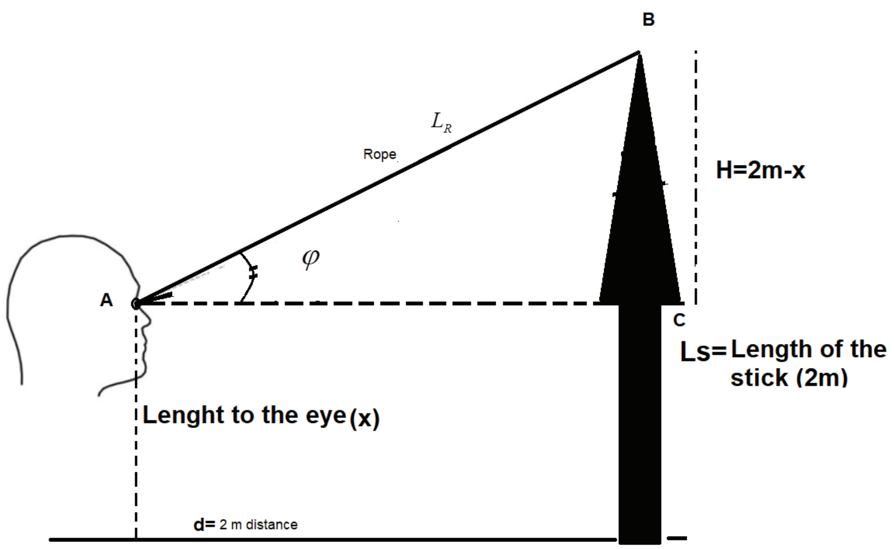
Step 8: Ask the observer on the chair to look straight on the top end of the $2m$ long stick (or more) positioned at 2m distance from him/her. He/she should make sure to be siting straight up with his/her shoulder tight to maintain the position of eyes for correct measurement.

Step 9: Ask two more volunteers to measure the distance between the eye of the observer and the top end of the stick positioned at two meters. To measure this, place a rope from the eye of the observer and the top end of the stick. Make sure the rope is held in a straight line between the eye and the top end of the stick and mark both end of the rope for measurement

Step 10: Measure the length of the rope using a meter rule or other measuring instrument and note the length as L_R

Step 11: On a manila paper, draw a triangle ΔABC made of the height H, base d and hypotenuse L_R

Step 12: Using protractor measure the angle $\widehat{BAC}$ and note it as φ



Step 13: Repeat step 7 to step 13 for the $4m$, $6m$, $8m$, $10m$ and $12m$ and record the result in the table of results.

Step 14: How does the angle obtained relate to the distance from the observer and the position of the stick?

Step 15: What is your interpretation of large or small angle φ in terms of visibility and magnification of the object observed by the observer?

Step 16: Which technical name is given to this angle? On the Manila paper, draw what should be the opposite angle to this and give its technical name.

Step 17: Taking reference to how the data changed, predict the distance s and speed of an airplane initially pictured 15km at an angle 45^o from the observer and pictured at 10^o after 10 second. Consider that the height of the plane did not change.

$$s = a\left(\frac{\tan\alpha}{\tan\beta} - 1\right) \text{ with } \begin{cases} a = \mathit{Initial} \text{ position} \\ \alpha = \mathit{Initial} \text{ angle} \\ \beta = \mathit{Final} \text{ angle} \end{cases}$$

Sn	Distance (d)/m	Length (L)	Angle φ
1	2		
2	4		
3	6		
4	8		
5	10		
6	12		

f) Interpretation of the results

i. Referring to table of result, the angles measured reduce in size as the distance of the object observed get far away from the observer.

ii. A large angle would explain that the object is closer to the observer, and it is bigger in size compared to how the object reduce in size when the angle is small, or object is reduced.

iii. The angle discussed in this activity is name the angle of elevation. The angle of elevation has different application including but not limited to its application in aviation to calculate the distance moved by the airplane and the speed used.

If you look up at an object, the angle your line of sight makes with a horizontal line is called the angle of elevation.

Note: The angle of elevation is formed when it is between the line of sight and the horizontal line.

Taking reference to how the data changed, predict the distance s and speed of an airplane initially pictured at an angle 45^o from 15km and 10^o after 10 second. at an position of and airplane initially

$$s = a\left(\frac{\tan\alpha}{\tan\beta} - 1\right) \text{ with } \begin{cases} a = Initial \text{ position} \\ \alpha = Initial \text{ angle} \\ \beta = Final \text{ angle} \end{cases}$$

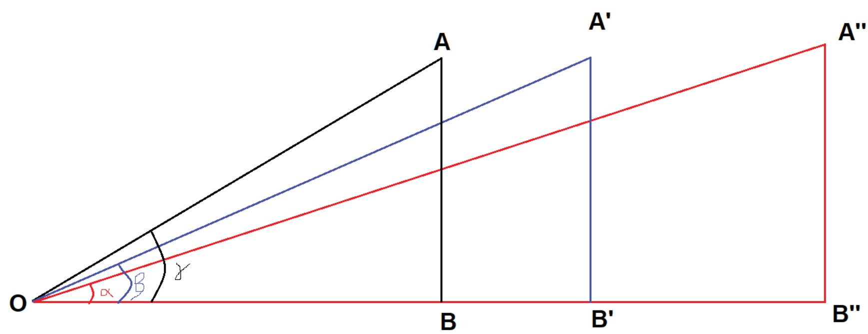
The distance covered by the airplane can be calculated using the same procedure. To explain it well, let us use the illustration below. This is a simplified figure with o being the eye of the observer. If an airplane is at point A, it is at a distance B from the observer. As the airplane move far away keeping its height, the elevation angle reduce and the image start diming down as it get even far.

From the triangle OAB, let $\tan\alpha = \frac{AB}{OA}$.

After time t, the airplane move to A'. from the triangle OA'B' $\tan\beta = \frac{A'B'}{OA'}$

considering that $AB = A'B'$ then, we can deduce that the distance moved by the airplane from A to A' is

$$s = AB' = OA\left(\frac{\tan\alpha}{\tan\beta} - 1\right) \text{ with } \begin{cases} OA = Initial \text{ position} \\ \alpha = Initial \text{ angle} \\ \beta = Final \text{ angle} \end{cases}$$



Note: The angle of elevation is formed when it is between the line of sight and the horizontal line.

Practical activity 8: Exploring the angle of depression in the air navigation.

a) Rationale:

This activity is done when teaching the Concept or topic related to Triangle and its applications. This idea is used in air navigation. Calculating the speed an airplane utilized to go from one location to another is attainable using the angle of elevation concept in air navigation.

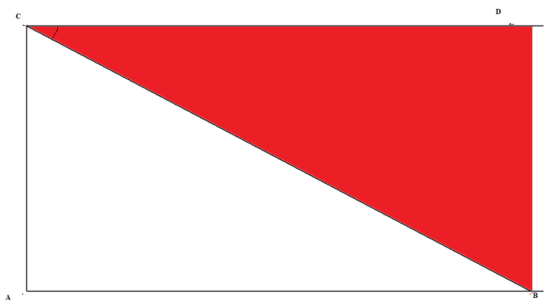
b) Objective:

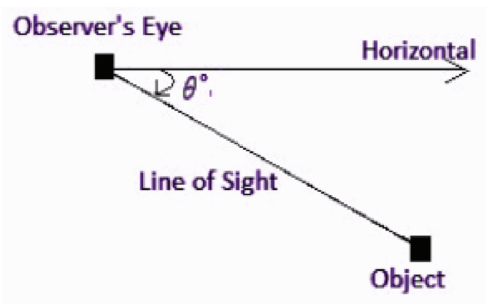
To explore the angle of depression in the air navigation.

c) List of required materials:

Manilla paper, ruler, protractor, and scientific calculator

d) Illustration of the activity:





e) Procedures

Step1: Fix the position point C of the airplane at a certain given height above the point A from the ground. The plane is moving horizontally towards the point D.

Step 2: Measure the angle of depression $B\hat{C}D$

Step 3: Using trigonometric ratios in the right -angled triangle ABC determine the distance AB covered by the airplane. Note that $B\hat{C}D$ =

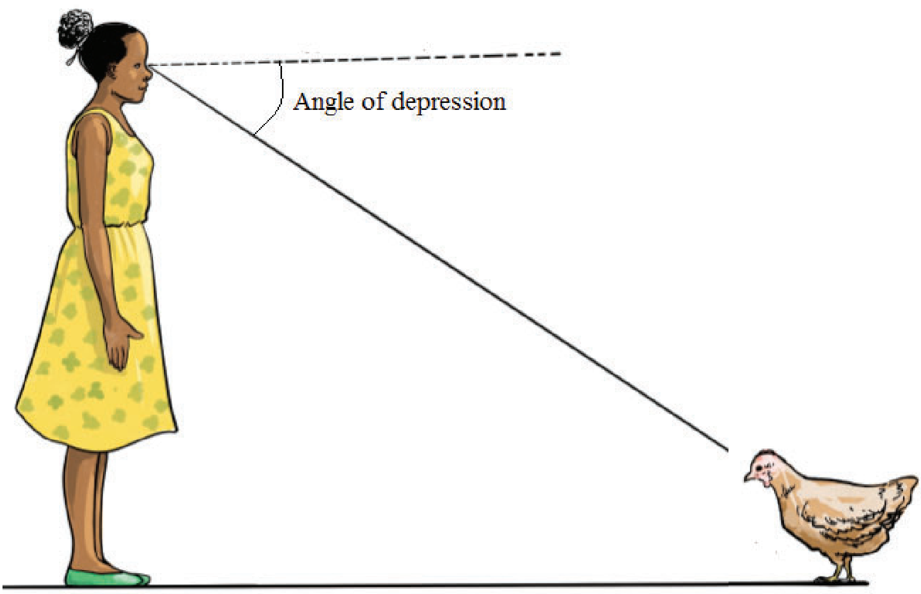
f) Results, interpretation, and conclusion

$$\tan A\hat{B}C = \frac{AC}{AB}$$

$$AB = \frac{AC}{\tan A\hat{B}C}$$

Note: If you look down at an object, the angle your line of sight makes with a horizontal line is called the **angle of depression**.

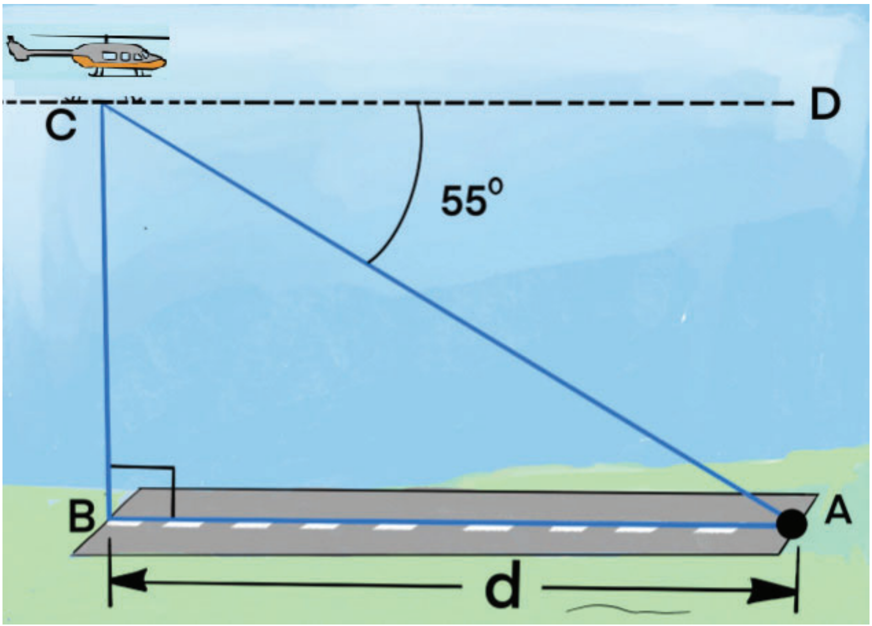
An angle of depression is the angle from the horizontal to a line of sight from the object. This angle goes "downwards" from the horizontal to a given point, as shown below.



g) Guidance on evaluation

Give Students a work related to this practical activity. For example,

A police helicopter hovering 200 metres directly above a squad car, observes the suspect running from the scene of a robbery. If the angle of depression of the suspect from the helicopter is 55Find the distance to the nearest metre, between the squad car and the suspect.



d = ?

Solution

BC=200m

$D\widehat{C}A = 55^0$

$Angles D\widehat{C}A = C\widehat{A}B$

$\tan C\widehat{A}B = \frac{BC}{AB}$

Thus, $d = AB = \frac{BC}{\tan 55^0} = \frac{200}{1.43}$ =139.86m ≈ 140m

The distance between the squad car and the suspect is thus at $140m$.

The distance between the squad car located at B and the suspect located at A is thus at

Practical activity 9: Calculating the height of a tree or a mountain

a) Rationale

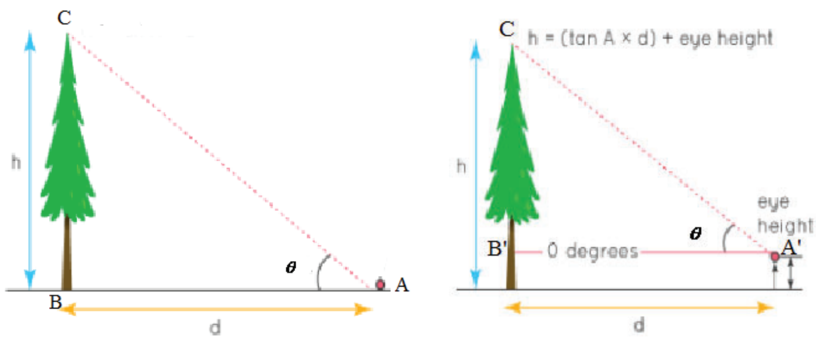
This activity is done when introducing the fundamentals of trigonometry. It is taught in the unit 1 of S4. In real life, we apply the results of such experiment when we need to calculate the breadth of the river and the height of the tree or a mountain. We note that it is also applicable in an inclined plane, also known as a ramp or a flat supporting surface titled at an angle with one end A higher than the other D, used as an aid for raising or lowering a road from D to A.

b) Objective:

To calculate the height of a tree.

c) Materials required:

A measuring tape, A calculator with cosine and tangent functions and An inclinometer to measure the angles.



d) Procedures

Step 1: Draw a right-angled triangle ABC rectangle in B, such that angle BDA is θ

Step 2: Fix a nail at a point A located at a horizontal distance from B (bottom of the tree).

Step 3: Use a measuring tape to record the distance d = d (A, B).

Step 4: Fix the eye on A and use an inclinometer to measure the angle θ = angle BAC made by the ray from the eye to the top of the tree and the horizontal line from the eye to the tree.

Step 4: Use the basic trigonometric ratios in a right-angled triangle to determine the height h = d (B, C) of the tree.

Step 5: Change the position of the nail A by increasing or reducing the distance d and complete your observations in the table of data recording.

Step 6: Use the same steps and calculate the height h when the eye of the observer is at A' such that the distance eye-ground is h'.

Does the value of h change?

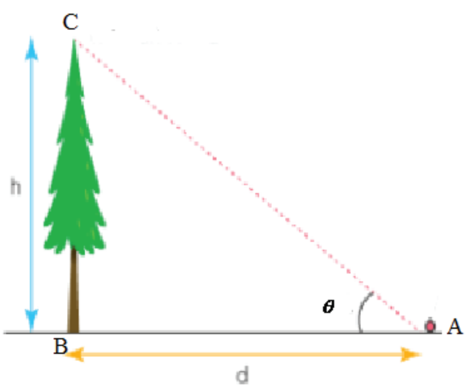
e) Data recording

Distance d	**d1**	**d2**	**d3**	**d4**
Angle θ				
$BC = h = d\tan\theta$				
$B'C = d\tan\theta$				
h = B'C+h' if h'=1.8m				

Reflective questions: How can you find the height of a tree? Is it possible to find the height of the tree when the observer is observing at a certain height from the ground?

f) Results, interpretation, and conclusion

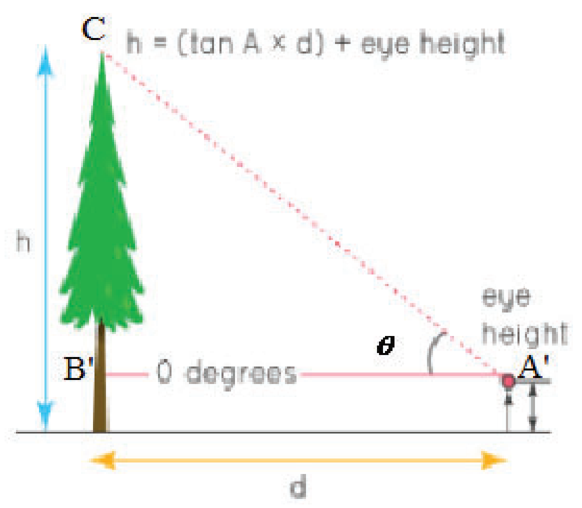
When the observer is at the ground, we can use the distance from tree to the observer and the inclination angle to find the height of the tree.



$$Tan\theta^0 = \frac{BC}{AB} = \frac{h}{d} \text{ and}$$

$$h = d\tan\theta$$

When the observer is at the height h' from the ground, we can use the distance from tree to the observer, the inclination angle and the height h' to find the height of the tree.



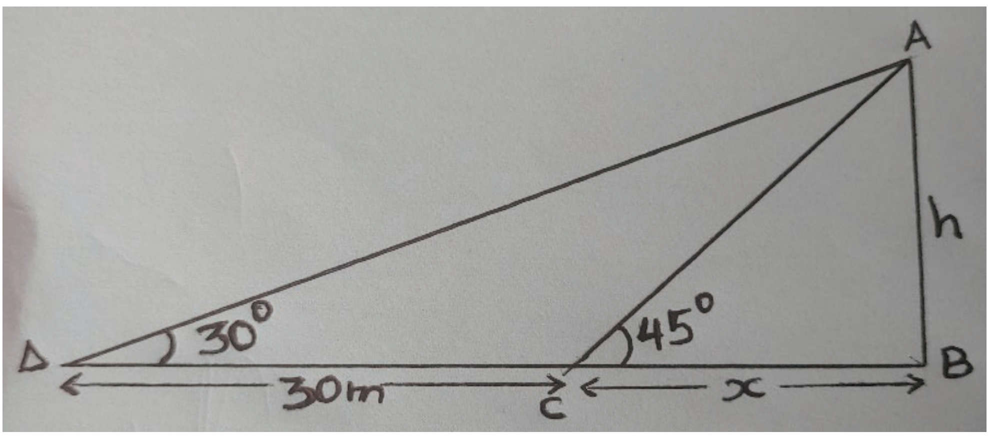
$B'C = d\tan\theta$

And $h = h' + d\tan\theta$

g) Guidance on evaluation

Give Students activities to be worked out. For example.

1. Find the value of h



Consider the point C on the triangle as the first position and D as the second position.

$$\angle BDA = 30^0$$

$$\angle ABC = \angle ABD = 90^0$$

$$Sin45^0 = \frac{AB}{AC} and \cos 45^0 = \frac{BC}{AC}$$

$$Tan45^0 = \frac{\sin 45^0}{co\,45^0} = \frac{AB}{BC}$$

where

$AB = h$

and

$BC = x$

$$\frac{h}{x} = \tan 45^0$$

$$\Leftrightarrow \frac{h}{x} = 1$$

$$h = x..........(1)$$

$$\frac{AB}{BD} = \tan 30^0$$

$$\Leftrightarrow \frac{h}{30+x} = \tan 30^0$$

$$\Leftrightarrow \frac{h}{30+x} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}h = 30 + x...........(2)$$

Putting (1) into (2), we get

$$\sqrt{3}(x) = 30 + x$$

$$\Leftrightarrow \sqrt{3}(x) - x = 30$$

$$\Leftrightarrow x\left(\sqrt{3}-1\right) = 30$$

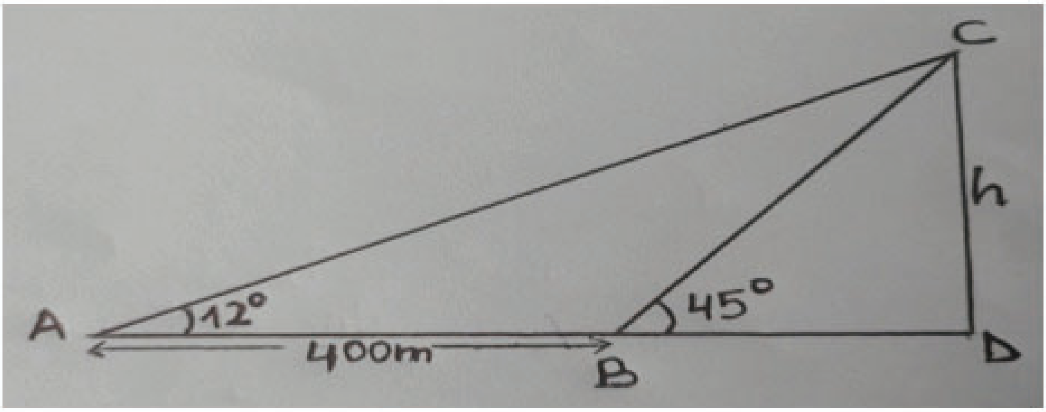
$$x = \frac{30}{\sqrt{3}-1} \approx 40.98m$$

$$x = 40.98m$$

2. From a ship, the angle of elevation of a point A to the top of a cliff is 12^0. After the ship has sailed 400m directly towards the foot of the cliff, the angle of elevation of A is 45^0.

Find the height of the cliff.

Solution



$$\frac{h}{AD} = \tan 12^0$$

$$\frac{h}{BD} = \tan 45^0 = 1 \Rightarrow h = BD$$

$$\frac{h}{400+h} = \tan 12^0$$

$$\Leftrightarrow h = (400+h)\tan 12^0$$
$$h(1-\tan 12^0) = 400\times\tan 12^0$$

$$h = \frac{400\times\tan 12^0}{1-\tan 12^0} \approx 106.33m$$

Then, the height of a cliff is $106.33m$

Practical activity 10:	Using trigonometric ratios to explore the rules used in bearing

a) Rationale:

This activity is done when teaching the concept or topic related to Triangles and applications. It is taught in the unit 1 of S4. The bearings are often used for calculation of the length of the path of moving bodies when the course is a sequence of segments of straight lines. In real life, there are two similar ways of indicating direction. On the left below is a kind of bearing which uses compass points. The bearing S34°E means the direction is 34° away from due south directed towards the east. The other way, on the right below, measures the angle clockwise from due north.

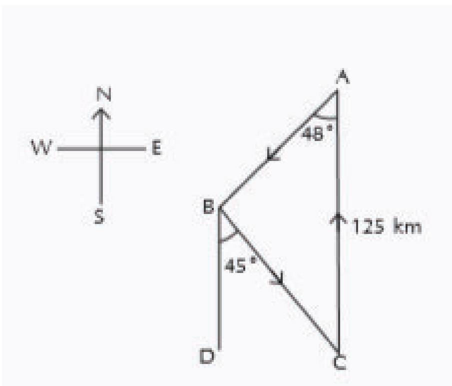
b) Objective:

Determine the length of displacement of a mobile when initial and final positions are given, and the directions of displacement indicated.

c) List of required materials:

Manilla paper, ruler, protractor, and scientific calculator

d) Illustration of one activity



e) Procedures

Step1: A body moves from A to B in direction S48°W

Step 2: Then it moves from B to C in direction S45°E

Step 3: Then it moves from C to A such that CA=125km

Step 4: Then determine the route AB+BC+CA

Express the meaning for each step and then, draw the related illustration.

f) Recording of data

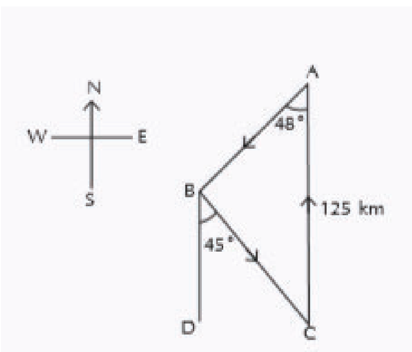
Expression	Meaning
A to B in direction S48°W	
B to C in direction S45°E	
from C to A such that CA=125km	

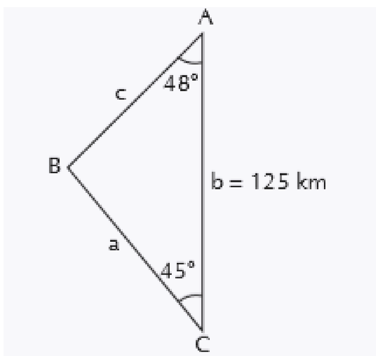
Then, determine the route AB+BC+C and explain how trigonometry is involved in bearing.

Expected answer:

Expression	Meaning
A to B in direction S48°W	From A move down, then make an angle of 48 degrees from the vertical to west
B to C in direction S45°E	From B move down, then make an angle of 45 degrees from the vertical line to East.
from C to A such that CA=125km	The distance from C to A is 125km

Because lines BD and AC are parallel it follows that $\angle DBC = \angle BCA = 45^0$.





Because lines BD and AC are parallel it follows that $\angle DBC = \angle BCA = 45^0$.

For angle B, we have B=$180° - (48° + 45°) = 87°$.

Using the law of sine:

$$\frac{\sin 48^0}{a}=\frac{\sin 87^0}{b}=\frac{\sin 45^0}{c}.$$

$$\frac{\sin 48^0}{a}=\frac{\sin 87^0}{b}\Leftrightarrow\frac{\sin 48^0}{a}=\frac{\sin 87^0}{125}\Leftrightarrow 125\sin 48^0=a\sin 87^0$$

$$\Leftrightarrow a=\frac{125\sin 48^0}{\sin 87^0}=\frac{92.89}{0.99}=93.83$$

Distance BC=93.83Km

$$\frac{\sin 87^0}{b}=\frac{\sin 45^0}{c}\Leftrightarrow\frac{\sin 87^0}{125}=\frac{\sin 45^0}{c}\Leftrightarrow 125\sin 45^0=c\sin 87^0$$

$$\Leftrightarrow c=\frac{125\sin 45^0}{\sin 87^0}=\frac{88.39}{0.99}=89.28$$

Distance AB=89.28Km

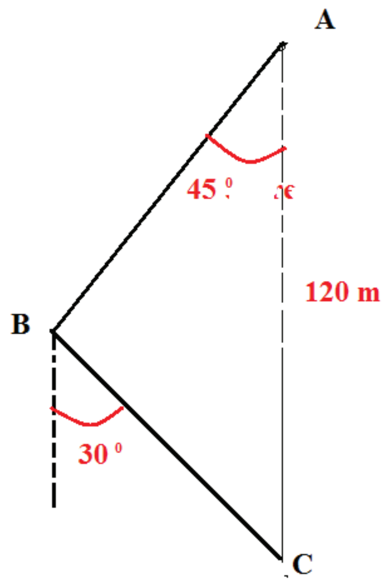
The total length of the course is $125km+93.83km+89.28km=308.11km$

g) Interpretation and conclusion

The bearing deals with directions taken by moving bodies. It is clear that trigonometry is applicable in bearing. The calculation displacement made in a certain direction by a moving body is done by the help of considering the angle made and the trigonometric ratio. The length is taken as the side of the triangle and the direction is considered as the angle.

h) Guidance on evaluation

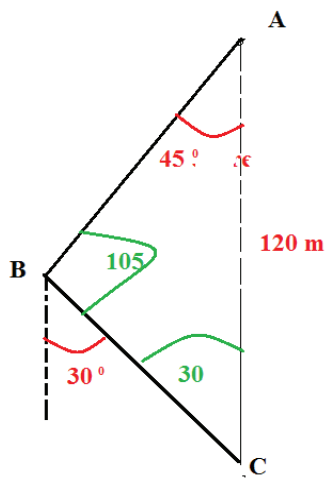
Give students the application activity. For example, Kagabo moved from the house A to B and took the direction S 45W to the house B. Then, in the direction S 30 to the house C, and finally back to the house A. The house C is located at 120m directly south of the house A. Approximate the total distance covered by Kagabo.



Solution

Expression	Meaning
A to B and took the direction S 45W	From A move down, then make an angle of 45 degrees from the vertical to west
B to C in direction S 30	From B move down, then make an angle of 30 degrees from the vertical line to East.
from C to A such that CA=120m	The distance from C to A is 120m

Drawing:



$$\frac{\sin 105^\circ}{120} = \frac{\sin 45^\circ}{BC} = \frac{\sin 30^\circ}{AB}$$

Thus, BC= $\frac{120 \times \sin 45^\circ}{\sin 105^\circ} = 87.5m$

Similarly, AB= $\frac{120 \times \sin 30^\circ}{\sin 105^\circ} = 62.5m$

The total distance covered=120m+87.5m+62.5m=270m

UNIT: 2 PROPOSITIONAL AND PREDICATE LOGIC

Practical activity 11: **Use of Venn diagrams to represent the truth of a statement**

a) Rationale:

This activity is done when teaching the concept or topic related to application of Set theory to Mathematical logic. It is taught in the unit 2 of S4. In real life, Venn diagrams are used to illustrate relationship between sets, and they facilitate to examine the validity of given statements related to member of families, groups or sets.

b) Objective:

To represent the truth of statement using Venn diagrams

c) List of required Materials:

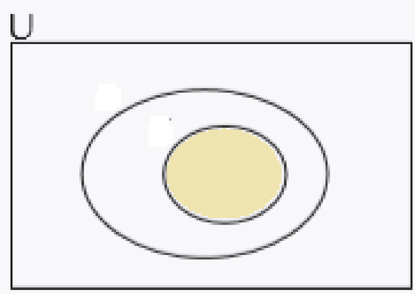
Manila paper, Marker, triangle cards, 2D shapes, etc.

d) Illustration of the activity

Let U = The set of all triangles

E = The set of equilateral triangles, and

I = The set of isosceles triangles.



e) Procedures

Step1: Represent by Venn diagram the set U of all triangles

Step2: Draw in the same diagram the set I of isosceles triangles

Step **3**: Draw in the same diagram the set E of all equilateral triangles

Step 4: What do you observe?

Step 5: Basing on your observation, determine if each of the following is true or false. In each case, explain your answer in detail or correct the question if it is false

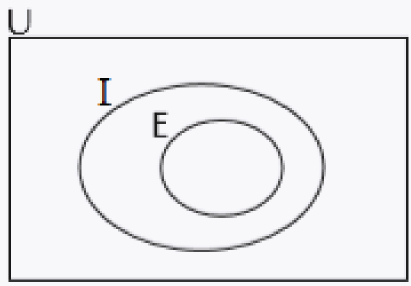
	Statement	True or false
1	All isosceles triangles are equiangular	
2	All equiangular triangles are isosceles	
3	All equilateral triangles are isosceles.	
4	All equilateral triangles are equiangular	
5	All equiangular triangles are equilateral	
6	All equiangular triangles are acute	
7	All equilateral triangles are acute	
8	All acute triangles are equilateral	
9	All acute triangles are equiangular	
10	A right triangle cannot be isosceles	
11	A right triangle cannot be equilateral	
12	An obtuse triangle cannot be isosceles	
13	An obtuse triangle cannot be equilateral.	

Expected answers:

Let U = The set of all triangles

E = The set of equilateral triangles, and

I = The set of isosceles triangles.



This diagram can help us to complete the truth value

	Statement	True (T) or false (F)
1	All Isosceles triangles are equilateral	F
2	All equilateral triangles are isosceles	T
3	All isosceles triangles are equiangular	F
4	All equiangular triangles are isosceles	T
5	All equilateral triangles are isosceles.	T
6	All equilateral triangles are equiangular	T
7	All equiangular triangles are equilateral	T
8	All equiangular triangles are acute	T
9	All equilateral triangles are acute	T
10	All acute triangles are equilateral	F
11	All acute triangles are equiangular	F
12	A right triangle cannot be isosceles	F
13	A right triangle cannot be equilateral	T

e) Results, interpretation, and conclusion

We observe that $E \subset I \subset U$. This means that:

- All equilateral triangles are isosceles triangles
- Some isosceles triangles are not equilateral
- Equilateral triangles and isosceles triangles are all triangles.

f) Additional information for the teacher

Examples to be used are not only limited to geometric shapes. The teacher should let students explore other examples from their real-life experiences. Examples:

- Relationship between animals: all domestic animals are edible (true or false)
- All Rwandans are Africans (True or false).

g) Guidance on evaluation

Give students an activity related to the above experiment. For example, ask them to do the following activity.

Use a Venn diagram to examine the truth of the following statements:

	Statement	True (T) or false (F)
1	All squares are rectangles	
2	All rectangles are squares	
3	All quadrilaterals are rectangles	
4	All parallelograms are rectangles	
5	All squares are quadrilaterals	
6	All parallelograms are quadrilaterals	
7	All quadrilaterals are squares	
8	All parallelograms are squares	

Expected answer

Let us assume that:

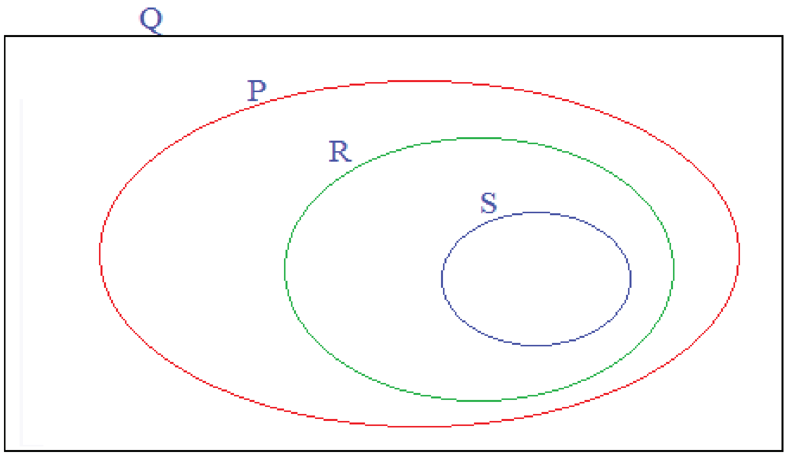
S: set of all squares

R: Set of all rectangles

P: Set of all parallelograms

Q: Set of all Quadrilaterals.

We have the following diagram:



From the above Venn diagram, the truth or falsity of statements is as follows:

	Statement	True (T) or false (F)
1	All squares are rectangles	T
2	All rectangles are squares	F
3	All quadrilaterals are rectangles	F
4	All parallelograms are rectangles	F
5	All squares are quadrilaterals	T
6	All parallelograms are quadrilaterals	T
7	All quadrilaterals are squares	F
8	All parallelograms are squares	F

Practical activity 12: Application of propositional and predicate logic on electrical circuit

a) Rationale:

This activity can be conducted when teaching a topic related to the application of Mathematical logic on electrical circuits. It is taught in unit 2 of S4.

In real life, the aim of logic is to develop a system of methods and principles that we may use as criteria for evaluating the arguments of others and as guides in constructing arguments of our own. An argument, as it occurs in logic, is a group of statements, one or more of which (the premises) are claimed to provide support for, or reasons to believe, one of the others (the conclusion). All arguments may be placed in one of two basic groups: those in which the premises really do support the conclusion and those in which they do not, even though they are claimed to. The former is said to be good arguments (at least to that extent), the latter bad arguments. The purpose of logic, as the science that evaluates arguments, is thus to develop methods and techniques that allow us to distinguish good arguments from bad arguments. The good arguments are the one applicable in other science such as in electricity as it is given here below.

b) Objective:

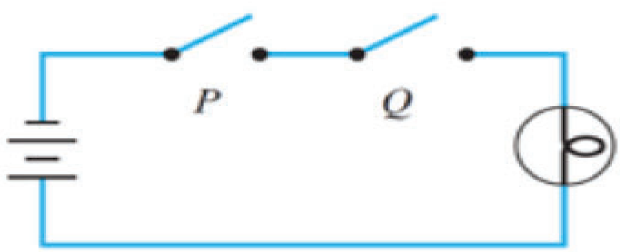
Obtain truth values of compound statements of the types $[P \wedge Q]$ and $[P \vee Q]$ using switch connections in series and in parallel respectively.

c) List of required Materials:

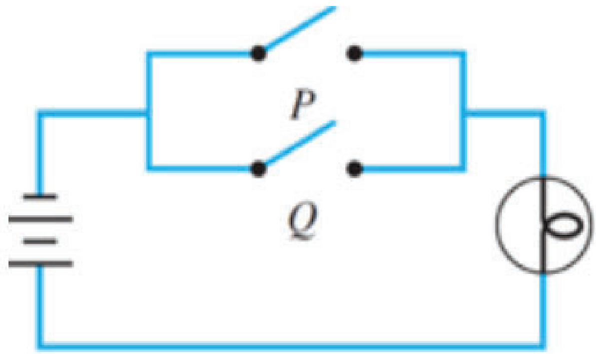
Battery (dry cell), electric wires, switches and two lamps/ bulbs.

d) Illustrations:

i. In series:

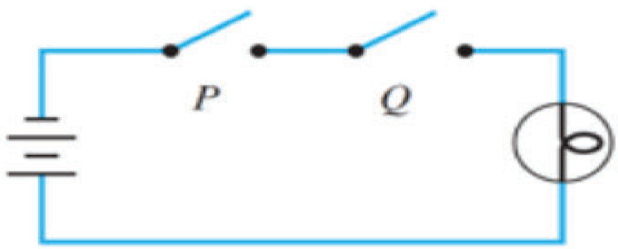


ii. In parallel:



e) Procedures

Step 1: Connect switches P and Q in series (see the following figure)



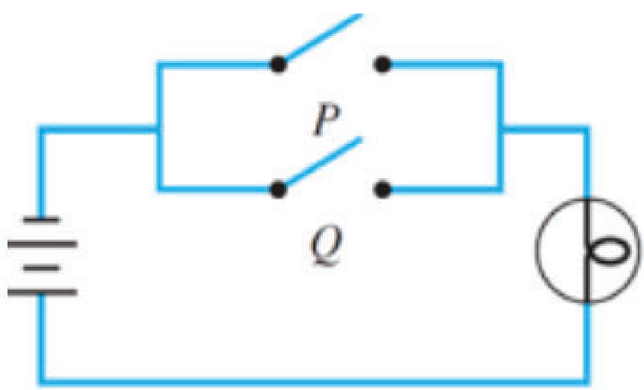
Step 2: Connect battery and lamp to complete the circuits.

Switch P on and Switch Q off and vice verso, What do you see on the lamp?

Switch P on and Switch Q on, what do you see on the lamp?

Can you relate your observation with the conjunction $[P \wedge Q]$seen in propositional logic?

Step 3: Connect switches P and Q in parallel (see the following figure)



Step 4: Connect battery and lamp to complete the circuit.

Switch P on or Switch Q off and vice verso, what do you see on the lamp?

Switch P on and Switch Q on, what do you see on the lamp?

Can you relate your observation with the disjunction $[P \vee Q]$ seen in propositional logic?

Record your findings in the table of data recording.

f) Recording of data

In series:

Switch P	Switch Q	Status of lamp
on	off	
on	on	
off	on	
off	off	

What is your observation?

In parallel:

Switch P	Switch Q	Status of lamp
on	off	
off	on	
off	off	
on	on	

What is your observation?

Expected answers:

In series:

Switch P	Switch Q	Status of lamp
on	off	Not glow
on	on	glow
off	on	Not glow
off	off	Not glow

The lamp will glow if both the switches P and Q together are on. This gives the following results:

In parallel:

Switch P	Switch Q	Status of lamp
on	off	glow
off	on	glow
off	off	Not glow
on	on	glow

In parallel: The lamp will glow if at least one of switches P and Q is on. This gives the following results:

Let p and q represent the statements as follows:

p: P is on, truth value of p is T.

~p: P is off, truth value of p is F.

q: Q is on, truth value of q is T

~q: Q is off, truth value of q is F.

In series: When the lamp glows, truth value of $p \wedge q$ is T. When the lamp does not glow, truth value of $p \wedge q$ is F. Thus, from the circuit, the following table gives the truth values of $p \wedge q$.

p	q	$p \wedge q$
T	T	T
F	T	F
T	F	F
F	F	F

In parallel: When the lamp glows, truth value of $p \vee q$ is T. When the lamp does not glow, the truth value of $p \vee q$ is F. Thus, from the circuit, the following table gives the truth value of $p \vee q$:

p	q	$p \vee q$
T	F	T
F	T	T
F	F	F
T	T	T

g) Results, interpretation, and conclusion

Let p and q represent the statements as follows:

p: P is on, truth value of p is T.

~p: P is off, truth value of p is F.

q: Q is on, truth value of q is T

~q: Q is off, truth value of q is F.

In series: When the lamp glows, truth value of $p \wedge q$ is T. When the lamp does not glow, truth value of $p \wedge q$ is F. Thus, this is related to the results obtained from the table of the truth values for $p \wedge q$.

In parallel: When the lamp glows, truth value of $p \vee q$ is T.

When the lamp does not glow, the truth value of $p \vee q$ is F. Thus, from the circuit, the following table gives the truth value of $p \vee q$

g) Guidance on evaluation

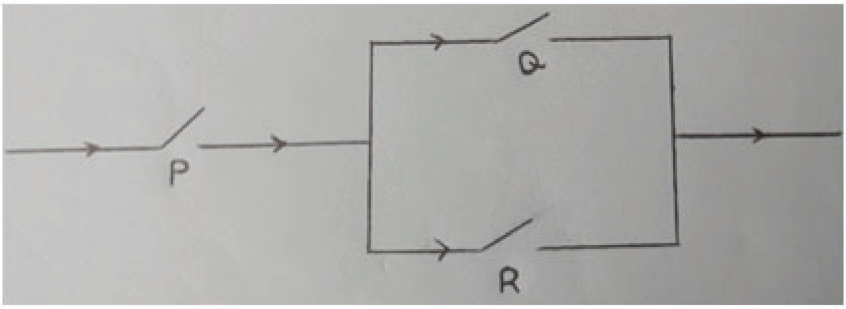
Do the following activity

Construct a switch for the following statement and deduce the case in which it is true.

$p \wedge (q \vee r)$

Solution

The related circuit is the following:



The current will flow when the switch p is closed and at least one of the switches q and r is closed.

Truth table of truth values

p	q	r	$q \wedge r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

Conclusion: The proposition $p \wedge (q \vee r)$ is true when p is true and at least one of the propositions q and r is true.

UNIT: 4 SET OF REAL NUMBERS

Practical activity 13: **Use properties of powers and explore the growth rate of the population for Rwanda**

a) Rationale:

This activity is done when teaching the concept or topic related to Powers. It is taught in the unit 4 of S4. In real life, the constant rate of population growth can be used to estimate the population of a country in the given year. Also, it can be used to determine the year in which a specific population target will be reached. It is used in decay of radioactive materials. It is also used in the compound interest.

b) Objective:

To explore the growth rate of the population for Rwanda using properties of powers.

c) List of required Materials:

Gridded Manilla paper or graph paper, Ruler, scientific calculator,

d) Illustration of activity

The following table illustrates the population of Rwanda in different years and the related rate of growth population $\left(r = \left(\frac{Current - last}{last} \right) \times 100 \right)\%$.

Where:

r = rate of population in %

Current = today's number of populations

Last = the number of populations at the beginning

Rwanda - Historical Population Data		
Year	**Population**	**Growth Rate**
2022	13,600,464	2.44%
2021	13,276,513	2.50%
2020	12,952,218	2.58%
2019	12,626,950	2.64%
2018	12,301,970	2.68%
2017	11,980,961	2.67%
2016	11,668,827	2.64%
2015	11,369,071	2.58%
2014	11,083,630	2.52%
2013	10,811,538	2.48%
2012	10,549,673	

For instance,

i. The population of Rwanda in 2022 is 13,600,464, at 2.44% increase from 2021.

ii. The population of Rwanda in 2021 was 13,276,513, at 2.5% increase from 2020.

iii. The population of Rwanda in 2020 was 12,952,218, at 2.58% increase from 2019.

iv. The population of Rwanda in 2019 was 12,626,950, at 2.64% increase from 2018.

It is possible to construct a growth model of population, which begins with the assumption that the rate of population growth is proportional to the current population. This assumption produces a model of the following form $P_n = P_0(1+r)^n$ with P_n, the current population, P_0 the population at the beginning of a given period, r the rate of population growth and n the number of periods.

Using data from two arbitrary sample points, for example 2012 and 2022, where the Rwandese population was $10\,549\,673$ and 13600464 respectively; we are required to determine the constant rate of population growth.

e) Procedures:

Step1: Write down the formula $P_n = P_0(1+r)^n$ where:

P_0 is the population at the beginning of period, n is the number of years of prediction, P_n is the population after n years and r is the constant rate of growth per year.

Step 2: Make $\frac{P_n}{P_0}$ as subject of formula

Step3: From step 2, express r in terms of $\frac{P_n}{P_0}$ and deduce the rate of population growth for Rwanda, from 2012 to 2022.

What are properties of powers did you use?

Step 4: Predict what will be the population of Rwanda in 2025.

Expected answers

Step 2: $P_n = P_0(1+r)^n \Leftrightarrow \frac{P_n}{P_0} = (1+r)^n$

Step3: $\frac{P_n}{P_0} = (1+r)^n \Leftrightarrow \sqrt[n]{\frac{P_n}{P_0}} = 1+r$ because of the following property of powers:

If $y = x^n$, $x = y^{\frac{1}{n}} = \sqrt[n]{y}$.

It implies that $\sqrt[n]{\frac{P_n}{P_0}} - 1 = r$

Or $r = \sqrt[n]{\frac{P_n}{P_0}} - 1 \Leftrightarrow r = \left(\frac{P_n}{P_o}\right)^{\frac{1}{n}} - 1$.

r is constant in a certain period of n years. From 2012 to 2022, n = 10.

Then, $r = \left(\frac{P_n}{P_o}\right)^{\frac{1}{n}} - 1 = \left(\frac{13\,600\,464}{10\,549\,673}\right)^{0.1} - 1 = 2.32\%$

Step 4: From 2012 to 2025, there are 13 years (that is n = 13); then, using

$P_n = P_0(1+r)^n$ with $P_0 =$ the population in 2012 equals 10549673,

n = 13, r =2.32, we have the population of Rwanda after 13 years given by

$P_{13} = 10\,549\,673(1+2.23)^{13} = 14\,216\,907$

The population of Rwanda in 2025 will be approximately 14216907.

f) Interpretation of results and conclusion

By the use of properties of powers and statistical data given in the table, we calculated the value of the growth rate r of the population and found that it is constant ($r = 2.32\%$) but the number n varies.

Using constant rate of population growth, we can estimate the population of a country in the given year. Also, it can be used to determine the year in which a specific population target will be reached. The properties of powers such the power of a quotient and the inverse of the power play a big role.

Note: The value of r is constant only in the period considered in calculation but can change in an ulterior period depending on the measure taken by the nation. This depends on the population policy taken by the nation. For example, in Rwanda in 1990 the rate of growth of population was more than 3% but in 2022 it is less than 3%.

g) Guidance on evaluation

Give students a work related to the practical activity done.

For example: The population of a town in 2013 was estimated to be 35,000 people. It has an annual rate of increase (growth) of about 2.4%.

Estimate the population in 2007 to the nearest hundred people.

Solution

$P_{2013} = P_{2007}(1+r)^n$

Where:

R=2.4% $n = 6\,years$

$p_{2007} =$ population in 2007

r= rate of growth

$n =$ number of years

$$p_{2007} = \frac{p_{2018}}{(1+r)^n}$$

Thus, $P_{2007} = \dfrac{35000}{(1+0.024)^6} \approx 30435$

The population in 2007 was approximately 30435 population.

UNIT: 7 POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS

Practical activity 14: Using properties of quadratic functions to explore the motion of a free-falling object

a) Rationale:

The activity on the motion of a free-falling object can be done when teaching the Concept or topic related to Application of polynomial functions. It is taught in the unit 7 of S4.

In real life, it is said that everybody on this earth is attracted by the gravitational pull of the earth. This is the reason why we stand up vertically on the soil and consequently, a freely falling object has weight W=mg, where W-weight, m-mass of the object and g-acceleration produced due to the earth's gravity.

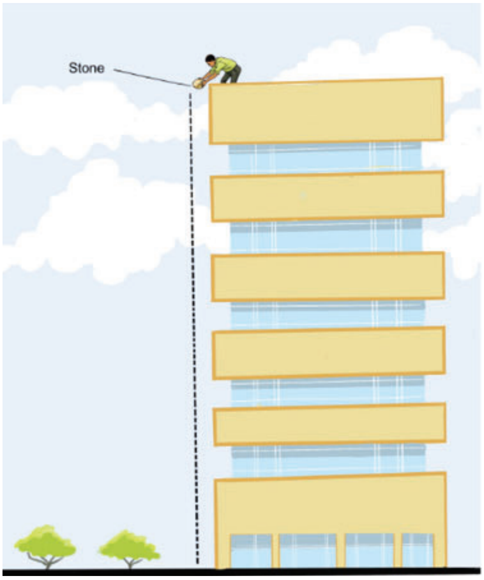
b) Objective:

To determine the time t used by the free-falling body from a given height to hit the ground.

c) List of required Materials:

Stone, chronometer, calculator, ruler, ladder.

d) Illustration of the activity

	A stone is released in free fall motion (downward) from a certain height as shown by the figure.

e) Procedures

Step1: Using a ladder, graduate the wall of the class (outside) with 1m of unit up to 4m of height at least.

Step 2: Standing on a ladder fixed on the wall of a class, let the stone fall in free fall motion on the different heights **h** from the top (suppose at 4m, 3.5m, 3m, 2.5m, 2m).

Step3: For each experiment, switch on the chronometer until the stone hits the ground. Read and record, in a table, the time t used by the stone to hit the ground.

Step 4: For each experiment, use the formula $h = f(t) = \frac{1}{2}gt^2$ for free fall motion of the stone to determine time using the heights given previously. Consider $g = 9.81m/s^2$ and complete the time in a table of data recording given below.

Step 4: Compare the results in Step 2 and 3.

f) Recording of data

Distance	10	8m	5m
Time recorded			
Time calculated			
Is the time recorded equal to the time calculated?			

- What can you say about the time used by the stone?
- Can you determine the distance covered by the stone from 4m with a free motion after 1 second? After 3 second? What is the total time to be used by the stone for hitting the ground from 10m of height?
- Can the stone and a sheet of paper fall in the same way? Explain your answer.
- Use the points $P(t,h)$ for t = 0, 1, 2, 3 4, 5, 6, 7, 8 and 10. Then, join them to find the graph of the function $h = f(t) = \frac{1}{2}gt^2$. Is the function linear or quadratic? Explain the answer.

Why does the graph of the function not look like the path followed by the stone?

Expected answers:

Distance (m)	10	8	5	4	3.5	3	2.5	2
Time recorded (tierce)								
Time calculated	1.43	1.28	1.01	0.90	0.84	0.78	0.71	0.64

g) Interpretation of result and conclusion

In free fall motion, the time t used by the moving body to hit the ground depends on the height h from where it comes from. This time is equal to the time we can observe when using the chronometer switched on immediately after the body leaves the top point and stopped immediately after the body hits the ground.

Note: It is better to use more than one chronometer to measure the time of motion and then compare the data because an error can be committed when reading with our proper eyes.

- It is better to repeat the activity many times for better comprehension.

h) Guidance on evaluation

Give an activity to be done by students. For example:

Peter climbed an avocado tree and then released an avocado that took 5m to reach the ground.

Calculate the time taken by the avocado to reach the ground.

Solution

$$y = \frac{1}{2}gt^2 \qquad t = \sqrt{\frac{2y}{g}}$$

Thus, t=1.01sec $\approx 1s$

It will take one second for an avocado to reach the ground.

UNIT: 8

LIMITS OF POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS

Practical activity 15: Representation and interpretation of limit of a polynomial function at a given point

a) Rationale:

This activity is done when teaching the lesson on the introduction of limits of functions. It is taught in the unit 8 of S4. Limits are used as real-life approximations of an effect when the cause (variable) is changing towards a given value.

Limits are not just restricted to calculus operations to define derivatives and integrals, but they also have a broad range of practical utility in physical sciences.

The real-life limits are used any time, a real-world application approaches a steady solution. One example of a limit is a chemical reaction started in a beaker in which two different compounds react to form a new compound. Now as time approaches infinity, the quantity of the new compound formed is a limit. Another typical example is that measuring the temperature of an ice cube sunk in a warm glass of water is a limit.

b) Objective:

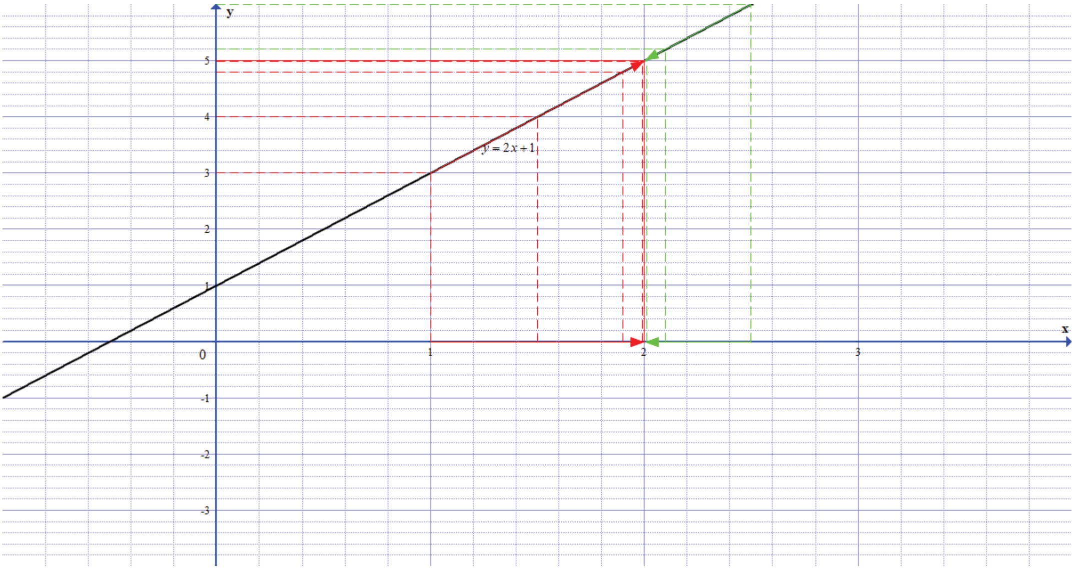
To represent and interpret the limit of a polynomial function at a given point.

c) List of required Materials:

Graph paper, ruler, scientific calculator.

d) Illustration of the activity

Determining graphically the limit of $f(x) = 2x + 1$ for x approaching 2.



The graph representing $f(x) = 2x + 1$

e) Procedures

Step 1: Calculate the value of $f(x)$ for some values of x chosen closer to 2 at the left side.

Step 2: Record the results in the table on a sheet of a paper

Step 3: Calculate the value of $f(x)$ for some values of x chosen and closer to 2 at the right side.

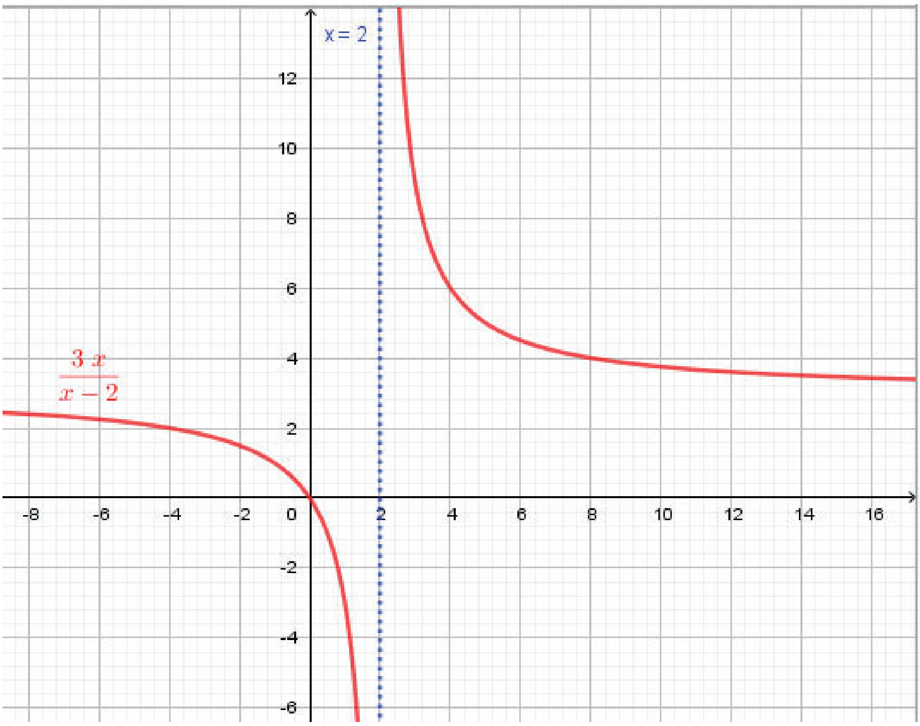
Step 4: Record the results in the table on a sheet of a paper

Can you deduce the value of $f(x)$ as x approaches 2?

Step 5: Represent graphically $f(x)$ and verify if the answer you provided in step 4 is correct.

What is the meaning of the limit of a function $f(x)$ as x approaches the value 2. Deduce the meaning of the function $f(x)$ as x approaches a value x_0.

Step 6: Follow the same steps and find the limit of $g(x) = \frac{3x}{x-2}$ for x approaching 2 from the left side and from the right side.



f) Recording of data

From the left side of 2

x	1	1.5	1.9	1.99
$f(x)=2x+1$				

From the right side of 2

x	2.5	2.1	2.01	2.001
$f(x)=2x+1$				

Expected answers

From the left side of 2

x	1	1.5	1.9	1.99
$f(x)=2x+1$	3	4	4.8	4.98

From the right side of 2

x	2.5	2.1	2.01	2.001
$f(x)$	6	5.2	5.02	5.002

The values of $f(x)$ approaches $f(2)$ as x approaches 2.

- As x approaches 2 from the left, $f(x)$ approaches 5 from below.
- As x approaches 2 from the right, $f(x)$ approaches 5 from above.

Limit of $f(x)$ when x tends to 2 from left and from right are the same and equal to 5.

Then we conclude that the limit of $f(x)$ when x tends to 2 is equal to 5.

For $g(x)=\dfrac{3x}{x-2}$ when x is approaching 2 from the left side:

x	1	1.5	1.9	1.99
$g(x)=\dfrac{3x}{x-2}$				

From the right side of 2

x	2.5	2.1	2.01	2.001
$g(x)=\dfrac{3x}{x-2}$				

Expected answers

From the left side of 2

x	1	1.5	1.9	1.99
$g(x)=\dfrac{3x}{x-2}$	-3	-9	-57	-597

The values of $g(x)$ is getting smaller and smaller as x approaches 2 form the left side.

This means that the limit of $g(x)=\dfrac{3x}{x-2}$ for x approaching 2 from the left side is $-\infty$

From the right side of 2

x	2.5	2.1	2.01	2.001
$g(x)=\dfrac{3x}{x-2}$	15	63	603	6003

The values of $g(x)$ is getting bigger and bigger as x approaches 2 form the right side. This means that the limit of $g(x)=\dfrac{3x}{x-2}$ for x approaching 2 from the right side is $+\infty$

g) Results interpretation, and conclusion

As x approaches 2 from the left, $f(x)$ approaches 5.

As x approaches 2 from the right, $f(x)$ approaches 5.

The number 5 is the value for $f(x)$ when x is approaching the value 2 from left and from right. In this case, we say that the limit of $f(x)$ when x tends to 2 is equal to 5 and we write:

$$\lim_{x\to 2}(2x+1)=5$$

For $g(x)=\dfrac{3x}{x-2}$, the limit at the left side is different from the limit at the right side because $-\infty \neq +\infty$. We say that the limit of $g(x)=\dfrac{3x}{x-2}$ as x is approaching 2 does not exist.

The limit of a function when the variable is approaching a given number is the value that function can take when the variable is that given number. That value should be the same from the left and from the right side. When the limit at the left side is different to the limit at the right side we say that the limit of that function does not exist.

h) Guidance on evaluation

Do the following activity

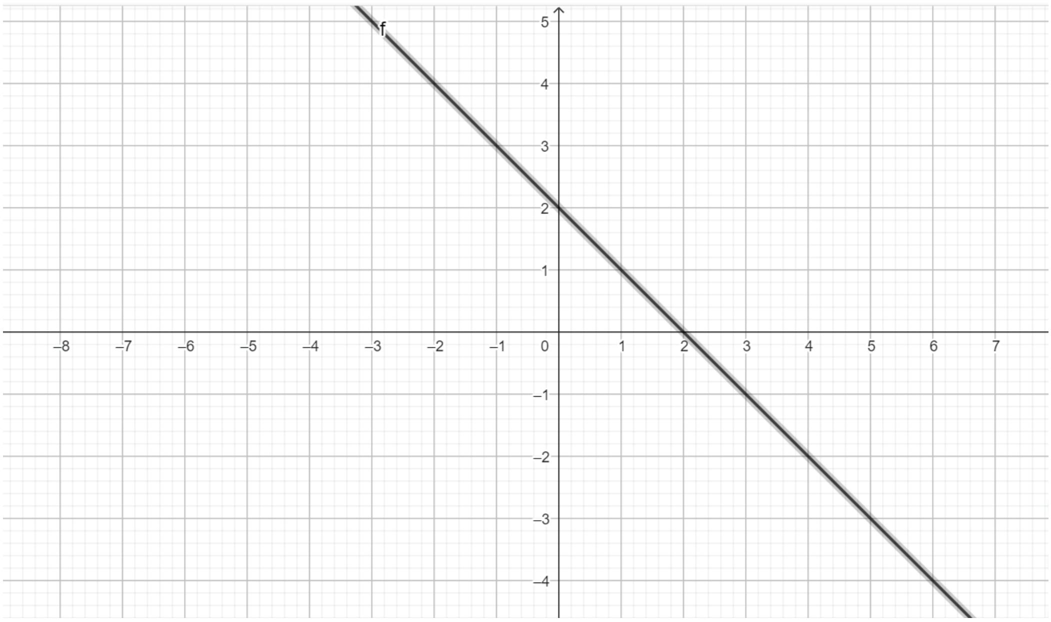
Given the function $f(x) = 2 - x$

Evaluate the limit of the function $f(x)$ as x approaches to 2 from the left and from the right.

Solution

Let us take randomly two values of x (x=3 and x=-1) and replace in the function. We get two points A (3, -1) and B (-1, 3)

Figure 12



From the left side of 2

x	1	1.5	1.7	1.9
$f(x)$				

From the right side of 2

x	2.7	2.5	2.3	2.1
$f(x)$				

Practical activity 16: Representation of a given rational function and its asymptotes on a graph paper

a) Rationale:

This activity is done when teaching the concept or topic related to application of limits**.** It is taught in the unit 8 of S4. An asymptote is a straight line that constantly approaches a given curve but does not meet at any infinite distance. In other words, Asymptote is a line that a curve approaches as it moves towards infinity. The curves visit these asymptotes but never overtake them. They are useful for graphing rational equations. They are relevant for Algebra: Rational functions and Calculus: Limits of function. The better teaching of this unit will facilitate the linkage of asymptote and its application in real life. For example, in psychology, the approach toward a full level of response or cure after many learning trials.

b) Objective:

Draw appropriately possible asymptotes of a given rational function.

This is an **inquiry based practical work**.

c) List of required Materials:

Graph paper, ruler, scientific calculator.

d) Illustration of the activity

Let us consider the rational function defined by $f(x) = \dfrac{2x+5}{x-1}$

e) Procedures

Step1: Determine the domain of definition for $f(x)=\dfrac{2x+5}{x-1}$

Step 2: From the obtained domain of definition of $f(x)=\dfrac{2x+5}{x-1}$, make the table of $f\left(x\right)$ using different values of x.

Step 3: On the graph paper, draw Cartesian coordinates and complete the points found in the table of values

Step 4: Determine if there is a number **b** that is equal to the limit of $f\left(x\right)$ when x tends to ∞.

How can you call the line with equation $y=b$ vis-a-vis the graph of $f\left(x\right)$?

Step 5: Observe the function and verify if there is a number "a" such that $\lim f(x)$ when x tends to "a" is ∞. If the number exists, how can you call the line with equation $x = a$?

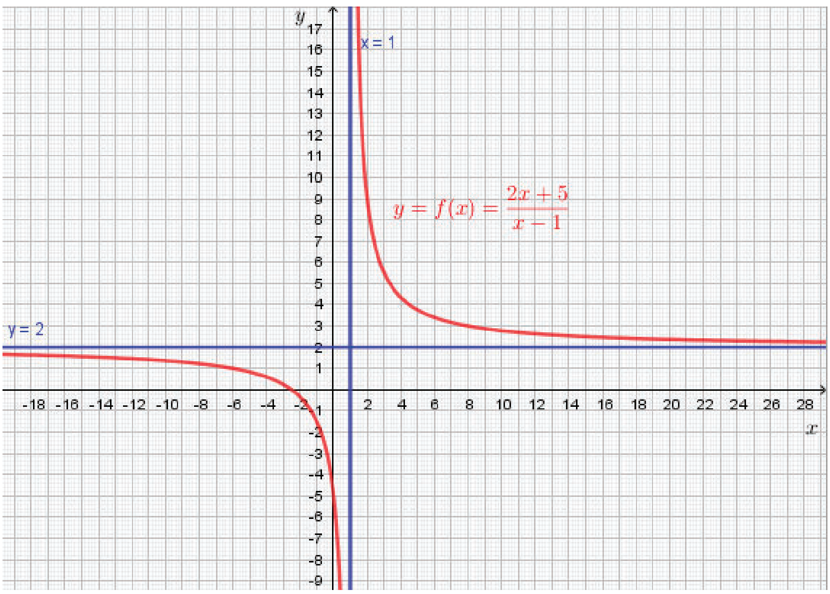
Step 6: Plot the function in the Cartesian coordinates, the lines found in step 4 and step 5.

What is the meaning of asymptotes to the graph of the function $f(x)$?

Does the graph of the function $f(x) = \frac{2x+5}{x-1}$ have more asymptotes? If yes, find them.

f) Results, interpretation, and conclusion

Graph of the function:



The equation of the horizontal asymptote is $y = 2$

The equation of the vertical asymptote is $x = 1$

The function $f(x) = \frac{2x+5}{x-1}$ admits horizontal and vertical asymptotes.

It has not an oblique asymptote.

g) Note for the teacher

The vertical asymptote is found to be the value which makes the denominator zero, but this condition is not enough; you must check if the limit of a given function for x approaching that value is infinite.

The horizontal asymptote is a special case of oblique asymptote.

h) Guidance on evaluation

Give Students an activity to be done. For example,

Find the horizontal and vertical asymptotes to the curve of the function defined by

$f(x)=\dfrac{3-4x}{2x+2}$ and sketch the graph.

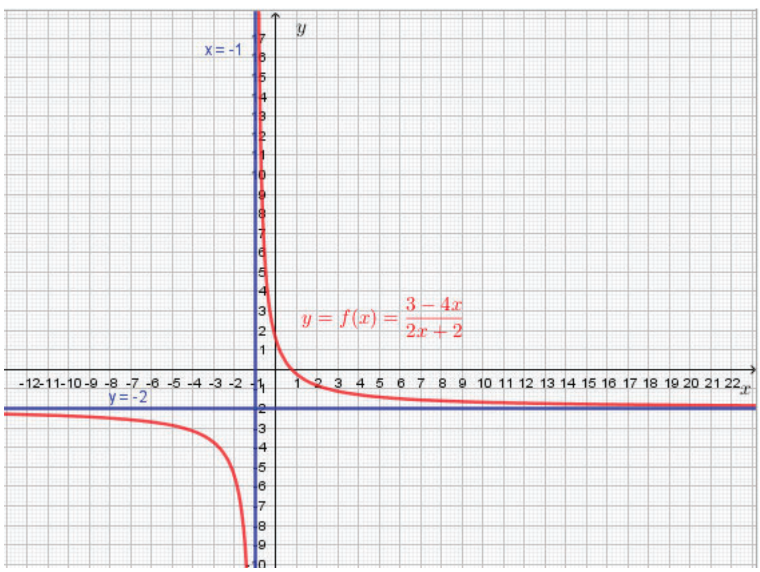
Solution

Table of values

x	0	1/2	1	2	-3/2	-2	-3	-4
$f(x)=\dfrac{3-4x}{2x+2}$	3/2	1/3	-1/4	-5/6	-9	-11/2	-15/4	-19/6

We plot the points given the above table and connect the dots to obtain the graph of the function

Graph:



$$\lim_{x\to\infty} f(x)=-2$$

Thus, $y=-2$ is the horizontal asymptote.

The function is not defined when $x=1$. Thus, the line with equation $x=-1$ is the equation of the vertical asymptote.

UNIT: 9

DIFFERENTIATION OF POLYNOMIAL, RATIONAL AND IRRATIONAL FUNCTIONS AND THEIR APPLICATION

Practical activity 17: Geometric interpretation of derivative of a function at a point

a) Rationale:

This activity is done when teaching the concept or topic related to derivative of functions. It is taught in the unit 9 of S4. Derivative of a function f(x) signifies the rate of change of the function f(x) with respect to x at a point lying in its domain. For a function to be differentiable at any point x = a in its domain, it must be continuous at that particular point but vice-versa is necessarily not always true. In everyday life, the derivative can tell you at which speed you are driving, or help you predict fluctuations on the stock market; in machine learning, derivatives are important for function optimization. The content of derivative of a function can be linked to the real-life situations such as in business, in transport, etc. For example, if you run a business (say selling ice creams), derivatives can help you decide what quantity you should sell.

b) Objective:

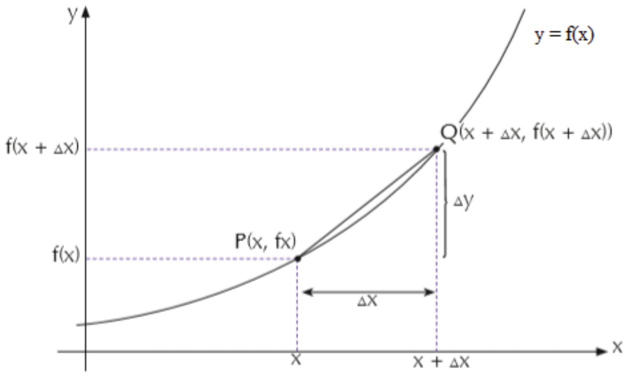
To illustrate derivative of the function $y = f(x) = x^2 - 6$ at $x_0 = 1$ by using a geometric interpretation.

This is an inquiry based practical work.

c) List of required Materials:

Graph paper, ruler, scientific calculator, Math software such as Geogebra.

d) Illustration of the activity



e) Procedures

Step 1: In a Cartesian plane draw the graph of the function $y = f(x) = x^2 - 6$.

Step 2: Take a point $P(x_0, f(x_0))$ for $x_0 = 1$ on the graph of the function f and draw the tangent of the graph at the point P.

Step 3: Consider a small change $\Delta x = 0.01$ on OX, identify the point whose abscise is $x_0 + \Delta x$. **Step 4**: Determine on the graph of the function f the corresponding point $Q(x_0 + \Delta x, f(x_0 + \Delta x))$

Step 5: Having two points from Step 2 and step 3, find the slope of the tangent of the graph at $P(x_0, f(x_0))$. (The slop is by definition $\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$).

Step 6: Find the derivative of the function $y = f(x) = x^2 - 6$ at $P(x_0, f(x_0))$ and complete your findings in the table of data recording.

Step 7: Compare the slop of the tangent of the graph at the point $P(x_0, f(x_0))$ and the derivative of the function $y = f(x) = x^2 - 6$ at the point $P(x_0, f(x_0))$.

How are they?

f) Data recording:

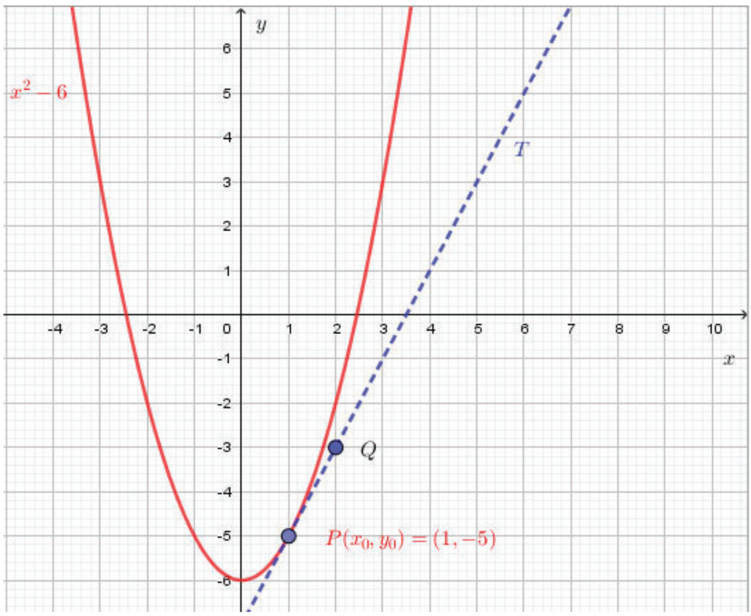
x_0	$f(x_0)$	Δx	$x_0+\Delta x$	$Q(x_0+\Delta x, f(x_0+\Delta x))$	$\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$	$f'(x_0)$
1		0.01				
1		0.001				

After comparing the slop of the tangent of the graph at the point $P(x_0, f(x_0))$ and the derivative of the function $y=f(x)=x^2-6$ at the point $P(x_0, f(x_0))$, try to explain the meaning of the derivative of any function $y=f(x)$ at a point $P(x_0, f(x_0))$.

Expected answer

The graph of the function $y=f(x)=x^2-6$

x_0	$f(x_0)$	Δx	$x_0+\Delta x$	$Q(x_0+\Delta x, f(x_0+\Delta x))$	$\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$	$f'(x_0)$
1	-5	0.01	1.01	(1.01,-4.9799)	2	2
1	-5	0.001	1.001		2	2



It is clear that the value of the slope $\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ is 2 and it is equal to the derivative $f'(x_0)$.

Therefore, the derivative of a function $y=f(x)$ at the point $P(x, f(x))$ equals the slope of the tangent line to the graph at that point.

g) Interpretation of result and conclusion

As conclusion, the derivative $f'(x_0)$ of a function $y=f(x)$ at the point $P(x_0, f(x_0))$ is the slope of the tangent line to the graph at that point $P(x_0, f(x_0))$.

g) Guidance on evaluation

Do the following activity

Find the slop of the tangent of the function $f(x)=\frac{1}{x}$ at $x=10$.

Solution

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}=\lim_{\Delta x\to 0}\frac{(\frac{1}{x+\Delta x}-\frac{1}{x})}{\Delta x}$$

$$=\lim_{\Delta x\to 0}\frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}=\lim_{\Delta x\to 0}\frac{x-(x+\Delta x)}{x\Delta x(x+\Delta x)}$$

$$=\lim_{\Delta x\to 0}\frac{-\Delta x}{x\Delta x(x+\Delta x)}=\lim_{\Delta x\to 0}\frac{-1}{x(x+\Delta x)}=\frac{-1}{x(x+0)}=-\frac{1}{x^2}$$

The slop of the tangent of the graph at the point P(10,0.1) is equal to

$$-\frac{1}{100}=-0.01$$

Practical activity 18: Verification of the validity of l' Hospital theorem

a) Rationale:

This activity is done when teaching the concept or topic related to limits of rational functions. It is taught in Unit 9 of S4. Hospital's rule is a theorem which provides a technique to evaluate limits of indeterminate forms. Hospital's rule is used primarily for finding the limit as $x \to a$ of a function of the form $\frac{f(x)}{g(x)}$, when the limits of f and g at a are such that $\frac{f(a)}{g(a)}$ results in an indeterminate form, such as $\frac{0}{0}$ or $\frac{\infty}{\infty}$. The experiment is conducted to foster students' conceptual understanding related to Hospital's rule. Hospital's rule has many applications in the real world, mostly in physics, and engineering.

b) Objective:

Verify l'Hospital theorem when finding the limit of a rational function at the value that make the denominator zero such as $\lim_{x \to 3} h(x)$ for $h(x) = \frac{9-x^2}{x^3-3x^2+2x-6}$.

It is an inquiry based practical work.

c) Procedures

Step 1: Given the function $h(x) = \frac{9-x^2}{x^3-3x^2+2x-6}$, what are values that make the denominator null? Can the function exist when $x^3-3x^2+2x-6=0$?

Step 2: Find $\lim_{x \to 3} h(x)$. What do you get?

Step 3: Let $f(x) = 9 - x^2$ and $g(x) = x^3-3x^2+2x-6$, find $\lim_{x \to 3} f(x)$ and $\lim_{x \to 3} g(x)$

Step 4: If you have found 0 in step 3, calculate $f'(x)$ and $g'(x)$ and $\lim_{x \to 3} \frac{f'(x)}{g'(x)}$

Step 5: Draw the graph of function $h(x)=\frac{9-x^2}{x^3-3x^2+2x-6}$ (use Math software such as Geogebra or use a table of values) and refer to the graph to give the approximate value of $\lim_{x\to 3} h(x)$.

Step 6: Compare the answer of $\lim_{x\to 3}\frac{f'(x)}{g'(x)}$ and the answer found in Step 5.

What can you say about the $\lim_{x\to 3}\frac{f(x)}{g(x)}$ and $\lim_{x\to 3}\frac{f'(x)}{g'(x)}$?

Expected answers:

$f'(x)=-2x$

$g'(x)=3x^2-6x+2$

$\lim_{x\to 3}\frac{-2x}{3x^2-6x+2}=\frac{-6}{27-18+2}$

$\lim_{x\to 3}\frac{f'(x)}{g'(x)}=-\frac{6}{11}$

Calculation of $\lim_{x\to 3}\frac{f(x)}{g(x)}$ by factorization

$f(x)=9-x^2=(3-x)(3+x)$

$g(x)=x^3-3x^2-2x-6=(x-3)(x^2+2)$

We have then:

$$\frac{f(x)}{g(x)}=\frac{(3-x)(3+x)}{(x-3)(x^2+2)}=\frac{-(3+x)}{x^2+2}$$

Thus, $\lim_{x\to 3}\frac{f(x)}{g(x)}=\lim_{x\to 3}\frac{-(3+x)}{x^2+2}=\frac{-(3+3)}{9+2}=-\frac{6}{11}$

d) Results, interpretation, and conclusion

On one hand, we can calculate $\lim_{x\to 3}\frac{f(x)}{g(x)}$ by factorization:

$$\frac{f(x)}{g(x)}=\frac{(3-x)(3+x)}{(x-3)(x^2+2)}=\frac{-(3+x)}{x^2+2}$$

Thus, $\lim_{x\to 3}\frac{f(x)}{g(x)}=\lim_{x\to 3}\frac{-(3+x)}{x^2+2}=\frac{-(3+3)}{9+2}=-\frac{6}{11}$

In the other hand,

$$\lim_{x\to 3}\frac{f'(x)}{g'(x)}=-\frac{6}{11}$$

Therefore,

$\lim_{x\to 3}\frac{f(x)}{g(x)}=\lim_{x\to 3}\frac{f'(x)}{g'(x)}=-\frac{6}{11}$ This illustrates that l' Hospital theorem is verified.

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

f) Guidance on evaluation

Give students activities related to the above one. For example;

Calculate the limit $\lim_{x\to 2}\frac{4-x^2}{x^2-3x+2}$ using the L'Hospital's theorem and factorization method.

What is the conclusion?

Solution

Using the L'Hospital's rule, we get:

$$(4-x^2)'=-2x$$

$$(x^2-3x+2)'=2x-3$$

$$\lim_{x\to 2}\frac{f(4-x^2)'}{(x^2-3x+2)'}=\lim_{x\to 2}\frac{-2x}{2x-3}=-4$$

Using the factorization method, we obtain

$$4-x^2=(2-x)(2+x)$$

$$x^2-3x+2=(x-2)(x-1)$$

$$\lim_{x\to 2}\frac{4-x^2}{x^2-3x+2}=\lim_{x\to 2}\frac{(2-x)(2+x)}{(x-2)(x-1)}=\lim_{x\to 2}\frac{-(2+x)}{x-1}=\frac{-4}{1}=-4$$

Practical activity 19: Using derivative to verify the maximum of a function

a) Rationale:

This activity is done when teaching the optimization or the use of Maximum or minimum of a function in real life. It is taught in Unit 9 of S4 Mathematics.

Local maximum and minimum points are quite distinctive on the graph of a function and are therefore useful in understanding the shape of the graph.

In real life, optimization methods are used in many areas of study to find the values of the variable that maximize or minimize some study parameters, such as minimizing costs in the production of a good or service, maximize profits, minimize raw material in the development of a good, or maximize production.

For instance, the concept of optimization problems can be used in economics while making packages of maximum volume with minimum cost.

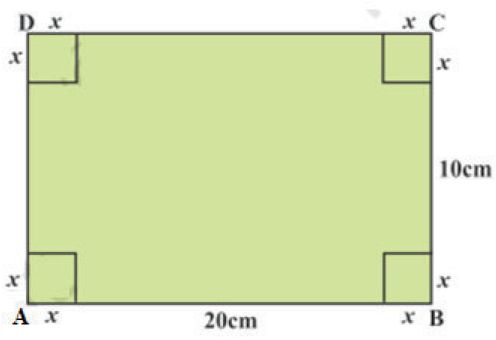
b) Objective:

Construct an open box of maximum volume V from a given rectangular sheet and using derivative to prove it.

c) List of required materials:

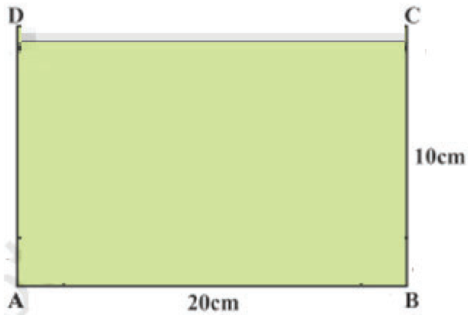
- Chart papers
- Pair of scissors
- Sellotape
- Scientific calculator

d) Illustration or set up

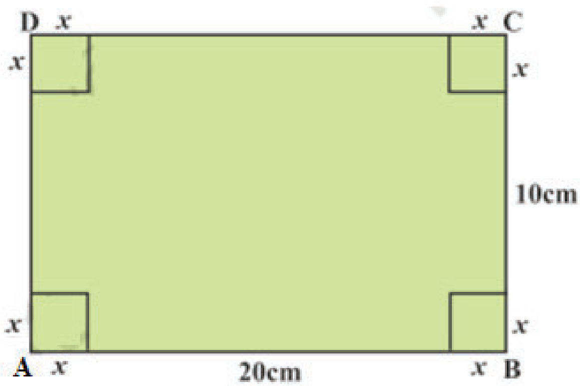


e) Procedures:

Step 1: Take a rectangular chart paper of size for example $20cm \times 10cm$ and name it as ABCD

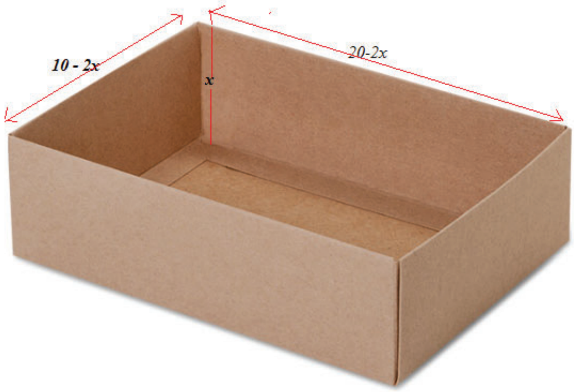


Step 2: From each corner A, B, C and D, cut off four squares of side xcm (see the Figure below).



Step 3: Repeat the process by taking the same size of chart papers and consider taking different values of xcm.

Step 4: Make open boxes by folding flaps using sell tape.



Step 5: Write down the function $V = f(x)$ for the volume of the open box in step 4.

Step 6: Using the function obtained in step 5, for each value of x provided in table below, find the volume of each open box and complete your findings in the table for data recording. For which value of x the volume V of the box will be maximum?

Step 7: Use the values of x provided in table below to construct and calculate the volume of each box. Compare the volumes of the boxes you constructed, which box has the maximum volume?

Step 8: Use the function $V = f(x)$ for the volume of the open box, find the first derivative of $V = f(x)$ and solve $f'(x) = 0$ and get the value of x. Compare the value of x and the value of x obtained in step 4.

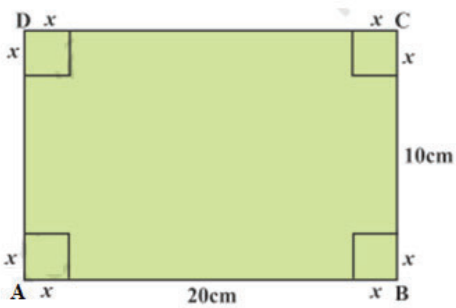
f) Recording of data

xcm	$Volume\ of\ the\ box$
$x = 1cm$	
$x = 1.5cm$	
$x = 2cm$	
$x = 2.1cm$	
$x = 2.5cm$	
$x = 3cm$	

For which value of x the volume V of the box will be maximum?

Expected answer:

Considering the length of 20 cm,



-The length L of the box will be $L = 20 - 2x$,

- The width of the box will be $W = 10 - 2x$

- The height of the box will be $H = x$

Then, the volume of the box

$V = L \times W \times H$

$= 200x - 60x^2 + 4x^3$

By considering an arbitrary volume of the box given by $V = 200x - 60x^2 + 4x^3$, and by taking xcm, we get the following table of values:

xcm	$Volume\ of\ the\ box$
$x = 1cm$	$144cm^3$
$x = 1.5cm$	$178.5cm^3$
$x = 2cm$	$192cm^3$
$x = 2.1cm$	$192.4cm^3$
$x = 2.5cm$	$187.5cm^3$
$x = 3cm$	168cm^3

After calculating the volume for different values of xcm, we find that the volume of the box is maximum when $x = 2.1cm$

Using the derivative, we have

$V = (20 - 2x)(10 - 2x)x$ or $V = 200x - 60x^2 + 4x^3$

$V'(x) = 200 - 120x + 12x^2$.

$V'(x) = 0$, *i.e.* $3x^2 - 30x + 50 = 0$

$$x = \frac{30 \pm \sqrt{900 - 600}}{6} = 7.9 \text{ or } 2.1$$

We reject $x = 7.9$ (while replacing the value of x in the expression defining the sides of a base of a rectangular prism, one side (width) will be negative, or having a negative distance does not make sense).

g) Interpretation of result and conclusion:

After calculating the volume for different values of xcm, we find that the volume of the box is maximum when $x = 2.1cm$

It is clear that we can get this value by using the derivative of the function $V(x) = 200x - 60x^2 + 4x^3$

As discussed above, we solve $V'(x) = 0$, *i.e.* $3x^2 - 30x + 50 = 0$

And find the value $x = 2.1$.

The obtained result implies that the first derivative of a function $y = f(x)$ can help us to get the value of x for which $y = f(x)$ is at the maximum value. This means that we need to solve $f'(x) = 0$

h) Additional note to the teacher:

For maxima or minima, we have,

$$\frac{dV}{dx} = 0, \text{ i.e. } 3x^2 - 30x + 50 = 0$$

$x = \dfrac{30 \pm \sqrt{900 - 600}}{6} = 7.9 \text{ or } 2.1$ we get 2 values to be analysed.

We reject $x = 7.9$ (while replacing the value of x in the expression defining the sides of a base of a rectangular prism, one side (width), one side (width) will be negative or having a negative distance does not make sense)

$$\frac{d^2V}{dx^2} = -120 + 24x$$

When $x = 2.1, \dfrac{d^2V}{dx^2}$ *is negative*. Hence, V is maximum at $x = 2.1cm$

The obtained result implies that the height of the box must be $x = 2.1cm$ to make a box of the maximum volume.

i) Guidance on evaluation:

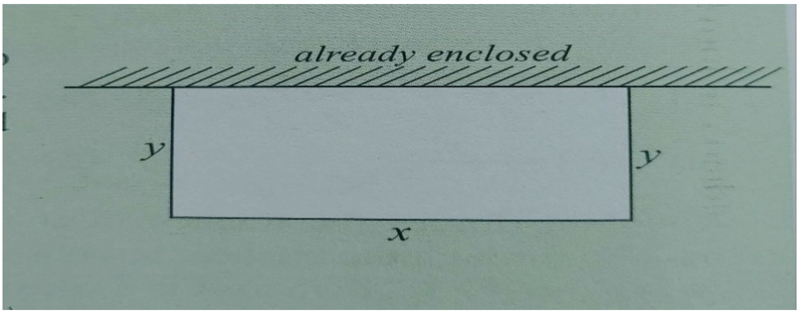
Ask students to do the following activities:

1. We need to enclose a small farm with a fence. We have 500 metres of fencing

materials, and one side of the farm is already enclosed by the neighbouring farm owner. Determine the dimensions of the farm that will be enclosed by the largest area.

Solution

The first idea in solving such a problem is to first draw a diagram that reflects the situation. We want to maximize the area of the farm and 500m of fencing materials.



The amount of the fencing is the constraint

Maximize: $A = xy$ ------------- (i)

Constraint: $500 = x + 2y$ ------------ (ii)

We need to express the area of function in a single variable from equation (ii).

$500 = x + 2y$

$x = 500 - 2y$

Therefore, the area will be:

$A = (500 - 2y)(y)$

$\Leftrightarrow A = 500y - 2y^2$

For maxima or minima, we have,

$\frac{dA}{dy} = 500 - 4y$

$500 - 4y = 0$

$y = 125m$

$x = 500 - 2y = 500 - 2(125) = 250m$

The dimensions of the farm are 250m to 125m.

Plugging this into the area gives:

$A = 500(125) - 2(125)^2$

$A = 31,250m^2$

Or,

$A = 125 \times 250 = 31250m^2$

The corresponding maximum area is $31,250m^2$

2. Use the derivative to find the maximum and the minimum height to be attained by a stone thrown vertically upwards with initial velocity of 20m/s.

UNIT: 13 POINTS, STRAIGHT LINES, AND CIRCLES IN 2D

Practical activity 20: Representation of a straight line given by its parametric equations

a) Rationale:

This activity is done when teaching the concept or topic related to lines in 2D**.** It is taught in unit 13 of S4. One of the advantages of parametric equations is that the parameter can be used to represent something useful and therefore provide us with additional information about the graph. For example, parametric equations can allow us to graph the complete position of an object over time.

All the details like the height of the ground, direction, and speed of spin can be modelled using parametric equations. For instance, if you have water flowing into a cylindrical tank at a known rate, you can express the volume in the tank as a function of height in the tank. But you can also express both the level in the tank and the volume of water in the tank as functions of a parameter t .

b) Objective:

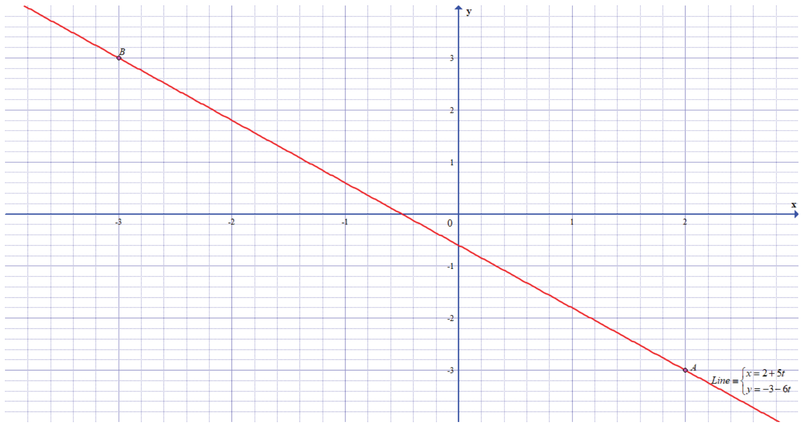
Represent, on a Cartesian plane, a straight line given by its parametric equations. This is a concept-based experiment.

c) List of required materials:

- Graph paper
- Ruler
- Scientific calculator

d) Illustration of the activity:

Let us consider a straight line defined by parametric equations $\begin{cases} x = 2 + 5t \\ y = -3 - 6t \end{cases}$



e) Procedures:

Step 1: Draw a table of values for $t=-1;0;0.5;and 1$ on row. On the column, write under t, x and y and then start filling the table of data recording

Step 2: By taking $t=0$, find the values of x and y

Step 3: By taking $t=-1$, find the values of x and y

Step 4: By taking $t=0.5$, find the values of x and y

Step 5: By taking $t=1$, find the values of x and y

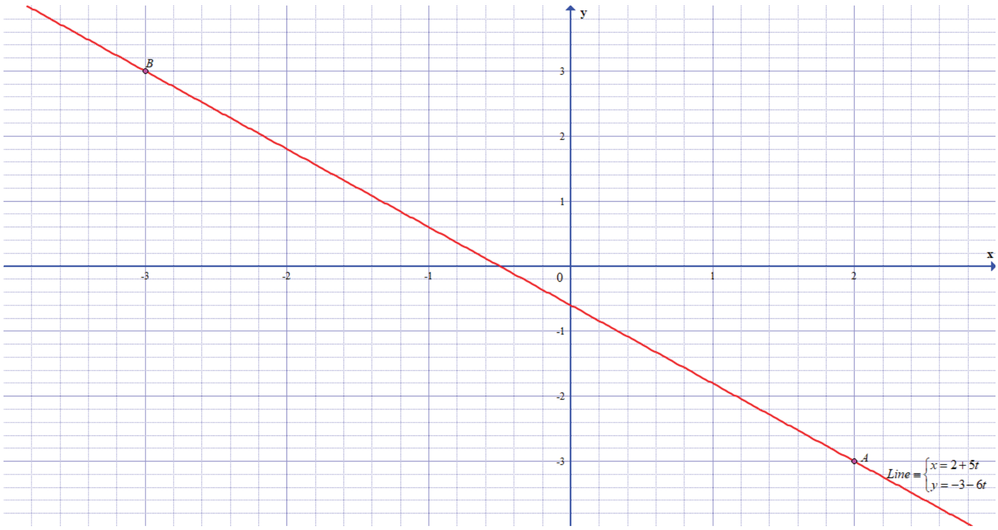
Step 6: On a Cartesian plane, locate all points (x,y) obtained from step 2 to step5 and use a ruler to join the points and trace the required straight line. What is your observation?

f) Data recording

t	-1	0	0.5	1
x				
y				

Expected answers

t	**-1**	**0**	**0.5**	**1**
x	-3	2	4.5	7
y	3	-3	-6	-9



After calculating x and y for different values of t and tracing a line joining the obtained points, we see that all the points are on the same straight line (see the Graph above).

g) Results interpretation and conclusion:

To get a line represented by its parametric equation, we have to take arbitraly some values of the parameter. For each value of t, find x and y which give the coordinate of a point $P(x, y)$ of the required line. Then , after finding at least 2 points $P(x, y)$ and $M(x, y)$, join them to find the required line in the XOY coordinates.

h) Information for the teacher:

A parametric equation is an equation depending on a parameter whose values vary. For example, a moving body in a plane. At different values of the time the position of the body changes.

h) Guidance on evaluation:

Ask students to do the following activity

Sketch the following parametric equation

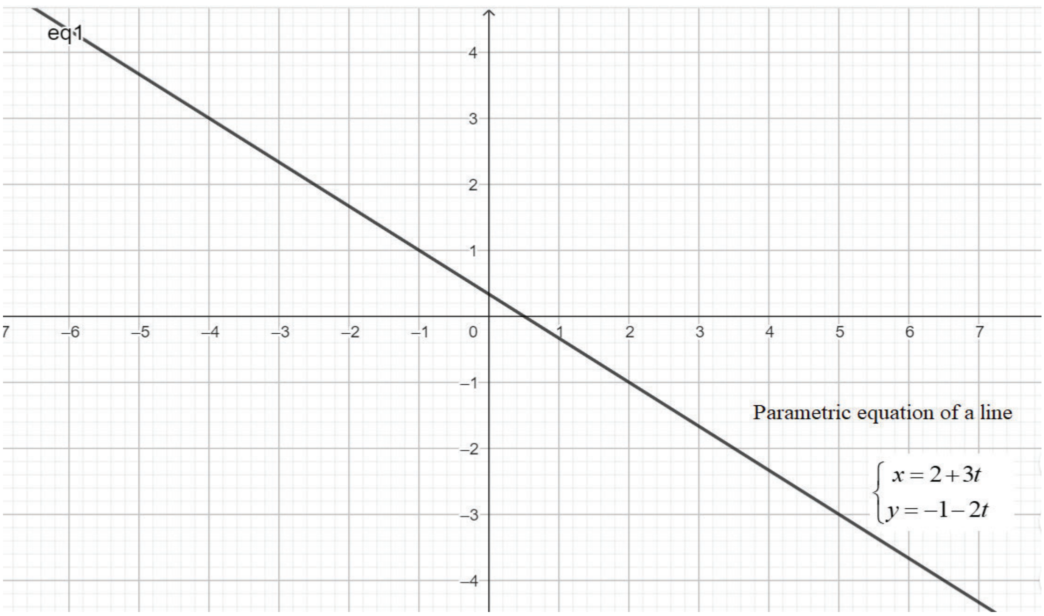
$$\begin{cases} x = 2 + 3t \\ y = -1 - 2t \end{cases}$$

Solution

Let us take two arbitrary values of $t=1$ and $t=-2$. We, therefore, find the values of x and y , as shown in the following table.

t	1	-2
x	5	-4
y	-3	3

By plotting two points namely A (5, -3) and B (-4, 3) on a cartesian plane, we get the following graph.



Practical activity 21: Representation of a circle given its Cartesian equation

a) Rationale:

This activity is done when teaching the concept or topic related to the equation of a circle. It is taught in unit 13 of S4. The concepts of the circle are used in real life. For instance, the **odometer** of an automobile calculates the distance travelled by counting the **number of rotations** and the **circumference** of the wheel, which is defined by its **radius.**

In addition, the focal length of a **camera lens** can be calculated using its **radius of curvature.** The **circle**, rather than squares, is preferred in **architecture.** Today's structures are constructed with round beams and curved surfaces.

b) Objective:

Represent a circle whose general equation is given in the Cartesian plane.

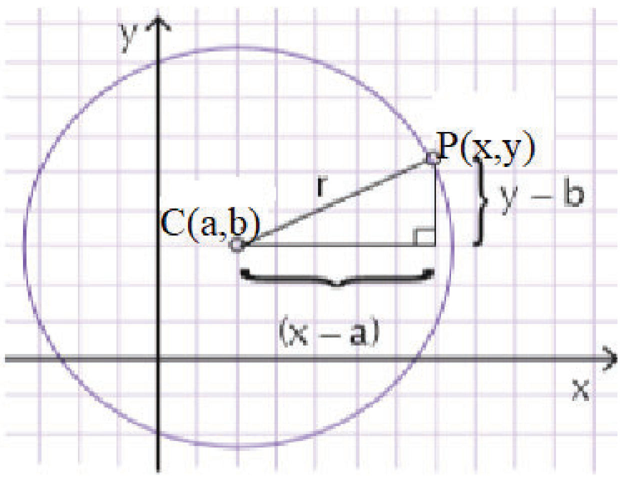
c) List of required materials:

Graph paper**,** ruler, pair of compasses, and scientific calculators.

d) Illustration of the activity:

If we have the standard equation of a circle in the form $x^2 + y^2 + mx + nx + p = 0$

for $m = -2a$, $n = -2b$ and $p = a^2 + b^2 - r^2$; It is easy to draw the graph of a circle:



e) Procedures

Step 1: Consider the function $x^2 - 2x + y^2 - 4y - 5 = 0$ and reduce it to the form $(x-a)^2 + (y-b)^2 = r^2$

Step 2: Consider the results from step 1; using a pair of compass needles, make an amplitude of radius r from the point $C(a, \mathrm{b})$ considered as the center to the point $P(\mathrm{x}, \mathrm{y})$ of your choice.

Step 3: Fix a compass needle in a center $\mathrm{C}(a,b)$ and draw the circle passing through the point $P(\mathrm{x}, \mathrm{y})$.

Step 4: Choose different points $P(\mathrm{x}, \mathrm{y})$ of the obtained circle and find the distance between C and P. What do you observe? Is it the same as the radius r?

Step 4: Observe the function given in the form $x^2 + y^2 + mx + ny + p = 0$

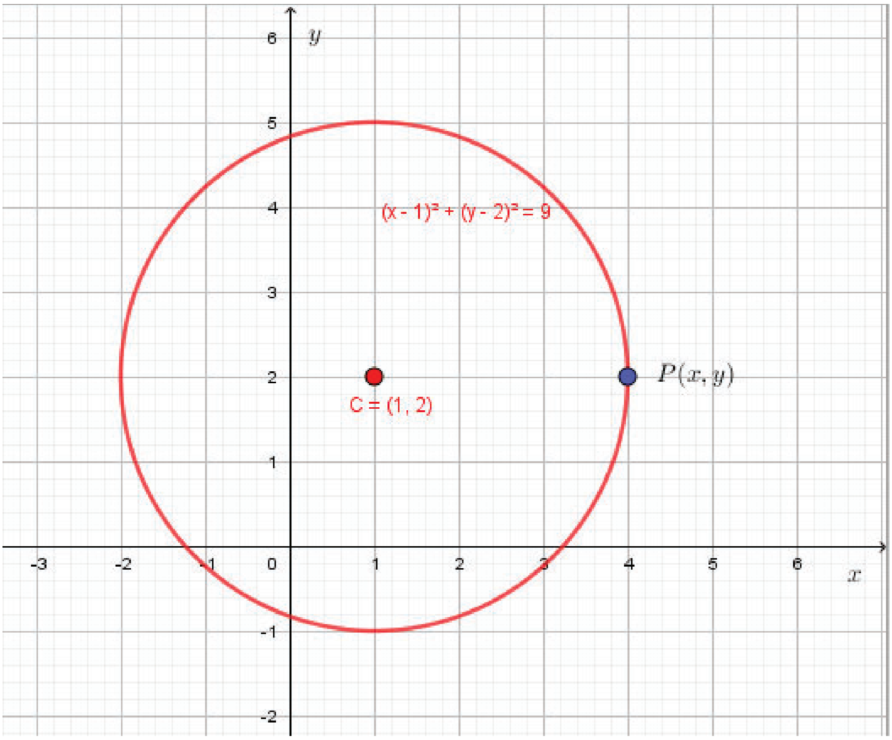
Step 5: By completing squares or using any appropriate method, reduce the given equation in the form $(x-a)^2 + (y-b)^2 = r^2$ where a, b and r^2 are to be determined. What does r represent? What does the point $\mathrm{C}(a,b)$ represent?

Expected answer:

From the equation $x^2 - 2x + y^2 - 4y - 5 = 0$ we find the equation is $(x-1)^2 + (y-2)^2 = 3^2$

This is the equation of a circle of center $C(a,b)=C(1,2)$ and radius $r=3$

The graph:



Distance between the center C (1,2) and a point P(x,y)

For example, we can take the point M (1,-1). We find that the distance MC=3 unit of length.

This length is the radius.

Generally, the equation of distance between C (1,2) and P(x,y) gives us the equation of the circle given above.

Reducing $x^2+y^2+mx+ny+p=0$ in the form $(x-a)^2+(y-b)^2=r^2$:

$$x^2+y^2+mx+ny+p=0 \Leftrightarrow x^2+y^2+mx+ny=-p$$

Grouping the terms containing the same variable, we get

$$\left(x^2+mx\right)+\left(y^2+ny\right)=-p$$

Completing squares gives

$$\left(x+\frac{m}{2}\right)^2+\left(y+\frac{n}{2}\right)^2=-p+\left(\frac{m}{2}\right)^2+\left(\frac{n}{2}\right)^2$$

$$\Leftrightarrow\left(x-\left(-\frac{m}{2}\right)\right)^2+\left(y-\left(-\frac{n}{2}\right)\right)^2=-p+\left(\frac{m}{2}\right)^2+\left(\frac{n}{2}\right)^2$$

Finally, we get that $x^2+y^2+mx+ny+p=0$ in the form $(x-a)^2+(y-b)^2=r^2$:

is $\left(x-\left(-\frac{m}{2}\right)\right)^2+\left(y-\left(-\frac{n}{2}\right)\right)^2=-p+\left(\frac{m}{2}\right)^2+\left(\frac{n}{2}\right)^2$

where $a=-\frac{m}{2}, b=-\frac{n}{2}$ and $r^2=-p+\left(\frac{m}{2}\right)^2+\left(\frac{n}{2}\right)^2$

$r=\sqrt{-p+\left(\frac{m}{2}\right)^2+\left(\frac{n}{2}\right)^2}$ and $C(a,b)=C\left(-\frac{m}{2},-\frac{n}{2}\right)$ represent respectively the radius and centre of circle defined by Cartesian equation $x^2+y^2+mx+ny+p=0$.

f) Results, interpretation, and conclusion:

When the circle has the equation $(x-a)^2+(y-b)^2=r^2$, its centre is the point $C(a,b)$ and its radius is r. For example, if $(a,b)=(-2,1)$ and $r=2$, its equation is $(x+2)^2+(y-1)^2=2^2$ or $x^2+2x+y^2-2y+1=0$.

When the circle has the centre $(a,b)=(0,0)$ and $r=1$ we get the trigonometric circle. Its equation is therefore $x^2+y^2=1$.

From Cartesian equation of the circle $x^2+y^2+mx+ny+p=0$, centre and radius are $C(a,b)=C\left(-\frac{m}{2},-\frac{n}{2}\right)$ and $r=\sqrt{-p+\left(\frac{m}{2}\right)^2+\left(\frac{n}{2}\right)^2}$ respectively, which can be found by completing squares of given Cartesian equation.

g) Information for the teacher:

When asked to draw a circle given its general form $x^2+y^2+mx+ny+p=0$, first transform it into the standard form $(x-a)^2+(y-b)^2=r^2$ which has the centre $C(a,b)$ and radius r, then draw the circle in Cartesian plane.

h) Guidance on evaluation:

Ask students to do the activity of drawing a circle. For example,

Given a general equation of the circle $x^2+y^2+2x-2y-2=0$

a) Find the standard equation of the circle

b) Draw the circle

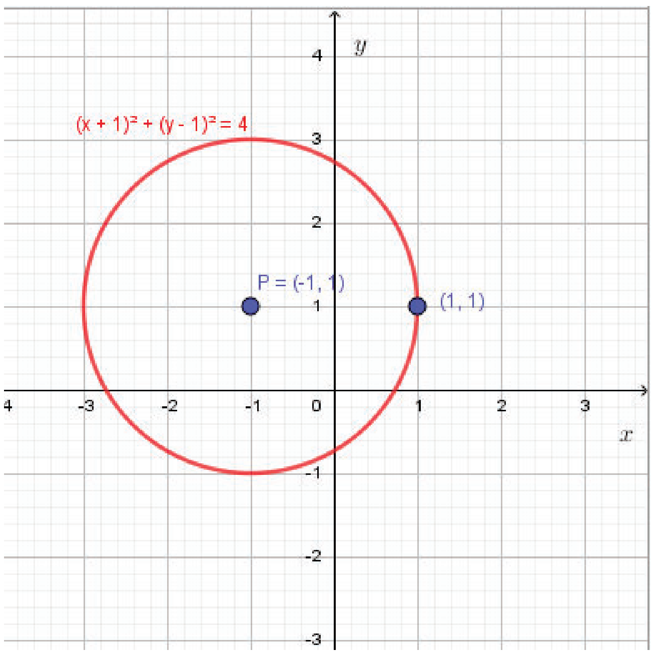
Solution

a) Transform the given equation into the standard form of the circle equation.

$$x^2+y^2+2x-2y-2=0$$
$$x^2+2x+y^2-2y=2$$
$$(x^2+2x+1)+(y^2-2y+1)=2+1+1$$
$$(x+1)^2+(y-1)^2=4$$
$$(x+1)^2+(y-1)^2=2^2$$

The standard form of the equation is $(x+1)^2+(y-1)^2=2^2$. It is shown from the equation of the circle that the circle has the centre $P(-1,1)$ and the radius $r=2$.

b) The graph



UNIT: 14 MEASURES OF DISPERSION

Practical activity 22: Solving a problem illustrating the coefficient of variation as the measure of dispersion

a) Rationale:

This activity is done when teaching the concept or topic related to descriptive statistics. It is taught in unit 17 of S4.

The coefficient of variation is also very useful when comparing two or more sets of data that are measured in the same units. Thus, the bigger the coefficient of variation, the bigger the dispersion.

b) Objective:

Compare two data sets using the coefficients of variation.

This is an Inquiry-based experiment.

c) List of required Materials:

- Paper
- Scientific calculator

d) Procedures:

Step1: Observe the data given in a problem as follows:

A researcher was interested in finding out the differences in the prices of soft drinks before Christmas and after Christmas. A sample of 5 soft drinks was selected and the data is shown below.

Drinks	Coke	Pepsi	Fanta	Mirinda	Banana juice
Before Christmas	300	300	320	280	280
After Christmas	350	310	400	300	320

Step 2: Separate the two sets of data and calculate the mean of each data set.

Step 3: Calculate the standard deviation for each data set.

Step 4: Calculate the coefficient of variation of each data set. How are the data for each set spread from the corresponding mean?

Step 5: Compare the CVs of the two sets of data and then make a conclusion. How does the greater CV illustrate related data?

Expected answer

a) Before Christmas

i. The mean of price is given by $(\bar{x}) = \frac{300+300+320+280+280}{5} = 296$

The set standard deviation (σ) is given by $\sigma = \frac{\sqrt{(x_i - \bar{x})^2}}{n}$

ii. Thus, the standard deviation=

$$\sqrt{\frac{(300-296)^2+(300-296)^2+(320-296)^2+(280-296)^2+(280-296)^2}{5}} \simeq 16.73$$

iii. The coefficient of variation is given by CV= (Standard Deviation/Mean) × 100%

Thus, the coefficient of variation (CV) $= \frac{16.73}{296}\% = 5.6\%$

b) After Christmas

i. The mean of prices $= \frac{350+310+400+300+320}{5} = 336$

ii. The set standard deviation (σ) is given by $\sigma = \frac{\sqrt{(x_i - \bar{x})^2}}{n}$

Thus, the standard deviation $(\sigma) =$

$$\sqrt{\frac{(350-336)^2+(310-336)^2+(400-336)^2+(300-336)^2+(320-336)^2}{5}} = 40.3$$

iii. The coefficient of variation is given by CV= (Standard Deviation/Mean) × 100%

Thus, the coefficient of variation (CV) $= \frac{40.3}{336}\% \simeq 12\%$

e) Interpretation of result and conclusion:

The CV of prices after Christmas is greater than the CV before Christmas.

The prices of soft drinks are dispersed after Christmas.

g) Information for the teacher:

The mean is a value that gives a global idea of the whole population.

The standard deviation gives an idea of the spread of the data.

The coefficient of variation shows the extent of variability in relation to the mean of the population.

h) Guidance on evaluation:

Ask students to do the following activity. An environmentalist was interested in measuring the soil PH along Sebeya River and Koko River. He used 10 different places on each river to obtain samples. The following table shows different soil pH on these points on each river.

Points	pH on River Koko	pH on River Sebeya
1	6.5	4.5
2	8.9	4.3
3	10.5	9.3
4	7.4	6.9
5	6.7	7.0
6	6.9	8.9
7	7.0	6.5
8	4.3	7.1
9	9.1	3.2
10	10	4.6

Find the CV of each soil sample. Compare the CV of the two samples and interpret the data.

Solution

a) On River Koko

$$\text{Mean of pH} = \frac{1}{10}[6.5+8.9+10.5+7.4+6.7+6.9+7.0+4.3+9.1+10]$$
$$= \frac{1}{10}\times 77.3 = 7.73$$

b) $\text{Variance} = \frac{1}{10}\left[(-1.23)^2+(1.17)^2+(2.77)^2+(-0.33)^2+(-1.03)^2+(0.83)^2+(0.73)^2+(-3.43)^2+(1.37)^2+(2.27)^2\right]$

$$= \frac{1}{10}\times 31.7 = 3.17$$

c) Standard deviation (SD) $= \sqrt{3.17} = 1.78$

d) The Coefficient of Variation (CV) is given by $\frac{\sigma}{\overline{x}}$

Thus, the Coefficient of Variation (CV) $= \frac{178}{7.73} = 23\%$

a) On River Sebeya

Mean of pH

$$= \frac{1}{10}(4.5+4.3+9.3+6.9+7.0+8.9+6.5+7.1+3.2+4.6)$$
$$= \frac{1}{10}\times 62.3 = 6.23$$

b) Variance

$$= \frac{1}{10}\left[(-1.73)^2+(1.93)^2+(3.07)^2+(0.67)^2+(0.77)^2+(2.67)^2+(0.27)^2+(0.87)^2+(-3.03)^2+(-1.63)^2\right]$$

$$= \frac{1}{10}(2.9+3.7+4.9+0.4+0.6+7.1+0.07+0.8+9.1+2.8)$$
$$= \frac{1}{10}\times 36.27 = 3.627$$

c) Standard deviation $= \sqrt{3.627} = 1.90$

d) The Coefficient of Variation (CV) $= \frac{190}{6.23} = 30.49\%$

The Coefficient of Variation of the sample from River Sebeya is greater than the Coefficient of Variation of the sample from River Koko. Thus, the data concerning the soil pH along the Sebeya River are more dispersed than the data concerning the soil pH along the Koko River.

Practical activity 23: **Explore and illustrate different ways of arranging the sits for a group of Students on a bench**

a) Rationale:

This activity is done when teaching the combinatory specifically in the technics of counting. It is taught in unit 15 of S4. The arrangement which involves the concepts of combinatorics is often described briefly as being about counting. As the name suggests, however, it is broader than this: it is about combining things.

In real life, questions that arise include counting problems: "How many ways can these elements be combined?"

b) Objective:

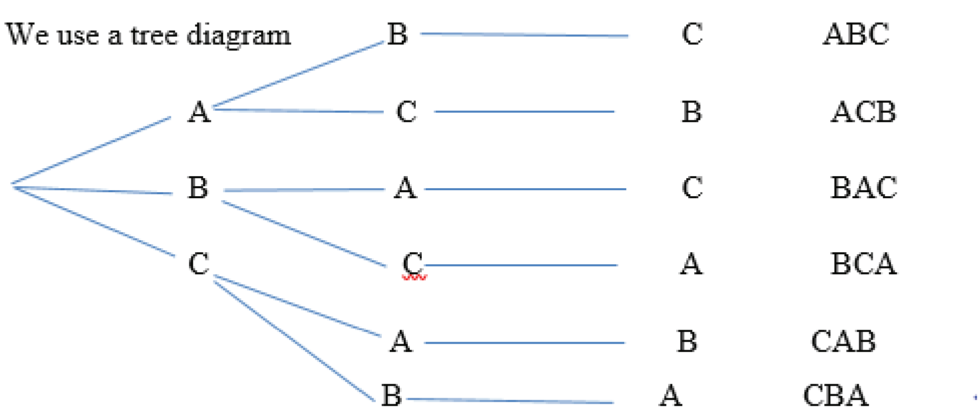
To calculate the number of different ways in which objects can be arranged on a line.

c) List of required Materials:

- A bench having three seats A, B, and C
- Manilla paper
- Marker
- Objects of different colors.

d) Illustration of the activity:

Sitting of 3 Students namely A, B and C on a bench in different possible ways.



e) Procedures:

Step 1: Choose 3 Students at random and name them A, B, C and ask them to sit on a bench of three places A, B, and C

Step 2: By occupying the first place**:** There are 3 ways to occupy the first place (make all possible ways that these three Students can sit on a bench and then record the results)

Step 3: By occupying the second place**:** Since the first place is already occupied, there are 2 ways to occupy the second place (make all possible ways that these two Students can sit on a bench and then record the results).

Step 4: Occupying the third place**:** Since the two first places are already occupied, there is only one way to occupy the third place.

Step 5: Ask Students to use a tree diagram and draw all the obtained arrangements on a manila paper

f) Interpretation of result and conclusion

The tree diagram shows that the number of ways to place 3 Students on a bench is $3 \times 2 \times 1 = 6$

We denote $3!$ the number of ways to place 3 Students on a bench.

Then, $3! = 3 \times 2 \times 1 = 6$

g) Information for the teacher

3! is read "factorial of three".

The factorial notation of n integers denoted by n! is the product of the first consecutive integers and is read as 'factorial of n'.

Thus, $n! = n \times (n-1) \times (n-2) \times \ldots \times 4 \times 3 \times 2 \times 1$.

For example $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

Note: $0! = 1$

h) Guidance on evaluation

Ask students to do the following activity

How many ways can four Students sit on a bench?

Answer

Since there are four Students to sit on a bench, we calculate the value of 4!

Thus, 4!=4x3x2x1=24

There are 24 ways of arranging four Students on a bench.

UNIT: 16 ELEMENTARY PROBABILITY

Practical activity 24: **Arranging 3 men and 4 women at random in a row and calculate the probability that all the men sit together**

a) Rationale:

This activity is done when teaching the sum and product laws of probability. It is taught in unit 16 of S4 mathematics.

The probability of events can be calculated in many situations of daily life such as production in a factory by given machines operating independently or no, in agriculture, and in medical area.

a) Objective:

Determine the different ways of sitting 3 men and 4 women on a bench and calculate the probability of related events.

This is a concept-based experiment.

c) List of required Materials:

Bench, 3 objects 1, 2 and 3 of the same colour standing for men and 4 objects 1, 2, 3 and 4 of another colour standing for women.

d) Illustration of the activity

$$\text{Probability for an event happenning} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

e) Procedures

Step1: Let 7 students (3 boys and 4 girls) sit on the bench in different ways and explain some ways that can happen.

Step 2: Arrange 7 objects on a row in different ways and calculate the number of ways of sitting 7 people on a bench.

Step 3: Arrange 3 ordered objects (M_1, M_2, M_3) of the same colour representing 3 men in different possible ways. Then, calculate the number of different ways of sitting all 3 men on the bench.

Step 4: Arrange 4 ordered (W_1, W_2, W_3, W_4) objects of one colour representing 4 women in different possible ways. Then, calculate the number of different ways of sitting all 4 women on the bench.

Step 5: Calculate the probability of sitting 3 men and 4 women on the same bench, such that all men sit together on the right hand of all women.

Expected answer

1. There are different arrangements of sitting on bench
2. The number of ways of sitting 7 people on a bench is $7! = 5040$
3. The number of different ways of sitting all 3 men on the bench is $3! = 6$

4. The number of different ways of sitting all 4 women on the bench is $4! = 24$

5. The probability of sitting 3 men and 4 women on the same bench such that all men sit together on the right side of all women is:

$$P = \frac{4!3!}{7!} = \frac{3!}{7\times6\times5} = \frac{1}{35}$$

f) Results, interpretation, and conclusion

The probability of sitting 3 men and 4 women such that all men and all women are together is:

$$P = \frac{4!3!}{7!} = \frac{3!}{7\times6\times5} = \frac{1}{35}$$

The required probability is calculated by division of the number found in step 3 and 4 by the number found in step 2.

Note: Multiplication rule in combinatorics:

- If an operation is composed of k successive steps which may be performed in $n_1, n_2, n_3, \cdots, n_k$ distinct ways, respectively, then the operation may be performed in $n_1 \times n_2 \times n_3 \times \cdots \times n_k$ distinct ways.
- Given that the total number $n = n_1 + n_2$ of people made by n_1 men and n_2 women, the probability of sitting n_1 men and n_2 women on a bench such that men are sit together, women sit together on the right side for men is

  $$P = \frac{n_1! \times n_2!}{n!}$$

g) Information to the teacher

Given that the total number $n = n_1 + n_2$ of people made by n_1 men and n_2 women, the probability of sitting n_1 men and n_2 women on a bench such that men are sit together, women sit together is

$$P = 2\times\frac{n_1! \times n_2!}{n!}$$

This is because the two groups can interchange the position: either all men are sitting at the left side or at the right side of women.

h) Guidance on evaluation

Do the following activity

If 4 people A, B, C, D sit in a row on a bench, what is the probability that A and B sit next to each other?

Solution

Since AB are next to each other, there are 3! ways of sitting AB, C and D.

Alternatively, there are 3! ways of sitting BA, C, and D.

Thus, there are $2\times3!$ or 12 ways of sitting A, B, C, D in a row on a bench such that A and B are next to each other.

It means, $|X| = 2\times3! = 12$

These are:

ABCD	ABDC	CABD	DABC	CDAB	DCAB
BACD	BADC	CBAD	DBAC	CDBA	DCBA

The are 4! of sitting 4 people in a row on bench. It means, $|S| = 4! = 24$

Thus, P (A and B sit next to each other) is $\frac{|X|}{|S|} = \frac{12}{24} = \frac{1}{2}$

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- *Ministry of Education and REB (2015). Mathematics Syllabus for Advanced Level (S4-S6), Kigali, Rwanda.*
- *UR-CE (2020). Continuous professional development certificate in innovative teaching mathematics and science (CPD ITMS), Module 1: Innovative Teaching methods for Mathematics and Science (CMS1141), Kigali: UR-CE.*
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Annex 1: Name of commonly hazard symbols useful in the laboratory

S/N	Name	Hazard Symbol	Explanation
1	Flammable and combustible		The flammable and combustible symbol signifies substances that will ignite and continue to burn in air. Substances in this category may be gases, aerosols, liquids, or solids, and include many solvents and cleaning materials that are commonly used in the laboratory.
2	Oxidizing agents		The symbol for oxidizing materials indicates the presence of chemicals that readily give off oxygen or other oxidizing substances. Oxidizing materials may intensify fires and cause explosions, and also may be toxic or corrosive. Some common oxidizing liquids and solids found in laboratories are bromine, chlorates, nitrates, perchloric acid, and peroxides.

3	Toxic		A substance known to pose that is classified as posing skin corrosion or irritation; serious eye damage or eye irritation; respiratory or skin sensitization; germ cell mutagenicity; carcinogenicity; reproductive toxicity, and other toxicity is classified as hazardous or toxic substance. These substances can cause death or damage to health by inhalation, ingestion or skin absorption. Example: acid
4	Irritants		Irritants are substances that cause reversible inflammatory effects on living tissue at the site of contact.

5	Magnetic Field		Certain pieces of laboratory equipment generate strong magnetic fields. The strong magnetic field sign alerts lab members to the dangers that this type of equipment can pose. The risks are especially imminent for people wearing pacemakers and implants, which will tend to align themselves with the magnetic field lines, as will watches, clipboards, and certain tools. Magnetic fields result from the flow of current through wires or electrical devices. Examples of sources: machines, electrical wiring (such as power lines)
6	Exit		It is good to know where all of the exits are located, especially when working in a laboratory environment where you may need to get out quickly. Labs are required to mark exits routes from the area with clearly identifiable signs.

7	Fire extinguisher		Fires can happen anywhere, but lab fires can be even more dangerous due to Bunsen burners, flammable liquids, research documents, laptops, and lab equipment that might be present at any given time. It is essential that the occupants of a laboratory are fully aware of the risks and the appropriate extinguishing media. A fire extinguisher safety sign indicates the exact location of a lab's fire extinguisher.
8	Electrical hazard		The electrical hazard safety symbol, which typically includes a frayed wire and a hand with a lightning bolt across it, indicates any electrical hazards in the lab. If an electrical hazard is suspected, the device in question should be disconnected immediately and the cause determined by a qualified technician. Equipment should always be turned off and unplugged when any work is being done on it.